Solar System and Pulsar Tests *

Parametrized post-Newtonian (PPN) formalism, parametrized post-Keplerian (PPK) formalism, and a theory-space approach

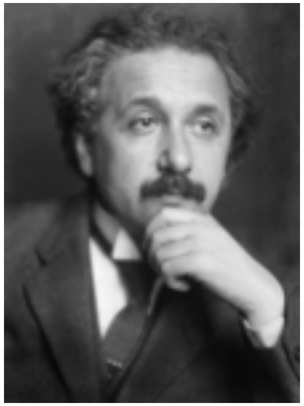
Norbert Wex

Max Planck Institute for Radio Astronomy, Bonn, Germany

Unifying Tests of General Relativity, Caltech, July 19th, 2016

* Partly modified compared to slides in the lecture. Some unpublished material has been removed.

November 1915 - the completion of general relativity



Nov. 4th, 1915

Zur allgemeinen Relativitätstheorie.

Von A. Einstein.

Nov. 11th, 1915

Zur allgemeinen Relativitätstheorie (Nachtrag).

Von A. Einstein.

Nov. 18th, 1915

Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie.

Nov. 25th, 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

$$G_{im} = -\varkappa \left(T_{im} - \frac{1}{2} g_{im} T \right),$$

Damit ist endlich die allgemeine Relativitätstheorie als logisches Gebäude abgeschlossen. Das Relativitätspostulat in seiner allgemeinsten Fassung, welches die Raumzeitkoordinaten zu physikalisch bedeutungslosen Parametern macht, führt mit zwingender Notwendigkeit zu einer ganz bestimmten Theorie der Gravitation, welche die Perihelbewegung des Merkur erklärt. Dagegen vermag das allgemeine Re-

The first experimental verification - November 18th, 1915

Gesamtsitzung vom 18. November 1915

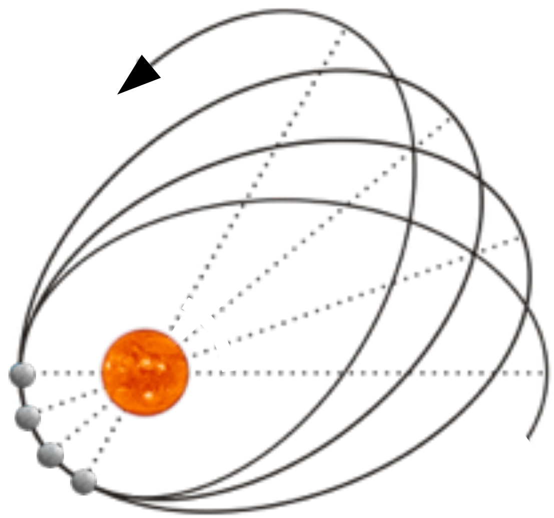
Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie.

Von A. Einstein.

Anomalous precession of the Mercury orbit

$$\varepsilon = 24\pi^3 \frac{a^2}{T^2 c^2 (1 - e^2)}. \qquad (14)$$

Die Rechnung liefert für den Planeten Merkur ein Vorschreiten des Perihels um 43″ in hundert Jahren, während die Astronomen 45″ ± 5″ als unerklärten Rest zwischen Beobachtungen und Newtonscher Theorie angeben. Dies bedeutet volle Übereinstimmung.



Albert Einstein to Arnold Sommerfeld (Dec 9th, 1915):

Wie kommt uns da die pedantische Genauigkeit der Astronomie zu Hilfe, über die ich mich im Stillen früher of lustig machte!"

[*"How helpful to us here is astronomy's pedantic accuracy, which I often used to ridicule secretly!"*]

The first experimental verification - November 18th, 1915

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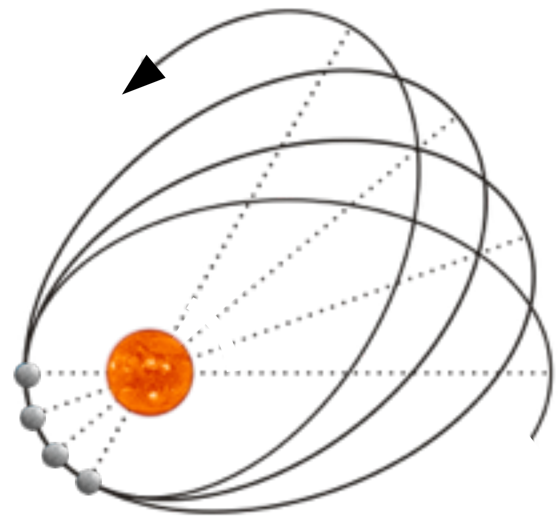
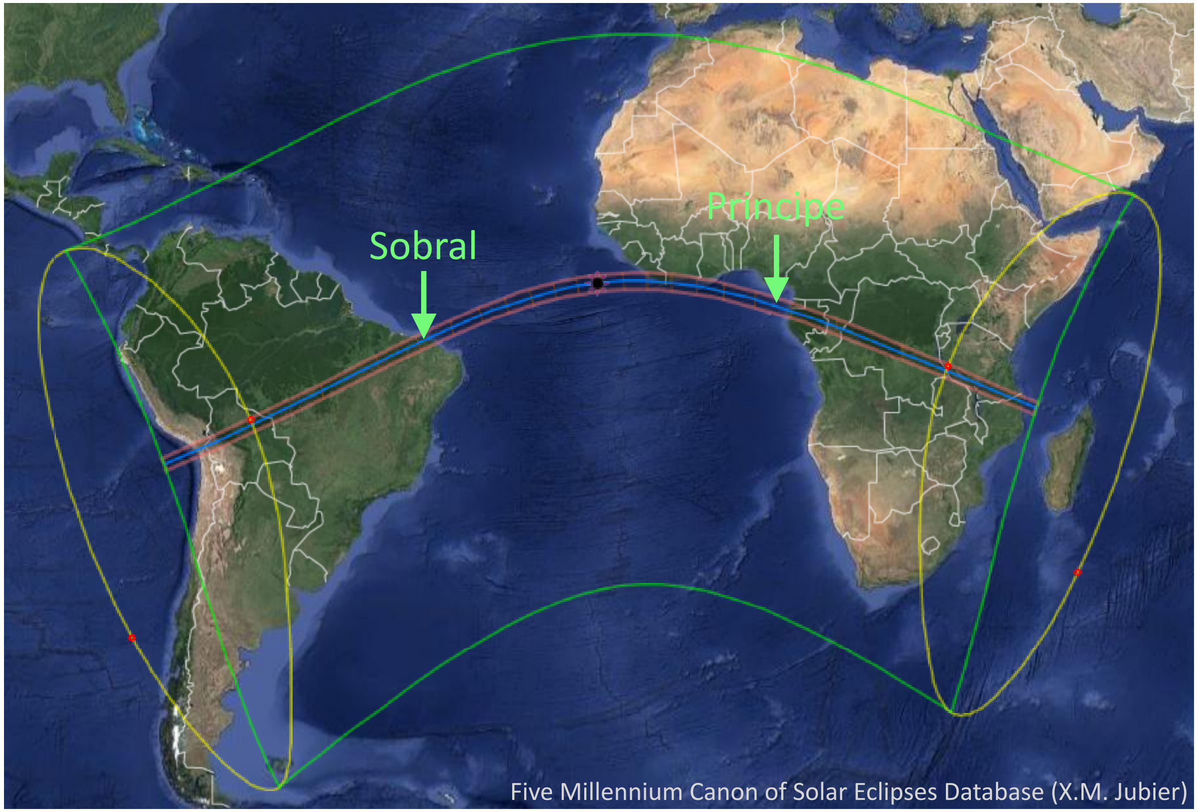
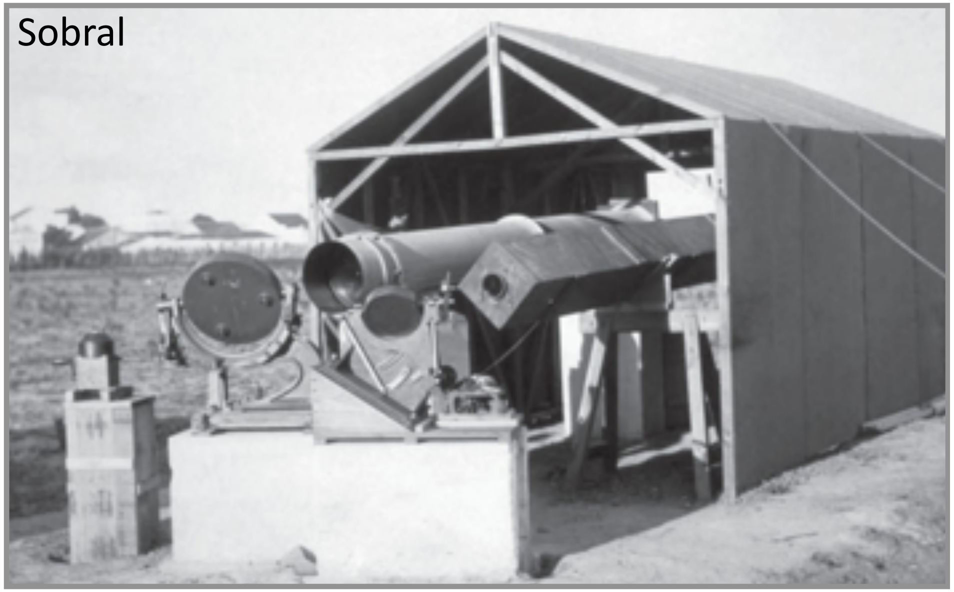


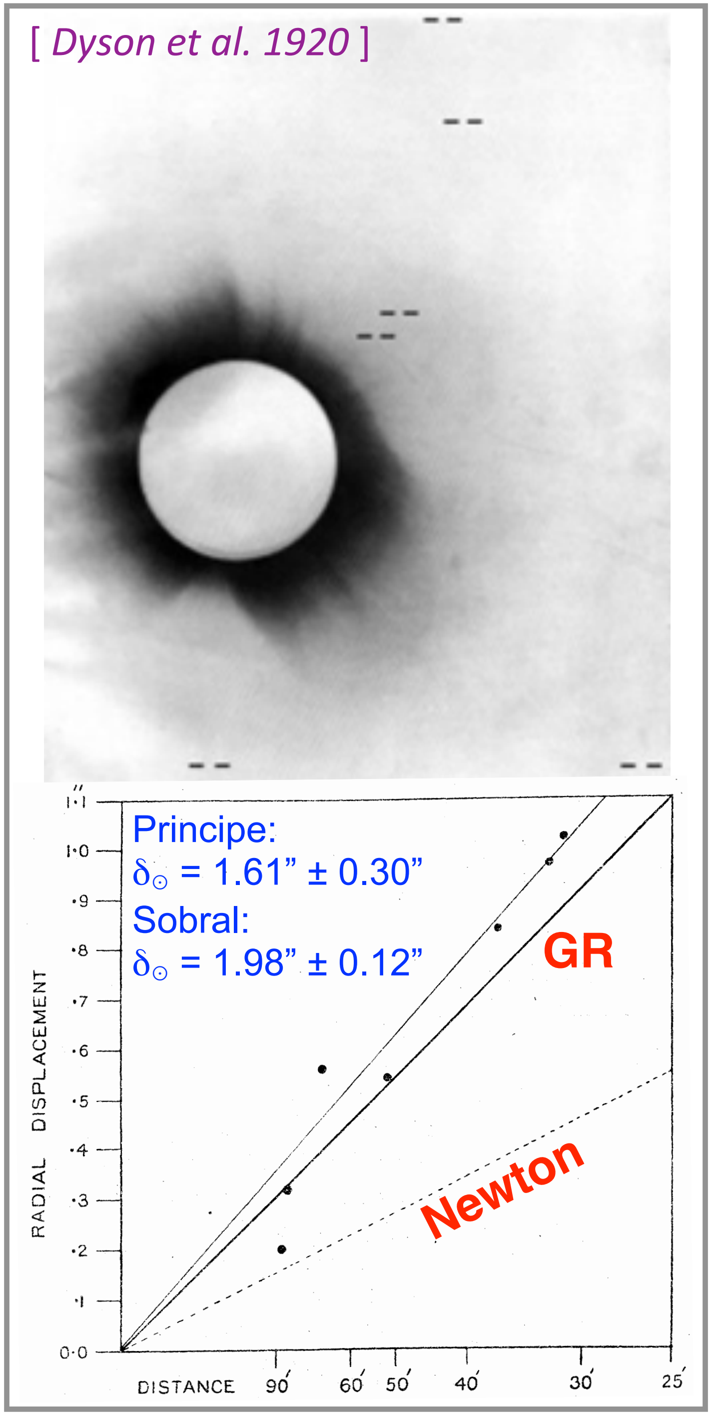
Deflection of light by the Sun, gravitational redshift

ergeben hatten. Ein an der Oberfläche der Sonne vorbeigehender Lichtstrahl soll eine Ablenkung von $1.7''$ (statt $0.85''$) erleiden. Hingegen bleibt das Resultat betreffend die Verschiebung der Spektrallinien durch das Gravitationspotential, welches durch Herrn Freundlich an den Fixsternen der Größenordnung nach bestätigt wurde, ungeändert bestehen, da dieses nur von g_{44} abhängt.

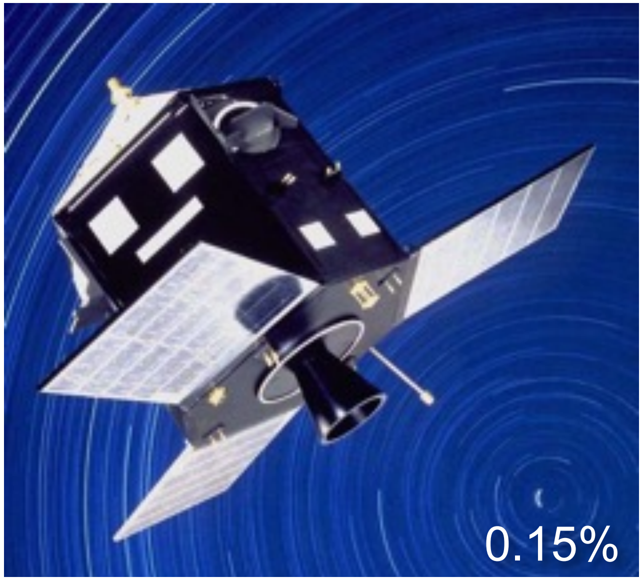
The first light deflection experiment - May 29th, 1919

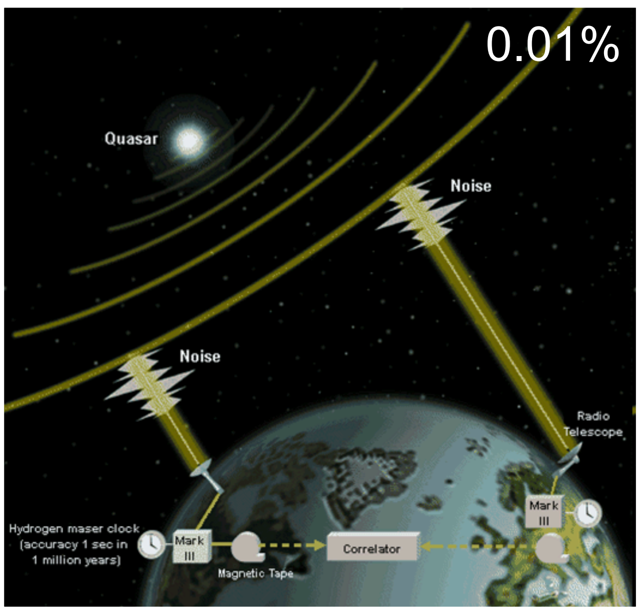


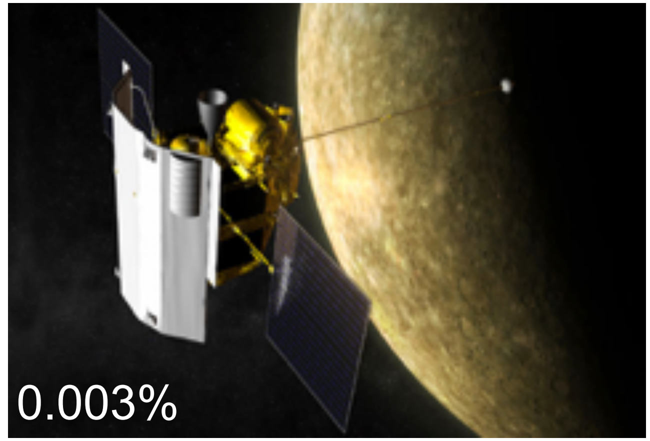


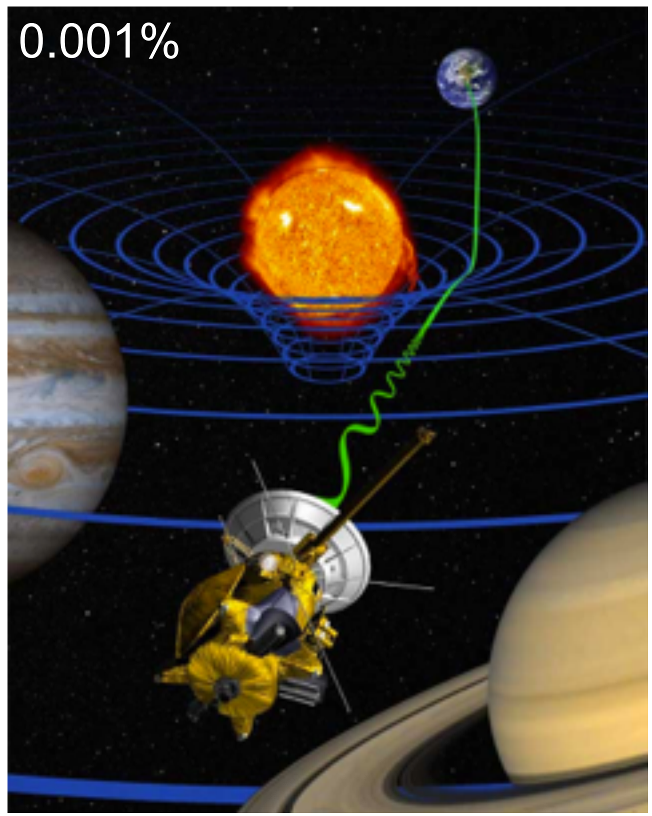


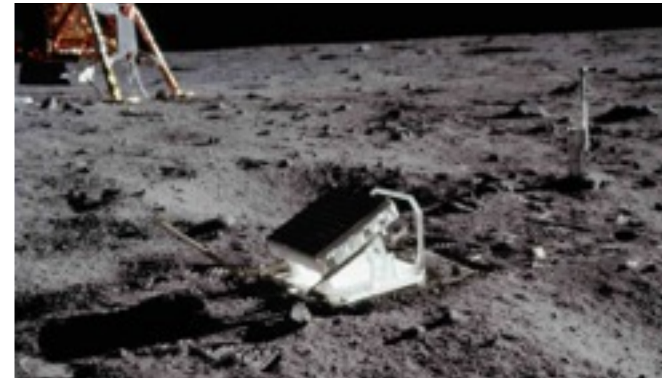
Modern Solar system experiments

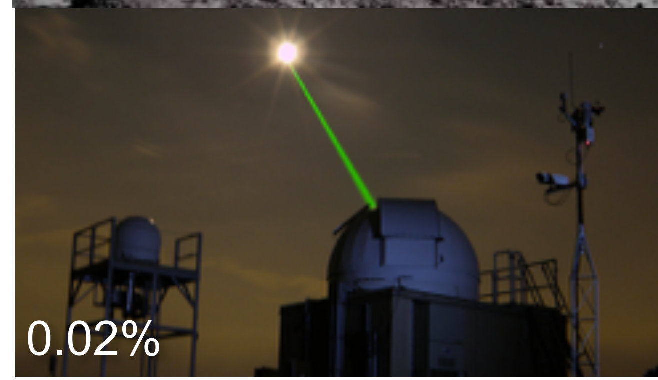


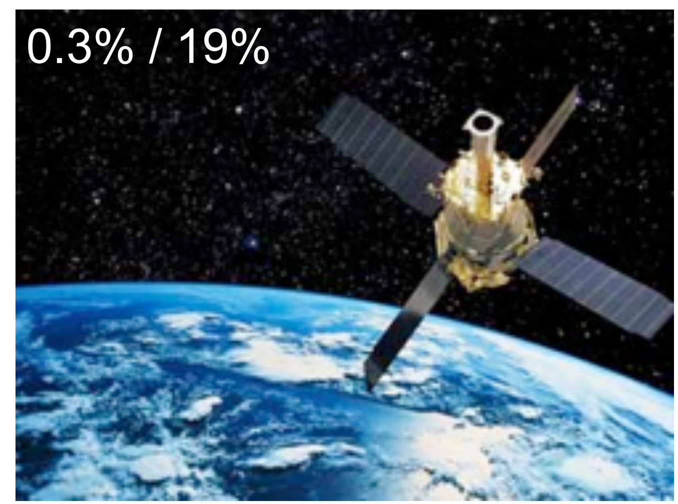












Parametrized post-Newtonian (PPN) formalism

PPN formalism for metric theories of gravity

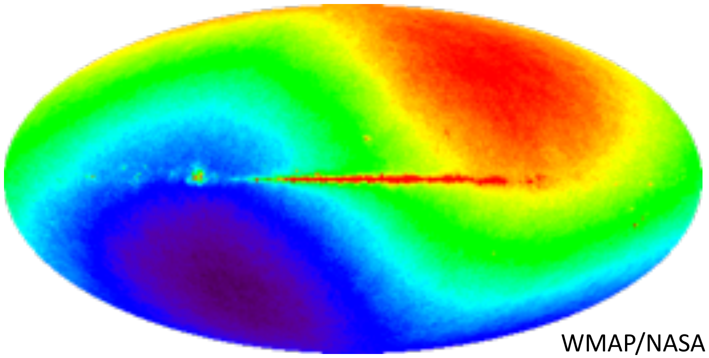
Metric:

$$g_{00} = -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 + 2(1+\zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij} + (2\alpha_3 - \alpha_1)w^i V_i + \mathcal{O}(\epsilon^3),$$

$$g_{0i} = -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U - \alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}),$$

$$g_{ij} = (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(\epsilon^2).$$

w: motion w.r.t. preferred reference frame



Metric potentials:

$$U = \int \frac{\rho'}{|\mathbf{x} - \mathbf{x}'|}\, d^3x', \quad \text{(Newtonian potential)}$$

$$U_{ij} = \int \frac{\rho'(x-x')_i(x-x')_j}{|\mathbf{x} - \mathbf{x}'|^3}\, d^3x',$$

$$\Phi_W = \int \frac{\rho'\rho''(\mathbf{x}-\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \cdot \left(\frac{\mathbf{x}'-\mathbf{x}''}{|\mathbf{x} - \mathbf{x}''|} - \frac{\mathbf{x}-\mathbf{x}''}{|\mathbf{x}' - \mathbf{x}''|}\right) d^3x'\, d^3x'',$$

$$\mathcal{A} = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x}-\mathbf{x}')]^2}{|\mathbf{x} - \mathbf{x}'|^3}\, d^3x',$$

$$\Phi_1 = \int \frac{\rho' v'^2}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',$$

$$\Phi_2 = \int \frac{\rho' U'}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',$$

$$\Phi_3 = \int \frac{\rho' \Pi'}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',$$

$$\Phi_4 = \int \frac{p'}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',$$

$$V_i = \int \frac{\rho' v'_i}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',$$

$$W_i = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x}-\mathbf{x}')](x-x')_i}{|\mathbf{x} - \mathbf{x}'|^3}\, d^3x'.$$

[Will 1993, *Will 2014, Living Reviews in Relativity*]

PPN formalism for metric theories of gravity

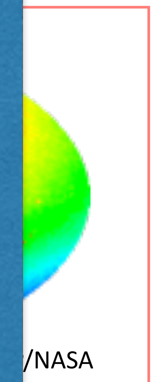
Metric:

$$g_{00} = -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2$$
$$+2(1+\zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij}$$
$$+(2\alpha_3 - \alpha_1)w^i V_i + \mathcal{O}(\epsilon^3),$$

$g_{0i} =$

$g_{ij} =$

frame



- Terms should be of Newtonian or post-Newtonian order.
- Terms should tend to zero as distance to source becomes large (asymptotically Minkowskian).
- Matter can be idealized as perfect fluid.
- The metric functionals should be generated by rest mass, energy, pressure, and velocity, not by their gradients.
- The functionals should be "simple", and should not contain any explicite scale.
- …

Metric

$U = \int$

$$U_{ij} = \int \frac{\rho'(x-x')_i(x-x')_j}{|\mathbf{x}-\mathbf{x}'|^3}\, d^3x', \qquad \Phi_2 = \int \frac{\rho' U'}{|\mathbf{x}-\mathbf{x}'|}\, d^3x', \qquad W_i = \int \frac{\rho'[\mathbf{v}'\cdot(\mathbf{x}-\mathbf{x}')](x-x')_i}{|\mathbf{x}-\mathbf{x}'|^3}\, d^3x'.$$

$$\Phi_W = \int \frac{\rho'\rho''(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}\cdot\left(\frac{\mathbf{x}'-\mathbf{x}''}{|\mathbf{x}-\mathbf{x}''|} - \frac{\mathbf{x}-\mathbf{x}''}{|\mathbf{x}'-\mathbf{x}''|}\right) d^3x'\, d^3x'', \qquad \Phi_3 = \int \frac{\rho'\Pi'}{|\mathbf{x}-\mathbf{x}'|}\, d^3x',$$

$$\mathcal{A} = \int \frac{\rho'[\mathbf{v}'\cdot(\mathbf{x}-\mathbf{x}')]^2}{|\mathbf{x}-\mathbf{x}'|^3}\, d^3x', \qquad \Phi_4 = \int \frac{p'}{|\mathbf{x}-\mathbf{x}'|}\, d^3x',$$

[Will 1993, *Will 2014, Living Reviews in Relativity*]

PPN parameters

Parameter	What it measures relative to GR	Value in GR	Value in semi-conservative theories	Value in fully conservative theories
γ	How much space-curvature produced by unit rest mass?	1	γ	γ
β	How much "nonlinearity" in the superposition law for gravity?	1	β	β
ξ	Preferred-location effects?	0	ξ	ξ
α_1	Preferred-frame effects?	0	α_1	0
α_2		0	α_2	0
$\boxed{\alpha_3}$		0	0	0
$\boxed{\alpha_3}$	Violation of conservation	0	0	0
ζ_1	of total momentum?	0	0	0
ζ_2		0	0	0
ζ_3		0	0	0
ζ_4		0	0	0

[*Will 2014, Living Reviews in Relativity*]

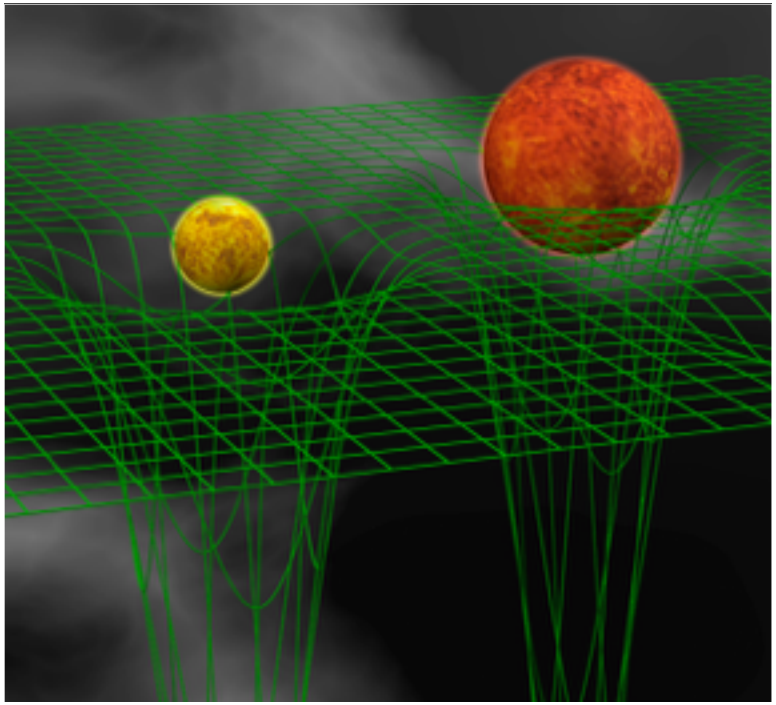
Limits on PPN parameters from Solar system and pulsars

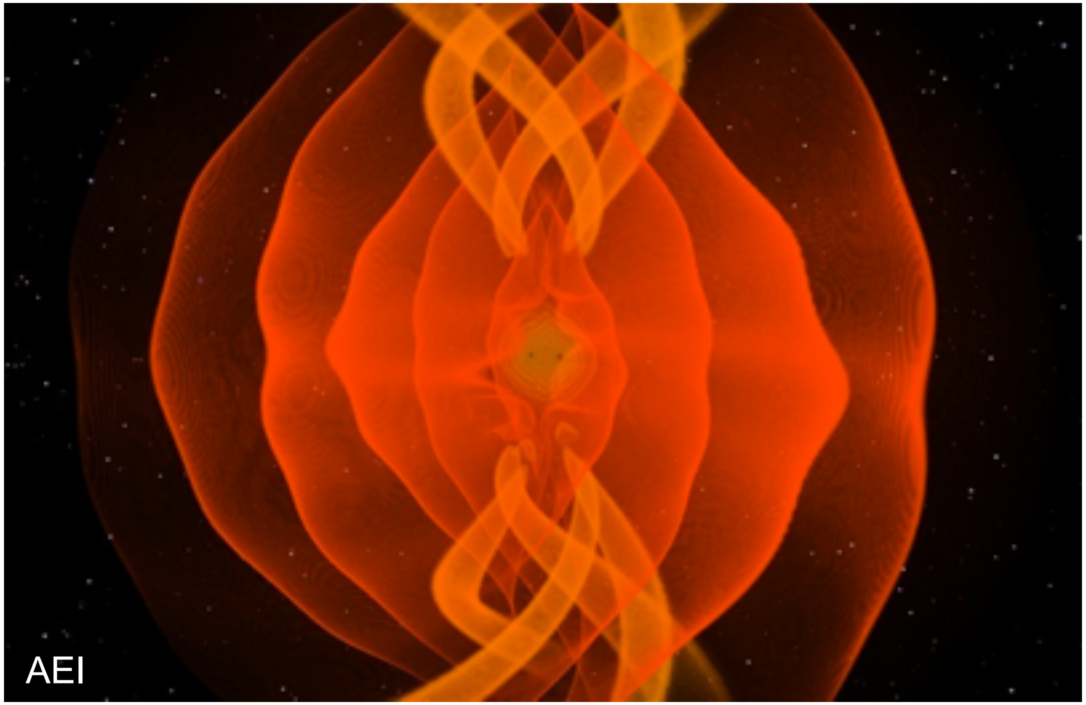
Parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	2.3×10^{-5}	Cassini tracking
	light deflection	2×10^{-4}	VLBI
$\beta - 1$	perihelion shift	8×10^{-5}	$J_{2\odot} = (2.2 \pm 0.1) \times 10^{-7}$
	Nordtvedt effect	2.3×10^{-4}	$\boxed{\eta_\mathrm{N} = 4\beta - \gamma - 3}$ assumed (Nordtvedt parameter)
ξ	spin precession	4×10^{-9}	millisecond pulsars
α_1	orbital polarization	10^{-4}	Lunar laser ranging
		4×10^{-5}	PSR J1738+0333
α_2	spin precession	2×10^{-9}	millisecond pulsars
α_3	pulsar acceleration	3×10^{-21}	PSR J1713+0747 [*Zhu et al., in prep.*]
ζ_1	—	2×10^{-2}	combined PPN bounds
ζ_2	binary acceleration	4×10^{-5}	$\ddot{P}_\mathrm{p}$ for PSR 1913+16
ζ_3	Newton's 3rd law	10^{-8}	lunar acceleration
ζ_4	—	—	not independent

[*Will 2014, Living Reviews in Relativity*]

Do strongly self-gravitating bodies move as predicted by GR?

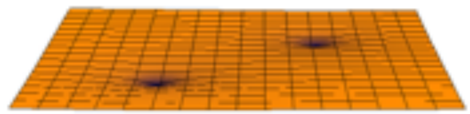
Do gravitational waves exist?





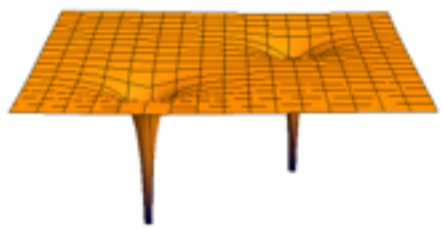
Gravity regimes relevant for this talk

(1) Quasi-stationary weak-field regime

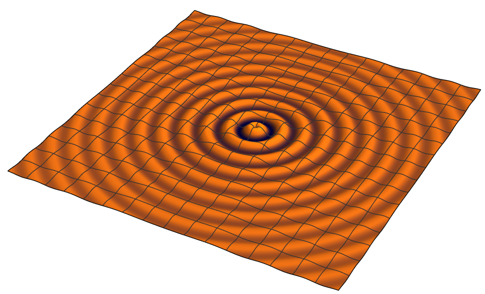


Solar system experiments

(2) Quasi-stationary strong-field regime

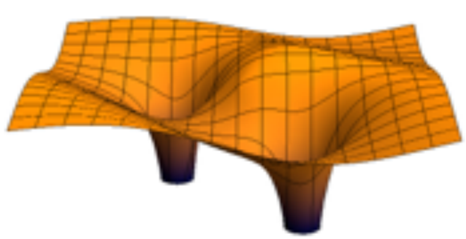


(3) Radiative regime



Binary pulsar experiments

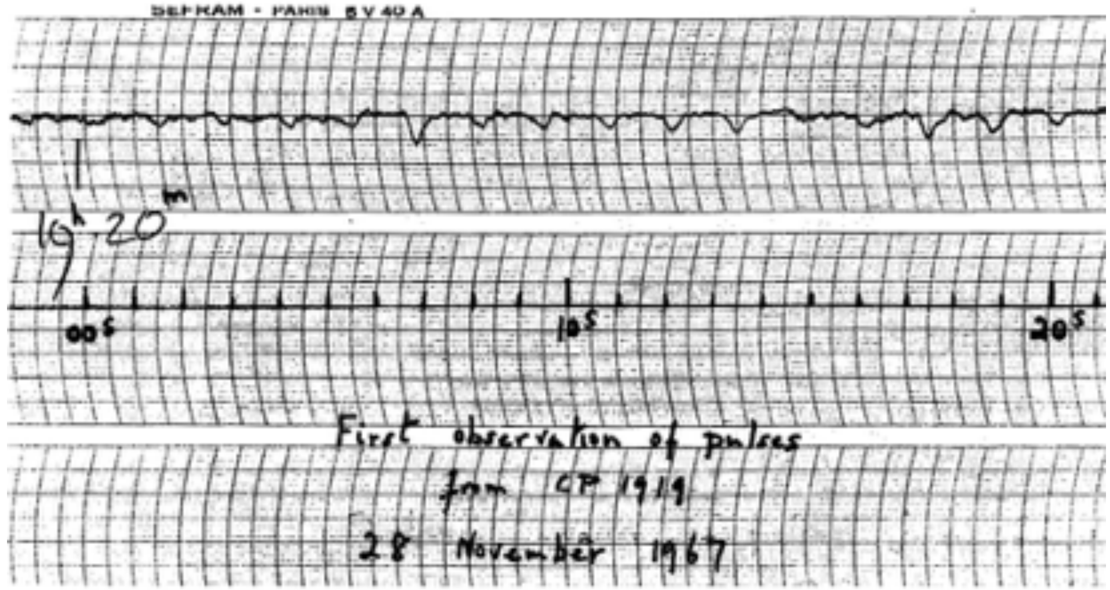
(4) Highly relativistic regime

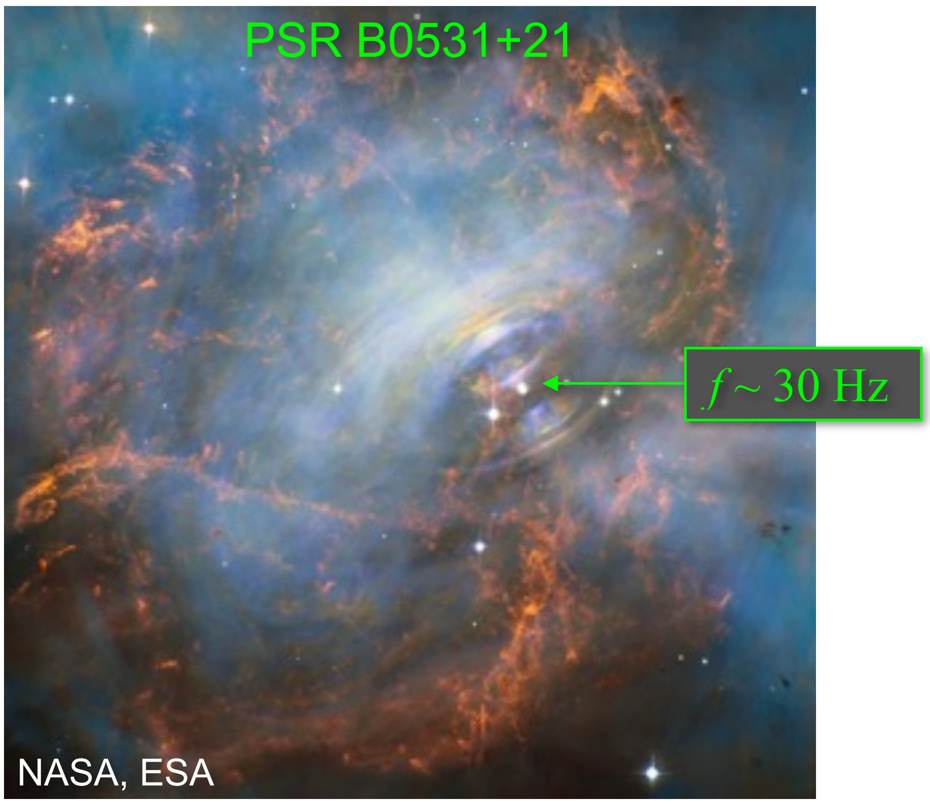


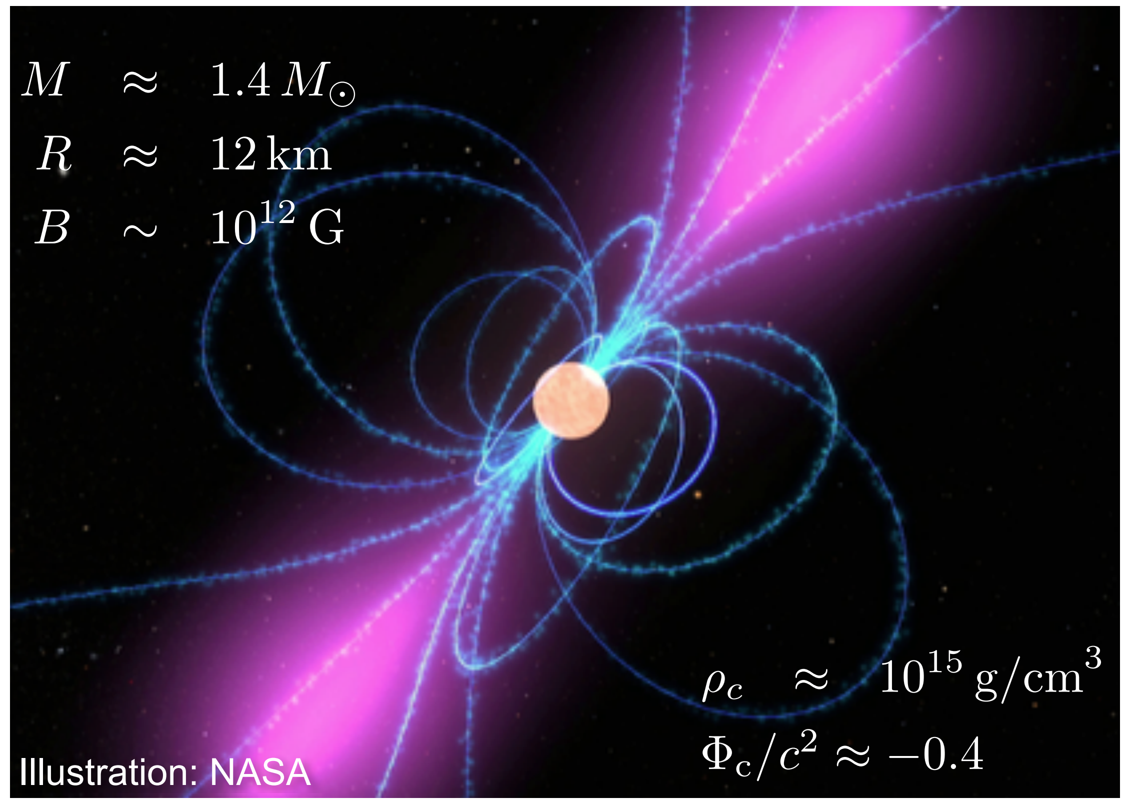
GW astronomy

Pulsars and pulsar timing

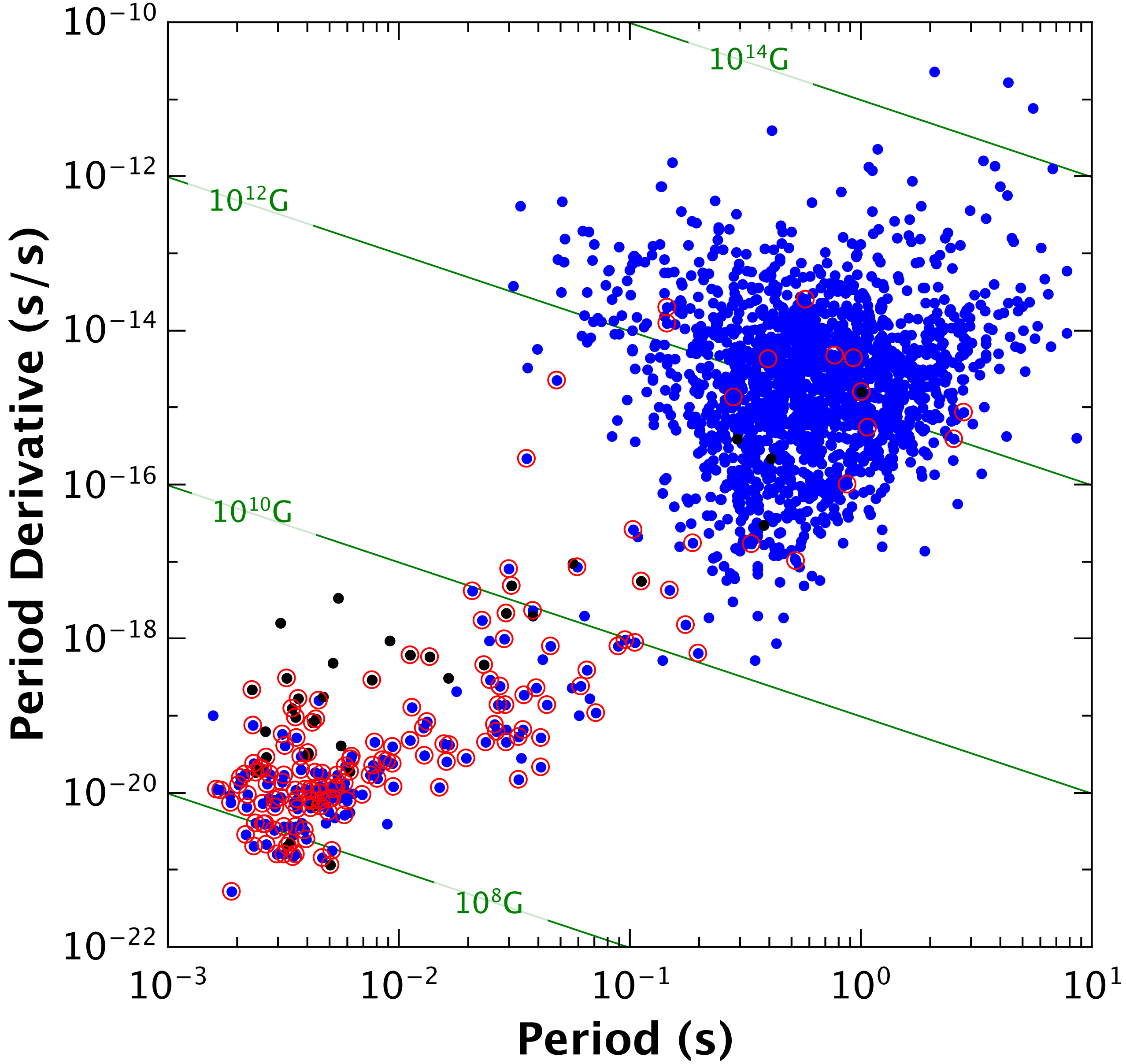
The discovery of pulsars - 1967







The radio pulsar population



~ 2500 radio pulsars

1.4 ms (PSR J1748-2446ad)

8.5 s (PSR J2144-3933)

~ 10% in binary systems

Orbital period range

95 min (PSR J0024-7204R)

>200 yr (PSR J1024−0719)

Companions

ordinary stars,

white dwarfs,

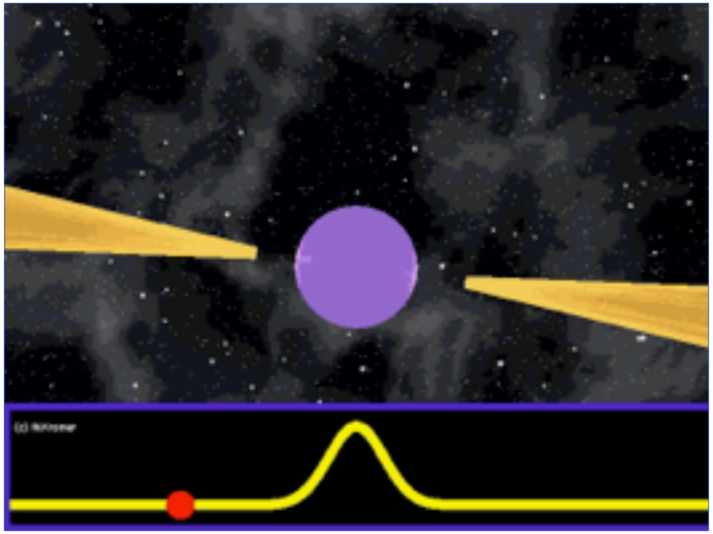
neutron stars,

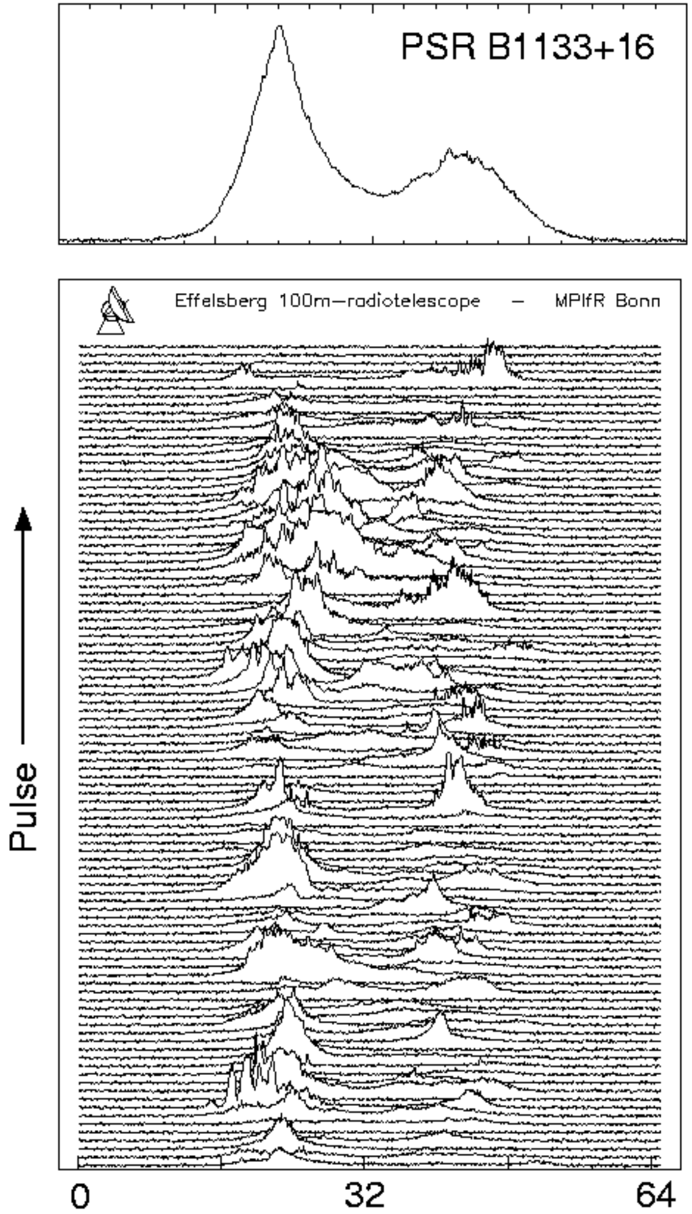
planets

Still missing: black hole

[ATNF pulsar catalogue]

Pulsar timing – time of arrival (TOA)

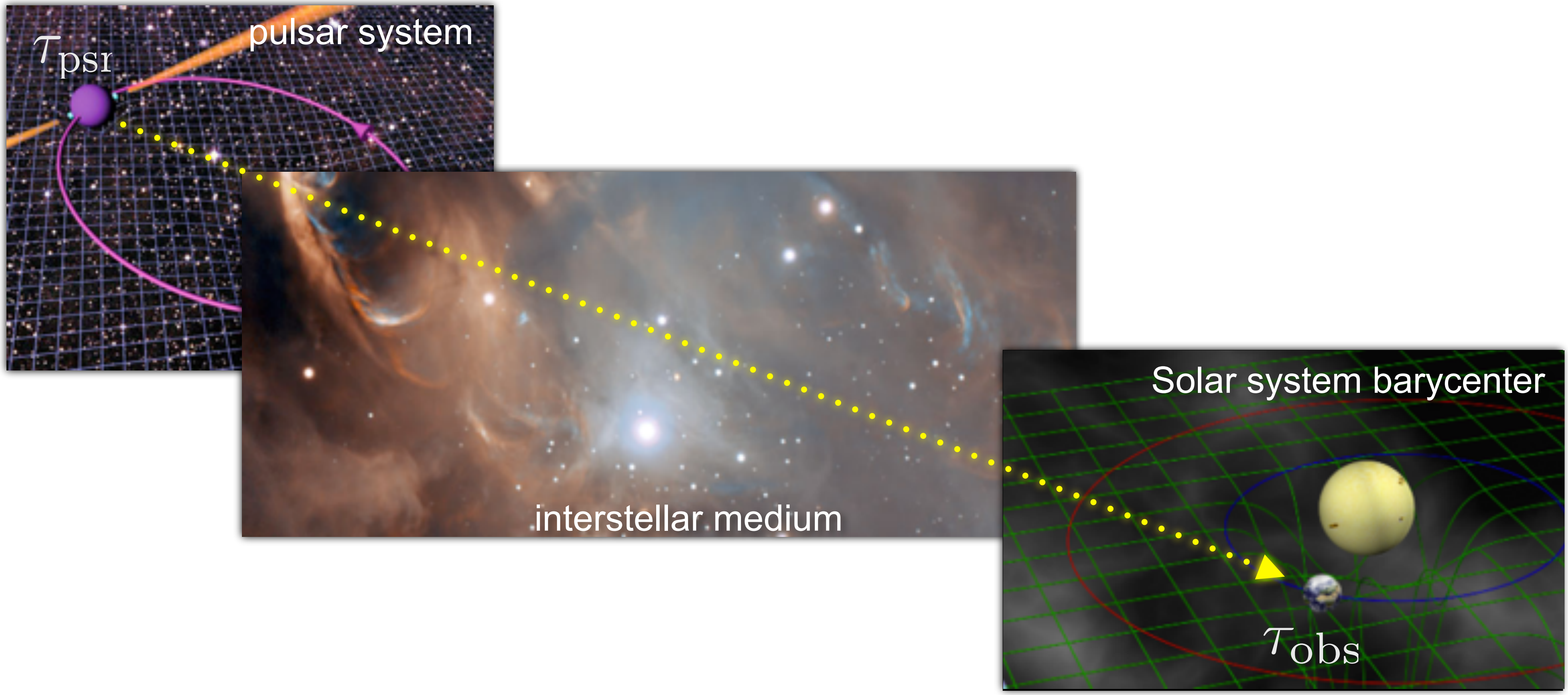




Timing precision for some millisecond pulsars < 100 ns → < 30 m

The timing model

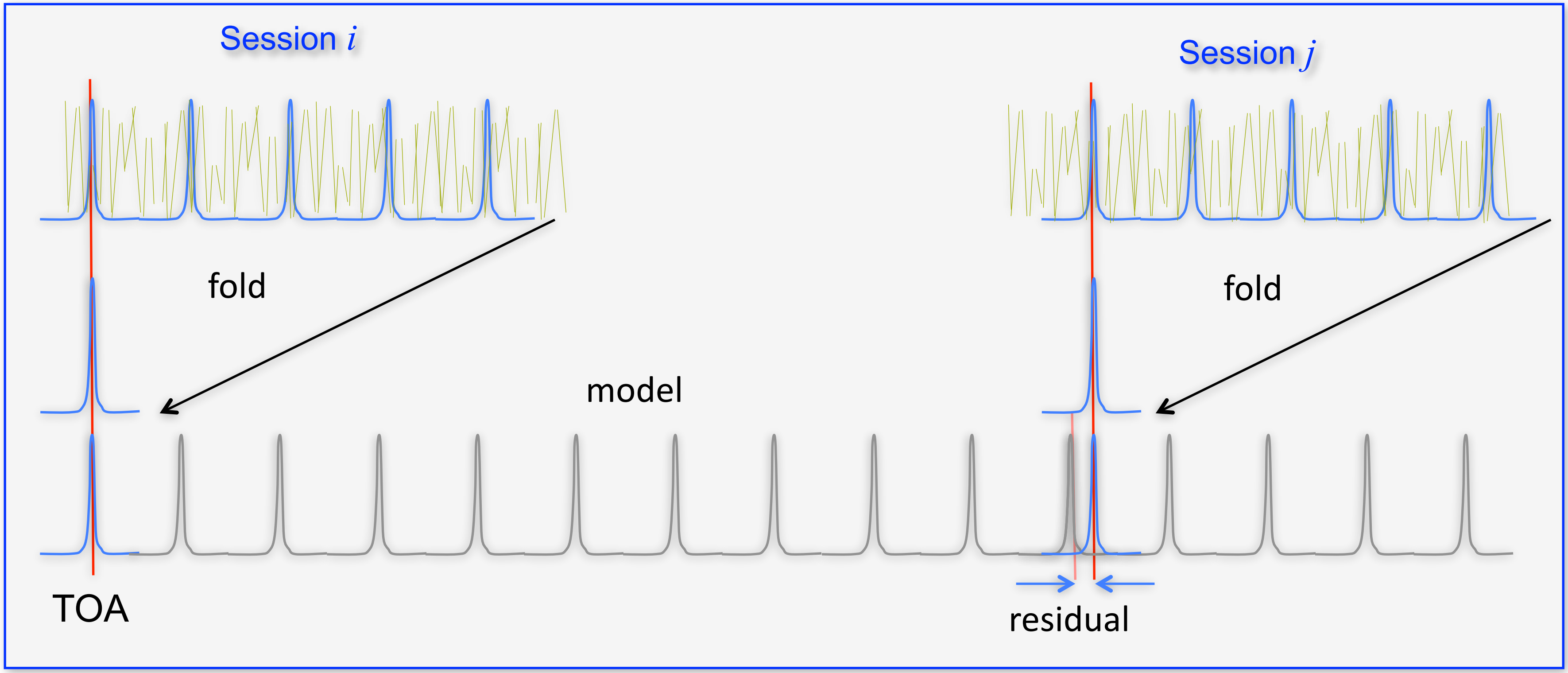
$$\tau_{\rm psr} \propto T\ , \quad \phi = \phi_0 + \nu T + \tfrac{1}{2}\dot{\nu} T^2$$

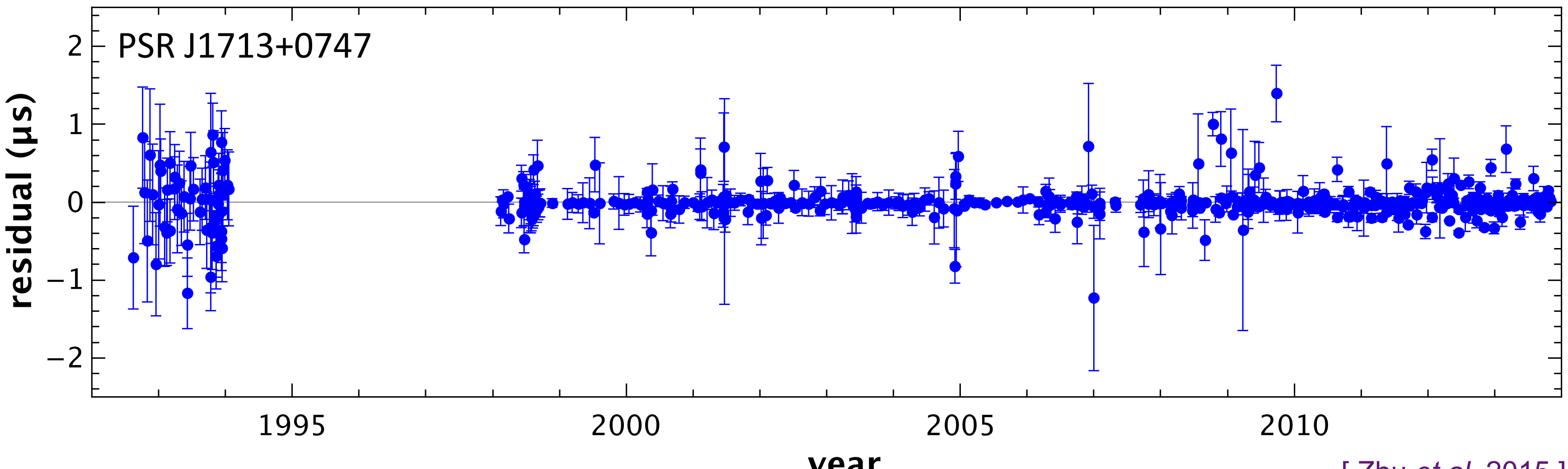


$$\tau_{\rm psr} = \tau_{\rm obs} - D/f^2 + \Delta_{\rm R\odot}(\lambda, \beta, \mu_\lambda, \mu_\beta, \pi) + \Delta_{\rm E\odot} + \Delta_{\rm S\odot}(\lambda, \beta) + \Delta_{\rm B}(K, PK)$$

Pulsar timing - parameter estimation

Phase-connected timing solution:





[Zhu *et al.* 2015]

What do we mean by precision timing? Best of…

Spin parameters:

Period:	2.947108069160717(3) ms	(Reardon et al. 2015)

3 atto seconds uncertainty!

Astrometry:

Position in the sky:	0.6 μas	(Reardon et al. 2015)
Proper motion:	140.911(3) mas/yr	(Reardon et al. 2015)
Distance:	156.79 ± 0.25 pc	(Reardon et al. 2015)

0.1 μs uncertainty!

Orbital parameters:

Orbital period:	0.102251562472(1) days	(Kramer et al. in prep.)
Projected semi-major axis	31,656,123.76(15) km	(Freire et al. 2011)
Eccentricity:	0.0000749402(6)	(Zhu et al. 2015)

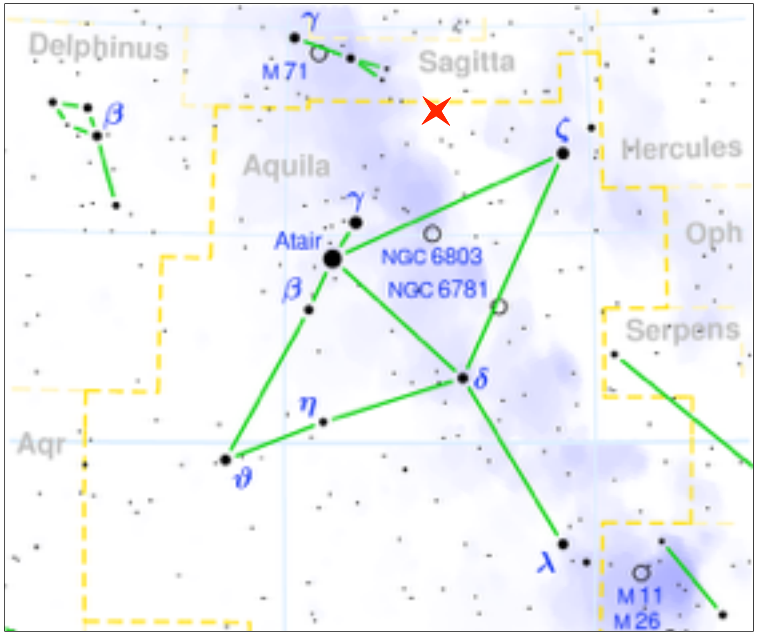
Masses:

Masses of neutron stars:	1.33816(2) / 1.24891(2) $M_\odot$	(Kramer et al. in prep.)
Mass of low-mass WD:	0.207(2) $M_\odot$	(Reardon et al. 2015)
Mass of millisecond pulsar:	1.667(7) $M_\odot$	(Freire et al. 2011)
Main sequence star companion:	1.029(3) $M_\odot$	(Freire et al. 2011)

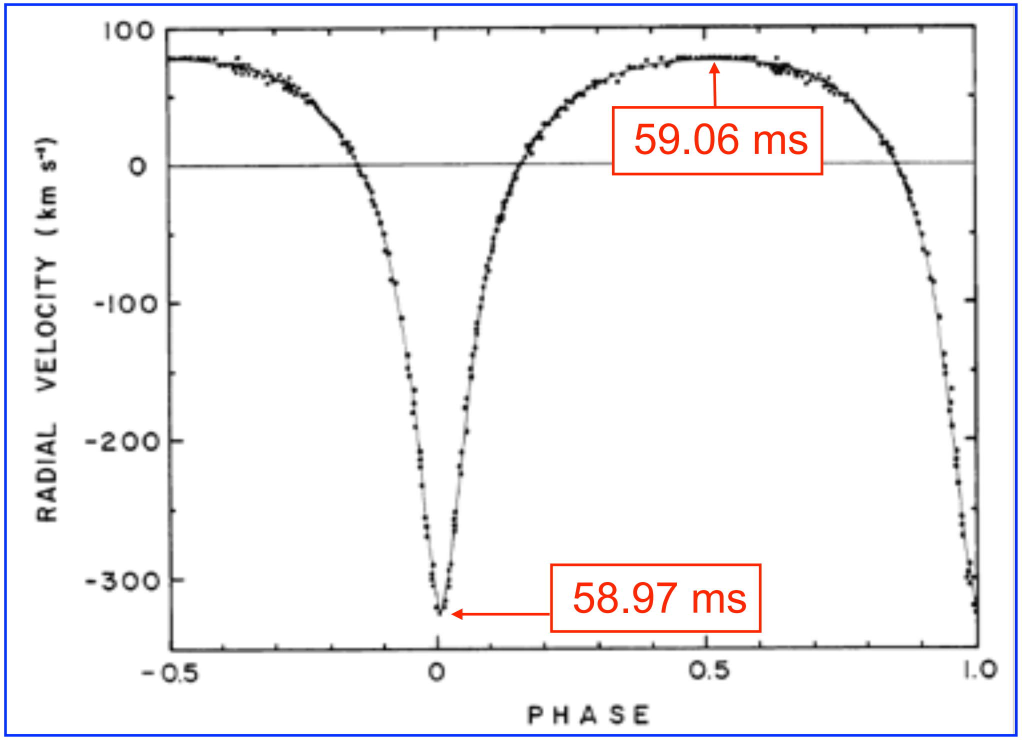
GR effects:

Periastron advance:	4.226598(5) deg/yr	(Weisberg et al. 2010)
Einstein delay:	4.2992(8) ms	(Weisberg et al. 2010)
Orbital GW damping:	-39.384(6) μs/yr	(Kramer et al. in prep.)

The first binary pulsar - 1974



PSR B1913+16



Pulse period: 59.0 ms

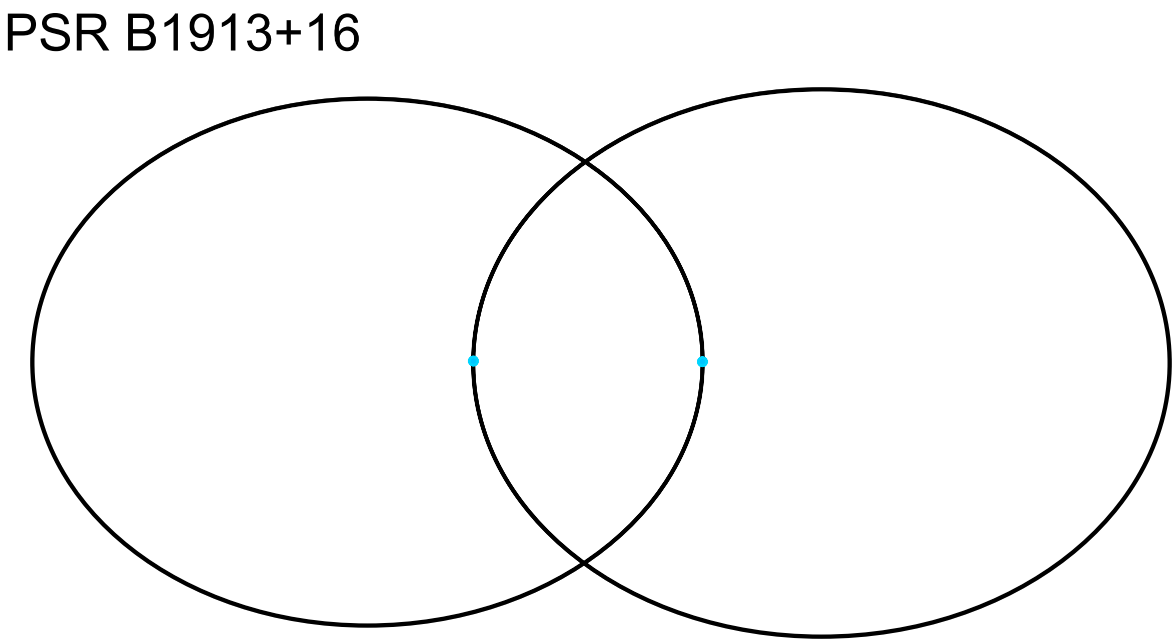
Orbital period: 7.75 h

Eccentricity: 0.617

Companion: neutron star

[Hulse & Taylor 1975]

PSR B1913+16 orbit



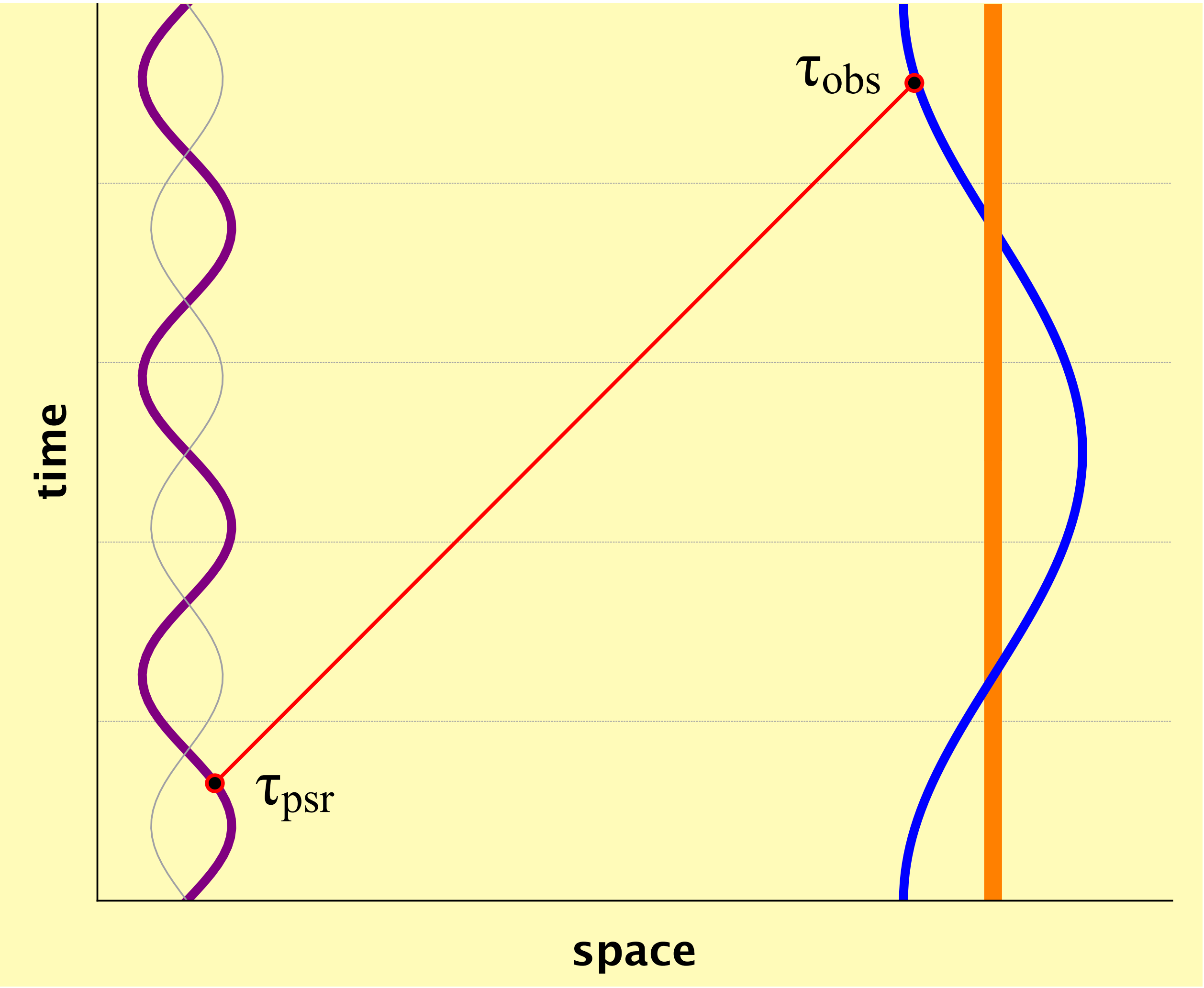
$$m_p = 1.44\,M_\odot \qquad a = 1.95 \times 10^{11}\,\mathrm{cm}$$
$$m_c = 1.39\,M_\odot \qquad e = 0.617$$

$$r_{\min} = 1.1\,R_\odot$$
$$v_{\max} = 900\,\mathrm{km/s} = 0.003\,c$$

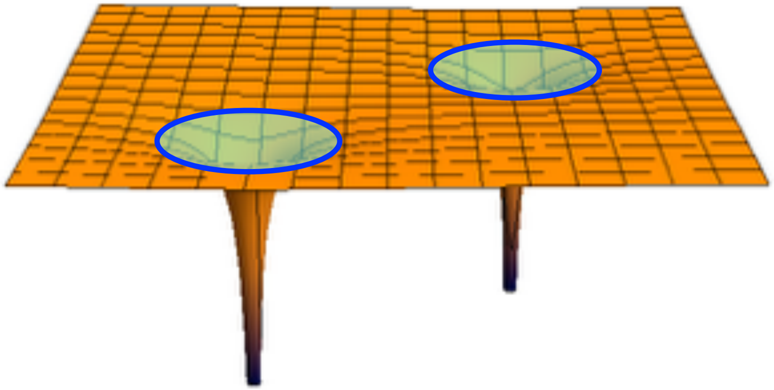
$$L_{\mathrm{GW}} = 7.8 \times 10^{24}\,\mathrm{W}$$

Binary Pulsars and GR

Pulsar timing - a spacetime view



The effacement principle in GR



Multi-chart approach to solve Einstein's field equations

—> one global coordinate chart x_μ: $g_{\mu\nu}(x^\lambda) = \eta_{\mu\nu} + h^{(1)}_{\mu\nu} + h^{(2)}_{\mu\nu} + \cdots$

—> two local coordinate charts $X_\mu{}^a$: $G_{\alpha\beta}(X^\gamma_a) = G^{(0)}_{\alpha\beta}(X^\gamma_a; m_a) + H^{(1)}_{\alpha\beta}(X^\gamma_a; m_a, m_b) + \cdots$

—> expansions are then 'matched' in some overlapping domain of common validity

In GR, the internal structure of a compact (R ~ few Gm/c^2) quasi-static body is effaced to a very high degree —> absence of any explicit strong-field-gravity effect in the orbital dynamics.

The masses are always defined such that the Lagrangian for non-interacting compact objects reads

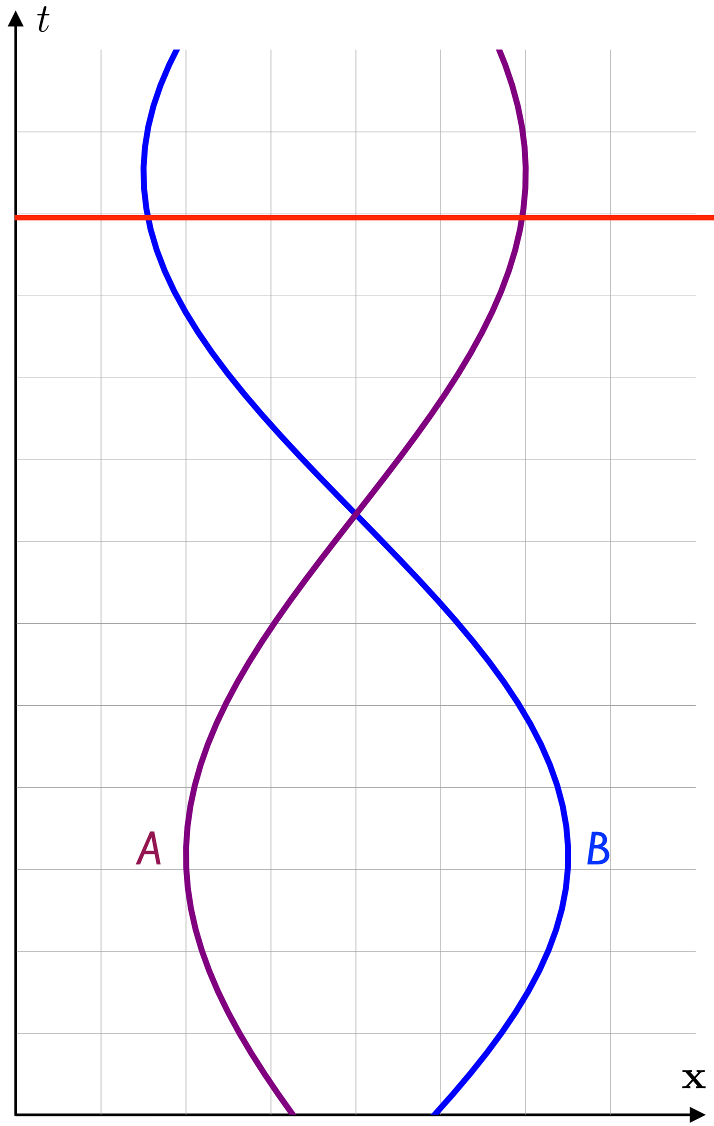
$$\mathcal{L}_0 = -\sum_a m_a \left(1 - \frac{v_a^2}{c^2}\right)^{1/2}$$

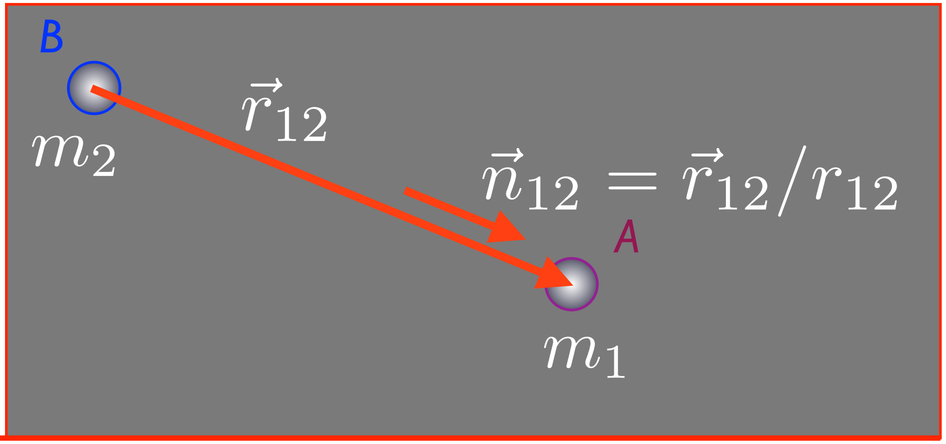
hence $m_a c^2$ represents the total energy of body *a.*

[Damour 1987, 2009 (SIGRAV lecture)]

Binary motion - Newtonian dynamics

Determining the world line of the pulsar





$$a_1^i = -\frac{G m_2 n_{12}^i}{r_{12}^2}$$

<u>Relative motion (Keplerian parametrization)</u>

Eccentric anomaly

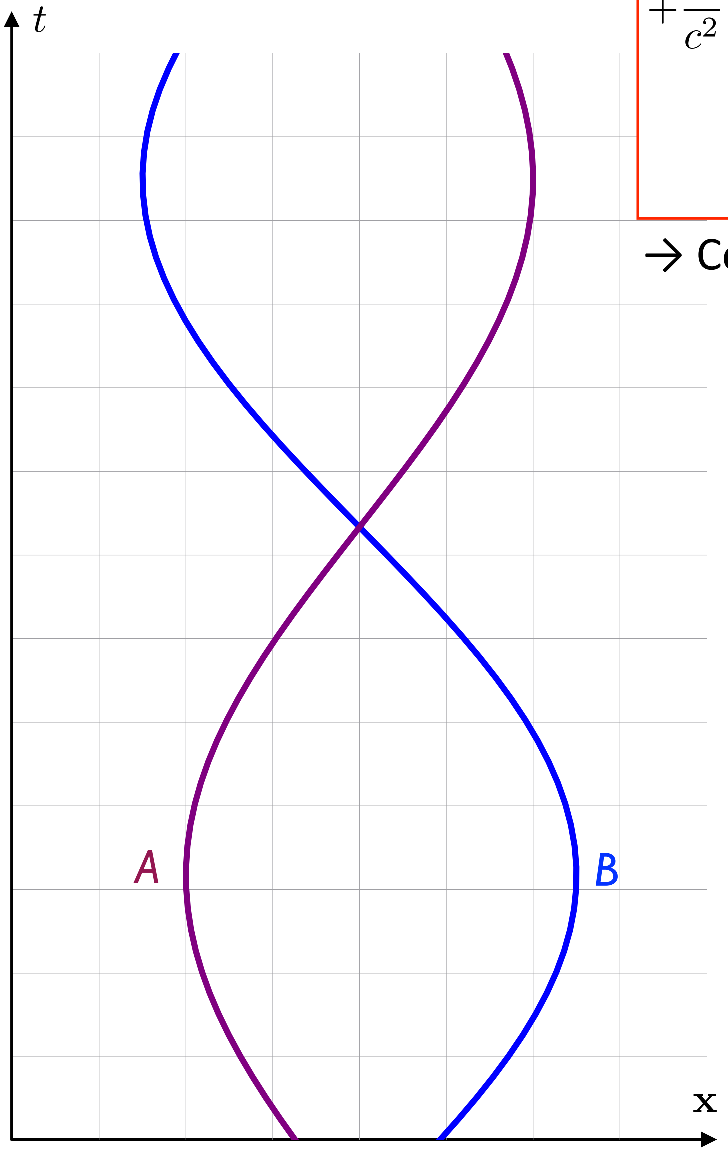
$$\frac{2\pi}{P_b}(t - t_0) = U - e \sin U \qquad \text{(Kepler's equation)}$$

$$r_{12} = a(1 - e\cos U)$$

$$\varphi - \varphi_0 = 2\arctan\left[\left(\frac{1+e}{1-e}\right)^{1/2} \tan\frac{U}{2}\right]$$

Binary Motion - First post-Newtonian Dynamics

harmonic coordinates



$$a_1^i = -\frac{Gm_2 n_{12}^i}{r_{12}^2}$$

$$\boxed{+\frac{1}{c^2}\left\{\left[\frac{5G^2 m_1 m_2}{r_{12}^3} + \frac{4G^2 m_2^2}{r_{12}^3} + \frac{Gm_2}{r_{12}^2}\left(\frac{3}{2}(n_{12}v_2)^2 - v_1^2 + 4(v_1 v_2) - 2v_2^2\right)\right] n_{12}^i + \frac{Gm_2}{r_{12}^2}\left(4(n_{12}v_1) - 3(n_{12}v_2)\right) v_{12}^i\right\}}$$

→ Conservation of orbital energy and angular momentum

Quasi-Keplerian parametrization [Damour & Deruelle 1985]

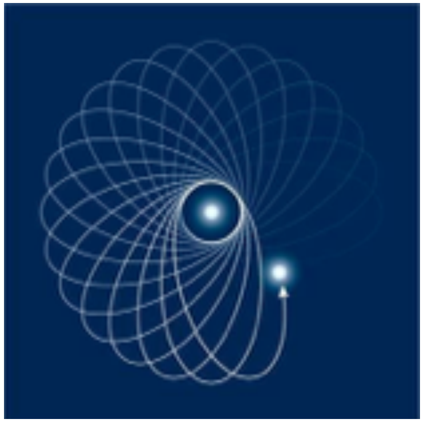
$$\begin{aligned}
\frac{2\pi}{P_b}(t - t_0) &= U - e_t \sin U \\
r_{12} &= a(1 - e_r \cos U) \\
\varphi - \varphi_0 &= 2(1 + k) \arctan\left[\left(\frac{1 + e_\varphi}{1 - e_\varphi}\right)^{1/2} \tan\frac{U}{2}\right]
\end{aligned}$$

Relativistic advance of periastron

$$\dot{\omega} = \frac{2\pi}{P_b} k$$

$$k = 3\frac{G^{2/3}}{c^2}\left(\frac{2\pi}{P_b}\right)^{2/3} \frac{(m_1 + m_2)^{2/3}}{1 - e^2}$$

Hulse-Taylor pulsar: 4.2 deg/yr



Second post-Newtonian motion - EoM

$$
\begin{aligned}
a_1^i = & -\frac{Gm_2 n_{12}^i}{r_{12}^2} \\
& + \frac{1}{c^2}\left\{\left[\frac{5G^2 m_1 m_2}{r_{12}^3} + \frac{4G^2 m_2^2}{r_{12}^3} + \frac{Gm_2}{r_{12}^2}\left(\frac{3}{2}(n_{12}v_2)^2 - v_1^2 + 4(v_1 v_2) - 2v_2^2\right)\right] n_{12}^i \right. \\
& \qquad \left. + \frac{Gm_2}{r_{12}^2}\left(4(n_{12}v_1) - 3(n_{12}v_2)\right) v_{12}^i \right\} \\
& + \frac{1}{c^4}\left\{\left[-\frac{57G^3 m_1^2 m_2}{4r_{12}^4} - \frac{69G^3 m_1 m_2^2}{2r_{12}^4} - \frac{9G^3 m_2^3}{r_{12}^4} \right.\right. \\
& \qquad + \frac{Gm_2}{r_{12}^2}\left(-\frac{15}{8}(n_{12}v_2)^4 + \frac{3}{2}(n_{12}v_2)^2 v_1^2 - 6(n_{12}v_2)^2(v_1 v_2) - 2(v_1 v_2)^2 + \frac{9}{2}(n_{12}v_2)^2 v_2^2 \right. \\
& \qquad\qquad \left. + 4(v_1 v_2)v_2^2 - 2v_2^4\right) \\
& \qquad + \frac{G^2 m_1 m_2}{r_{12}^3}\left(\frac{39}{2}(n_{12}v_1)^2 - 39(n_{12}v_1)(n_{12}v_2) + \frac{17}{2}(n_{12}v_2)^2 - \frac{15}{4}v_1^2 - \frac{5}{2}(v_1 v_2) + \frac{5}{4}v_2^2\right) \\
& \qquad \left. + \frac{G^2 m_2^2}{r_{12}^3}\left(2(n_{12}v_1)^2 - 4(n_{12}v_1)(n_{12}v_2) - 6(n_{12}v_2)^2 - 8(v_1 v_2) + 4v_2^2\right)\right] n_{12}^i \\
& \qquad + \left[\frac{G^2 m_2^2}{r_{12}^3}\left(-2(n_{12}v_1) - 2(n_{12}v_2)\right) + \frac{G^2 m_1 m_2}{r_{12}^3}\left(-\frac{63}{4}(n_{12}v_1) + \frac{55}{4}(n_{12}v_2)\right) \right. \\
& \qquad\quad + \frac{Gm_2}{r_{12}^2}\left(-6(n_{12}v_1)(n_{12}v_2)^2 + \frac{9}{2}(n_{12}v_2)^3 + (n_{12}v_2)v_1^2 - 4(n_{12}v_1)(v_1 v_2) \right. \\
& \qquad\qquad \left.\left.\left. + 4(n_{12}v_2)(v_1 v_2) + 4(n_{12}v_1)v_2^2 - 5(n_{12}v_2)v_2^2\right)\right] v_{12}^i \right\}
\end{aligned}
$$

→ Conservation of orbital energy and angular momentum

Generalized quasi-Keplerian parametrization

$$\begin{aligned}
\frac{2\pi}{P_b}(t - t_0) &= U - e_t \sin U + \boxed{\frac{f_t}{c^4}\,\sin v + \frac{g_t}{c^4}\,(v - U)} \\
v &= 2\arctan\left[\left(\frac{1+e_\varphi}{1-e_\varphi}\right)^{1/2} \tan\frac{U}{2}\right] \\
r_{12} &= a(1 - e_r \cos U) \\
\varphi - \varphi_0 &= (1+k)\,v + \boxed{\frac{f_\varphi}{c^4}\,\sin 2v + \frac{g_\varphi}{c^4}\,\sin 3v}
\end{aligned}$$

Relevant for present day pulsar timing

$$k = \frac{3\beta_O^2}{1-e_T^2}\left\{1 + \frac{\beta_O^2}{1-e_T^2}\left[\left(\frac{13}{2} - \frac{7}{3}\eta\right) + \left(\frac{1}{4} + \frac{5}{6}\eta + 3\frac{m_1}{M}\right) e_T^2\right]\right\}$$

$$\beta_O = \left(\frac{2\pi G M}{P_b c^3}\right)^{1/3} \quad \text{and} \quad \eta = m_1 m_2 / M^2$$

Hulse-Taylor pulsar:	$\beta_O = 0.0015$	
Observed periastron advance:	4.226585(4)	deg/yr
2pN contribution:	0.000098	deg/yr

[Damour & Schäfer 1988, Schäfer & Wex 1993]

Binary motion in Einstein's gravity: 2.5 post-Newtonian

$$
\begin{aligned}
a_1^i &= -\frac{Gm_2 n_{12}^i}{r_{12}^2} \\
&+\frac{1}{c^2}\left\{\left[\frac{5G^2m_1m_2}{r_{12}^3}+\frac{4G^2m_2^2}{r_{12}^3}+\frac{Gm_2}{r_{12}^2}\left(\frac{3}{2}(n_{12}v_2)^2-v_1^2+4(v_1v_2)-2v_2^2\right)\right]n_{12}^i\right. \\
&\qquad\left.+\frac{Gm_2}{r_{12}^2}\left(4(n_{12}v_1)-3(n_{12}v_2)\right)v_{12}^i\right\} \quad + \quad \text{2pN}
\end{aligned}
$$

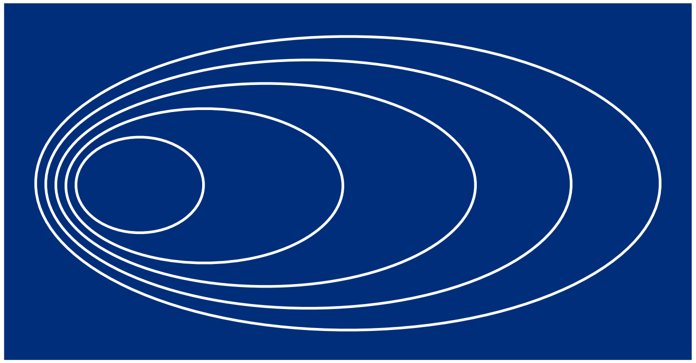
$$
\boxed{
\begin{aligned}
&+\frac{1}{c^5}\left\{\left[\frac{208G^3m_1m_2^2}{15r_{12}^4}(n_{12}v_{12})-\frac{24G^3m_1^2m_2}{5r_{12}^4}(n_{12}v_{12})+\frac{12G^2m_1m_2}{5r_{12}^3}(n_{12}v_{12})v_{12}^2\right]n_{12}^i\right. \\
&\qquad\left.+\left[\frac{8G^3m_1^2m_2}{5r_{12}^4}-\frac{32G^3m_1m_2^2}{5r_{12}^4}-\frac{4G^2m_1m_2}{5r_{12}^3}v_{12}^2\right]v_{12}^i\right\}
\end{aligned}
}
$$

→ Loss of orbital energy and angular momentum

$$
\frac{dE}{dt} = -\frac{G}{5c^5}\sum_{i,j}\left\langle \frac{d^3Q_{ij}}{dt^3}\,\frac{d^3Q_{ij}}{dt^3}\right\rangle
$$

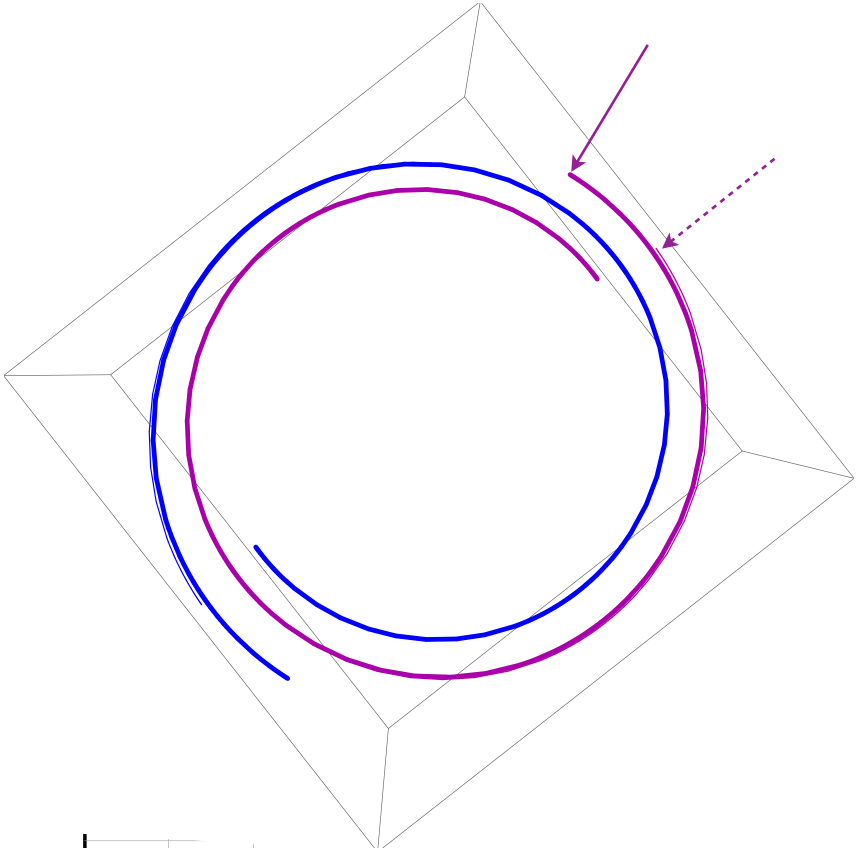
$$
\frac{dJ^i}{dt} = \frac{2G}{5c^5}\sum_{k,l,a}\epsilon^{ikl}\left\langle \frac{d^2Q_{ka}}{dt^2}\,\frac{d^3Q_{la}}{dt^3}\right\rangle
$$

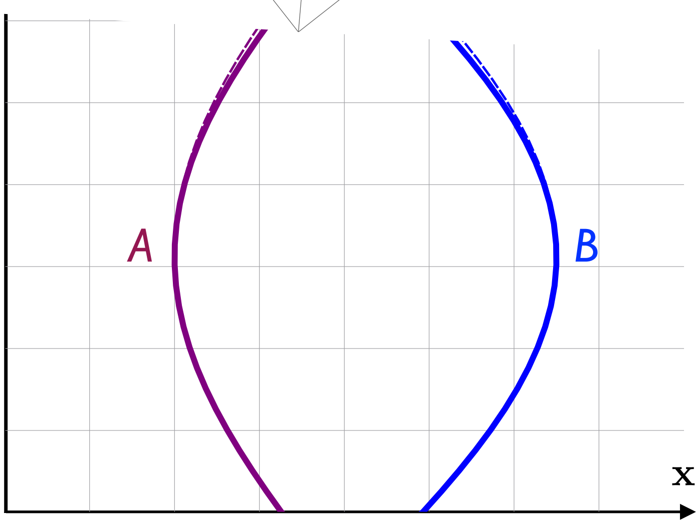
"radiation reaction **quadrupole formula**"



Hulse-Taylor pulsar: 3.5 m/yr

Gravitational-wave emission and orbital dynamics





Kepler's 3rd law:

$$a^3 \left(\frac{2\pi}{P_b}\right)^2 = G(m_1 + m_2) \quad \Longrightarrow \quad \frac{\dot{P}_b}{P_b} = \frac{3}{2}\frac{\dot{a}}{a}$$

In GR:

$$\boxed{\dot{P}_b^{\rm GR} = -\frac{192\,\pi\,G^{5/3}}{5\,c^5}\left(\frac{P_b}{2\pi}\right)^{-5/3}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) \times (1-e^2)^{-7/2}\, m_1\, m_2\, (m_1 + m_2)^{-1/3}}$$

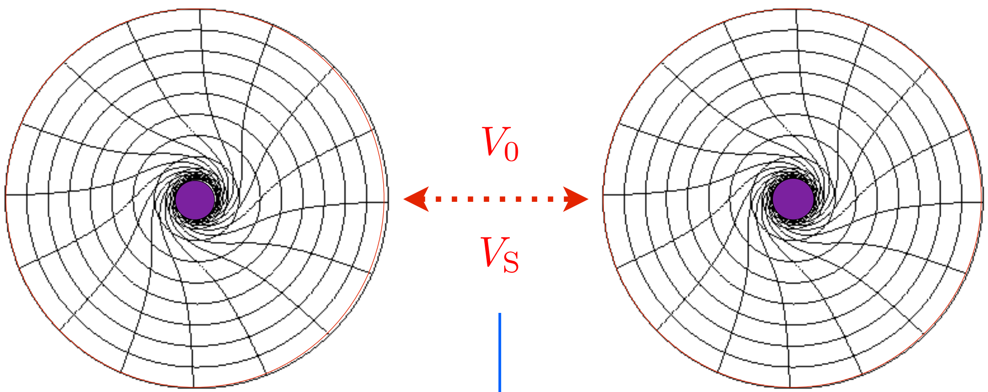
[Peters 1964]

Hulse-Taylor pulsar: 76 µs/yr

Orbital phase evolution:
modification of Kepler's equation

$$U - e\sin U = 2\pi\left[\left(\frac{t-t_0}{P_b}\right) - \frac{\dot{P}_b}{2}\left(\frac{t-t_0}{P_b}\right)^2\right]$$

Hulse-Taylor pulsar after 40 years:
At periastron ΔU = 2.3 deg
-> 30000 km -> 0.1 lt-s



[Andersen 1996]

Potential-energy terms for the interaction (S_1-L, S_2-L and S_1-S_2, to leading order)

$$V_{S_1,L} = \frac{G}{c^2 r_{12}^3}\left(2 + \frac{3m_2}{2m_1}\right)\left(\vec{S}_1 \cdot \vec{L}\right)$$

$$\cancel{V_{S_2,L} = \frac{G}{c^2 r_{12}^3}\left(2 + \frac{3m_1}{2m_2}\right)\left(\vec{S}_2 \cdot \vec{L}\right)}$$

$$\cancel{V_{S_1,S_2} = \frac{G}{c^2 r_{12}^3}\left(3(\vec{S}_1 \cdot \vec{n}_{12})(\vec{S}_2 \cdot \vec{n}_{12}) - \vec{S}_1 \cdot \vec{S}_2\right)}$$

Binary pulsars:

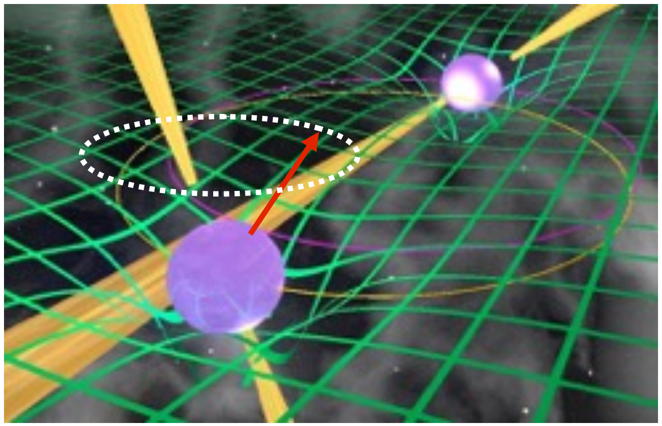
$|\boldsymbol{S}_\mathrm{p}| >> |\boldsymbol{S}_\mathrm{c}|$ [**HT**: >20]

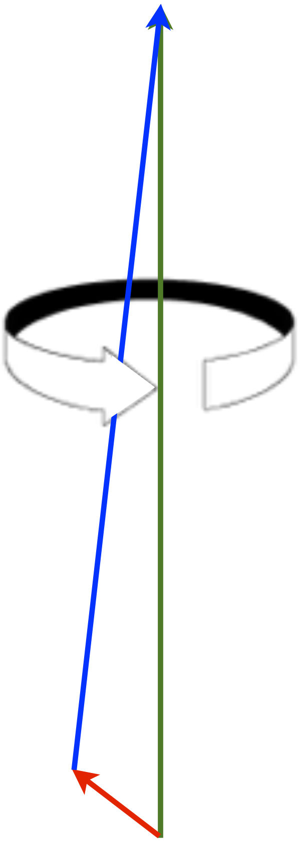
$|\boldsymbol{L}| >> |\boldsymbol{S}_\mathrm{p}|$ [**HT**: 70000]

$\boldsymbol{L} + \boldsymbol{S} = \boldsymbol{J} = \mathrm{const.}$

$|\boldsymbol{S}| = \mathrm{const.}$

$|\boldsymbol{L}| = \mathrm{const.}$

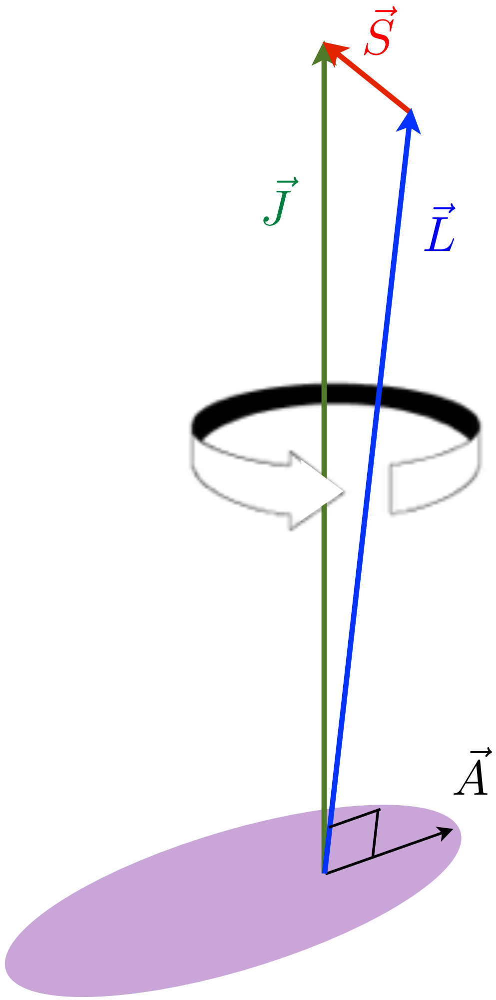




$$\Omega_p^{\rm geod} = \frac{G^{2/3}}{c^2}\left(\frac{2\pi}{P_b}\right)^{5/3}\frac{m_c(4m_p + m_c)}{2(m_p + m_c)^{4/3}}\,\frac{1}{1-e^2}$$

Hulse-Taylor pulsar: 1.2 deg/yr

[Barker & O'Connell 1975]



Spin of the pulsar causes a precession of the orbit:
→ Lense-Thirring precession of orbital angular momentum and Laplace-Runge-Lenz vector

$$\left.\begin{aligned}\left\langle\frac{d\vec{L}}{dt}\right\rangle &= \vec{\Omega}_{\rm SO}\times\vec{L}\\ \left\langle\frac{d\vec{A}}{dt}\right\rangle &= \vec{\Omega}_{\rm SO}\times\vec{A}\end{aligned}\right\}\quad \vec{\Omega}_{\rm SO} = \left(\frac{2\pi}{P_b}\right)^2\frac{m_c(4m_p+m_c)}{2c^2(m_p+m_c)}\,\frac{1}{(1-e^2)^{3/2}}\times\left(\vec{S}-3(\vec{S}\cdot\vec{L})\,\vec{L}/L^2\right)$$

Consequently, we have a change in the orbital inclination and an <u>additional</u> change in the longitude of periastron:

$$k = k_{\rm 1pN} + k_{\rm 2pN} + k_{\rm SO}$$

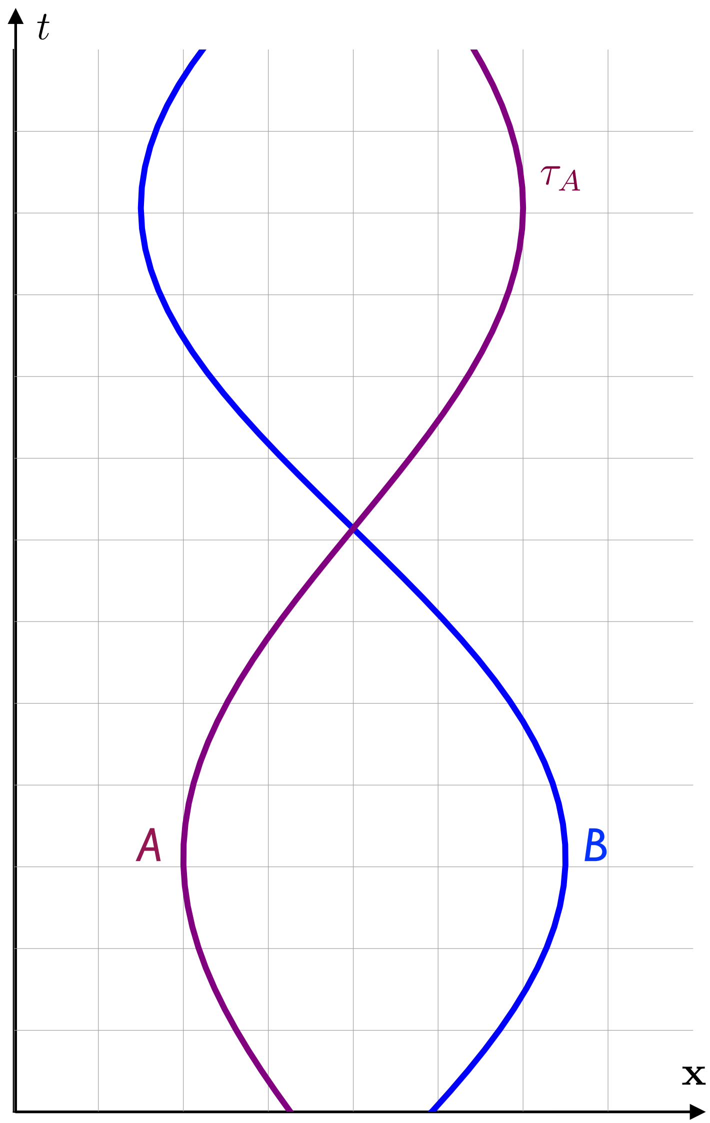
$$k_{\rm SO} = \frac{3\Gamma}{(1-e^2)^{3/2}}\,\beta_O^3\,\beta_{\rm S}\quad\left\{\begin{aligned}\Gamma &= \Gamma(m_1,m_2,i,\delta,\lambda)\\ \beta_O &= \left[G(m_1+m_2)n_b\right]^{1/3}/c\\ \beta_{\rm S} &= 2\pi\,\frac{c}{G}\,\frac{\nu_p I_p}{m_p^2}\end{aligned}\right.$$

Hulse-Taylor pulsar: -0.00004 deg/yr ~ 40% of 2pN

[Barker & O'Connell 1975, Damour & Schäfer 1988]

Time dilation

$$ds^2 = -c^2 d\tau^2 \approx -\left(1 + 2\frac{\Phi}{c^2}\right) dt^2 + \left(1 - 2\frac{\Phi}{c^2}\right)(dx^2 + dy^2 + dz^2)$$



where $\Phi(t, \mathbf{r}) = -\frac{Gm_A}{|\mathbf{r} - \mathbf{r}_A(t)|} - \frac{Gm_B}{|\mathbf{r} - \mathbf{r}_B(t)|}$

$$\frac{d\tau_A'}{dt} = 1 - \frac{Gm_A}{c^2 R} - \frac{v_A^2}{2c^2} + \mathcal{O}\left(\frac{v^4}{c^4}\right) \qquad \frac{d\tau_A}{dt} \propto \frac{d\tau_A'}{dt} \propto \frac{dT_A}{dt}$$

$$v_A^2 = G\frac{m_B^2}{M}\left(\frac{2}{R} - \frac{1}{a}\right)$$

$$R = a(1 - e\cos u)$$

$$n_b(t - t_0) = u - e\sin u$$

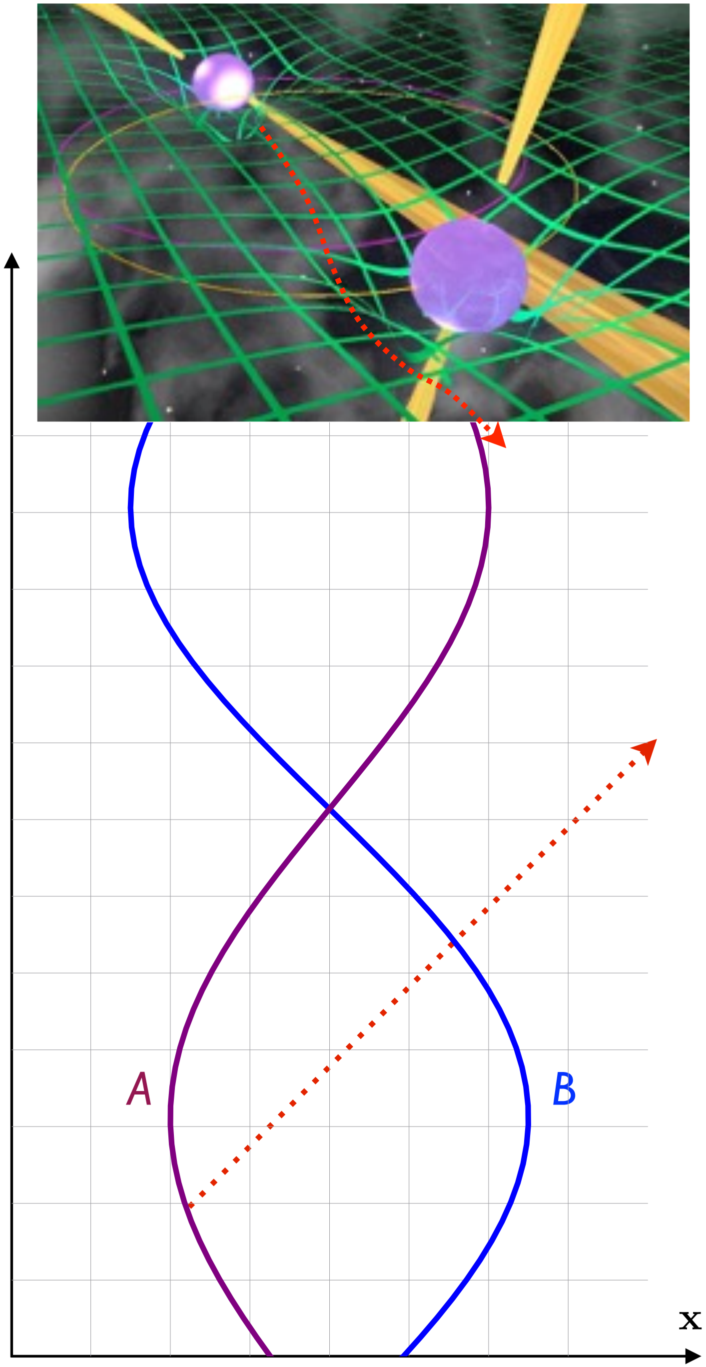
$$\boxed{T_A = t - \gamma_A \sin u \qquad \gamma_A = \frac{e}{n_b}\left(\frac{GMn_b}{c^3}\right)^{2/3}\frac{m_B}{M}\left(1 + \frac{m_B}{M}\right)}$$

"Einstein delay"

Hulse-Taylor pulsar: 4.3 ms

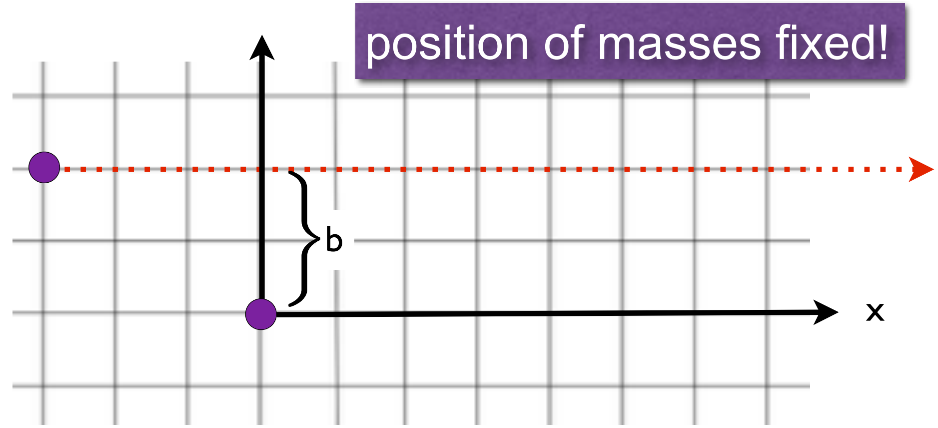
[Blandford & Teukolsky 1976]

Shapiro delay



$$0 = ds^2 \approx -\left(1 + 2\frac{\Phi}{c^2}\right) dt^2 + \left(1 - 2\frac{\Phi}{c^2}\right) (dx^2 + dy^2 + dz^2)$$

$$\left|\frac{d\mathbf{x}}{dt}\right| \approx c\left(1 + 2\frac{\Phi}{c^2}\right)$$



$$t_{\rm arr} - t_{\rm em} \approx \frac{1}{c}\int_{x_{\rm em}}^{x_{\rm arr}} \left(1 - 2\frac{\Phi}{c^2}\right) dx \qquad \Phi = -\frac{Gm_B}{\sqrt{x^2 + b^2}}$$

$$t_{\rm arr} - t_{\rm em} \approx \frac{|x_{\rm arr} - x_{\rm em}|}{c} \boxed{+ \frac{2Gm_B}{c^3} \ln\left(\frac{1 + e\cos\varphi}{1 - \sin i \sin(\omega + \varphi)}\right)} + const.$$

[Blandford & Teukolsky 1976]

The Damour-Deruelle (DD) timing formula

$$\boxed{t_b - t_0 = F[T; \{p^{\rm K}\}; \{p^{\rm PK}\}; \{q^{\rm PK}\}]}$$

Keplerian parameters: $\{p^{\rm K}\} = \{P_b, T_0, e_0, \omega_0, x_0\}$

Separately measurable post-Keplerian parameters: $\{p^{\rm PK}\} = \{k, \gamma, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\}$

Not separately measurable post-Keplerian parameters: $\{q^{\rm PK}\} = \{\delta_r, A, B, D\}$

$$F(T) = D^{-1}[T + \Delta_R(T) + \Delta_E(T) + \Delta_S(T) + \Delta_A(T)]$$

Roemer delay: $\Delta_R = x \sin\omega[\cos u - e(1+\delta_r)] + x[1 - e^2(1+\delta_\theta)^2]^{1/2} \cos\omega \sin u$

Einstein delay: $\Delta_E = \gamma_{\rm E} \sin u$

Shapiro delay: $\Delta_S = -2r \ln\{1 - e\cos u - s[\sin\omega(\cos u - e) + (1-e^2)^{1/2}\cos\omega \sin u]\}$

Aberration delay: $\Delta_A = A\{\sin[\omega + A_e(u)] + e\sin\omega\} + B\{\cos[\omega + A_e(u)] + e\cos\omega\}$

$$\omega = \omega_0 + k\, A_e(u) \qquad x = x_0 + \dot{x}(T - T_0) \qquad e = e_0 + \dot{e}(T - T_0)$$

$$A_e(u) = 2\arctan\left[\left(\frac{1+e}{1-e}\right)^{1/2} \tan\frac{u}{2}\right]$$

$$u - e\sin u = 2\pi\left[\left(\frac{T-T_0}{P_b}\right) - \frac{1}{2}\dot{P}_b\left(\frac{T-T_0}{P_b}\right)^2\right]$$

$$\boxed{\begin{aligned} P_b^{\rm obs} &= D^{-1} P_b^{\rm intrinsic} \\ x^{\rm obs} &= D^{-1} x^{\rm intrinsic} \\ e^{\rm obs} &= e^{\rm intrinsic}\ , \ \text{etc.} \end{aligned}}$$

[Damour & Deruelle 1986, Damour & Taylor 1992]

DD timing model and GR

$$\dot\omega = nk = \frac{3n}{1-e^2}\left(\frac{GMn}{c^3}\right)^{2/3}$$

$$\gamma_{\rm E} = \frac{e}{n}\left(\frac{GMn}{c^3}\right)^{2/3} X_B(X_B+1)$$

$$r = \frac{Gm_B}{c^3}$$

$$s = xn\left(\frac{GMn}{c^3}\right)^{-1/3} X_B^{-1}$$

$$\dot P_b = -\frac{192\pi}{5}\,\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}}\left(\frac{GMn}{c^3}\right)^{5/3} X_A X_B$$

$$\delta_\theta = \left(\frac{GMn}{c^3}\right)^{2/3}\left(\tfrac{7}{2}X_A^2+6X_AX_B+2X_B^2\right)$$

$$\Omega_B = \frac{n}{2(1-e^2)}\left(\frac{GMn}{c^3}\right)^{2/3} X_A(3+X_B)$$

DDGR model

$P_b,\ e,\ x, T_0,\ \omega$ Keplerian

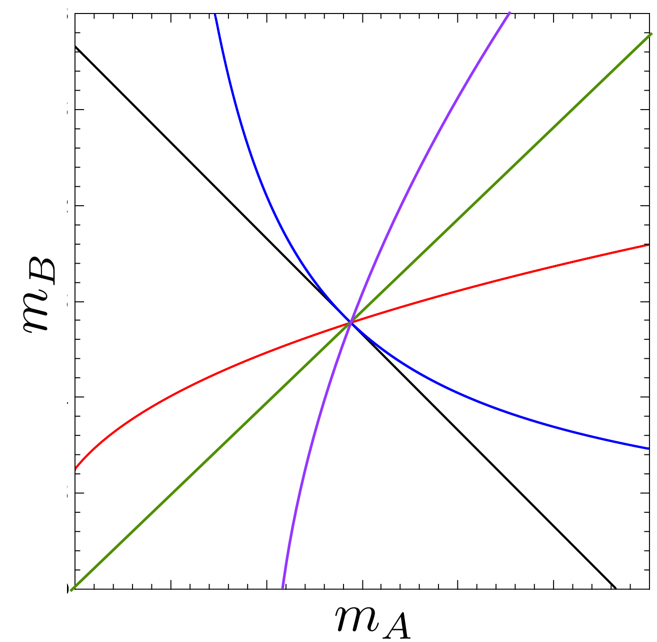
m_A, m_B (inertial) masses

DD model

$P_b,\ e,\ x, T_0,\ \omega$ Keplerian

$\dot\omega,\ \gamma,\ r,\ s,\ \dot P_{\rm b},\ \delta_\theta$ post-Keplerian

$$\boxed{p_i^{\rm PK} = f_i(p^{\rm K}; m_A, m_B)}$$

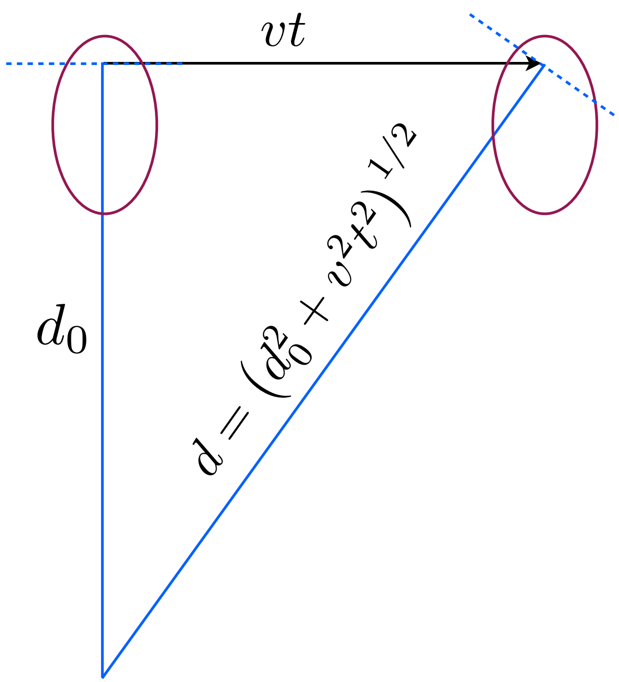


$$n = 2\pi/P_b,\ M = m_A+m_B,\ X_A = m_A/M,\ X_B = m_B/M = 1-X_A$$

External contributions

Proper motion effects

[Shklovskii 1970, Arzoumanian et al. 1996, Kopeikin 1996]



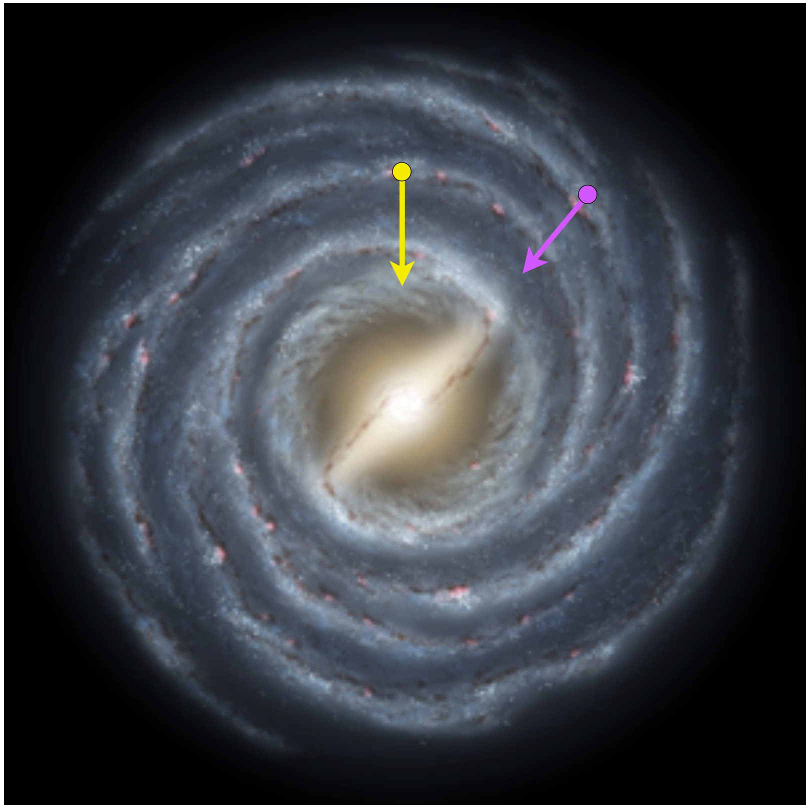
$$\frac{\dot{P}_b}{P_b} = \frac{v^2}{cd_0} = \frac{\mu^2 d_0}{c} \quad > 0$$

$$\dot{\omega} = \csc i(\mu_\alpha \cos\Omega_{\rm asc} + \mu_\delta \sin\Omega_{\rm asc})$$

$$\dot{x} = x \cot i(-\mu_\alpha \sin\Omega_{\rm asc} + \mu_\delta \cos\Omega_{\rm asc})$$

Galactic differential acceleration

[Damour & Taylor 1991]



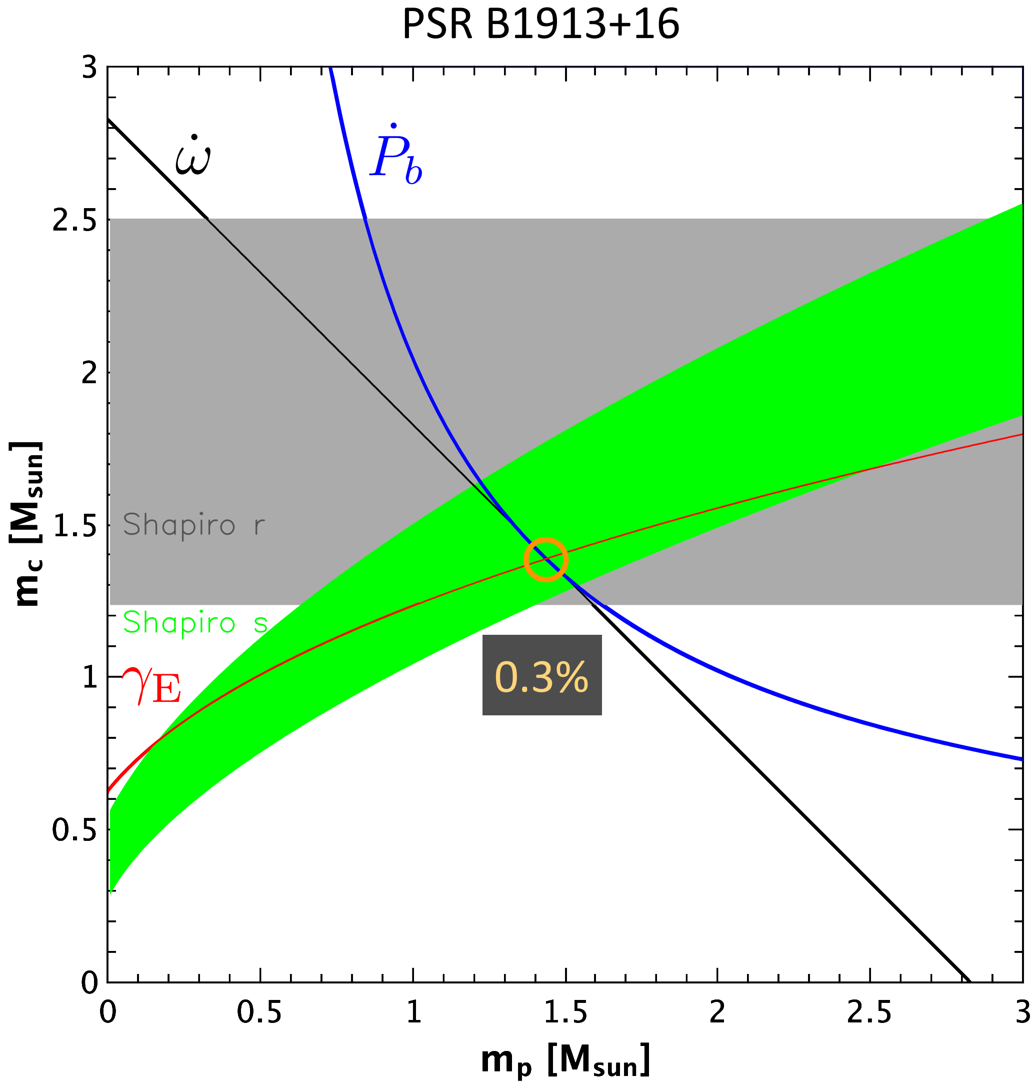
$$\ddot{d} = \vec{K}_0 \cdot (\vec{g}_{\rm PSR} - \vec{g}_{\rm SSB})$$

$$\frac{\dot{P}_b}{P_b} = \frac{\ddot{d}}{c}$$

Applications

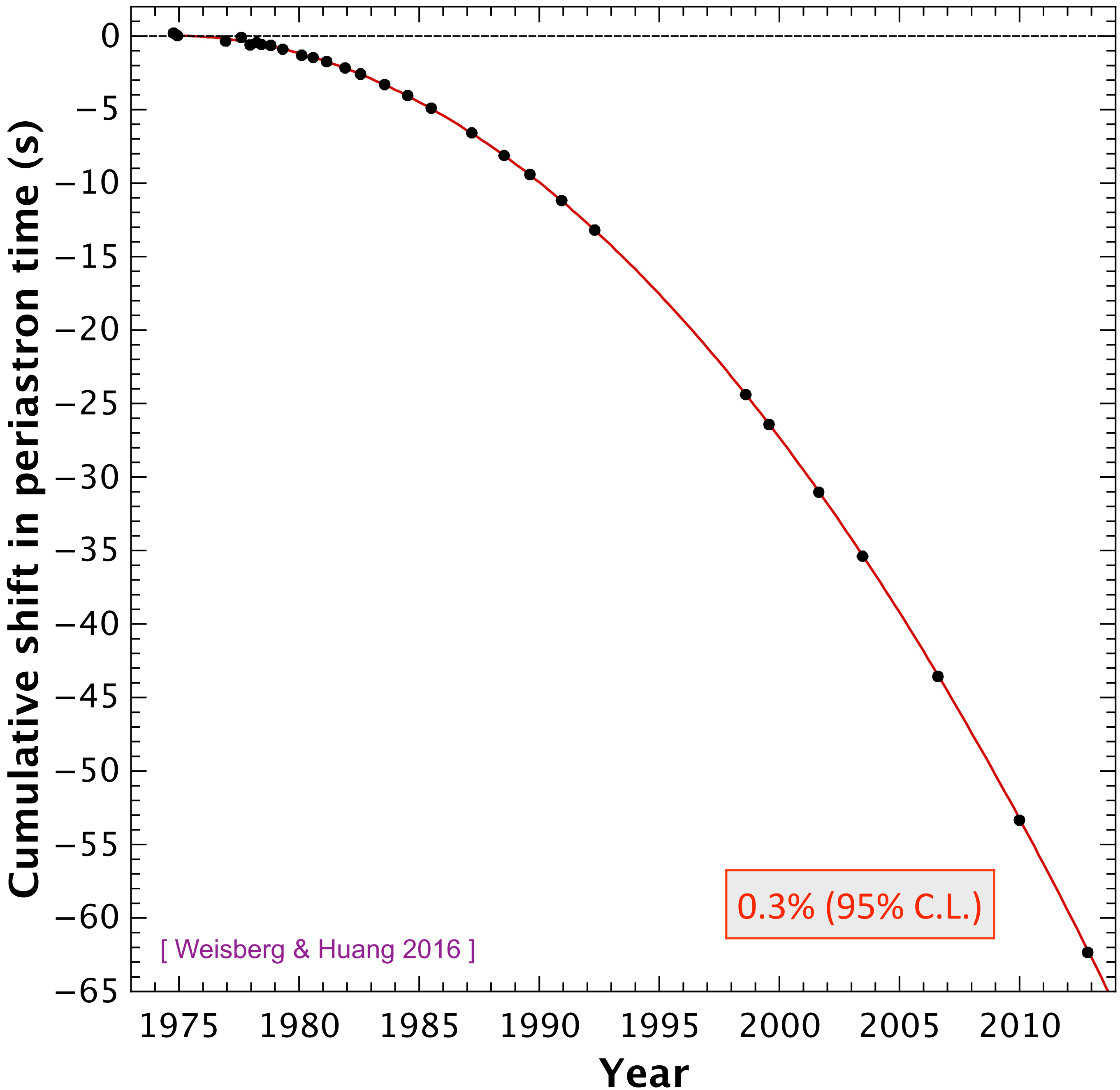
The mass-mass diagram for the Hulse-Taylor pulsar

Parameter	Value[a]
T_0 (MJD)	52144.90097849(3)
$x \equiv a_1 \sin i$ (s)...	2.341776(2)
e	0.6171340(4)
P_b (d)..........	0.322997448918(3)
ω_0 (deg)	292.54450(8)
$\langle\dot{\omega}\rangle$ (deg / yr) ...	4.226585(4)
γ (ms)	0.004307(4)
$\dot{P}_b^{\rm obs}$	$-2.423(1) \times 10^{-12}$
$\delta_\theta^{\rm obs}$	$4.0(25) \times 10^{-6}$
$\dot{x}^{obs}$	$-0.014(9) \times 10^{-12}$
$\dot{e}^{obs}$ (s^{-1}).......	$0.0006(7) \times 10^{-12}$
Shapiro Gravitational Propagation Delay Parameters	
Damour & Deruelle (1986) Parametrization	
s	$0.68^{+0.10}_{-0.06}$
r (μs)	$9.6^{+2.7}_{-3.5}$
Freire & Wex (2010) Parametrization	
ς	0.38(4)
h_3	$0.6(1) \times 10^{-6}$

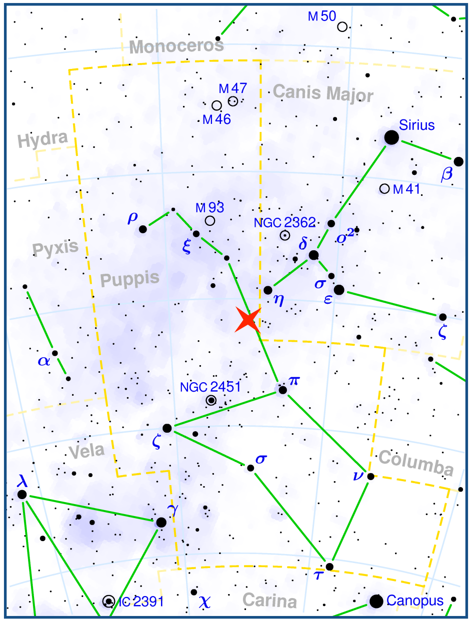


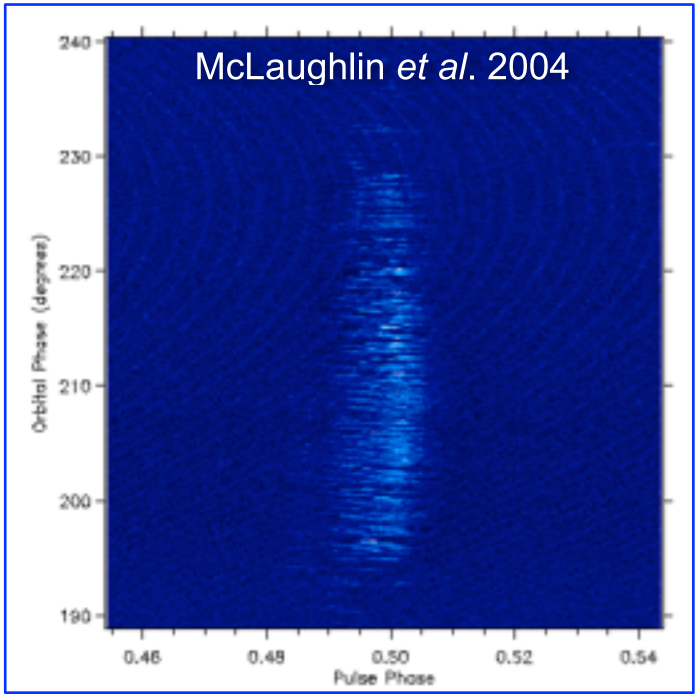
[Weisberg & Huang 2016]

Gravitational wave damping in the Hulse-Taylor pulsar

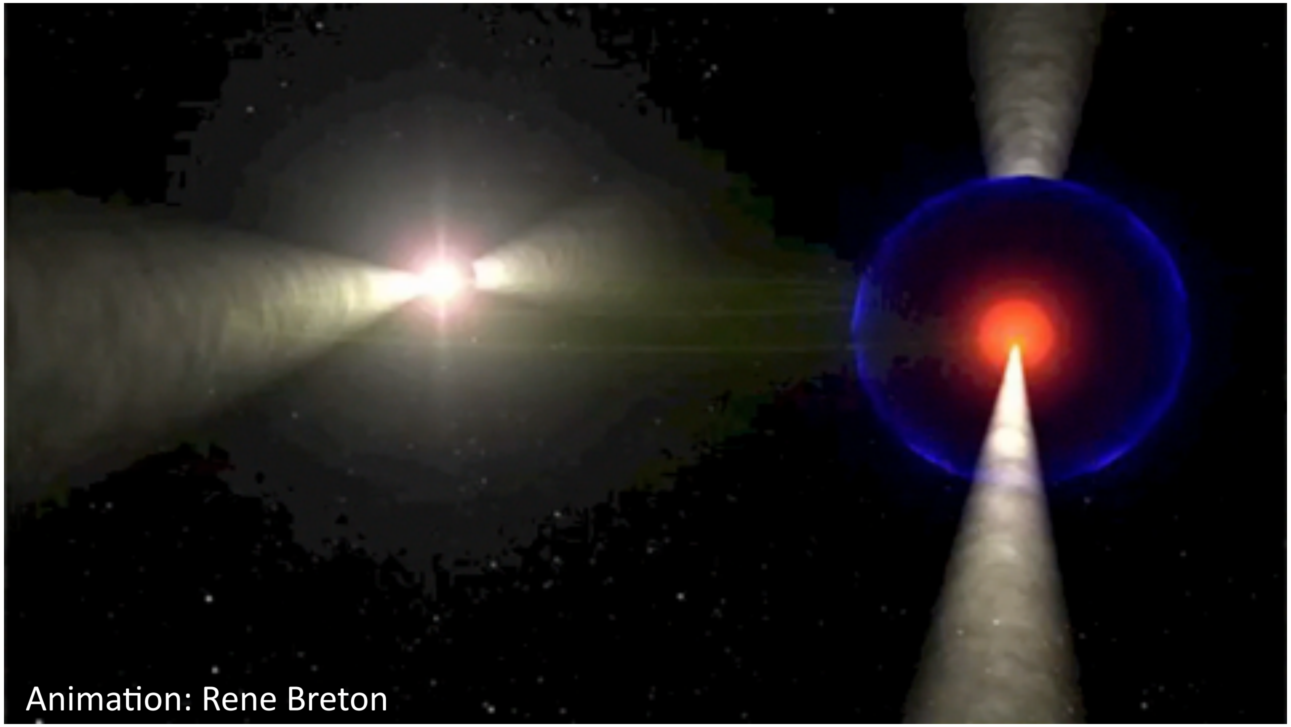


The Double Pulsar PSR J0737-3039A/B





Spin periods:	23 ms / 2.8 s
Orbital period:	2.45 h
Eccentricity:	0.088



[Burgay *et al.* 2003, Lyne *et al.* 2004]

Relativistic effects in the Double Pulsar

- **Binary parameters from timing**

Timing parameter	PSR J0737-3039A	PSR J0737-3039B
Orbital period P_b (day)	0.10225156248(5)	—
Eccentricity e	0.0877775(9)	—
Projected semimajor axis $x = (a/c)\sin i$ (s)	1.415032(1)	1.5161(16)
Longitude of periastron ω (°)	87.0331(8)	87.0331 + 180.0
Epoch of periastron T_0 (MJD)	53,155.9074280(2)	—
Advance of periastron $\dot{\omega}$ (°/year)	16.89947(68)	[16.96(5)]
Gravitational redshift parameter γ_E (ms)	0.3856(26)	—
Shapiro delay parameter s	0.99974(−39,+16)	—
Shapiro delay parameter r (μs)	6.21(33)	—
Orbital period derivative $\dot{P}_b$	$-1.252(17) \times 10^{-12}$	—

[Kramer *et al.* 2006]

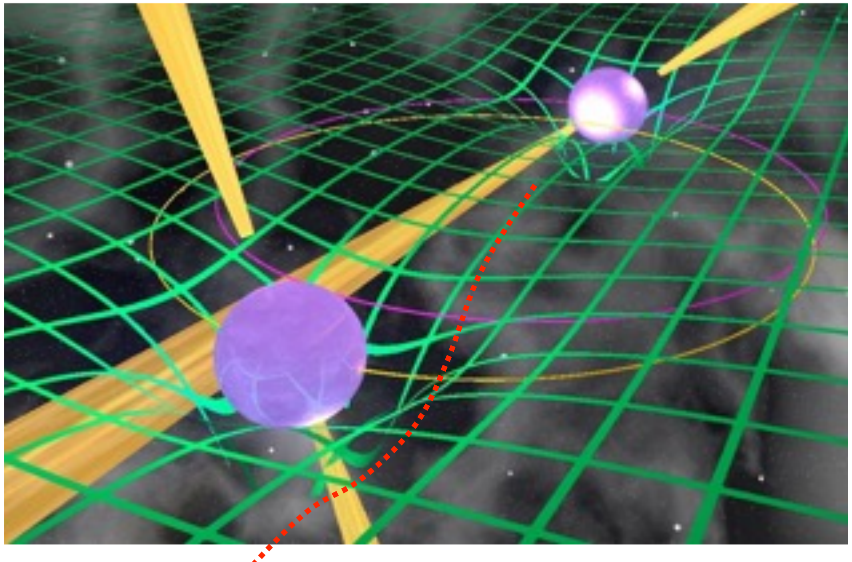
➥ mass ratio + 5 post-Keplerian parameters

- **Spin precession of B (from eclipses)**

$$\Omega_B = 4.77^{+0.66}_{-0.65} \, ^\circ/\text{year}$$

[Breton *et al.* 2008]

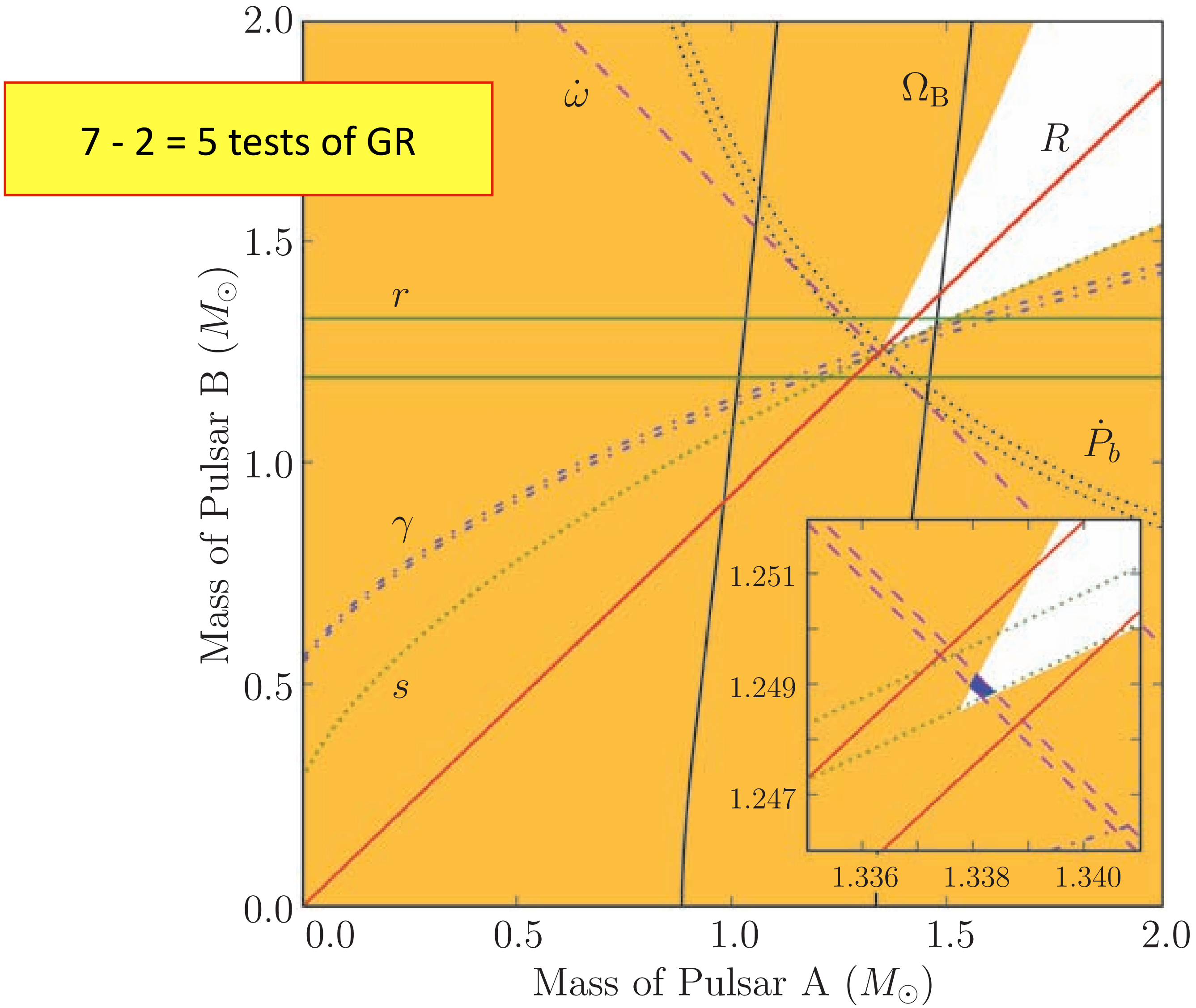
➥ 6th post-Keplerian parameter



Observing the Double Pulsar

~ 1.3×10^6 TOAs from five different radio telescopes

The GR mass-mass diagram of the Double Pulsar

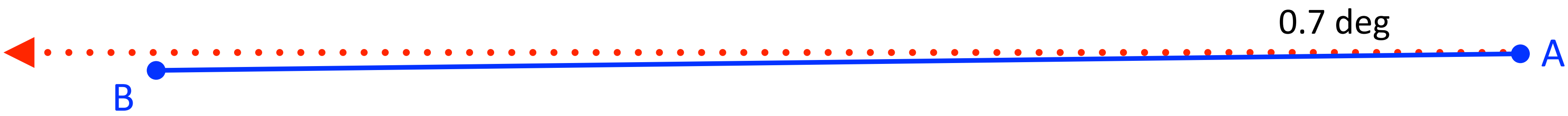


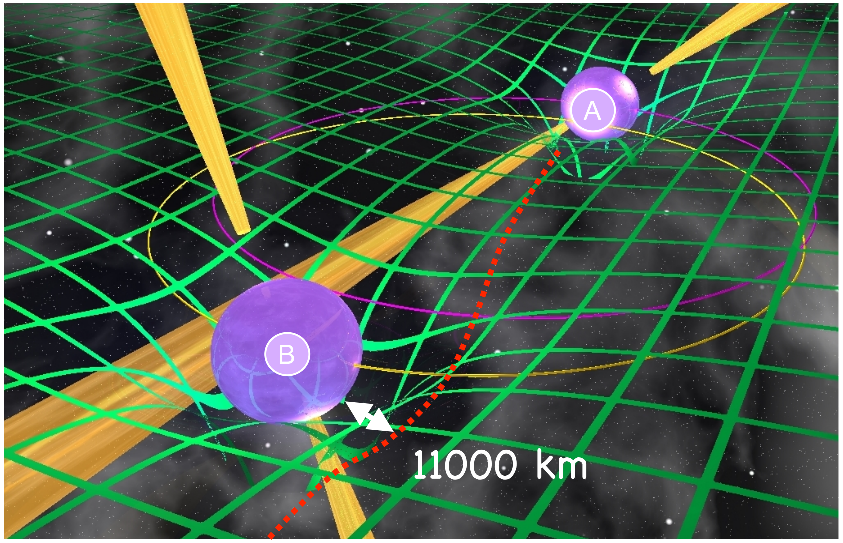
Kramer et al. 2006, Breton et al. 2008

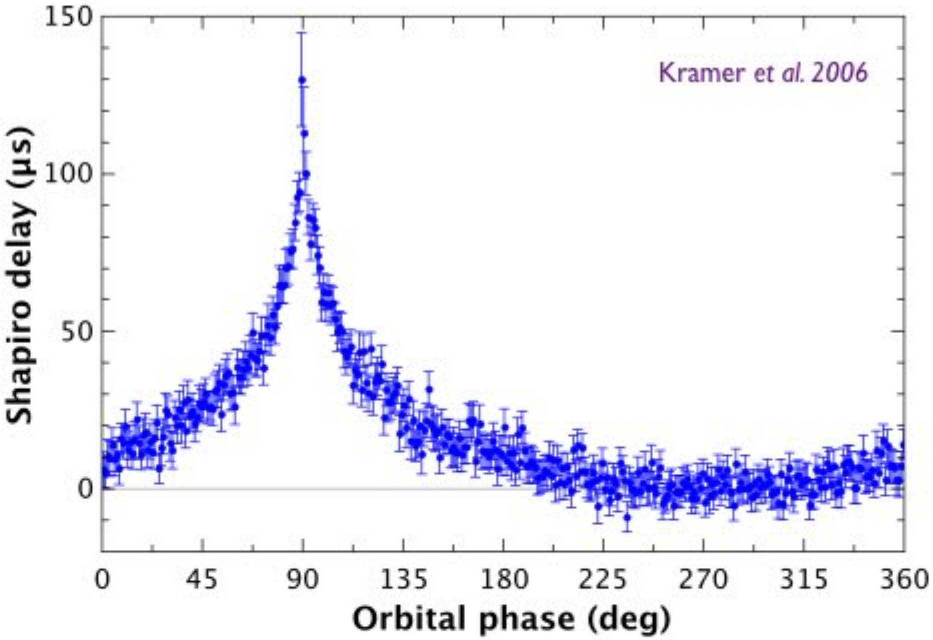
➡ New version by Kramer et al. with greatly improved precision should become available soon.

➡ GW damping in the Double Pulsar by now tested with a precision of significantly better than 0.1%.

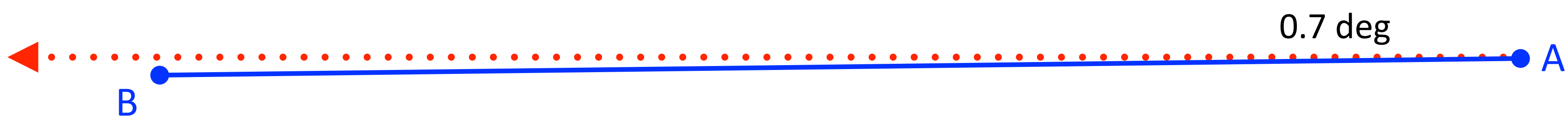
The Shapiro delay in the Double Pulsar

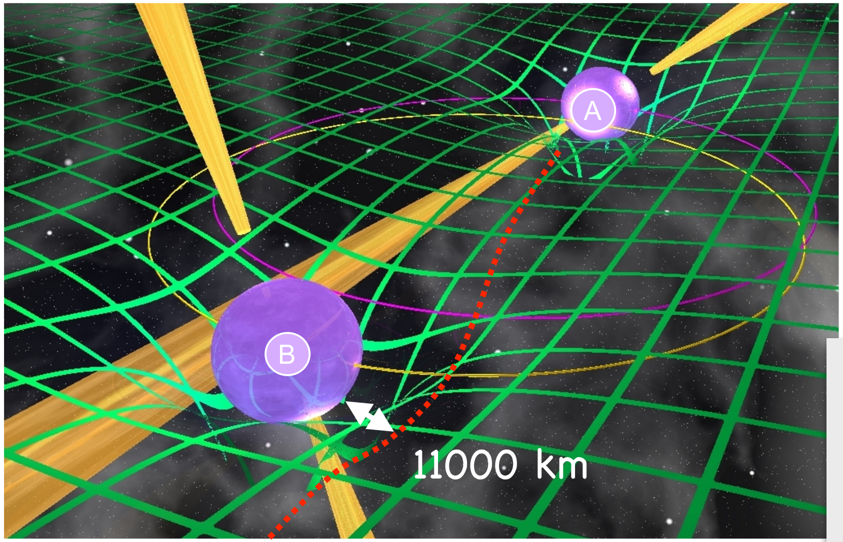


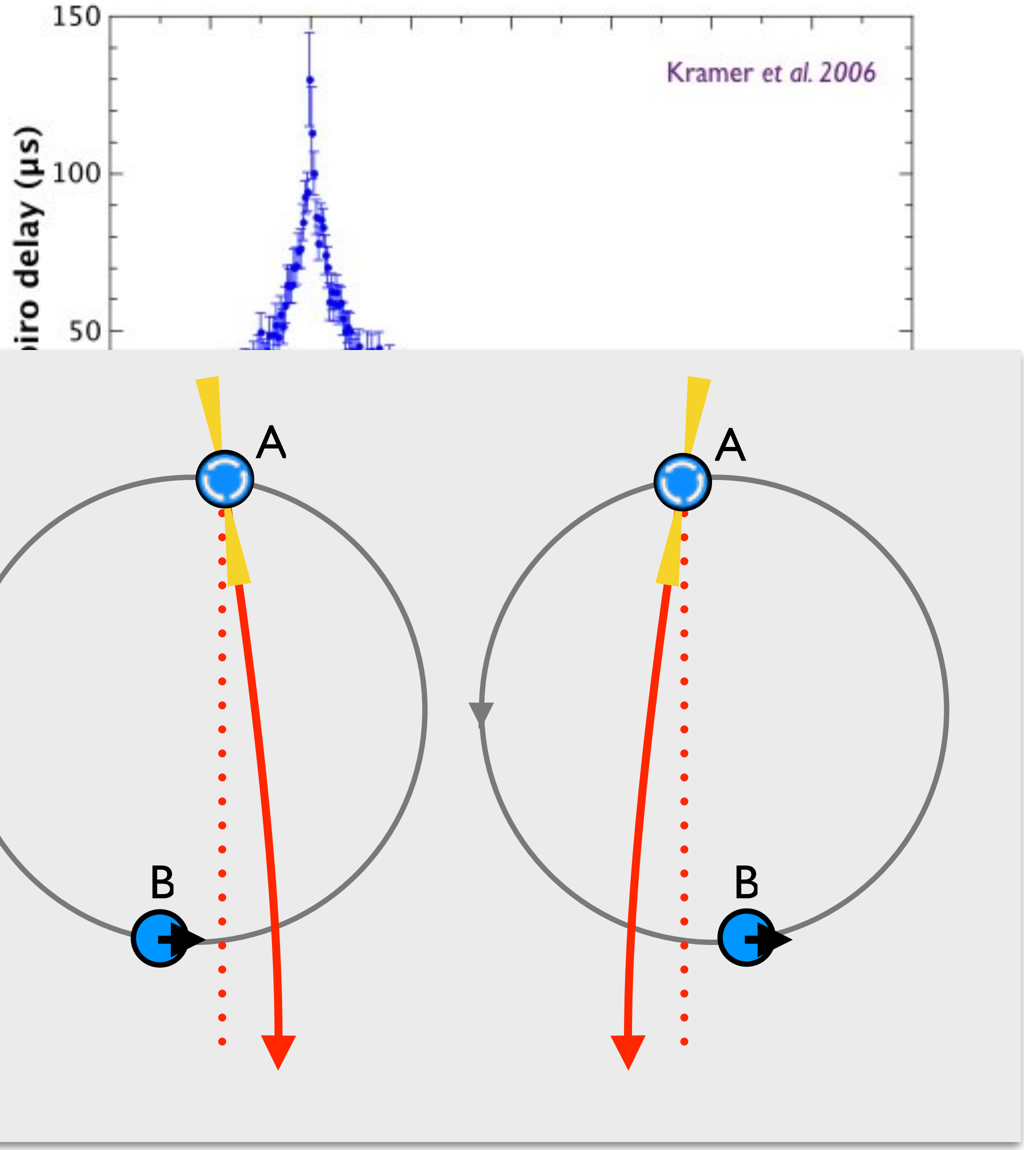




The Shapiro delay in the Double Pulsar



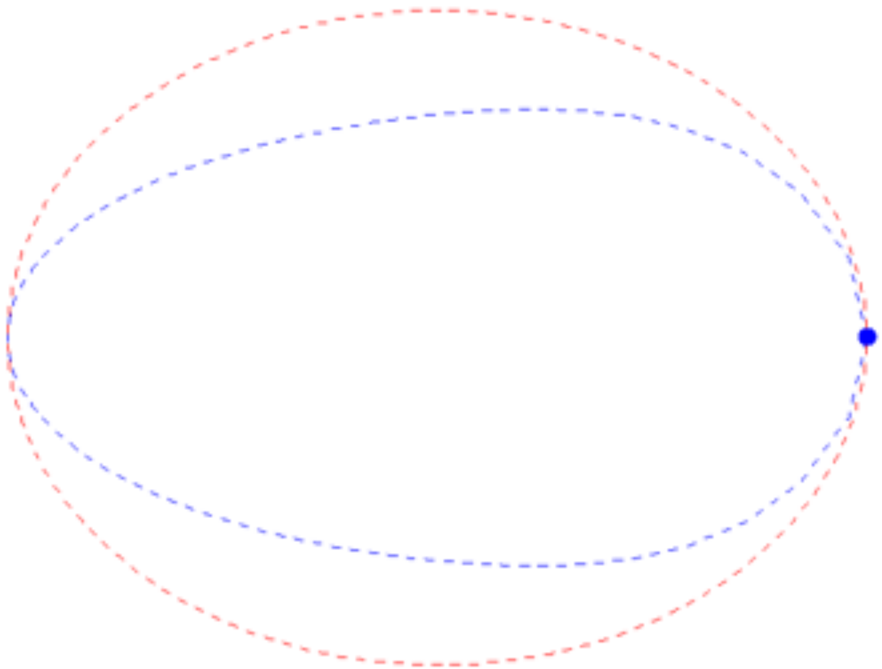




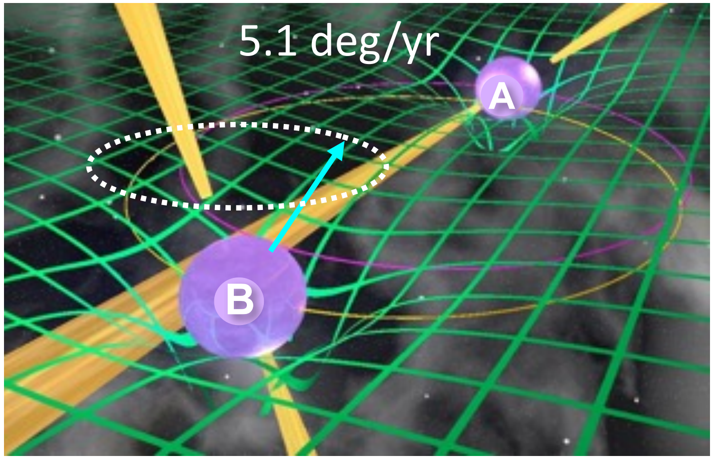
Relativistic deformation of the Double Pulsar orbit

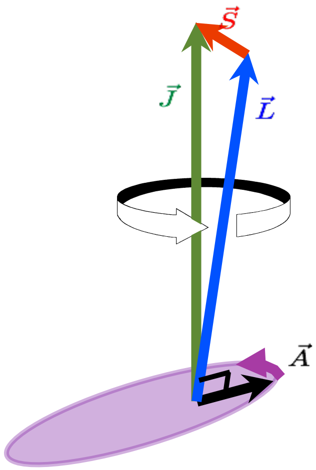
$$\begin{aligned}
\frac{2\pi}{P_b}(t - t_0) &= U - e\sin U \\
r &= a(1 - e_r\cos U) \\
\theta - \theta_0 &= 2(1+k)\arctan\left[\left(\frac{1+e_\theta}{1-e_\theta}\right)^{1/2}\tan\frac{U}{2}\right]
\end{aligned}$$

$$\begin{aligned}
e_\theta &= e(1+\delta_\theta) \\
e_r &= e(1+\delta_r)
\end{aligned}$$

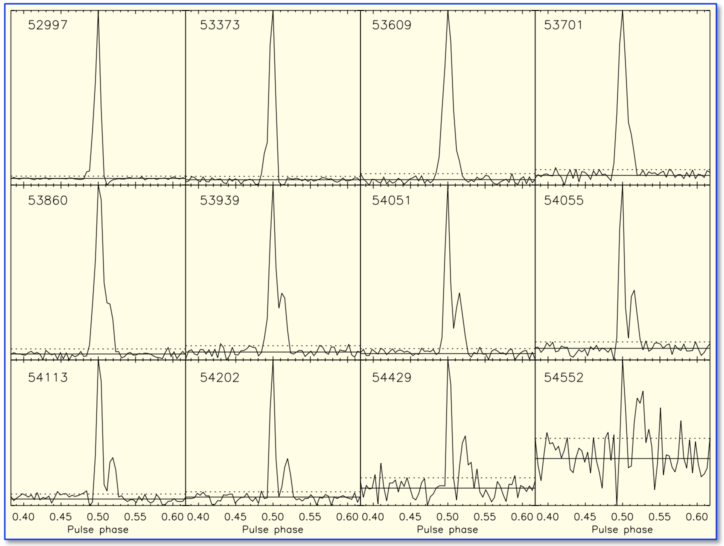


Relativistic spin-orbit coupling in the Double Pulsar

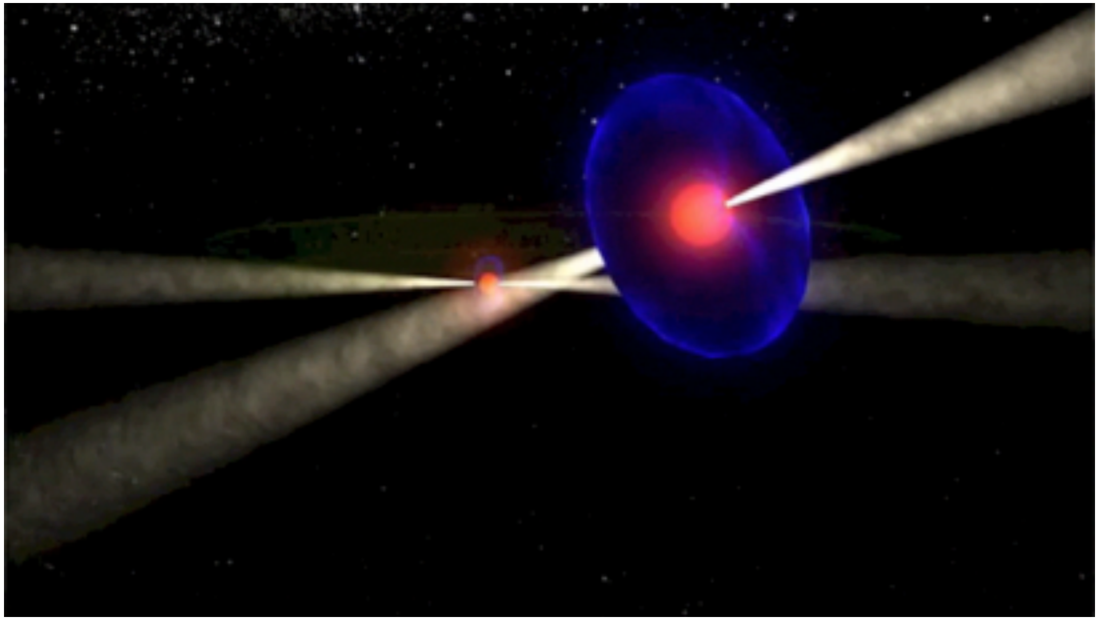




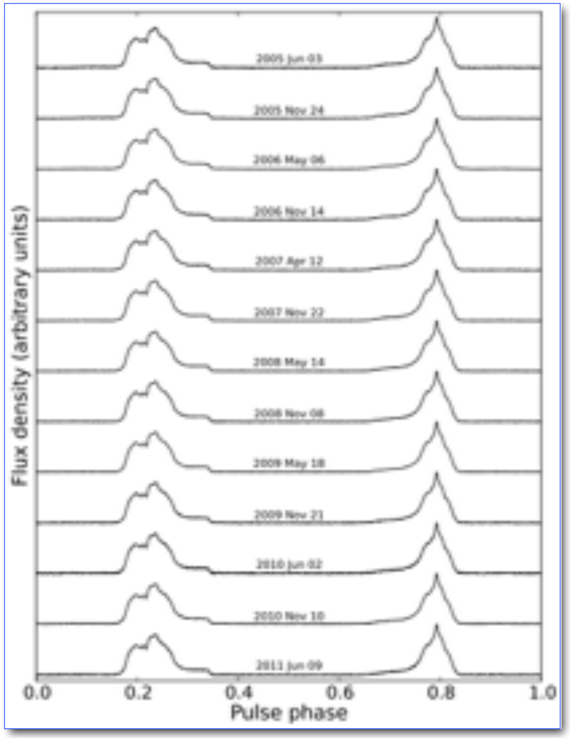
B



[Perera *et al.* 2010]

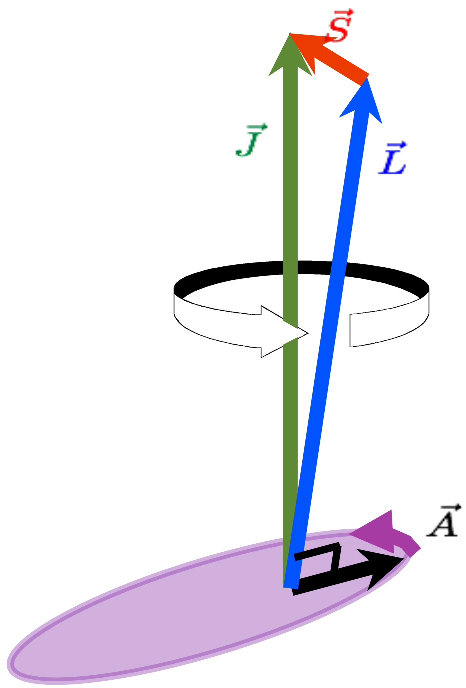


A



[Ferdman *et al.* 2013]

Lense-Thirring effect / moment of inertia of pulsar A



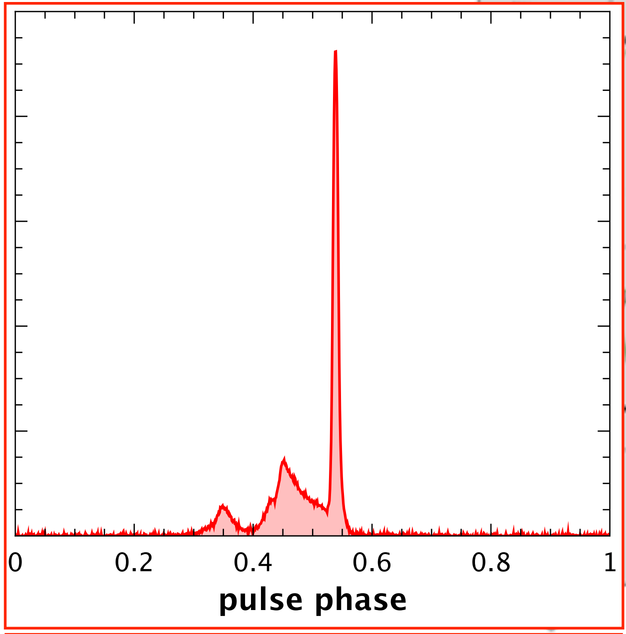
$$\boxed{\dot\omega = \dot\omega_{1\mathrm{pN}} + \dot\omega_{2\mathrm{pN}} + \dot\omega_{\mathrm{SO}}}$$

PSR J0737-3039

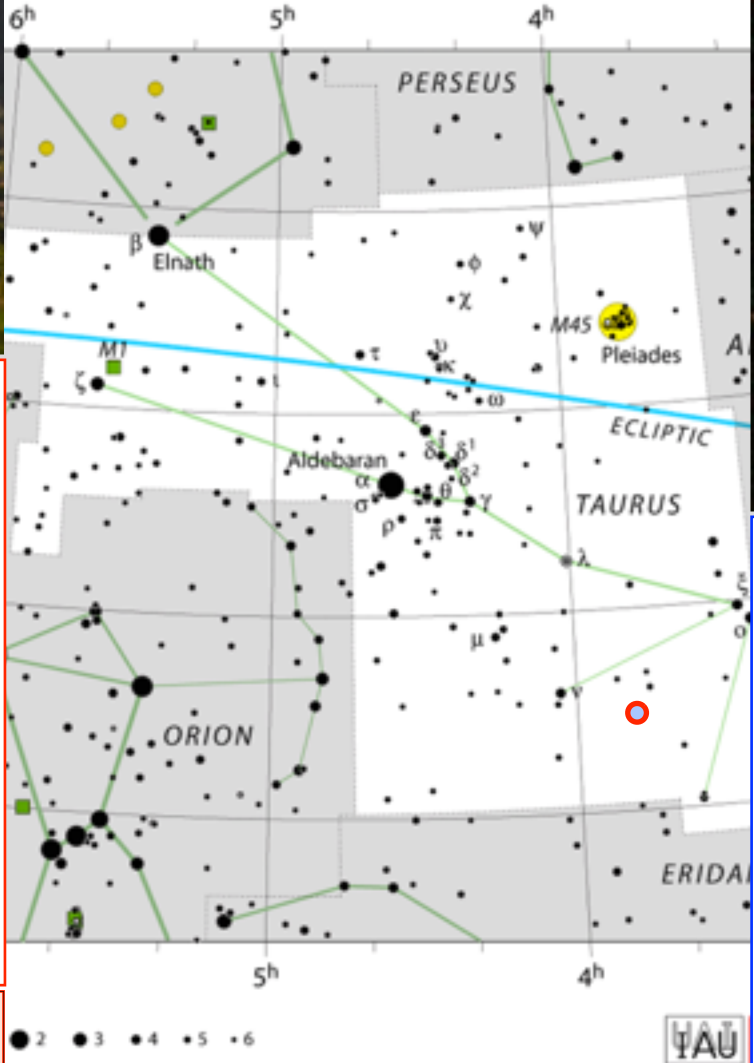
$\dot\omega_{1\mathrm{pN}}$	$=$	$16.89\ldots$	deg/yr
$\dot\omega_{2\mathrm{pN}}$	$=$	0.00044	deg/yr
$\dot\omega_{\mathrm{SO}}$	$=$	$-0.00038\, I_A/(10^{45}\,\mathrm{g\,cm^2})$	deg/yr
$\delta\dot\omega_{\mathrm{obs}}$	$=$	0.00002	deg/yr

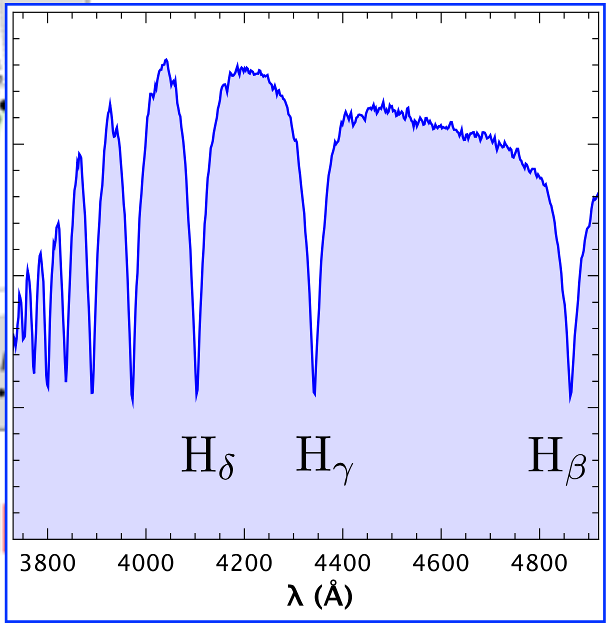
[Kramer *et al.,* in prep.; Kehl et al., in prep.]

PSR J0348+0432



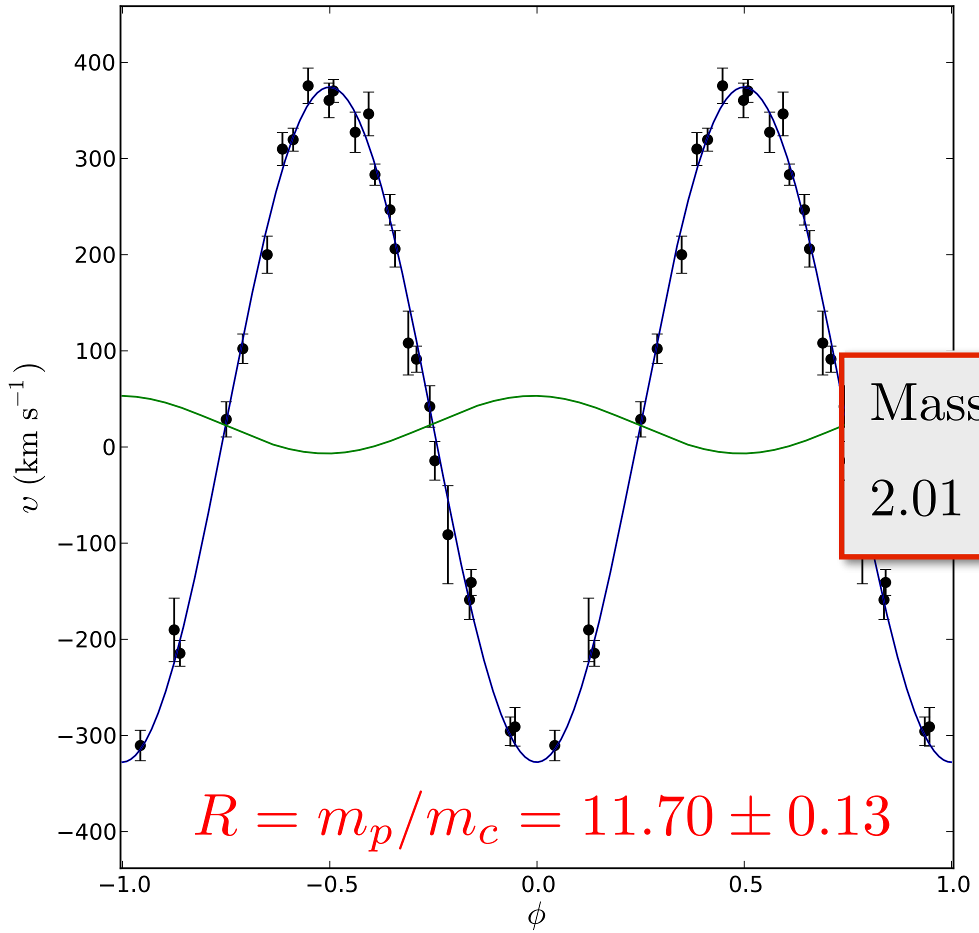
$$P = 39.1226569017806(5)\,\mathrm{ms}$$
$$P_\mathrm{b} = 2.45817750533(2)\,\mathrm{h}$$
$$e \lesssim 10^{-6}$$





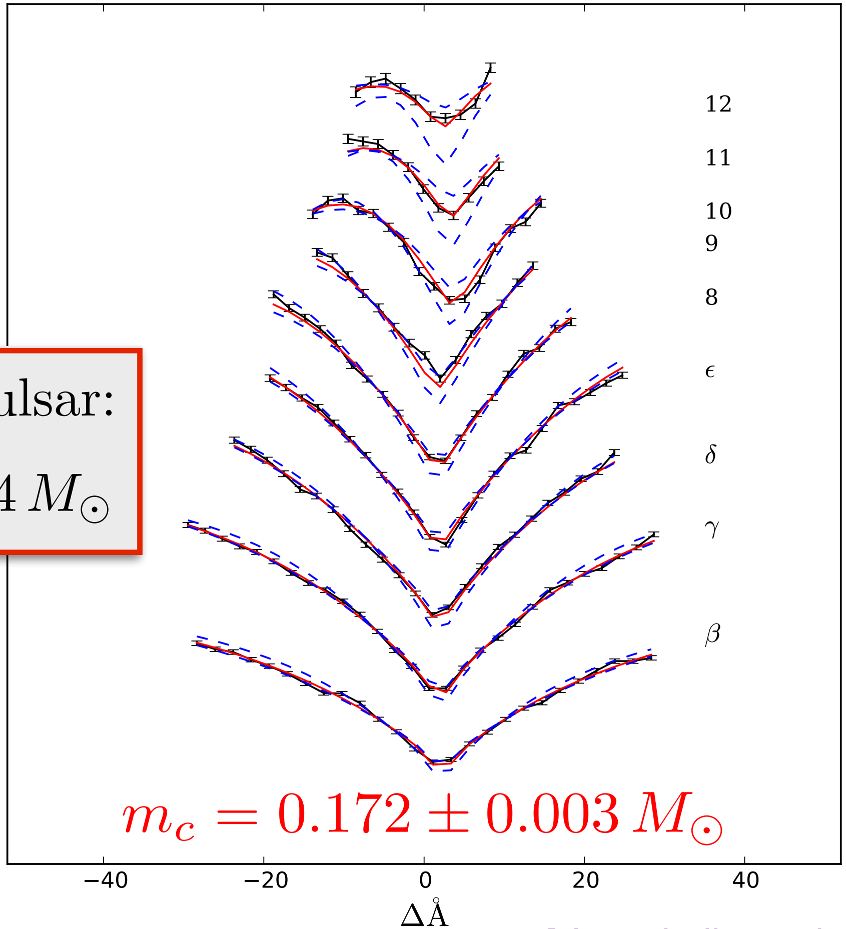
[Boyles *et al.* 2013, Lynch *et al.* 2013, Antoniadis et al. 2013]

High-resolution optical spectroscopy of the PSR J0348+0432 companion



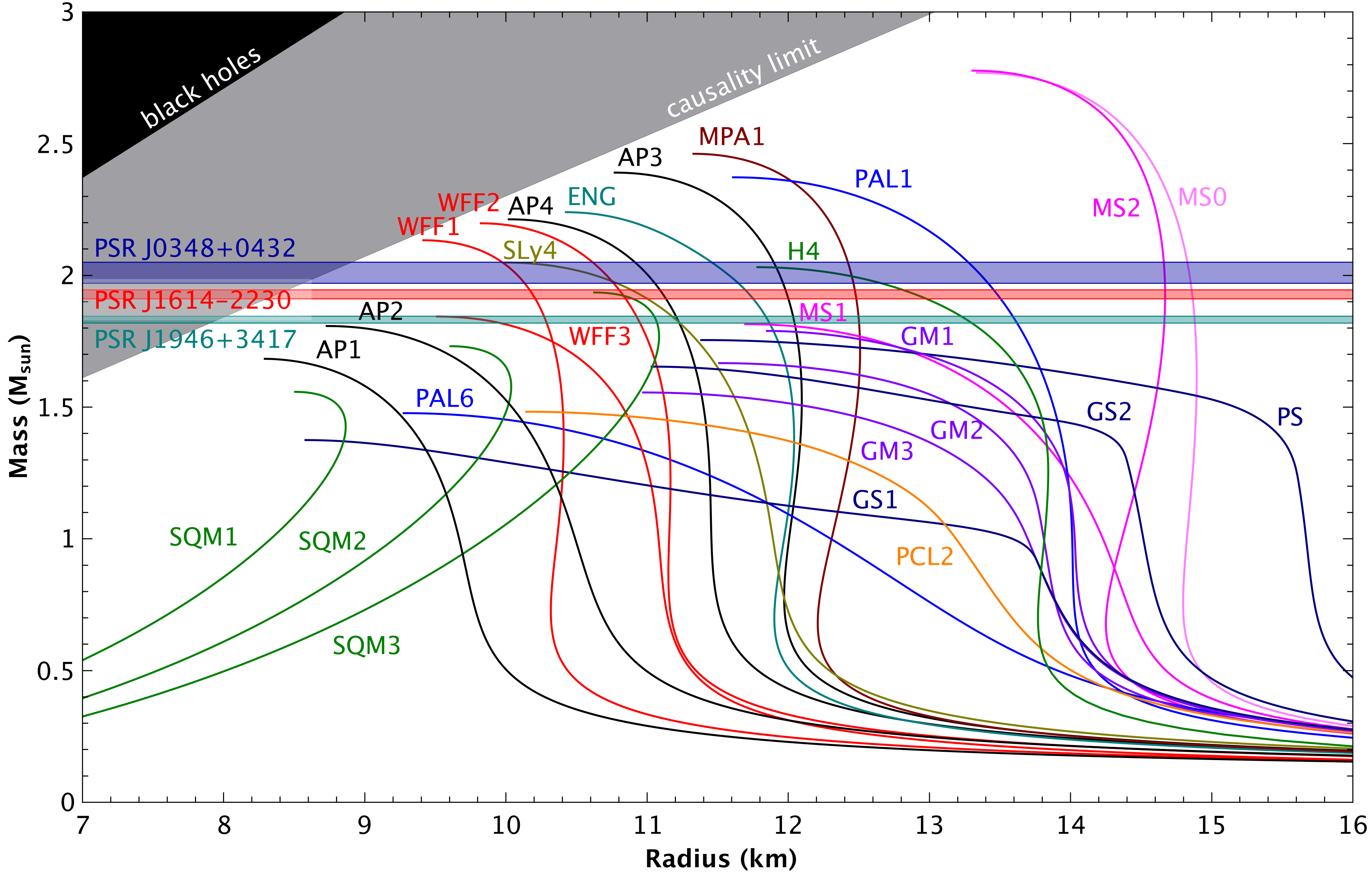
Mass of pulsar:

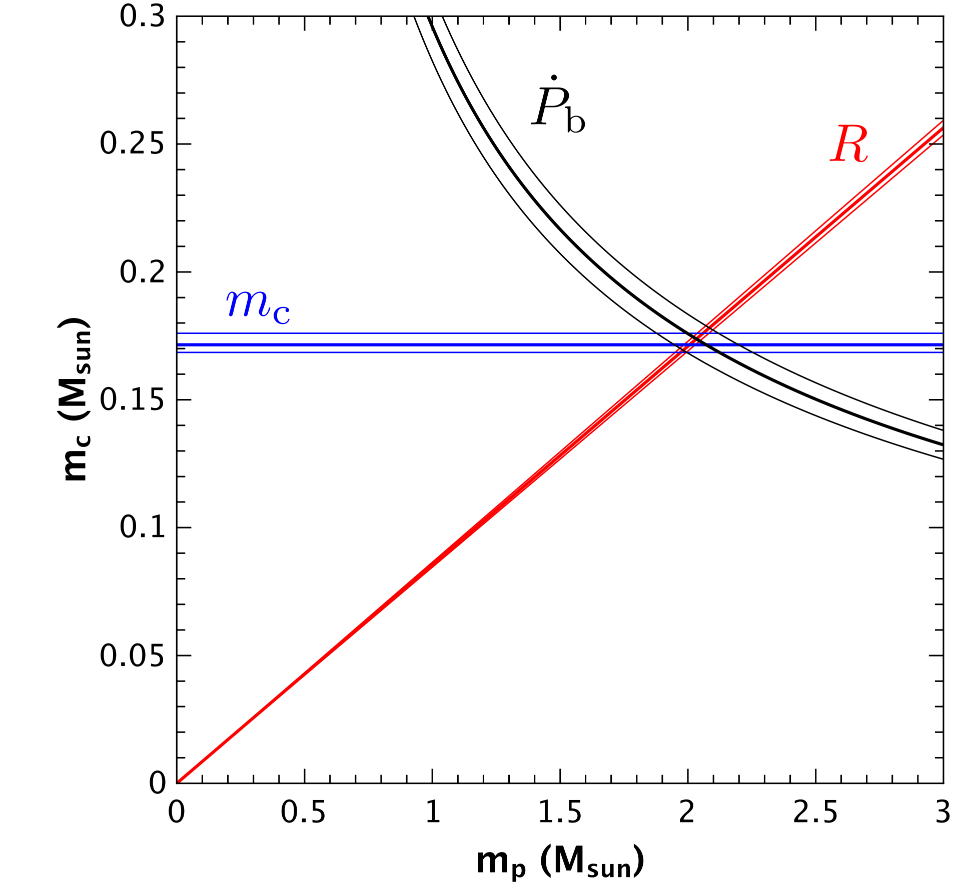
$2.01 \pm 0.04\, M_\odot$

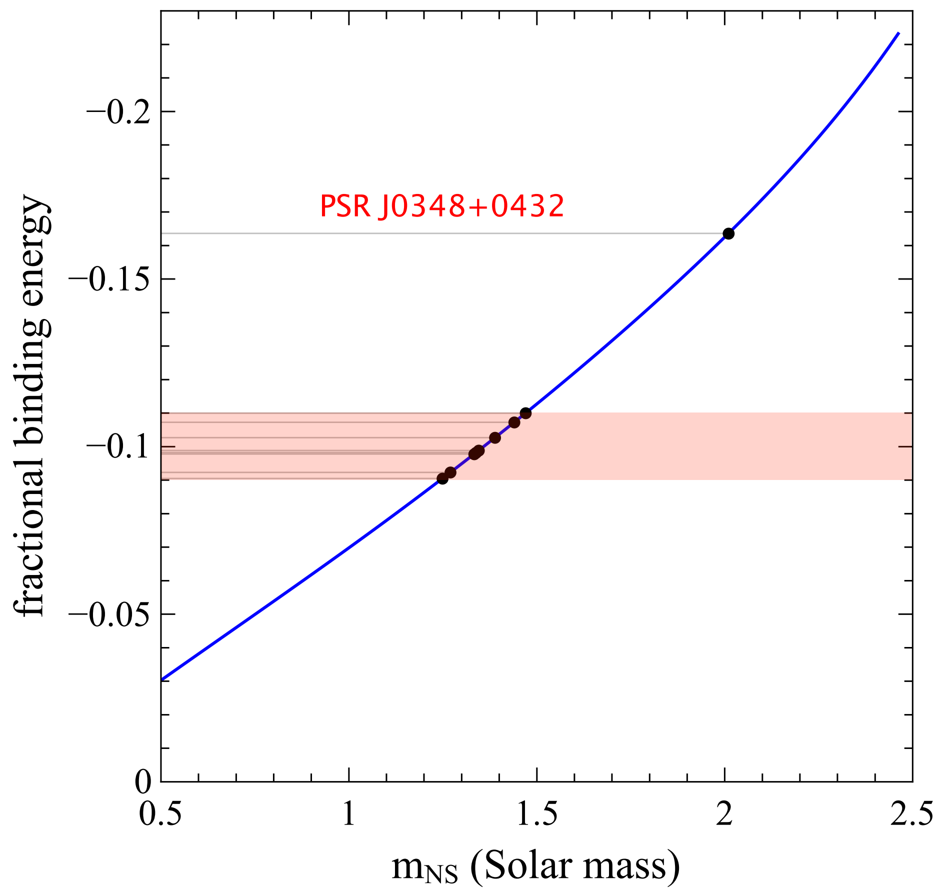


[Antoniadis *et al.* 2013]

Constraining equations of state at supranuclear densities



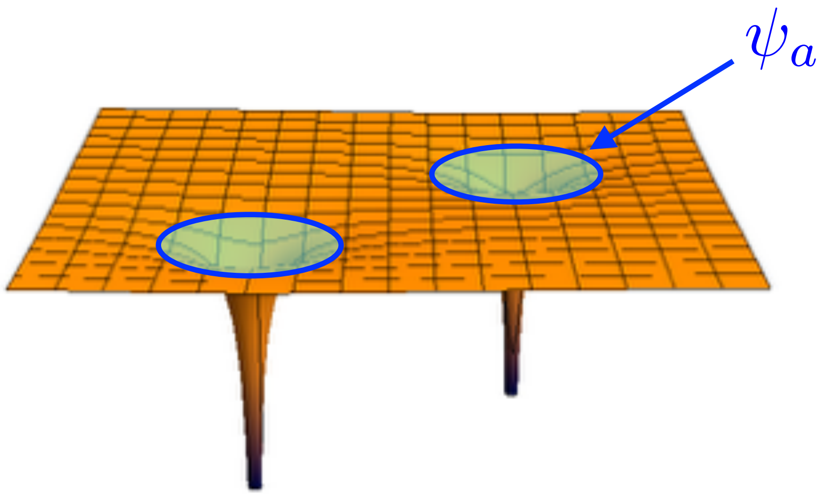




[Antoniadis *et al.* 2013]

Generic approach to PPK parameters in alternative gravity theories

Binary pulsars in alternative gravity



Additional fields influence the structure of each body, and in turn affect its motion
—> violation of the strong equivalence principle (SEP).

Properties of a body depend on the values of the additional fields in the matching region ψ_a.
Hence, one can write mass, moment of inertia, etc. as a function of the external auxiliary field(s) evaluated at the location of the body

$$m_A(\psi_a[\mathbf{x}_A(\tau_A)])$$

$$I_A(\psi_a[\mathbf{x}_A(\tau_A)])$$

Binary pulsars in alternative gravity - orbital motion and Shapiro delay

Restriction to boost-invariant gravity theories without Whitehead term in the post-Newtonian limit

1pN orbital dynamics (N-body system) - modified EIH formalism

$$
\begin{aligned}
L_{\rm O} \;=\;& -\sum_A m_A c^2 \left[1 - \frac{\mathbf{v}_A^2}{2c^2} - \frac{\mathbf{v}_A^4}{8c^4}\right] \\
&+ \sum_A \sum_{B\neq A} \frac{\mathcal{G}_{AB} m_A m_B}{2r_{AB}} \left[1 - \frac{\mathbf{v}_A\cdot\mathbf{v}_B}{2c^2} - \frac{(\mathbf{n}_{AB}\cdot\mathbf{v}_A)(\mathbf{n}_{AB}\cdot\mathbf{v}_B)}{2c^2} + (3+2\bar\gamma_{AB})\frac{(\mathbf{v}_A-\mathbf{v}_B)^2}{2c^2}\right] \\
&- \sum_A \sum_{B\neq A} \sum_{C\neq A} (1+2\bar\beta^A_{BC}) \frac{\mathcal{G}_{AB}\mathcal{G}_{AC} m_A m_B m_C}{2c^2 r_{AB} r_{AC}}
\end{aligned}
$$

$$
L_{\rm SO} = \frac{1}{c^2}\sum_A S_A^{ij}\left[\frac{1}{2} v_A^i a_A^j + \sum_{B\neq A} \frac{\Gamma_A^B m_B}{r_{AB}^2}(v_A^i - v_B^i) n_{AB}^j\right]
$$

$$
\boxed{m_A \equiv m_A^{(0)}}
$$

Strong-field modification of G: G_{AB}

Strong-field PPN parameters

$$
\left.
\begin{aligned}
\bar\gamma_{AB} &\equiv \gamma_{AB} - 1 \\
\bar\beta^A_{BC} &\equiv \beta^A_{BC} - 1
\end{aligned}
\right\} \text{0 in GR}
$$

Spin-orbit strong-field parameter Γ_A^B

PPN limit suggests: $\Gamma_A^B = (2+\bar\gamma_{AB})G_{AB}$

[Will 1993, Damour & Taylor 1992]

Post-Keplerian parameters for a two-body system

$$
\dot\omega = \left(3 + 2\bar\gamma_{AB} - \frac{m_A}{M}\bar\beta^B_{AA} - \frac{m_B}{M}\bar\beta^A_{BB}\right)\frac{\beta_b^2}{1-e^2}
$$

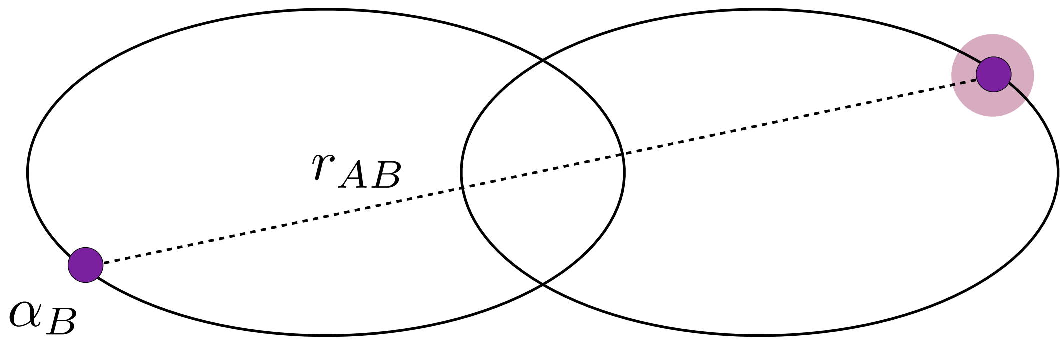
$$
r = \frac{G_{0B} m_B}{c^3}\left(1 + \frac{1}{2}\bar\gamma_{0B}\right) \qquad s = \frac{xn}{\beta_b}\frac{M}{m_B}
$$

$$
\beta_b \equiv \left(\frac{G_{AB} M n}{c^3}\right)^{1/3}
$$

Binary pulsars in alternative gravity - pulsar rotation

In alternatives to GR, the local gravitational constant at the location of the pulsar my depend on the gravitational potential of the companion star

$$G_A = G\left(1 - \eta_B^* \frac{Gm_B}{c^2 r_{AB}}\right)$$



$$\psi_A(t) = \psi_0 + \frac{q_B^{(\psi)}}{r_{AB}(t)}$$

If the companion is a weakly self-gravitating body: $\eta_B^* = \eta_\mathrm{N} = 4\beta - \gamma - 3$

$$S_A = \Omega_A I_A = \text{adiabatic invariant}$$

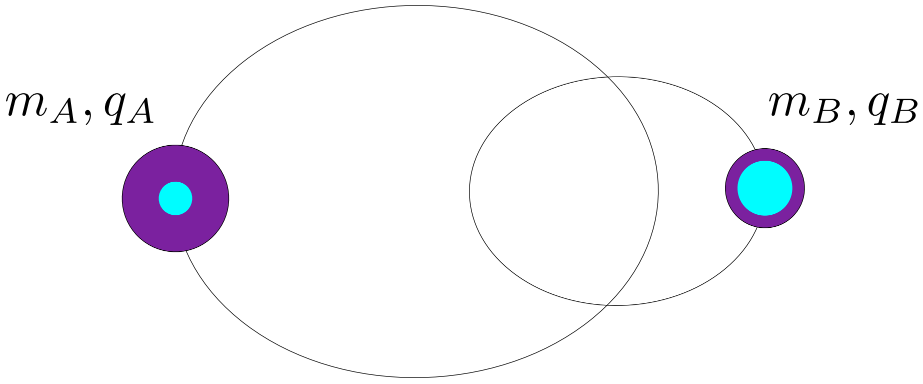
$$\Longrightarrow \frac{\Delta\nu_A}{\nu_A} = -\frac{\Delta I_A}{I_A} = \kappa_A \frac{\Delta G_A}{G} = -\kappa_A \eta_B^* \frac{Gm_B}{c^2 r_{AB}}$$

$\longrightarrow$ modification of γ_E: $\boxed{\gamma_\mathrm{E} = \frac{e}{n}\left(\frac{G_{AB}Mn}{c^3}\right)^{2/3} \frac{m_B}{M}\left(\frac{G_{0B}}{G_{AB}} + \frac{m_B}{M} + \kappa_A \eta_B^*\right)}$

[Will 1993]

Binary pulsars in alternative gravity - gravitational wave damping

In alternative gravity one generally expects radiation of **all multipoles**



$$\dot{E}_q \propto \frac{\dot{\Psi}^2}{c} + \frac{\ddot{\Psi}_i \ddot{\Psi}_i}{3c^3} + \frac{\dddot{\Psi}_{ij} \dddot{\Psi}_{ij}}{30c^5} + \mathcal{O}(c^{-7})$$

monopole radiation $$\Psi = \sum_A \left[\frac{dq_A}{dt} + \frac{q_A}{6c^2} \frac{d^3}{dt^3}(r_A^2) \right] + \mathcal{O}(c^{-3}) \qquad \frac{dq_A}{dt} = \mathcal{O}(c^{-2})$$

dipole radiation $$\Psi_i = \sum_A \left[\frac{d^2}{dt^2}(q_A r_A^i) + \frac{q_A}{10c^2} \frac{d^4}{dt^4}(r_A^2 r_A^i) \right] + \mathcal{O}(c^{-3})$$

quadrupole radiation $$\Psi_{ij} = \sum_A q_A \frac{d^3}{dt^3} \left(r_A^i r_A^j - \frac{1}{3} r_A^2 \delta_{ij} \right) + \mathcal{O}(c^{-2})$$

—> Leading contribution: **dipole radiation damping** at 1.5 post-Newtonian order $\propto \beta_b^3 (q_A - q_B)^2$

[Will 1993, Damour & Esposito-Farèse 1992]

Time-varying gravitational constant

In many alternatives to GR, the effective gravitational constant gets promoted to a dynamical field $G \longrightarrow \Phi$.

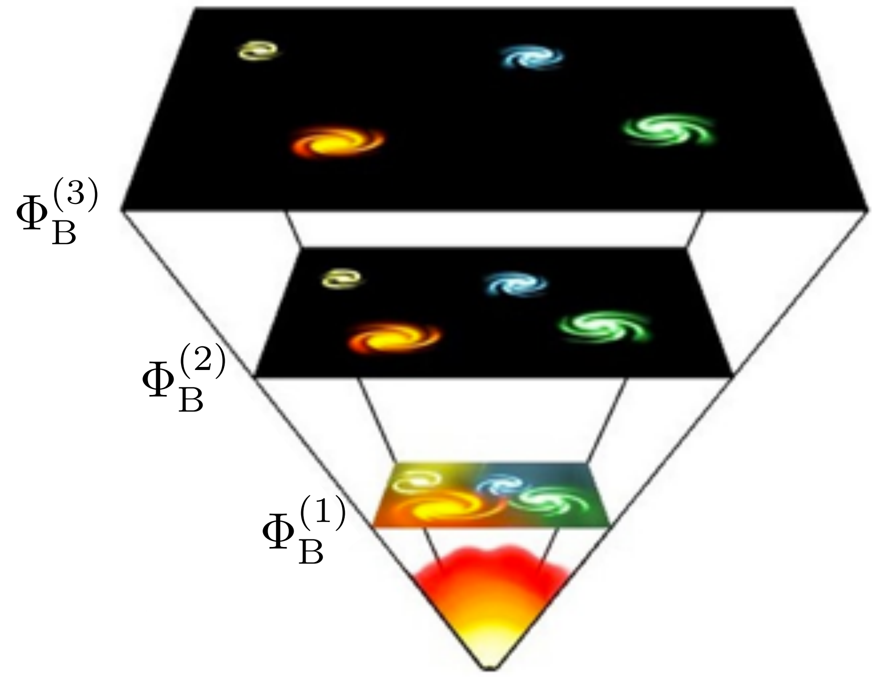
Expansion of the universe then generally leads to changes in the background value of $\Phi_{\rm B}$ and consequently to a time varying G.

A change of G leads to a change in the orbital period of a binary pulsar according to

$$\frac{\dot{P}_{\rm b}}{P_{\rm b}} = -2\left[1 + \frac{m_1 s_1 + m_2 s_2}{m_1 + m_2} + \frac{3}{2}\,\frac{m_1 s_2 + m_2 s_1}{m_1 + m_2}\right]\frac{\dot{G}}{G}$$

[Nordtvedt 1990]

"sensitivity": $s_i \equiv \dfrac{G}{m_i}\dfrac{\partial m_i}{\partial G}$



Binary pulsars limits (95% C.L.)

J1713+0747 (1.3 M$_\odot$): $\dot{G}/G = (-0.6 \pm 1.1) \times 10^{-12}\ \mathrm{yr}^{-1}$

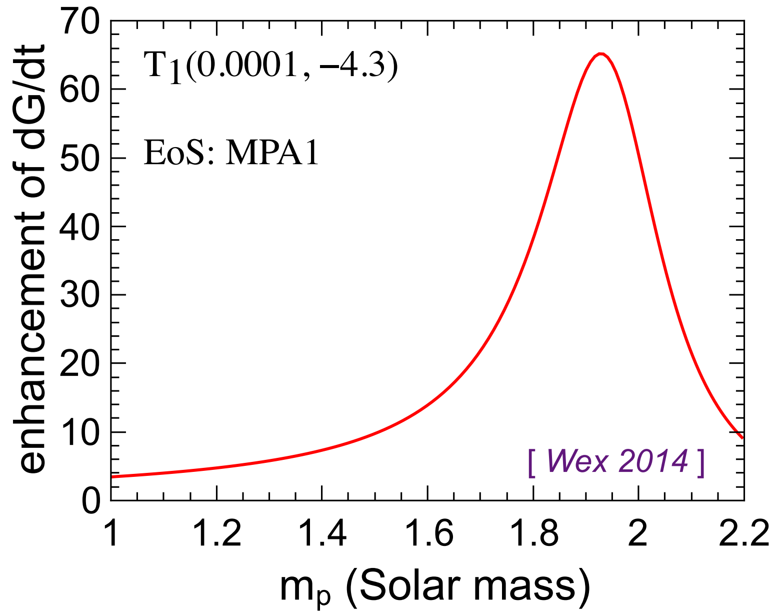
J0437-4715 (1.45 M$_\odot$): $\dot{G}/G = (-0.6 \pm 3.2) \times 10^{-12}\ \mathrm{yr}^{-1}$

J1614-2230 (1.93 M$_\odot$): $|\dot{G}/G| \lesssim 10^{-11}\ \mathrm{yr}^{-1}$

Zhu, Stairs, et al. 2015; Freire et al. 2012; Deller et al. 2008; NANOGrav

Solar system: $|\dot{G}/G| < 3 \times 10^{-13}\ \mathrm{yr}^{-1}$

Konopliv et al. 2014



Theory-based approach and PPK parameters

Two parameter mono-scalar-tensor gravity - $T_1(\alpha_0,\beta_0)$

Tensor field $g^*_{\mu\nu}$ plus massless/low mass scalar field φ

Field equations in Einstein frame:

$$R^*_{\mu\nu} = \frac{8\pi G_*}{c^4}\left(T^*_{\mu\nu} - \frac{1}{2}T^* g^*_{\mu\nu}\right) + 2\partial_\mu\varphi\partial_\nu\varphi - 2\cancel{V(\varphi)}g^*_{\mu\nu}$$

$$\Box_{g_*}\varphi = -\frac{4\pi G_*}{c^4}\alpha(\varphi)T_* - \cancel{V'(\varphi)}$$

$V''(\varphi) \ll 1/(\text{size of system})^2$

$$a(\varphi) = \alpha_0(\varphi - \varphi_0) + \frac{1}{2}\beta_0(\varphi - \varphi_0)^2$$

["logarithmic coupling function"]

$$\alpha(\varphi) \equiv \frac{\partial\, a(\varphi)}{\partial\, \varphi} = \alpha_0 + \beta_0(\varphi - \varphi_0)$$

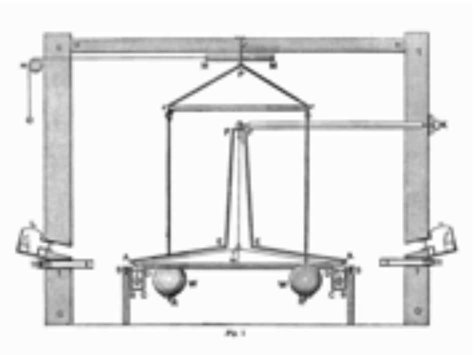
["coupling strength"]

Physical metric (Jordan frame)

$$g_{\mu\nu} = g^*_{\mu\nu}\,\exp 2a(\varphi)$$

Effective gravitational constant (Cavendish experiment)

$$G = G_*(1 + \alpha_0^2)$$



General Relativity: $\alpha_0=0$, $\beta_0=0$

Jordan-Fierz-Brans-Dicke: $\alpha_0\neq0$, $\beta_0=0$ [$\omega_{BD}=(1-3\alpha_0^2)/2\alpha_0^2$]

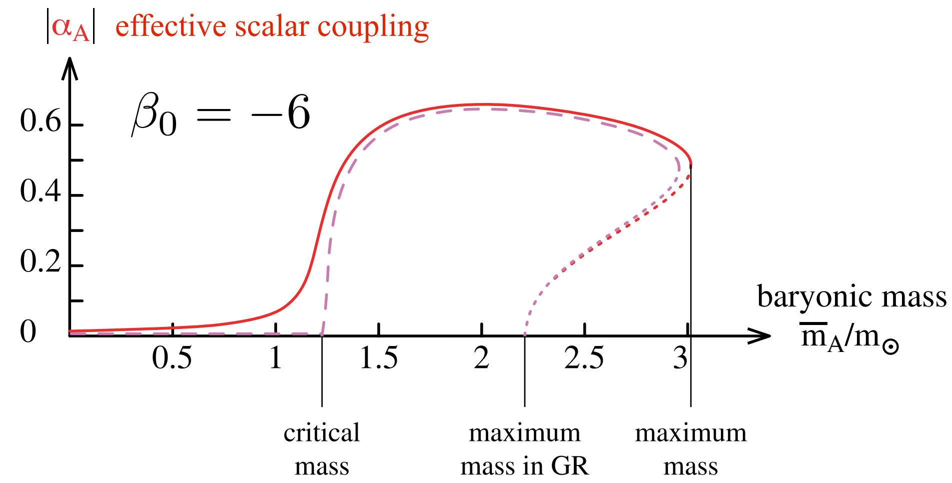
[Damour & Esposito-Farèse 1992, 1993, 1996]

No effacement of the internal structure
→ structure related parameters enter the equations of motion.

Already at **Newtonian level**:

$$G_{AB} = G_*(1 + \alpha_A \alpha_B)$$

$$\alpha_A(m_A, \alpha_0, \beta_0; \mathrm{EoS}) \equiv -\frac{\omega_A}{m_A} = \frac{\partial \ln m_A}{\partial \varphi_a}$$

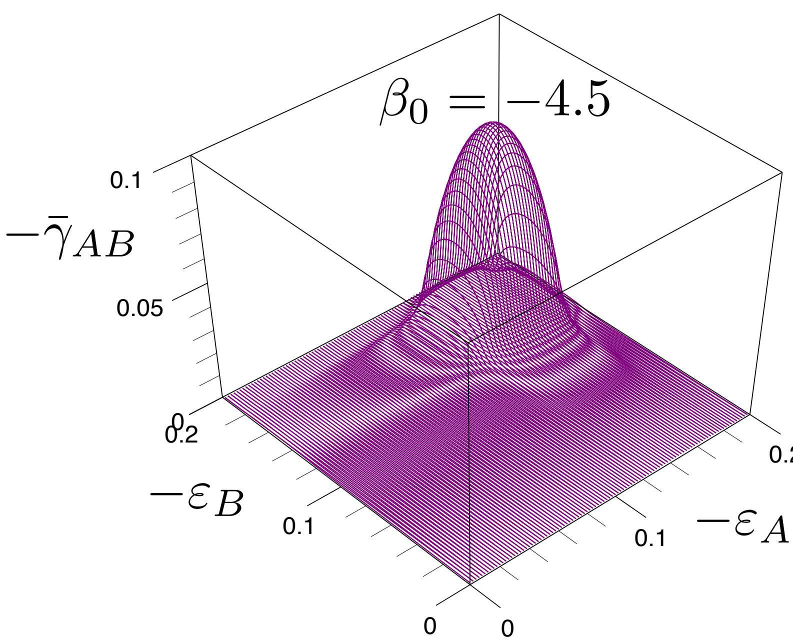


At **post-Newtonian level**:

$$\bar\gamma_{AB} = -\frac{2\alpha_A\alpha_B}{1 + \alpha_A\alpha_B}$$

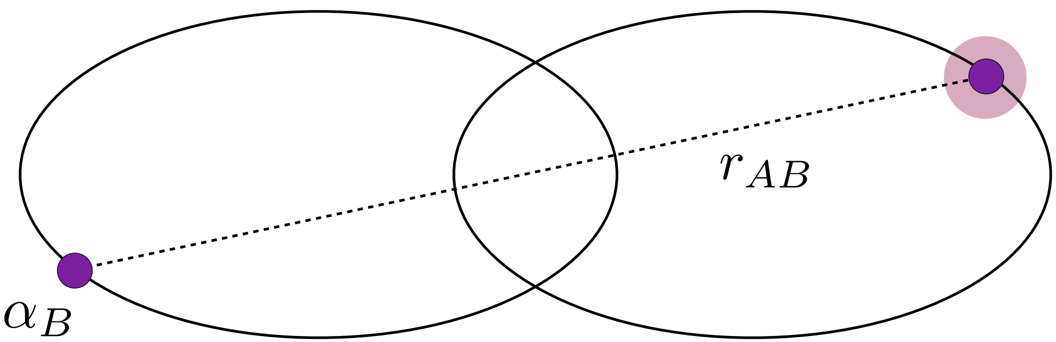
$$\bar\beta^A_{BC} = \frac{\beta_A\alpha_B\alpha_C}{2(1 + \alpha_A\alpha_B)(1 + \alpha_A\alpha_C)}$$

$$\beta_A \equiv \frac{\partial \alpha_A}{\partial \varphi_a}$$



[see e.g. Damour 2009 (SIGRAV lecture)]

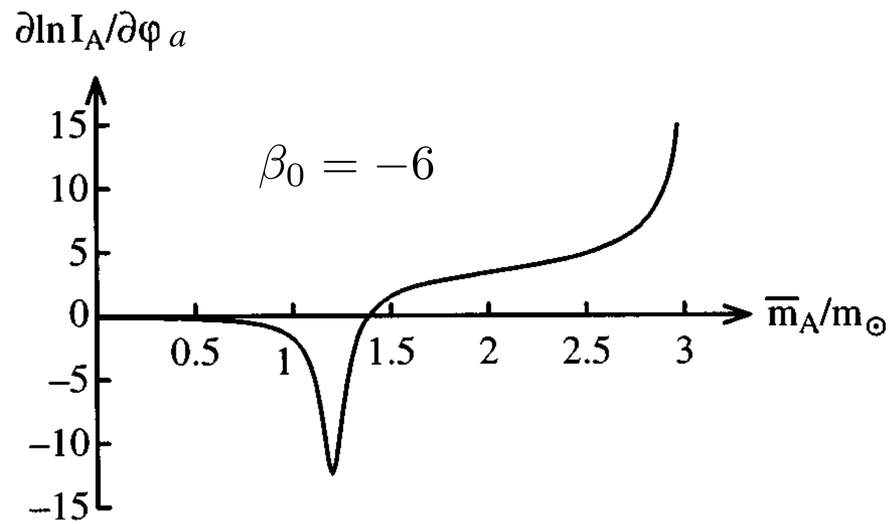
Modification of the moment-of-inertia



$$\varphi_A(t) = \varphi_0 - \frac{G_* m_B \alpha_B}{r_{AB} c^2}$$

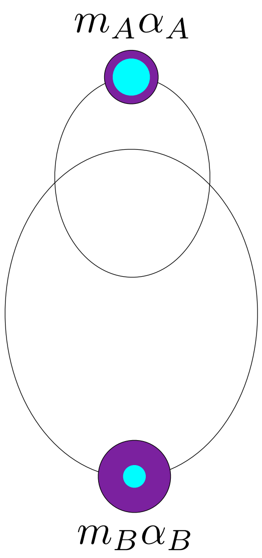
$$\boxed{\frac{I_A}{I_0} \simeq 1 + \frac{G_* m_B \alpha_B}{r_{AB} c^2} k_A}$$

$$-\frac{1}{I_A}\frac{\partial I_A}{\partial \varphi_a}$$



$$S_A = \Omega_A I_A = \text{adiabatic invariant} \quad \longrightarrow \quad \text{modification of } \gamma_{\rm E}$$

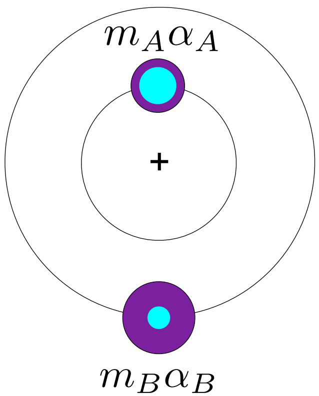
[Damour & Esposito-Farèse 1996]



STG: leading multipole moment = scalar monopole

$$\dot{P}_b^{\text{Monopole}} = -\frac{3\pi}{1+\alpha_A\alpha_B}\frac{e^2(1+\frac{1}{4}e^2)}{(1-e^2)^{7/2}}\frac{m_A m_B}{(m_A+m_B)^2}\beta_b^5 \times \left[\frac{3m_A+5m_B}{m_A+m_B}\alpha_A + \frac{5m_A+3m_B}{m_A+m_B}\alpha_B + \frac{\beta_B\alpha_A+\beta_A\alpha_B}{1+\alpha_A\alpha_B}\right]^2$$

$$\beta_b = \left(\frac{G_{AB}Mn}{c^3}\right)^{1/3}$$ << 1



STG: leading pN term = scalar dipole GW damping

$$\dot{P}_b^{\text{Dipole}} = -\frac{2\pi}{1+\alpha_A\alpha_B}\frac{1+\frac{1}{2}e^2}{(1-e^2)^{5/2}}\frac{m_A m_B}{(m_A+m_B)^2}\beta_b^3(\alpha_A-\alpha_B)^2$$

[Damour & Esposito-Farèse 1992]

DD timing model for scalar-tensor gravity

General relativity

$$\dot\omega = nk = \frac{3n}{1-e^2}\left(\frac{GMn}{c^3}\right)^{2/3}$$

$$\gamma_{\rm E} = \frac{e}{n}\left(\frac{GMn}{c^3}\right)^{2/3} X_B(X_B+1)$$

$$r = \frac{Gm_B}{c^3}$$

$$s = xn\left(\frac{GMn}{c^3}\right)^{-1/3} X_B^{-1}$$

$$\dot P_b = \dot P_b^{\rm Q}\left(m_A, m_B, \{p^K\}\right)$$

Scalar-tensor gravity

$$\dot\omega = \frac{3n}{1-e^2}\left(\frac{G_{AB}Mn}{c^3}\right)^{2/3}\left[\frac{1-\frac{1}{3}\alpha_A\alpha_B}{1+\alpha_A\alpha_B} - \frac{X_A\alpha_A^2\beta_B + X_B\alpha_B^2\beta_A}{6(1+\alpha_A\alpha_B)}\right]$$

$$\gamma_{\rm E} = \frac{e}{n}\left(\frac{G_{AB}Mn}{c^3}\right)^{2/3}\frac{X_B(X_B+1+X_B\alpha_A\alpha_B+\alpha_B k_A)}{1+\alpha_A\alpha_B}$$

$$r = \frac{G_* m_B}{c^3}$$

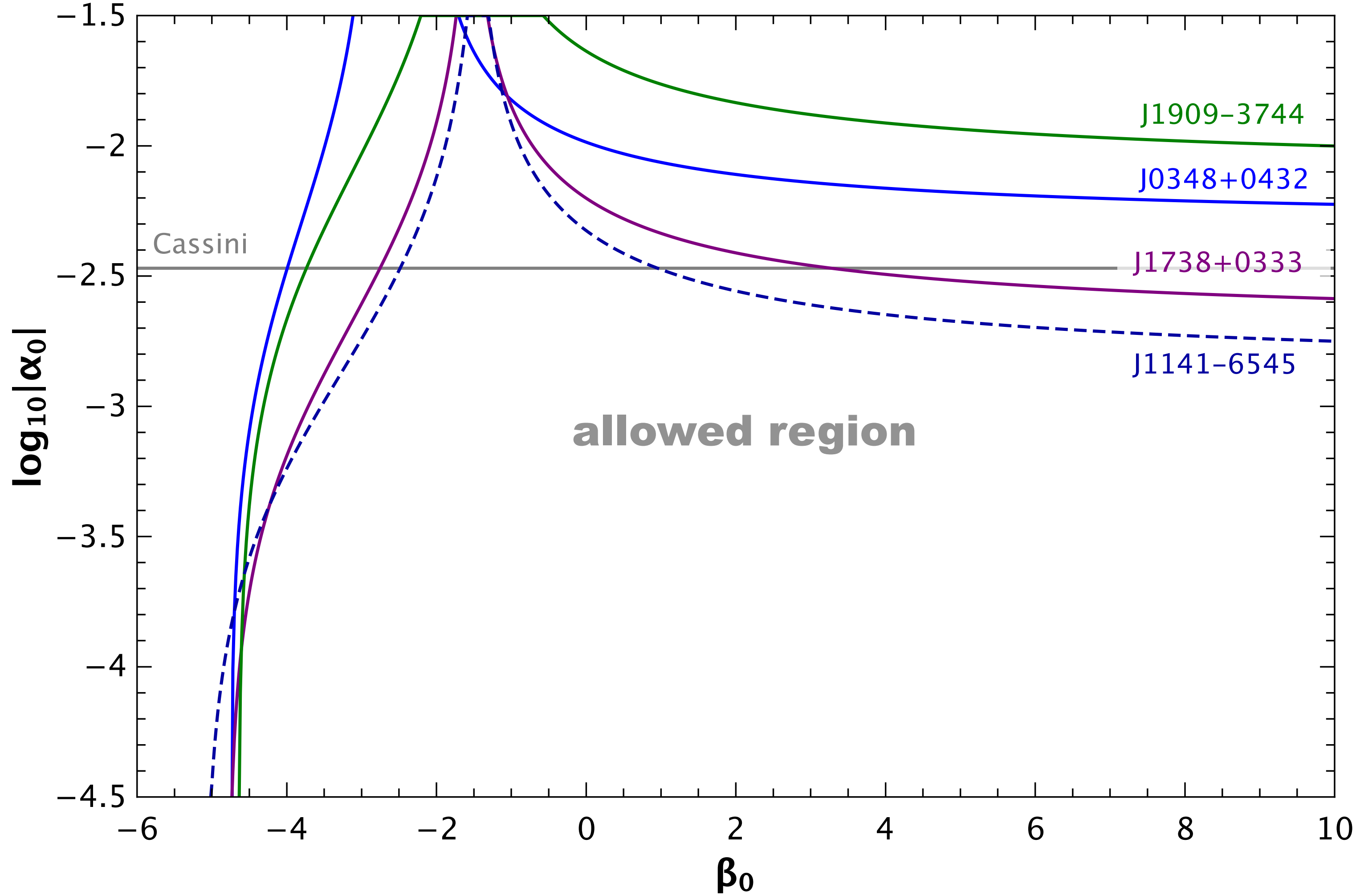
$$s = xn\left(\frac{G_{AB}Mn}{c^3}\right)^{-1/3} X_B^{-1}$$

$$\dot P_b = \dot P_b^{\rm M,\varphi} + \dot P_b^{\rm D,\varphi} + \dot P_b^{\rm Q,\varphi} + \dot P_b^{\rm Q,g_*}$$

$$= -\frac{2\pi G_* m_A m_B n}{(m_A+m_B)c^3}\,\frac{1+e^2/2}{(1-e^2)^{5/2}}\,(\alpha_A-\alpha_B)^2 + O\left(\frac{v^5}{c^5}\right)$$

$$n = 2\pi/P_b,\ M = m_A + m_B,\ X_A = m_A/M,\ X_B = m_B/M = 1 - X_A \qquad G_{AB} = G_*(1+\alpha_A\alpha_B)$$

Constraining scalar-tensor gravity

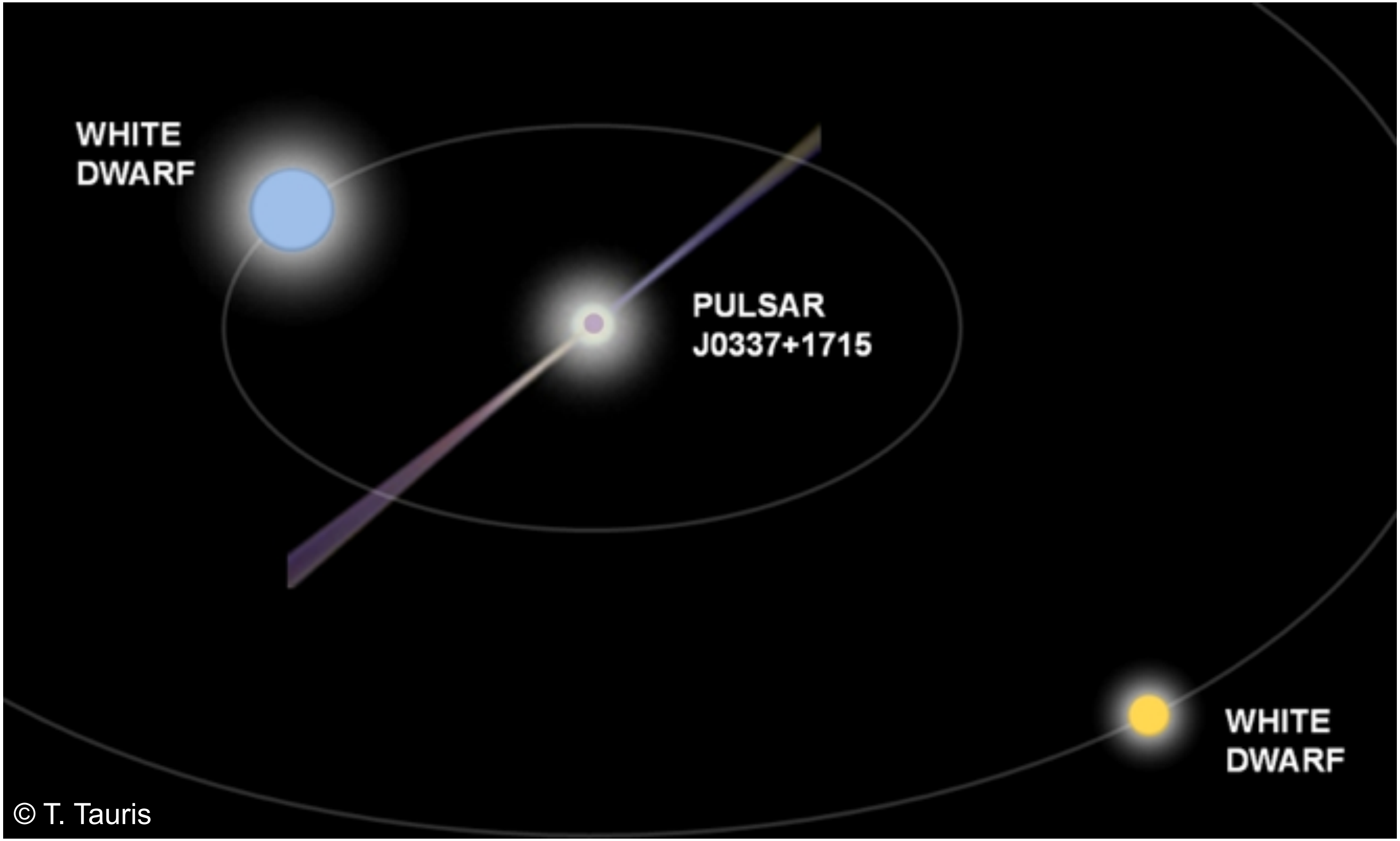


GR: $\alpha_0 = \beta_0 = 0$

Jordan-Fierz-Brans-Dicke: $\beta_0 = 0$

Triple system pulsar and the violation of SEP

PSR J0337+1715: P = 2.7 ms, $M_{PSR} = 1.44\ M_\odot$
Inner orbit: 1.6 d, $M_{WD} = 0.20\ M_\odot$
Outer orbit: 327 d, $M_{WD} = 0.41\ M_\odot$



[Ransom *et al.*, 2014]

Triple system pulsar and the violation of SEP

PSR J0337+1715: P = 2.7 ms, M_{PSR} = 1.44 $M_\odot$
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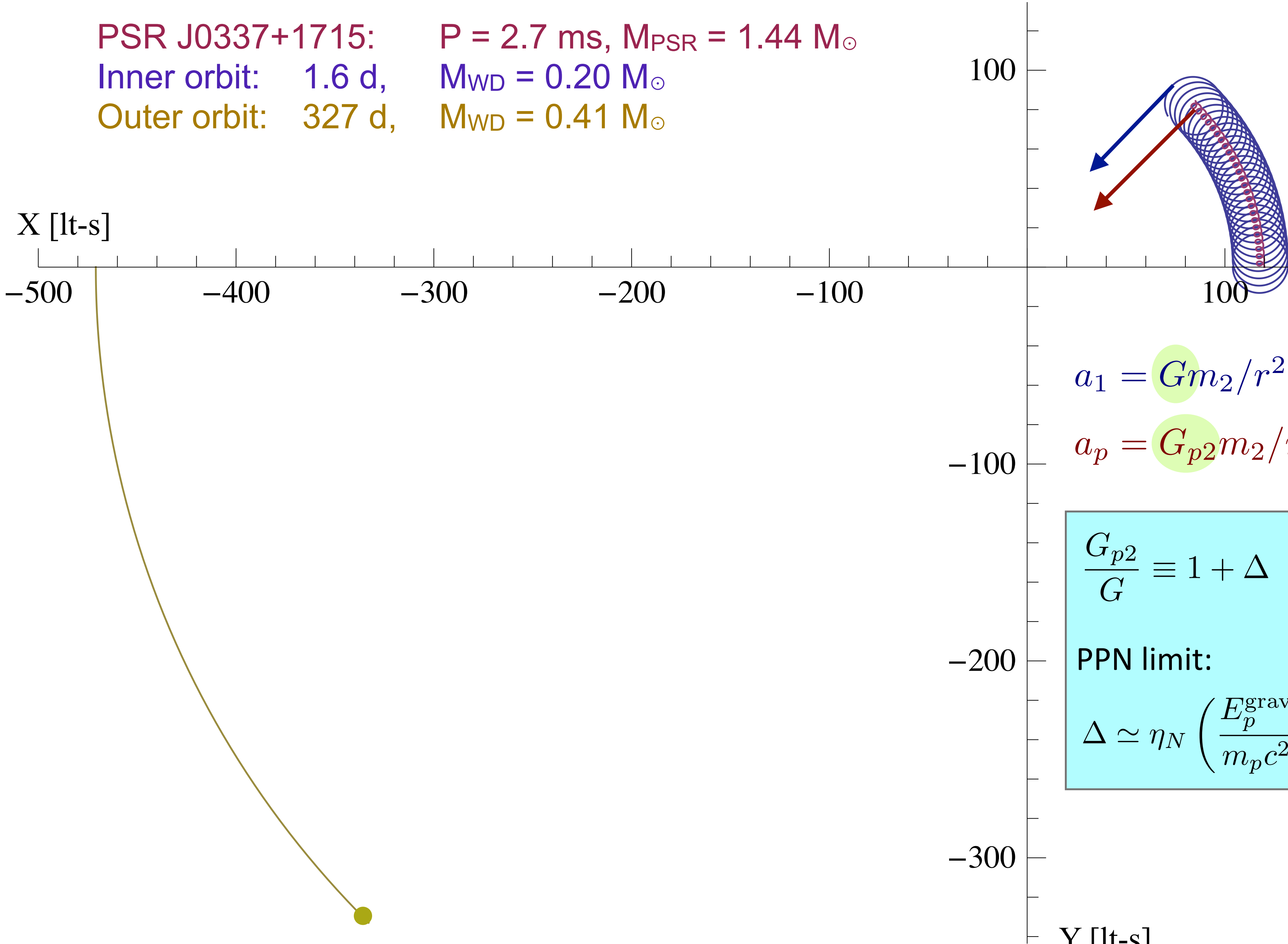


$$a_1 = Gm_2/r^2$$

$$a_p = G_{p2}m_2/r^2$$

$$\frac{G_{p2}}{G} \equiv 1 + \Delta$$

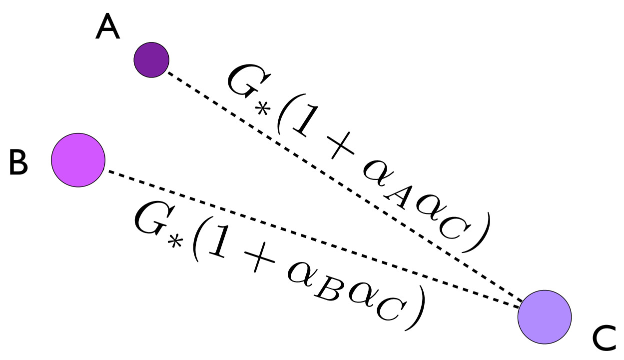
PPN limit:

$$\Delta \simeq \eta_N \left(\frac{E_p^{\rm grav}}{m_p c^2} - \frac{E_1^{\rm grav}}{m_1 c^2} \right)$$

[Ransom *et al.*, 2014]

Three-body system in scalar-tensor gravity

Effective gravitational constant: $\boxed{G_{AB} = G_*(1 + \alpha_A \alpha_B)}$



—> Violation of the universality of free fall for self-gravitating bodies

J0337+1715 (pulsar in the triple system)

$$\begin{aligned} G_{AC} &= G_*(1 + \alpha_0 \alpha_p) \\ G_{BC} &= G_*(1 + \alpha_0^2) \\ \Rightarrow & \\ \frac{G_{AC}}{G_{BC}} &\simeq 1 - \underbrace{\alpha_0(\alpha_p - \alpha_0)}_{\Delta} \end{aligned}$$

[see e.g. Damour 2009 (SIGRAV lecture)]

Expected limits on scalar-tensor gravity from PSR J0337+1715