Equilibrium Pricing and Optimal Hedging in Electricity Forward Markets

HENDRIK BESSEMBINDER and MICHAEL L. LEMMON

ABSTRACT

Spot power prices are volatile and since electricity cannot be economically stored, familiar arbitrage-based methods are not applicable for pricing power derivative contracts. This paper presents an equilibrium model implying that the forward power price is a downward biased predictor of the future spot price if expected power demand is low and demand risk is moderate. However, the equilibrium forward premium increases when either expected demand or demand variance is high, because of positive skewness in the spot power price distribution. Preliminary empirical evidence indicates that the premium in forward power prices is greatest during the summer months.

WHOLESALE POWER MARKETS, where producers trade electricity among themselves and with power-marketing and power-distribution companies, have grown rapidly in recent years.1 The U.S. Department of Energy (2000) reports that U.S. wholesale power transactions during 1999 amounted to approximately 2.6 billion megawatt hours (MWh), or about $85 billion. The U.S. wholesale power market is soon likely to comprise the world’s largest commodity market.

Electricity as a commodity has many interesting characteristics, most of which stem from the fact that it cannot be economically stored.2 Market-

1 The growth in power trading is partially attributable to ongoing deregulation of the industry. Most deregulation scenarios call for the separation of power production from transmission and retailing, with production and retailing opened to competition.

2 Though power cannot be stored, potential energy can be stored in the form of fuel stockpiles or water behind dams. The capacity to quickly convert potential energy to power remains limited, however, as evidenced by the differential between daytime and nighttime power prices.
clearing prices are volatile because inventories cannot be used to smooth supply or demand shocks. The absence of storage also allows for predictable intertemporal variation in equilibrium prices. Power prices for daytime delivery are typically more than twice as high as for nighttime delivery, and power prices during the summer are predictably higher than during the temperate months of spring and fall. In addition, power prices are subject to sudden, but generally temporary, upward spikes. Some of these features of wholesale power prices are evident in Figure 1, which displays average daily on-peak spot power prices in the Pennsylvania, New Jersey, Maryland (PJM) market over the period April 1997 through July 2000. The standard deviation of percentage changes in the average daily power prices displayed on the figure is 34 percent. By comparison, the standard deviation of daily returns on the S&P 500 Index during the most volatile single month in recent decades, October 1987, was 5.7 percent. Large, temporary, upward spikes in PJM prices are evident in Figure 1 during each summer season. In another dramatic and well-publicized example, power prices in

Figure 1. Average daily on-peak power prices in dollars per megawatt hour, for delivery at PJM from April 1997 through July 2000.
the U.S. Midwest briefly rose from near $30/MWh to over $7,000/MWh during June 1998.\textsuperscript{3}

Finally, and most important for purposes of this paper, the inability to store power means that the no-arbitrage approach to pricing derivative securities cannot be applied in the usual manner. The well-known cost-of-carry relationship links spot and forward prices as a no-arbitrage condition. However, the arbitrage strategies required to enforce the cost-of-carry relationship include purchasing the asset at the spot price and storing it for subsequent sale at the forward price.\textsuperscript{4} Since this strategy cannot be executed in power markets, forward prices for electricity need not conform to the cost-of-carry relationship.

Pirrong and Jermakyan (1999) and Eydeland and Geman (1999) also observe that the no-arbitrage approach does not apply to power derivatives. Pirrong and Jermakyan note that electricity forward prices will differ from expected delivery date spot prices due to an endogenous market price of power demand risk. However, they do not attempt to model the determinants of the market price of power risk, while we do. Eydeland and Geman focus on electricity options, noting (p. 72) that:

there is another important consequence of non-storability: using the spot price evolution models for pricing power options is not helpful, since hedges involving the underlying asset, i.e., the famous delta hedging, cannot be implemented, as they require buying and holding power for a period of time.

Eydeland and Geman present a pricing model for power options that relies on assumptions regarding the evolution of forward power prices. We adopt an equilibrium approach and explicitly model the economic determinants of market clearing forward power prices.

The equilibrium model presented here relies on the assumptions that prices are determined by industry participants rather than outside speculators, and that power companies are concerned with both the mean and the variance of their profits. We evaluate power producers’ and retailers’ net demands for forward contracts, and obtain closed form solutions for the equilibrium forward power price and for optimal forward positions. The implications of the model are illustrated with a set of simulations.

The model generates the testable implication that the forward risk premium is a function of the difference between two covariance terms that can be related to the variance and skewness of spot power prices. Positive skewness in wholesale power prices is attributable to the nonstorability of power

\textsuperscript{3} For example, see “Staff Report to the Federal Energy Regulatory Commission on the Causes of Wholesale Electric Pricing in the Midwest During June 1998,” available at http://www.ferc.fed.us.

\textsuperscript{4} For a description of the standard cost-of-carry forward pricing model see, for example, MacKinlay and Ramaswamy (1988) or Kawaller, Koch, and Koch (1987).
combined with convexity in the industry supply curve. When expected power demand is low and demand variability is modest (as might be expected during the temperate months of spring and fall), there is little skewness in spot prices, and power retailers' desire to hedge their revenues leads to a downward bias in equilibrium forward prices. In contrast, when expected demand is high relative to capacity or when demand is more variable, the distribution of spot power prices becomes positively skewed. Short forward positions incur large losses if upward spikes in spot prices occur, and the equilibrium forward price is bid up to compensate for skewness in the spot price distribution. The U.S. demand for power is largest and most variable in the summer. The model, therefore, predicts an upward bias in U.S. forward power prices for summer delivery.

We also evaluate optimal forward positions. Power-producing firms' optimal forward market positions depend on forecast output and on the skewness of power demand. Power retailing firms' optimal forward positions depend on forecast usage, and on the interaction between local and system demand as measured by power demand betas.

Finally, we conduct some preliminary empirical analysis using the available short time series of electricity forward and spot delivery prices. The evidence is generally supportive of the model's predictions. Forward prices for summer delivery exceed forecast spot prices, while forward prices for spring and fall delivery are similar to, or slightly less than, forecast spot prices. Regression analysis confirms that the premium in forward power prices as compared to expected spot prices is positively related to the level of power demand.

Routledge, Seppi, and Spatt (1999) also consider the equilibrium pricing of electricity contracts. They focus on linkages between the natural gas and electricity markets (i.e., the "spark spread") that arise from the fact that natural gas can be stored or converted into electricity. They obtain several implications, including predictions of mean reverting spot prices and unstable electricity/fuel price correlations. Their model also implies that electricity prices will be positively skewed, while we obtain the implication that skewness will affect the equilibrium forward premium and optimal forward positions.

Despite the unique features of power markets that arise from nonstorability, studying power markets is likely to lead to broader insights regarding market learning and financial innovation. As noted above, the model presented here is one of limited participation, with prices determined by the trades of those who produce and deliver power rather than by speculators from outside the power industry. The limited participation assumption appears reasonably accurate at present, as discussed in Section I.C below. How-

6 There are also several papers that consider pricing of derivatives on storable energy commodities such as oil and natural gas. These include Amin, Ng, and Pirrong (1995), Gabillion (1995), and Routledge, Seppi, and Spatt (2000).
ever, the existence of a nonzero premium in forward power prices as implied by the model and the preliminary data provides incentives for financial intermediaries to create instruments, for example, power-indexed bonds, to allow outside speculators to include power positions in their portfolios. The presence of outside speculators would then be expected to decrease the magnitude of the forward premium. A similar evolution has been described by Froot (1998, 2001) in the case of catastrophe insurance, an industry which evolved during the 1990s from a limited participation market with premiums well above actuarial value, to a broader market with more modest premiums, after insurers issued catastrophe-linked bonds. It will be of interest to observe whether financial contracts that allow outside speculators to take positions in power markets are introduced, and if power forward prices then converge toward average spot prices.

Finally, although we focus explicitly on hedging decisions and equilibrium in wholesale electricity markets, we believe that some insights obtained in our analysis will prove useful to a broader set of hedging problems. Previous studies concerning forward hedging strategies, such as Anderson and Danthine (1980) and Hirshleifer and Subramanyam (1993), among others, develop expressions showing that optimal forward positions depend on the covariance of revenues (price times quantity) with prices. We extend the analysis by making endogenous both market prices and output decisions, obtaining the implication that price skewness is relevant to hedging decisions. Our conjecture is that equilibrium approaches like ours will also prove relevant in a broader array of commodity hedging applications.

This paper is organized as follows. Section I outlines the general setup of the model and discusses the demand for risk reduction on the part of power industry participants. Section II derives the equilibrium forward price and the optimal forward positions of power producers and retailers. Section III presents some preliminary empirical evidence, and Section IV concludes.

I. Power Production, Wholesale Markets, and the Demand for Risk Reduction

A. The General Setup

The inability to store power implies that traditional cost-of-carry models for pricing forward contracts do not readily apply to power markets. We therefore use an equilibrium approach. Our primary goal is to assess equilibrium forward power prices and optimal hedge positions for power firms. To do so requires a model of the underlying spot market and of the transactions that participants will optimally make there. We analyze power production during a single future time period, and assume that there is no uncertainty in spot markets, that is, that power companies are able to forecast demand in the immediate future with precision, and are able to enter contracts in the wholesale market at known prices. Actual spot power markets operate with short delivery horizons: The most active markets are for
next-hour and next-day delivery. In practice, power demand over a short
horizon such as the next hour can be forecast with substantial, though not
complete, precision.

The model includes \( N_P \) power producers, each producing power using iden-
tical technology and selling it to power retailers in a competitive wholesale
market.\(^7\) We assume that power can be transmitted without cost.\(^8\) There are
\( N_R \) power retailing (or power marketing) firms that purchase power in the
wholesale market and sell it to final consumers at a fixed unit price, de-
noted \( P_R \). Retail customers consume as much power as they desire at that
price.\(^9\) The realized demand for retailer \( i \) is denoted by the exogenous ran-
dom variable \( Q_{Ri} \).

Each producer \( i \) is assumed to have a power production cost function of the form

\[
TC_i = F + \frac{a}{c} (Q_{Pi})^c,
\]

where \( F \) represents fixed costs, \( Q_{Pi} \) is the output of producer \( i \), and \( c \) is a
constant greater than or equal to two. This simple cost function captures
many features relevant to the production of electricity. It implies that mar-
ginal production costs increase with output, which reflects the fact that the
industry employs an array of production technologies and fuel sources with
differing out-of-pocket costs, including hydro, nuclear, coal, oil, and natural
gas. Increasing marginal costs are also consistent with empirical regulari-
ties such as higher daytime than nighttime power prices.

\(^7\) Whether wholesale power markets are in fact competitive has been the subject of consid-
there are an insufficient number of suppliers in the British power markets. Borenstein and
Bushnell (1999) conclude that the California electricity markets have some potential for market
power, but view the resulting costs to be small relative to the potential gains from deregulation.
Joskow and Kahn (2001, p. 30) conclude that recent prices in the California market “far ex-
cceeded competitive levels.” See also the survey by Joskow (1997). In relying on the assumption
that wholesale power markets are competitive, our analysis can be viewed as the limiting case
obtained when market mechanisms are developed to mitigate possible producer market power.

\(^8\) We abstract from analyzing transmission costs in order to focus on an alternate set of
issues. The pricing of transmission rights is itself an interesting and challenging issue, made
complicated by the presence of externalities within the distribution system. For a discussion of
possible market-oriented solutions to the allocation of transmission capacity, see Joskow and

\(^9\) Note that this approach does not rule out predetermined seasonal variation in retail prices—
the assumption is that the retail price does not respond to real-time shocks. Despite the famili-
arity of the fixed-price retail contract, it is actually a complex derivative: a variable quantity
forward contract. The main argument in favor of the fixed price contract is that retail custom-
ers do not have a comparative advantage in bearing price risk. However, a fixed retail price
fails to provide incentives to curtail demand at times when wholesale prices are high. Hybrid
contracts that allow for some variation in retail prices in response to wholesale prices have
begun to evolve, being offered mainly to larger and more sophisticated final customers.
If the cost parameter $c$ is greater than two, marginal costs increase at an increasing rate with (are convex in) output. This allows some flexibility to account for complexities that are not formally modeled, including the use of relatively inefficient peaking plants to meet high levels of demand, as well as the possibility that constraints on production and transmission capacity will become binding when demand is high. The rapid increases in marginal costs implied by production levels that approach capacity can be approximated by considering the effect of increasing the cost convexity parameter, $c$. Note also that if $c$ is greater than two, the distribution of power prices will be positively skewed even when the distribution of power demand is symmetric.

B. The Demand for Risk Reduction

The literature on corporate risk management argues that firms can benefit from hedging market risks, because excessive volatility increases the expected costs of financial distress and can lead to suboptimal investment. For example, Smith and Stulz (1985) show that risk hedging can reduce expected tax liabilities, expected bankruptcy costs, and equilibrium wages paid to risk-averse managers. Stulz (1990), Bessembinder (1991), and Froot, Scharfstein, and Stein (1993) present models incorporating contracting costs and costly external financing, and show that a policy of hedging market risks can lead to more efficient capital investment outcomes.

Relying on the corporate hedging literature, we argue that companies in the power industry are likely to benefit from reducing the risk of their cash flows. The extreme volatility of wholesale power prices implies that even well-capitalized power firms may have power price exposures sufficiently large that adverse price changes could lead to corporate default or bankruptcy. Indeed, two major California power retailers, Southern California Edison and Pacific Gas and Electric, defaulted on scheduled payments to creditors and suppliers during January 2001, attributing the defaults to the high costs of purchasing power on the wholesale markets. Further, the large capital investments involved in power production and distribution increase the relevance of risk-related changes in investment incentives. In practice, power producers have used bilateral forward contracts for decades, and there is an active brokered market in power forward and option contracts. The New York Mercantile Exchange and the Chicago Board of Trade have each introduced power futures contracts, and there is an emerging market in weather-related derivatives targeted to power producers.

We assume that power firms consider both expected profits and the volatility of profits. Higher expected profits from power transactions improve firm value, while higher volatility imposes costs due to increases in the likelihood of financial distress and/or effects on future investment incentives. We assume that power firms' objectives are linear in expected profit and the

---

10 See also Anderson (2000) for a description of how events in June 1998 caused power prices in the U.S. Midwest to increase from $30/MWh to over $7,000, leading to contract defaults and the near bankruptcy of some power firms.
variance of profit. Optimal hedging strategies when the objective is linear in expected profit and variance of profit have been studied by Rolfo (1980) and Hirshleifer and Subramanyam (1993), among others.

C. Forward Market Participation

We model the forward market as a closed system, where only producers and retailers (power marketing firms) can take positions. While this oversimplifies the actual situation, it seems a reasonable starting point. Since power cannot be stored, each power marketer must arrange to deliver the power it purchases to an ultimate retail customer who will consume it. Outside speculators, that is, those who neither produce power nor have delivery contracts with final customers who will consume the power, cannot take positions in contracts that require physical power delivery. Futures contracts allow for participation by outside speculators, so long as they offset their positions prior to the delivery date. Although power futures contracts are traded, activity levels are extremely low. Outside speculators can take positions in cash-settled contracts, but prices of cash-settled contracts remain linked to prices of physical-delivery contracts only by the trades of those participants who can accomplish physical delivery. In practice, trading in cash-settled electricity contracts has not developed rapidly. Krapels (2000, pp. 13 and 72) observes that:

regional over-the-counter markets are the centers of price dynamics, with very limited potential for participation in trading by organized or individual speculators . . . [and] . . . electricity markets have not yet developed effective mechanisms for speculator participation.

Ong (1996) reports the market for cash-settled trades in electricity (mainly options and swaps) to be only about five percent as large as the physical delivery market. He attributes the slow growth in cash-settled trading to the lack of definitive spot power price indices.

11 A power-marketing firm that contracts to purchase power must also arrange a “sink” for the power. Conversations with industry participants indicate that some power marketing firms have defaulted on agreements to purchase power, not for financial reasons, but because they were unable to arrange for a sink.

12 For example, on October 2, 2000, the New York Mercantile Exchange listed futures prices for power delivery at the Pennsylvania, New Jersey, Maryland (PJM) market for each month from October 2000 through November 2001. However, open interest in every contract except November 2000 was zero, and only four November contracts were open.

13 Eydeland and Geman (1999) also consider the role of physical production capacity, noting that the safest way to hedge the risks involved in trading power options is to operate a power plant.

14 The best publicized spot power indices are those disseminated by Dow Jones, & Company, Inc. However, the Dow Jones data reflect the average price paid for power delivered at a location on a given day. Since many deliveries result from previously arranged forward or option contracts, the data does not accurately represent actual spot prices.
Though we do not formally present an alternative model with costless participation by unlimited numbers of outside speculators, it is easy to envision the possible equilibria that would result. With risk-neutral outside speculators, the forward price would converge to the expected delivery date spot price. If the outside speculators are risk-averse but hold diversified portfolios, then a CAPM-style result will be obtained, with the bias in the forward price as a predictor of the spot price dependent on the covariance between power prices and overall market returns. If (as might be expected) power prices are not significantly correlated with aggregate market returns, then frictionless models with unlimited numbers of either risk-averse or risk-neutral speculators will imply a zero risk premium for power contracts.

An alternative to assuming either the complete absence of outside speculators or costless participation by an unlimited number of outside speculators is to consider the case where a finite number of outsiders endogenously choose to speculate, after bearing an entry cost. Hirshleifer (1988) presents a model where outside speculators bear a fixed setup cost to trade in the forward market. In the resulting equilibrium, the sign of the forward premium is determined by systematic risk and by the net hedging pressure of producers and consumers of the underlying good. This suggests that the sign (though not the magnitude) of the forward risk premium in a market without systematic risk is not altered by the introduction of limited outside speculation, and that the model’s testable implications are unchanged. Section III below presents results of testing the implications of the model developed here against the zero-risk-premium alternative.

II. Wholesale Power Trading, Equilibrium Hedging, and the Forward Risk Premium

We begin by assessing real-time trading in the wholesale spot market while taking into account previously selected forward positions. We then work back to assess optimal strategies and market-clearing conditions in the forward market, given that each producer and retailer will behave optimally in the spot market.

A. The Wholesale Spot Market

In the wholesale spot market, producers sell to retailers, who in turn distribute power to their customers. Let $P_W$ denote the wholesale spot power price, $Q_{Pi}^W$ denote the quantity sold by producer $i$ in the wholesale spot market, and $Q_{Fi}^F$ denote the quantity that producer $i$ has previously agreed to deliver (purchase if negative) in the forward market at the fixed forward price $P_F$. The ex post profit of producer $i$ is given as

$$\pi_{Pi} = P_W Q_{Pi}^W + P_F Q_{Fi}^F - F - \frac{a}{c} (Q_{Pi})^c,$$

(2)
where each producer’s physical production, $Q_{Pi}$, is the sum of its spot and forward sales, $Q_{Pi}^W + Q_{Pi}^F$.

Retailers simply buy in the real-time wholesale markets the difference between realized retail demand and their previous forward purchases. Letting $Q_{Rj}^F$ denote the quantity sold (purchased if negative) forward by retailer $j$, and $P_R$ denote the fixed retail price per unit, the ex post profits of each retailer $j$ are given by

$$\pi_{Rj} = P_R Q_{Rj} + P_F Q_{Rj}^F - P_W (Q_{Rj} + Q_{Rj}^F).$$  (3)

The profit-maximizing quantity sold in the spot market by producer $i$ is

$$Q_{Pi}^W = \left( \frac{P_W}{a} \right)^x - Q_{Pi}^F,$$  (4)

where $x = 1/(c - 1)$. Equating the total physical production of producers to total retail demand, and using the fact that forward contracts are in zero net supply, the market-clearing wholesale price can be expressed as

$$P_W = a \left( \frac{Q_D}{N_P} \right)^{c-1}.$$  (5)

where $Q_D = \sum_{j=1}^{N_R} Q_{Rj}$ denotes total system retail demand. Note that if $c > 2$, as would be expected if marginal productive efficiency decreases with output or if capacity constraints are potentially binding, then this expression can readily account for some of the important characteristics of observed power prices. These include disproportionately large increases in spot prices when demand rises, and positive skewness in the probability distribution of spot prices.

Using expressions (4) and (5), each producer’s sales in the wholesale spot market can be written as

$$Q_{Pi}^W = \frac{Q_D}{N_P} - Q_{Pi}^F.$$  (6)

B. The Demand for Forward Positions

We next step back in time to determine the optimal forward positions taken by retailers and producers and the equilibrium forward price. It is useful to consider the interaction between profits in the absence of any hedge positions and wholesale prices. Restating (2) and (3) and using expression (6) for $Q_{Pi}^W$, define the “but-for-hedging” profits of producers and retailers as
Define $A/2$ to be the coefficient on the variance of profit in the objective functions of power marketers and producers. If $A$ is zero, then volatility risk is not relevant to corporate objectives, while $A$ greater than zero implies that volatility risk is viewed negatively. The optimal forward position when the objective function is linear in expected profit and the variance of profit is known (e.g., Anderson and Danthine (1980) and Hirshleifer and Subramaniam (1993)) to be

$$Q_{(P,R)}^F = \frac{P_F - E(P_W)}{A \text{Var}(P_W)} + \frac{\text{Cov}(\rho_{P,R}, P_W)}{\text{Var}(P_W)}.$$  \hfill (9)

The optimal forward position contains two components. The first term on the right side of (9) reflects the position taken in response to the bias in the forward price as compared to the expected spot price. The second term is the quantity sold forward to minimize the variance of profits. In evaluating (9), it is useful to further consider the covariance between “but-for-hedging” profits and the wholesale price. Forward hedging can reduce risk precisely because this covariance is nonzero. Using expressions (7) and (8) along with expression (5) for $P_W$ derived above (to reflect that firms will optimize in the spot markets), these covariance terms can be written as

$$\text{Cov}(\rho_{P_i}, P_W) = \frac{1}{a^x} \text{Cov}(P_W^{x+1}, P_W) - \frac{1}{ca^x} \text{Cov}(P_W^{x+1}, P_W)$$  \hfill (10)

and

$$\text{Cov}(\rho_{R_i}, P_W) = P_R \text{Cov}(Q_{R_i}, P_W) - \text{Cov}(P_W Q_{R_i}, P_W).$$  \hfill (11)

Expressions (10) and (11) reveal the four types of risk that power forward positions can potentially hedge. The first term on the right side of (11) reflects that, despite fixed retail prices, retail revenues covary positively with wholesale prices when the quantity demanded locally is positively correlated with wholesale price. Because the wholesale price is an increasing function of system demand, this correlation is necessarily positive on average, implying that the industry’s retail revenue is risky. Second, retailers bear risk in
their costs of acquiring power on the wholesale markets, reflected by the second term on the right side of expression (11). Third, producers bear risk in their sales revenue, reflected by the first term on the right side of expression (10). Finally, producers bear risk in production costs, reflected by the second term on the right side of expression (10). Each of these risks will affect the forward positions that individual firms take. Note though, that producer’s revenues and retailer’s costs are zero-sum, implying that these risks are diversifiable within the system. The equilibrium forward price will depend only on the risks borne by the industry as a whole: variability in retail revenues and in production costs.

C. The Equilibrium Forward Price

The forward price that yields a zero net supply of forward contracts is shown in Appendix A to be

\[
P_F = E(P_W) - \frac{N_P}{N_{CA}} \left[ cP_R \cdot \text{Cov}(P_W, P^+_W) - \text{Cov}(P^+_W, P_W) \right], \tag{12}
\]

where \( N = (N_R + N_P)/A \) is a measure that reflects the number of firms in the industry and the degree to which they are concerned with risk. The forward price converges to the expected spot price if the number of firms in the industry approaches infinity, or if risk is irrelevant to firms’ objectives \((A = 0)\).\(^{15}\) With finite \( N \), the equilibrium forward price differs from the expected wholesale price by the difference between two risk-related terms that reflect the net hedging pressure of producers and retailers. The forward price will be less than the expected spot price if the first term in brackets, which reflects retail revenue risk, is greater than the second term, which reflects production cost risk.

To gain additional intuition about the characteristics of the forward risk premium, it is useful to restate equation (12) in terms of the central moments of the distribution of wholesale spot prices. Appendix A shows that when the functions \( P^z \) and \( P^{z+1} \) are approximated using second-order Taylor series expansions, the equilibrium forward price can be restated as

\[
P_F = E(P_W) + \alpha \text{Var}(P_W) + \gamma \text{Skew}(P_W), \tag{13}
\]

where \( \text{Var}(P_W) \) and \( \text{Skew}(P_W) \) denote the variance and skewness of the wholesale spot price, respectively, and where

\(^{15}\) We assume that producers and retailers are equally concerned with risk. If retailers and producers view risk differently, then the term \( N \) in expression (12) is restated as \( N = (N_P/A_P + N_R/A_R) \), where \( A_P \) and \( A_R \) are the coefficients on the variance of profits in the objective functions of power producers and retailers, respectively. Thus the sign of the forward premium is unaffected by differential degrees of concern with risk, but the magnitude of the premium will typically be altered.
To induce risk-averse retailers to enter the industry, the fixed retail price must exceed the expected wholesale spot price, implying that $\alpha$ and the second term on the right side of (13) are strictly negative. If the distribution of spot prices is not skewed, the forward price is downward biased relative to the expected spot price. The downward bias in the forward price in this case reflects retailers’ net hedging demand. The profits of power retailers are positively exposed, on average, because more retail power is sold when the wholesale price is high. This positive revenue exposure leads to a net demand to sell (to create an offsetting exposure) in the forward market to reduce risk. The downward bias in the forward price stimulates an offsetting demand for forward purchases, so that the market can clear. Ceteris paribus, the magnitude of the bias increases with the expected variance of prices.

Recall that $c > 2$ and $x = 1/(c - 1)$, implying that $0 < x < 1$. The term $\gamma$ is therefore positive, implying that, ceteris paribus, the forward price increases with the skewness of spot prices. The distribution of wholesale power prices will be positively skewed if marginal production costs are convex ($c > 2$) or if the demand distribution itself is positively skewed. Positive skewness in wholesale prices reflects the possibility of large upward spikes in marginal production costs which, given fixed retail prices, reduce industry profitability. The industry as a whole would prefer to hedge against production cost spikes by purchasing power at fixed forward prices. As a consequence, the equilibrium forward price must be increased to induce offsetting forward sales to allow the market to clear.

To illustrate the sign and economic determinants of the equilibrium forward risk premium as expressed in equation (12), we conduct a series of simulations. In interpreting the simulation results, note that the magnitude of the simulated premium can be made arbitrarily large or small by varying the number of firms or the risk relevance parameter, $A$. The simulation is useful in terms of clarifying the determinants of the sign of the forward bias.

Initially, we assume that power demand is distributed normally with mean $E(Q^D) = 100$. We consider demand standard deviations ($SD$) ranging from $SD = 1$ to $SD = 40$. The numbers of firms are set at $N_R = N_P = 20$, and cost functions ranging from quadratic ($c = 2$) to quintic ($c = 5$) are considered. To maintain comparability across cost structures, the variable cost parameter is set as $a = 30(N_P/100)^{(c-1)}$, which ensures that the wholesale price conditional on demand of 100 is $30$, regardless of $c$. The retail price is set at 1.2 times the expected wholesale price conditional on $c$. Finally, to keep risk premia comparable across $c$, the risk-relevance parameter is set as

$$\alpha = \frac{N_P(x + 1)}{Nca^x} \left( [E(P_W)]^x - P_R[E(P_W)]^{x-1} \right)$$

and

$$\gamma = \frac{N_P(x + 1)}{2Nca^x} \left( x[E(P_W)]^{x-1} - (x - 1)P_R[E(P_W)]^{x-2} \right).$$
For each value of $c$ and SD, we randomly generate 1,000 demand realizations. For each realization, the spot price is computed according to expression (5), and from the 1,000 realizations, the covariance measures and the forward price in expression (12) are computed.

Figure 2 displays the bias in the forward price implied by expression (12), as a percentage of the expected spot price, for differing degrees of cost function convexity and levels of demand risk. For quadratic costs ($c = 2$), the bias is always negative, with the forward price less than the expected delivery date spot price, and increases in absolute magnitude as demand risk increases. Quadratic costs imply a linear supply schedule and, given normally distributed demand, that the skewness of wholesale prices is zero. As a consequence, only the variance of prices is relevant, and the equilibrium forward price decreases as demand becomes more variable.

With convex marginal costs ($c > 2$) and normally distributed demand, the distribution of spot power prices becomes positively skewed, and the skewness increases rapidly with demand variability. Figure 2 shows that the bias in equilibrium forward prices is then negative for low levels of demand risk, but reaches a minimum and becomes positive at higher levels of demand.
risk. The minimum is reached sooner and the equilibrium bias is larger if the production cost function is more convex. This reflects that retail revenue risk dominates when demand variance is small, but that the production cost risks increase with skewness and dominate when demand risk is large.

Mean power demand varies substantially across times of day and across seasons. To investigate the implications of the model with respect to variation in expected demand, we conduct simulations in which $E(D)$ is varied from 75 to 125, while the standard deviation of demand is varied from 1 to 40. For brevity, and in light of the empirical evidence presented in Section III below, the simulations reported are limited to cost convexity of $c = 4$. The retail price is set as $P_R = 1.2E(P_W|c)$. The risk relevance parameter is set at $A = 0.8/2^c$.

![Figure 3. Bias in forward power price as a percentage of the expected spot price, for varying mean demand and demand risk. Demand is normally distributed with mean and standard deviations as indicated on the figure. The numbers of firms are set at $N_F = N_P = 20$. Variable costs for each producer are given by $(a/c)(Q_D)^c$, where $a = 30(N_F/E(D))^{c-1}$ with $c$ fixed at 4. The retail price is set as $P_R = 1.2E(P_W|c)$. The risk relevance parameter is set at $A = 0.8/2^c$.](image-url)
D. Testable Hypotheses

The model developed here makes the following testable predictions regarding the forward premium (i.e., the bias in the forward as a predictor of the delivery-date spot) in power prices.

**Hypothesis 1:** The equilibrium forward premium decreases in the anticipated variance of wholesale prices, ceteris paribus.

**Hypothesis 2:** The equilibrium forward premium increases in the anticipated skewness of wholesale prices, ceteris paribus.

Though our primary testable implications are stated in terms of power prices, the underlying state variable is power demand. The simulations conducted above demonstrate that the model also supports the following implications.

**Hypothesis 3:** The equilibrium forward premium is convex, initially decreasing and then increasing, in the variability of power demand, ceteris paribus.

**Hypothesis 4:** The equilibrium forward premium increases in expected power demand, ceteris paribus.

The equilibrium premium in forward power prices is likely to vary in sign and magnitude on a seasonal as well as a geographic basis. The probability distribution of spot power prices during the temperate climates of spring and fall is likely to be characterized by lower mean demand, relatively low volatility, and low skewness. In contrast, the distribution of spot prices during winter and summer is likely to involve higher mean demand, more price variability, and greater price skewness. Further, the model predicts higher power forward prices (relative to also higher average spot prices) in geographic regions where the power system typically operates nearer to system capacity.

E. Optimal Forward Positions

We next present expressions for producers' and retailers' optimal forward positions. These expressions are useful in evaluating which firms have a comparative advantage in selling into the forward market as compared to buying forward or abstaining from forward transactions.

**E.1. Optimal Forward Positions for Producers**

In Appendix B, we show that the optimal forward sale by a power producer is given, to a second-order approximation, by

\[
Q_{F_i}^P = \frac{E(Q_D)}{N_p} + \frac{\text{PREM}}{A \text{Var}(P_W)} + \left(\frac{x[E(P_W)]^{x-1}}{2a^x}\right)\left(\frac{\text{Skew}(P_W)}{\text{Var}(P_W)}\right),
\]

where \(\text{PREM}\) is the excess of the forward price defined by (12) over the expected spot price.
A starting point for considering the power producer’s optimal forward position is its expected physical production, \( \frac{E(Q^D)}{N_p} \). The optimal producer forward sale will equal expected output only if the forward price is an unbiased predictor of the expected spot price and the distribution of wholesale prices is symmetric. Power producers optimally respond to a positive premium in the forward price by increasing forward sales and vice versa. Finally, the optimal producer forward sale increases with the skewness of prices, which serves to hedge its wholesale revenue stream.

Figure 4 illustrates the implications of expression (14), with \( c = 4 \) and normally distributed demand. The optimal forward positions displayed in Figure 3 have been scaled by the producer’s expected output, and hence can be interpreted as its optimal hedge ratio. When expected demand and demand risk are low, the producer responds to the resulting downward bias in the forward price by reducing forward sales, and its hedge ratio is less than one. However, as either demand risk or expected demand increases, the producer optimally increases its forward sales, in response to both the increasing equilibrium forward price and to hedge its revenue stream.

**E.2. Optimal Forward Positions for Power Retailers**

In this model, producers are homogeneous, reflecting the fact that each shares the same production technology and sells into the same wholesale
market. Retailers, in contrast, can be heterogeneous if the probability distribution of demand load varies across firms. To investigate the potential role of cross-sectional variation in local demand characteristics for optimal retailer hedge positions, consider the ordinary least squares regression of local demand on the average of system demand:

\[ Q_{Ri} = \rho_i + \beta_i \frac{Q^D}{N_R} + \varepsilon_i, \]  

where \( \beta_i = \text{Cov}(Q_{Ri}, Q^D)/N_R \text{Var}(Q^D) \). This “power demand beta” measures the sensitivity of retailer \( i \) demand to average system-wide demand. We assume that the errors from this regression are independent of total system demand, \( Q^D \).

Using (15), Appendix B shows that the optimal retailer forward position can be expressed, to a second-order approximation, as

\[ Q^F_{Rj} = -E(Q_{Rj}) + \frac{\text{PREM}}{\text{Var}(P_W)} + \beta^*_j \left( Z + Y \left[ \frac{\text{Skew}(P_W)}{\text{Var}(P_W)} \right] \right), \]  

where

\[ Z = (P_R x[E(P_W)]^{x-1} - x[E(P_W)]^x), \]

\[ Y = \left( P_R \frac{x(x-1)}{2} [E(P_W)]^{x-2} - \frac{x(x+1)}{2} [E(P_W)]^{x-1} \right), \]

and

\[ \beta^*_j = \left( \frac{\beta_j}{a^x} \right) \left( \frac{N_p}{N_R} \right). \]

Note that since \( x \) is less than or equal to one, the term \( Y \) is strictly negative.

Expression (16) shows that power retailers faced with different power demand betas will optimally take differing forward positions. However, demand load characteristics may not vary across retailers in long-run equilibrium. In the absence of contracting costs, risk-averse retailers have incentives to enter contracts where each agrees to service a portion of the demand load in every geographic region, thereby diversifying demand risk across retailers, and pushing each retailer’s power demand beta towards

---

\(^{16}\) The regression errors are, by construction, uncorrelated with system demand. Our stronger assumption assures that the regression errors are also uncorrelated with the wholesale price, which is a nonlinear function of demand. In the absence of this assumption expression (16) contains two nuisance terms involving correlations between the regression errors and functions of the wholesale price.
The optimal forward positions taken by such fully diversified retailers can be assessed by setting the power beta coefficient in (16) equal to one. We do not present simulations of the resulting forward positions in this case for two reasons. First, the position that a power retailer with a power demand beta of 1.0 would take to reduce its risk is simply the opposite of the position taken by a representative power producer (Figure 4). Second, contracting costs and remnant regulation of access to local markets, though not formally included in our model, are likely to prevent actual power retailers from immediately sharing demand risk. It will likely be years before most retail markets are shared among power marketing companies that operate on a system-wide basis. The analysis of optimal hedging for retailers with heterogeneous demand is likely to be relevant to actual retailers in the interim.

The first term on the right side of (16) is the forecast demand load. However, the retailer optimally purchases forward a quantity that matches its expected demand only when the premium in the forward price is zero and local demand is uncorrelated with system demand. The second term on the right side of (16) reflects the response to the bias in the forward price. The third term on the right side of (16) reflects systematic demand risk. If demand risk is not systematic ($\beta_j = 0$), the third term is zero. Within the third term, $Z$ is positive when the retail price exceeds the expected spot price. A positive value of $Z$ reflects retail revenue risk, which the retailer hedges by increasing its quantity sold forward (i.e., decreasing the quantity purchased forward), if the demand risk is systematic. In contrast, any positive skewness in the price distribution increases the importance of risk in the costs of purchasing power, and leads to a reduction in the optimal quantity sold forward (i.e., an increase in forward purchases), if the risk is systematic.

Figures 5 and 6 present the results of simulations, again assuming normally distributed demand and $c = 4$, that illustrate the implications of expression (16) for the optimal forward position of power retailers. Figure 5 presents optimal hedge ratios (forward positions relative to expected demand) for a retailer whose demand risk is nonsystematic, that is, its power beta is zero. Figure 6 presents optimal hedge ratios for a retailer with a high degree of systematic power demand risk, as indicated by a power beta of 2.0.

The retailer with nonsystematic demand risk chooses a hedge ratio that differs from (negative) one only as a response to the bias in the forward price. As a consequence, its purchases forward an amount greater (less than) its expected sales if the forward price is downward (upward) biased.

The retailer with systematic demand risk optimally deviates further from a hedge ratio of (negative) one. When demand risk is small and expected demand is low, this retailer is more concerned with revenue risk, reducing its forward purchase, and in extreme cases may even sell forward. As expected demand or demand risk increases, the retailer with systematic demand risk rapidly increases its forward purchases, reflecting an increased desire to hedge purchase costs against the possibility of wholesale price spikes.
III. Preliminary Empirical Evidence on the Pricing of Power Forward Contracts

A. Issues in Testing Forward Pricing Hypotheses in Power Markets

Empirical testing of the hypotheses developed here is constrained by the fact that the markets are new and data is scarce. The newness and uniqueness of the wholesale power markets raise the possibility that observed forward prices will reflect the inexperience of industry participants, and may differ from the pricing structure that will be observed in a longer-run equilibrium. A learning phenomenon of this type was documented by Figlewski (1984), who reported that market prices for stock index futures initially deviated significantly from theoretical values, but converged toward predicted values after a few months of trading. The pricing structure is also likely to change if financial contracts that facilitate the sharing of power price risk with outside investors are developed.

There is substantial empirical literature that tests forward and futures pricing theories in stock, bond, foreign exchange, and commodity markets. The research designs employed typically seek to identify the ex ante premium in the forward price by measuring the ex post differential between
forward prices and realized delivery date spot prices, which equals the ex ante premium plus random noise. A common difficulty, however, is that random shocks to asset prices are large compared to any premium in the forward price, so that tests conducted in smaller samples lack statistical power. For example, Fama and French (1987) conduct tests of whether futures risk premiums are nonzero using between 9 and 18 years of data from 22 commodity markets. They conclude (p. 73) that “the large variances of realized premiums mean that average premiums that seem economically large are usually insufficient to infer that expected premiums are nonzero” and “the evidence is not strong enough to resolve the long-standing controversy about the existence of nonzero expected premiums.” Difficulties related to a lack of statistical power are compounded in the case of electricity markets because of the short time series of available data and due to the unusually large variability of realized prices. As a consequence, the results reported here should be viewed as preliminary; to obtain definitive statistical results regarding the pricing of power forward contracts may well require numerous years of additional data.

**B. Research Design and Data Description**

In addition to the traditional approach of comparing forward prices to subsequently realized spot prices, we adopt an alternative empirical approach with potential to improve statistical power in the small sample. We
exploit the fact that spot electricity prices are determined by demand for consumption and supply from production at each point in time. Specifically, we use realized daily demand (load) and price data to estimate parameters of the power production function, and then obtain estimates of expected spot prices by calendar month using the fitted cost curve and realized load data. Forward premia can then be estimated simply as the difference between observed forward prices and cost-based estimates of expected delivery date spot prices.

We examine power pricing for the Pennsylvania, New Jersey, Maryland (PJM), and the California Power Exchange (CALPX) markets. We focus on these two markets because they are the first markets to implement centralized dispatch and real-time market pricing of power, and because daily load, spot, and forward price data are available for these markets. Each market is governed by an independent system operator (ISO), who assigns generation to meet power demand. The ISO uses supply schedules submitted by producers to dispatch generation assets to minimize the cost of meeting power demand while maintaining system reliability. The available PJM data cover the period April 1997 through July 2000. For CALPX, spot price data is available from April 1998 through July 2000.

Figures 7 and 8 display monthly average spot prices and power demand (load) for PJM and CALPX, respectively. The PJM price spike during July 1999 is apparent, as is a notable spike in CALPX power prices during summer 2000. Some distinctions in the two price spikes are also apparent, however. First, the PJM price spike quickly reverted and affected prices during only a single month, while the CALPX price spike persisted until the end of our sample period. Second, the PJM price spike was accompanied by the highest observed demand, and is potentially attributable to convexity in the production cost function. The CALPX price spike occurred even though demand levels were actually less than in corresponding months of the previous two summers, and must therefore be attributed to either a leftward shift of the competitive supply function, or to the exercise of market power. We do not attempt to distinguish between these explanations, but observe that nei-

---

17 Prices for storable commodities, in contrast, are determined not only by supply from current production and demand for current consumption, but also by supply from or demand for inventory.

18 Spot price and load data are obtained from the web sites http://www.caiso.com and http://www.pjm.com, which also contain additional information about the markets. Standardized futures contracts for delivery at several locations, including Palo Verde (Arizona), California–Oregon Border, Cinergy (U.S. Midwest), Entergy (South-central U.S.) and PJM, are traded on the New York Mercantile Exchange and the Chicago Board of Trade. However, trading activity in the futures has been thin, and load data is not readily available for these markets.

19 The U.S. Department of Energy (on the web page http://www.eia.doe.gov/cneaf/electricity/california/subsequentevents.html) has attributed the “high level of planned and unplanned (power plant) outages” in California to “the extended use of power plants during the previous exceptionally hot summer months” and that producers “had used their allotted emission allowances.” Joskow and Kahn (2001, p. 30) in contrast attribute reduction in output to “the withholding of supplies from the market.”
ther is allowed for in our model. Although we report evidence for both markets, these considerations suggest that the results from PJM potentially provide more information as to the validity of our model of forward pricing in competitive electricity markets.

We begin by estimating the parameters of the power production function, using daily price and load data. Our model implies that spot prices are given by equation (5). Taking logarithms, parameters of the production cost function can be estimated by the following linear regression:

$$\ln(P_t) = a + (c - 1)\ln(Q_t + d_t) + e_t,$$

where $P_t$ is the daily average on-peak spot price, $Q_t$ is daily average load, and $a$ and $c$ are parameters to be estimated. We use indicator variables denoted $d_t$ to designate each calendar month to account for the fact that the power supply function varies seasonally, as producers schedule planned maintenance outages during periods of low expected power demand. We also empirically accommodate the CALPX summer 2000 data by use of additional indicator variables for June and July of 2000. In this specification, $c$ estimates the degree of convexity in the cost function. A value of $c$ greater than two implies that the spot prices will be positively skewed, even if the power demand distribution is symmetric.

Figure 7. Monthly average electricity demand (load) and realized wholesale spot prices in PJM. The data cover April 1997 to July 2000.
Applied to daily PJM data from April 1997 through July 2000, the estimated value for \( c \) is 4.80 (\( t \)-statistic = 37.9), indicating a significant degree of convexity in the cost function. The regression \( R \)-squared is approximately 60 percent, indicating that variation in load and the simple cost function in expression (1) capture a substantial portion of the variation in observed spot prices. For CALPX over the period April 1998 to July 2000, the estimated value for \( c \) is 5.81 (\( t \)-statistic = 23.3), and the model explains almost 70 percent of the variation in spot power prices.

The indicator variables used to account for the structural break in prices in the CALPX market that occurred during summer 2000 are positive and statistically significant. One potential reason for the change in the pricing function during this period is rising prices for natural gas, which is often the marginal input fuel in the California market. To explore this possibility, we obtain data on the nearby natural gas futures price for delivery at the Southern California border.\(^{20}\) There is an upward trend in gas prices over the sample period, with the mean price increasing from $2.30/MMBtu in August 1998 to $3.88/MMBtu in July 2000. We again estimate (17), but use the

\(^{20}\) We thank the referee for suggesting that we control for fuel prices and Alexander Eydeland of Mirant Corporation for the natural gas pricing data. The natural gas futures prices are only available starting in August 1998. We therefore lose the first four months of our sample when we control for fuel prices.
natural log of the ratio of the electricity price to that of the nearby gas futures price as the dependent variable. The resulting estimate of $c$ is 5.30 ($t$-statistic = 17.23), and the model explains about 60 percent of the variation in spot power prices. More importantly, the summer 2000 indicator variables remain significantly positive, which indicates that rising fuel prices do not fully explain the structural shift in CALPX prices during summer 2000.

We report our remaining results for CALPX using the pricing function that does not control for fuel prices. This pricing function fits the data slightly better and allows us to use a longer sample period. The results obtained after controlling for variation in gas prices are qualitatively similar, and can be obtained from the authors on request.

Having estimated the cost function based on daily realizations of price and load, we next estimate expected spot prices by month, using a bootstrap approach to generate average fitted values from (17). We generate a series of daily load outcomes for each calendar month by randomly drawing (with replacement) 2,000 observations from the sample of actual load outcomes observed in that calendar month, during any year of the sample. This procedure yields a bootstrapped empirical distribution of power demand, under the assumption that the distribution of power demand varies across calendar months, but is stationary across years. From each daily load observation, we compute the fitted spot price from (17), and then estimate the expected spot price for each calendar month as the mean of the 2,000 fitted values.\footnote{We also repeated the estimations of expected spot prices when drawing load observations from hourly, rather than daily, data. The resulting estimated spot prices were similar to those reported. Additionally, the inferences regarding the bias in forward prices were unchanged under this methodology.}

The fitted prices for CALPX do not, however, include the effect of the summer 2000 indicator variable, which occurred for reasons not modeled and that presumably surprised industry participants.

Table I displays both the realized average on-peak spot power price by month, as well as the resulting cost-based estimates of expected spot prices by calendar month in both markets. Both the realized and expected spot prices exhibit clear seasonality, with average spot prices highest in the summer months. Note that average realized spot prices are higher than our cost-based estimates of expected spot prices during the summer months, indicative of the significant spikes in prices that have occurred in both markets that are not accounted for by our estimated production function.

The model implies that the volatility and skewness in spot prices will be largest in the summer months when expected demand is high, and smallest in the spring and fall. Table II reports expected spot prices, as well as the standard deviation and the standardized (by dividing by the standard deviation cubed) skewness in unexpected spot prices. The standard deviation and standardized skewness of unexpected spot prices are estimated on a monthly basis, using differences between actual daily prices and the cost-based estimates of expected prices for the calendar month. Results reported
are averages across the months in each season, with winter defined as the months December, January, and February, and the other seasons defined similarly using consecutive three-month periods.

Expected PJM spot prices are highest in the summer, while CALPX prices are actually slightly higher during the fall. For PJM, expected prices are similar across the other three seasons, while in CALPX, fall prices are somewhat higher than winter or spring prices. The standard deviation of spot prices peaks during the summer in each market. As would be expected given convexity in the cost function, the standardized skewness coefficient is positive and greatest during the spring and summer months.

Since the standardized skewness coefficient is the third moment of spot power prices normalized by the cube of the standard deviation in spot power prices, the seasonal increase in standardized skewness implies that unstandardized price skewness increases during the summer more rapidly than price variance. The model therefore implies an upward bias in power forward prices for summer delivery. The model predicts that the bias in the forward price should be small or negative when price variability is moderate and price skewness is low. In light of the estimates reported in Table II, we anticipate small or negative bias in power forward prices for spring and fall delivery. We test whether the data is consistent with these broad predictions by examining seasonal variation in the bias of forward prices.

Table I

Monthly Average On-Peak Spot Power Prices in the PJM and CALPX Power Markets

Actual spot prices are equally weighted means of daily prices in dollars per megawatt hour observed during the indicated calendar months. Fitted spot prices by month are average fitted values of the estimated cost function (17), obtained based on 2,000 random draws from the calendar months from the sample distribution of loads. The PJM data covers April 1997 through July 2000. The CALPX data covers April 1998 through July 2000.

<table>
<thead>
<tr>
<th>Month</th>
<th>Actual Spot Prices</th>
<th>Fitted Spot Prices</th>
<th>Actual Spot Prices</th>
<th>Fitted Spot Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>25.91</td>
<td>25.49</td>
<td>30.08</td>
<td>27.37</td>
</tr>
<tr>
<td>February</td>
<td>22.54</td>
<td>20.94</td>
<td>27.71</td>
<td>22.89</td>
</tr>
<tr>
<td>March</td>
<td>24.08</td>
<td>23.77</td>
<td>26.97</td>
<td>21.12</td>
</tr>
<tr>
<td>April</td>
<td>25.46</td>
<td>25.30</td>
<td>29.39</td>
<td>27.59</td>
</tr>
<tr>
<td>May</td>
<td>30.89</td>
<td>26.81</td>
<td>37.96</td>
<td>23.12</td>
</tr>
<tr>
<td>June</td>
<td>38.23</td>
<td>33.03</td>
<td>79.36</td>
<td>28.07</td>
</tr>
<tr>
<td>July</td>
<td>71.40</td>
<td>45.80</td>
<td>73.05</td>
<td>36.82</td>
</tr>
<tr>
<td>August</td>
<td>36.66</td>
<td>30.93</td>
<td>47.37</td>
<td>45.53</td>
</tr>
<tr>
<td>September</td>
<td>27.89</td>
<td>28.44</td>
<td>41.45</td>
<td>38.09</td>
</tr>
<tr>
<td>October</td>
<td>27.08</td>
<td>26.13</td>
<td>42.43</td>
<td>40.98</td>
</tr>
<tr>
<td>November</td>
<td>23.57</td>
<td>23.18</td>
<td>37.40</td>
<td>37.35</td>
</tr>
<tr>
<td>December</td>
<td>20.72</td>
<td>21.78</td>
<td>32.97</td>
<td>32.19</td>
</tr>
</tbody>
</table>
C. Evidence on Forward Power Pricing

Daily observations on one-month-ahead power forward contracts are obtained from Bloomberg and from the CALPX Web site. Each forward contract calls for the continuous delivery of power throughout the delivery month, rather than delivery at any particular time during the delivery month.\footnote{More specifically, the contract calls for delivery during each weekday, on-peak, hour of the delivery month.} Bloomberg reports forward prices for PJM delivery from April 1997, but does not report forward prices for CALPX delivery. CALPX began trading block forward contracts in August 1999. We use CALPX block forward data from that date on, but augment it with Bloomberg data on forward prices for Palo Verde, Arizona, delivery from April 1998 through July 1999. Power delivered to Palo Verde is often routed into the California market. Using the Palo Verde forward prices undoubtedly introduces additional measurement error into the analysis, but allows examination of a longer sample period.

Figures 9 and 10 display on a calendar month basis the average bias in the one-month-forward price for the PJM and CALPX markets, respectively. Results are reported both when the forward price is compared to the cost-based estimate of the expected delivery date price and when the forward price is compared to the average realized price during the delivery month. In each market, the bias in the forward power price is relatively small for delivery months outside of summer. Relatively large biases occur during summer months. In the PJM market, the summer bias in the forward price is positive regardless of benchmark, but is greater when forward prices are

<table>
<thead>
<tr>
<th>Season</th>
<th>Expected Spot Price</th>
<th>Standard Deviation of Spot Price</th>
<th>Coefficient of Skewness of Spot Price</th>
<th>Expected Spot Price</th>
<th>Standard Deviation of Spot Price</th>
<th>Coefficient of Skewness of Spot Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter</td>
<td>23.02</td>
<td>7.10</td>
<td>0.18</td>
<td>27.36</td>
<td>7.55</td>
<td>0.65</td>
</tr>
<tr>
<td>Spring</td>
<td>26.03</td>
<td>10.87</td>
<td>0.72</td>
<td>24.72</td>
<td>13.84</td>
<td>1.19</td>
</tr>
<tr>
<td>Summer</td>
<td>37.63</td>
<td>51.04</td>
<td>1.87</td>
<td>35.22</td>
<td>59.09</td>
<td>1.30</td>
</tr>
<tr>
<td>Fall</td>
<td>25.88</td>
<td>7.85</td>
<td>0.23</td>
<td>39.01</td>
<td>16.95</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Equilibrium Electricity Forward Pricing

Table II
Seasonal Average Estimates of Electricity Spot Price Parameters

Expected spot prices in dollars per megawatt hour are computed on a monthly basis as average fitted values from text equation (17). Standard deviations and standardized skewness are computed on a monthly basis using deviations of daily prices from the expected monthly prices. Results reported are averaged across the three calendar months per season and across years. Winter is defined as months December, January, and February. The other seasons are defined using consecutive three-month periods. The PJM data covers April 1997 through July 2000. The CALPX data covers April 1998 through July 2000.
compared to cost-based estimates of expected spot prices than when compared to average realized spot prices. For CALPX, the sign of the bias depends on the benchmark. Forward prices for July delivery exceed cost-based estimates of expected spot prices by as much as $40 MWh, or over 100 percent. However, forward prices for June delivery are less than average realized spot prices by a similar magnitude. This difference in inference is attributable to the impact of the large and sustained price spike in summer 2000 CALPX prices. It will likely require many more summers before we can assess whether these data points reflect price skewness that should have been anticipated and priced in the forward market, or represent an influential but nonforecastable event.23

Figure 9. The bias in one-month-forward power prices as predictors of subsequently realized spot prices, and relative to cost-based estimates of expected spot prices. Price data are for power delivery at PJM from April 1997 to July 2000, and have been averaged across all days in the indicated calendar month.

23 Although not reported, there is also some evidence of a learning effect in the PJM market. In the summer of 1997, the one-month forward price was below the expected spot price. The first highly publicized spikes in wholesale power prices occurred during June of 1998. Subsequently, one-month forward power prices for summer delivery lay considerably above expected spot prices, while forward prices for non-summer delivery were little changed relative to 1997. This is consistent with the reasoning that market participants learned of the degree to which summer power prices are positively skewed, and that the consequences of price skewness at times of high mean demand then became embedded in forward prices.
Table III reports the average bias in the forward price in each market by season, where the bias is computed as the difference between forward prices and both expected spot prices and realized spot prices. Each seasonal mean is computed from monthly observations. Also reported are simple $t$-statistics for the hypothesis that each seasonal mean is zero. In assessing statistical significance, it should be kept in mind that each seasonal mean is estimated from between 6 and 11 monthly observations.

When the forward price is compared to cost-based estimates of the forward price, a positive bias in summer prices is observed. The bias exceeds $200/MWh in each market, and is statistically significant ($t$-statistics of 2.25 and 1.84 for PJM and CALPX, respectively). A significant positive bias is also observed for winter PJM forward prices while a significant negative bias is observed for fall CALPX prices. The picture is somewhat different when forward prices are compared to average realized spot prices. The bias in PJM summer prices remains positive but is reduced in magnitude by more than half and is no longer statistically significant. The bias in CALPX...
summer prices becomes negative and statistically insignificant. There remains evidence of a positive bias in winter PJM forward prices, and there is evidence of a negative bias in forward prices for spring delivery at PJM as well as for spring and fall delivery at CALPX.

The evidence of a positive premium in summer forward power prices is consistent with the implications of the model developed here, and with the results reported by Pirrong and Jermakyan (1999), who also find that PJM forward prices for summer delivery contain a substantial risk premium. A negative bias in power forwards for delivery during the temperate months of spring and fall is also consistent with the model’s predictions.

Finally, we provide some more specific evidence on the relationship between the bias in forward prices and variations in the demand for power across seasons. In each market, we estimate the following regression:

$$PREM_{it} = \alpha_0 + \alpha_1 MEAN_{it} + \alpha_2 STD_{it} + \alpha_3 VAR_{it} + \eta_{it},$$

(18)
where $\text{PREM}_i$ is the difference between the one-month-forward price for delivery in month $t$ and the cost-based estimate of expected spot price in month $t$ for market $i$, $\text{MEAN}_i$ is mean daily load for month $t$ in market $i$, $\text{STD}_i$ is the standard deviation of daily load during month $t$ in market $i$, and $\text{VAR}_i$ is the square of $\text{STD}_i$. Units for $\text{PREM}$ are dollars per megawatt hour. For the PJM market, the data cover the period from April 1997 through July 2000. For the CALPX market, the data cover the period from April 1998 through July 2000.

### Table IV
Regression of Monthly Bias in Forward Power Price on Mean Demand, Standard Deviation of Demand, and Variance of Demand, by Month

Reported are coefficient estimates obtained when estimating the following regression:

$$\text{PREM}_i = \alpha_0 + \alpha_1 \text{MEAN}_i + \alpha_2 \text{STD}_i + \alpha_3 \text{VAR}_i + \eta_i,$$

where $\text{PREM}_i$ is the difference between the one-month-forward price for delivery in month $t$ and the cost-based estimate of expected spot price in month $t$ for market $i$, $\text{MEAN}_i$ is mean daily load for month $t$ in market $i$, $\text{STD}_i$ is the standard deviation of daily load during month $t$ in market $i$, and $\text{VAR}_i$ is the square of $\text{STD}_i$. Units for $\text{PREM}$ are dollars per megawatt hour.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PJM</td>
<td>Coefficient</td>
<td>49.03</td>
<td>39.31</td>
</tr>
<tr>
<td></td>
<td>$t$-statistic</td>
<td>(3.46)</td>
<td>(0.37)</td>
</tr>
<tr>
<td>CALPX</td>
<td>Coefficient</td>
<td>37.81</td>
<td>145.87</td>
</tr>
<tr>
<td></td>
<td>$t$-statistic</td>
<td>(2.08)</td>
<td>(0.51)</td>
</tr>
</tbody>
</table>

where $\text{PREM}_i$ is the difference between the one-month-forward price for delivery in month $t$ and the cost-based estimate of expected spot price in month $t$ for market $i$, $\text{MEAN}_i$ is the average load for month $t$ in market $i$, $\text{STD}_i$ is the standard deviation of daily market $i$ load during month $t$, and $\text{VAR}_i$ is the square of $\text{STD}_i$. The theory predicts that the forward premium should increase with mean demand ($\alpha_1 > 0$). It also predicts that, ceteris paribus, the forward premium should be convex, initially decreasing ($\alpha_2 < 0$) and then increasing ($\alpha_3 > 0$) in demand risk.

The results of estimating (18) are reported in Table IV. In the PJM market, the coefficient estimate on mean load is positive and statistically significant ($t$-statistic $= 3.46$). For CALPX, the coefficient estimate on mean load is also positive and statistically significant ($t$-statistic $= 2.08$). These results are consistent with the model implication that the premium in forward power prices for summer delivery is associated with high mean power demand. Given the convexity of the cost function, high mean demand, in turn, gives rise to positive skewness in spot power prices. The coefficient estimates on the standard deviation and variance of load are not statistically significant. In view of the small sample size and the presence of substantial multicollinearity among the predictive variables, however, we view these results as providing a degree of support for the model presented here. Definitive evidence will not be available until considerable time passes and the sample size is increased.
IV. Conclusions

Forward contracts for electricity cannot be priced by typical “cost-of-carry” relationships, because power is not storable. We take an equilibrium approach, and obtain the implication that the forward power price will generally be a biased forecast of the future spot price, with the forward premium decreased by the anticipated variance of wholesale spot prices and increased by the anticipated skewness of wholesale spot prices. Simulations confirm that the model implies that forward prices will exceed expected spot prices when either expected demand or demand volatility are high, due to the positive skewness induced in the spot power price distribution.

Though we focus on the pricing of forward contracts, there is anecdotal evidence that positive price skewness (or price spikes) affect the pricing of electricity options as well. For example, Krapels (2000, p. 24) asserts that

\[ \text{[it is common knowledge, however, that traders in many OTC electricity options markets have become so fearful of being physically “net short” (having agreed to deliver electricity in the future at an earlier agreed-upon price) when one of the price spikes occurs that they place extremely high volatility assumptions into the pricing of OTC electricity call options.}\]

We also analyze producers’ and retailers’ optimal forward positions to gain insights into optimal risk bearing, showing that optimal positions depend on statistical properties of power demand and spot prices, including forecast demand and measures of “power demand betas” and price skewness. Finally, we provide some preliminary empirical evidence on wholesale power prices. Consistent with the implications of the model, we document a positive bias in forward power prices for summertime delivery, while the bias in forward prices for spring and fall delivery is zero or negative. Regression analysis also indicates that the forward premium increases with mean demand, as predicted.

The summertime forward premia documented here for the power markets, though estimated imprecisely, appear to be large relative to those that have been documented for other commodities (e.g., Fama and French (1987)). This suggests that the power markets are not well-integrated with the broader financial markets, that is, that outside speculators are not a significant presence in these markets. A lack of integration may be due to informational setup costs (as discussed by Hirshleifer (1988)) associated with learning about power markets, or due to the lack of good benchmark price indices on which to base cash-settled derivative contracts, as suggested by Ong (1996). It will be of interest to see if mechanisms are developed to facilitate the sharing of power price risk with outside speculators, and if risk premia decline as a consequence.
Appendix

A. Forward Pricing

Substituting the expressions for the covariance between the “but-for-hedging” profits and wholesale spot price from text equations (10) and (11) into the expressions for the optimal forward positions taken by retailers and producers from text equation (9), the optimal quantities sold forward can be rewritten as

\[
Q_{Pi}^F = \frac{P_F - E(P_W)}{A \text{Var}(P_W)} + \frac{1}{a^x} \left[ 1 - \frac{1}{c} \right] \frac{\text{Cov}(P_W^{t+1}, P_W)}{\text{Var}(P_W)}, \tag{A1}
\]

for producers and

\[
Q_{Rj}^F = \frac{P_F - E(P_W)}{A \text{Var}(P_W)} + P_R \frac{\text{Cov}(Q_{Rj}, P_W)}{\text{Var}(P_W)} - \frac{\text{Cov}(P_W Q_{Rj}, P_W)}{\text{Var}(P_W)}, \tag{A2}
\]

for retailers. The market clearing forward price can be determined by equating the sum of the forward positions across retailers and producers to zero

\[
\sum_{j=1}^{N_R} Q_{Rj}^F + \sum_{i=1}^{N_P} Q_{Pi}^F = N_R \frac{P_F - E(P_W)}{A \text{Var}(P_W)} + P_R \frac{\text{Cov}(Q_{Rj}^D, P_W)}{\text{Var}(P_W)} - \frac{\text{Cov}(P_W Q_{Rj}^D, P_W)}{\text{Var}(P_W)}
+ N_P \frac{P_F - E(P_W)}{A \text{Var}(P_W)} + \frac{N_P}{a^x} \left[ 1 - \frac{1}{c} \right] \frac{\text{Cov}(P_W^{t+1}, P_W)}{\text{Var}(P_W)} = 0, \tag{A3}
\]

where we have used the properties of covariances and the fact that \(\sum_{j=1}^{N_R} Q_{Rj} = Q^D\) in writing the expression. Solving (A3) for \(P_F\), using the result that \(P_W = a(Q^D/N_P)^{c-1}\), and letting \(N = (N_R + N_P)/A\), the market clearing forward price can be written as

\[
P_F = E(P_W) - \frac{N_P}{N_{ca^x}} \left[ cP_R \text{Cov}(P_W^x, P_W) - \text{Cov}(P_W^{x+1}, P_W) \right], \tag{A4}
\]

which is text equation (12). Also, define

\[
\text{PREM} = P_F - E(P_W) = -\frac{N_P}{N_{ca^x}} \left[ cP_R \text{Cov}(P_W^x, P_W) - \text{Cov}(P_W^{x+1}, P_W) \right], \tag{A5}
\]

to denote the premium in the forward price as compared to the expected delivery date spot price.
The function \( P^z \) (where \( z \) is a constant) can be approximated, by using a Taylor’s series expansion around the point \( E(P) \), as

\[
P^z \approx [E(P)]^z \left( 1 - z + \frac{z(z - 1)}{2} \right) + z(2 - z)[E(P)]^{z-1}P + \frac{z(z - 1)}{2} [E(P)]^{z-2}P^2.
\]

(A6)

Substituting expression (A6), letting \( z = x \) and \( z = x + 1 \) in turn, into (A4), and using the properties of covariances, we obtain

\[
Q^F_{Pi} = E(P_W) + \left( \frac{N_P}{Nc a^x} \right)(x + 1)([E(P_W)]^x - P_R [E(P_W)]^{x-1}) \text{Var}(P_W)
\]

\[
+ \left( \frac{N_P}{Nc a^x} \right) \left( \frac{x + 1}{2} \right) \times (x[E(P_W)]^{x-1} - (x - 1)P_R [E(P_W)]^{x-2}) \text{Skew}(P_W),
\]

which is equivalent to text equation (13).

B. Optimal Forward Positions

Using expression (A6) with \( z = x + 1 \) in combination with text equations (9) and (10) and the definition (A5), we obtain

\[
Q^F_{Pi} = \frac{[E(P_W)]^x}{a^x} + \text{PREM} \text{Var}(P_W) \left( \frac{x[E(P_W)]^{x-1}}{2a^x} \right) \left( \frac{\text{Skew}(P_W)}{\text{Var}(P_W)} \right)
\]

(A8)

for the optimal quantity sold forward by a power producer. Using a second-order Taylor series expansion of text equation (6) at \( E(Q^D) \) and the properties of expectations, we obtain the result that

\[
[E(P_W)]^x \approx a^x \frac{E(Q^D)}{N_P}.
\]

(A9)

Using (A9) with (A8) gives text (14).

Combining text equations (6), (9), (11), and (15), and using the properties of covariances along with the assumption that the regression errors in (15) are independent of system demand, the optimal quantity sold forward by a power retailer can be expressed as

\[
Q^F_{Rj} = -\rho_j + \text{PREM} \left( \frac{\beta_j N_P}{N_R a^x \text{Var}(P_W)} \right) (P_R \text{Cov}(P_W, P_R) - \text{Cov}(P^{x+1}_W, P_W))
\]

(A10)
where $\rho_j$ is the intercept from the regression equation (15). Using expression (A6) with (A10) this can be approximated as

$$Q_{Rj}^* = -\rho_j + \frac{\text{PREM}}{\text{A Var}(P_W)} + \beta_j^* \left( W + Y \left[ \frac{\text{Skew}(P_W)}{\text{Var}(P_W)} \right] \right),$$

(A11)

where

$$W = (P_R x[E(P_W)]^{x-1} - (x + 1)[E(P_W)]^x),$$

$$Y = \left( P_R \frac{x(x - 1)}{2} [E(P_W)]^{x-2} - \frac{x(x + 1)}{2} [E(P_W)]^{x-1} \right),$$

and

$$\beta_j^* = \left( \frac{\beta_2}{\alpha^2} \right) \left( \frac{N_P}{N_R} \right).$$

Finally, taking the expectation of text equation (15), and using the result along with (A9) in (A11), we obtain

$$Q_{Rj}^* = -E(Q_{Ri}) + \frac{\text{PREM}}{\text{A Var}(P_W)} + \beta_j^* \left( Z + Y \left[ \frac{\text{Skew}(P_W)}{\text{Var}(P_W)} \right] \right),$$

(A12)

where $Z = (P_R x[E(P_W)]^{x-1} - x[E(P_W)]^x)$, which is text equation (16).

REFERENCES


