1. Introduction

We now turn our attention to the low-density diffuse phases that fill most of the volume of the ISM. These phases are:

- The **cold neutral medium** (CNM): Low temperature ($T \sim 20$ K), moderate density ($n \sim$ few cm$^{-3}$) gas cooled by the [C ii] 158 µm line.

- The **warm neutral medium** (WNM): Moderate temperature ($T \sim$ few $\times 10^3$ K) gas in approximate pressure equilibrium with the CNM, but cooled by electronic excitations (e.g. forbidden lines; Lyman-α) rather than fine structure lines.

- The **warm ionized medium** (WIM): Diffuse ionized gas; presumably ionizing radiation has eaten its way out of star-forming regions and floods the diffuse ISM.

- Hot gas (up to a few $\times 10^7$ K), heated mechanically by shocks (e.g. supernova explosions, accretion onto galaxies, etc.) and generally not in thermal equilibrium (i.e. the heating source is transient, and the gas subsequently cools).

We will investigate the physical properties (ionization and temperature) of each phase.

**Introductory Note:**
The topology and life cycle of the ISM phases are still not fully understood. It is clear that the hot phase is produced by shocks and ultimately destroyed by cooling. Also
changes in pressure can interconvert the cold and warm phases: decreases in $P$ can destabilize the CNM, leading to runaway heating and conversion to WNM, while increases in $P$ can destabilize the WNM and convert it to CNM (and at sufficiently high pressure to molecular clouds). Stars are formed from dense, self-gravitating clumps in molecular clouds, and the radiation they produce gives rise to H II regions. Where these interface with molecular clouds, they lead to PDRs, and where they break out of the clouds in which stars form they flood the ISM with ionizing radiation, possibly giving rise to the WIM. But the quantitative flow of mass between each phase remains an area of active research. Even the 3D structure of the phases is often not known: while there is abundant evidence (from e.g. cooling and recombination radiation) for all of these phases, there is often not enough information to reconstruct a full 3D structure.

References:
Tielens, Ch. 8
Dopita & Sutherland, Chs. 5 & 7

2. Diffuse Cold and Warm Gas

A. IONIZATION

The ionization of the diffuse phases can be broken down into two subjects: the ionization state of the metals (e.g. C I vs. C II) and of the hydrogen (H I vs. H II). Clearly, in order to be “neutral” H I must be the dominant form of hydrogen. At typical values of the density and radiation field, the carbon will be mostly C II (just as for PDRs).

The ionization balance of carbon is dominated by photoionization,

$$C^0 + \gamma \rightarrow C^+ + e^-.$$

Recombination of carbon may occur either directly,

$$C^+ + e^- \rightarrow C^0 + \gamma,$$

or by PAH-catalyzed recombination. The latter involves the formation of PAH anions, determined by the balance of electron captures:

$$PAH^0 + e^- \rightarrow PAH^- + n\gamma_{IR},$$
$$PAH^- + \gamma_{UV} \rightarrow PAH^0 + e^-.$$

The PAH$^-$ abundance depends on the UV/electron ratio $G_0/n_e$, with lower $G_0/n_e$ favoring the formation of PAH$^-$. There can then be a charge transfer since the electron affinity of a PAH is less than the ionization energy of carbon:
C\(^+\) + PAH\(^-\) → C\(^0\) + PAH\(^0\).

This dominates under typical diffuse ISM conditions. The rate of formation of C\(^0\) is proportional to the electron fraction (due to the need for PAH\(^-\)), to the inverse of the UV intensity (since UV destroys PAH\(^-\)), and to the square of the density (since two collisions are required). The rate of photoionization is determined by the UV intensity, so overall the abundance of C\(^0\) is proportional to \(x_e(n/G_0)^2\) (where \(x_e=n_e/n_H\)). The coefficient is\(^1\)

\[
\frac{n(C^0)}{n_C} = 0.0022 \left(\frac{n/50\text{cm}^{-3}}{G_0}\right)^2 \left(\frac{100\text{K}}{T}\right)^{1/2}.
\]

The \(T^{-1/2}\) reflects the greater efficiency of reactions between oppositely charged reactants (C\(^+\) and PAH\(^-\)) at low temperature due to Coulomb focusing. In the practical diffuse cases, \(n(C^0)/n_C << 1\) and the C\(^0\) will be concentrated in the densest regions.

Hydrogen ionization cannot be driven by UV radiation in neutral regions since the hydrogen is shielded. However, some H\(^+\) can be created by cosmic ray ionization:

\[
\text{H} \rightarrow \text{H}^+ + e^-,
\]

with a rate of \(\zeta_{\text{CR}} \sim 10^{-16} \text{ s}^{-1}\). Recombination of H may occur radiatively as well as via the PAH-catalyzed channel. If the balance involves primarily radiative recombination, then we expect \(\zeta_{\text{CR}}\) to equal the rate of recombinations per H nucleus, \(\alpha_B(T)nx_e^2\):

\[
x_e = \sqrt{\frac{\zeta_{\text{CR}}}{n\alpha_B(T)}} = 1.4 \times 10^{-3} \zeta_{\text{CR},-16}n_2^{-1/2}T_0^{0.35}.
\]

PAH catalyzed recombination will reduce this, possibly by factors of a few. It should be clear that in regions of high temperature and low density, this leads to high ionization fractions; and in regions of low temperature and high density, the hydrogen ionization may drop to low values and C\(^+\) may become the main electron donor (this is the case in the CNM).

**B. COOLING**

We have already discussed the heating and cooling of PDRs. The heating was dominated by the photoelectric effect; this also appears to be the case for the general diffuse ISM. The cooling was dominated by the \([\text{C}\ II]\ 158\) and \([\text{O}\ I]\ 63\ \mu\text{m}\) lines; this will require some modification for diffuse conditions.

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\(^1\) Tielens, Eq. (8.5).
In general, the cooling rate in the low-density limit (appropriate here) is given by the cooling function $\Lambda(T)$:

$$\frac{d(\text{energy emitted})}{d(\text{volume}) \, d(\text{time})} = n_H^2 \Lambda(T).$$

The cooling function has units of erg cm$^3$ s$^{-1}$, and in general it can depend on ionization state and composition; here we will assume neutral gas, but for $x_e > 10^{-3}$ electron impact excitation can significantly enhance the cooling rate. We have already discussed the cooling function for the [O I] 63 μm line,

$$\Lambda_{63\mu m}(T) = 2.5 \times 10^{-29} \, T^{0.67} \, e^{-230 \, K/T} \, \text{erg cm}^3 \, \text{s}^{-1}.$$  

A similar result holds for the [C II] 158 μm line,

$$\Lambda_{158\mu m}(T) = 3 \times 10^{-27} \, e^{-92 \, K/T} \, \text{erg cm}^3 \, \text{s}^{-1}.$$  

(For reactions between a charged and neutral species, there is no temperature exponent.) At low temperatures (CNM), the [C II] dominates.

At higher temperatures ($\sim 5000$ K), excitation of electronic states becomes possible. One low-lying excitation is to raise O I into the $2p^4 1D^e_2$ state. The decay back to $2p^4 3P^e_1$ (and ultimately to $2p^4 3P^e_2$) emits in the [O I] 6300,6364 Å doublet (3:1 line ratio; the latter is also accompanied by a 63 μm photon). The cooling rate is:

$$\Lambda_{6300\AA}(T) = 1.8 \times 10^{-24} \, e^{-22800 \, K/T} \, \text{erg cm}^3 \, \text{s}^{-1}.$$  

Other metals have closed shells (Ne, Na+, Ar), have no optical forbidden line excitations due to np$^1$ structure (C+, Mg+, Si+), or are rarer (N, S+, Fe+); so O dominates.

At very high temperatures ($\sim 10^4$ K), Lyman-α can be collisionally excited:

$$\Lambda_{\text{Ly}a}(T) = 7.8 \times 10^{-19} \, x_e \, e^{-118 \, K/T} \, \text{erg cm}^3 \, \text{s}^{-1}.$$  

This cooling curve is very steeply rising. At higher temperatures still, collisions would ionize the hydrogen, e.g.

$$H + e^- \rightarrow H^+ + 2e^-.$$  

This will of course result in ionized gas, which is not (yet) our concern.

**C. THERMAL EQUILIBRIUM**

Our discussion of the thermal equilibrium of the diffuse phases follows that of H II regions and PDRs. There is a heating rate $\Gamma$ (units: ergs/s/H atom), which
depends weakly on gas temperature, and a cooling rate $\Lambda$ that depends strongly on gas temperature. The gas heats up if $\Gamma > n\Lambda$ and cools if $\Gamma < n\Lambda$. We will actually work in terms of the pressure rather than the density because pressure (related to density via $P = nkT$) is more likely to be equal in different coexisting phases (although if turbulent motions approach the sound speed, as often happens, or shocks are important, this may not be true; and we have not considered magnetic or cosmic ray pressure).

At a given pressure $P$ the condition for heating is $\Gamma > (P/k)(\Lambda/T)$, and for cooling it is $\Gamma < (P/k)(\Lambda/T)$. Therefore the thermal equilibrium depends critically on the behavior of the function $\Lambda(T)/T$: when it is high the gas cools and when it is low the gas heats. Equilibria occur when $\Lambda/T$ crosses the values $k\Gamma/P$. If $\Lambda/T$ is increasing, we have a stable thermal equilibrium: a slight increase in the temperature leads to an increase in the cooling rate and the gas returns to its initial state. If $\Lambda/T$ is decreasing, we have an unstable thermal equilibrium: a slight increase in the temperature leads to runaway heating and a slight decrease to runaway cooling.

Each of the above-described cooling processes ($\text{C}^+/\text{O}$ fine structure, and $\text{O}/\text{H}$ electronic excitation) have maxima in $\Lambda/T$, with a minimum in between. Stable equilibria can occur on the rising parts of either. The rising part of the fine structure cooling curve gives rise to the CNM, and the rising part of the $\text{O}/\text{H}$ electronic excitation cooling curve gives rise to the WNM. Theory predicts that since $\Lambda/T$ must equal $k\Gamma/P$, there is a maximum pressure at which the WNM can exist (given by the minimum of $\Lambda/T$) and a minimum pressure at which the CNM can exist (given by the maximum of the fine structure cooling peak). In practice, pressures in the Galactic ISM do not get low enough to destabilize the WNM (Lyman-$\alpha$ can supply enormous amounts of cooling!) and the intergalactic medium is controlled by different physics (it is ionized).

Most interestingly, there is a range of pressures over which the CNM and WNM can co-exist in pressure equilibrium. This range is roughly $P/k = 1000—5000\text{ K cm}^{-3}$. This range also happens to coincide (roughly) with the pressure in the Galactic disk.

D. THE WARM IONIZED MEDIUM

The WIM consists of ionized gas that – like the WNM – is at temperatures of a few thousand K. It is observed in diffuse Hα emission and microwave (free-free) emission, and can also be probed using electron column density measurements from pulsar dispersion (coming later). The ionization source is presumed to come from young stars whose ionizing radiation has burned through the dense environment in which they formed and escaped into the general ISM, but its global structure and the mass flux into and out of the WIM phase is unclear.

3. Hot Gas
Thus far we have discussed diffuse gas heated by radiation from stars. It is difficult for gas to be heated above a few \( \times 10^4 \) K by this mechanism. However, much hotter gas is indeed seen – most notably by its X-ray emission. We now turn our attention to this gas.

In almost all cases, hot gas in the ISM is heated by mechanical energy (shocks). The most common such mechanism locally is supernova explosions. In other cases (e.g. galaxy clusters), infall of gas into a deep potential well leads to high velocities (>1000 km/s), shocks, and ultimately very hot gas. Roughly speaking, a shock at velocity \( v \) can heat gas to temperatures of order \( kT \sim mv^2/2 \), where \( m \) is the mean molecular weight (typically \( \sim m_H/2 \) since fast shocks will ionize the hydrogen). After the shock has passed, the gas cools at a rate determined by the density, composition, and temperature. Hot gas is (usually) not in equilibrium – its heating source is transient. In the most extreme cases, gas can have cooling timescales comparable to the age of the Universe.

We will discuss the generation of shocks later, when we consider hydrodynamics. However, we are prepared now to discuss what happens to shock-heated gas. In particular, we can investigate the ionization state of the metals, and the timescale for cooling of the gas.

A. IONIZATION

The ionization of very hot gas is usually driven by collisions, rather than by radiation, since at \( T \gg 10^5 \) K the collisions can supply far more energy. That is, the ionization balance is determined by comparing collisional ionization:

\[
X^i + e^- \rightarrow X^{i+1} + 2e^- ,
\]

where \( X \) is any element and \( i \geq 0 \) is an ionization stage, to radiative recombinations (either direct or dielectronic):

\[
X^{i+1} + e^- \rightarrow X^i + n\gamma .
\]

Any ionization balance that can be described by equating these two processes is called coronal equilibrium (since it is applicable to the solar corona) or collisional ionization equilibrium (CIE).

Collisional ionization equilibrium depends only on the temperature \( T \) because both the changes in ionization state \( i \rightarrow i+1 \) and \( i+1 \rightarrow i \) have rates proportional to density. The ratio of \( X^{i+1}:X^i \) is equal to the ratio of collisional ionization rate, \( C_i \), to the recombination rate, \( \alpha_i \). The former has a factor of \( \exp(-E_i/kT) \), where \( E_i \) is the ionization energy of \( X^i \), so there is a suppression of the more highly ionized stages at low temperature. However, this is not Saha equilibrium because the dominant ionization and recombination processes are not inverses of each other. Saha equilibrium would require a blackbody radiation field at temperature \( T \).
Charge exchange may modify the simplistic CIE picture described above (due to collisions of ions with neutrals). It is also a two-body reaction and hence does not change the fact that the equilibrium state depends only on $T$ and not $n$.

As a simple example, we consider the CIE of hydrogen (i.e. H$^0$ versus H$^+$). The collisional ionization rate may be approximated to zeroth order as the geometric collision rate times the Boltzmann factor; approximating the first by a circular cross section of radius $\sim 2a_0$, we find $\sim 3\times 10^{-16} v e^{-13.6 \text{eV}/kT} \text{cm}^2$, where $v = 6 \times 10^5 \ T^{1/2} \text{cm s}^{-1}$ is the typical electron velocity. The recombination rate, on the other hand, is $\alpha_A \sim 4 \times 10^{-13} T_4^{-0.72} \text{cm}^3/\text{s}$. (Hot gas is often optically thin to H continuum radiation in which case the Case A coefficient is applicable.) The ratio is H$^0$:H$^+=1:1$ at $T=1.4 \times 10^4$ K. The fact that recombination cross sections are far less than geometric generally implies that the collisional ionization temperature (as defined by when the ratio of adjacent ionization stages is unity) is usually an order of magnitude less than $E_i/k$.

The ionization temperatures for heavy elements and high ionization stages are larger. For example, the values of log $T$ at the collisional ionization temperature are:

**Hydrogen**
- I $\rightarrow$ II $\quad 4.19$

**Helium**
- I $\rightarrow$ II $\quad 4.47$
- II $\rightarrow$ III $\quad 4.89$

**Carbon**
- I $\rightarrow$ II $\quad 4.13$
- II $\rightarrow$ III $\quad 4.67$
- III $\rightarrow$ IV $\quad 5.03$
- IV $\rightarrow$ V $\quad 5.00$
- V $\rightarrow$ VI $\quad 5.88$
- VI $\rightarrow$ VII $\quad 6.08$

Notice that the C III $\rightarrow$ C IV transition temperature is slightly above C IV $\rightarrow$ C V. This is because if the gas is hot enough to collisionally ionize one of the 2s electrons, it is hot enough to ionize the other. Thus C IV is not the majority ionization stage in CIE at any temperature – it only exists as a trace species. A similar result applies to other 3-electron ions (N V, O VI) and to some of the 11-electron ions (S VI).

Unfortunately, we will see that these same ions are also the easiest to observe via UV absorption lines (e.g. in the intergalactic medium). Their abundances may be sensitive to non-equilibrium effects (e.g. rapid cooling or heating; photoionization) and despite common assumption, they may not trace gas at their theoretical CIE-implied temperature.

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The heaviest of the major metals, iron, reaches the 50% in Fe XXV stage (2 electrons; 1s²) at \( T = 17 \) MK, and 50% Fe XXVII (0 electrons) at 160 MK. However, most of the hot regions in the ISM do not reach these extreme temperatures (they occur at very high shock velocities, >3000 km/s, which do occur in young supernova remnants).

**B. COOLING PROCESSES**

Hot gas cools primarily by the excitation of atomic/ionic lines, and by free-free radiation. The latter is more important when it is available. We consider two cases: the simpler case of primordial composition, and then the more complex case with metals of roughly solar composition.

**Primordial composition:**

At low temperatures (~1.4×10⁴ K), excitation of the \( \text{H}^0 \) Lyα (1216Å) line is dominant, reaching a peak of \( \Lambda \sim 1.3 \times 10^{-22} \text{ erg cm}^3 \text{ s}^{-1} \). This rate is sharply peaked since at lower temperatures Lyα is not excited and at higher temperatures the H is ionized. Excitation to 2s followed by two-photon decay is present but subdominant. At higher temperature, there is a second peak in the cooling function corresponding to the excitation of the He⁺ Lyα (304Å) line.

At temperatures in excess of a few \( \times 10^5 \) K, the gas is fully ionized, and cooling switches over to free-free radiation, with \( \Lambda \sim T^{1/2} \).

**Solar composition:**

Again, at low temperatures, \( \text{H}^0 \) Lyα is the dominant cooling mechanism. As the temperature is increased, however, and \( \text{H}^0 \) is destroyed, cooling switches to the ns and ns² ions: C III/IV, O V/VI, Ne VII/VIII, and Fe VII/VIII (in the range of \( 10^5 - 10^6 \) K). At higher temperatures, the Fe n=2 shell is partially filled and its resonance lines can be excited. There is a final peak in the cooling function at ~10⁷ K corresponding to the coolants Fe XXIII/XXIV. As one goes to higher temperatures, free-free radiation takes over. At temperatures far in excess of a Rydberg (158 kK) the free-free contribution becomes

\[
\Lambda_{\text{ff}} = 1.4 \times 10^{-27} T^{1/2} \text{ erg cm}^3 \text{ s}^{-1}.
\]

**C. COOLING TIMESCALES**

We may define the **cooling timescale** to be the ratio of the thermal energy of the gas \( (3nkT/2) \) to the cooling rate \( n^2 \Lambda \):

\[
\tau = \frac{3kT}{2n\Lambda}.
\]

The cooling time is clearly shorter for higher density gas. Additionally, since for very hot gas (>10⁷ K) the free-free cooling becomes dominant, we see that \( T/\Lambda \) is an
increasing function of temperature ($\sim T^{1/2}$). To be specific, in the free-free dominated regime,

$$\tau = 16 n^{-1} T_{7/2}^{1/2} \text{ Myr}.$$ 

Thus we see that low-density gas (e.g. in the hot halo of a galaxy cluster, with $T_7 \sim$ few and $n \sim 10^{-3} \text{ cm}^{-3}$) may cool only on cosmological timescales. Hot gas in the Galactic disk heated by supernovae will be denser and cool faster, but the cooling is often slow compared to the timescale for motion of the gas (which we will discuss later).

At lower temperatures ($T \sim 10^5 – \text{few} \times 10^6 \text{ K}$), the cooling of the gas becomes much faster. Moreover, the recombination and cooling times (both scaling as $n^{-1}$) can become similar, in which case CIE does not hold. The ionization state and cooling function $\Lambda$ then depend on the full history of the gas, and numerical simulations that model ionization as well as hydrodynamics are required for a full understanding.