Colliding black holes

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Holographic vistas on Gravity and Strings
Kyoto, 26th May 2014

Overview

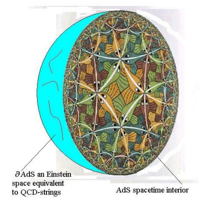
- Introduction, motivation
- Numerical tools
- $D = 4$ vacuum
- $D = 4$ matter
- $D \geq 5$ collisions
- Conclusions and outlook

1. Introduction, motivation

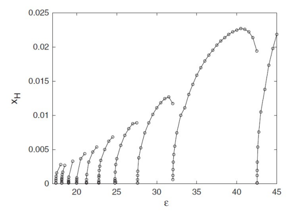
Research areas: BHs are (almost) everywhere

Astrophysics

Gauge-gravity duality

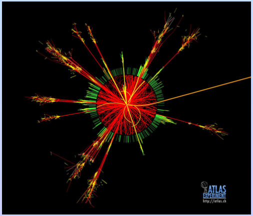


Fundamental studies

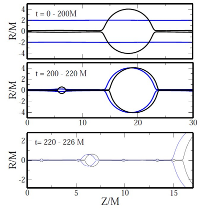


GW physics

High-energy physics



Fluid analogies



BH collisions

- Astrophysics: Kicks, structure formation,...
- GW physics: LIGO, VIRGO, LISA,... sources
- Focus here: HE, HD collisions
  - Cosmic censorship
  - Hoop conjecture
  - Matter does not matter
  - Trans-Planckian scattering
  - Probing GR

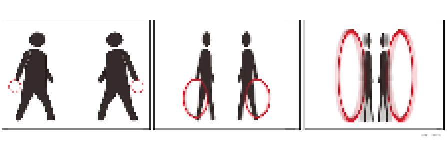
Cosmic censorship

- Singularities hidden inside horizon
- GR's protection from itself

Hoop conjecture

- Hoop conjecture: Hoop with $c = 2\pi r_S$ fits around object $\Rightarrow$ BH

  Thorne '72



- Especially relevant for trans-Planckian scattering!
  - de Broglie wavelength: $\lambda = \frac{hc}{E}$
  - Schwarzschild radius: $r = \frac{2GE}{c^4}$
  - BH will form if $\lambda < r \quad \Leftrightarrow \quad E \gtrsim \sqrt{\frac{hc^5}{G}} \equiv E_{\text{Planck}}$

Trans-Planckian scattering

- Matter does not matter at energies well above the Planck scale

  ⇒ Model particle collisions by black-hole collisions

  Banks & Fischler '99;   Giddings & Thomas '01

- TeV-gravity scenarios

  ⇒ The Planck scale might be as low as TeVs due to extra dimensions

  Arkani-Hamed, Dimopulos & Dvali '98,   Randall & Sundrum '99

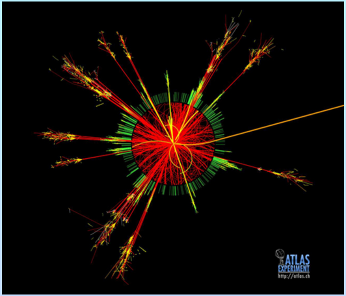
  ⇒ Black holes could be produced in colliders

  Eardley & Giddings '02,   Dimopoulos & Landsberg '01,...

Experimental signature at the LHC

Black hole formation at the LHC could be detected by the properties of the jets resulting from Hawking radiation.

- Multiplicity of partons: Number of jets and leptons
- Large transverse energy
- Black-hole mass and spin are important for this!



ToDo:

- Exact cross section for BH formation
- Determine loss of energy in gravitational waves
- Determine spin of merged black hole

2. Numerical tools

Summary of the numerical methods

- 3+1 numerical relativity
  - BSSN moving punctures
  - Generalized Harmonic Gauge
- Higher dimensions: Reduced to 3+1 plus extra fields
  - $SO(D-3)$ isometric spacetimes
  - Reduction by isometry

    Geroch 1970, Cho 1986, Zilhão et al 2010
  - Modified Cartoon

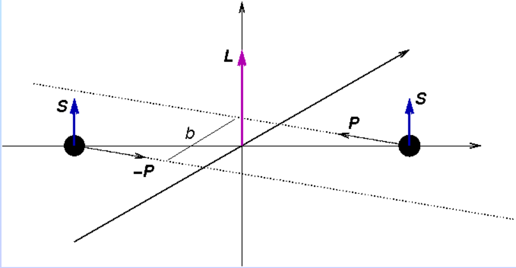
    Alcubierre 1999, Shibata & Yoshino 2009, 2010
- Energy-momentum: Standard treatment when present

3. Four-dimensions, vacuum

Initial setup: 1) Aligned spins

- Orbital hang-up   Campanelli et al. '06
- 2 BHs: Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

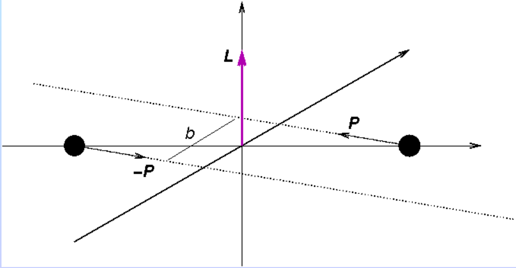
  Boost: $\gamma = 1/\sqrt{1-v^2}$, $M = \gamma M_0$
- Impact parameter: $b \equiv \frac{L}{P}$



Initial setup: 2) No spins

- Orbital hang-up   Campanelli et al. '06
- 2 BHs: Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

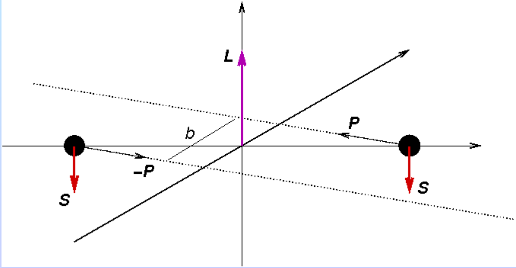
  Boost: $\gamma = 1/\sqrt{1-v^2}$, $M = \gamma M_0$
- Impact parameter: $b \equiv \frac{L}{P}$



Initial setup: 3) Anti-aligned spins

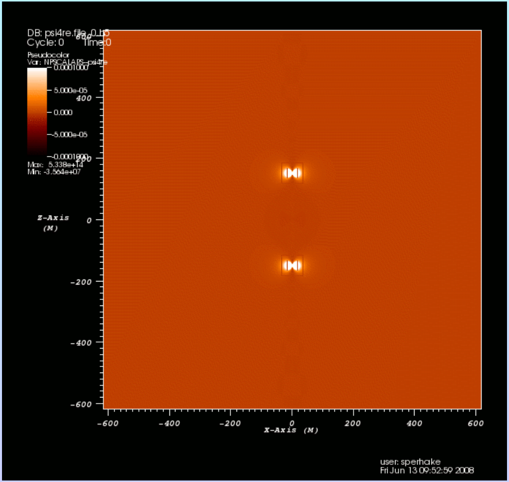
- Orbital hang-up   Campanelli et al. '06
- 2 BHs: Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

  Boost: $\gamma = 1/\sqrt{1 - v^2}$, $M = \gamma M_0$
- Impact parameter: $b \equiv \frac{L}{P}$



Head-on: $b = 0$, $\vec{S} = 0$

$$\gamma = 2.93\,, \qquad v = 0.94\,c$$



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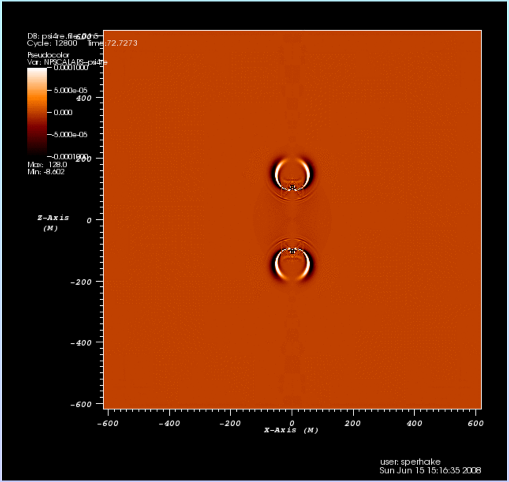

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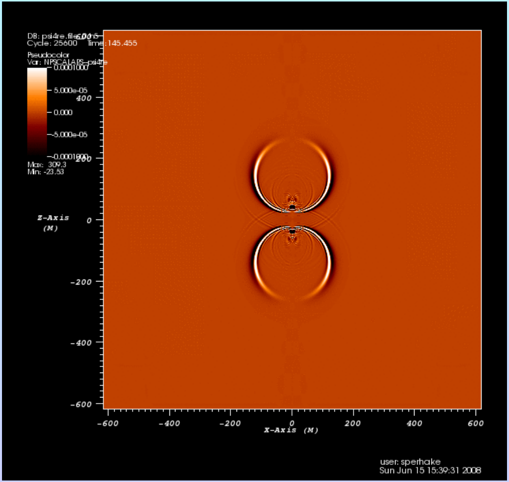

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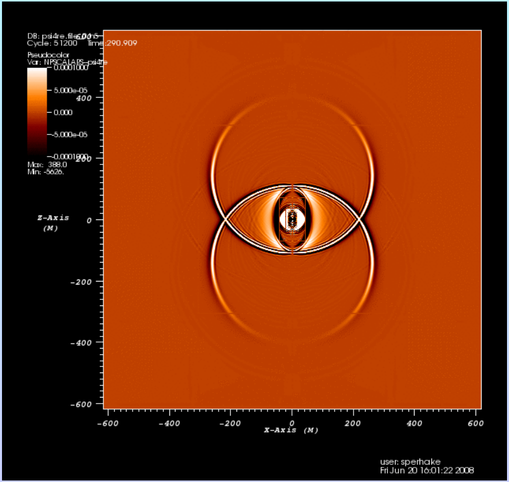

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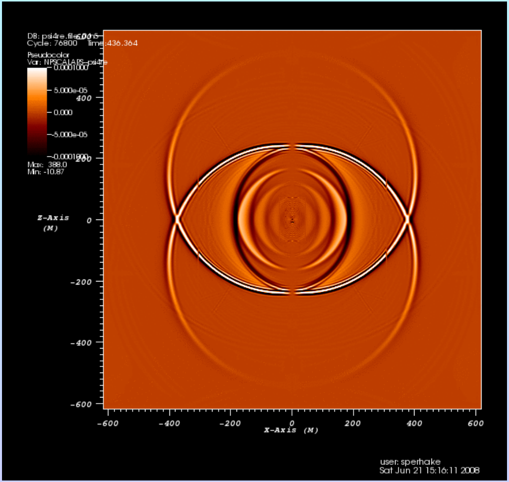

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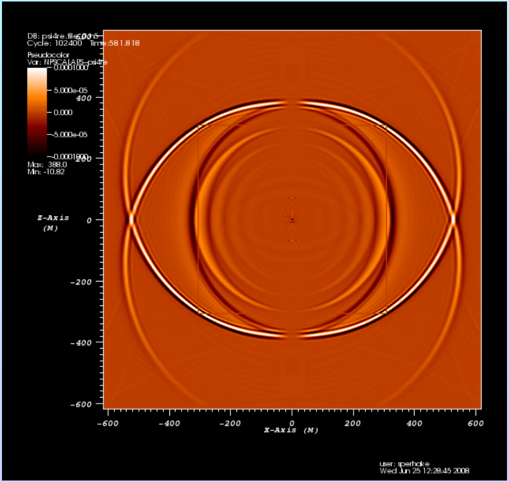
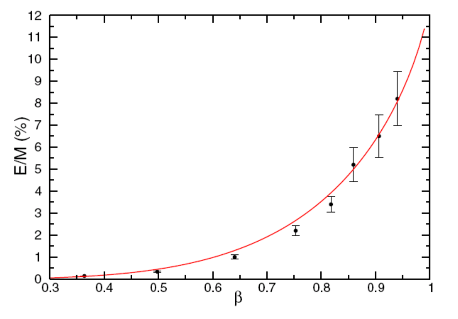


Head-on: $b = 0, \quad \vec{S} = 0$

- Total radiated energy: 14 ± 3 % for $v \to 1$ US *et al.* '08

  About half of Penrose '74

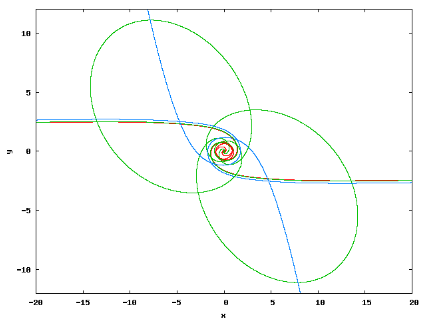


- Agreement with approximative methods

  Flat spectrum, multipolar GW structure Berti *et al.* '10

Grazing: $b \neq 0, \quad \vec{S} = 0, \quad \gamma = 1.52$

- Radiated energy up to at least 35 % M
- Immediate vs. Delayed vs. No merger
- Zoom-whirl like behaviour: $N_{orb} \propto \ln |b^* - b|$



US, Cardoso, Pretorius, Berti et al 2009

Scattering threshold b_{scat} for $D = 4$, $\vec{S} = 0$

- $b < b_{\rm scat}$ $\Rightarrow$ Merger

  $b > b_{\rm scat}$ $\Rightarrow$ Scattering

- Numerical study: $b_{\rm scat} = \frac{2.5 \pm 0.05}{v} M$

  Shibata, Okawa & Yamamoto '08

- Independent study by US, Pretorius, Cardoso, Berti *et al.* '09, '13

  $\gamma = 1.23 \ldots 2.93$:

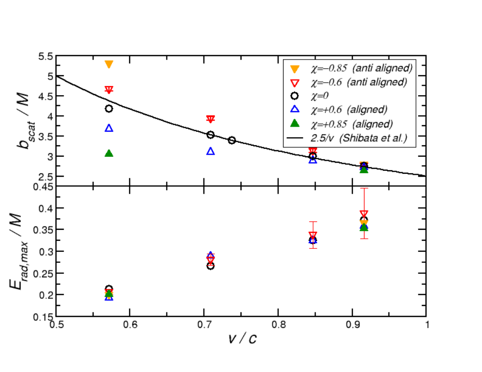
  $\chi = 0, \pm 0.6, \pm 0.85$ (anti-aligned, nonspinning, aligned)

- Limit from Penrose construction: $b_{\rm crit} = 1.685\ M$

  Yoshino & Rychkov '05

Scattering threshold and radiated energy

US, Berti, Cardoso & Pretorius '12

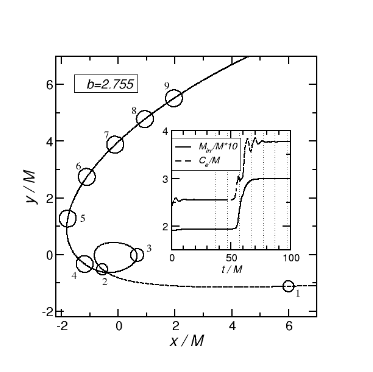


- At speeds $v \gtrsim 0.9$ spin effects washed out
- E_{rad} always below $\lesssim 50\ \%\ M$

Absorption

- For large γ: $E_{kin} \approx M$
- If E_{kin} is not radiated, where does it go?
- Answer: ~ 50 % into E_{rad}, ~ 50 % is absorbed

US, Berti, Cardoso & Pretorius '12

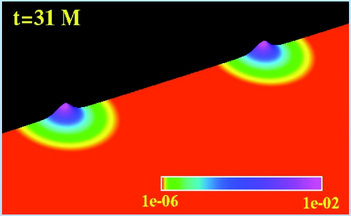


4. Four dimensions, matter

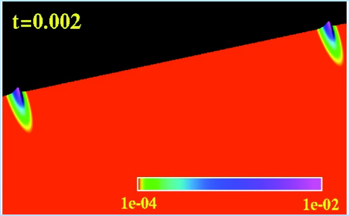
Does matter "matter"?

- Hoop conjecture ⇒ kinetic energy triggers BH formation
- Einstein plus minimally coupled, massive, complex scalar filed

"Boson stars" Pretorius & Choptuik '09



$\gamma = 1$



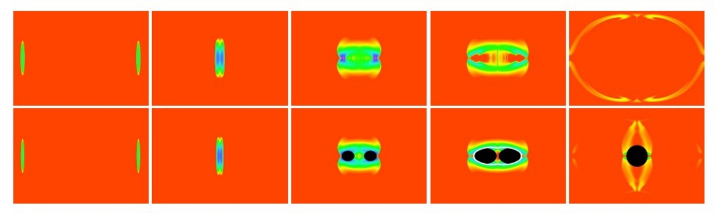
$\gamma = 4$

- BH formation threshold: $\gamma_{\rm thr} = 2.9 \pm 10\ \% \sim 1/3\ \gamma_{hoop}$
- Model particle collisions by BH collisions

Does matter "matter"?

- Perfect fluid "stars" model
- $\gamma = 8 \ldots 12$; BH formation below Hoop prediction

  East & Pretorius '12
- Gravitational focusing $\Rightarrow$ Formation of individual horizons



- Type-I critical behaviour
- Extrapolation by 60 orders would imply no BH formation at LHC

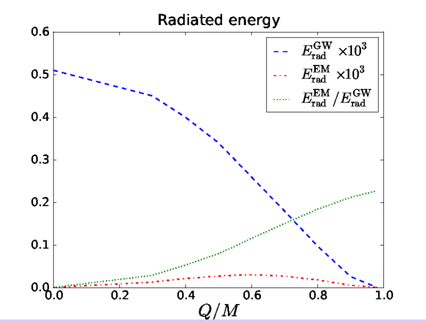
  Rezzolla & Tanaki '12

Collisions of equally charged BHs in $D = 4$

- Electro-vacuum Einstein-Maxwell Eqs.; Moesta et al. '10
- Brill-Lindquist construction for equal mass, charge BHs
- Wave extraction $\Phi_2 := F_{\mu\nu}\bar{m}^{\mu}k^{\nu}$

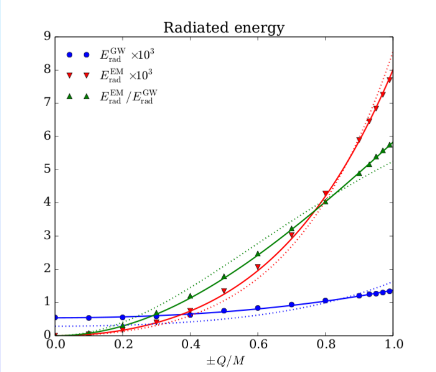
- $E_{EM} < E_{GW}$
- E_{GW} decreases with Q
- E_{GW} max. at $Q \approx 0.6\ M$

Zilhão et al. 2012



Collisions of oppositely charged BHs in $D = 4$

- Electro-vacuum Einstein-Maxwell Eqs.; Moesta et al. '10
- Brill-Lindquist construction for equal mass, charge BHs
- Wave extraction $\Phi_2 := F_{\mu\nu}\bar{m}^{\mu}k^{\nu}$



- E_{EM}, E_{GW} increase with Q
- E_{EM} dominates at $Q \gtrsim 0.4\ M$
- Good agreement with PP

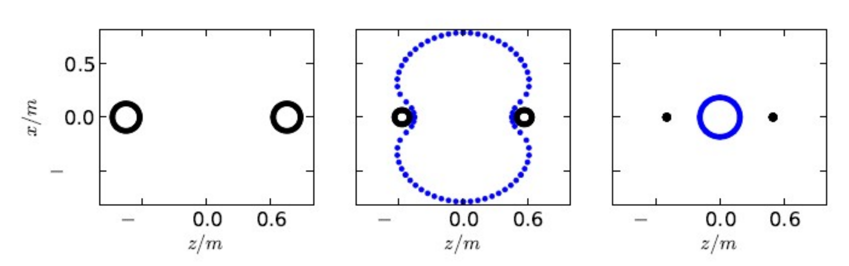
  Zilhão et al. 2014

Cosmic Censorship in $D = 4$ de Sitter

Zilhão et al. '12

- Two parameters: MH, d
- Initial data: McVittie type binaries McVittie '33
- "Small BHs": $d < d_{crit} \Rightarrow$ merger

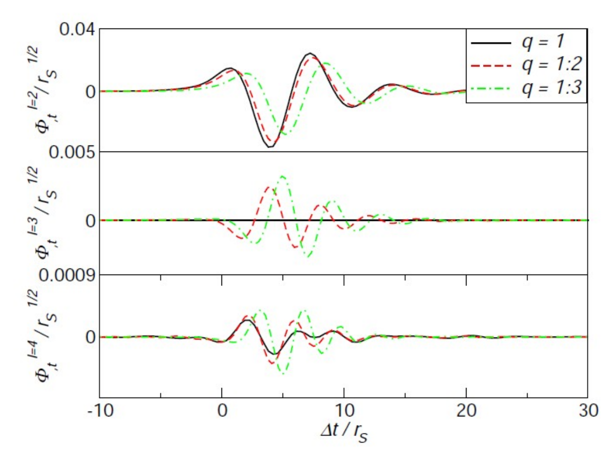
  $d > d_{crit} \Rightarrow$ no common AH
- "Large" holes at small d: Cosmic Censorship holds



5. Higher D collisions

GWs in $D = 5$ head-on from rest

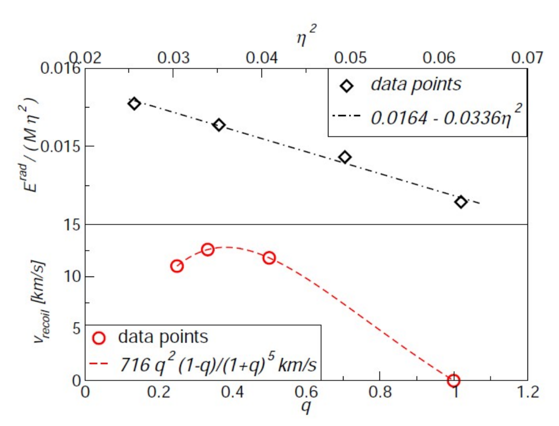
Wave extraction based on Kodama & Ishibashi '03



Witek *et al.* 2010

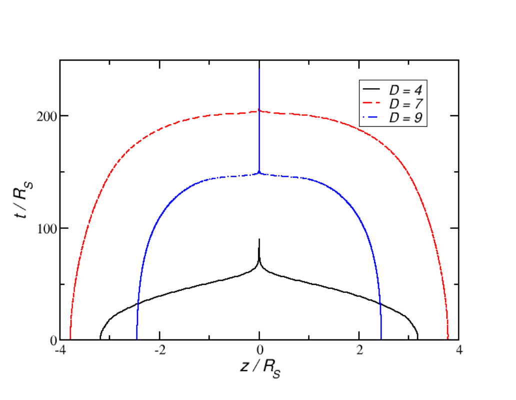
Unequal-mass head-on in $D = 5$

Radiated energy and momentum



Agreement within < 5 % with extrapolated point particle calculations

Head-on from rest in $D = 4 \ldots 10$

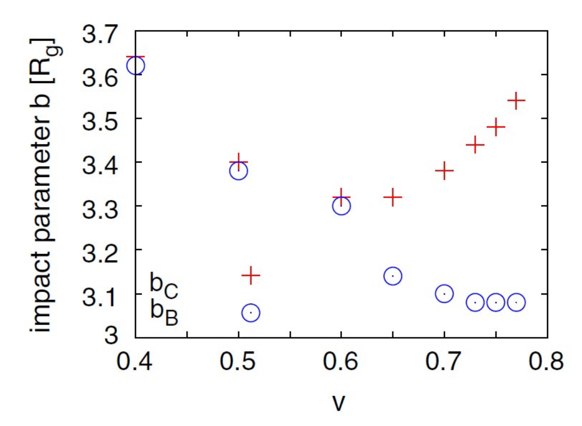


- Puncture trajectories
- Brute force exploration of gauge parameter space

US et al, work in progress

Scattering threshold in $D = 5$

Okawa, Nakao & Shibata 2011



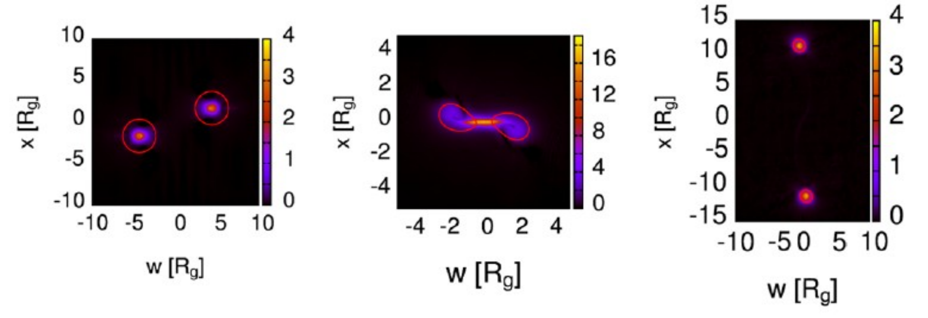
Numerical stability still an issue...

Super Planckian regime in $D = 5$

Okawa, Nakao & Shibata '11

- Take Tangherlini metric; boost and translate
- Superpose two of those

$$\frac{\sqrt{R^{abcd}R_{abcd}}}{6\sqrt{2}E_P^2}$$



Code comparison

- SACRAND Yoshino & Shibata 2009
  - Einstein $\rightarrow$ ADM $\rightarrow$ BSSN $\rightarrow$ $3+1$ + add. fields
  - GWs from Landau-Lifshitz pseudo tensor
- HD-LEAN Zilhão et al 2010
  - Einstein $\rightarrow$ $3+1$ + add. fields $\rightarrow$ ADM $\rightarrow$ BSSN
  - GWs from Kodama & Ishibashi 2003
- $E_{D=4} \approx 0.55 \times 10^{-4}\ M$

  $E_{D=5} \approx 0.90 \times 10^{-4}\ M$

  $E_{D=6} \sim 0.8 \times 10^{-4}\ M$
- Codes in agreement within numerical accuracy: $\sim 5\ \%$

  Witek, Okawa et al 2014, in preparation

6. Conclusions and outlook

Conclusions and outlook

- Collisions in $D = 4$ rather well understood
  - Cosmic censorship supported
  - $b_{\rm scat} = \frac{2.5 \pm 0.05}{v} M$
  - $E_{rad} \leq \sim 50\ \%$
- Structure does not seem to matter
- Matter collisions in agreement with Hoop conjecture
- $D > 4$ head-on from rest in reach
- $D = 5$: bracketing of $b_{\rm scat}$
- Super Planckian regime in $D = 5$
- Numerical stability: gauge?, Z4c?

BH collisions: Computational framework

- Focus here: $D = 4$ dimensions
- "Moving puncture" technique

  Goddard '05, Brownsville-RIT '05
- BSSN formulation; Shibata & Nakamura '95, Baumgarte & Shapiro '98
- $1 + \log$ slicing, Γ-driver shift condition
- Puncture ini-data; Bowen-York '80; Brandt & Brügmann '97; Ansorg et al. '04
- Mesh refinement `Cactus, Carpet`
- Wave extraction using Newman-Penrose scalar
- Apparent Horizon finder; e.g. Thornburg '96