The role of black-hole simulations in fundamental physics

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Overview

- Introduction, Numerical relativity
- BHs in GW physics
- BHs in astrophysics
- High-energy collisions of BHs
- BH holography
- Fundamental properties of BHs

1. Introduction, motivation

Evidence for astrophysical black holes

- X-ray binaries

  e. g. Cygnus X-1 (1964)

  MS star + compact star

  $\Rightarrow$ Stellar Mass BHs

  $\sim 5 \ldots 50\ M_{\odot}$

- Stellar dynamics

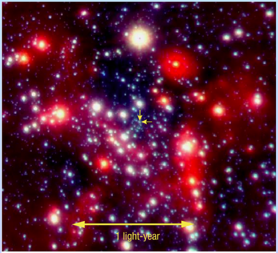
  near galactic centers,

  iron emission line profiles

  $\Rightarrow$ Supermassive BHs

  $\sim 10^6 \ldots 10^9\ M_{\odot}$

  AGN engines



The Centre of the Milky Way
(VLT YEPUN + NACO)
ESO PR Photo 23a/02 (9 October 2002)
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Conjectured BHs

- Intermediate mass BHs

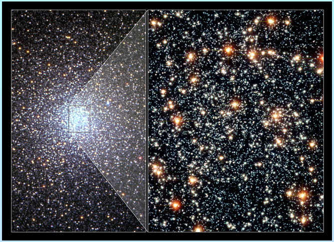
  $\sim 10^2 \dots 10^5\ M_\odot$

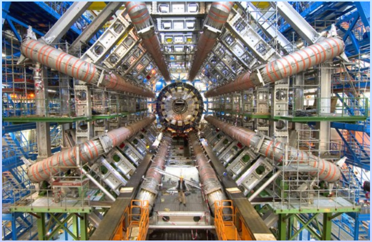
- Primordial BHs

  $\leq M_{Earth}$

- Mini BHs, LHC

  $\sim TeV$



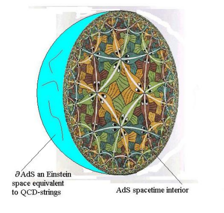


Note: BH solution is scale invariant!

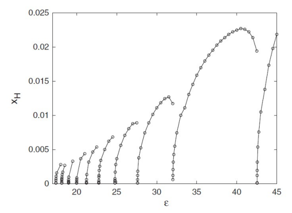
Research areas: Black holes have come a long way!

Astrophysics

Gauge-gravity duality

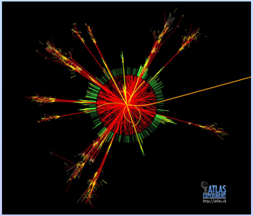


Fundamental studies

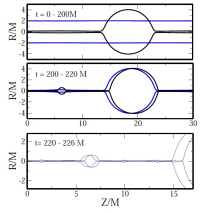


GW physics

High-energy physics



Fluid analogies



How to get the metric?

Train cemetery
Uyuni, Bolivia

- Solve for the metric $g_{\alpha\beta}$

Solving Einstein's equations: Different methods

- Analytic solutions
  - Symmetry assumptions
  - Schwarzschild, Kerr, FLRW, Myers-Perry, Emparan-Reall,...
- Perturbation theory
  - Assume solution is close to known solution $g_{\alpha\beta}$
  - Expand $\hat{g}_{\alpha\beta} = g_{\alpha\beta} + \epsilon h^{(1)}_{\alpha\beta} + \epsilon^2 h^{(2)}_{\alpha\beta} + \ldots \;\Rightarrow$ linear system
  - Regge-Wheeler-Zerilli-Moncrief, Teukolsky, QNMs, EOB,...
- Post-Newtonian Theory
  - Assume small velocities $\Rightarrow$ expansion in $\frac{v}{c}$
  - N^{th} order expressions for GWs, momenta, orbits,...
  - Blanchet, Buonanno, Damour, Kidder, Will,...
- Numerical Relativity

A list of tasks

- Target: Predict time evolution of BBH in GR
- Einstein equations: 1) Cast as evolution system
  2) Choose specific formulation
  3) Discretize for computer
- Choose coordinate conditions: Gauge
- Fix technical aspects: 1) Mesh refinement / spectral domains
  2) Singularity handling / excision
  3) Parallelization
- Construct realistic initial data
- Start evolution...
- Extract physics from the data

A brief history of BH simulations

- Pioneers: Hahn & Lindquist ’60s, Eppley, Smarr *et al.* ’70s
- Grand Challenge: First $3D$ Code Anninos *et al.* ’90s
- Further attempts: Bona & Massó, Pitt-PSU-Texas

  AEI-Potsdam, Alcubierre *et al.*

  PSU: first orbit Brügmann *et al.* ’04

Codes unstable!

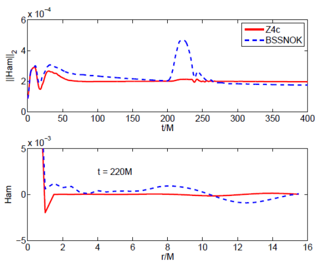
- Breakthrough: Pretorius ’05 — GHG

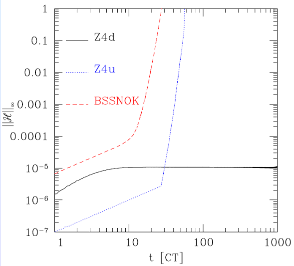
  UTB, Goddard’05 — Moving Punctures
- ∼10 codes world wide

Formulations

- Formulations mostly used: GHG, BSSN
- Combine advantages from both through conformal Z4 formulation
  - Z4 system   Bona et al, PRD 67 (2003), PRD 69 (2004)
  - Conformal decomposition ⇒ Z4c, CCZ4
    Alic et al, PRD 85 (2011), Cao et al, PRD 85 (2011)

    Hilditch et al, 1212.2901   Weyhausen et al, PRD 85 (2012)
- Advantages: constraint damping, constraint preserving BCs

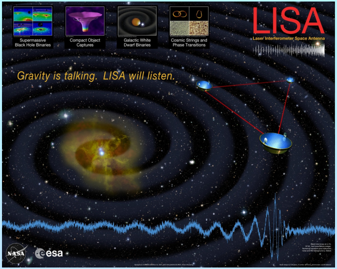


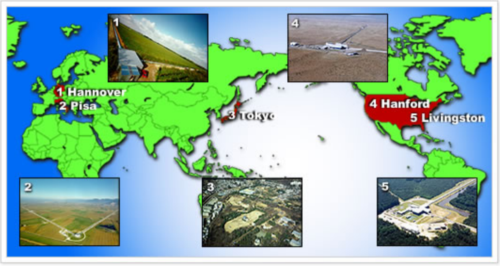


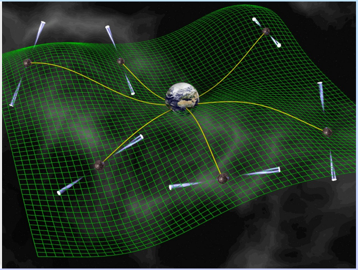
2. BHs in GW physics

Gravitational wave detectors

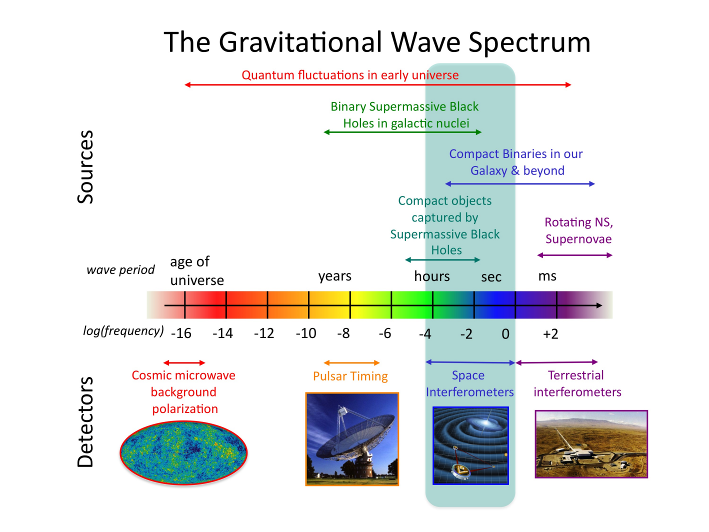
- Accelerated masses $\Rightarrow$ GWs
- Weak interaction!
- Laser interferometric detectors







The gravitational wave spectrum



Free parameters of BH binaries

- Total mass M

  Relevant for GW detection: Frequencies scale with M

  Not relevant for source modeling: trivial rescaling

- Mass ratio $q \equiv \frac{M_1}{M_2}$, $\eta \equiv \frac{M_1 M_2}{(M_1+M_2)^2}$

- Spin: $\vec{S}_1$, $\vec{S}_2$ (6 parameters)

- Initial parameters

  Binding energy E_b Separation

  Orbital ang. momentum L Eccentricity

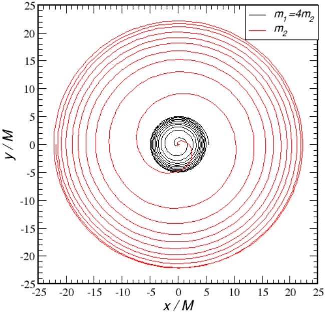
  Alternatively: frequency, eccentricity

BBH trajectory and waveform

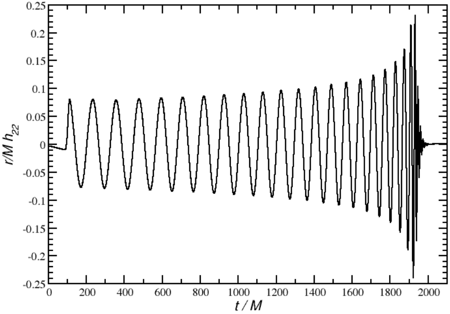
- $q = 4$, non-spinning binary; ~ 11 orbits

  US et al, CQG 28 (2011)

Trajectory

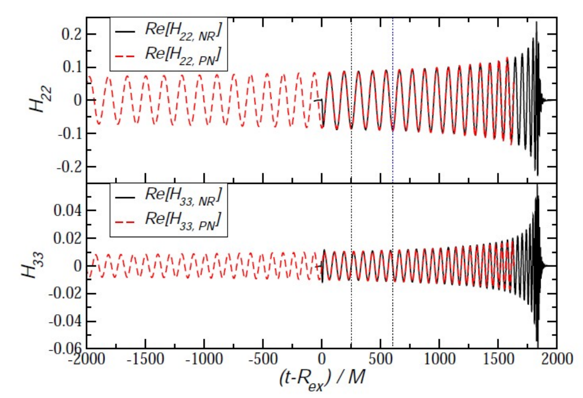


Quadrupole mode



Template construction

- Stitch together PN and NR waveforms
- EOB or phenomenological templates for $\geq$ 7-dim. par. space



Template construction

- Phenomenological waveform models
  - Model phase, amplitude with simple functions $\rightarrow$ Model parameters
  - Create map between physical and model parameters
  - Time or frequency domain

    Ajith et al, CQG 24 (2007), PRD 77 (2008), CQG 25 (2008), PRL 106 (2011); Santamaria et al, PRD 82 (2010), Sturani et al, 1012.5172
- Effective-one-body (EOB) models
  - Particle in effective metric, PN, ringdown model
    Buonanno & Damour PRD 59 (1999), PRD 62 (2000)
  - Resum PN, calibrate pseudo PN parameters using NR
    Buonanno et al, PRD 77 (2008); Damour et al, PRD 77 (2008), PRD 78 (2008), PRD 83 (2011); Pan et al, PRD 81 (2010), PRD 84 (2011)

The Ninja project

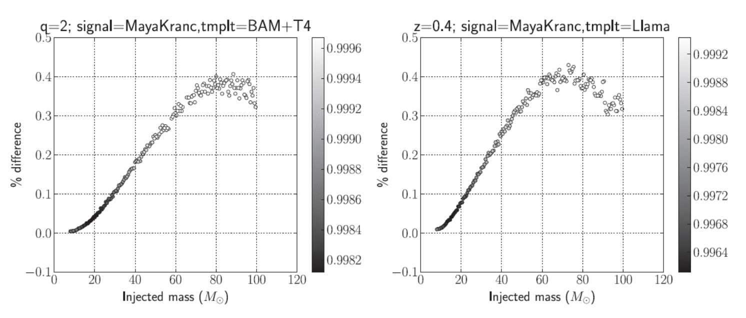
`https://www.ninja-project.org/`

Aylott et al, CQG 26 (2009), CQG 26 (2009)

Ajith et al, CQG 29 (2012)

- Use PN/NR hybrid waveforms in GW data analysis
- Ninja2: 56 hybrid waveforms from 8 NR groups
- Details on hybridization procedures
- Overlap and mass bias study:
  - Take one waveform as signal, fixing M_{tot}
  - Search with other waveform (same config.) varying $t_0,\ \phi_0,\ M_{tot}$

The Ninja project



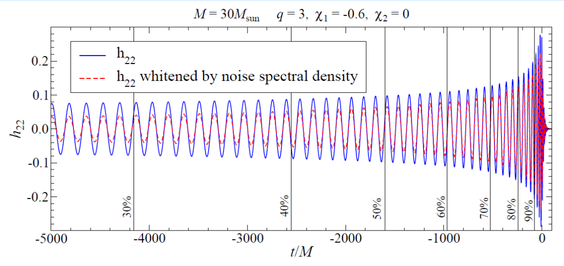
- Left: $q = 2$, non-spinning waveforms, MAYAKRANC, BAM + T4
- Right: $q = 1,\ \chi_1 = \chi_2 = 0.4$ waveform, MAYAKRANC, LLAMA + T4
- Mass bias < 0.5 %

The NRAR project

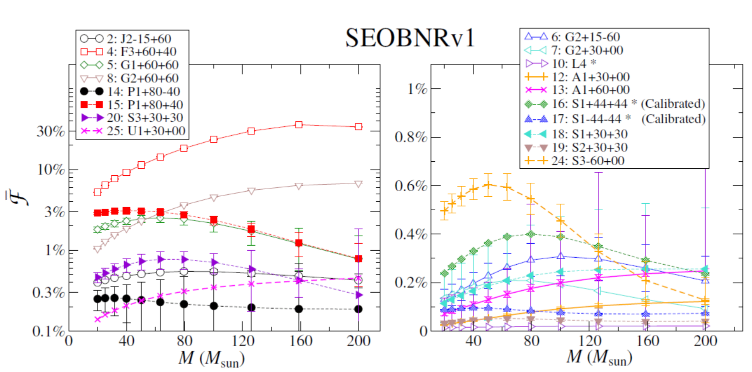
https://www.ninja-project.org/doku.php?id=nrar:home

Hinder, Buonanno et al, 1307.5307

- Pool efforts from 9 NR groups
- 11M core hours on XSEDE Kraken
- $22 + 3$ waveforms, including precessing runs
- Standardize analysis, comparison with analytic models



The NRAR project

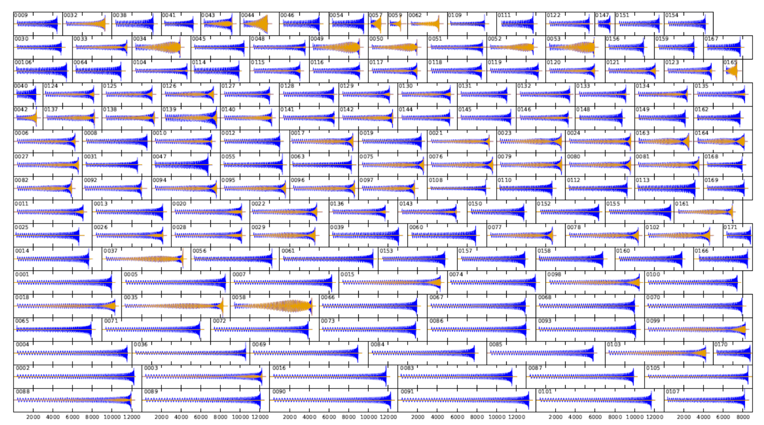


- Unfaithfulness $\bar{\mathcal{F}} = 1-$ best overlap varying $t_0, \ \phi_0$
- $\bar{\mathcal{F}}$ between SEOBNRv1 and NR waveforms

Tools of mass production

- SpEC catalog: 171 waveforms: $q \le 8$, 90 precessing, ≤ 34 orbits

  Mroué et al, 1304.6077



Strategies in parameter space

- SpEC: 16 orbits in 40 hours
- Still, 7-dimensional parameter space $\rightarrow$ $N \sim 10^7$ waveforms?
- Probably too many... Accuracy needed...
- Reduce # of parameters describing dominant spin effects

  Ajith et al, PRL 106 (2011), PRD 84 (2011),

  Pürrer et al, 1306.2320
- Spin-robit resonances $\Rightarrow$ preferred regions in parameter space?

  Gerosa et al, PRD 87 (2013) [gr-qc]
- Trade-off: Quantity or quality of waveforms?

  Both affects parameter estimation!

Limits in the parameter space

- Mass ratio $q = 100$

  Lousto & Zlochower, PRL 106 (2011)

  Head-on case: US et al, PRD 84 (2011)

- Spin magnitude $\chi = 0.97$

  Superposed Kerr-Schild data (non-conformally flat)

  Lovelace et al, CQG 29 (2012)

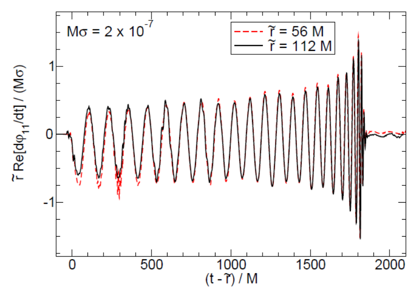
- Separations $D = 100\ M$; few orbits

  Lousto & Zlochower, PRD 88 (2013) [gr-qc]

Going beyond GR: Scalar-tensor theory of gravity

- Brans-Dicke theory: 1 parameter ω_{BD}; well constrained
- Bergmann-Wagoner theories: Generalize $\omega = \omega(\phi),\ V = V(\phi)$
- No-hair theorem: BHs solutions same as in GR
  e.g. Hawking, Comm.Math.Phys. 25 (1972)
  Sotiriou & Faraoni, PRL 108 (2012)

- Circumvent no-hair theorem: Scalar bubble
  Healy et al, 1112.3928
- Circumvent no-hair theorem: Scalar gradient
  Horbatsch & Burgess, JCAP 1205 (2012), Berti et al, PRD 87 (2013)



3. BHs in Astrophysics

Gravitational recoil

- Anisotropic GW emission $\Rightarrow$ recoil of remnant BH

  Bonnor & Rotenburg, Proc.Roy.Soc. 265 (1961) Peres, PR 128 (1962), Bekenstein, ApJ 183 (1973)

- Escape velocities:

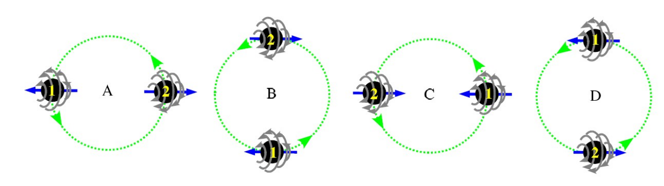
Globular clusters	30 $\mathrm{km/s}$
dSph	$20 - 100$ $\mathrm{km/s}$
dE	$100 - 300$ $\mathrm{km/s}$
Giant galaxies	~ 1000 $\mathrm{km/s}$

Ejection / displacement of BH $\Rightarrow$

- Growth history of SMBHs
- BH populations, IMBHs
- Structure of galaxies

Spinning BHs: Superkicks

- Superkick configuration:



- Kicks up to $v_{\mathrm{max}} \approx 4\,000\ \mathrm{km/s}$

  Campanelli *et al.*, PRL 98 (2007)

  González *et al.* PRL 98 (2007)

- Suppression via spin alignment and Resonance effects in inspiral

  Schnittman, PRD 70 (2004)

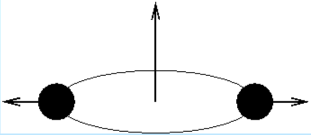
  Bogdanovicź et al, ApJ 661 (2007)

  Kesden et al, PRD 81 (2010), ApJ 715 (2010)

Even larger kicks: superkick and hang-up

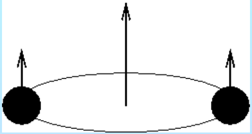
Lousto & Zlochower, PRL 107 (2011)

Superkicks

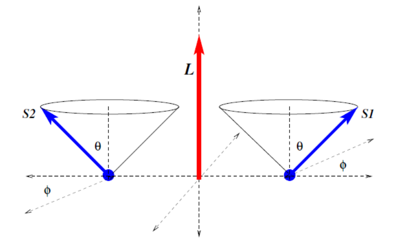


- Moderate GW generation
- Large kicks

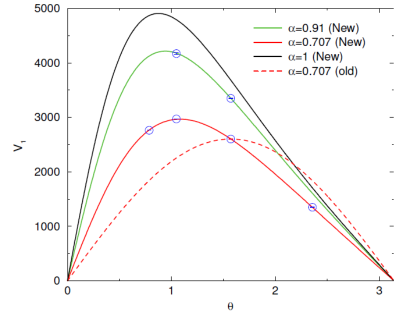
Hangup



- Strong GW generation
- No kicks



Superkicks and orbital hang-up



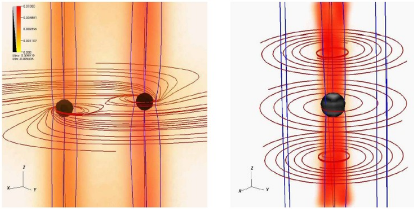
- Maximum kick about 25 % larger: $v_{\mathrm{max}} \approx 5\,000$ km/s
- Distribution asymmetric in θ; v_{max} for partial alignment
- Supression through resonances still works

Berti et al, PRD 85 (2012)

EM counterparts generated by binary BHs

Palenzuela et al, PRL 103 (2009), Science 329 (2010)
PRD 81 (2010), PRD 82 (2010)

- Non-spinning BH binary
- Einstein-Maxwell equtions with “force free” plasma
- Electromagnetic field extracts energy from **L** $\Rightarrow$ jets
- Optical signature: double jets



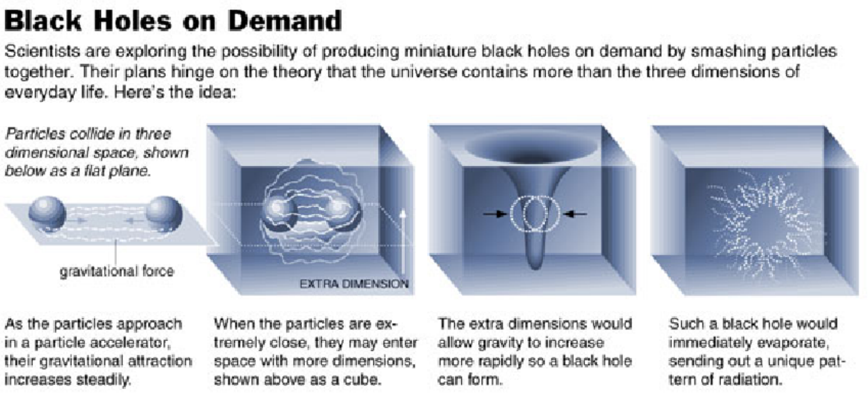
(a) $-8.2\,M_8$ hrs (b) $4.6\,M_8$ hrs

4. High-energy BH collisions

The Hierarchy Problem of Physics

- Gravity $\approx 10^{-39}\times$ other forces
- Higgs field $\approx \mu_{obs} \approx 250\ GeV = \sqrt{\mu^2 - \Lambda^2}$
  where $\Lambda \approx 10^{16}\ GeV$ is the grand unification energy
- Requires enormous finetuning!!!
- Finetuning exist: $\frac{987654321}{123456789} = 8.0000000729$
- Or E_{Planck} much lower? Gravity strong at small r?
  $\Rightarrow$ BH formation in high-energy collisions at LHC
- Gravity not measured below 0.16 mm! Diluted due to...
  - Large extra dimensions   Arkani-Hamed, Dimopoulos & Dvali '98
  - Extra dimension with warp factor   Randall & Sundrum '99

Stages of BH formation



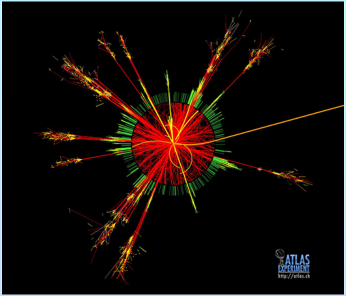
- Matter does not matter at energies well above the Planck scale

$\Rightarrow$ Model particle collisions by black-hole collisions

Banks & Fischler, gr-qc/9906038; Giddings & Thomas, PRD 65 (2002)

Experimental signature at the LHC

Black hole formation at the LHC could be detected by the properties of the jets resulting from Hawking radiation. `BlackMax`, `Charybdis`



- Multiplicity of partons: Number of jets and leptons
- Large transverse energy
- Black-hole mass and spin are important for this!

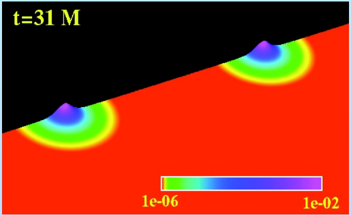
ToDo:

- Exact cross section for BH formation
- Determine loss of energy in gravitational waves
- Determine spin of merged black hole

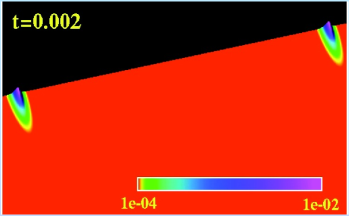
Does matter "matter"?

- Hoop conjecture $\Rightarrow$ kinetic energy triggers BH formation
- Einstein plus minimally coupled, massive, complex scalar filed

  "Boson stars"   Choptuik & Choptuik, PRL 104 (2010)



$\gamma = 1$



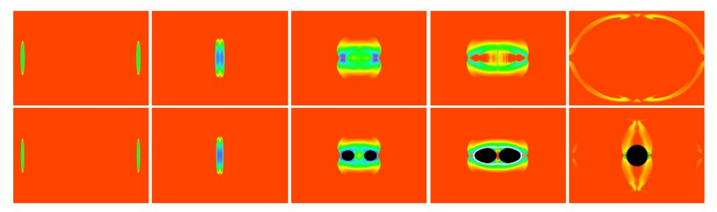
$\gamma = 4$

- BH formation threshold: $\gamma_{\rm thr} = 2.9 \pm 10\ \% \sim 1/3\ \gamma_{hoop}$
- Model particle collisions by BH collisions

Does matter "matter"?

- Perfect fluid "stars" model
- $\gamma = 8 \ldots 12$; BH formation below Hoop prediction

  East & Pretorius, PRL 110 (2013)
- Gravitational focussing $\Rightarrow$ Formation of individual horizons



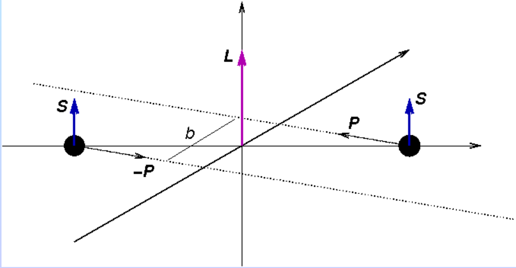
- Type-I critical behaviour
- Extrapolation by 60 orders would imply no BH formation at LHC

  Rezzolla & Tanaki, CQG 30 (2013)

$D = 4$: Initial setup: 1) Aligned spins

- Orbital hang-up   Campanelli et al, PRD 74 (2006)
- 2 BHs: Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

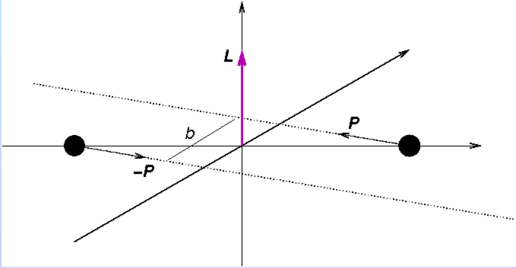
  Boost: $\gamma = 1/\sqrt{1-v^2}$, $M = \gamma M_0$
- Impact parameter: $b \equiv \frac{L}{P}$



$D = 4$: Initial setup: 2) No spins

- Orbital hang-up   Campanelli et al, PRD 74 (2006)
- 2 BHs: Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

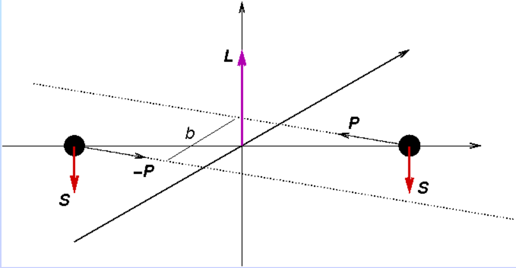
  Boost: $\gamma = 1/\sqrt{1 - v^2}$, $M = \gamma M_0$
- Impact parameter: $b \equiv \frac{L}{P}$



$D = 4$: Initial setup: 3) Anti-aligned spins

- Orbital hang-up   Campanelli et al, PRD 74 (2006)
- 2 BHs: Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

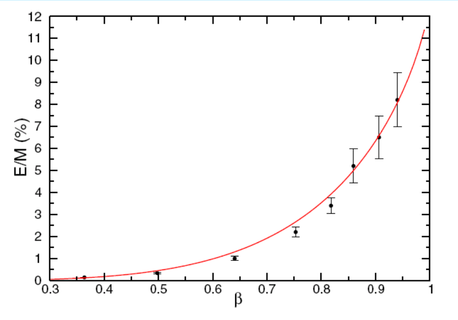
  Boost: $\gamma = 1/\sqrt{1-v^2}$, $M = \gamma M_0$
- Impact parameter: $b \equiv \frac{L}{P}$



$D = 4$: Head-on: $b = 0, \quad \vec{S} = 0$

- Total radiated energy: 14 ± 3 % for $v \to 1$
  US et al, PRL 101 (2008)

  About half of Penrose '74



- Agreement with approximative methods

  Flat spectrum, GW multipoles Berti et al, PRD 83 (2011)

$D = 4$: Scattering threshold b_{scat} for $\vec{S} = 0$

- $b < b_{\rm scat}$ $\Rightarrow$ Merger

  $b > b_{\rm scat}$ $\Rightarrow$ Scattering

- Numerical study: $b_{\rm scat} = \frac{2.5 \pm 0.05}{v} M$

  Shibata et al, PRD 78 (2008)

- Independent study US et al, PRL 103 (2009), PRL 111 (2013)

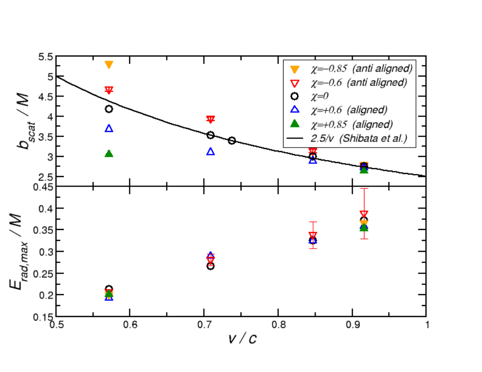
  $\gamma = 1.23 \ldots 2.93$:

  $\chi = \pm 0.85,\ \pm 0.6,\ 0$ (anti-aligned, nonspinning, aligned)

- Limit from Penrose construction: $b_{\rm crit} = 1.685\ M$

  Yoshino & Rychkov, PRD 74 (2005)

$D = 4$: Scattering threshold and radiated energy $\vec{S} \neq 0$

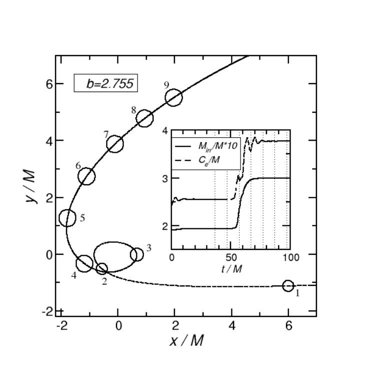


US et al, PRL 111 (2013)

- At speeds $v \gtrsim 0.9$ spin effects washed out
- E_{rad} always below $\lesssim 50\ \%\ M$

$D = 4$: Absorption

- For large γ: $E_{kin} \approx M$
- If E_{kin} is not radiated, where does it go?
- Answer: $\sim$ 50 % into E_{rad}, $\sim$ 50 % is absorbed

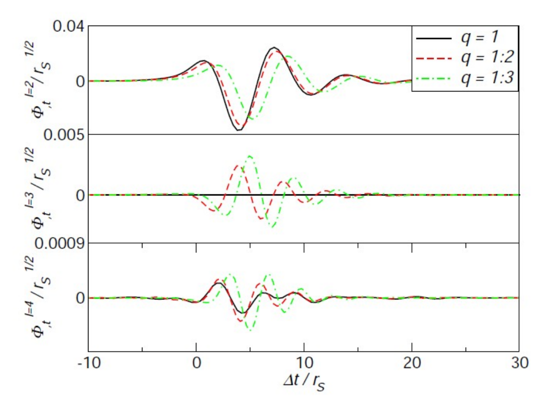


US et al, PRL 111 (2013)

$D = 5$: Unequal-mass head-on

Dimensional reduction: Zilhão et al, PRD 81 (2010)

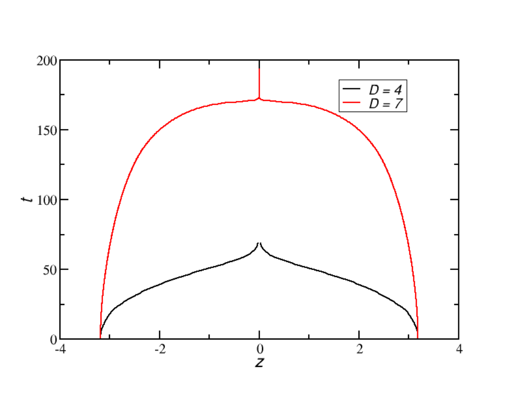
Wave extraction: Kodama & Ishibashi PTP 110 (2003), Witek et al, PRD 83 (2011)



Witek *et al.*, PRD 83 (2011)

$D = 7$ Head-on collisions

Work in progress...



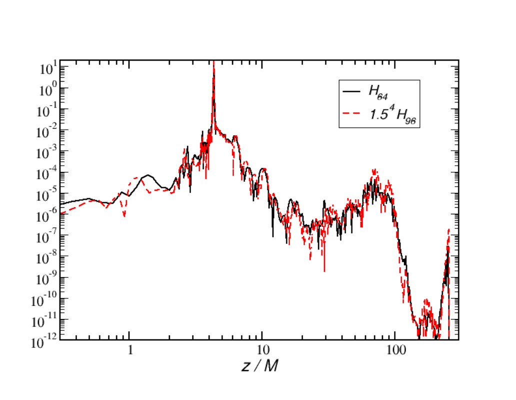
$D = 4$ and $D = 7$ spacetime dimensions

Collisions of charged BHs

- Equal Q/M ratio

  Zilhão et al, PRD 85 (2012)

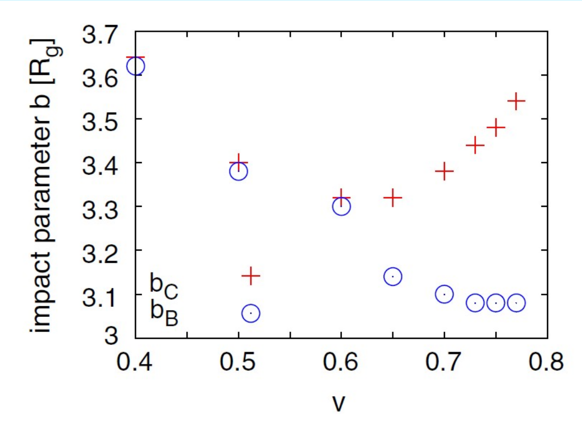
- Opposite charges in progress...



$D = 5$: Scattering threshold

Modified Cartoon: Yoshino & Shibata, PRD 80 (2009)

First boosted collisions in $D > 4$: Okawa et al, PRD 83 (2011)



Numerical stability still an issue...

5. BH Holography

The AdS/CFT conjecture

Maldacena, Adv.Theor.Math.Phys. 2 (1997)

- "strong form": Type IIb string theory on $AdS_5 \times S^5$

  $\Leftrightarrow \mathcal{N} = 4$ super Yang-Mills in $D = 4$

  Hard to prove; non-perturbative Type IIb String Theory?

- "weak form": low-energy limit of string-theory side

  $\Rightarrow$ Type IIb Supergravity on $AdS_5 \times S^5$

- Some assumptions, factor out S^5

  $\Rightarrow$ General Relativity on AdS_5

- Corresponds to limit of large N, $g^2 N$ in the field theory

- E. g. Stationary AdS BH $\Leftrightarrow$ Thermal Equil. with T_{Haw} in dual FT

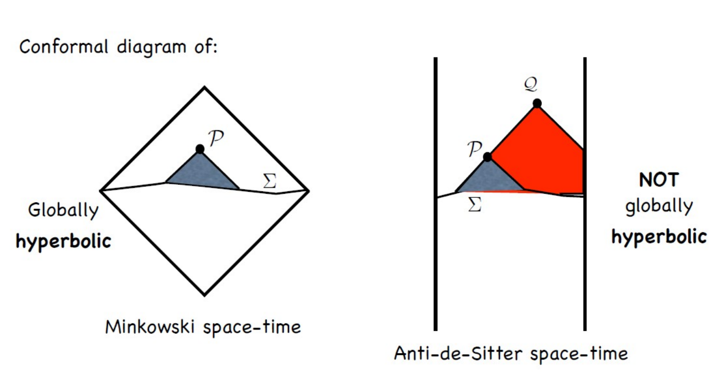
  Witten, Adv.Theor.Math.Phys. 2 (1998)

The boundary in AdS

- Dictionary between metric properties and

  vacuum expectation values of CFT operators.

  E. g. $T_{\alpha\beta}$ operator of CFT $\leftrightarrow$ transverse metric on *AdS* boundary.
- The boundary plays an active role in *AdS*! Metric singular!



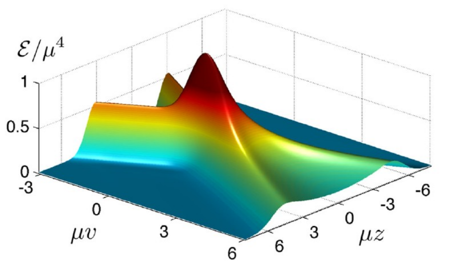
Collision of planar shockwaves in $\mathcal{N} = 4$ SYM

- Heavy-ion collisions (RHIC, LHC) $\Rightarrow$ Strongly coupled QGP
- Dual to colliding gravitational shock waves in AADS
- Characteristic study with translational invariance

  Chesler & Yaffe PRL 102 (2009), PRD 82 (2010), PRL 106 (2011)

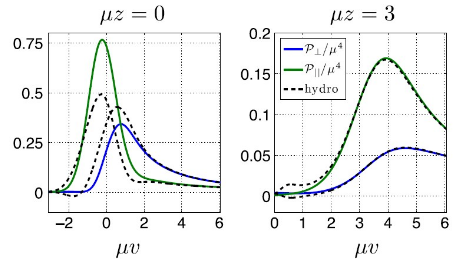
- Initial data: 2 superposed shockwaves

  $$ds^2 = r^2[-dx_+dx_- + d\mathbf{x}_\perp] + \frac{1}{r^2}[dr^2 + h(x_\pm)dx_\pm^2]$$



Collision of planar shockwaves in $\mathcal{N} = 4$ SYM

- Initially system far from equilibrium
- Thermalization after $\Delta v \sim 4/\mu \sim 0.35\ fm/c$
- Confirms hydro sims. of QGP $\sim 1\ fm/c$ Heinz, nucl-th/0407067



- Non-linear vs. linear Einstein Eqs. agree within ~ 20 %
  Heller et al, PRL 108 (2012)
- Thermalization in ADM formulation Heller et al, PRD 85 (2012)

Cauchy ("4+1") evolutions in asymptotically AdS

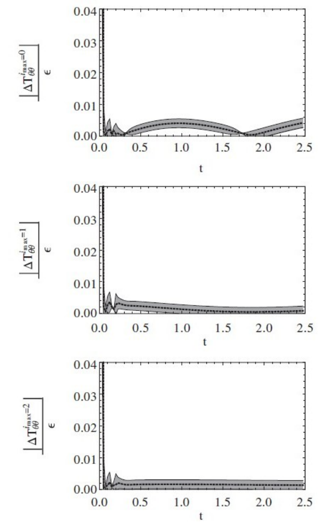
- Characteristic coordinates successful numerical tool in AdS/CFT
- But: restricted to symmetries, caustics problem...
- Cauchy evolution needed for general scenarios? Cf. BBH inspiral!!
- Cauchy scheme based on generalized harmonic formulation

  Bantilan & Pretorius, PRD 85 (2012)

  - $SO(3)$ symmetry
  - Compactify "bulk radius"
  - Asymptotic symmetry of AdS_5: $SO(4,2)$
  - Decompose metric into AdS_5 piece and deviation
  - Gauge must preserve asymptotic fall-off

Cauchy (“4+1”) evolutions in asymptotically AdS

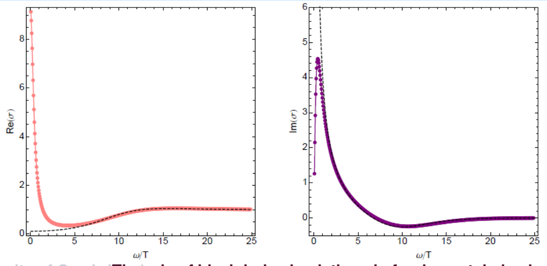
- Scalar field collapse
- BH formation and ringdown
- Low order QNMs $\sim$ perturbative studies, but mode coupling
- CFT stress-energy tensor consistent with thermalized $\mathcal{N} = 4$ SYM fluid
- Difference of CFT $T_{\theta\theta}$ and hydro ($+1^{st}$, 2^{nd} corrs.)



Conductivity and holography

Horowitz, Santos & Tong, JHEP 1207 (2012) JHEP 1211 (2012)

- Goal: AC conductivity of cuprates (strange metals)
- Einstein-Maxwell in $D = 4$ with negative Λ plus scalar field
- Perturbed Reissner-Nordström AdS BH
- Conductivity in frequency space:
  - Drude's result at low ω, QFT plateau at high ω
  - Intermediate $\sim \omega^{-2/3}$ fall-off; cf. experiment!



6 Fundamental properties of BHs

Stability of AdS

- $m = 0$ scalar field in as. flat spacetimes   Choptuik, PRL 70 (1992)

  $p > p^* \Rightarrow$ BH, $p < p^* \Rightarrow$ flat

- $m = 0$ scalar field in as. AdS Bizoń & Rostworowski, PRL 107 (2011)

- Similar behaviour for "Geons"

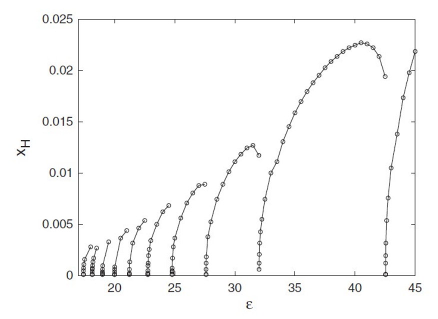
  Dias et al, CQG 29 (2012)

- $D > 4$ dimensions

  Jałmużna et al, PRD 84 (2011)

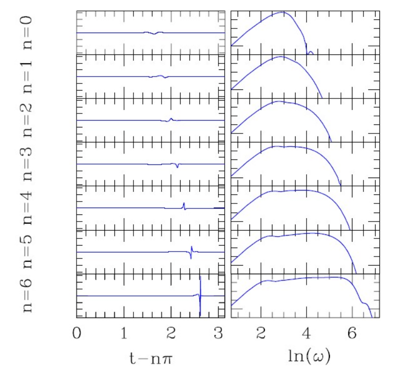
- $D = 3$: Mass gap: smooth solutions

  Bizoń & Jałmużna, 1306.0317



Stability of AdS

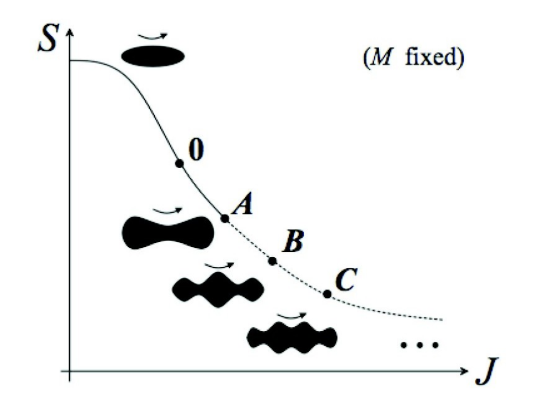
- Pulses narrow under successive reflections
  Buchel et al, PRD 86 (2012)



- ∃ Non-linearly stable solutions in AdS
  Dias et al, CQG 29 (2012), Buchel et al, PRD 87 (2013),
  Maliborski & Rostworowski PRL 111 (2013)

Bar mode instability of Myers-Perry BH

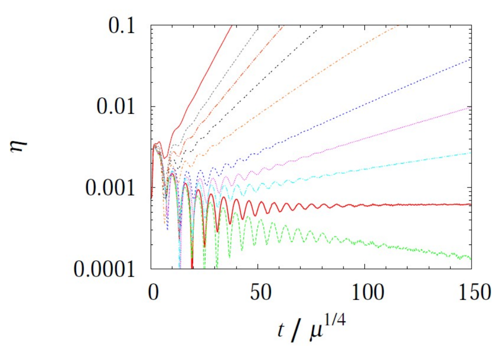
- MP BHs (with single ang.mom.) should be unstable.
- Linearized analysis Dias et al, PRD 80 (2009)



Non-linear analysis of MP instability

Shibata & Yoshino, PRD 81 (2010)

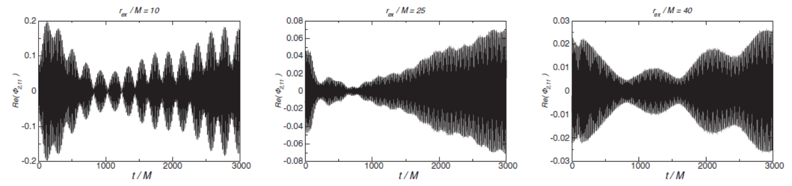
- Myers-Perry metric; transformed to Puncture like coordinate
- Add small bar-mode perturbation
- Deformation $\eta := \frac{2\sqrt{(l_0 - l_{\pi/2})^2 + (l_{\pi/4} - l_{3\pi/4})^2}}{l_0 + l_{\pi/2}}$



Superradiant instability

- Scattering of waves with $\mathrm{Re}[\omega]$ off BH with ang. horizon velocity Ω_H

  $\Rightarrow$ amplification $\Leftrightarrow$ $\mathrm{Re}[\omega] < m\Omega_H$
- Measure photon mass? Pani et al, PRL 109 (2012)
- Numerical simulations Dolan, PRD 87 (2013) Witek et al, PRD 87 (2013)
- Instability of spinning BHs, Beating effects



Witek et al, PRD 87 (2013)

Cosmic Censorship in $D = 5$

Lehner & Pretorius, PRL 105 (2010)

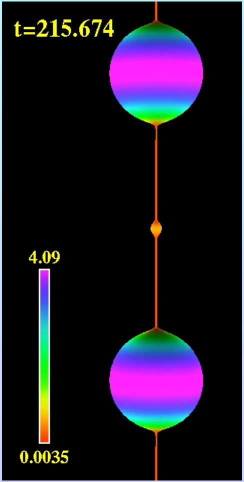
- Axisymmetric code
- Evolution of black string...
- Gregory-Laflamme instability

  cascades down

  in finite time

  until string has zero width

  $\Rightarrow$ naked singularity

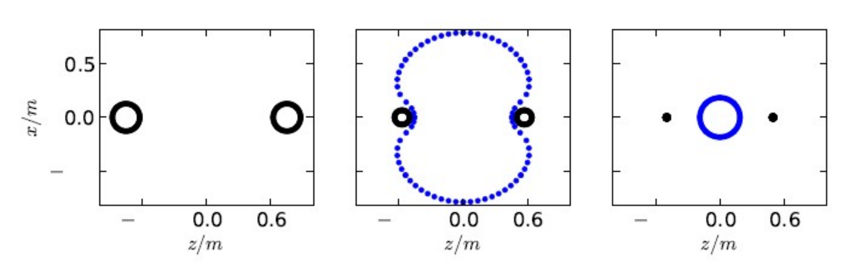


Cosmic Censorship in $D = 4$ de Sitter

Zilhão et al, PRD 85 (2012)

- Two parameters: MH, d
- Initial data: McVittie type binaries McVittie, MNRAS **93** (1933) 325
- "Small BHs": $d < d_{crit} \Rightarrow$ merger

  $d > d_{crit} \Rightarrow$ no common AH
- "Large" holes at small d: Cosmic Censorship holds



Conclusions

- Nearly 10 years after breakthroughs, codes matured
- GW template bank within reach
- Kicks still getting bigger
- High-energy collisions understood in $D = 4$, higher $D \rightarrow$ stability
- Applications to AdS/CFT exploding...
- NR reveals new insight into BH stability