Numerical simulations of coalescing binaries

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Very High Energy Phenomena in the Universe

Quy Nhon, 8$^{\text{th}}$ August 2014

Overview

- Introduction
- Modelling of NSs, BHs in GR
- Gravitational Wave Physics
- Kicks and electromagnetic counterparts
- Conclusions

1. Introduction

Neutron stars and stellar-mass BHs

NSs

- Progenitors stars

  $M_\star \sim 8 \ \ldots \ 40 \ (80?) \ M_\odot$

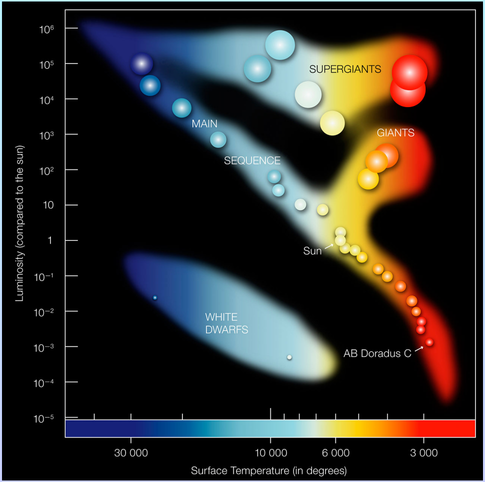
- $M_{NS} \gtrsim 1.4 \ \ldots \ 2 \ M_\odot$

BHs

- Progenitor stars

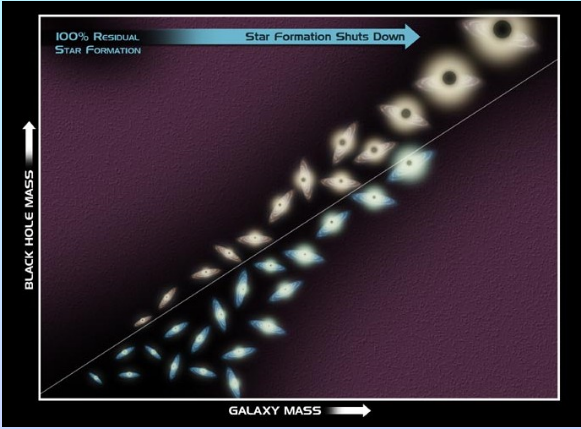
  $M_\star \gtrsim 20 \ M_\odot$

- $M_{BH} \sim 3 \ \ldots \ 50 \ M_\odot$



Supermassive BHs

- Galaxies ubiquitously harbor SMBHs
- $M_{BH} \sim 10^6 \ \dots \ 10^{10} \ M_\odot$
- BH properties correlated with bulge properties



Evidence for astrophysical black holes

- X-ray binaries

  e. g. Cygnus X-1 (1964)

  MS star + compact star

  ⇒ Stellar Mass BHs

  $\sim 5 \ldots 50\ M_{\odot}$

- Stellar dynamics

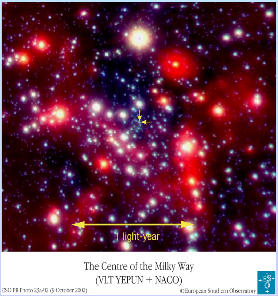
  near galactic centers,

  iron emission line profiles

  ⇒ Supermassive BHs

  $\sim 10^{6} \ldots 10^{10}\ M_{\odot}$

  AGN engines



Conjectured BHs

- Intermediate mass BHs

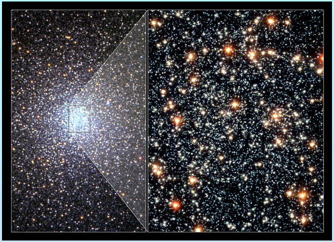
  $\sim 10^2 \ldots 10^5\ M_\odot$

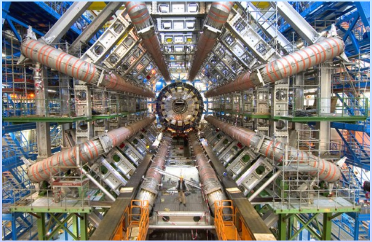
- Primordial BHs

  $\leq M_{Earth}$

- Mini BHs, LHC

  $\sim TeV$



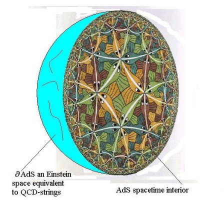


Note: BH solution is scale invariant!

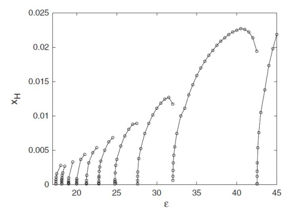
Research areas of compact stars

Astrophysics

Gauge-gravity duality

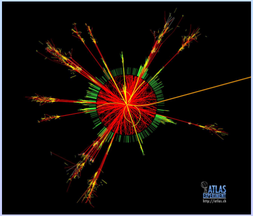


Fundamental studies

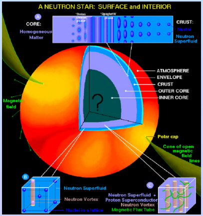


GW physics

High-energy physics



Equation of state



Luminosities

- Lasers: $\lesssim 10^{18}$ W
- Tsar Bomba: $\sim 10^{26}$ W
- GRB: $\sim 10^{45}$ W
- Universe in electromagnetic radiation: $\sim 10^{49}$ W
- Planck luminosity: 3.7×10^{52} W
- One BH binary can outshine the entire electromagnetic universe
- Energy from $10^9\ M_{\odot}$ BH binary: $E_{GW} \sim 10^{61}$ erg

2. Modelling of NSs, BHs

General Relativity: Curvature

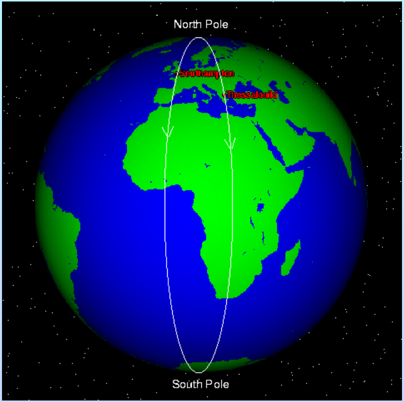
- Curvature generates acceleration

  "geodesic deviation"

  No "force"!!

- Description of geometry

Metric	$g_{\alpha\beta}$
Connection	$\Gamma^{\alpha}_{\beta\gamma}$
Riemann Tensor	$R^{\alpha}{}_{\beta\gamma\delta}$



How to get the metric?

Train cemetery
Uyuni, Bolivia

- Solve for the metric $g_{\alpha\beta}$

How to get the metric?

- Ricci-Tensor, Einstein Tensor, Matter Tensor

  $R_{\alpha\beta} \equiv R^{\mu}{}_{\alpha\mu\beta}$

  $G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R^{\mu}{}_{\mu}$ "Trace reversed" Ricci

  $T_{\alpha\beta}$ "Matter"

- Equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$, $\nabla_{\mu} T^{\mu}{}_{\alpha} = 0$

  2^{nd} order PDEs for $g_{\alpha\beta}$, matter Eqs.

- Solutions: Easy! Take metric

  $\Rightarrow$ Calculate $G_{\alpha\beta}$

  $\Rightarrow$ Use that as matter tensor

- Physically meaningful solutions: Difficult!

Solving Einstein's equations: Different methods

- Analytic solutions
  - Symmetry: Schwarzschild, Kerr, FLRW, Oppenheimer-Snyder dust
- Perturbation theory
  - Assume solution is close to known solution $g_{\alpha\beta}$
  - Expand $\hat{g}_{\alpha\beta} = g_{\alpha\beta} + \epsilon h^{(1)}_{\alpha\beta} + \epsilon^2 h^{(2)}_{\alpha\beta} + \ldots \;\Rightarrow$ linear system
  - Regge-Wheeler-Zerilli-Moncrief, Teukolsky, QNMs, EOB,...
- Post-Newtonian Theory
  - Assume small velocities $\Rightarrow$ expansion in $\frac{v}{c}$
  - N^{th} order expressions for GWs, momenta, orbits,...
  - Blanchet, Buonanno, Damour, Kidder, Will,...
- Numerical Relativity
  - Breakthroughs: Pretorius '05, UT Brownsville '05, NASA Goddard '05

A list of tasks

- Matter: High-resolution shock capturing, Microphysics
- Einstein equations: 1) Cast as evolution system

  2) Choose specific formulation: BSSN, GHG

  3) Discretize for computer
- Choose coordinate conditions: Gauge
- Fix technical aspects: 1) Mesh refinement / spectral domains

  2) Singularity handling / excision

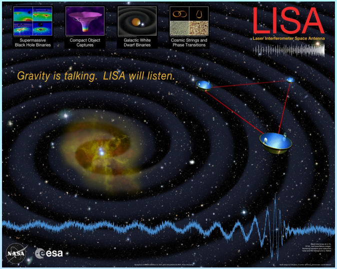
  3) Parallelization

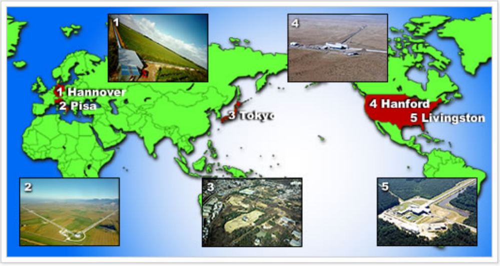
- Construct realistic initial data
- Start evolution...
- Extract physics from the data

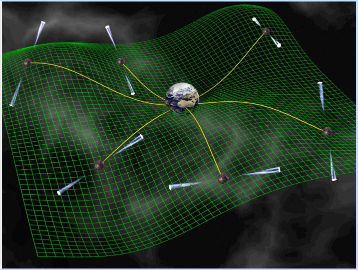
3. GW physics

Gravitational wave detectors

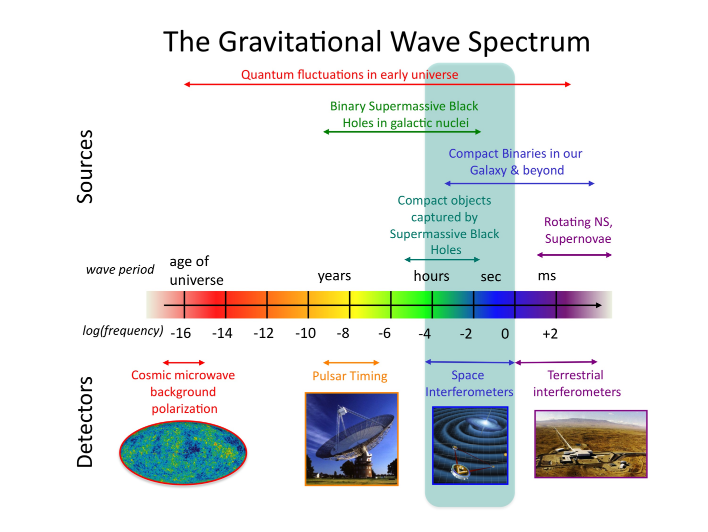
- Accelerated masses $\Rightarrow$ GWs
- Weak interaction!
- Laser interferometric detectors







The gravitational wave spectrum



Some targets of GW physics

- Tests of GR

  Hulse & Taylor 1993 Nobel Prize

- Parameter determination of BHs: M, $\vec{S}$

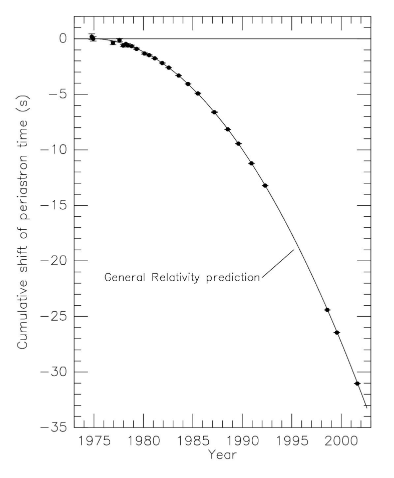
- Optical counter parts

  Standard sirens (candles)

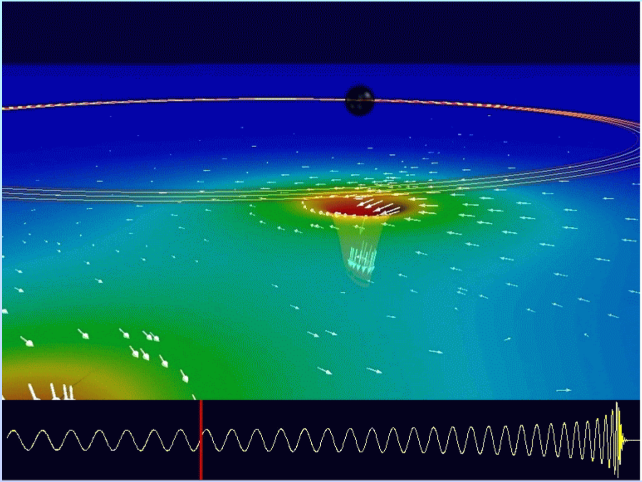
- Test Kerr Nature of BHs

- Neutron stars: EOS

- BH formation scenarios



Morphology of a BBH inspiral



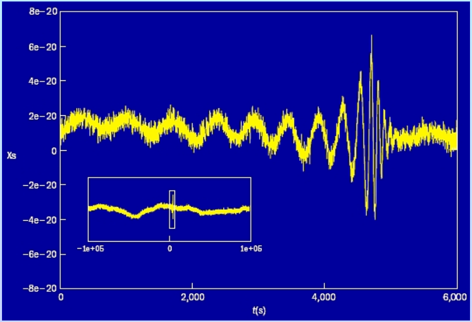
Thanks to Caltech, CITA, Cornell

Matched filtering

- BH binaries have 7 parameters: 1 mass ratio, 2×3 for spins
- Sample parameter space, generate waveform for each point

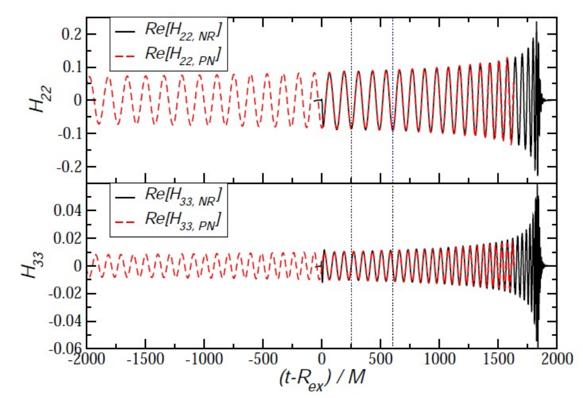
  - NR + PN
  - Effective one body
  - GEO 600 noise

    chirp signal



Template construction

- Stitch together PN and NR waveforms
- EOB or phenomenological templates for $\geq$ 7-dim. par. space



Template construction

- Phenomenological waveform models
  - Model phase, amplitude with simple functions $\rightarrow$ Model parameters
  - Create map between physical and model parameters
  - Time or frequency domain

    Ajith et al. 0704.3764, 0710.2335, 0712.0343, 0909.2867, Santamaria et al. 1005.3306, Sturani et al. 1012.5172, Hannam et al. 1308.3271
- Effective-one-body (EOB) models
  - Particle in effective metric, PN, ringdown model
    Buonanno & Damour PRD '99, gr-qc/0001013
  - Resum PN, calibrate pseudo PN parameters using NR
    Buonanno et al. 0709.3839, Pan et al. 0912.3466, 1106.1021, 1307.6232, Damour et al. 0712.202, 0803.3162, 1009.5998, 1406.6913

The Ninja project

https://www.ninja-project.org/

Aylott et al, CQG **26** 165008, CQG **26** 114008

Ajith et al, CQG **29** 124001

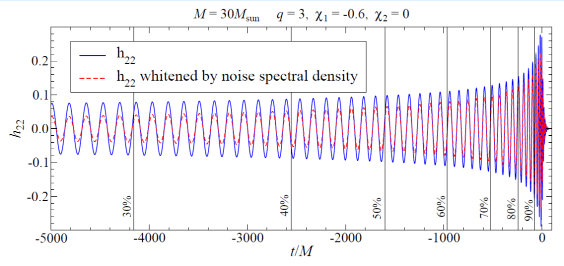
- Use PN/NR hybrid waveforms in GW data analysis
- Ninja2: 56 hybrid waveforms from 8 NR groups
- Details on hybridization procedures
- Overlap and mass bias study:
  - Take one waveform as signal, fixing M_{tot}
  - Search with other waveform (same config.) varying $t_0,\ \phi_0,\ M_{tot}$
  - Mass bias < 0.5 %

The NRAR project

https://www.ninja-project.org/doku.php?id=nrar:home

Hinder, Buonanno et al. 1307.5307

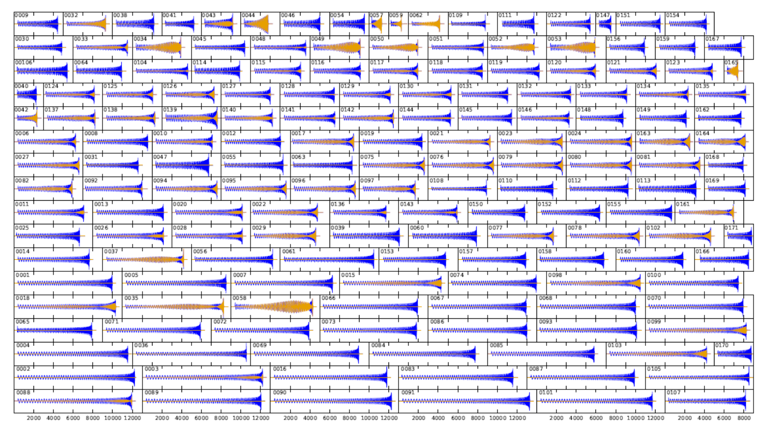
- Pool efforts from 9 NR groups
- $22 + 3$ waveforms, including precessing runs
- Standardize analysis, comparison with analytic models
- Test EOB models with NR



Tools of mass production

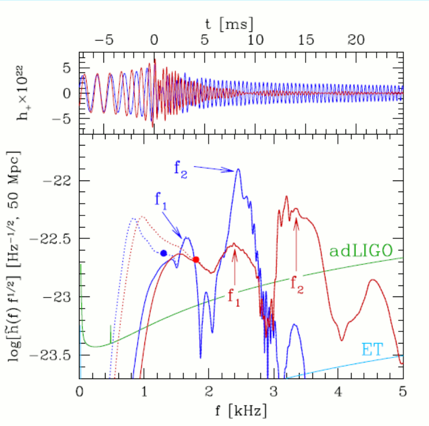
- SpEC catalog: 171 waveforms: $q \leq 8$, 90 precessing, ≤ 34 orbits

  Mroué et al. 1304.6077



Constraining the EOS of NSs with GWs

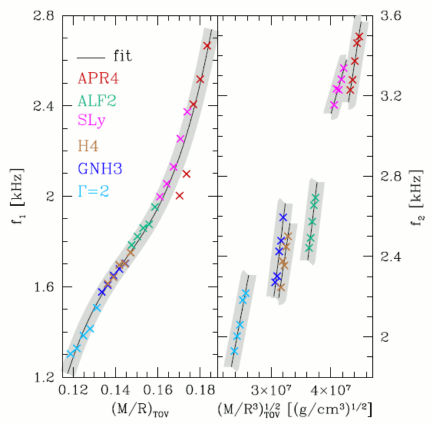
Step 1: Binary NS coalescence $\rightarrow$ char. frequency peaks



Takami et al. 1403.5672, Bauswein & Janka 1106.1616

Constraining the EOS of NSs with GWs

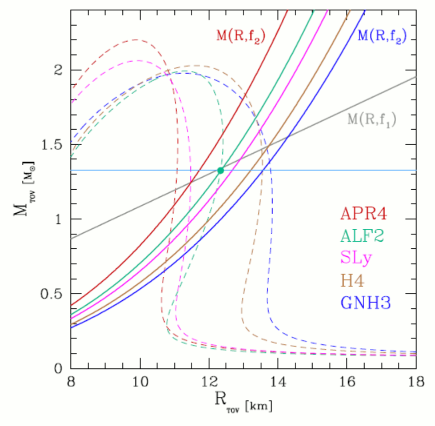
Step 2: Relations f_1 vs. M/R and f_2 vs. M/R^3



Takami et al. 1403.5672, Bauswein & Janka 1106.1616

Constraining the EOS of NSs with GWs

Step 3: Combine with M_{TOV} / R_{TOV} curve and measured M

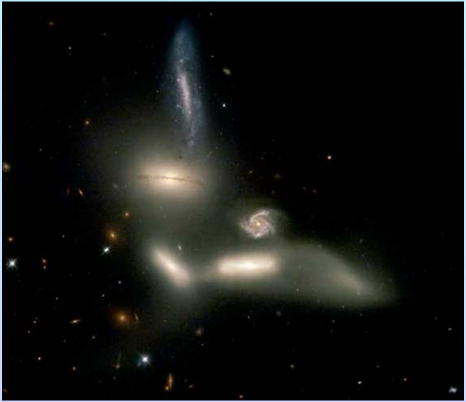


Takami et al. 1403.5672, Bauswein & Janka 1106.1616

4. Kicks and electromagnetic counterparts

SMBH formation

- Most widely accepted scenario for galaxy formation: hierarchical growth; “bottom-up”
- Galaxies undergo frequent mergers $\Rightarrow$ BH merger



Gravitational recoil

- Anisotropic GW emission $\Rightarrow$ recoil of remnant BH

  Bonnor & Rotenburg '61, Peres '62, Bekenstein '73

- Escape velocities:

Globular clusters	30 $\mathrm{km/s}$
dSph	20 $-$ 100 $\mathrm{km/s}$
dE	100 $-$ 300 $\mathrm{km/s}$
Giant galaxies	$\sim$ 1000 $\mathrm{km/s}$

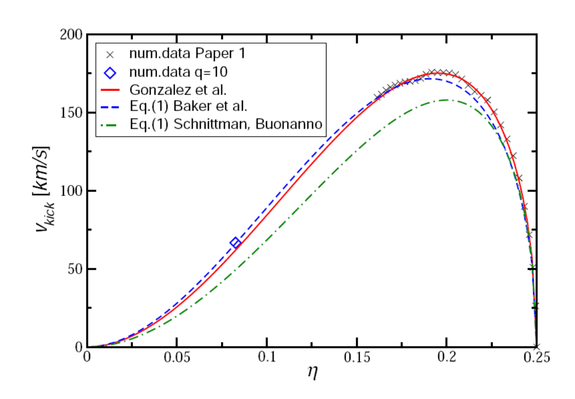
Ejection / displacement of BH $\Rightarrow$

- Growth history of SMBHs
- BH populations, IMBHs
- Displaced QSOs

Kicks from non-spinning BHs

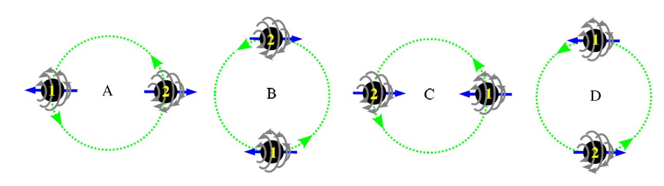
- Max. kick: $\sim 180\ km/s$, harmless!

  González et al., PRL 98, 091101 (2009)



Spinning BHs: Superkicks

- Kidder ’95, UTB-RIT ’07: maximum kick expected for



- Kicks up to $v_{\text{max}} \approx 4\,000\ \text{km}/\text{s}$

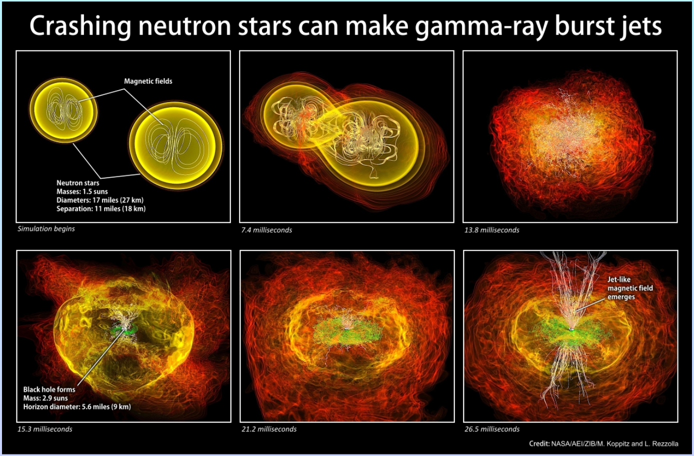
  González *et al.* ’07, Campanelli *et al.* ’07

- “Hang-up kicks” of up to $5\,000\ km/s$ Lousto & Zlochower ’12

- Suppression via spin alignment and Resonance effects in inspiral
  Schnittman ’04, Bogdanović et al. ’07, Kesden, US & Berti ’10, ’10a, ’12

Neutron stars and γ-ray bursts

NS binaries can generate GRBs

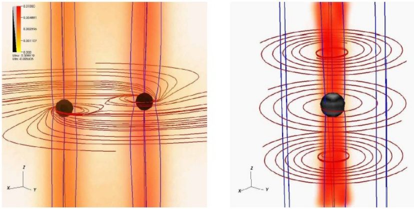


Search for coincidence GRB + GW events

Double jets and spin flips

- BH binary with plasma
- Jets driven by **L**
- Optical signature: double jets

- Spin re-alignment
  - $\Rightarrow$ new + old jet
  - $\Rightarrow$ X-shaped radio sources



(a) $-8.2\ M_8$ hrs (b) $4.6\ M_8$ hrs

Palenzuela, Lehner & Liebling '10

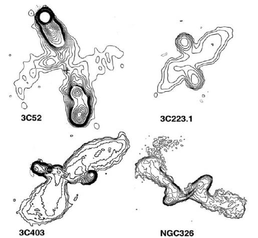


Fig. 2. A composite of four clear examples of the X-shape morphology. VLA radio observations of 3C52 (28), 3C223.1 (29), 3C403 (29), and NGC 326 (12).

Campanelli et al. '06

5. Conclusions

Conclusions

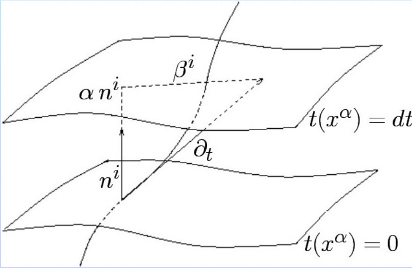
- NR can reliably evolve compact binaries
- PN results good for early inspiral of BHs, NSs
- NR needed for merger
- BHs, NSs important source of GWs
- Some goals of GW physics:
  EOS of NS matter, Standard Sirens, BH formation
- Astrophysical studies: BH Kicks, Elm. signatures
- Other physics: *TeV* Gravity, AdS/CFT, Fundamental physics

Additional material for questions

3+1 Decomposition

- GR: “Space and time exist as a unity: Spacetime”
- NR: ADM 3+1 split — Arnowitt, Deser & Misner ’62; York ’79, Choquet-Bruhat & York ’80

$$g_{\alpha\beta} = \left(\begin{array}{c|c} -\alpha^2 + \beta_m \beta^m & \beta_j \\ \hline \beta_i & \gamma_{ij} \end{array} \right)$$



- 3-Metric γ_{ij}
  Lapse α
  Shift β^i
- lapse, shift $\Rightarrow$ Gauge

ADM Equations

The Einstein equations $R_{\alpha\beta} = 0$ become

- 6 Evolution equations

  $$(\partial_t - \mathcal{L}_\beta)\gamma_{ij} = -2\alpha K_{ij}$$

  $$(\partial_t - \mathcal{L}_\beta)K_{ij} = -D_i D_j \alpha + \alpha[R_{ij} - 2K_{im}K^m{}_j + K_{ij}K]$$

- 4 Constraints

  $$R + K^2 - K_{ij}K^{ij} = 0$$

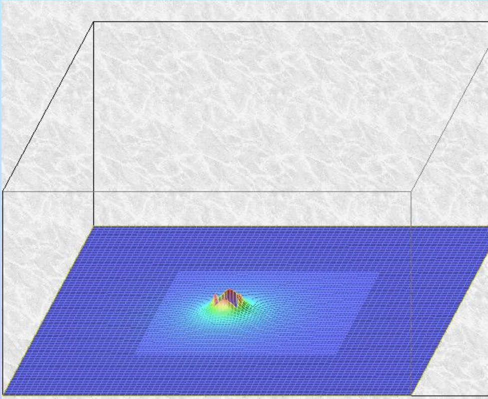
  $$-D_j K^{ij} + D^i K = 0$$

  preserved under evolution!

- Evolution

  1) Solve constraints

  2) Evolve data



Formulations I: BSSN

- One can easily change variables. E. g. wave equation

$$\partial_{tt} u - c \partial_{xx} u = 0 \qquad \Leftrightarrow \qquad \begin{aligned} \partial_t F - c\partial_x G &= 0 \\ \partial_x F - \partial_t G &= 0 \end{aligned}$$

- BSSN: rearrange degrees of freedom

$$\chi = (\det \gamma)^{-1/3} \qquad \tilde{\gamma}_{ij} = \chi \gamma_{ij}$$

$$K = \gamma_{ij} K^{ij} \qquad \tilde{A}_{ij} = \chi \left(K_{ij} - \tfrac{1}{3} \gamma_{ij} K \right)$$

$$\tilde{\Gamma}^i = \tilde{\gamma}^{mn} \tilde{\Gamma}^i_{mn} = -\partial_m \tilde{\gamma}^{im}$$

Shibata & Nakamura '95, Baumgarte & Shapiro '98

- BSSN strongly hyperbolic, but depends on details...

Sarbach *et al.*'02, Gundlach & Martín-García '06

Formulations I: BSSN

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\phi = \tfrac{1}{12}\ln\gamma \qquad \hat{\gamma}_{ij} = e^{-4\phi}\gamma_{ij}$$

$$K = \gamma_{ij}K^{ij} \qquad \hat{A}_{ij} = e^{-4\phi}\left(K_{ij} - \tfrac{1}{3}\gamma_{ij}K\right)$$

$$\hat{\Gamma}^i = \gamma^{ij}\hat{\Gamma}^i_{jk} = -\partial_j\hat{\gamma}^{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\hat{\gamma}_{ij} = -2\alpha\hat{A}_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\phi = -\tfrac{1}{6}\alpha K$$

$$(\partial_t - \mathcal{L}_\beta)\hat{A}_{ij} = e^{-4\phi}(-D_iD_j\alpha + \alpha R_{ij})^{\mathrm{TF}} + \alpha(K\hat{A}_{ij} - 2\hat{A}_{ik}\hat{A}^k{}_j)$$

$$(\partial_t - \mathcal{L}_\beta)K = -D^iD_i\alpha + \alpha\left(\hat{A}_{ij}\hat{A}^{ij} + \tfrac{1}{3}K^2\right)$$

$$\partial_t\hat{\Gamma}^i = 2\alpha\left(\hat{\Gamma}^i_{jk}\hat{A}^{jk} + 6\hat{A}^{ij}\partial_j\phi - \tfrac{2}{3}\hat{\gamma}^{ij}\partial_jK\right) - 2\hat{A}^{ij}\partial_j\alpha + \hat{\gamma}^{jk}\partial_j\partial_k\beta^i$$

$$+\tfrac{1}{3}\hat{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\hat{\Gamma}^i + \tfrac{2}{3}\hat{\Gamma}^i\partial_j\beta^j \underbrace{- \left(\chi + \tfrac{2}{3}\right)\left(\hat{\Gamma}^i - \hat{\gamma}^{jk}\hat{\Gamma}^i_{jk}\right)\partial_l\beta^l}_{\text{Yo et al. (2002)}}$$

Formulations II: Generalized harmonic (GHG)

- Harmonic gauge: choose coordinates such that

  $\nabla_\mu \nabla^\mu x^\alpha = 0$

- 4-dim. version of Einstein equations

  $R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \ldots$

  Principal part of wave equation

- Generalized harmonic gauge: $H_\alpha \equiv g_{\alpha\nu} \nabla_\mu \nabla^\mu x^\nu$

  $\Rightarrow R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \ldots - \frac{1}{2} \left(\partial_\alpha H_\beta + \partial_\beta H_\alpha \right)$

  Still principal part of wave equation !!! Manifestly hyperbolic

  Friedrich '85, Garfinkle '02, Pretorius '05

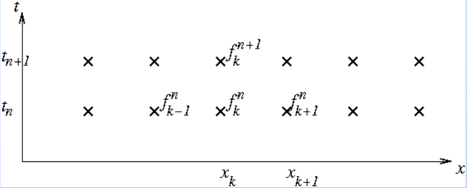
- Constraint preservation; constraint satisfying BCs

  Gundlach et al. '05, Lindblom et al. '06

Discretization of the time evolution

- Finite differencing (FD)

  Pretorius, RIT, Goddard, Georgia Tech, LEAN, BAM, UIUC,...
- Spectral Caltech-Cornell-CITA
- Parallelization with MPI, $\sim$ 128 cores, $\sim$ 256 *Gb* RAM
- Example: advection equation $\partial_t f = \partial_x f$, FD
- Array f_k^n for fixed n



$$f_k^{n+1} = f_k^n + \Delta t \frac{f_{k+1}^n - f_{k-1}^n}{2\Delta x}$$

Initial data

Two problems: Constraints, realistic data

- Rearrange degrees of freedom

  York-Lichnerowicz split: $\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$

  $$K_{ij} = A_{ij} + \frac{1}{3}\gamma_{ij} K$$

  York & Lichnerozwicz, O'Murchadha & York,

  Wilson & Mathews, York

- Make simplifying assumptions

  Conformal flatness: $\tilde{\gamma}_{ij} = \delta_{ij}$, and $K = 0$

- Find good elliptic solvers, e. g. Ansorg et al. '04

Mesh refinement

3 Length scales :	BH	$\sim 1\ M$
	Wavelength	$\sim 10...100\ M$
	Wave zone	$\sim 100...1000\ M$

- Critical phenomena

  Choptuik '93

- First used for BBHs

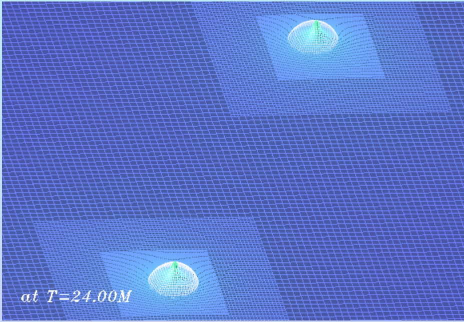
  Brügmann '96

- Available Packages:

  Paramesh MacNeice *et al.* '00

  Carpet Schnetter *et al.* '03

  SAMRAI MacNeice *et al.* '00



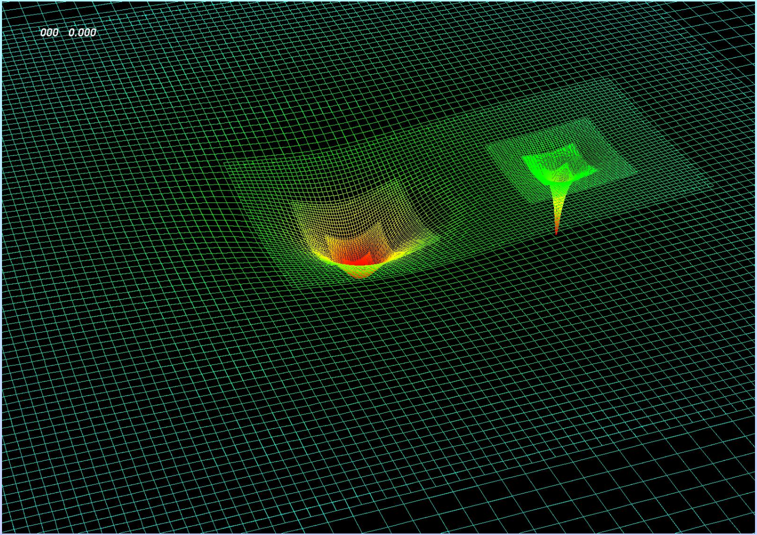
The gauge freedom

- Remember: Einstein equations say nothing about $\alpha, \quad \beta^i$
- Any choice of lapse and shift gives a solution
- This represents the coordinate freedom of GR
- Physics do not depend on $\alpha, \quad \beta^i$

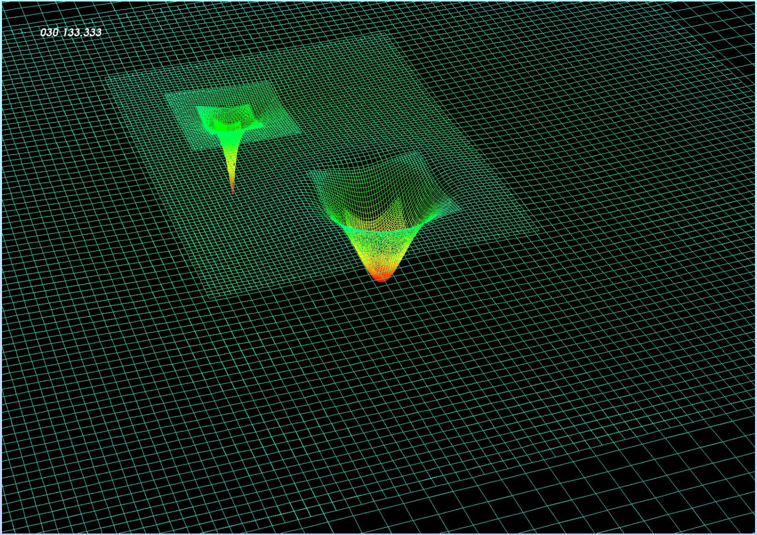
  So why bother?
- The performance of the numerics DO depend strongly on the gauge!
- How do we get good gauge?

  Singularity avoidance, avoid coordinate stretching, well posedness

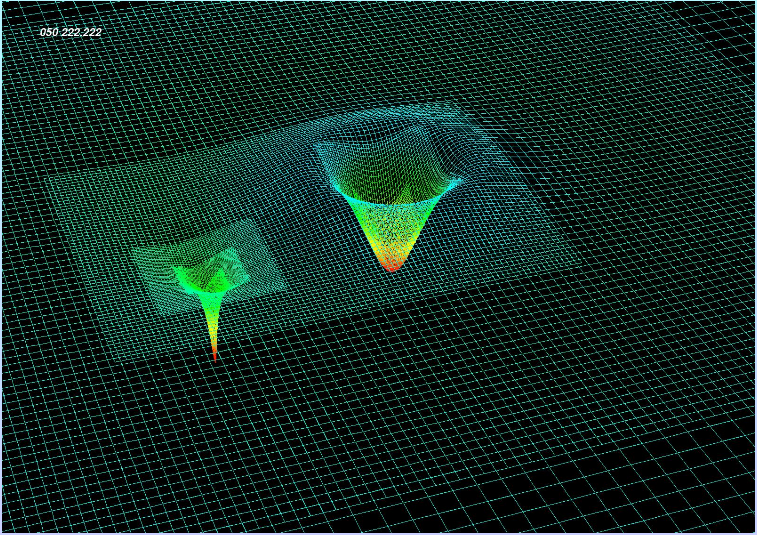
What goes wrong with bad gauge?



What goes wrong with bad gauge?



What goes wrong with bad gauge?



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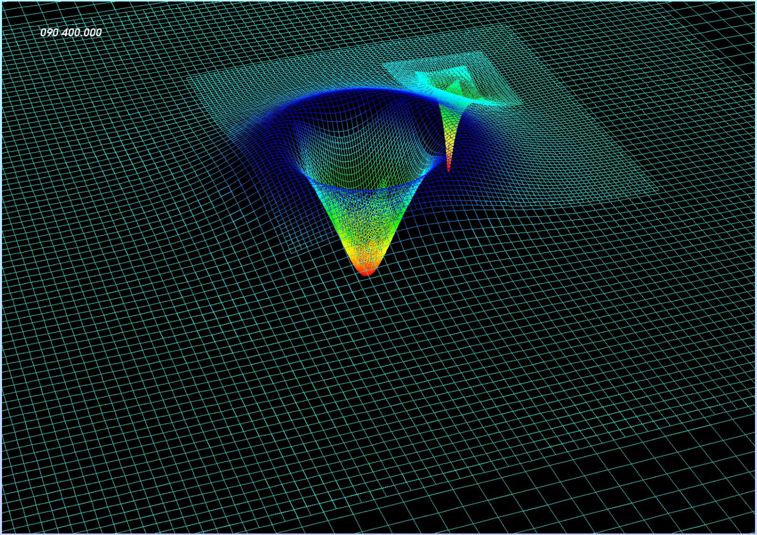

A brief history of BH simulations

- Pioneers: Hahn & Lindquist '60s, Eppley, Smarr *et al.* '70s
- Grand Challenge: First $3D$ Code Anninos *et al.* '90s
- Further attempts: Bona & Massó, Pitt-PSU-Texas

  AEI-Potsdam, Alcubierre *et al.*

  PSU: first orbit Brügmann *et al.* '04

  Codes unstable!

- Breakthrough: Pretorius '05 — GHG

  UTB, Goddard'05 — Moving Punctures

- Currently about 10 codes world wide