Testing Gravity with Black Holes and the Universe

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- Cosmology and BH as strong field tests of GR. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

  $$\mathcal{L}_{\rm GR} \sim M_P^2(\cancel{\partial^2 h} + \partial^2 h^2 + \partial^2 h^3 + ...) + hT$$

  $$\text{Neither tests : } \Delta\mathcal{L} \sim (\partial^2 h + \partial^2 h^2 + ...)^2$$

- Necessity of new d.o.f. e.g. ϕ.

  Weinberg's theorem: L.I. massless spin 2 particle, at low energy $\rightarrow$ GR.

  $\mathcal{L}_\phi \sim M_P^2(\partial^2\phi^2 + \mathcal{L}_{\rm int.}(\phi, h)) + \alpha\phi T$ e.g. massive gravity.

- Solar system test: photons deflected by h but not by ϕ.

  Options: 1. $\alpha \ll 1$ (at least for baryons).

  2. $\alpha \sim 1$ but screening non-negligible. $\mathcal{L}_{\rm int.} \sim \frac{1}{m^2}(\partial\phi)^2\partial^2\phi$

  bonus: self-acceleration / degrav. c.c. $\text{galileon inv. : } \phi \rightarrow \phi + c + b\cdot x$

- Robustness of E.P. for h, but not for ϕ.

  Possibilities - different values of α for different particles.

  - even if α is universal, (weak) E.P. can be broken by $\mathcal{L}_{\rm int.}(\phi)$ e.g. chameleon.

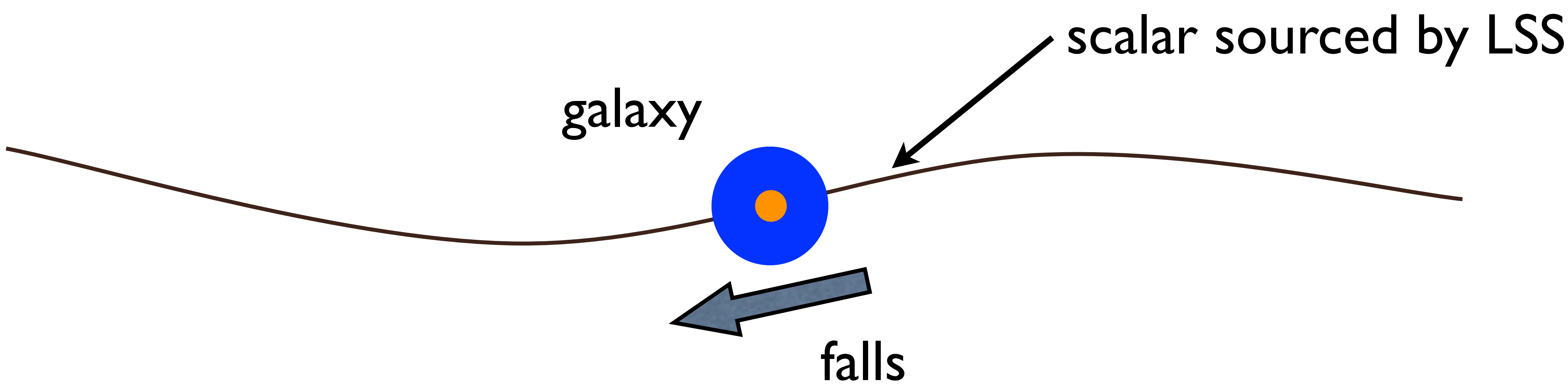
    galileon is robust against this, by charge conservation.

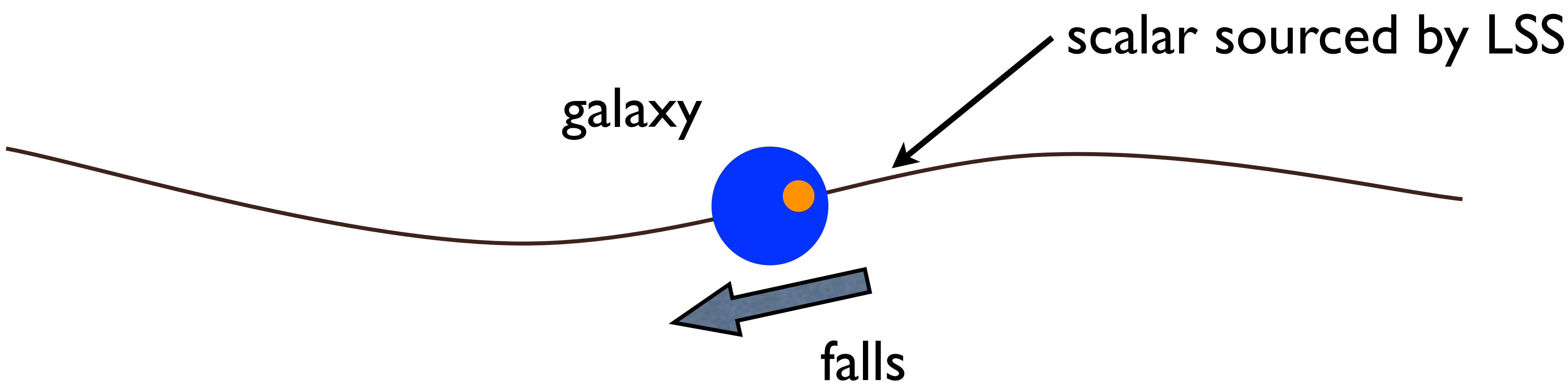
  - strong E.P. violation in general.

- Incompleteness of screening.

  - given $\phi_{\rm nonlinear}$, $\phi = \phi_{\rm nonlinear} + b\cdot x$ is also a solution.
  - consider $g^J_{\mu\nu} = \mathcal{C}(\phi) g^E_{\mu\nu}$ $\quad G^J_{\rm eff.} = G^E\mathcal{C}\left[1 + \left(\frac{1}{2}\frac{d\ln\mathcal{C}}{d\phi}\right)^2\right]$

- Consistency relations and soft theorems.


scalar sourced by LSS
galaxy
falls

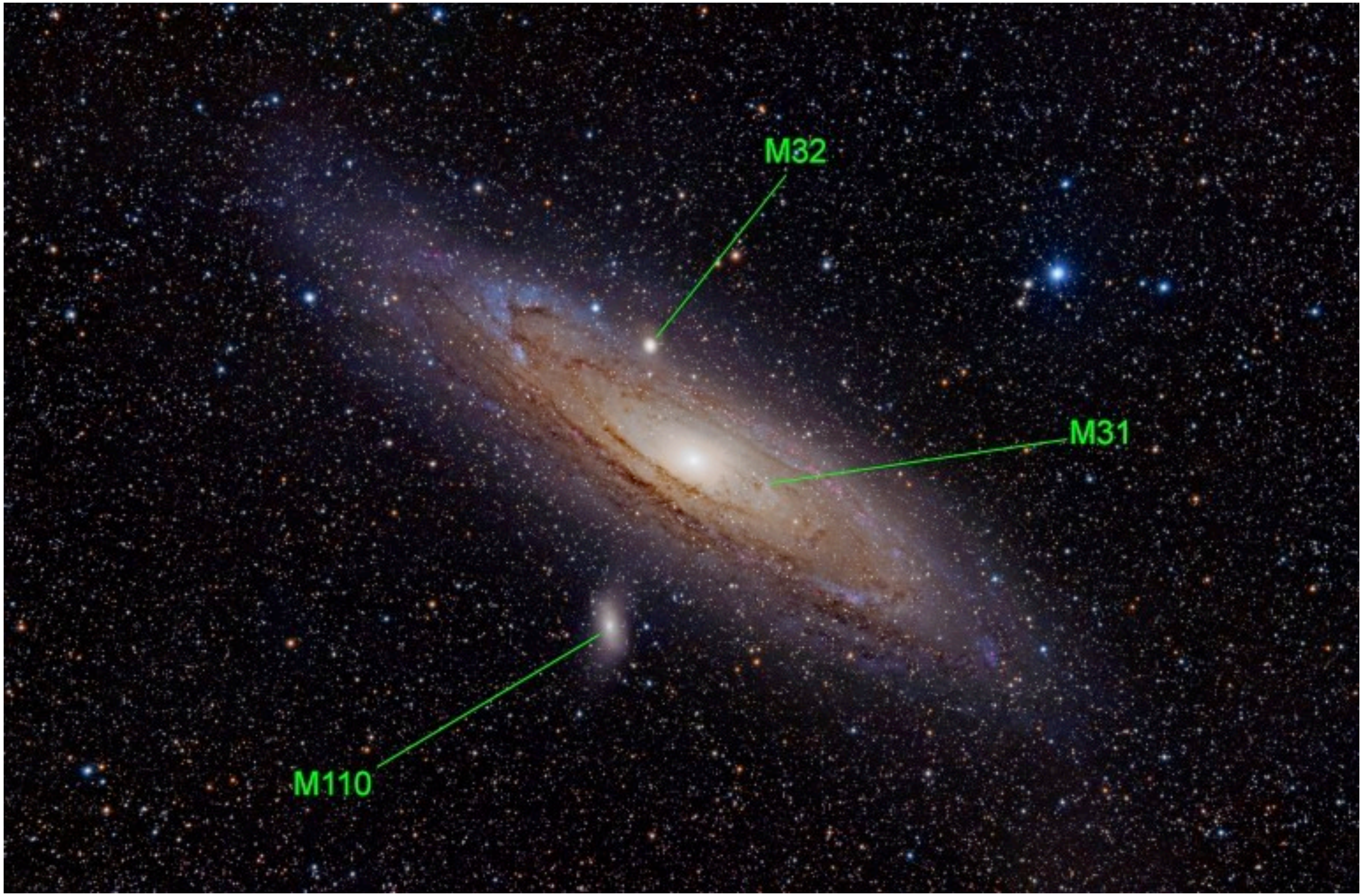

scalar sourced by LSS
galaxy
falls

- The idea is to look for the offset of massive black holes from the centers of galaxies (bottom of the gravitational potential well). Look at Seyfert galaxies where we can see both the stars and the black hole (active nucleus).

  The offset is estimated to be up to 0.1 kpc, for small galaxies.

- Sources of confusion:
  - asymmetric jets (case of M87: 7 pc offset, Batcheldor et al. 2010).
  - binary merger recoil.
  - Brownian motion.
  - disturbed galaxies.

- Distiniguishing feature: the spatial offset should be correlated with the direction of the streaming motion. Also: small velocity offset.


M32
M31
M110

From M31-M32 system:

- Observation: displacement $< 0.03\,\mathrm{pc} \to \alpha < 0.3$

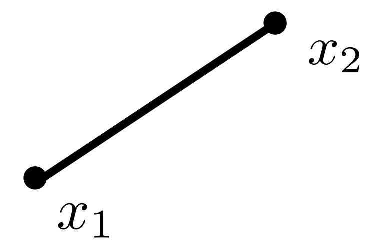
- Vainshtein mechanism (cubic galileon) gives $\alpha \sim 10^{-3}$.

  Reachable with better astrometry (?)

Idea 1: non-perturbative consistency relations in LSS

- 1. Consider a familiar example of symmetry: spatial translation.

$$x \to x + \Delta x \quad , \quad \text{where } \Delta x = \text{const.}$$



Its consequence for correlation function is well known:

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\rangle = \langle \phi(x_1 + \Delta x)\phi(x_2 + \Delta x)\phi(x_3 + \Delta x)\rangle$$

For small Δx , we have:

$$\langle \phi(x_1 + \Delta x)\phi(x_2 + \Delta x)\phi(x_3 + \Delta x)\rangle \sim \langle \phi(x_1)\phi(x_2)\phi(x_3)\rangle + \Delta x \cdot \partial_1 \langle \phi(x_1)\phi(x_2)\phi(x_3)\rangle + \text{perm.}$$

Thus, alternatively, we say:

$\langle \phi_1\phi_2\phi_3\rangle$ is invariant under $\quad \phi \to \phi + \Delta x \cdot \partial\phi \quad \text{i.e.} \quad \Delta x \cdot \partial_1 \langle \phi_1\phi_2\phi_3\rangle + \text{ perm.} = 0$

- 2. Consider a different symmetry: shift in gravitational potential.

$$\phi \to \phi + c \quad , \quad \text{where } c = \text{ const.}$$

For small c , we have:

$$\langle (\phi_1 + c)(\phi_2 + c)(\phi_3 + c)\rangle \sim \langle \phi_1\phi_2\phi_3\rangle + c\langle \phi_1\phi_2\rangle + c\langle \phi_2\phi_3\rangle + c\langle \phi_1\phi_3\rangle$$

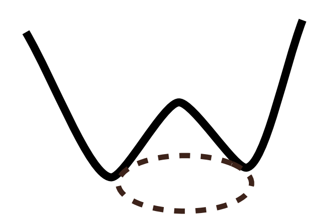
Thus, saying $\quad \langle \phi_1\phi_2\phi_3\rangle = \langle (\phi_1 + c)(\phi_2 + c)(\phi_3 + c)\rangle \quad$ is equiv. to saying :

$$c(\langle \phi_1\phi_2\rangle + \langle \phi_2\phi_3\rangle + \langle \phi_1\phi_3\rangle) = 0 \longleftarrow$$ clearly false!

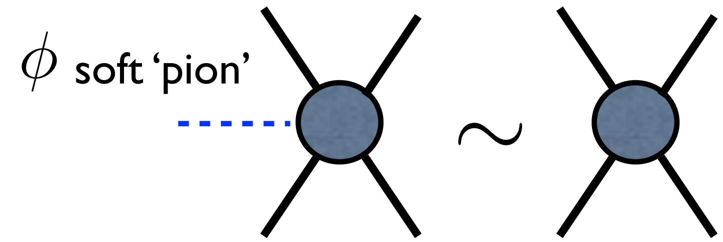
Conclude : $\quad \langle \phi_1\phi_2\phi_3\rangle \quad$ is not invariant under $\phi \to \phi + c$

- What makes the second case so different? We generally choose some expectation value for ϕ e.g. $\langle \phi \rangle = 0$. The choice breaks the shift symmetry i.e. spontaneous symm. breaking.

1. Unbroken symmetries $\longrightarrow$ invariant correlation functions.
2. Spontaneously broken symmetries $\longrightarrow$ consistency relations.

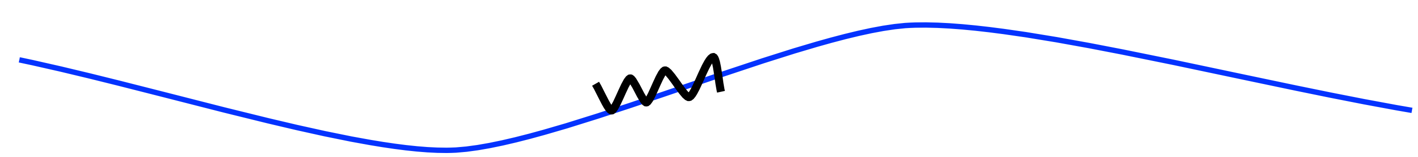


Consistency relations from SSB



- Schematic form: $$\lim_{q\to 0}\frac{1}{P_\phi(q)}\langle\phi(q)\mathcal{O}(k_1)...\mathcal{O}(k_N)\rangle \sim \langle\mathcal{O}(k_1)...\mathcal{O}(k_N)\rangle$$

  They are (momentum space) statements about how correlations of observables $\mathcal{O}$ behave in the presence of a long wave-mode Nambu-Goldstone boson/pion.



Relativistic symmetries and consistency relations

comoving gauge $\delta\rho = 0$ $\qquad ds^2_{\rm spatial} = a^2 e^{2\zeta}[e^\gamma]_{ij} dx^i dx^j$

dilation symm. $\quad x \to e^{-2\lambda} x \quad , \quad \zeta \to \zeta + \lambda$

$$\lim_{q\to 0} \frac{1}{P_\zeta(q)} \langle \zeta(q)\zeta_{k_1}...\zeta_{k_m}\rangle' \sim k\cdot\partial_k \langle \zeta_{k_1}...\zeta_{k_m}\rangle'$$

Maldacena

generalization $\quad x \to x + M\cdot x^{N+1} \quad , \quad \zeta \to \zeta + M\cdot x^N \quad , \quad \gamma \to \gamma + M\cdot x^N$

$$\lim_{q\to 0} \partial_q^N \left(\frac{1}{P_\zeta(q)} \langle \zeta(q)\zeta_{k_1}...\zeta_{k_m}\rangle' + \frac{1}{P_\gamma(q)} \langle \gamma(q)\zeta_{k_1}...\zeta_{k_m}\rangle' \right) \sim k\cdot\partial_k^{N+1} \langle \zeta_{k_1}...\zeta_{k_m}\rangle'$$

$N = 0$ dilation , $N = 1$ special conformal , etc.

Note:

1. The symmetries originate as diff. But consistency relations are not empty statements i.e. they can be violated (e.g. curvaton); they are a test of initial conditions (e.g. single clock, etc).

2. They are non-perturbative, derived from Ward identities.

3. Testing these requires seeing general relativistic effects, but there exists 2 Newtonian consistency relations (Peloso & Pietroni; Kehagias & Riotto).

Newtonian limit

$$\delta' + \nabla \cdot (1+\delta)v = S$$ mass/number conservation (or lack thereof)

$$v' + v \cdot \nabla v + \mathcal{H} v = -\nabla\phi + F$$ equation of motion

$$\nabla^2 \phi = 4\pi G a^2 \delta\rho_m$$ Poisson equation

Symmetries:

1. $\phi \to \phi + c$
2. $x \to x + n\,,\ v \to v + n'\,,\ \phi \to \phi - (n'' + \mathcal{H} n') \cdot x$

Newtonian consistency relations

no consistency relation statement

$$\lim_{q\to 0} \frac{\langle \delta_q \mathcal{O}_k \ldots\rangle}{P_\delta(q)\langle \mathcal{O}_k \ldots\rangle} \quad \sim \quad \left(\frac{q}{k}\right)^{-2} + \left(\frac{q}{k}\right)^{-1} + \left(\frac{q}{k}\right)^{0} + \ \ldots$$

vanishes by $\phi \to \phi + c$

vanishes by $v \to v + n'$ (equal time)
(non − vanishing at unequal times)

Note: consistency relation simplifies in Lagrangian space.

Why are consistency relations interesting?

1. These are symmetry statements, and are therefore exact, non-perturbative i.e. they hold even if the observables $\mathcal{O}$ are highly nonlinear, and even if they involve astrophysically complex objects, such as galaxies. The main input necessary is how they transform under the symmetry of interest (robust against galaxy mergers, birth, etc.)

2. In the fully relativistic context, there is an infinite number of consistency relations. Two of them have interesting Newtonian limits (shift and time-dependent translation).

3. Two assumptions go into these consistency relations, which can be experimentally tested (using highly nonlinear observables!): Gaussian initial condition (or more precisely, single-clock initial condition such as provided by inflation), and the equivalence principle (that all objects fall at the same rate under gravity).

4. Non-trivial constraints on analytic models.

An open issue:

Connection with asymptotic symmetries (e.g. BMS in the case of scattering amplitudes).