#1. Neutral ISM phases. [13 points]

(a) Using the formulae in the notes, write down an expression for the total cooling (in erg s$^{-1}$ per H nucleus) of neutral gas as a function of temperature $T$ (in K) and pressure $P/k$ (in units of K cm$^{-3}$, with Boltzmann’s constant divided out). Include the cooling in the fine structure lines of C II and O I, the optical forbidden doublet [O I] 6300, 6364 Å, and H I Lyα. Assume that we are below the critical density for all of these lines; in which case is this most restrictive?

(b) Plot this on a log-log scale as a function of temperature and compare to the typical photoelectric heating rate. What is the range of pressures over which you predict that two phases are allowed?

#2. Energy levels of molecules. [12 points]

In dense environments, it is a common occurrence for molecules to reach a thermal distribution of rotational energy levels.

(a) At $T = 20$ K – a typical temperature deep inside a molecular cloud – compute the probability distribution for $J$ in the $^{12}$C$^{16}$O molecule.

(b) Draw qualitatively the equilibrium ratio of ortho-H$_2$:para-H$_2$ as a function of $T$. At what temperature is the ratio 1:1?

#3. Deuterium. [15 points]

Deuterium is rare in the ISM – the primordial D/H ratio was $\sim 2 \times 10^{-5}$ and D has only been destroyed by stars. Nevertheless, deuterated molecules have been detected. Assuming that the H–X bond (where X is a heavy atom or radical) oscillates at angular frequency $\omega$, compute:

(a) The frequency of oscillation of the D–X bond.

(b) The difference in ground state energies of the hydrogenated versus deuterated molecule.

(c) The equilibrium constant of the reaction:$^1$

$$HX + D^* \rightleftharpoons DX + H^*.$$  

$^1$ For the purposes of this simplified problem (but not necessarily the purposes of real life) you may ignore any possible H or D atoms in the radical X and the consequent wave function (anti)symmetrization requirements.
(d) The value of this equilibrium constant for a typical H–X bond (which emits radiation at ~3000 waves/cm) and a temperature of $T = 20$ K.

The strong preference for deuterating molecules can lead to some amazing molecules, including fully deuterated ammonia, ND$_3$! (Lis et al, ApJ 571, L55, 2002)