1. Introduction

In the previous lecture, we studied in great detail the absorption and scattering properties of dust particles. In this lecture, we will put the pieces together to derive the properties of the dust population (particularly the size distribution for both carbonaceous and silicate grains). The key diagnostics that we will use for this are the extinction curve and the emission spectra from various regions of the ISM.

The ultimate goal of this lecture is to understand the grain size distribution for a two-component model (carbonaceous + silicate), and the observational constraints that drive us to the chosen model.

The key references used for preparing these notes are:


Tielens, Ch. 6

Although the papers go into substantially more detail than we need in class, they are classics and worth reading.

2. The Extinction Curve and Abundance Constraints
We first consider the **extinction** of starlight due to dust (which includes both absorption and scattering). This is a function of wavelength. First we will go through the formal description of extinction, and then consider how the grain size distribution affects the extinction curve.

### A. FORMALITIES

For historical reasons, extinction is measured in magnitudes. The number of magnitudes of extinction to a star at a particular wavelength is:

\[ A_\lambda = m_{\text{obs}} - m_{\text{int}} = 1.086 \tau_\lambda, \]

where \( m_{\text{obs}} \) is the observed magnitude, \( m_{\text{int}} \) is the intrinsic magnitude (i.e. without dust), and \( \tau_\lambda \) is the optical depth. We may also write the extinction in particular bands: \( A_V, A_R \), etc.

The wavelength dependence usually leads to **reddening** – the tendency of extincted objects to appear redder because \( A_\lambda \) (usually) decreases with wavelength. The reddening (or color excess) between two bands is e.g. \( E_{B-V} = A_B - A_V \). Finally, the ratio of total to color-dependent extinction is defined by:

\[ R_V \equiv \frac{A_V}{E_{B-V}} = \frac{1}{(A_B - A_V) - 1}. \]

Typical values of \( R_V \) are \(~3\) in the diffuse ISM, but larger values \((~5)\) occur in dense regions, indicating a change in grain populations.

Generally we expect more dust along denser or longer lines of sight, so it is common to normalize extinction to the hydrogen column density \( N_H \) (units: cm\(^{-2}\)). (In theory this is the total hydrogen density, atomic, ionized, or molecular; although often only \( N_{HI} \) is known directly.) Then the extinction curve is given by:

\[ \frac{A_\lambda}{N_H} = 1.086 \int \frac{1}{n_H} \frac{dn_{\text{gr}}}{da} \left[ \sigma_{\text{abs}}(a) + \sigma_{\text{sc}}(a) \right] da. \]

Here \( dn_{\text{gr}}/da \) is the grain size distribution, and we have normalized it to the H density. In the presence of a multi-component model for the grain composition, we should include a summation over the components.

The total amount of element X in dust grains is given by

\[ \frac{n_{X,\text{dust}}}{n_H} = \int \frac{1}{n_H} \frac{dn_{\text{gr}}}{da} \frac{4}{3} \pi a^3 \rho f_{X,\text{mass}} \frac{m_x}{m_X} da, \]

where \( \rho \) is the grain density and \( f_{X,\text{mass}} \) is the fraction by mass of the grain material in X. This represents an important constraint on grain models, since otherwise one
could place most of the dust in “boulders” \((a \gg 1 \mu m)\) that have no observable effect on extinction or thermal emission.

## B. FEATURES

We now consider the basic features of the extinction curve and their uses. The model shown below indicates the major features of an extinction curve.

The two most prominent aspects of this curve are (i) the broadband slope \((A_\lambda\text{ decreases as something between } \lambda^{-1} \text{ and } \lambda^{-2})\), and (ii) the bump at 2200Å. The rollover of the broadband slope across the optical bands (i.e. where there are no features in the dielectric constant) is suggestive of a break in the grain size distribution at a few tenths of a micron.

As described in the previous lecture, the 2200Å bump is believed to be due to an electronic absorption band in the carbonaceous grains. Thus its strength is indicative of the amount of carbonaceous material. As we found in the last lecture, the strength of absorption (assuming grain sizes \(a<<\lambda\) is proportional to \(a^3\) (i.e. to grain volume), whereas for \(a>>\lambda\) the absorption scales as \(\sim a^2\) and the strength of spectral features is even weaker. Therefore one might naively guess that the 2200Å bump is telling us about the amount of carbon in small grains (radius of a few \(0.01\mu m\) or smaller). The amount of such carbon required is (by number) \(C:H\sim6\times10^{-5}\), or roughly a quarter of the solar abundance (with weak dependence on how this is distributed among the range of sizes from PAH to several tens of nm). A further \(\sim30\%\) of solar abundance \((C:H\sim10^{-4})\) can be found in the gas phase (as
traced by C I and C II UV absorption lines – see later lectures). The rest of the carbon must be found in large dust grains. Their specific size distribution is related to the extinction curve in the optical/NIR; however silicate grains also contribute here so modeling is necessary.

For the silicate grains, we know from UV absorption line spectroscopy that Si, Mg, and Fe are heavily depleted in the ISM gas phase so that they must be mostly locked up in dust. This suggests a dust mass in silicates comparable to that in carbon (volume \( \sim 3 \times 10^{-27} \) cm\(^3\) per H atom). One would therefore expect both to be of comparable importance in the optical/NIR.

Weingartner & Draine (2001) performed \( \chi^2 \) fits of dust models to the observed interstellar extinction curves. Their key findings for diffuse Milky Way ISM dust are:

- The silicate size distribution can be fit with a peak at \( a \sim 0.2 \) \( \mu \)m.
- The peak in the silicate size distribution cannot be made too wide without putting more mass in larger grains and underproducing the UV extinction. (This UV extinction could be compensated by increasing the small carbonaceous grains, but this would overproduce the 2200Å feature.)
- A variety of carbonaceous grain size distributions are possible, but they are needed to contribute to the NIR extinction, so a substantial portion of the mass must be placed at \( a \sim \) few \( \times 10^{-1} \) \( \mu \)m. The tail to small sizes is not constrained by the extinction curve.

It is clear that the extinction curve fits are underconstrained, as one might expect. Therefore we turn to the emission from dust.

### 3. Infrared Emission

The thermal emission from dust is a powerful diagnostic of its properties, including the grain size distribution.

#### A. EQUILIBRIUM TEMPERATURE

The equilibrium temperature of a dust grain can be derived by balancing the emitted and absorbed radiation. Specifically, we recall the luminosity per unit frequency of a grain:

\[
L_\nu = \frac{192\pi^3 h a^3 \nu^4}{c^3} \left( \frac{1}{e^{h\nu/kT_a} - 1} \right) \Im \epsilon \left| \epsilon + 2 \right|^2.
\]

If we assume that the dielectric constant at low frequencies (where the grain emits; far below the main vibrational resonances of the material) is given by \( \Re \epsilon \rightarrow \epsilon_{\text{DC}} = \) constant, while the imaginary part is given by \( \Im \epsilon = (\nu/\nu_0)^{\beta-1} \), then we find an emitted spectrum
The total luminosity is

\[ L = \frac{192\pi^3 h a^3}{c^3 \nu_0^{\beta-1}(\epsilon_{DC} + 2)^2} \left( \frac{\nu}{\epsilon_{DC} + 2} \right)^{4+\beta} e^{h\nu/kT_d} - 1. \]

This must balance the absorbed flux:

\[ L = 4\pi c \int_0^\infty J_\nu \sigma_{\text{abs}}(\nu) d\nu. \]

It is clear from these considerations that the equilibrium temperature of the dust scales weakly with the intensity of the ambient radiation; for example, if the radiation field normalization is \( G_0 \), then \( T_d \sim G_0^{1/(4+\beta)} \). For silicate grains, if we model the dielectric constant by a superposition of damped harmonic oscillators,

\[ \chi(\omega) \propto \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}, \]

then we have \( \beta=2 \) and we expect \( T_d \sim G_0^{1/6} \).

For an \( a=0.1 \) \( \mu \)m grain, and parameters \( \epsilon_{DC}=11 \) and \( \nu_0 \sim 1 \) THz the above numbers give \( L \approx 8 \times 10^{-20} (T_d/K)^6 \) erg/s. The flux of starlight in the far UV is \( 1.2 \times 10^{-4} G_0 \) erg/cm²/s/sr, where \( G_0 \) is a normalization (of order \( \sim 1.7 \) in the diffuse ISM). Assuming a geometric cross section for absorption (good to an order of magnitude in the FUV), we find that the absorbed power by the above grain is \( 5 \times 10^{-13} \) erg/s. This suggests \( T_d = 14 G_0^{1/6} K \). This is roughly correct, although values of \( \sim 18 \) K are more typical. Note that this is far greater than what one would obtain for a blackbody in the ISM due to the steep fall-off of emission with frequency.

For graphite grains, which are conductors, the expected behavior at low frequencies is that

\[ \frac{\text{Im}\epsilon}{|\epsilon + 2|^2} \rightarrow \frac{\rho \omega}{4\pi} \sim \omega, \]

so once again we expect the behavior that the total grain luminosity scales as \( L \sim T_d^6 \).

The thermal emission spectrum from the dust peaks at \( \sim 150 \) \( \mu \)m. At lower frequencies (longer wavelengths) we expect to see a modified blackbody, i.e. \( j_\nu \) should be a blackbody times \( \nu^2 \). However, the actual spectrum observed by COBE/FIRAS deviates from a modified blackbody at \( \nu < 500 \) GHz, with excess emission observed. The FIRAS team suggested the fit:

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1 Tielens, Ch. 1.
This is suggestive of an additional “very cold dust” component. This component has attracted much attention, by the FIRAS team and others. Explanations have fallen into two major categories. One is that there really is very cold dust, e.g. due to material with low optical/UV absorption or very high millimeter wave emissivity, or due to a population of very large grains (although the expected dependence of the second blackbody temperature on $G_0$ is not observed). The other is that the emissivity law (equivalently, the frequency dependence of the susceptibility) we have used is wrong, and an enhancement of $\text{Im} \varepsilon$ above the $\nu^4$ dependence occurs for $\nu < 500$ GHz. This explanation is most commonly invoked today but the case is not closed.

B. SMALL GRAINS AND THERMAL SPIKES

If a dust grain is very small the above analysis does not apply. To see this, we consider the thermal energy in a dust grain. The vibrational energy density in a material at low temperature ($T_d << T_{\text{Debye}}$) is given by

$$U = \frac{\pi^2(kT_d)^4}{10(hc_s)^3} V,$$

where $c_s$ is the sound speed in the grain. If we take $T_d = 20$ K, this can be written as

$$U = 0.002 \left( \frac{T_d}{20K} \right)^4 \left( \frac{4\text{km/s}}{c_s} \right)^3 \left( \frac{a}{\text{nm}} \right)^3 \text{eV}.$$

A typical far ultraviolet photon that is absorbed by a dust grain has an energy of $\sim 10$ eV. This is equal to the equilibrium thermal energy of the grain if $a \sim a_c = 17$ nm (with the critical size depending somewhat on the intensity of the radiation field, $G_0$). Thus for grains larger than $a_c$ the grains should be treated as being in equilibrium at a constant temperature. In contrast, grains smaller than $a_c$ can be treated as having discrete heating events every time they absorb a UV photon. Following such absorptions, they rapidly cool to below their equilibrium temperature. These discrete events are called thermal spikes. The maximum temperature reached during a thermal spike can be obtained by taking the Debye formula, setting the total energy to $\sim 10$ eV, and solving for the temperature:

$$T_{\text{max}} = 170 \left( \frac{c_s}{4\text{km/s}} \right)^{3/4} \left( \frac{a}{\text{nm}} \right)^{-3/4} \text{K}.$$

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For the very smallest grains (the PAHs, which may have only several tens of atoms and \(a \sim 0.4 \text{ nm}\)) the maximum temperature may reach \(\sim 1000 \text{ K}\).

Thermal spikes are important because they enable dust grains to radiate at frequencies larger than \(\sim 10k(20K)/h \sim 4 \text{ THz}\). This enables us to distinguish the small tail of the size distribution: we look beyond the Wien tail of the thermal dust emission and look for an extension that is inconsistent with any kind of equilibrium blackbody. The smaller the grains, the hotter they can get, and the farther into the mid-IR one will observe their emission. Furthermore, one can observe the vibrational modes of these smallest grains in emission and hence directly probe their composition. For example, the non-detection of the Si–O stretch band at 10 \(\mu\text{m}\) in emission limits the abundance of small silicate grains.

### C. POLYCYCLIC AROMATIC HYDROCARBONS

The major emission features seen in the mid-IR are the **PAH bands**. The strongest of these (and their identifications) are:

- 3.3 \(\mu\text{m}\) C–H stretch
- 6.2, 7.7 \(\mu\text{m}\) C–C stretch
- 8.6 \(\mu\text{m}\) C–H in-plane bend
- 11.3, 12.7 \(\mu\text{m}\) C-H out-of-plane bend

Some weaker features can be observed in some sources; these are sometimes attributed to other modes involving the carbon skeleton, overtones, and non-PAH carriers.

The shorter-wavelength bands (3.3 and 6.2 \(\mu\text{m}\)) will be enhanced for the smallest dust grains because they get hotter during thermal spikes. However, the interpretation is slightly complicated by charging effects: remember that the strength of an emission feature depends on whether that vibrational mode of the molecule results in an oscillating dipole moment. The C–H modes generally have larger dipole moments than modes of the carbon skeleton (C–C stretch) in neutral PAHs. For ionized PAHs, the C–C stretch modes radiate more efficiently. Thus some ratios (e.g. 6.2/7.7) are very sensitive to sizes, while others (e.g. 11.3/7.7) are very sensitive to charge. PAH charge is determined by a balance between photoionization and collisions with charges (electrons or ions), just as for the ionization states of atoms; but of course the actual mechanisms are different and the ionization potential is lower (\(\sim 5 \text{ eV}\) for large PAHs). PAHs also have large electron affinities and in regions of high electron/UV ratio may form anions. A full analysis of size distribution fitting of PAHs (as done in the series by Draine & Li) must therefore simultaneously model the charging of PAHs.

The Li & Draine (2001) model uses an abundance of carbon C:H = \(6 \times 10^{-5}\) in small carbonaceous grains and PAHs down to a minimum size of \(a \sim 0.35 \text{ nm}\) (20 carbon atoms!).

The smallest grains, as we have already discussed, are really molecules, and a variety of effects may become important. Since they reach very high temperatures during thermal spikes, they are susceptible to hydrogen atoms evaporating off,
leaving only the carbon skeletons, or even being destroyed by evaporation of \( \text{C}_2 \), \( \text{C}_3\text{H}_2 \), etc. units. This sets a lower limit of the order of 20 carbons. Small PAHs (at least in the laboratory) can actually form specific molecular structures, often based on interlocking benzene rings. Specific examples include naphthalene (\( \text{C}_{10}\text{H}_8 \)), pyrene (\( \text{C}_{16}\text{H}_{10} \)), coronene (\( \text{C}_{24}\text{H}_{12} \)), ovalene (\( \text{C}_{32}\text{H}_{14} \)), and circumcoronene (\( \text{C}_{54}\text{H}_{18} \)).

Despite the compelling evidence from multiple bands that PAHs are abundant in the ISM, and are a dominant contributor to the 2200Å feature in the extinction curve, not one specific molecule of this family has ever been unambiguously identified in the ISM.

4. Microwave Emission

We have now discussed at length the absorption by dust in the UV/optical/NIR and its emission in the MIR/FIR. One of the surprises from cosmic microwave background experiments\(^4\) was the discovery of an additional, “anomalous” component to the Galactic emission at \( \nu = 14.5 \) and 32 GHz. This was in addition to the already-known components of the Galactic microwave emission: synchrotron radiation (to be discussed later; roughly \( j_\nu \sim \nu^{-1} \), with some variation), free-free radiation from ionized and partially ionized regions (\( j_\nu \sim \nu^{-0.15} \) in the microwave), and thermal emission from dust (typically \( j_\nu \sim \nu^{3.7} \)). Its spatial morphology has been well-mapped by WMAP and is strongly correlated with the FIR emission seen by IRAS and COBE/DIRBE, suggesting a common origin from dust grains. Additionally, the signal is weakly polarized or unpolarized (suggesting that it is not synchrotron) and the non-observation of \( \text{H}\alpha \) rules out free-free from warm gas. The WMAP bands cover \( \nu = 22—94 \) GHz, and find a red spectrum, but some other experiments (e.g. ARCADE) find evidence that the spectrum is actually blue at lower frequencies (i.e. a peak at \( \sim 20 \) GHz).

This leaves the question: how do dust grains emit microwaves? Two major mechanisms have been suggested.

- **Spinning dust**: Here the idea is that rotating asymmetric ultrasmall grains (e.g. PAHs) with permanent electric dipole moments could act as microwave transmitters. Thus the anomalous emission would be attributed to a forest of rotational lines of small molecules.\(^5\) The spinning dust spectrum is determined by the electric dipole moments, and by the processes that control grain rotation (collisions, interactions with radiation and the plasma, etc.)

- **Magnetic dust**: An alternative is that ferromagnetic materials such as Fe could be present in dust grains. The thermal fluctuations of the magnetic dipole moment of the grain can radiate.\(^6\)

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Both of these mechanisms naturally produce microwave emission: the former due to the rotation speed of the smallest interstellar PAHs, and the latter due to the spin precession frequency in a permanent magnet ($B \sim$ few kG). At present the spinning dust explanation is more commonly invoked, but again the case is not closed.