Parameterized Post-Einsteinian (PPE) Formalism

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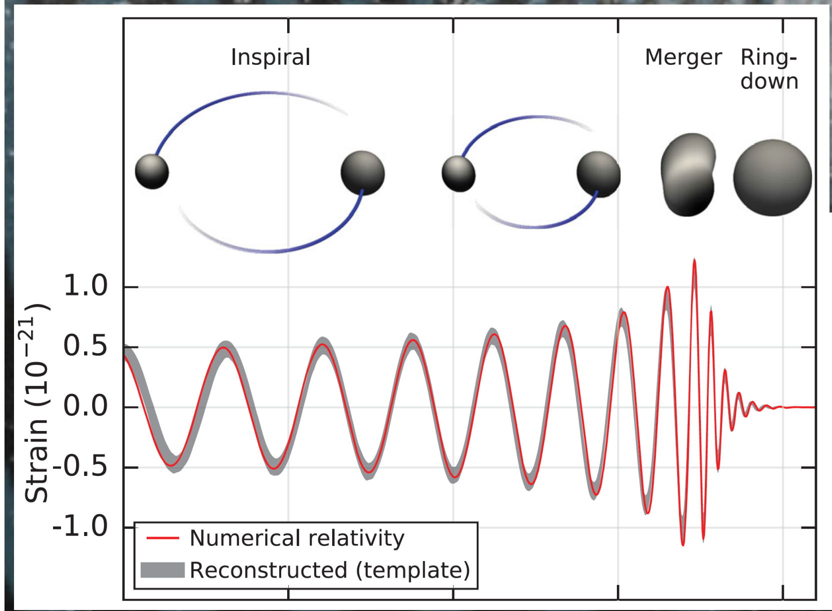
Unifying Tests of GR
Caltech, July 20th 2016

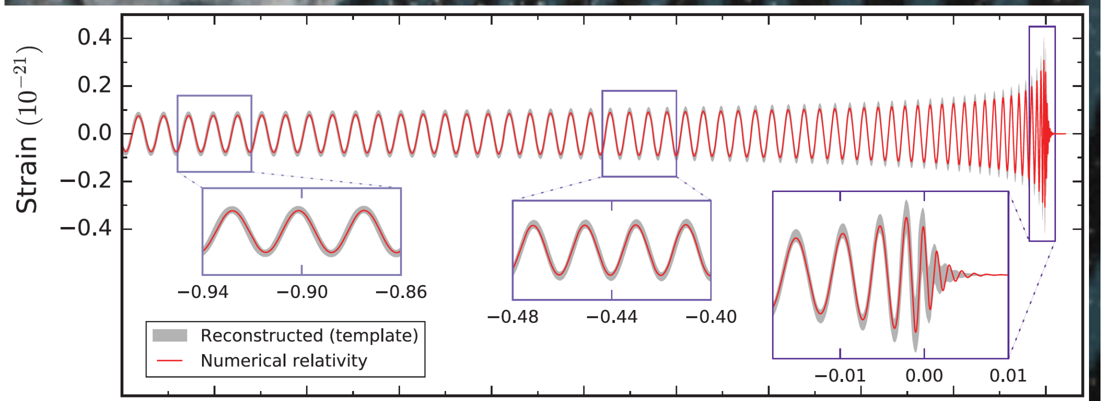
Direct GW detection, at last!

GW150914



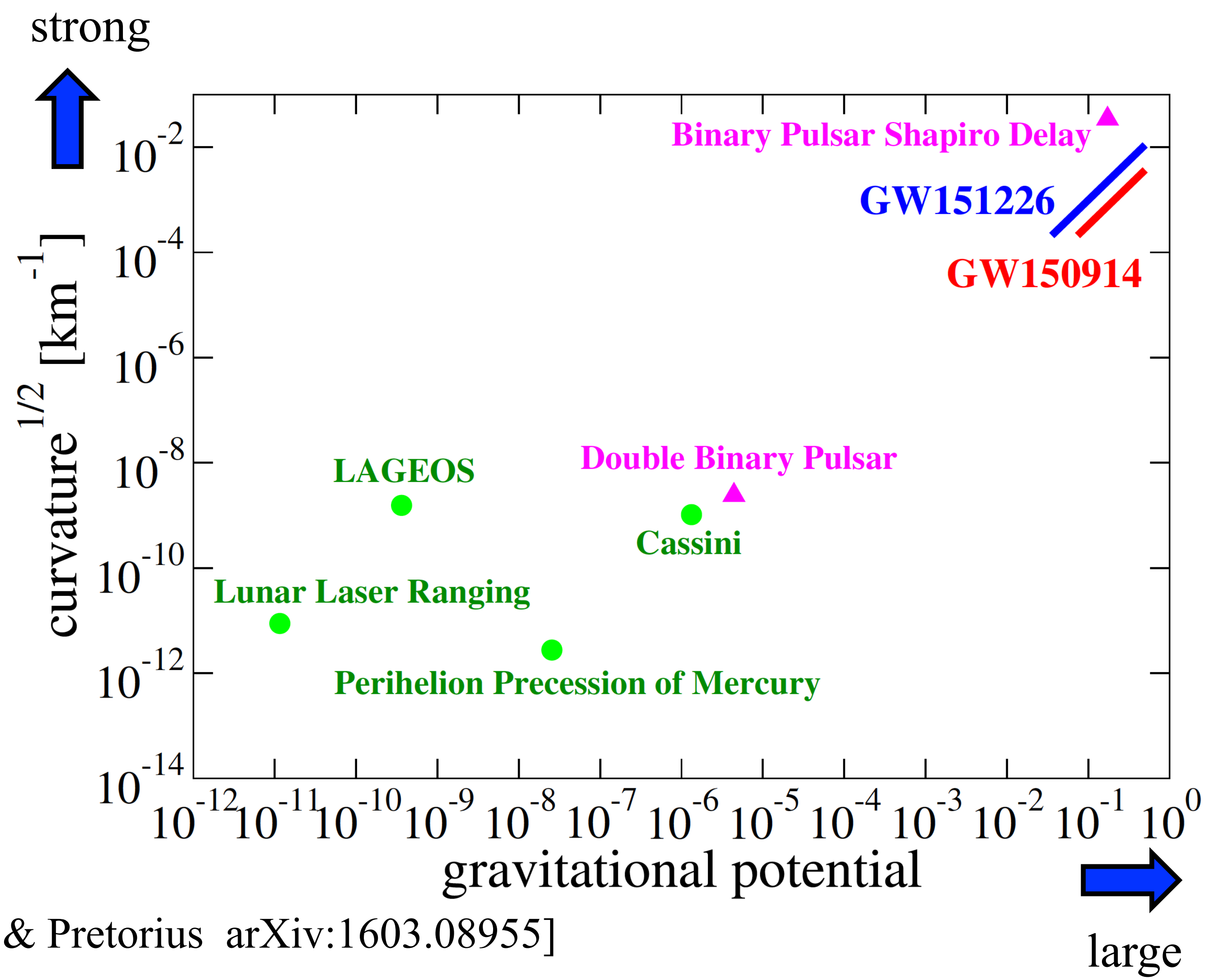
[Abbott et al. PRL 116 061102 (2016)]

GW151226



[Abbott et al. PRL 116 241103 (2016)]

Strong/Dynamical Nature of GW Sources

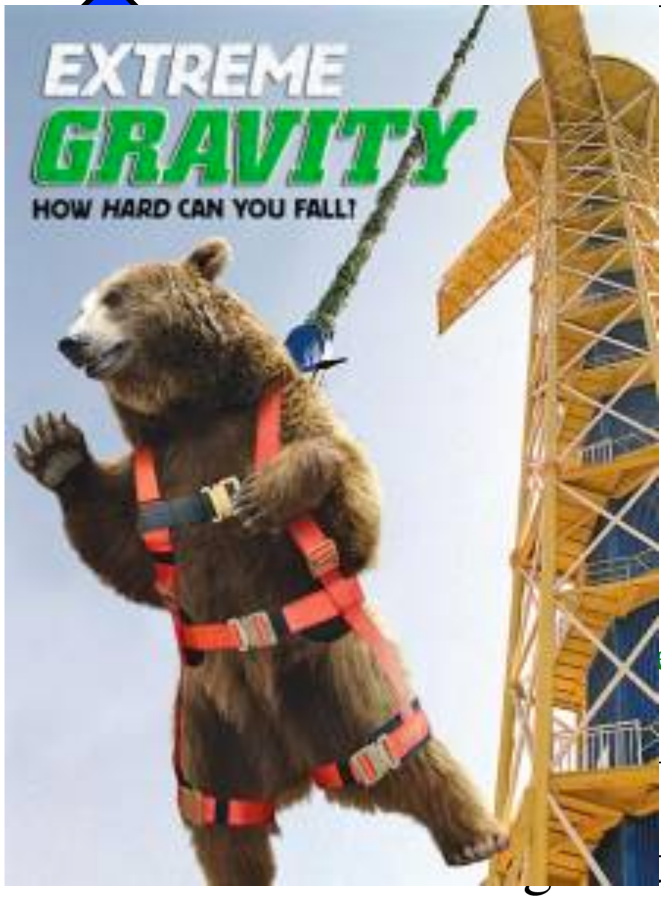


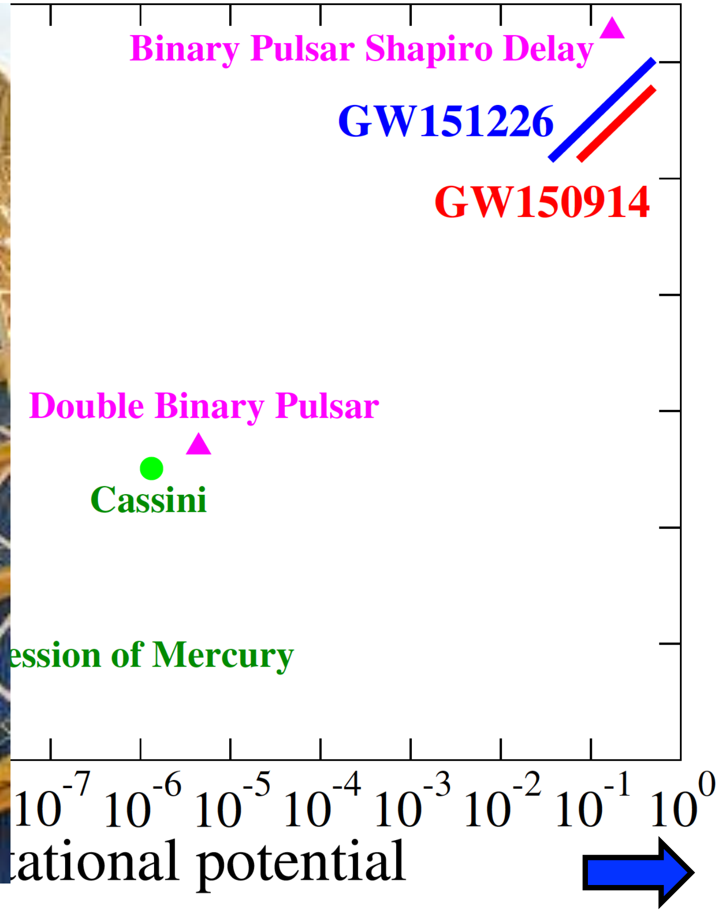
[Yunes, KY & Pretorius arXiv:1603.08955]

Strong/Dynamical Nature of GW Sources

Extreme Gravity = Strong & Dynamical Gravity

strong

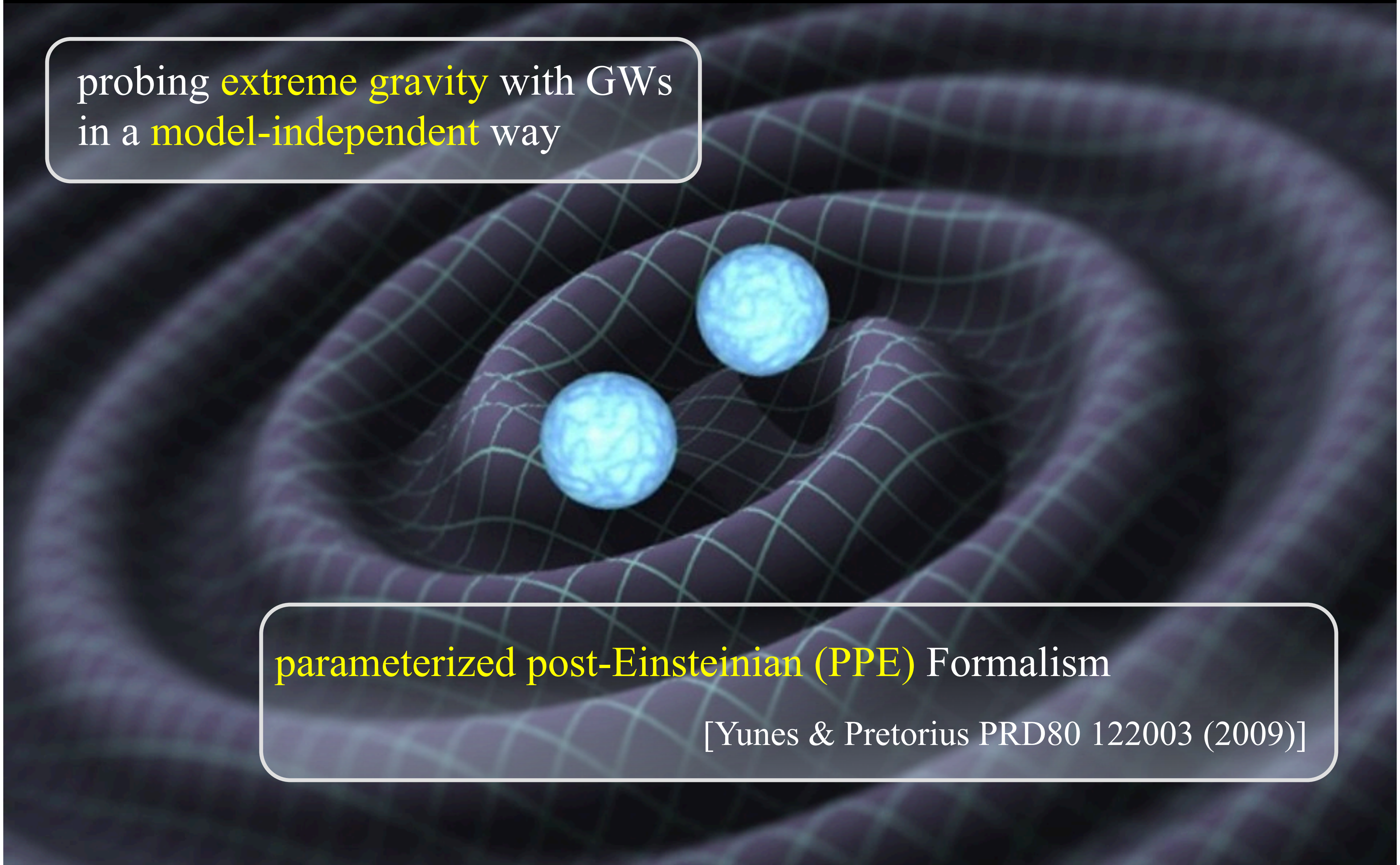




[Yunes, KY & Pretorius arXiv:1603.08955]

large

Model-independent Tests of GR with GWs

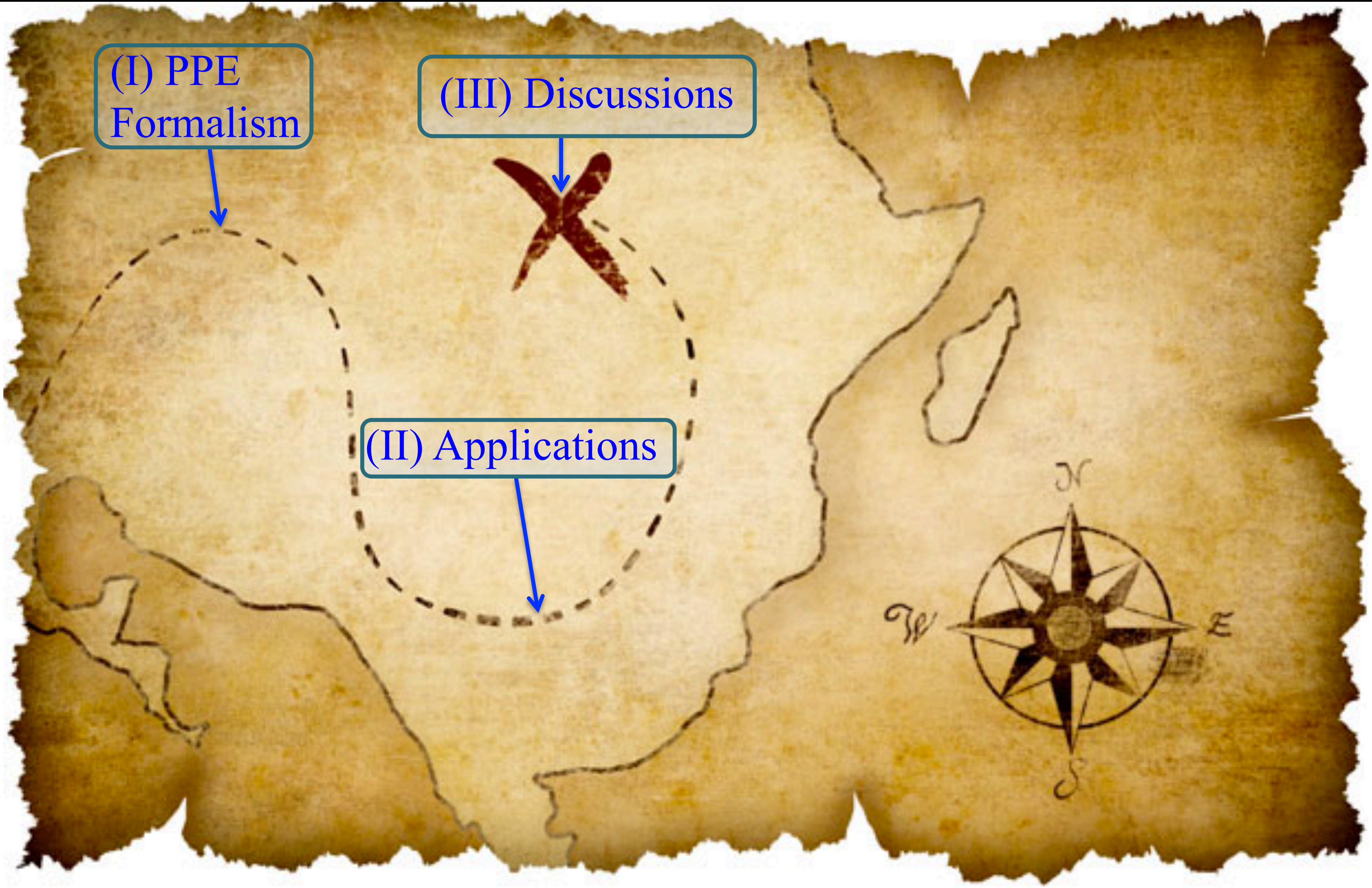


probing extreme gravity with GWs
in a model-independent way

parameterized post-Einsteinian (PPE) Formalism

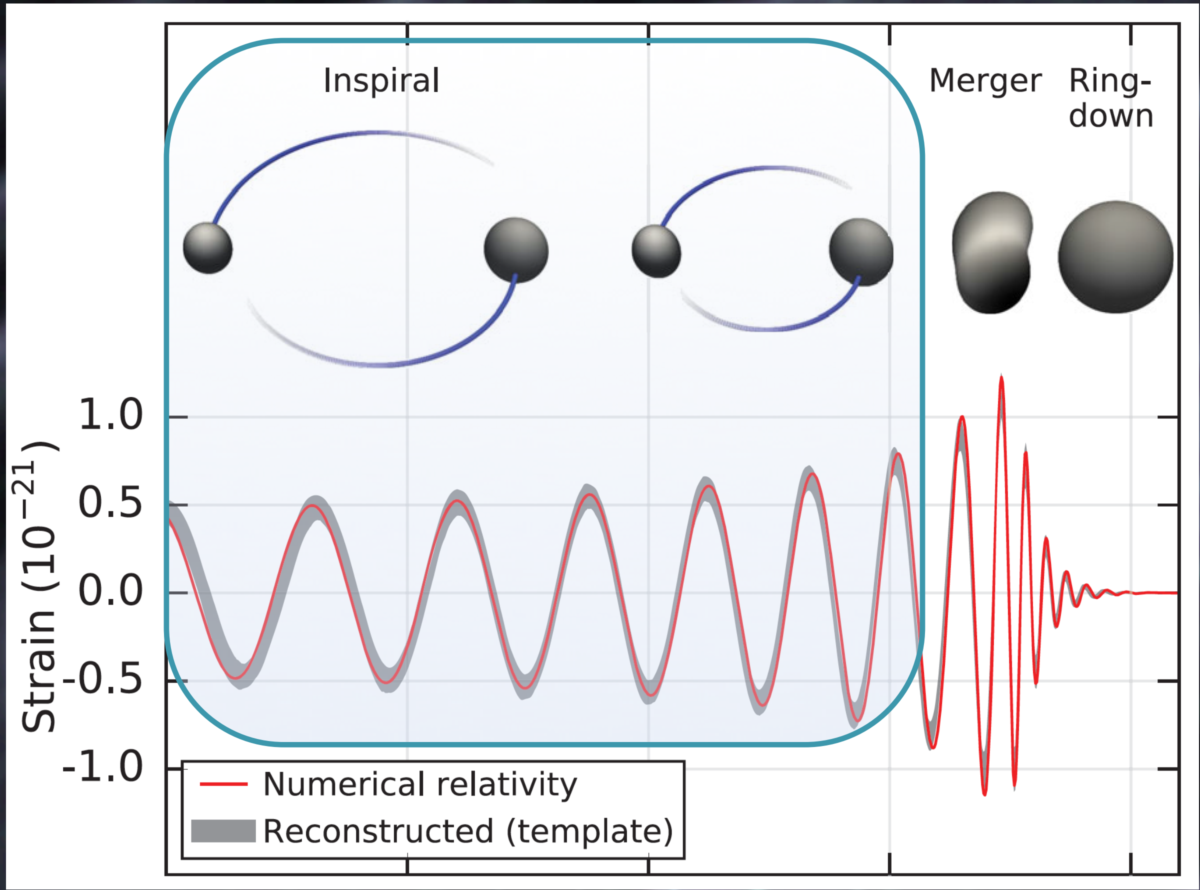
[Yunes & Pretorius PRD80 122003 (2009)]

Roadmap



PPE Formalism

Where to start…?



[Abbott et al. PRL 116 061102 (2016)]

Matched filtering more sensitive to phase than amplitude

Inspiral Waveform Phase in GR

$$\Psi(f) = 2\pi \int^f t(f')\, df' = 2\pi \int^f \int^{f'} \frac{dt}{df''}\, df''\, df'$$

$$= 2\pi \int^f \int^{f'} \frac{dt}{dE} \frac{dE}{dr} \frac{dr}{df''}\, df''\, df'$$

$$\sim (\pi \mathcal{M} f)^{-5/3}$$

chirp mass: $\mathcal{M} \equiv \left(\frac{m_1^3 m_2^3}{m} \right)^{1/5}$

Quadrupolar Radiation: reduced mass

$$\frac{dE}{dt} \sim \dddot{Q}_{ij} \dddot{Q}^{ij} \sim \mu^2 r^4 f^6$$

separation

Binding Energy:

$$E \sim \frac{\mu\, m}{r}$$

total mass

Kepler's Law:

$$f^2 \sim \frac{m}{r^3}$$

PPE-modified Inspiral Waveform Phase

[Yunes & Pretorius PRD80 122003 (2009)]

$$\Psi(f) = 2\pi \int^{f} \int^{f'} \frac{dt}{dE} \frac{dE}{dr} \frac{dr}{df''} \, df'' \, df'$$

$$\sim \Psi(f)_{\mathrm{GR}} + \beta \, u^{b}$$

PPE parameters:

$$\beta = \beta(A, B)$$

$$b = \min(2p - 5, 2q - 5)$$

Quadrupolar Radiation:

q PN

$$\frac{dE}{dt} = \left(\frac{dE}{dt}\right)_{\mathrm{GR}} \left(1 + B \, u^{2q}\right) \qquad u \equiv (\pi \mathcal{M} f)^{1/3}$$

Binding Energy:

$$E = E_{\mathrm{GR}} \left(1 + A \, u^{2p}\right)$$

Kepler's Law:

$$r(f) \sim r(f)_{\mathrm{GR}} \left(1 + A \, u^{2p}\right)$$

(Simplest) Full PPE Waveform

$u \equiv (\pi \mathcal{M} f)^{1/3}$

[Yunes & Pretorius PRD80 122003 (2009)]

$$\tilde{h}^{(\mathrm{IMR})}(f) = \begin{cases} \tilde{h}^{(\mathrm{I})}_{\mathrm{GR}}(f)\,(1 + \alpha\, u^{a})\exp\left(i\,\beta\, u^{b}\right) & \text{(inspiral)} \\ & \\ & \end{cases}$$

GR limit: $(\alpha, \beta) = (0, 0)$,

PPE Dictionary: Inspiral

$$\tilde{h}^{(\mathrm{I})}_{\mathrm{GR}}(f)\,(1+\alpha\,u^{a})\exp\left(i\,\beta\,u^{b}\right)$$

Theories	GR Pillars	Theoretical Mechanism	PPE a	PPE b	PN Order	PPE (α,β)
time-varying G	Strong Equivalence Principle	Anomalous Acceleration	-8	-13	-4 PN	$(\alpha_{\dot{G}},\beta_{\dot{G}})$
RS-II Braneworld	4D	Anomalous Acceleration	-8	-13	-4 PN	$(\alpha_{\mathrm{ED}},\beta_{\mathrm{ED}})$
Scalar-Tensor (including Brans-Dicke)	Strong Equivalence Principle	(Monopole) Scalar Field	-2	-7	-1 PN	$(\alpha_{\mathrm{ST}},\beta_{\mathrm{ST}})$
Einstein-dilaton Gauss-Bonnet	Strong Equivalence Principle	(Monopole) Scalar Field	-2	-7	-1 PN	$(\alpha_{\mathrm{EdGB}},\beta_{\mathrm{EdGB}})$
dynamical Chern-Simons	Parity Invariance	(Dipole) Scalar Field	$+4$	-1	$+2$ PN	$(\alpha_{\mathrm{dCS}},\beta_{\mathrm{dCS}})$
Einstein-Æther, Hořava-Lifshitz	Lorentz Invariance	Vector Field	-2	-7	-1 PN	$\left(\alpha^{(-1)}_{\text{Æ}},\beta^{(-1)}_{\text{Æ}}\right)$
			0	-5	0 PN	$\left(\alpha^{(0)}_{\text{Æ}},\beta^{(0)}_{\text{Æ}}\right)$

$$(\alpha_{\dot{G}}\,,\beta_{\dot{G}}) = \left(-\frac{5}{512}\dot{G}\mathcal{M}\,,-\frac{25}{65536}\dot{G}\mathcal{M}\right)$$

[Yunes, Pretorius & Spergel PRD80 122003 (2010)]

(Simplest) Full PPE Waveform

$u \equiv (\pi \mathcal{M} f)^{1/3}$

[Yunes & Pretorius PRD80 122003 (2009)]

$$\tilde{h}^{(\mathrm{IMR})}(f) = \begin{cases} \tilde{h}^{(\mathrm{I})}_{\mathrm{GR}}(f)\,(1+\alpha\, u^a)\exp\left(i\,\beta\, u^b\right) & \text{(inspiral)} \\ \gamma\, u^c \exp\left[i\,(\delta + \epsilon\, u)\right] & \text{(merger)} \\ \zeta \dfrac{\tau_{\mathrm{damp}}}{1+4\pi^2\,\tau_{\mathrm{damp}}^2\,(f - f_{\mathrm{RD}})^d} & \text{(ringdown)} \end{cases}$$

GR limit: $(\alpha, \beta) = (0,0), \quad (c, \epsilon) \approx (-2/3, \epsilon_{\mathrm{GR}}), \quad d = 2$

γ , δ & ζ determined from continuity of the waveform at each interface.

No known mapping of merger PPE parameters in non-GR theories (due to the lack of numerical simulations).

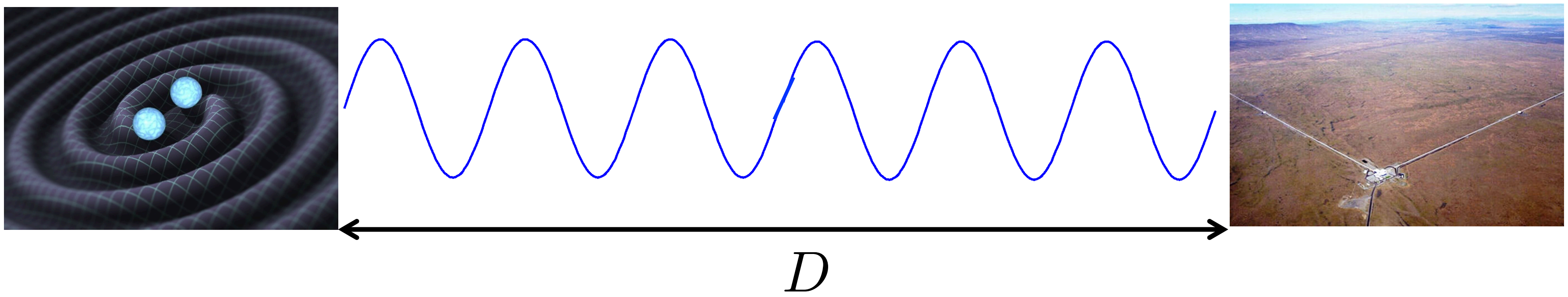
$d = 2$ in well-behaved theories [Vitor]

PPE for Modified GW Propagation

[Will PRD57 2061 (1998), Mirshekari et al. PRD85 024041 (2012)]

-graviton dispersion relation

$$E^2 = p^2 + A\,p^{\gamma} \quad \Rightarrow \quad v_g^2 \approx 1 + (\gamma - 1)\,A\,E^{\gamma-2}$$



$$\Psi \sim 2\pi f \frac{D}{v_g} \quad \Rightarrow \quad \tilde{h}^{(\mathrm{IMR})}(f) = \tilde{h}^{(\mathrm{IMR})}_{\mathrm{GR}}(f)\, \exp\left(i\,\beta\,u^{b}\right)$$

$$\beta \sim A\,D\,\mathcal{M}^{1-\gamma}, \quad b = 3(\gamma - 1)$$

PPE Dictionary: GW Propagation

$$\tilde{h}_{\rm GR}^{\rm (IMR)}(f)\,(1+\alpha\,u^{a})\exp\left(i\,\beta\,u^{b}\right)$$

Theories	PPE a	PN Order	PPE b	PN Order	PPE (α, β)
Massive Gravity	—	—	-3	$+1$ PN	
Double Special Relativity	—	—	$+6$	$+5.5$ PN	
Extra Dimension, Hořava-Lifshitz	—	—	$+9$	$+7$ PN	$\left(0, A\,D\,\mathcal{M}^{-b/3}\right)$
Multifractional Spacetime	—	—	3–6	4–5.5 PN	
Standard Model Extension ($d = 4, 5, \ldots$)	—	—	$3(d-3)$	$(3d-4)/2$ PN	
Parity-violating Theories	$+3$	$+1.5$PN	$+6$	$+5.5$ PN	$(\alpha_{\rm PV}, \beta_{\rm PV})$

PPN vs PPK vs PPE

$$\beta^{(\mathrm{PPE})}_{1\mathrm{PN}} \approx \left(\beta^{(\mathrm{PPN})} - 1\right) - \alpha_1^{(\mathrm{PPN})} + \alpha_2^{(\mathrm{PPN})}$$

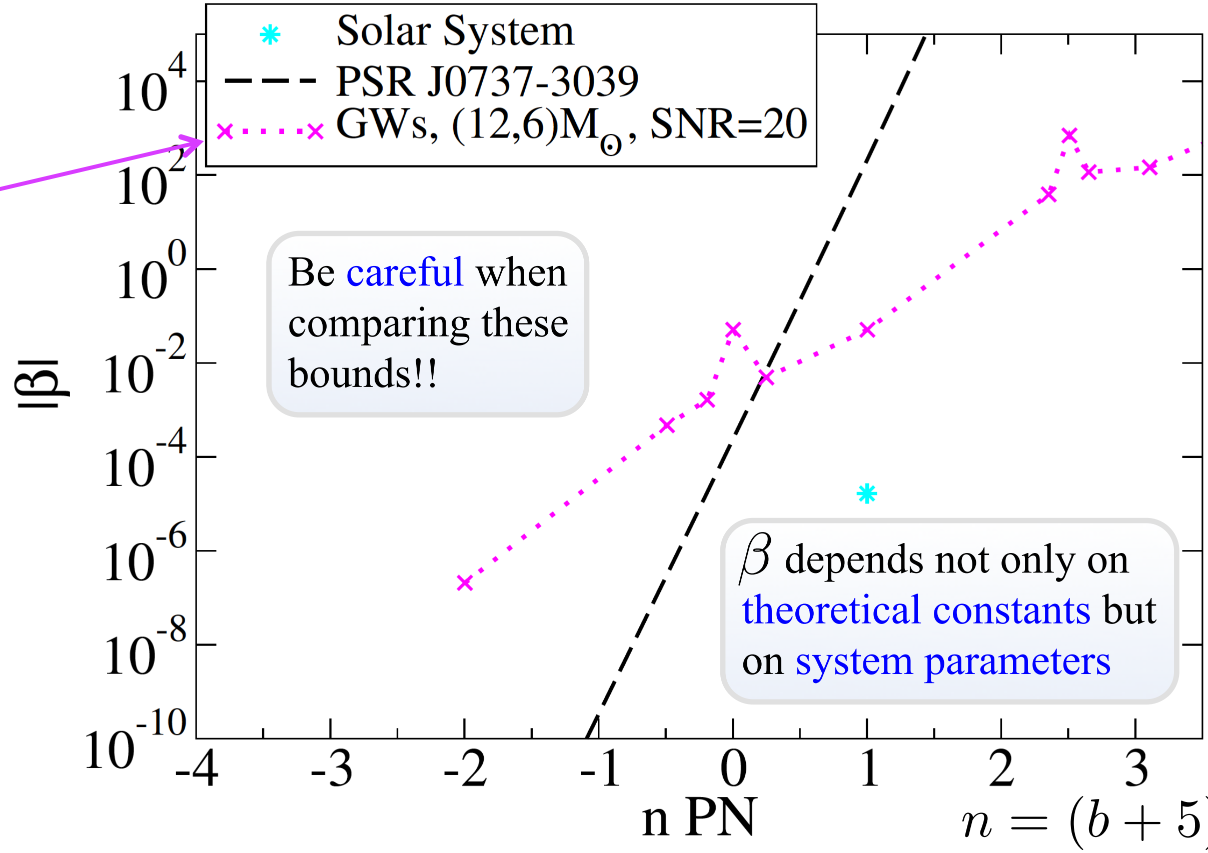
[Sampson et al. PRD88 064058 (2013)]

$$|\beta^{(\mathrm{PPE})}| \lesssim \frac{1}{|b||b-3|}\frac{1}{u^{b+5}}\frac{\delta \dot{P}}{\dot{P}}$$

[Yunes & Hughes PRD82 082002 (2010)]

[Cornish et al. PRD84 062003 (2011)]

GWs place stronger constraints than binary pulsars for positive PN corrections.



$n = (b+5)/2$

PPE Extensions

- *Stealth Bias* [Cornish et al. (2011), Vallisneri & Yunes (2013), Sampson et al. (2014), Vitale & Del Pozzo (2014)]
- *Precession, Higher Harmonics* [Huwyler et al. (2012)]
- *Non-tensorial Polarizations* [Chatziioannou et al. (2012)]
- *Test Infrastructure for GEneral Relativity (TIGER)* [Li et al. (2012), Agathos et al. (2014)]
- *Multiple PPE Parameters* [Sampson et al. (2013)]
- *GW bursts (eccentric binaries)* [Loutrel et al. (2014)]
- *Spontaneous Scalarization / Massive Scalar* [Sampson et al. (2014)]
- *Time-domain Waveform* [Huwyler et al. (2015)]
- *GW Background* [Maselli et al. (2016)]

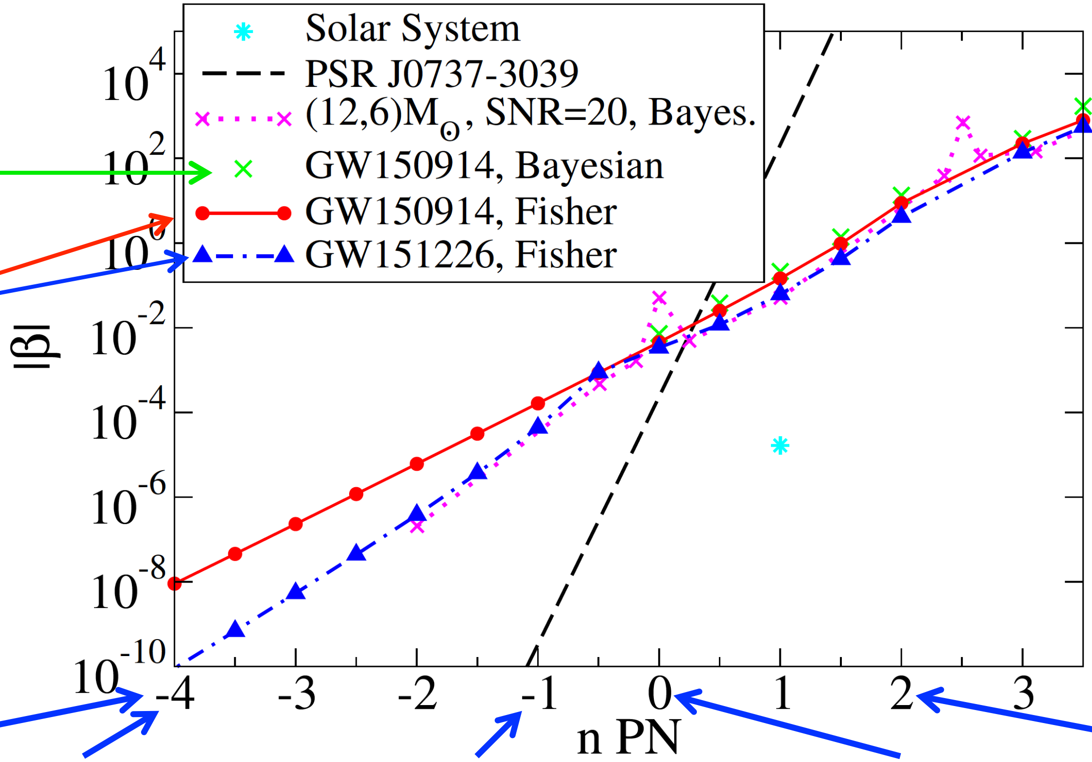
Applications to GW150914 & GW151226

[Yunes, KY & Pretorius arXiv:1603.08955]

Constraints on GW Generation

[LVC (2016)]

[Yunes, KY & Pretorius (2016)]



PPE introduced only in inspiral

extra dimension	time varying G	scalar monopole activation	vector field activation	scalar dipole activation
4D	strong equivalence principle (SEP)	strong equivalence principle (SEP)	Lorentz invariance	parity invariance

GW vs Current Bounds

Example Theories	GR Pillar	PN	Example Theory Constraints			
			Repr. Parameters	**GW150914**	**GW151226**	Current
Einstein-dilaton Gauss-Bonnet	SEP	-1	$\sqrt{\lvert\alpha_{\rm EdGB}\rvert}$ [km]	—	—	10^7, 2
scalar-tensor	SEP	-1	$\lvert\dot{\phi}\rvert$ [1/sec]	—	—	10^{-6}
dynamical Chern-Simons	Parity Inv.	$+2$	$\sqrt{\lvert\alpha_{\rm dCS}\rvert}$ [km]	—	—	10^8
Einstein-Æther	Lorentz Inv.	0	(c_+, c_-)	$(\mathbf{0.9}, \mathbf{2.1})$	$(\mathbf{0.8}, \mathbf{1.1})$	$(0.03, 0.003)$
RS-II Braneworld	4D	-4	ℓ [μm]	$\mathbf{5.4 \times 10^{10}}$	$\mathbf{2.0 \times 10^{9}}$	10–10^3
time-varying G	SEP	-4	$\lvert\dot{G}\rvert/G$ [10^{-12}/yr]	$\mathbf{5.4 \times 10^{18}}$	$\mathbf{1.7 \times 10^{17}}$	0.1–1

Einstein-dilaton Gauss-Bonnet (EdGB)
Scalar-Tensor
dynamical Chern-Simons (CS)

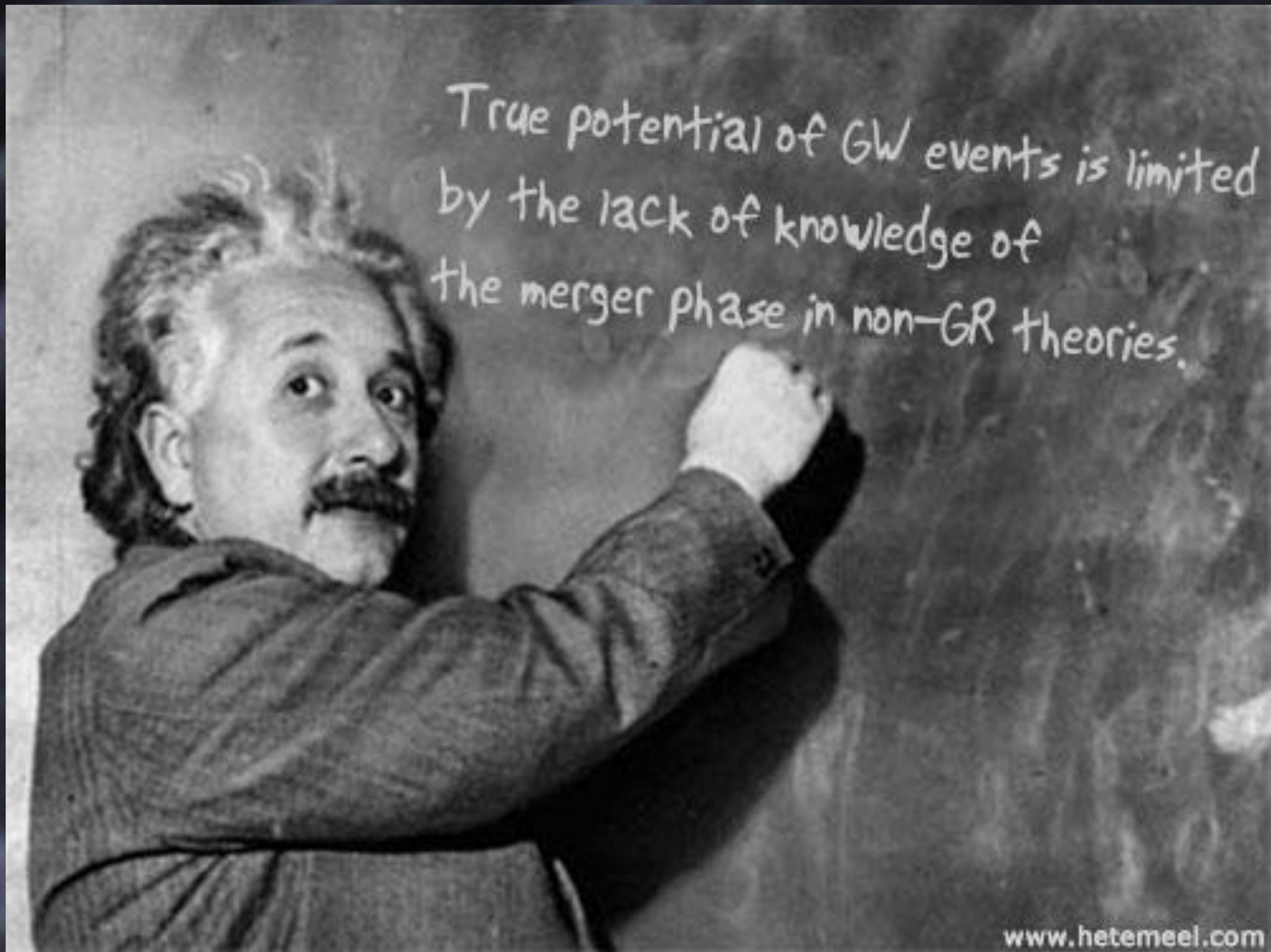
➡ no meaningful constraints (beyond small-coupling approximation)

Einstein-AEther
RS-II Braneworld
Time-varying G

➡ -weaker than current bounds

-first constraint in the extreme gravity regime

Important Message



Constraints on GW Propagation

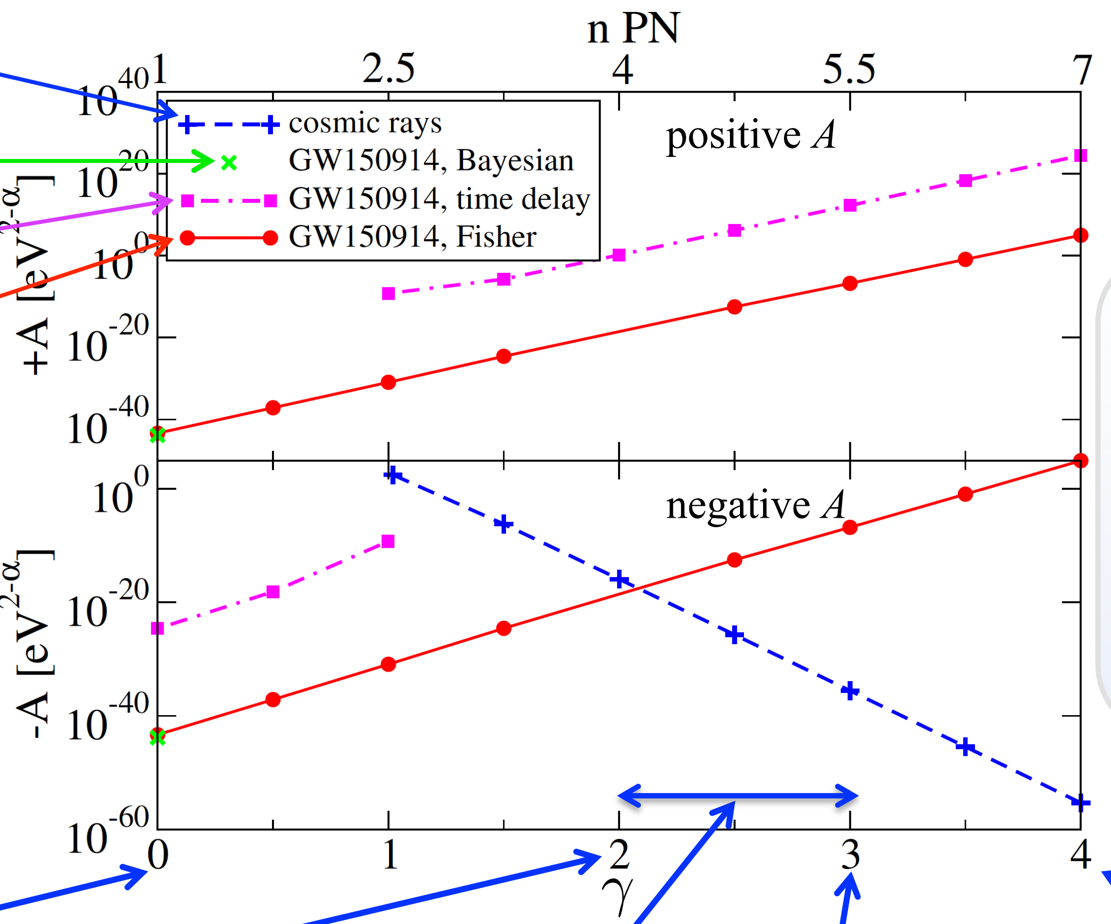
$$E^2 = p^2 + A\,p^\gamma$$

[Kiyota & Yamamoto (2015)]

[LVC (2016)]

[Blas et al. (2016)]

[Yunes, KY & Pretorius (2016)]



-GW151226 bound similar to GW150914

-strong constraints from GWs

-complementary to cosmic ray bounds

-massive gravity

-Lorentz violation

-multifractional spacetime

-Lorentz violation
-mod. special relativity

-Lorentz violation
-extra dimension

Questions / Discussions

What's next…?

- *Improving merger PPE*… need numerical relativity simulations in non-GR theories
- *Improving PPE vs PPK*…correlations between PPE and binary parameters, including conservative PPK (currently on going)
- *PPE for cosmologically-interesting theories*…f(R), Horndeski, DGP, MONDian, etc.
- *Screening effect*…PPE is still valid for cubic galileon theories [de Rham et al. PRD87 044025 (2013)]
  Other screening (chameleon, symmetron)?
  Propagation effect?