Screening mechanism

Kazuya Koyama
University of Portsmouth

Reading list

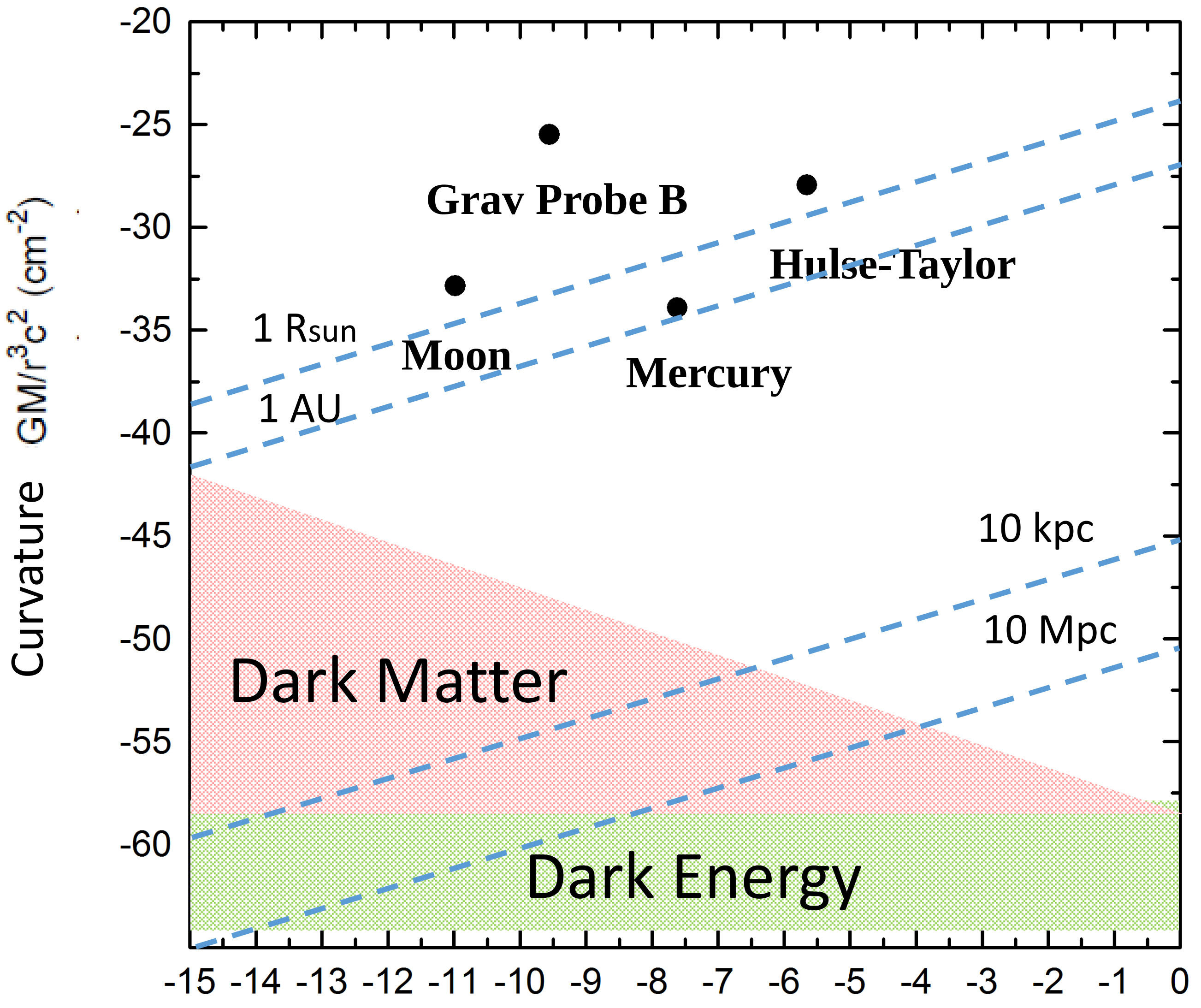
- Beyond the Cosmological standard model” Joyce, Jain, Khoury and Trodden, arXiv:1407.0059
- “Cosmological Tests of Modified Gravity” KK, arXiv:1504.04623
- “Chameleon Field Theories” Khoury, arXiv:1306.4326
- “An introduction to the Vainshtein mechanism” Babichev, Deffayet, arXiv:1304.7240

Questions

- Departures from spherical symmetry - still not well understood? - **Jeremy Sakstein**
- Is it possible to make a parametrisation of screening mechanisms? – **Phil B**
- How does screening work inside composite objects like galaxies? If the galaxy is screened, does that mean everything inside the galaxy is also screened? – **Phil B**
- Do screening mechanisms that rely on spontaneous symmetry breaking produce topological defects or other exotic objects? – **Phil B**
- Can PPN/PPK/PPF/EFT/etc. parameters be related to *any* properties of screening mechanisms? Or does screening depend on essentially different properties of the relevant theories? – **Phil B**
- Vainshtein mechanism in curved ST (i.e. broken Galilean symmetry)? **Miguel Z.**
- The current status of N-body cosmological simulations of non-GR theories, and their importance in understanding screening effects on structure formation. **Alex Barreira**
- I would personally be interested in a (quick) review on the different screening mechanisms. **Davide G.**

Assuming GR

Psaltis Living Rev. Relativity 11 (2008), 9
Baker et.al. 1412.3455

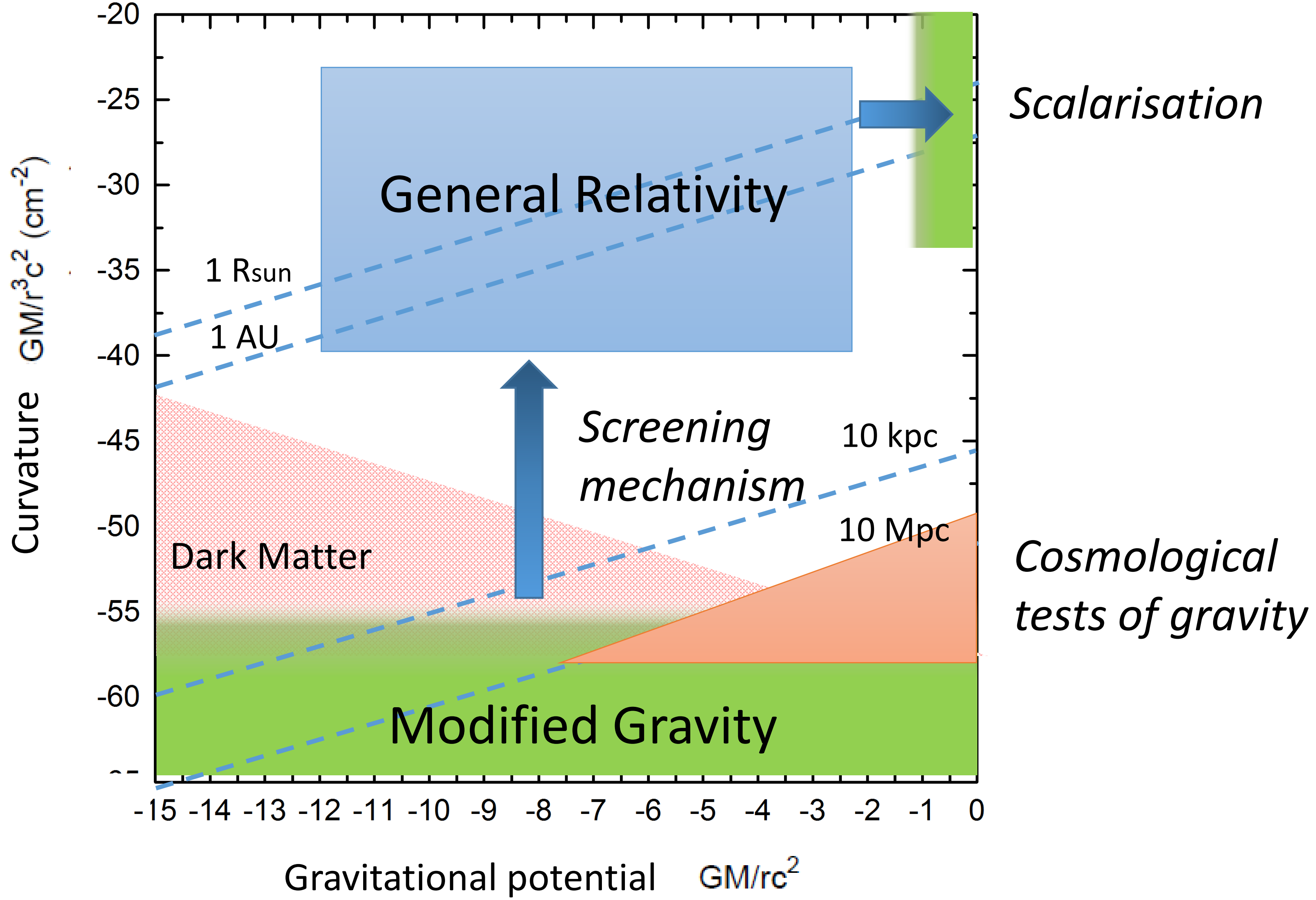


curvature

$$R = \frac{GM}{r^3c^2}$$

potential

$$\Phi = \frac{GM}{rc^2}$$



General Relativity
Scalarisation
1 Rsun
1 AU
Screening
mechanism
10 kpc
10 Mpc
Dark Matter
Cosmological
tests of gravity
Modified Gravity
-20
-25
-30
-35
-40
-45
-50
-55
-60
-15 -14 -13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0
Curvature GM/r^3c^2 (cm^-2)
Gravitational potential GM/rc^2

Why we need screening mechanism?

- Brans-Dicke gravity

$$S=\int d^4x\left(\psi R-\frac{\omega_{BD}}{\psi}(\nabla\psi)^2\right)+\int d^4x\,L_{matter}$$

quasi-static approximations (neglecting time derivatives)

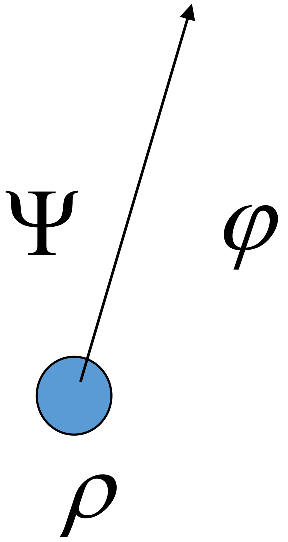
$$ds^2=-(1+2\Psi)dt^2+a(t)^2(1-2\Phi)d\vec{x}^2 \qquad \psi=\psi_0+\varphi$$

$$(3+2\omega_{BD})\nabla^2\varphi=-8\pi G\rho$$

$$\nabla^2\Psi=4\pi G\rho-\frac{1}{2}\nabla^2\varphi$$

$$\Phi-\Psi=-\varphi$$

φ : fifth force



Constraints on BD parameter

- Solutions

$$(3+2\omega_{BD})\nabla^2\varphi = -8\pi G\rho$$

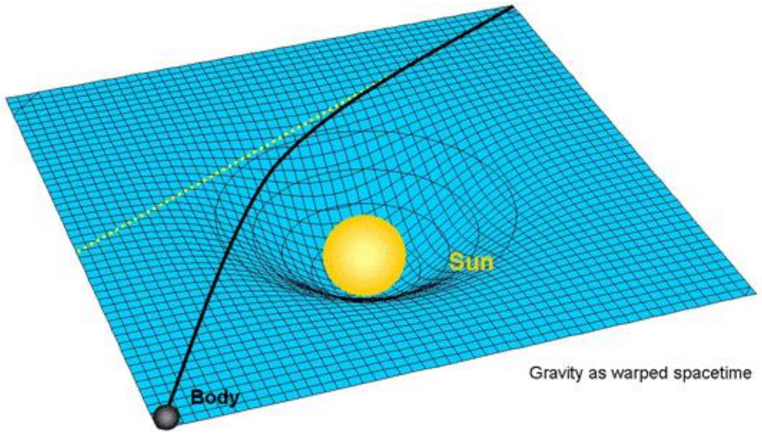
$$\nabla^2\Psi = -4\pi G\left(\frac{4+2\omega_{BD}}{3+2\omega_{BD}}\right)\rho, \quad G_{eff} = \left(\frac{4+2\omega_{BD}}{3+2\omega_{BD}}\right)G$$

$$\Psi = \frac{2+\omega_{BD}}{1+\omega_{BD}}\Phi \equiv \gamma^{-1}\Phi$$

- PPN parameter

$$\gamma = \frac{1+\omega_{BD}}{2+\omega_{BD}}$$

$$\gamma - 1 = (2.1 \pm 2.3)\times 10^{-5} \qquad \omega_{BD} \geq 40,000$$



This constraint excludes any detectable modifications in cosmology

Screening mechanism

- Require screening mechanism to restore GR

$$S = \int d^4x \left(\psi R - \frac{\omega_{BD}(\psi)}{\psi} (\nabla\psi)^2 + V(\psi) + N(\nabla\psi, \nabla^2\psi) \right)$$

recovery of GR must be environmental dependent

- make the scalar short-ranged using $V(\psi)$ (*chameleon*)
- make the kinetic term large to suppress coupling to matter

  using $\omega_{BD}(\psi)$ (*dilaton/symmetron*) or $N(\nabla\psi)$(*k-mouflage*)

  $N(\nabla^2\psi)$ (*Vainshtein)*

Break equivalence principle

$$\int d^4x \left(B(\psi) L_{baryon} + L_{CDM} \right)$$

remove the fifth force from baryons
(*interacting DE models* in Einstein frame)

Chameleon mechanism

- Scalar is coupled to matter

  Depending on density, the mass can change

  It is easier to understand the dynamics in Einstein frame

$$S = \int d^4x \left[\sqrt{-g} \left(\psi R - \frac{\omega_{BD}}{\psi} (\nabla \psi)^2 + V(\psi) \right) + L_m[g_{\mu\nu}] \right]$$

$$g_{\mu\nu} = \exp\left(-\frac{\alpha\phi}{M_{pl}} \right) \bar{g}_{\mu\nu},$$

$$\log \psi = 2 \frac{\alpha\phi}{M_{pl}}$$

$$\Downarrow$$

$$S_E = \int d^4x \left[\sqrt{-\bar{g}} \left(\bar{R} - \frac{1}{2} (\bar{\nabla} \phi)^2 + \bar{V}(\phi) \right) + L_m[e^{-\alpha\phi/M_{pl}} \bar{g}_{\mu\nu}] \right]$$

$$\boxed{\alpha = \sqrt{\frac{1}{3 + 2\omega_{BD}}}}$$

$$\nabla^2 \phi = \partial_\phi V + \frac{\alpha}{M_{pl}} \rho e^{\alpha\phi/M_{pl}} \qquad V_{eff}(\phi) = V + \rho e^{\alpha\phi/M_{pl}}$$

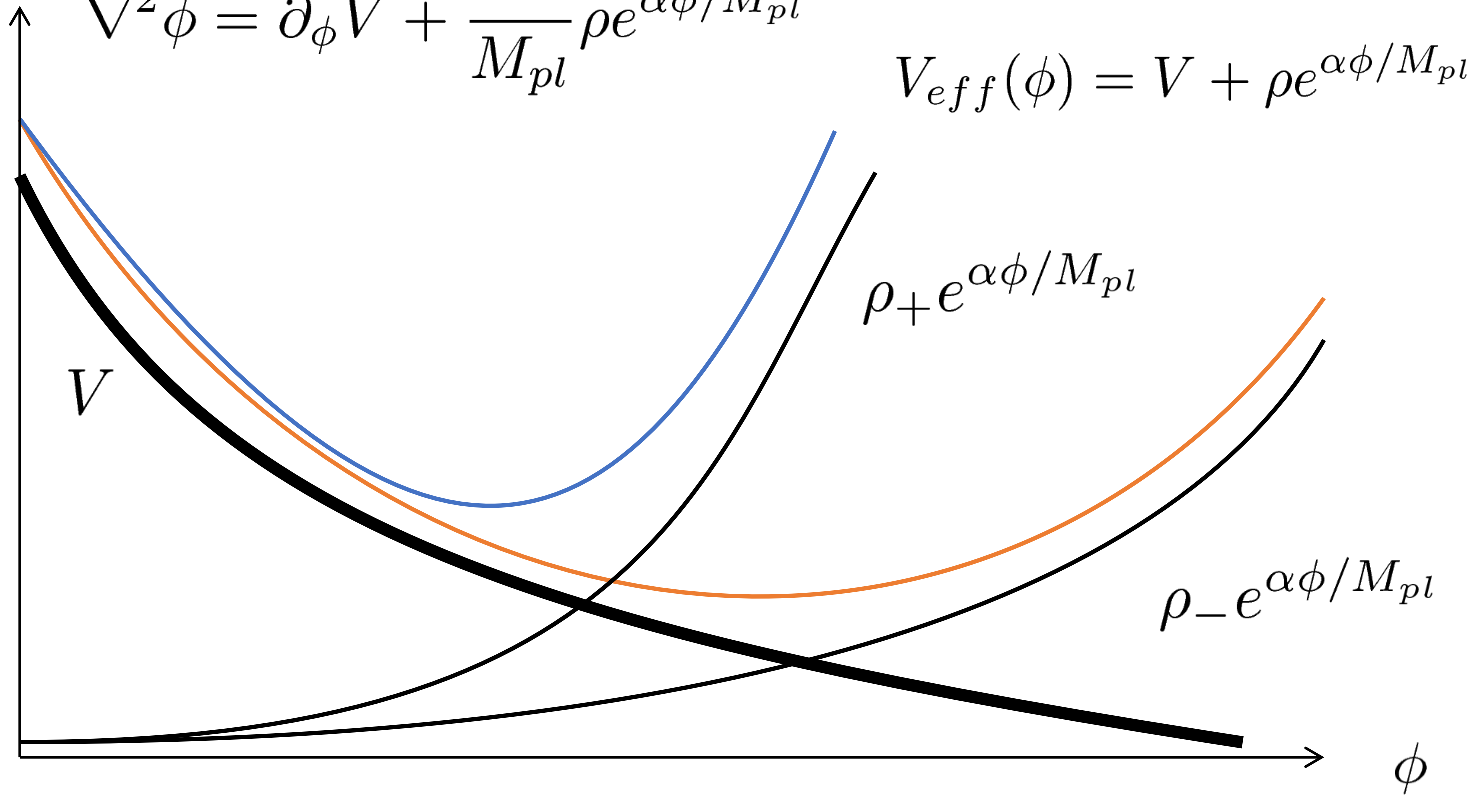
Chameleon mechanism

Khoury & Weltman astro-ph/0309300

$$S=\int d^4x\sqrt{-\tilde{g}}\left(\frac{M_{pl}^2}{2}R-\frac{1}{2}(\nabla\phi)^2-V(\phi)\right)+S_m[A^2(\phi)g_{\mu\nu}]\qquad A(\phi)=\exp\left(\alpha\frac{\phi}{M_{pl}}\right)$$

$$\nabla^2\phi=\partial_\phi V+\frac{\alpha}{M_{pl}}\rho e^{\alpha\phi/M_{pl}}$$

$$V_{eff}(\phi)=V+\rho e^{\alpha\phi/M_{pl}}$$

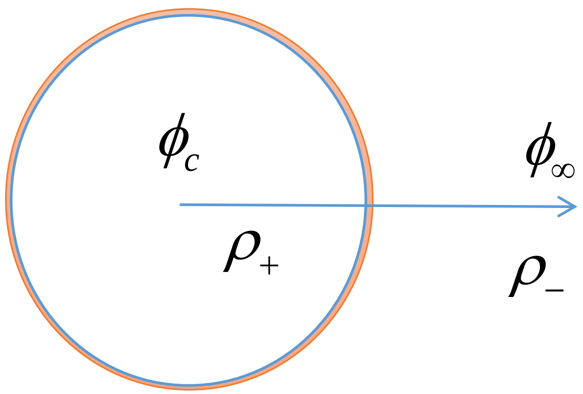


Thin shell condition

- If the thin shell condition is satisfied, only the shell of the size ΔR_c contributes to the fifth force

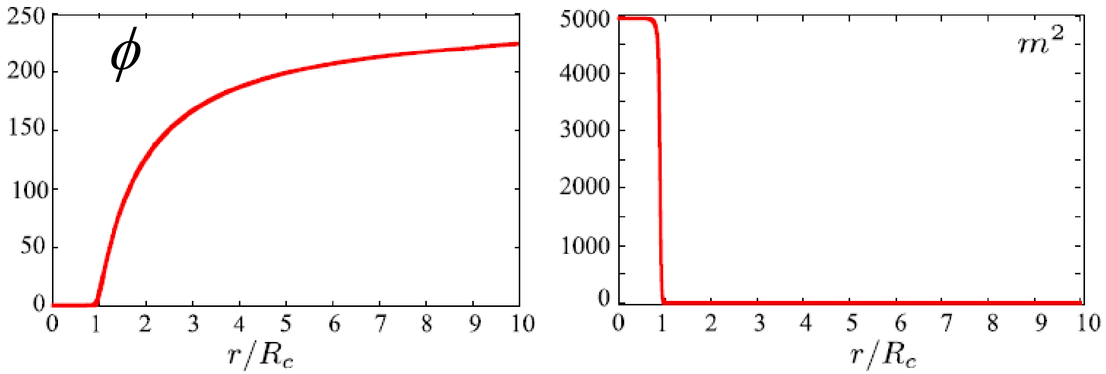
Khoury & Weltman astro-ph/0309411

$$V_{eff}(\phi) = V + \rho e^{\alpha\phi/M_{pl}}$$



$$\phi(r) = -\left(\frac{\alpha}{4\pi M_{pl}}\right)\left(\frac{3\Delta R_c}{R_c}\right)\frac{M\exp\left(-m_\infty(r-R_c)\right)}{r} + \phi_\infty$$

$$\frac{\Delta R_c}{R_c} = \frac{(\phi_\infty - \phi_c)/M_{pl}}{6\alpha\Psi_c} << 1 \qquad \Psi_c = \frac{GM}{R_c}$$



Silvestri 1103.4013

Screening is determined by the gravitational potential of the object

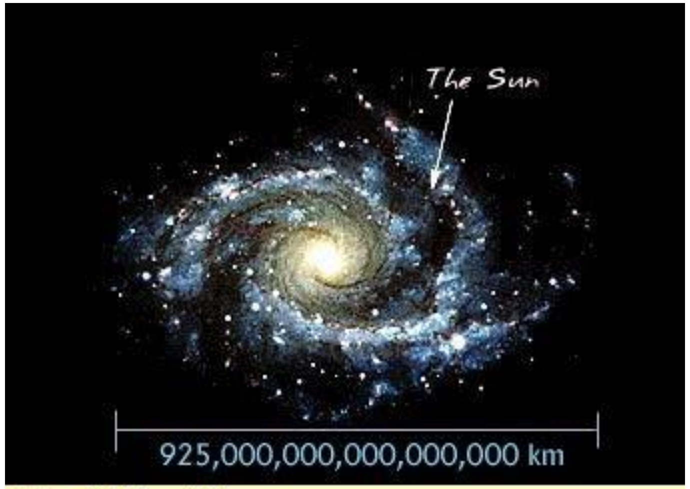
Solar system constraints

- Solar system constraints

$$\rho_\odot \sim 10 \text{ g cm}^{-3}$$

$$\rho_{gal} \sim 10^{-24} \text{g cm}^{-3}$$

$$\frac{\Delta R_c}{R_c} = \frac{\phi_{gal} - \phi_\odot}{6\alpha M_{pl}\Psi_\odot} \sim \frac{\phi_{gal}}{6\alpha M_{pl}\Psi_\odot}$$



The Milky Way

The sun has a potential $\Psi_\odot \sim 10^{-6}$

The thin shell suppression eases the constraints $\alpha = O(1)$

$$\frac{\Delta R_c}{R_c} \leq 10^{-5} \quad \Rightarrow \quad \frac{\phi_{gal}}{M_{pl}} < 5\times 10^{-11}$$

This is a model (potential) independent constraint

From galaxy to cosmology

- Example

$$V = V_0 - M^4 \left(\phi / M_{pl}\right)^{1/2} \qquad \frac{\phi_{gal}}{M_{pl}} = \left(\frac{M^4}{\alpha\rho_{gal}}\right)^2 \qquad \rho_{gal} \sim 10^{-24}\,\mathrm{g\ cm^{-3}}$$

$$\rho_{crit} \sim 10^{-29}\,\mathrm{g\ cm^{-3}}$$

Solar system constraints

$$\frac{\phi_{gal}}{M_{pl}} < 10^{-11} \qquad \frac{\phi_{\cos mo}}{M_{pl}} = \left(\frac{M^4}{\alpha\rho_{crit}}\right)^2 \simeq 10^{10}\frac{\phi_{gal}}{M_{pl}} \leq 10^{-1} \qquad M \simeq 10^{-3}\mathrm{eV}$$

Galaxy $$\frac{\Delta R_c}{R_c} = \frac{\phi_{\cos mo} - \phi_{gal}}{6\alpha M_{pl}\Psi_{gal}} \sim \frac{\phi_{\cos mo}}{6\alpha M_{pl}\Psi_{gal}}$$

The Milky way galaxy $\Psi_{Milk} \sim 10^{-6}$

in order to screen the Milky way, we need $\frac{\phi_{\cos mo}}{M_{pl}} < 10^{-6}$

Screening of isolated objects $\frac{\phi_{\cos mo}}{M_{pl}} \approx 10^{-6}$

Object	Φ_N	Screening Status
Earth	10^{-9}	Screened by the Milky Way
The Sun	2×10^{-6}	Screened by the Milky Way
Main-sequence stars	10^{-6}–10^{-5}	Screened
Local group	10^{-4}	Screened
Milky Way	$\mathcal{O}(10^{-6})$	Screened
Spiral and elliptical galaxies	10^{-6}–10^{-5}	Screened
Post-main-sequence stars	10^{-7}–10^{-8}	Unscreened in dwarf galaxies in cosmic voids
Dwarf galaxies	$\mathcal{O}(10^{-8})$	Screened in clusters, unscreened in cosmic voids

Sakstein, arXiv:1502.04503

Chameleon/Symmetron/dilaton

- Einstein frame

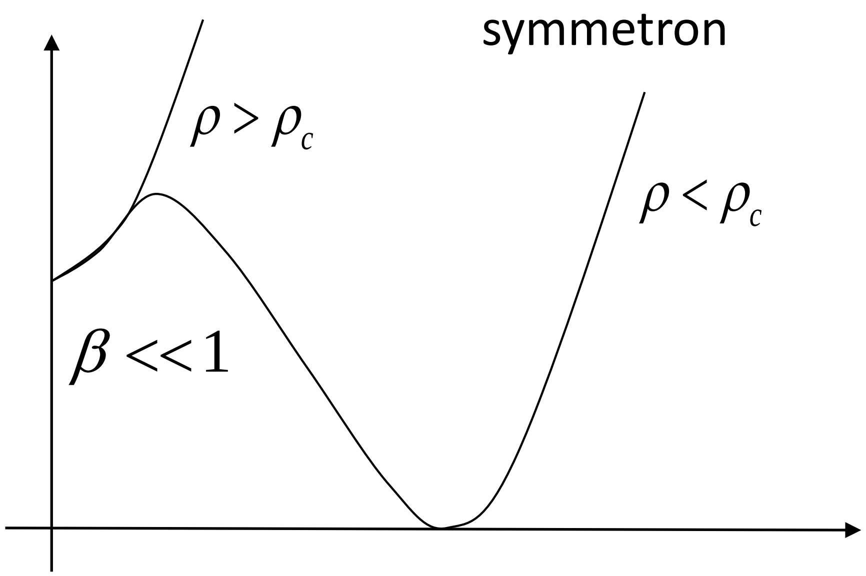
$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] + S_m(A^2(\phi)g_{\mu\nu}) \qquad V_{\rm eff} = V(\phi) - [A(\phi) - 1]T^\mu_\mu$$

$$m^2 = V''_{\rm eff}(\bar\phi) \quad \beta = M_{\rm Pl}\frac{d\ln A}{d\phi}\bigg|_{\phi=\bar\phi}$$

$$A(\phi) = 1 + \xi\frac{\phi}{M_{\rm pl}}, \quad V(\phi) = \frac{M^{4+n}}{\phi^n} \quad \text{chamaleon,}$$

$$A(\phi) = 1 + \frac{1}{2M}(\phi - \bar\phi)^2, \quad V(\phi) = V_0 e^{-\phi/M_{\rm pl}} \quad \text{dilaton,}$$

$$A(\phi) = 1 + \frac{1}{2M^2}\phi^2, \quad V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 \quad \text{symmetron}$$



Hinterbichler, Khoury, arXiv:1001.4525

Vainshtein mechanism

- Vainshtein mechanism

originally discussed in massive gravity

rediscovered in DGP brane world model

linear theory $\omega_{BD} = 0$

$$3\nabla^2\varphi = -8\pi G\rho$$

$$\nabla^2\Psi = 4\pi G\rho - \frac{1}{2}\nabla^2\varphi$$

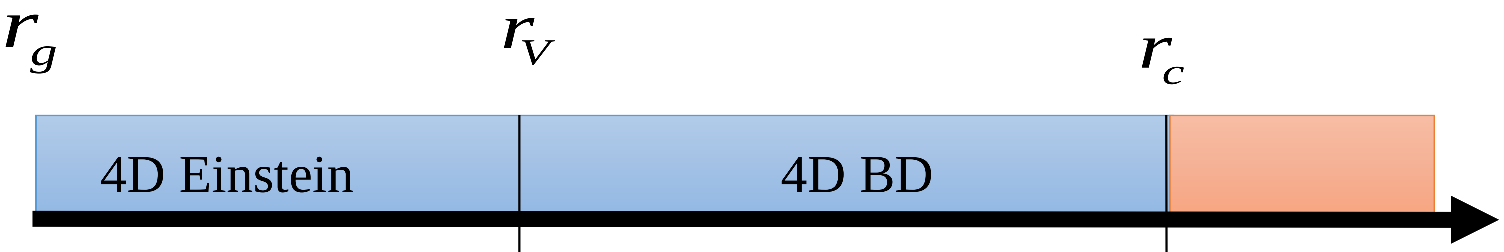
even if gravity is weak, the scalar can be non-linear $r_c \sim m^{-1} \sim H_0^{-1}$

$$3\nabla^2\varphi + r_c^2\left\{\left(\nabla^2\varphi\right)^2 - \partial_i\partial_j\varphi\,\partial^i\partial^j\varphi\right\} = 8\pi G a^2\rho$$

Vainshtein radius

- Spherically symmetric solution for the scalar

$$\frac{d\varphi}{dr} = \frac{r_g}{r^2}\Delta(r), \quad \Delta(r) = \frac{2}{3}\left(\frac{r}{r_V}\right)^3\left(\sqrt{1+\left(\frac{r_V}{r}\right)^3} - 1\right) \qquad r_V = \left(\frac{8r_c^2 r_g}{9}\right)^{\frac{1}{3}}, \quad r_g = 2GM$$



Screening is determined by the curvature of the object

$$\Phi = \frac{r_g}{2r} + \sqrt{\frac{r_g r}{2r_c^2}}, \qquad \Psi = -\frac{r_g}{2r} + \sqrt{\frac{r_g r}{2r_c^2}}$$

$$\Phi = \frac{r_g}{2r}\left(\frac{2}{3}\right), \qquad \Psi = -\frac{r_g}{2r}\left(\frac{4}{3}\right)$$

$$\left(\frac{r_V}{r}\right)^{-3/2} \propto \left(\frac{GM}{r^3} r_c^2\right)^{-3/2}$$

2.95km 0.1 kpc 3000Mpc for the Sun

Vainshtein radius $r_c \sim m^{-1} \sim H_0^{-1}$

Object	R	r_s	r_*
Universe	$\sim 1.2 \times 10^{26}$	$\sim 4.5 \times 10^{25}$	$\sim 8.6 \times 10^{25}$
Milky Way	$\sim 0.9 \times 10^{21}$	$\sim 2 \times 10^{15}$	$\sim 3 \times 10^{22}$
Sun	$\sim 0.7 \times 10^{9}$	$\sim 3 \times 10^{3}$	$\sim 3.5 \times 10^{18}$
Earth	$\sim 6 \times 10^{6}$	$\sim 9 \times 10^{-3}$	$\sim 5 \times 10^{16}$
Atom	$\sim 5 \times 10^{-11}$	$\sim 1.8 \times 10^{-54}$	$\sim 3 \times 10^{-1}$

$[m]$

Li, Zhao, KK, arXiv:1303.0008

$$r_g = 2GM, \quad r_V = \left(r_c^2 r_g\right)^{\frac{1}{3}}$$

Solar system constraints

- The fractional change in the gravitational potential $\varepsilon = \dfrac{\delta\Psi}{\Psi}$

  The anomalous perihelion precession

$$\delta\phi = \pi r \frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\left(\frac{\varepsilon}{r}\right)\right]$$

The vainshtein radius is shorter for a smaller object

Lunar laser ranging: the Erath-moon distance $r_{E-M} = 4.1 \times 10^5 \mathrm{km}$

$$\delta\phi = \frac{3\pi}{4}\left(\frac{r_{E-M}{}^3}{2GM_\oplus r_c^2}\right)^{1/2} < 2.4 \times 10^{-11} \quad \Rightarrow \quad r_c > H_0^{-1}$$

Dvali, Gruzinov, Zaldarriaga, hep-ph/0212069

breaking of Vainshtein mechanism

- "beyond Horndeski" Gleyzes, Langlois, Piazza, Vernizzi arXiv:1303.0008, 1408.1952

$$S = \int d^4x \sqrt{-g} \left[M_{\rm pl}^2 \left(\frac{R}{2} - \Lambda \right) - k_2 \mathcal{L}_2 + f_4 \mathcal{L}_{4,\rm bH} \right]$$

$$\mathcal{L}_2 = \phi_\mu \phi^\mu \equiv X$$

$$\mathcal{L}_{4,\rm bH} = -X \left[(\Box\phi)^2 - (\phi_{\mu\nu})^2 \right] + 2\phi^\mu \phi^\nu \left[\phi_{\mu\nu} \Box\phi - \phi_{\mu\sigma} \phi^\sigma{}_\nu \right]$$

$$\mathrm{d}s^2 = (-1 + 2\Phi)\,\mathrm{d}t^2 + (1 + 2\Psi)\,\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j$$

$$k_2 = -2\frac{M_{\rm pl}^2 H^2}{v_0^2}(1-\sigma^2), \qquad f_4 = \frac{M_{\rm pl}^2}{6v_0^4}(1-\sigma^2)$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{G_{\rm N} M}{r^2} + \frac{\Upsilon_1 G_{\rm N} M''}{4}$$

$$\frac{\mathrm{d}\Psi}{\mathrm{d}r} = \frac{G_{\rm N} M}{r^2} - \frac{5\Upsilon_2 G_{\rm N} M'}{4r^2}$$

$$\sigma^2 \equiv \Lambda/(3M_{\rm pl}^2 H^2)$$

$$\phi(r,t) = v_0 t + \frac{v_0}{2H} \ln\left(1 - H^2 r^2\right)$$

$$G_{\rm N} = \frac{3G}{5\sigma^2 - 2}$$

$$\Upsilon_1 = \Upsilon_2 \equiv \Upsilon = -\frac{1}{3}(1-\sigma^2)$$

Kobayashi, Watanabe, Yamauchi, arXiv:1411.4130
KK, Sakstein, arXiv:1502.06872
Saito, Yamauchi, Mizuno, Gleyze, Langlois, arXiv:1503.01448
Babichev, KK, Langlois, Saito, Sakstein, arXiv:1606.06627

Toy models

- Two representative (toy) models
  - The background expansion is the same as LCDM
  - One parameter to describe deviations from LCDM

Chameleon: f(R) $|f_{R0}| = 10^{-4}, 10^{-5}, 10^{-6}$

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + L_m \right] \qquad f(R) = R - 2\Lambda - |f_{R0}| \frac{\bar{R}}{R^2}$$

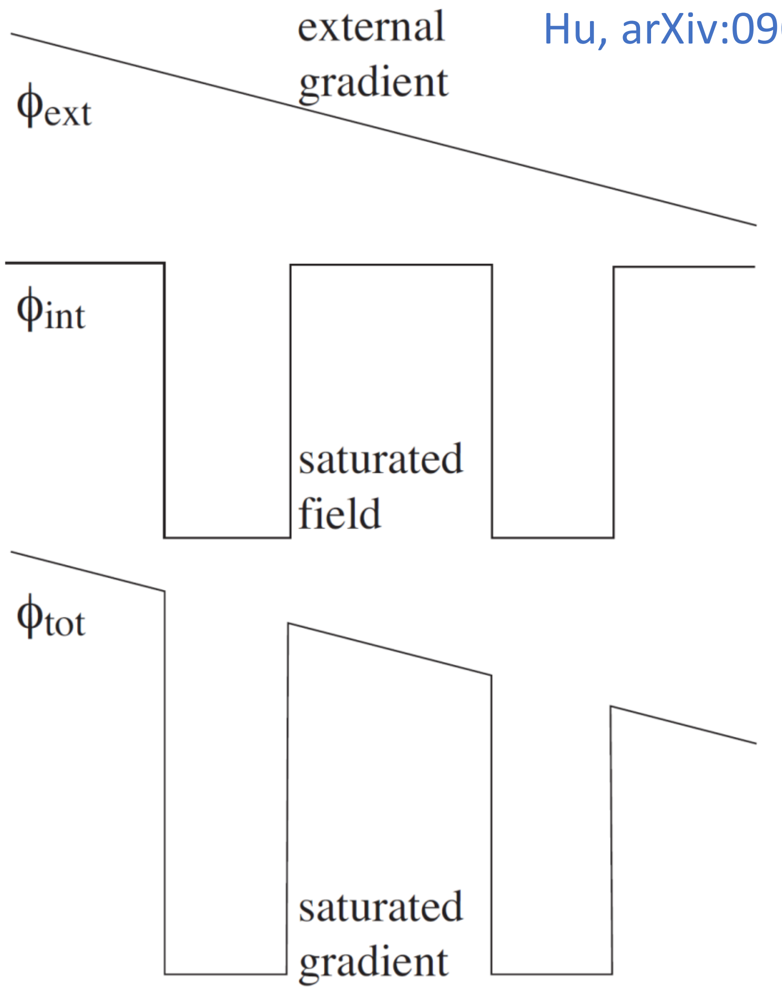
Vainshtein: normal branch DGP $H_0 r_c = 0.57, 1.2, 5.6$

$$S = \frac{1}{32\pi G r_c} \int d^5x \sqrt{-g_{(5)}} R_{(5)} + \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left(R - 2\Lambda \right)$$

1. Beyond a static spherically symmetric solution

- How does screening work inside composite objects like galaxies? If the galaxy is screened, does that mean everything inside the galaxy is also screened? – **Phil B**
- Departures from spherical symmetry - still not well understood? - **Jeremy Sakstein**

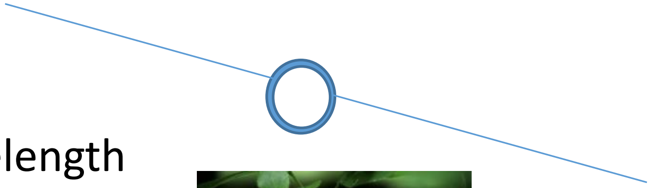
No superposition – chameleon



Hu, arXiv:0906.2024

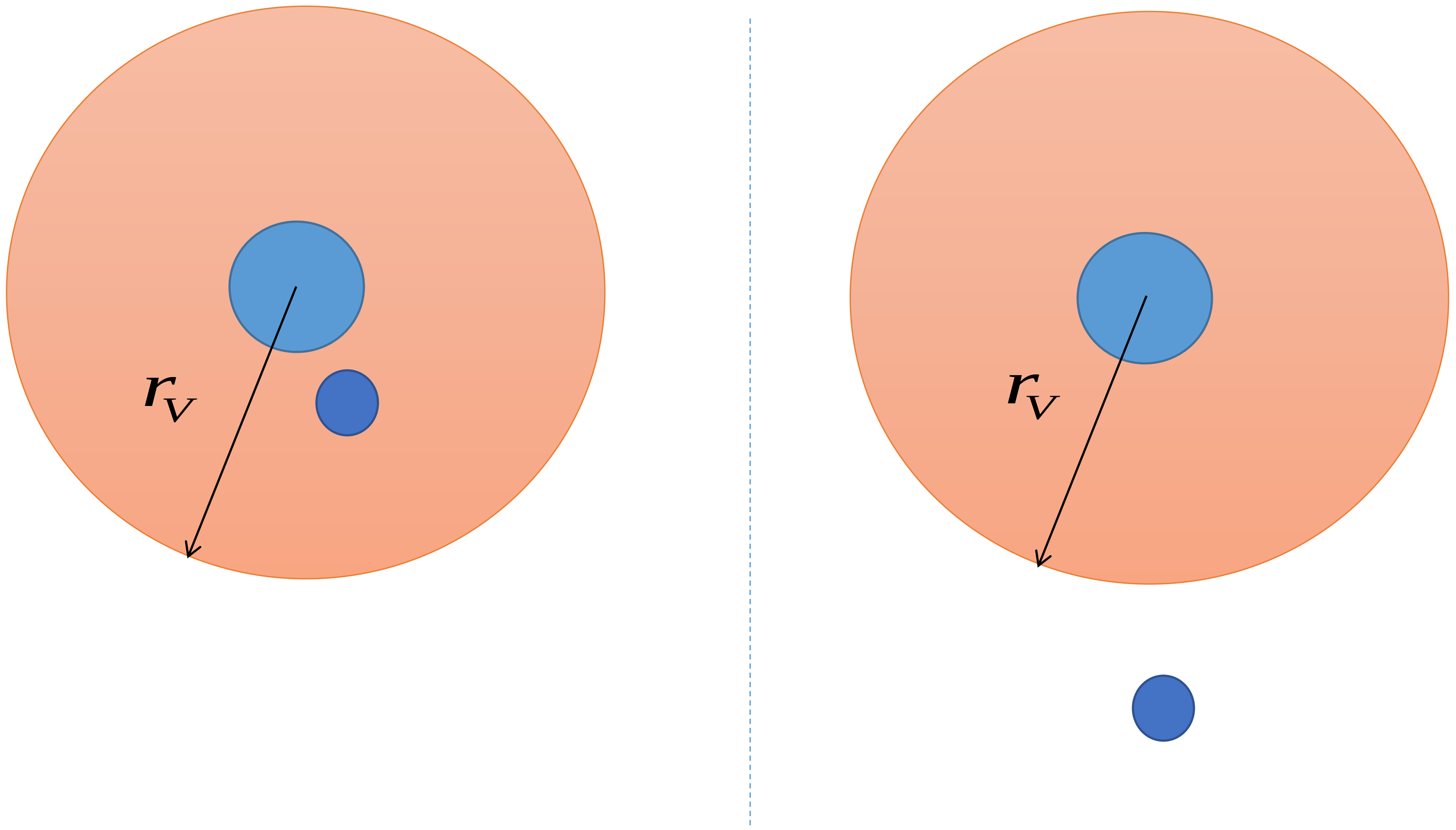
The field inside the screened objects loses knowledge of any exterior gradient

long
wavelength
mode



$$\nabla \varphi_{\text{total}} \sim 0$$

No superposition– Vainshtein



Superposition rule

- Newton gravity

$$\Psi = -\frac{M_A + M_B}{r} = \Psi_A + \Psi_B$$

- The fifth force within the Vainshtein radius $\varphi(r) = C\sqrt{M\,r}$

$$\varphi = \sqrt{(M_A + M_B)r} \neq \varphi_A + \varphi_B$$

$$\varphi = \varphi_A + \varphi_B + \varphi_\Delta$$

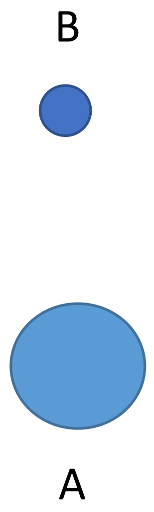
Apparent violation of equivalence principle

- Anomalous angle of perihelion advance

$$\cancel{\frac{\Delta\varphi_{\rm DGP}}{P} = \frac{3}{8}\frac{1}{r_c} = 7.91\left(\frac{h}{H_0 r_c}\right)\mu\text{arcsec/yr}}$$

$$\frac{\Delta\varphi_{\rm DGP}}{P} = \frac{3}{8}\frac{1}{r_c}(1+Q_1)$$

$$Q_1(0) \approx -0.56\left(\frac{M_B}{M_A}\right)^{0.6}\left[1 - 0.13\left(\frac{M_A}{M_B}\right)^{1/2}\left(\frac{r_{sB}}{d}\right)^{3/2}\right]$$



different mass bodies will precess at different rates

cf. Earth-moon

$$M_B / M_A = 1/80 \rightarrow Q_1 = 0.04$$

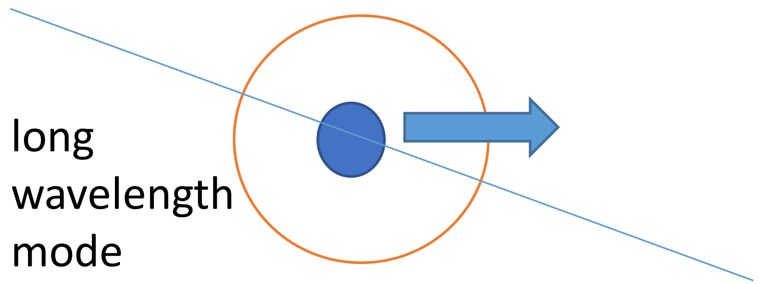
Hiramatsu, Hu, Koyama, Schmidt arXiv:1209.3364

Linearisation

- If body B is outside of the Vainshtein radius of body A

body B still feels the force from body A as we can add a constant gradient to the solution (Galileon symmetry)

$$\nabla\varphi = \nabla\varphi_B + \nabla\varphi_A \qquad \nabla\varphi_A \sim \text{const. near body B}$$



Hui, Nicolis, Syubbs arXiv:0905.2966, Hui, Nicolis, arXiv:1201.1508

Shape dependences

- Vainshtein   Bloomfield, Burrage, Davis, arXiv: 1408.4759

Vainshtein mechanism does not work for one dimensional object

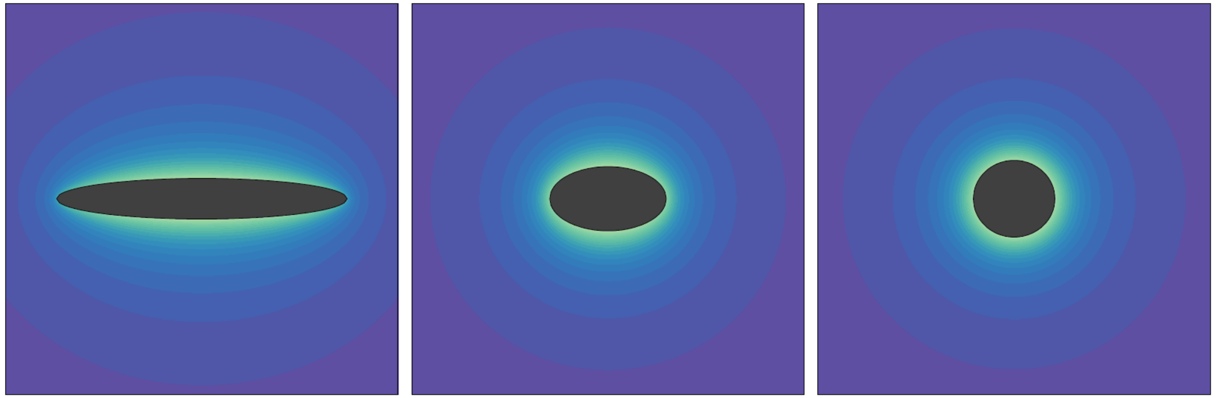
$$\left(\partial^2\pi\right)^2-\left(\partial_i\partial_j\pi\right)\left(\partial^i\partial^j\pi\right)=0$$

$$\frac{F_\phi}{F_G}=4\beta^2\frac{r}{r_v}$$:cylindrical

$$\frac{F_\phi}{F_G}=4\beta^2\left(\frac{r}{r_v}\right)^{3/2}$$:spherical

- Chameleon   Burrage, Copeland, Stevenson arXiv:1412.6373

Variation of the Chameleon Force F_φ



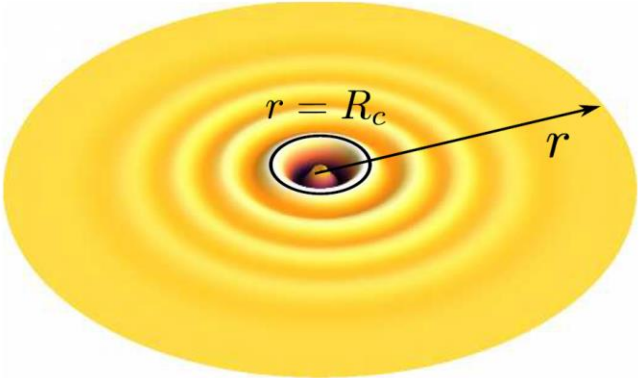
$F_\varphi/F_G=0.3056$ $F_\varphi/F_G=0.22795$ $F_\varphi/F_G=0.22093$

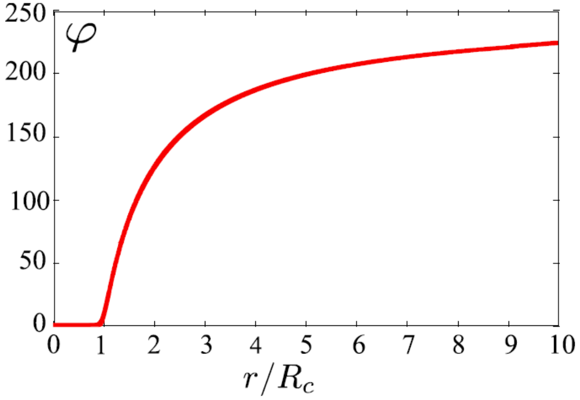
Fifth force is enhanced for a large ellipticity because the gravitational force suffers far more from the deformation

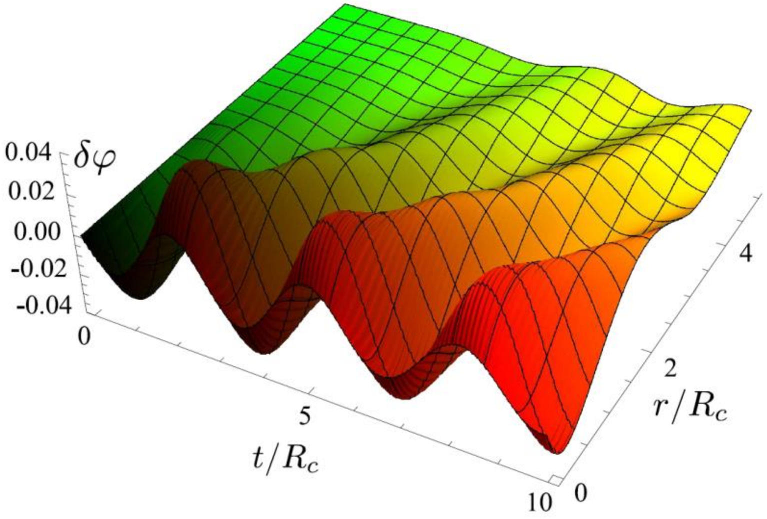
Time dependence – chameleon

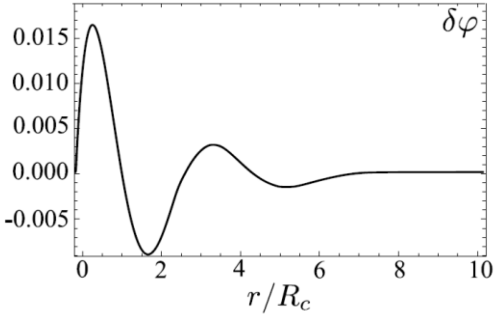
- Pulsating matter Silvestri arXiv:1103.4013

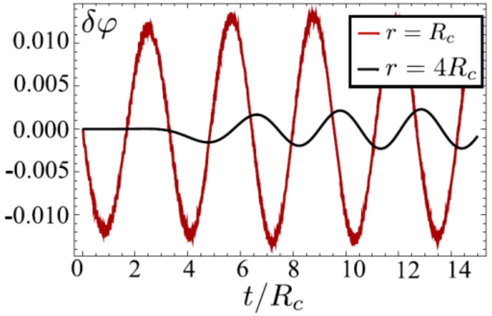
$$\delta(\tau, x) = 3\frac{\delta R_c}{R_c}\sin(\omega_0 R_c \tau)\theta(\tau)\theta(1-x)$$









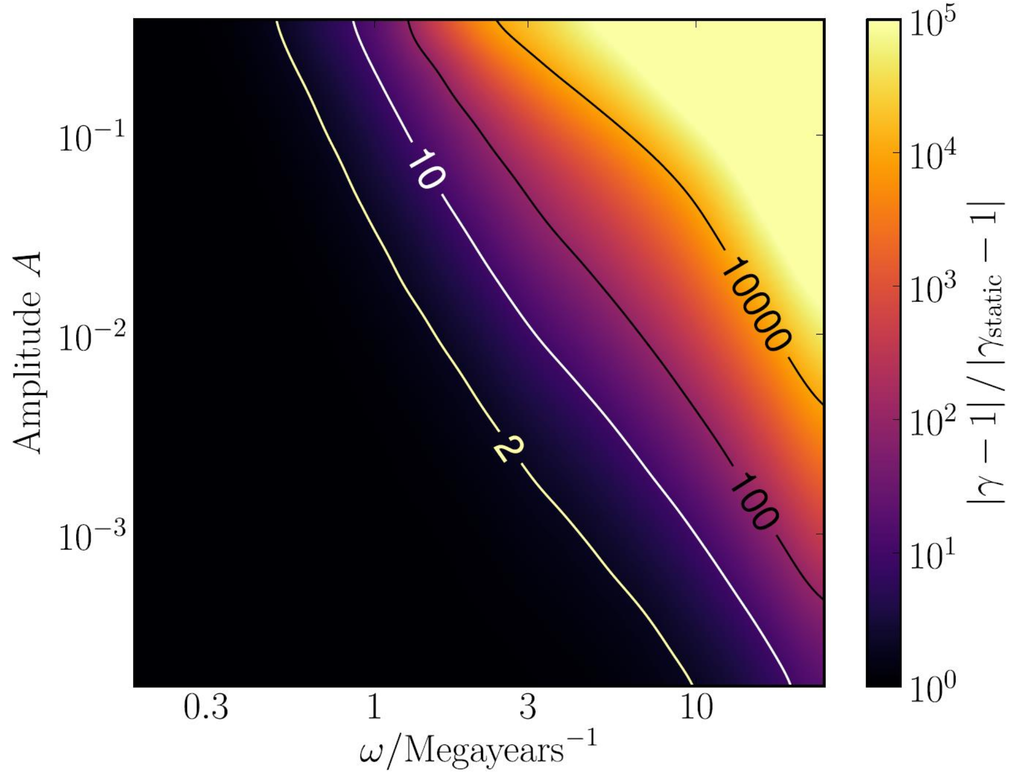


Time dependence – symmetron

- Spherically symmetric non-static solution Hagala,Llinares, Mota arXiv: 1607.02600

the effect of adding incoming waves
to the screening profile
on the PPN parameter

$$\phi(r_{\max}, t) = \phi_0(r_{\max}) + A\sin(\omega t)$$



Time dependence – Vainshtein

Brito, Terrana, Johnson, Cardoso, arXiv: 1409.0886

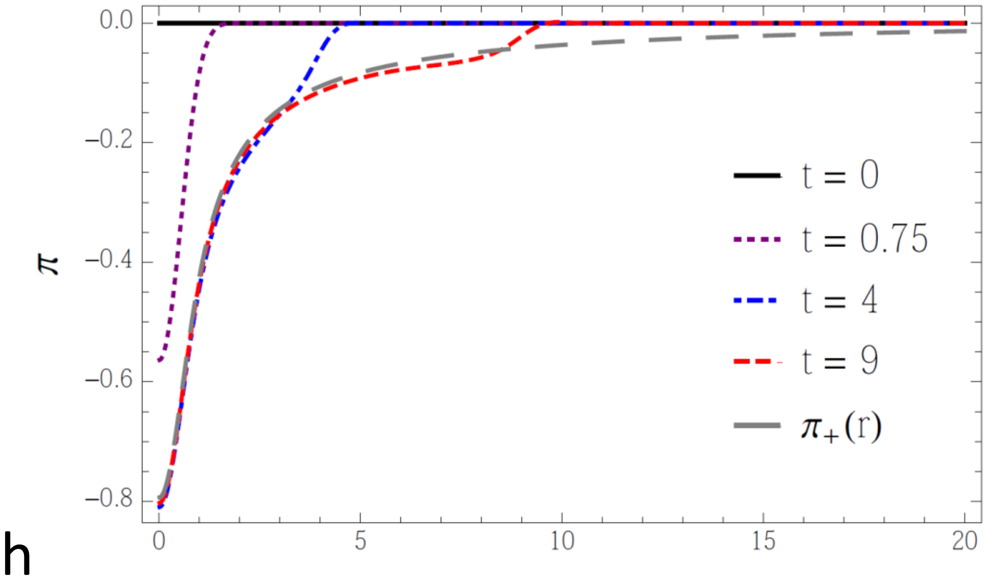
- Dynamical evolution

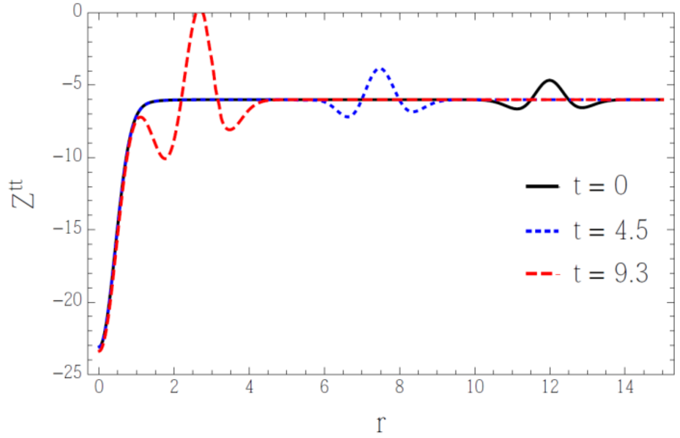
  static screening solutions are attractors and they are stable against small perturbations

- Cauchy problem

  perturbations around a screening profile with an incoming wave packet

$$S_{\delta\pi} = \int d^4x \left[\frac{1}{2} Z^{\mu\nu} \partial_\mu \delta\pi \partial_\nu \delta\pi + \frac{1}{2M_4} \delta\pi T\right]$$

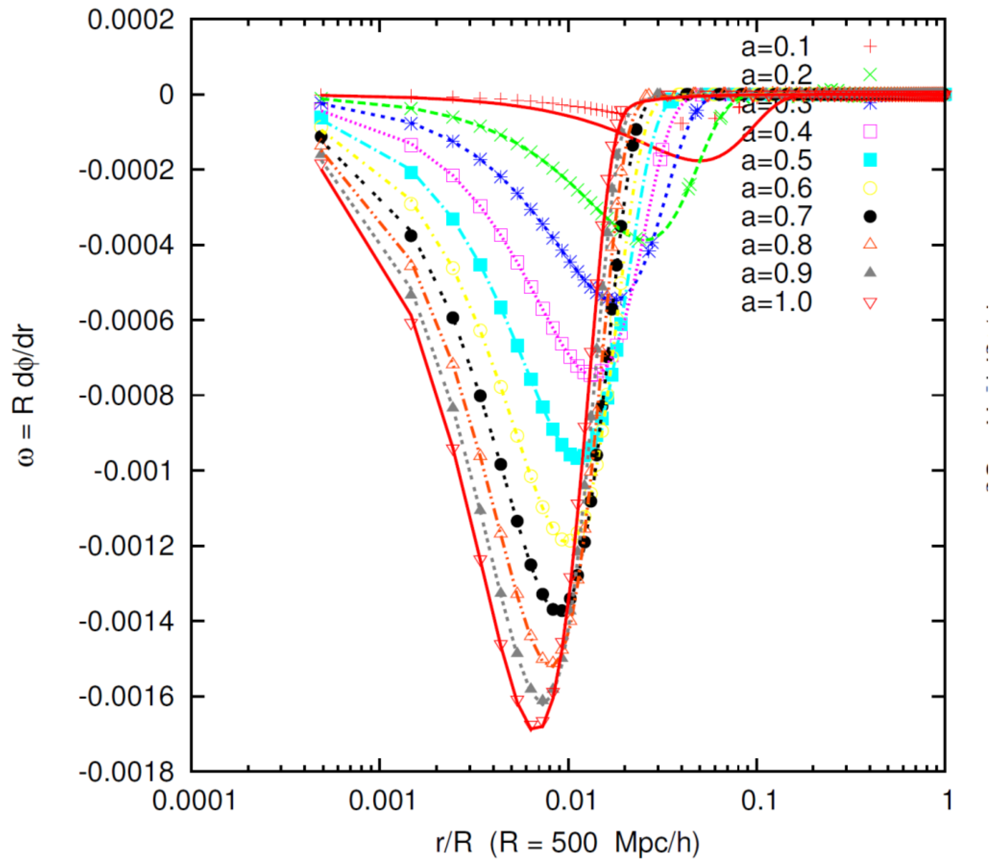




Time dependence – Vainshtein

- Spherically symmetric non-static solution Winther, Ferreira, arXiv: 1505.03539

  If the simulation starts with the profile of the scalar away from static solution, it will quickly start to evolve towards it



- Problems with voids

  solutions for the scalar cease to exist deep inside the void as $r_V < 0$

$$\frac{d\varphi}{dr} = \frac{r_g}{r^2}\Delta(r), \quad \Delta(r) = \frac{2}{3}\left(\frac{r}{r_V}\right)^3\left(\sqrt{1+\left(\frac{r_V}{r}\right)^3} - 1\right)$$

  this problem persists with time dependence

2. N-body simulations

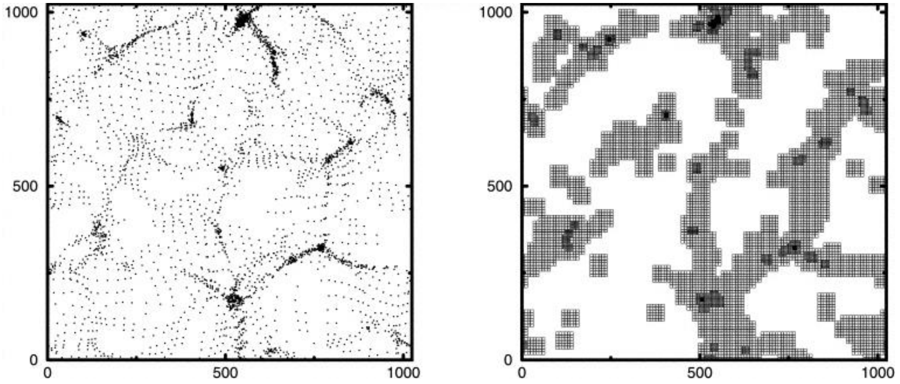
- The current status of N-body cosmological simulations of non-GR theories, and their importance in understanding screening effects on structure formation. **Alex Barreira**

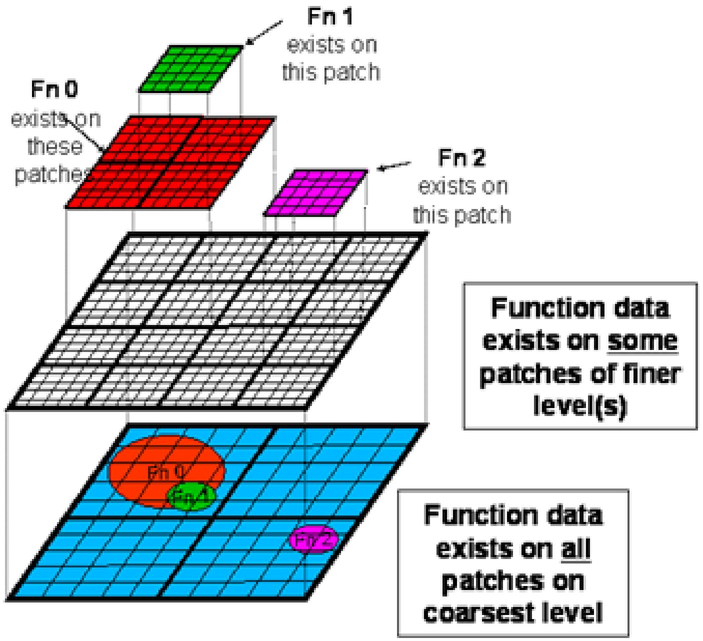
N-body Simulations for MG

- Multi-level adaptive mesh refinement
- solve Poisson equation using a linear Gauss-Seidel relaxation
- add a scalar field solver using a non-linear Gauss Seidel relaxation

ECOSMOG Li, Zhao, Teyssier, KK JCAP1201 (2012) 051
MG-GADGET Puchwein, Baldi, Springel MNRAS (2013) 436 348
ISIS Llinares, Mota, Winther A&A (2014) 562 A78
DGPM, Schmidt PRD80, 043001

Modified Gravity Simulations comparison project
Winther, Shcmidt, Barreira et.al. arXiv: 1506.06384





Models $n = 1, |f_{R0}| = 10^{-4}, 10^{-5}, 10^{-6}$

- Full f(R) simulations

  solve the non-linear scalar equation

$$\nabla^2 \delta f_R = -\frac{a^2}{3}[\delta R(f_R) + 8\pi G \delta \rho_{\mathrm{M}}]$$

- Non-Chameleon simulations

  artificially suppress the Chameleon by linearising the scalar equation to remove the Chameleon effect

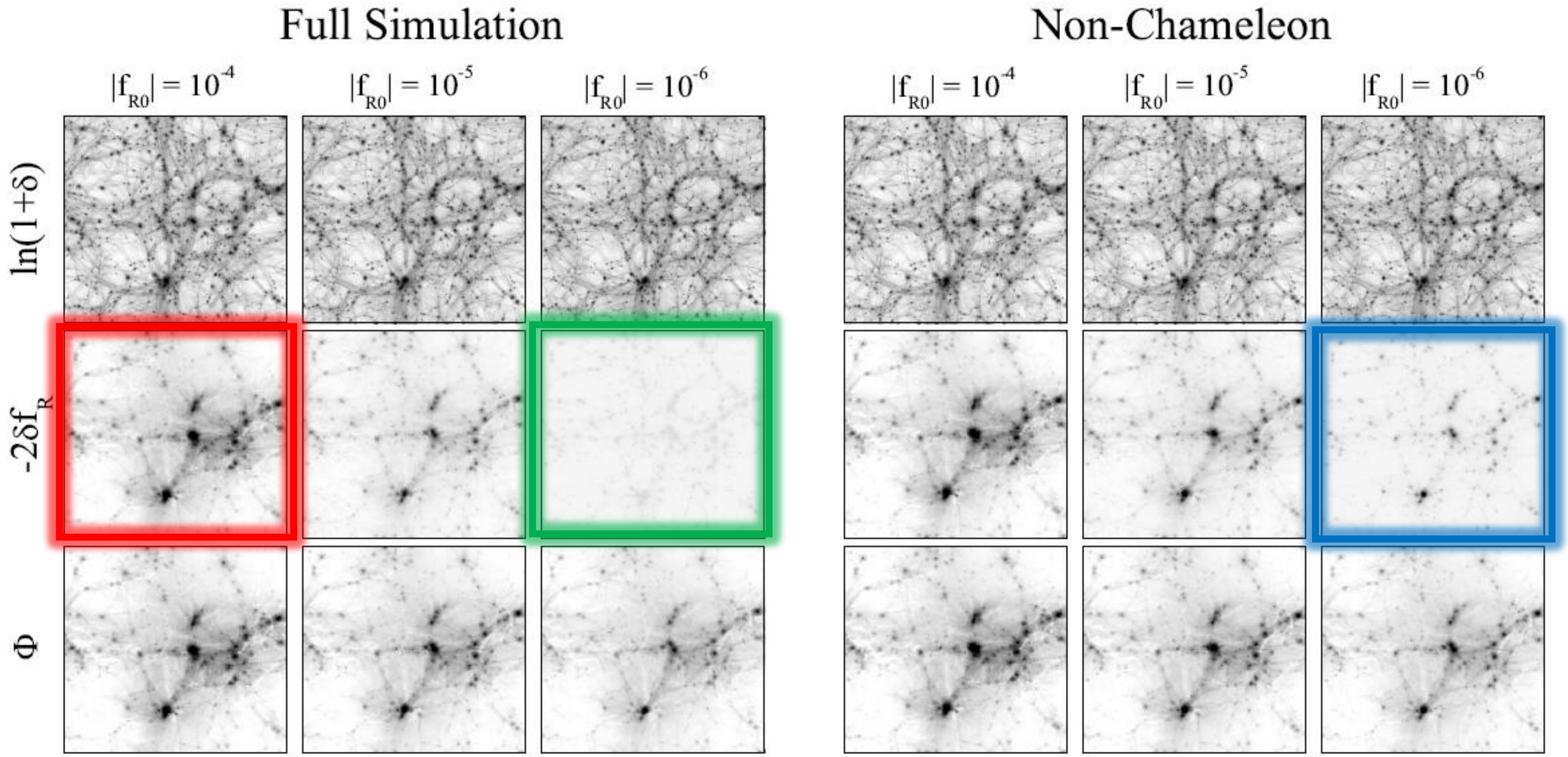
$$\nabla^2 \delta f_R = a^2 \bar{\mu}^2 \delta f_R - \frac{8\pi G}{3} a^2 \delta \rho_{\mathrm{M}}$$

Oyaizu et.al. PRD78 123524 2008, Schmidt et.al. PRD79 083518 2009

Snapshots at z=0

Zhao, Li, KK, arXiv:1011.1257

- If the fifth force is not suppressed, we have $-2\delta f_R = \Phi$.



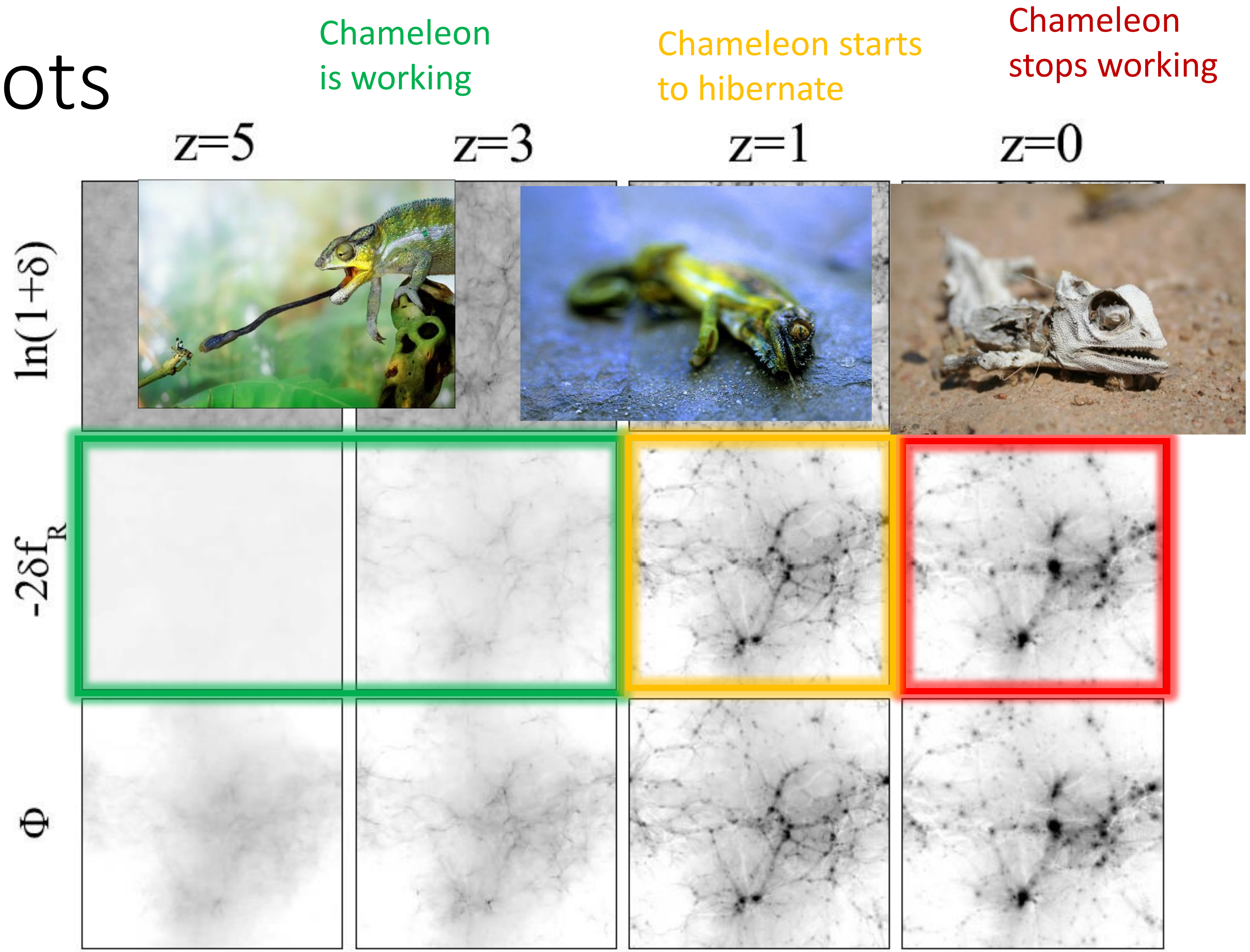
Fifth force is not suppressed

Chameleon is working

Compton wavelength is short

Snapshots

$|f_{R0}| = 10^{-4}$



Zhao, Li, KK, arXiv:1011.1257

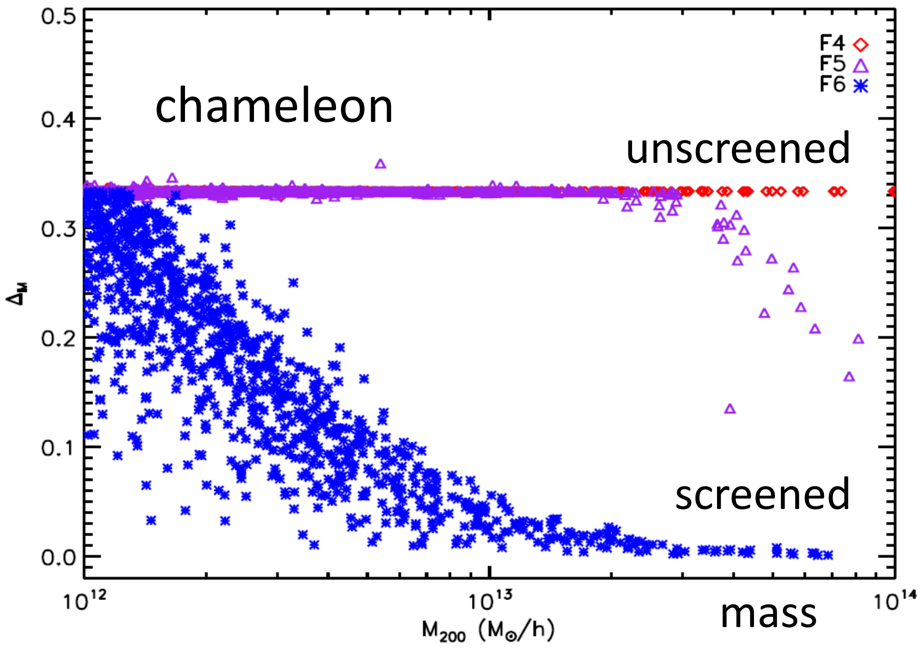
Screening of dark matter halo

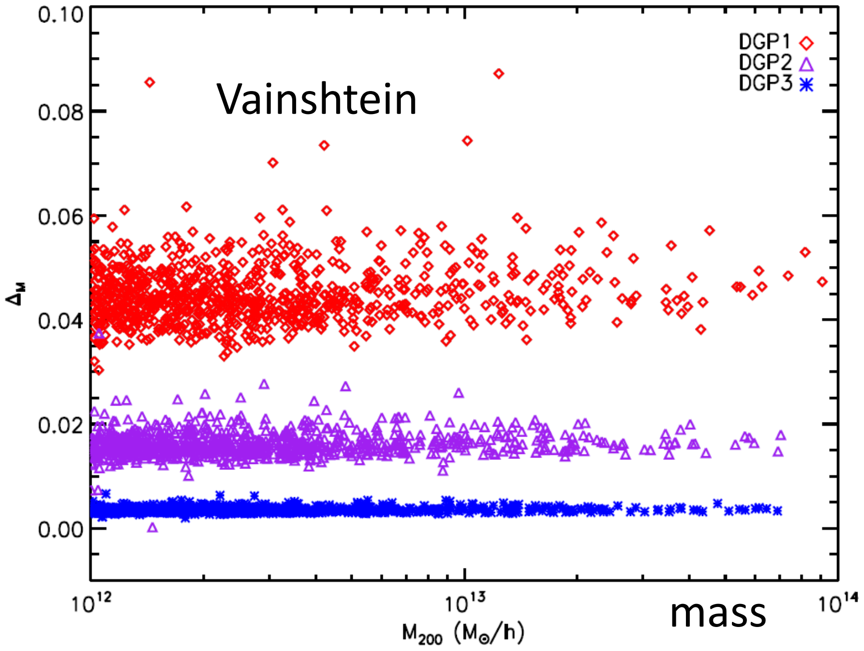
Schmidt, arXiv:1003.0409
Falck, KK, Zhao, arXiv:1503.06673

- Dynamical mass/lensing mass

$$\Delta_M(r) = \frac{d\Psi(r)/dr}{d\Psi_+(r)/dr} - 1, \quad \Psi_+ = \frac{\Phi+\Psi}{2}$$

$\Delta_M = d$





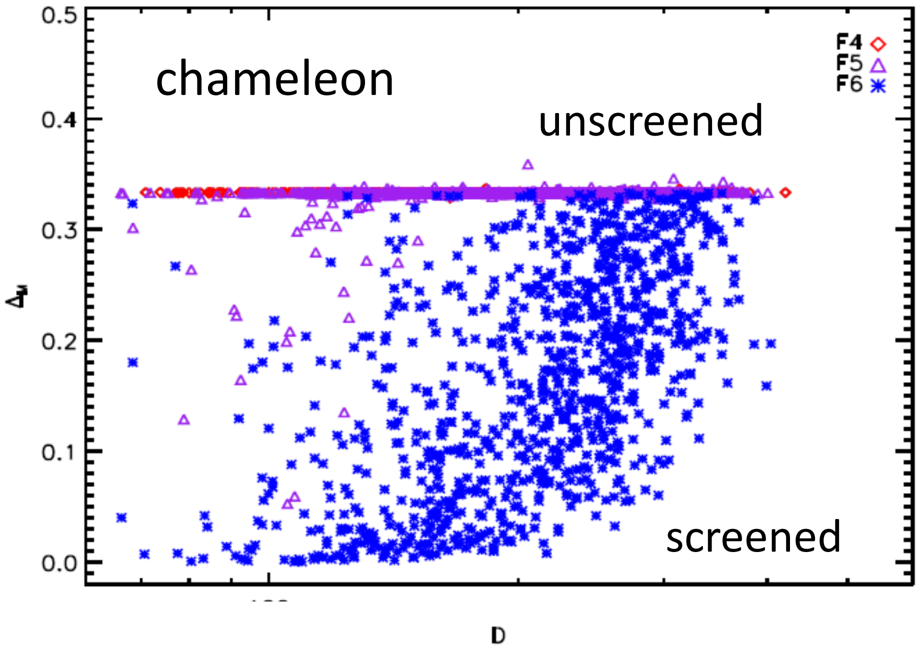
- Screening depends on mass of dark matter halos
- Massive halos with a deeper potential are more screened

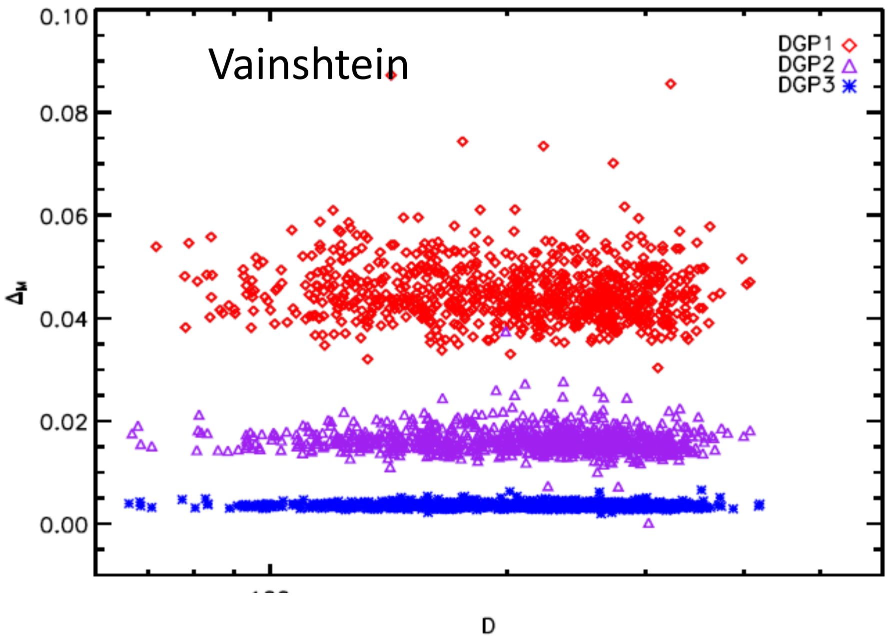
- Screening does not depend on mass of dark matter halos
- The Vainshtein radius is always larger than the size of halos

Screening of dark matter halo

Falck, KK, Zhao, arXiv:1503.06673

- Environment $D = d / r_{NB}$, d : distance to the nearest halo with $M_{NB} > M$





- Screening depends on environment
- Halos in “dense” environment are more screened

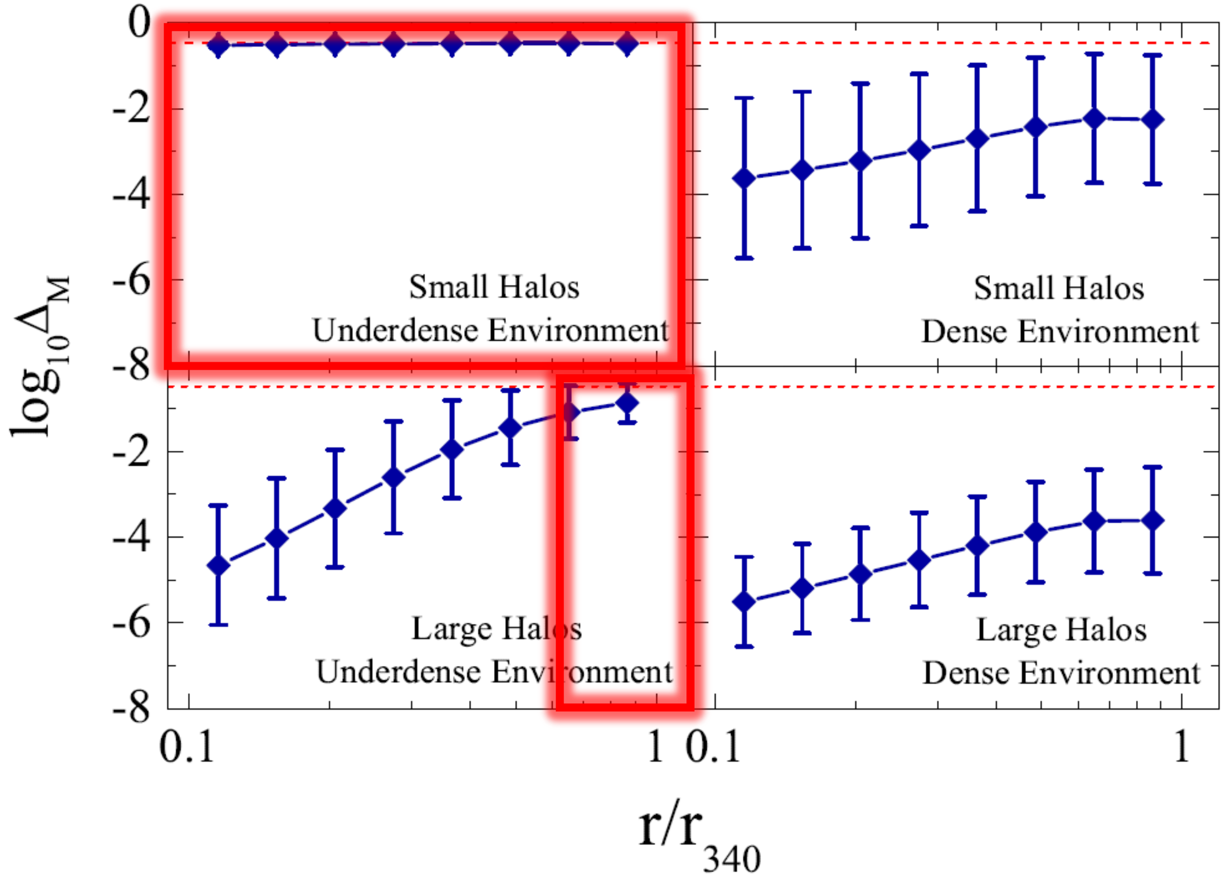
- Screening does not depend on environment

Environmental dependence Zhao, Li, Koyama 1011.1257

- Difference between lensing and dynamical mass

$$\Delta_M(r) = \frac{d\Psi(r)/dr}{d\Psi_+(r)/dr} - 1, \quad \Psi_+ = \frac{\Phi+\Psi}{2}$$

environment: $D = d / r_{NB}$



O(1) difference

no difference

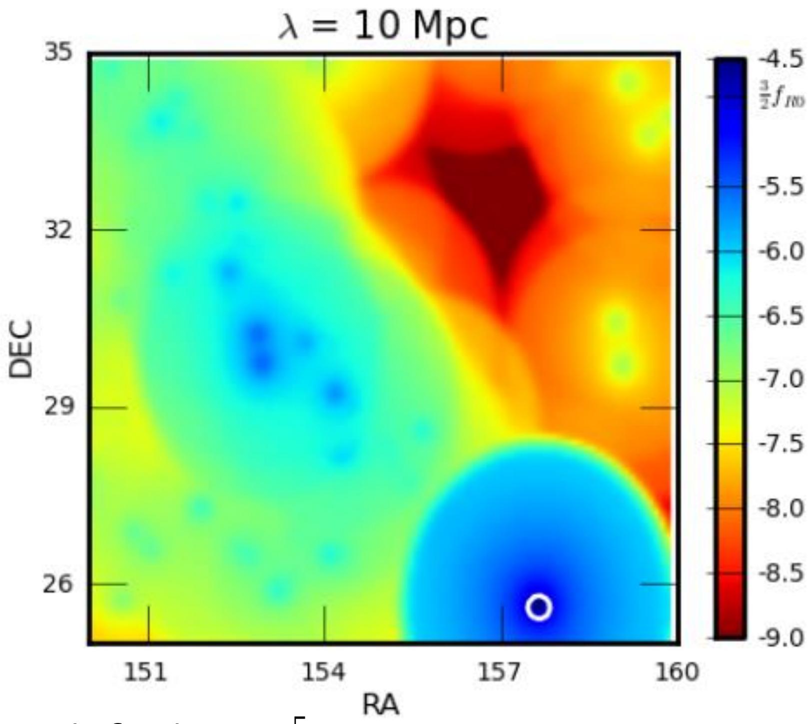
O(1) difference

no difference

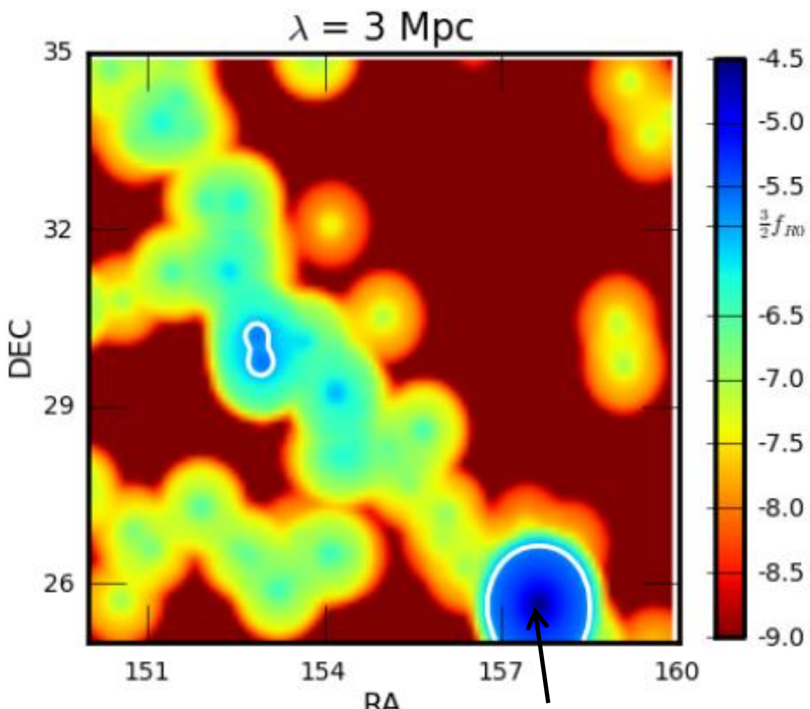
Creating a screening map

- It is essential to find places where GR is not recovered
  - Small galaxies in underdense regions
  - SDSS galaxies within 200 Mpc

Cabre, Vikram, Zhao, Jain, KK
arXiv:1204.6046



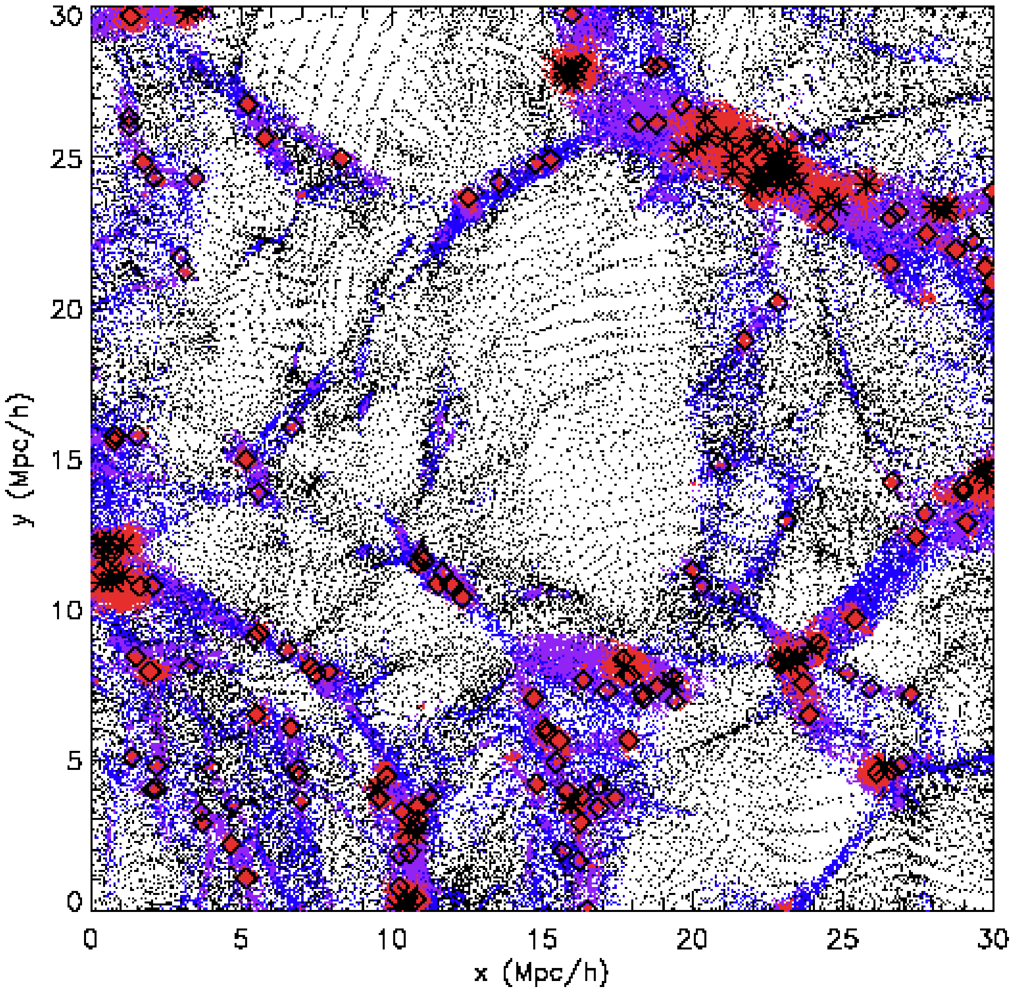
$|f_{R0}| = 10^{-5}$



$|f_{R0}| = 10^{-6}$

GR is recovered

Morphology



ORIGAMI finds shell-crossing by looking for particles out of order with respect to their original configuration

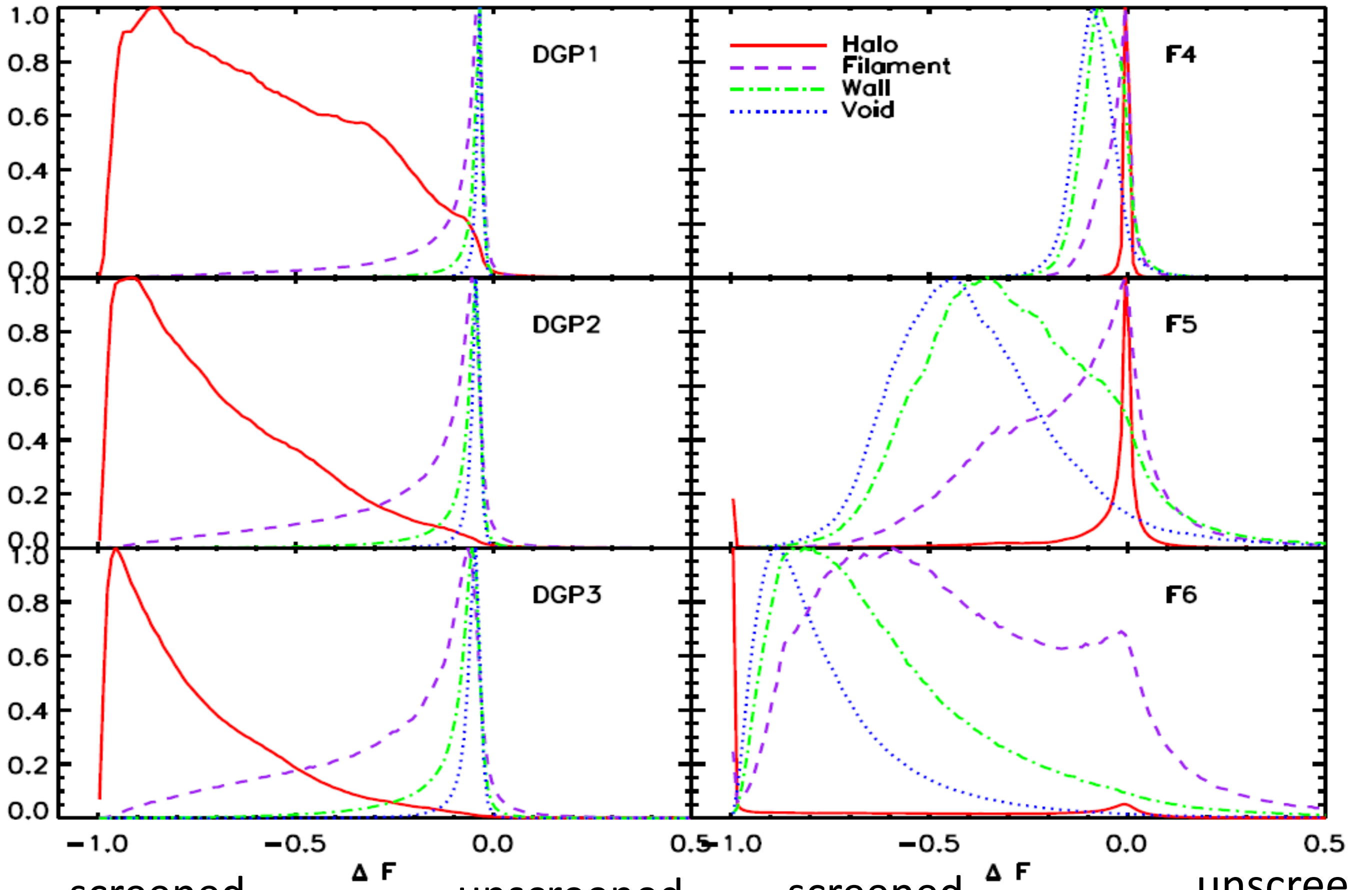
Halo particles have undergone shell-crossing along 3 orthogonal axes, filaments along 2, walls 1, and voids 0

Neyrinck, Falck & Szalay 1309.4787

Morphology dependence

Falck, Koyama, Zhao and Li 14004.2206

- Vainshtein
- Chameleon

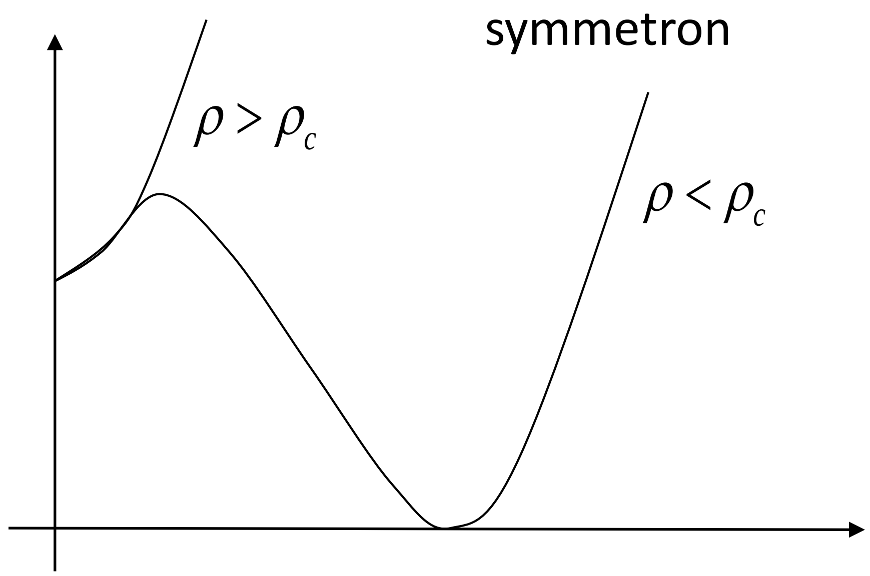


screened unscreened screened unscreened

Code	DGPM	ECOSMOG	MG-GADGET	ISIS	ISIS-NONSTATIC
Code paper	Schmidt (2009a)	Li et al. (2012, 2013a)	Puchwein et al. (2013)	Llinares et al. (2014)	Llinares & Mota (2014)
Base code	Oyaizu (2008)	RAMSES	P-GADGET3	RAMSES	RAMSES
Density assignment	CIC	CIC/TSC	CIC	CIC	CIC
Force assignment	CIC	CIC/TSC	effective mass	CIC	CIC
Adaptive refinement?	No	Yes	Yes	Yes	No
Timestep	Fixed	Adaptive	Adaptive	Adaptive	Adaptive
MG solver	Multigrid	Multigrid	Multigrid	Multigrid	Leapfrog
Gravity solver	Multigrid	Multigrid	TreePM	Multigrid	Multigird
Parallelisation	OpenMP	MPI	MPI	MPI	MPI
Programming language	C++	Fortran	C	Fortran	Fortran
Models simulated	DGP	$f(R)$/DGP	$f(R)$	$f(R)$/Symmetron	Symmetron

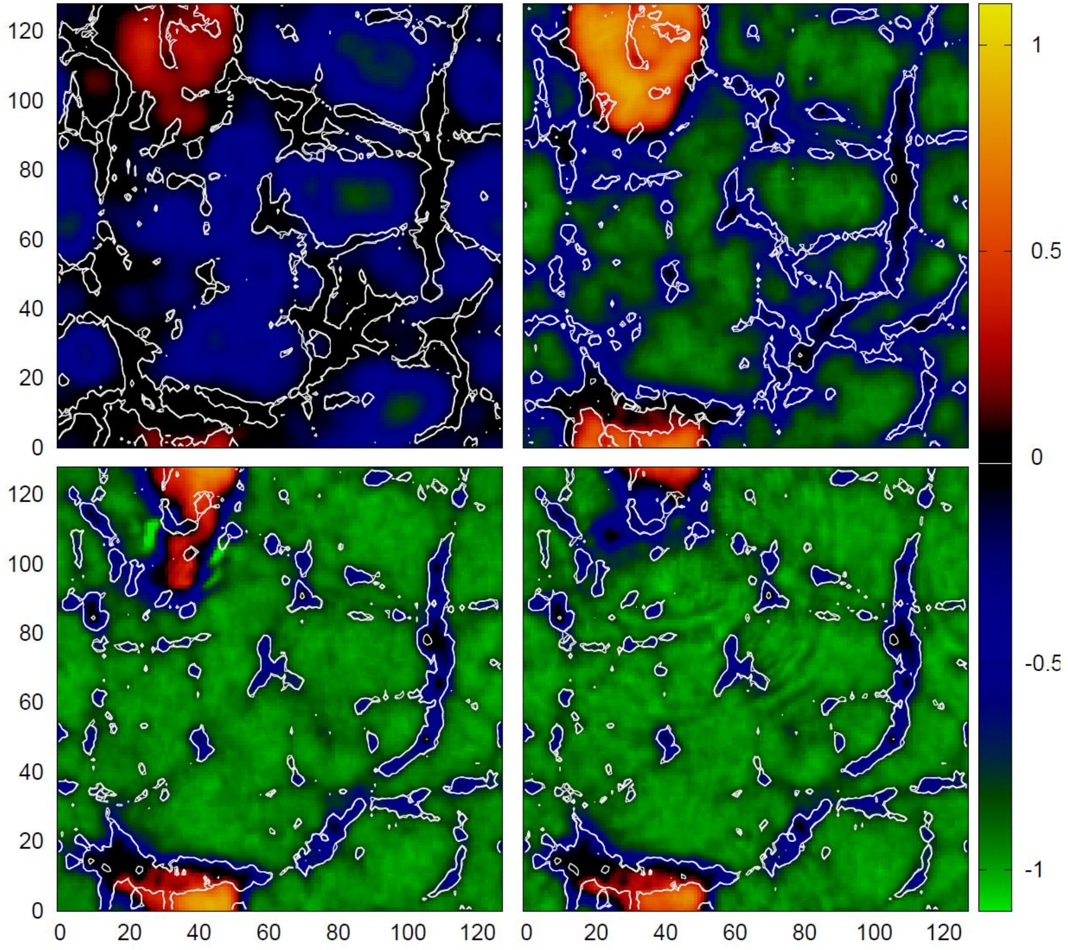
3. Topological defects

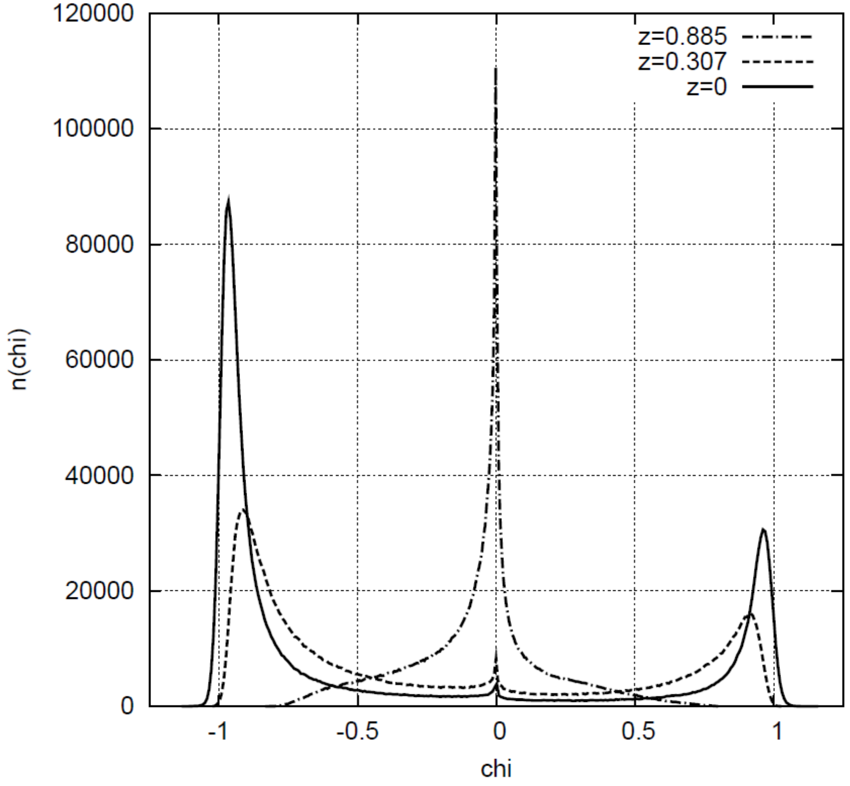
- Do screening mechanisms that rely on spontaneous symmetry breaking produce topological defects or other exotic objects? – **Phil B**



Yes - non-static simulations

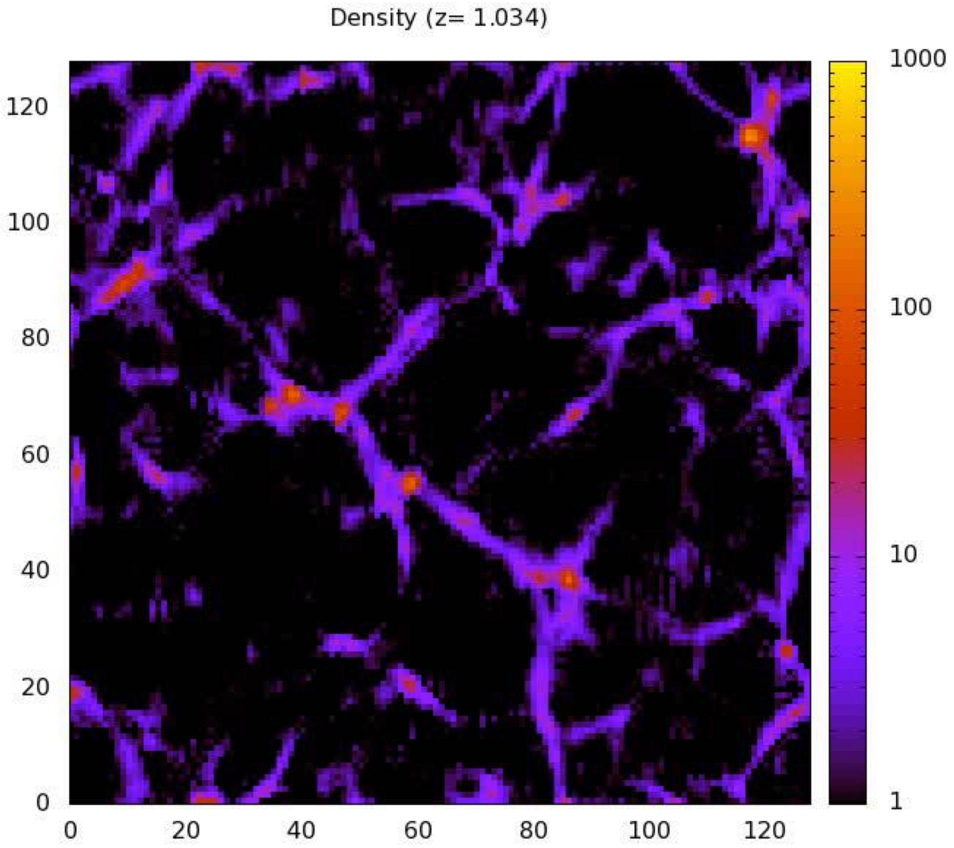
Linares, Mota, arXiv:1302.1774
Llinares, Pogosian, arXiv:1401.2857

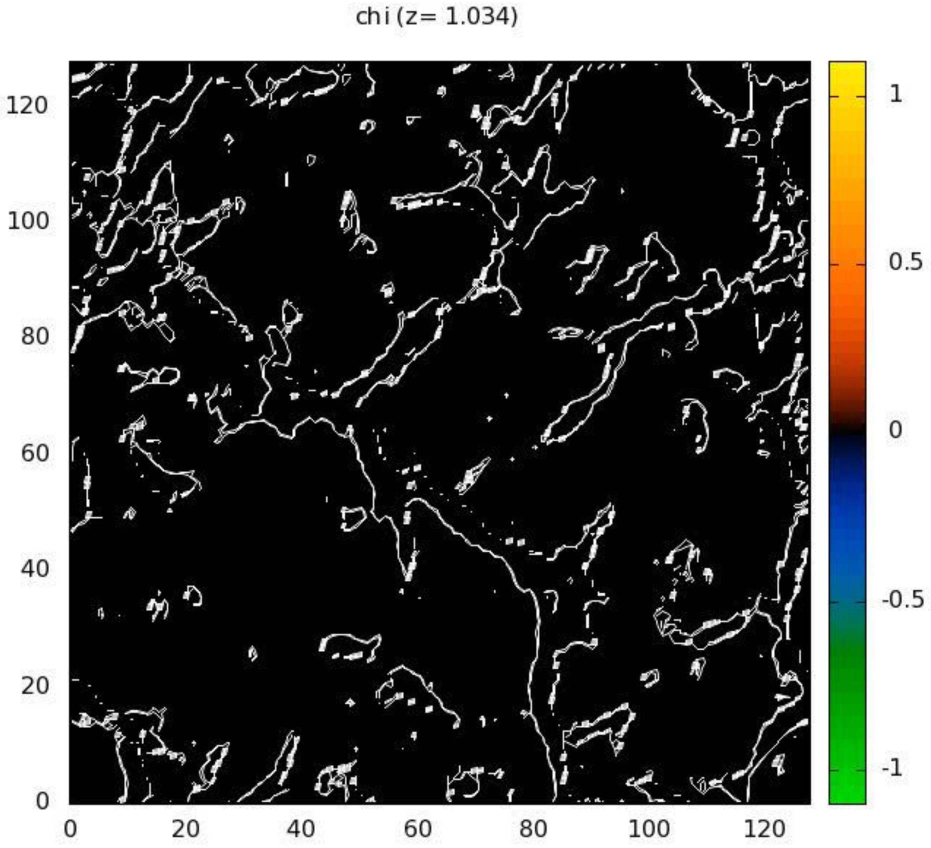




$z = 0.807, 0.366, 0.03$ and 0.008

©Claudio Linares





4. Parametrisation

- Is it possible to make a parametrisation of screening mechanisms? – **Phil B**
- Can PPN/PPK/PPF/EFT/etc. parameters be related to *any* properties of screening mechanisms? Or does screening depend on essentially different properties of the relevant theories? – **Phil B**
- Vainshtein mechanism in curved ST (i.e. broken Galilean symmetry)? **Miguel Z.**

Chameleon/Symmetron/dilaton

- Einstein frame

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] + S_m(A^2(\phi)g_{\mu\nu})$$

$$m^2 = V''_{\rm eff}(\bar{\phi}) \quad \beta = M_{\rm Pl}\frac{d\ln A}{d\phi}\bigg|_{\phi=\bar{\phi}}$$

Brax, Davis, Li, Winther arXiv:1203.4812

assuming the cosmological scale field is in the minimum of the effective potential, we can describe full non-linear dynamics of the theory

$$\frac{\mathrm{d}V}{\mathrm{d}\phi}\Big|_{\phi_{min}} = -\beta A\frac{\rho_m}{m_{\rm Pl}}$$

$$\phi(a) = \frac{3}{m_{\rm Pl}}\int_{a_{\rm ini}}^{a}\frac{\beta(a)}{am^2(a)}\rho_m(a)\mathrm{d}a + \phi_c$$

$$V = V_0 - \frac{3}{m_{\rm Pl}^2}\int_{a_{\rm ini}}^{a}\frac{\beta^2(a)}{am^2(a)}\rho_m^2(a)\mathrm{d}a,$$

Horndeski theory

$$\mathcal{L}_{\rm GG} = K(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$
$$+G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

- Quasi-static approximation Kimura, Kobayashi, Yamamoto arXiv:1111.6749

$$ds^2 = -(1+2\Phi(t,\mathbf{x}))dt^2 + a^2(t)(1-2\Psi(t,\mathbf{x}))d\mathbf{x}^2 \qquad \phi \to \phi(t) + \delta\phi(t,\mathbf{x}) \qquad Q = H\delta\phi/\dot{\phi}$$

$$\nabla^2\left(\mathcal{F}_T\Psi - \mathcal{G}_T\Phi - A_1 Q\right) = \frac{B_1}{2a^2H^2}\mathcal{Q}^{(2)} + \frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2 Q - \partial_i\partial_j\Phi\partial^i\partial^j Q\right)$$

$$\mathcal{G}_T\nabla^2\Psi = \frac{a^2}{2}\rho_{\rm m}\delta - A_2\nabla^2 Q - \frac{B_2}{2a^2H^2}\mathcal{Q}^{(2)}$$
$$-\frac{B_3}{a^2H^2}\left(\nabla^2\Psi\nabla^2 Q - \partial_i\partial_j\Psi\partial^i\partial^j Q\right) - \frac{C_1}{3a^4H^4}\mathcal{Q}^{(3)},$$

$$A_0\nabla^2 Q - A_1\nabla^2\Psi - A_2\nabla^2\Phi + \frac{B_0}{a^2H^2}\mathcal{Q}^{(2)} - \frac{B_1}{a^2H^2}\left(\nabla^2\Psi\nabla^2 Q - \partial_i\partial_j\Psi\partial^i\partial^j Q\right)$$
$$-\frac{B_2}{a^2H^2}\left(\nabla^2\Phi\nabla^2 Q - \partial_i\partial_j\Phi\partial^i\partial^j Q\right) - \frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2\Psi - \partial_i\partial_j\Phi\partial^i\partial^j\Psi\right)$$
$$-\frac{C_0}{a^4H^4}\mathcal{Q}^{(3)} - \frac{C_1}{a^4H^4}\mathcal{U}^{(3)} = 0,$$

$$\mathcal{Q}^{(2)} := \left(\nabla^2 Q\right)^2 - \left(\partial_i\partial_j Q\right)^2,$$
$$\mathcal{Q}^{(3)} := \left(\nabla^2 Q\right)^3 - 3\nabla^2 Q\left(\partial_i\partial_j Q\right)^2 + 2\left(\partial_i\partial_j Q\right)^3$$

$$\mathcal{U}^{(3)} := \mathcal{Q}^{(2)}\nabla^2\Phi - 2\nabla^2 Q\partial_i\partial_j Q\partial^i\partial^j\Phi + 2\partial_i\partial_j Q\partial^j\partial^k Q\partial_k\partial^i\Phi.$$

Galileon symmetry

$$Q \to Q + c + b_\mu x^\mu$$

- Quasi-static approximation
- Weak field limit $\Phi, \ \Psi, \ \text{and} \ Q \ \sim \ \epsilon$
- Keep non-linearity of the second derivatives $(\partial^2\epsilon)^2 \ \text{and} \ (\partial^2\epsilon)^3$
- All terms in e.o.m can be written as a total derivative

$$
\begin{aligned}
&\nabla^2\Phi\nabla^2 Q - \partial_i\partial_j\Phi\partial^i\partial^j Q = \partial_i\left(\partial^i\Phi\nabla^2 Q - \partial_j\Phi\partial^i\partial^j Q\right), \\
&\mathcal{U}^{(3)} = \partial_i\Big(\partial^i\Phi\mathcal{Q}^{(2)} - 2\partial_j\Phi\nabla^2 Q\partial^i\partial^j Q \\
&\qquad\qquad\qquad\qquad +2\partial^j\Phi\partial_k\partial_j Q\partial^k\partial^i Q\Big).
\end{aligned}
$$

$$
\boxed{\begin{aligned}
\mathcal{F}_T &= 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right], \\
\mathcal{G}_T &= 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right],
\end{aligned}}
$$

$$
\begin{aligned}
\Theta &= -\dot{\phi}XG_{3X} + 2HG_4 - 8HXG_{4X} - 8HX^2G_{4XX} + \dot{\phi}G_{4\phi} + 2X\dot{\phi}G_{4\phi X} \\
&\quad -H^2\dot{\phi}\left(5XG_{5X} + 2X^2G_{5XX}\right) + 2HX\left(3G_{5\phi} + 2XG_{5\phi X}\right), \\
\mathcal{E} &= 2XK_X - K + 6X\dot{\phi}HG_{3X} - 2XG_{3\phi} - 6H^2G_4 + 24H^2X(G_{4X} + XG_{4XX}) \\
&\quad -12HX\dot{\phi}G_{4\phi X} - 6H\dot{\phi}G_{4\phi} + 2H^3X\dot{\phi}(5G_{5X} + 2XG_{5XX}) \\
&\quad -6H^2X(3G_{5\phi} + 2XG_{5\phi X}), \\
\mathcal{P} &= K - 2X(G_{3\phi} + \ddot{\phi}G_{3X}) + 2(3H^2 + 2\dot{H})G_4 - 12H^2XG_{4X} - 4H\dot{X}G_{4X} \\
&\quad -8\dot{H}XG_{4X} - 8HX\dot{X}G_{4XX} + 2(\ddot{\phi} + 2H\dot{\phi})G_{4\phi} + 4XG_{4\phi\phi} + 4X(\ddot{\phi} - 2H\dot{\phi})G_{4\phi X} \\
&\quad -2X(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 3H^2\ddot{\phi})G_{5X} - 4H^2X^2\ddot{\phi}G_{5XX} + 4HX(\dot{X} - HX)G_{5\phi X} \\
&\quad +2\left[2(HX)^{\cdot} + 3H^2X\right]G_{5\phi} + 4HX\dot{\phi}G_{5\phi\phi}.
\end{aligned}
$$

Non-linear

$$
\boxed{\begin{aligned}
B_0 &= \frac{X}{H}\Big\{\dot{\phi}G_{3X} + 3\left(\dot{X} + 2HX\right)G_{4XX} + 2X\dot{X}G_{4XXX} - 3\dot{\phi}G_{4\phi X} + 2\dot{\phi}XG_{4\phi XX} \\
&\quad + \left(\dot{H} + H^2\right)\dot{\phi}G_{5X} + \dot{\phi}\left[2H\dot{X} + \left(\dot{H} + H^2\right)X\right]G_{5XX} + H\dot{\phi}X\dot{X}G_{5XXX} \\
&\quad -2\left(\dot{X} + 2HX\right)G_{5\phi X} - \dot{\phi}XG_{5\phi\phi X} - X\left(\dot{X} - 2HX\right)G_{5\phi XX}\Big\}, \\
B_1 &= 2X\left[G_{4X} + \ddot{\phi}\left(G_{5X} + XG_{5XX}\right) - G_{5\phi} + XG_{5\phi X}\right], \\
B_2 &= -2X\left(G_{4X} + 2XG_{4XX} + H\dot{\phi}G_{5X} + H\dot{\phi}XG_{5XX} - G_{5\phi} - XG_{5\phi X}\right), \\
B_3 &= H\dot{\phi}XG_{5X}, \\
C_0 &= 2X^2G_{4XX} + \frac{2X^2}{3}\left(2\ddot{\phi}G_{5XX} + \ddot{\phi}XG_{5XXX} - 2G_{5\phi X} + XG_{5\phi XX}\right), \\
C_1 &= H\dot{\phi}X\left(G_{5X} + XG_{5XX}\right),
\end{aligned}}
$$

$$
\boxed{\begin{aligned}
A_0 &= \frac{\dot{\Theta}}{H^2} + \frac{\Theta}{H} + \mathcal{F}_T - 2\mathcal{G}_T - 2\frac{\dot{\mathcal{G}}_T}{H} - \frac{\mathcal{E} + \mathcal{P}}{2H^2}, \\
A_1 &= \frac{\dot{\mathcal{G}}_T}{H} + \mathcal{G}_T - \mathcal{F}_T, \\
A_2 &= \mathcal{G}_T - \frac{\Theta}{H},
\end{aligned}}
$$

linear

- Linear solution

$$
\begin{aligned}
\delta_1(t,\mathbf{p}) &= D_+(t)\delta_{\rm L}(\mathbf{p}),\\
\theta_1(t,\mathbf{p}) &= -D_+(t)f(t)\delta_{\rm L}(\mathbf{p}),\\
\Phi_1(t,\mathbf{p}) &= -\frac{a^2H^2}{p^2}D_+(t)\kappa_\Phi(t)\delta_{\rm L}(\mathbf{p}),\\
\Psi_1(t,\mathbf{p}) &= -\frac{a^2H^2}{p^2}D_+(t)\kappa_\Psi(t)\delta_{\rm L}(\mathbf{p})\\
Q_1(t,\mathbf{p}) &= -\frac{a^2H^2}{p^2}D_+(t)\kappa_Q(t)\delta_{\rm L}(\mathbf{p})
\end{aligned}
$$

$$
\frac{d^2D_+(t)}{dt^2}+2H\frac{dD_+(t)}{dt}+L(t)D_+(t)=0,
$$

$$
L(t) = -\frac{(A_0\mathcal{F}_T-A_1^2)\rho_{\rm m}}{2(A_0\mathcal{G}_T^2+2A_1A_2\mathcal{G}_T+A_2\mathcal{F}_T)},
$$

$$
\begin{aligned}
\mathcal{R}(t) &= A_0\mathcal{F}_T-A_1^2,\\
\mathcal{S}(t) &= A_0\mathcal{G}_T+A_1A_2,\\
\mathcal{T}(t) &= A_1\mathcal{G}_T+A_2\mathcal{F}_T,\\
\mathcal{Z}(t) &= 2\left(A_0\mathcal{G}_T^2+2A_1A_2\mathcal{G}_T+A_2^2\mathcal{F}_T\right),\\
\kappa_\Phi(t) &= \frac{\rho_{\rm m}\mathcal{R}}{H^2\mathcal{Z}},\\
\kappa_\Psi(t) &= \frac{\rho_{\rm m}\mathcal{S}}{H^2\mathcal{Z}},\\
\kappa_Q(t) &= \frac{\rho_{\rm m}\mathcal{T}}{H^2\mathcal{Z}},
\end{aligned}
$$

Effective Field Theory

$$
\begin{aligned}
M_*^2 \equiv& 2\left(G_4 - 2XG_{4X} + XG_{5\phi} - \dot{\phi}HXG_{5X}\right)\\
HM_*^2\alpha_{\mathrm{M}} \equiv& \frac{\mathrm{d}}{\mathrm{d}t}M_*^2\\
H^2M_*^2\alpha_{\mathrm{K}} \equiv& 2X\left(K_X + 2XK_{XX} - 2G_{3\phi} - 2XG_{3\phi X}\right) +\\
&+ 12\dot{\phi}XH\left(G_{3X} + XG_{3XX} - 3G_{4\phi X} - 2XG_{4\phi XX}\right) +\\
&+ 12XH^2\left(G_{4X} + 8XG_{4XX} + 4X^2G_{4XXX}\right) -\\
&- 12XH^2\left(G_{5\phi} + 5XG_{5\phi X} + 2X^2G_{5\phi XX}\right) +\\
&+ 4\dot{\phi}XH^3\left(3G_{5X} + 7XG_{5XX} + 2X^2G_{5XXX}\right)\\
HM_*^2\alpha_{\mathrm{B}} \equiv& 2\dot{\phi}\left(XG_{3X} - G_{4\phi} - 2XG_{4\phi X}\right) +\\
&+ 8XH\left(G_{4X} + 2XG_{4XX} - G_{5\phi} - XG_{5\phi X}\right) +\\
&+ 2\dot{\phi}XH^2\left(3G_{5X} + 2XG_{5XX}\right)\\
M_*^2\alpha_{\mathrm{T}} \equiv& 2X\left(2G_{4X} - 2G_{5\phi} - \left(\ddot{\phi} - \dot{\phi}H\right)G_{5X}\right)
\end{aligned}
$$

To do list

- Extend EFT to higher orders

  keep relevant operators for the Vainshtein mechanism

- Study higher order interactions in beyond Horndeski (and its extension)

  cf. EFT contains one more parameter α_H

$$\nabla^2\Psi + \frac{5\epsilon}{4}\left[(\nabla^2\pi)^2 - (\nabla_i\nabla_j\pi)(\nabla^i\nabla^j\pi)\right] = 4\pi G\rho,$$

$$\nabla^2\Phi - \nabla^2\Psi - \frac{\epsilon}{4}\left[(\nabla^2\pi)^2 + 3(\nabla_i\nabla_j\pi)(\nabla^i\nabla^j\pi) + \boxed{4(\nabla_i\nabla^2\pi)(\nabla^i\pi)}\right] = 0,$$

$$\nabla^2\pi - \frac{2}{\Lambda^4}\left[(\nabla^2\pi)^3 - 3(\nabla^2\pi)(\nabla_i\nabla_j\pi)(\nabla^i\nabla^j\pi) + 2(\nabla_i\nabla_j\pi)(\nabla_k\nabla^i\pi)(\nabla^k\nabla^j\pi)\right] +$$
$$\varepsilon\left[5(\nabla^2\Phi)(\nabla^2\pi) - 5(\nabla_i\nabla_j\Phi)(\nabla^i\nabla^j\pi) + (\nabla^2\Psi)(\nabla^2\pi) + (\nabla_i\nabla_j\Psi)(\nabla^i\nabla^j\pi) + \boxed{2(\nabla_i\nabla^2\Psi)(\nabla^i\pi)}\right] = 0$$

KK, Sakstein, arXiv:1502.06872

5. Strong gravity

Theoretical Physics Implications of the Binary Black-Hole Merger GW150914

Nicolás Yunes,[1] Kent Yagi,[2] and Frans Pretorius[2]

GW150914 constrains the properties of exotic compact object alternatives to Kerr black holes. The true potential for GW150914 to both constrain exotic objects and physics beyond General Relativity is limited by the lack of understanding of the dynamical strong field in almost all modified gravity theories. GW150914 thus raises the bar that these theories must pass, both in terms of having

Black Hole solutions in Horndeski

- No hair theorem   Hui, Nicolis arXiv:1202.1296

  shift symmetry $\phi \to \phi + c$ $\nabla_\mu J^\mu = 0$ Babichev, Charmousis arXiv:1604.06402

Consider a shift symmetric galileon theory as (1) where G_2, G_3, G_4, G_5 are arbitrary functions of X. We now suppose that:

(i) spacetime is spherically symmetric and static (10) while the scalar field is also static ($q = 0$),

(ii) spacetime is asymptotically flat, $\phi' \to 0$ as $r \to \infty$ and the norm of the current J^2 is finite on the horizon,

(iii) there is a canonical kinetic term X in the action and the G_i functions are such that their X-derivatives contain only positive or zero powers of X.

Under these hypotheses, we conclude that ϕ is constant and thus the only black hole solution is locally isometric to Schwarzschild.

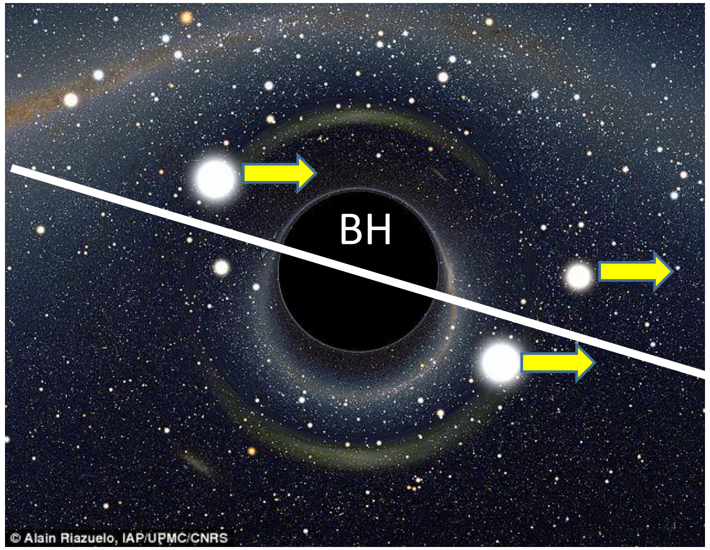
Test of Vainshetin mechanism

- Apparent equivalent principle violation

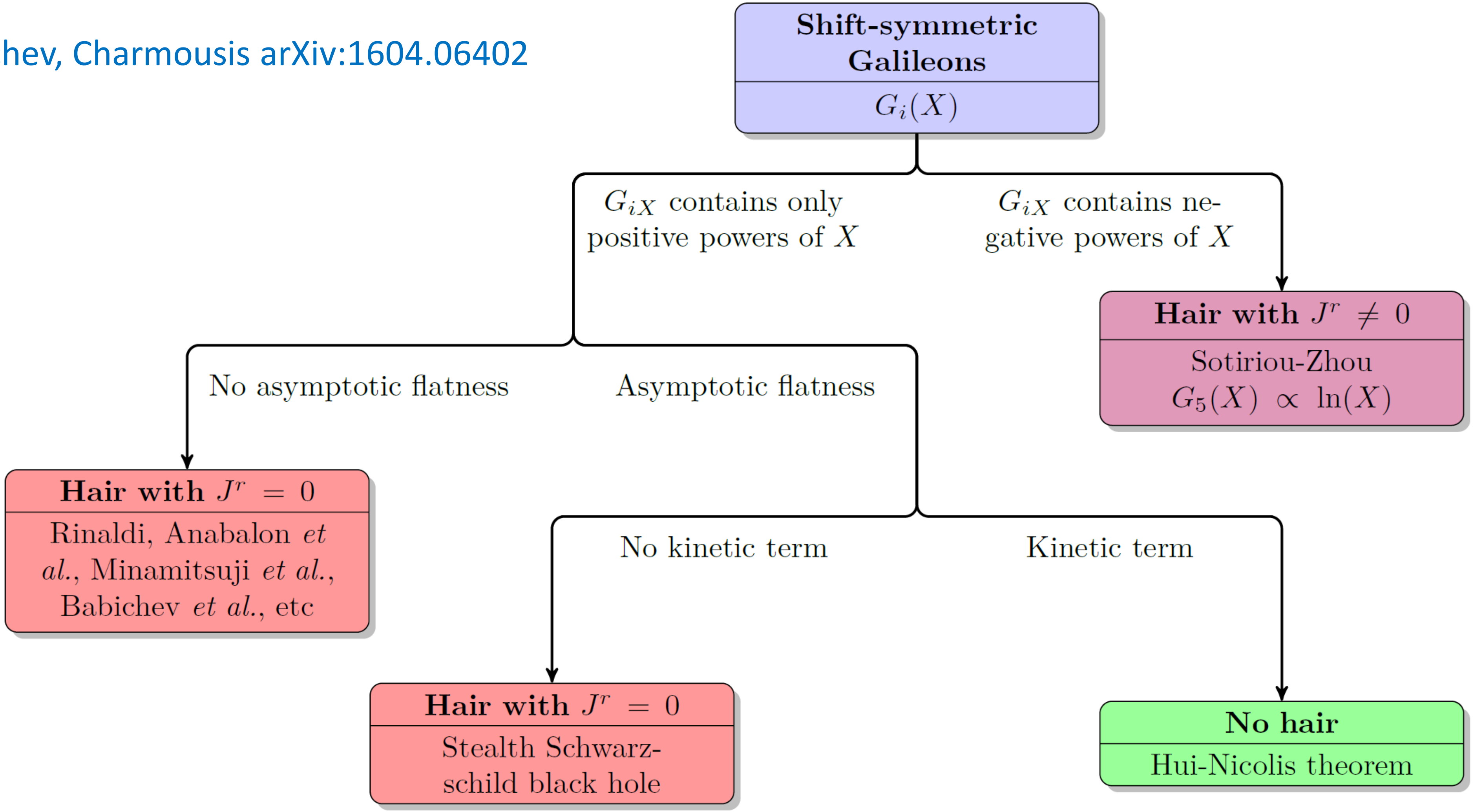
stars can feel an external field generated by large scale structure but a black hole does not due to no hair theorem   Hui, Nicolis arXiv:1201.1508

central BH lag behind stars

$$r = 0.1\,\mathrm{kpc}\left(\frac{2\alpha^2}{1}\right)\left(\frac{|\vec{\nabla}\Phi_{\rm ext}|}{20(\mathrm{km/s})^2/\mathrm{kpc}}\right)\left(\frac{0.01\mathrm{M}_{\odot}\mathrm{pc}^{-3}}{\rho_0}\right)$$

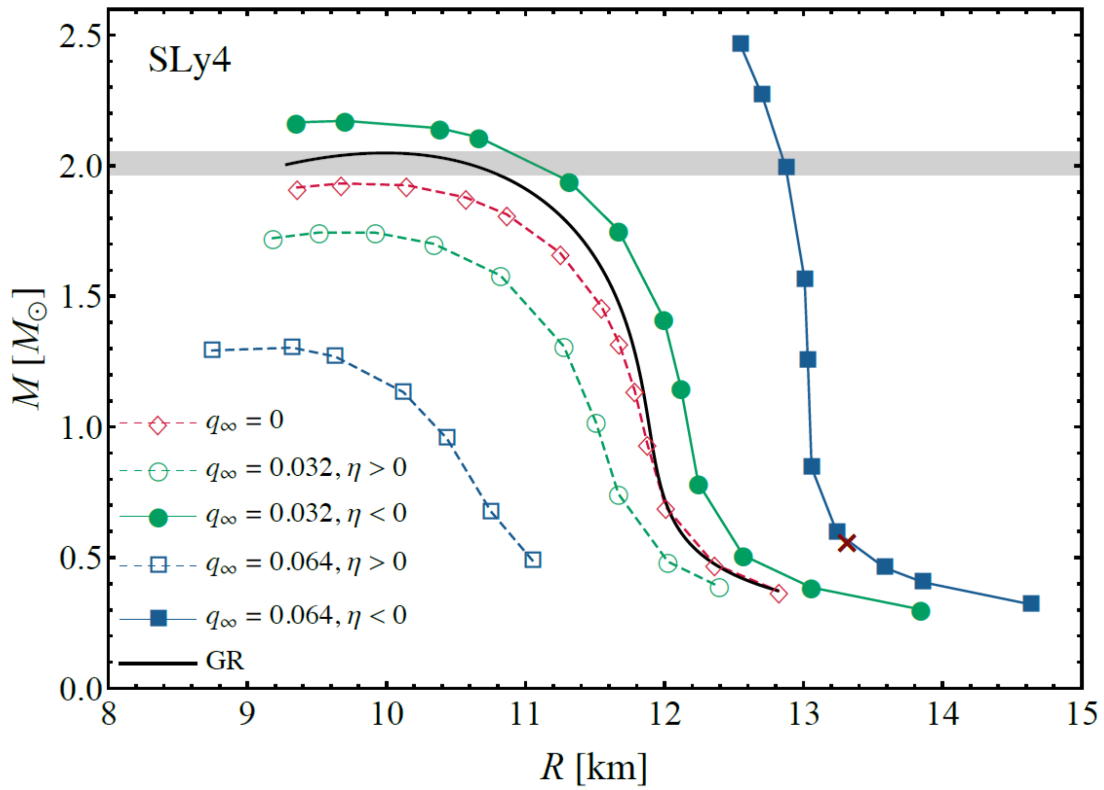


Babichev, Charmousis arXiv:1604.06402



Neutron stars

$$S_G = \int d^4x\sqrt{-g}\left[\kappa R - \frac{1}{2}(\beta g^{\mu\nu} - \eta G^{\mu\nu})\partial_\mu\phi\partial_\nu\phi\right] \qquad G_2 = \beta X, \quad G_4 = \kappa + \frac{\eta}{2}X\ , \quad G_3 = G_5 = 0$$



$$\phi(r,t) = qt + \psi(r)$$

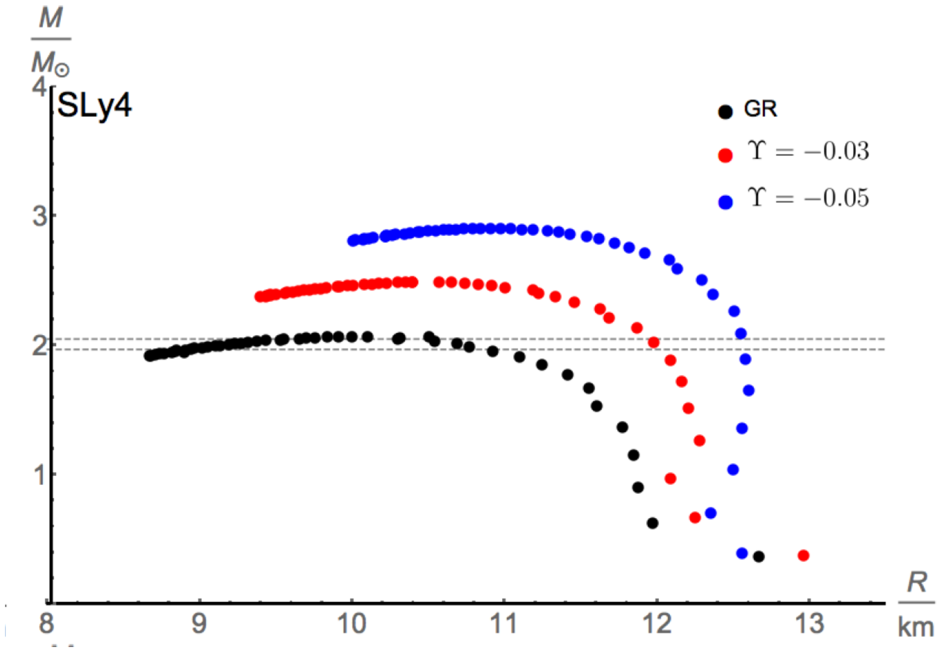
Maselli, Silva, Minamitsuji, Berti, arXiv:1603.04876

Neutron stars

$$S = \int d^4x \sqrt{-g} \left[M_{\rm pl}^2 \left(\frac{R}{2} - \Lambda \right) - k_2 \mathcal{L}_2 + f_4 \mathcal{L}_{4,\rm bH} \right]$$

$$\mathcal{L}_2 = \phi_\mu \phi^\mu \equiv X$$

$$\mathcal{L}_{4,\rm bH} = -X \left[(\Box\phi)^2 - (\phi_{\mu\nu})^2 \right] + 2\phi^\mu \phi^\nu \left[\phi_{\mu\nu} \Box\phi - \phi_{\mu\sigma} \phi^\sigma{}_\nu \right]$$



$$\phi(r,t) = v_0 t + \frac{v_0}{2H} \ln\left(1 - H^2 r^2\right) + \varphi(r)$$

Babichev, KK, Langlois, Saito, Sakstein, arXiv:1606.06627

Binary pulsars

de Rham, Tolley, Wesley arXiv: 1208.0580
Chu, Trodden, arXiv:1210.6651

Cubic Galileon

$$\text{Radiative Vainshtein suppression} = \frac{1}{(\Omega_P r_\star)^{3/2}} \qquad \Omega_P^2 = G(M_1 + M_2)/\bar{r}^3$$

	A	B	C	D	E
Pulsar	1913+16 Taylor-Hulse	B2127+11	B1534+12	J0737–3039 double pulsar	J1738+0333
$M_1/M_\odot$	1.386	1.358	1.345	1.338	1.46
$M_2/M_\odot$	1.442	1.354	1.333	1.249	0.181
T_P/days	0.323	0.335	0.420	0.102	0.355
e	0.617	0.681	0.274	0.088	3.4×10^{-7}
$\frac{dT_P}{dt}\vert_\pi$ Monopole	4.5×10^{-22}	8.3×10^{-22}	1.2×10^{-23}	8.1×10^{-25}	2.1×10^{-36}
$\frac{dT_P}{dt}\vert_\pi$ Dipole	10^{-30}	10^{-32}	10^{-33}	10^{-32}	10^{-31}
$\frac{dT_P}{dt}\vert_\pi$ Quadrupole	2.0×10^{-20}	2.2×10^{-20}	1.4×10^{-20}	9.7×10^{-21}	2.4×10^{-21}
$\frac{dT_P}{dt}\vert_{GR}$	2.4×10^{-12}	3.8×10^{-12}	1.9×10^{-13}	1.2×10^{-12}	2.2×10^{-14}
σ	5.1×10^{-15}	1.3×10^{-13}	2.0×10^{-15}	1.7×10^{-14}	10^{-15}
Ref.	[29–31]	[32]	[33, 34]	[35]	[36]