The impact of spin-orbit resonances on astrophysical black-hole populations

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Southampton Gravity Seminar
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Overview

- Introduction
- Spin orbit resonances
- Final BH spins
- Suppression of superkicks
- Stellar-mass BH binary formation

Kesden, Sperhake & Berti, PRD **81** (2010) 084054

Kesden, Sperhake & Berti, ApJ **715** (2010) 1006-1011

Berti, Kesden & Sperhake, PRD **85** (2012) 124049

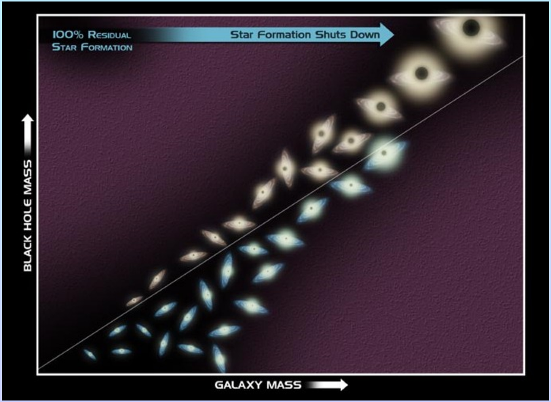
Gerosa, Kesden, Berti, O'Shaughnessy & Sperhake, arXiv:1302.4442 [gr-qc]

Schnittman, PRD **70** (2004) 124020

1. Introduction

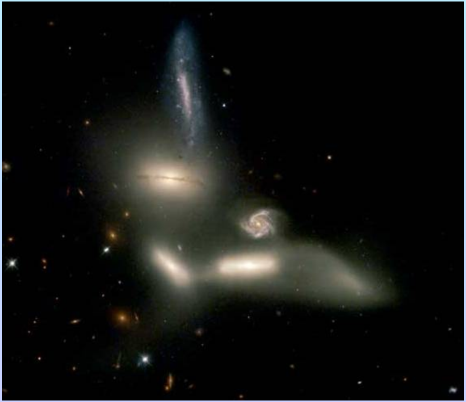
Introduction: Kicks

- Galaxies ubiquitously harbor BHs
- BH properties correlated with bulge properties
  e. g. J.Magorrian *et al.*, AJ 115, 2285 (1998)



Introduction

- Most widely accepted scenario for galaxy formation: hierarchical growth; “bottom-up”
- Galaxies undergo frequent mergers, especially elliptic ones



Superkicks

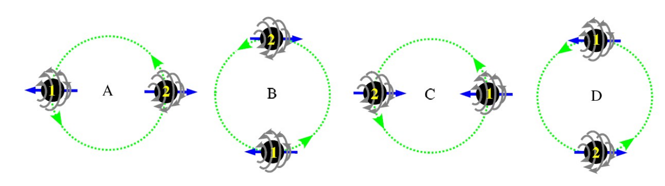
- Numerical relativity breakthroughs in 2005

  Pretorius '05, Goddard, RIT '06

- NR now able to accurately calculate kicks
- Superkicks: up to several 1000 km/s

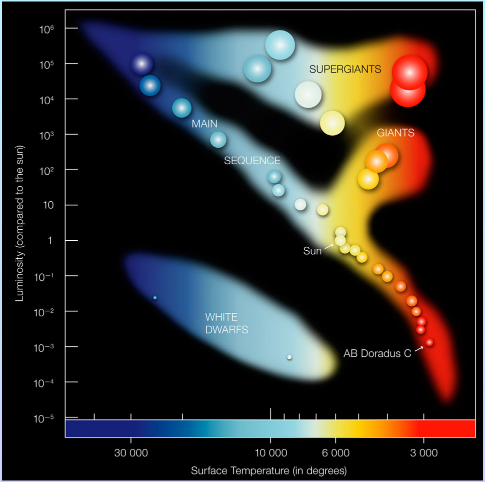
  González et al. '07, Campanelli et al. '07

- $>$ escape velocities from giant galaxies!



Introduction: BH binary formation

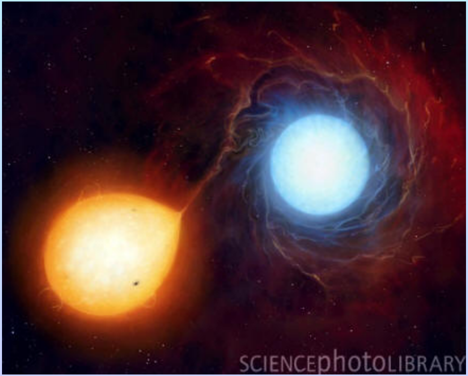
Evolution of single stars



Introduction: BH binary formation

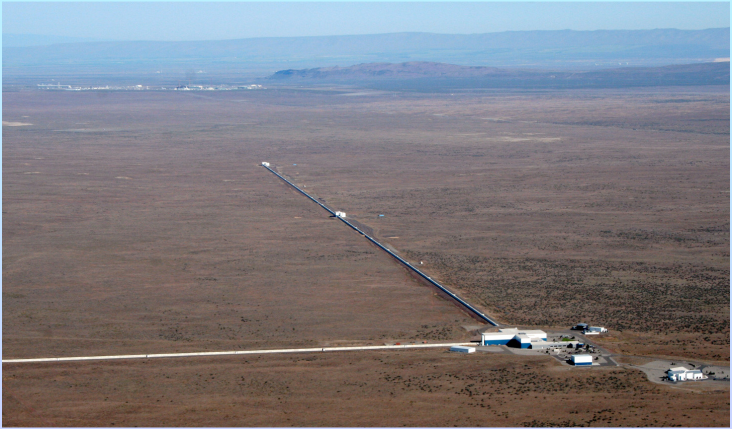
Stellar binaries

- Tides
- Roche lobe ⇒ mass transfer



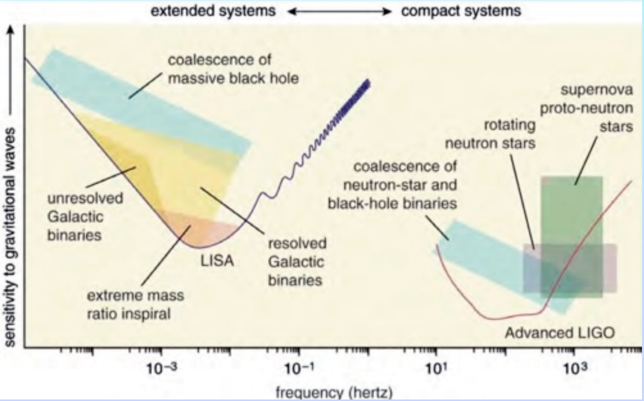
Gravitational wave detectors

LIGO, VIRGO upgraded; ET design studies



Gravitational wave detectors

GW sources



What can we learn from GW observations about BH binary formation?

2. Spin orbit resonances

Parameters of a black-hole binary

10 intrinsic parameters for quasi-circular orbits

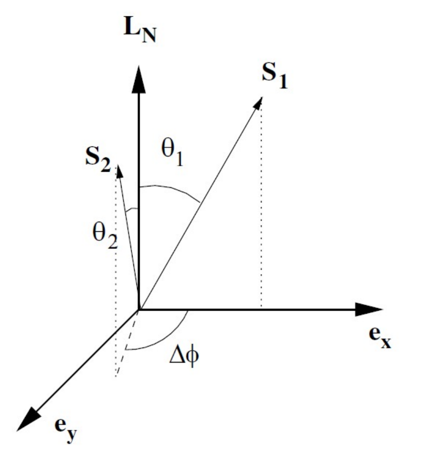
- 2 masses m_1, m_2
- 6 for two spins $\mathbf{S}_1$, $\mathbf{S}_2$
- 2 for the direction of the orbital ang. mom. $\hat{\mathbf{L}}$.

Elimination of parameters in PN inspiral

- 1 mass; scale invariance
- 2 for $\hat{\mathbf{L}}$; fix z axis
- 2 spin magnitudes, 1 mass ratio q; conserved
- 1 spin direction; fix x axis

Evolution variables

⇒ Three variables: θ_1, θ_2, $\Delta\phi$



Evolution equations

$$\frac{d\mathbf{S}_1}{dt} = \mathbf{\Omega}_1 \times \mathbf{S}_1, \qquad M\mathbf{\Omega}_1 = \eta v^5 \left(2 + \frac{3q}{2}\right)\hat{\mathbf{L}} + \frac{v^6}{2M^2}\left[\mathbf{S}_2 - 3\left(\hat{\mathbf{L}}\cdot\mathbf{S}_2\right)\hat{\mathbf{L}} - 3q\left(\hat{\mathbf{L}}\cdot\mathbf{S}_1\right)\hat{\mathbf{L}}\right];$$

$$\frac{d\mathbf{S}_2}{dt} = \mathbf{\Omega}_2 \times \mathbf{S}_2, \qquad M\mathbf{\Omega}_2 = \eta v^5 \left(2 + \frac{3}{2q}\right)\hat{\mathbf{L}} + \frac{v^6}{2M^2}\left[\mathbf{S}_1 - 3\left(\hat{\mathbf{L}}\cdot\mathbf{S}_1\right)\hat{\mathbf{L}} - \frac{3}{q}\left(\hat{\mathbf{L}}\cdot\mathbf{S}_2\right)\hat{\mathbf{L}}\right];$$

$$\frac{d\hat{\mathbf{L}}}{dt} = -\frac{v}{\eta M^2}\frac{d}{dt}(\mathbf{S}_1 + \mathbf{S}_2);$$

$$\begin{aligned}
\frac{dv}{dt} = \frac{32}{5}\frac{\eta}{M}v^9 \Bigg\{ & 1 - v^2\frac{743 + 924\eta}{336} + v^3\left[4\pi - \sum_{i=1,2}\chi_i(\hat{\mathbf{S}}_i\cdot\hat{\mathbf{L}})\left(\frac{113}{12}\frac{m_i^2}{M^2} + \frac{25}{4}\eta\right)\right] \\
& + v^4\left[\frac{34103}{18144} + \frac{13661}{2016}\eta + \frac{59}{18}\eta^2 + \frac{\eta\chi_1\chi_2}{48}\left(721(\hat{\mathbf{S}}_1\cdot\hat{\mathbf{L}})(\hat{\mathbf{S}}_2\cdot\hat{\mathbf{L}}) - 247(\hat{\mathbf{S}}_1\cdot\hat{\mathbf{S}}_2)\right)\right. \\
& \left. + \frac{1}{96}\sum_{i=1,2}\left(\frac{m_i\chi_i}{M}\right)^2\left(719(\hat{\mathbf{S}}_i\cdot\hat{\mathbf{L}})^2 - 233\right)\right] - v^5\pi\frac{4159 + 15876\eta}{672} \\
& + v^6\left[\frac{16447322263}{139708800} + \frac{16}{3}\pi^2 - \frac{1712}{105}(\gamma_E + \ln 4v) + \left(\frac{451}{48}\pi^2 - \frac{56198689}{217728}\right)\eta + \frac{541}{896}\eta^2 - \frac{5605}{2592}\eta^3\right] \\
& + v^7\pi\left[-\frac{4415}{4032} + \frac{358675}{6048}\eta + \frac{91495}{1512}\eta^2\right] + O(v^8)\Bigg\};
\end{aligned}$$

- 2.5 PN: precessional motion about $\hat{\mathbf{L}}$
- 3 PN: spin-orbit coupling

Schnittman's resonances

Schnittman '04

For a given separation r of the binary, resonances are

- $\mathbf{S}_1,\ \mathbf{S}_2,\ \hat{\mathbf{L}}_N$ lie in a plane $\Rightarrow \Delta\phi = 0^\circ, \pm 180^\circ$
- Resonance condition: $\ddot{\theta}_{12} = \dot{\theta}_{12} = 0$ Apostolatos '96, Schnittman '04
- $\Delta\phi = 0^\circ$ resonances: always $\theta_1 < \theta_2$

  $\Delta\phi = \pm 180^\circ$ resonances: always $\theta_1 > \theta_2$
- The resonance $\theta_1,\ \theta_2$ vary with r or $\mathbf{L}_N$

  $\Rightarrow$ Resonances sweep through parameter plane
- Time scales: $t_{\rm orb} \ll t_{\rm pr} \ll t_{\rm GW}$

  $\Rightarrow$ "Free" binaries can get caught by resonance

Evolution in $\theta_1, \ \theta_2$ plane for $q = 9/11$

$\theta_i := \angle(\vec{S}_i, \vec{L}_N)$

$\theta_1 = \theta_2$

$\mathbf{S} \cdot \mathbf{L}_N = \text{const}$

$\mathbf{S}_0 \cdot \mathbf{L}_N = \text{const}$

evolution

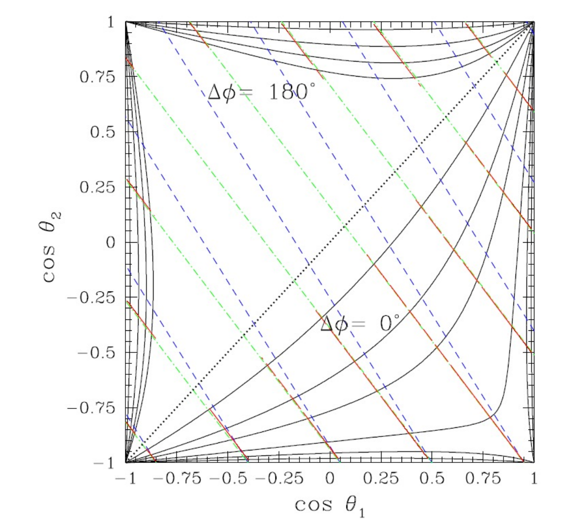
$\Rightarrow$ BHs approach

$\theta_1 = \theta_2$

$\Rightarrow$ $\mathbf{S}_1, \ \mathbf{S}_2$ align

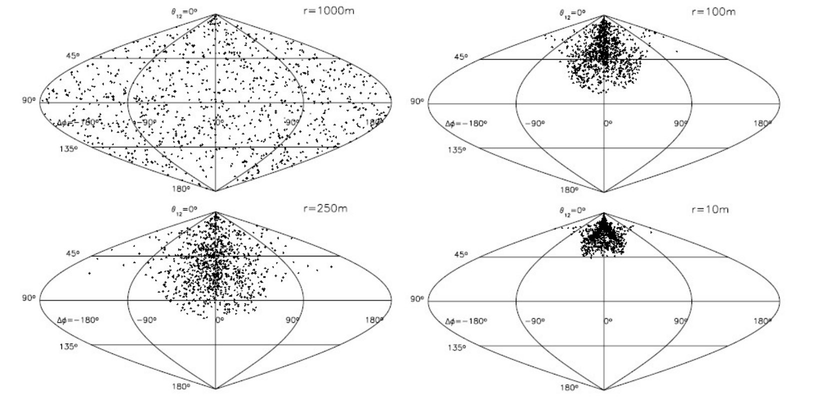
if θ_1 small

Kesden, Berti & US '10



Resonance capture: $\Delta\phi = 0^\circ$

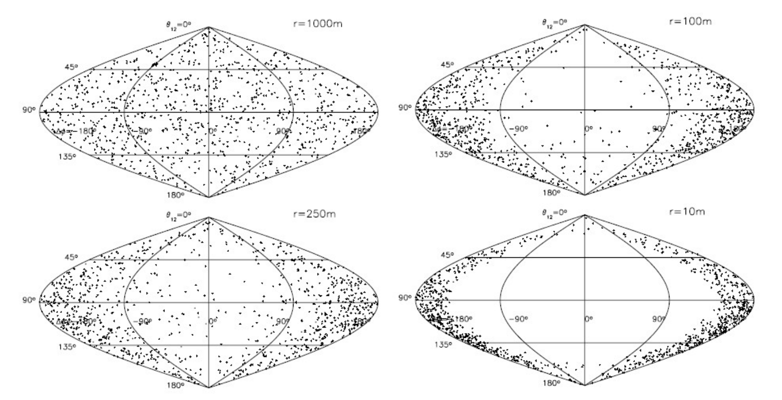
$q = 9/11$, $\chi_i = 1$, $\theta(t_0) = 10^\circ$, rest random



Schnittman '04

Resonance capture: $\Delta\phi = 180^\circ$

$q = 9/11$, $\chi_i = 1$, $\theta(t_0) = 170^\circ$, rest random



Schnittman '04

Consequences of resonances

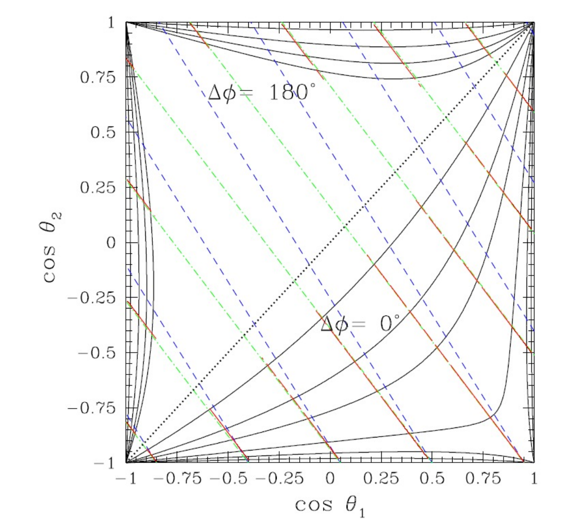
EOB spin

$\mathbf{S_0} = \frac{M}{m_1}\mathbf{S}_1 + \frac{M}{m_2}\mathbf{S}_2$

$\mathbf{S_0} \cdot \mathbf{L_N} = \mathrm{const}$

evolution

$\Rightarrow \mathbf{S}_0 \;\sim$conserved



Consequences of resonances

Total spin

$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$

$\vec{S} \cdot \vec{L}_N = \text{const}$

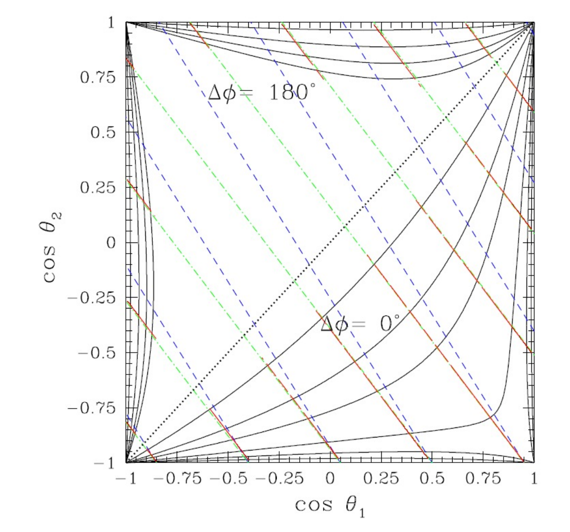
evolution

blue steeper red

$\Rightarrow$ $\mathbf{S}$, $\mathbf{L}_N$ become

antialigned; $\Delta\phi = 0°$

aligned; $\Delta\phi = 180°$



Consequences of resonances

r decreases

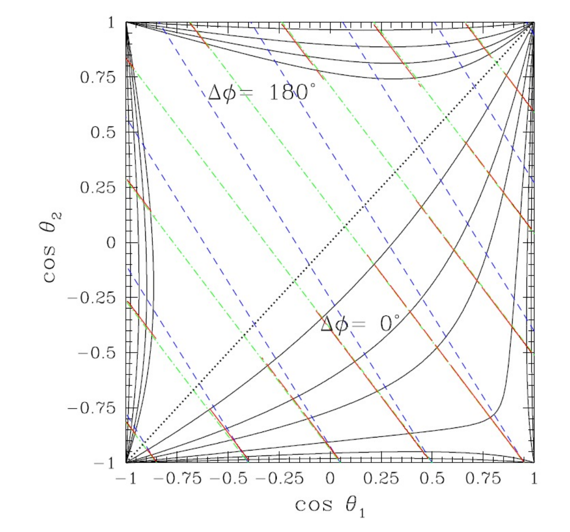
$\Rightarrow \theta_1, \theta_2 \rightarrow$ diagonal

i.e. $\theta_1 = \theta_2$

$\Rightarrow \mathbf{S}_1, \mathbf{S}_2$ become

aligned; $\Delta\phi = 0^\circ$

$\theta_{12} = \theta_1 + \theta_2$; $\Delta\phi = 180^\circ$

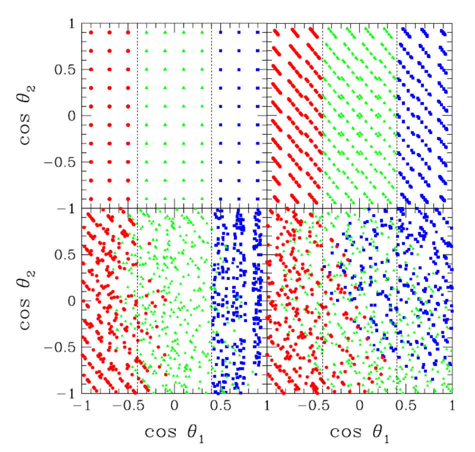


Summary: Resonances

- $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{L}_N$ precess in plane
- 2 types: I) $\Delta\phi = 0°$, II) $\Delta\phi = 180°$
- Free binaries can get caught by symmetries
- Consequences for $\Delta\phi = 0°$
  - $\mathbf{S}_1$, $\mathbf{S}_2$ aligned
  - $\mathbf{S}$, $\mathbf{L}_N$ antialigned
- Consequences for $\Delta\phi = 180°$
  - $\mathbf{S}_1$, $\mathbf{S}_2$ approach $\theta_{12} = \theta_1 + \theta_2$
  - $\mathbf{S}$, $\mathbf{L}_N$ aligned

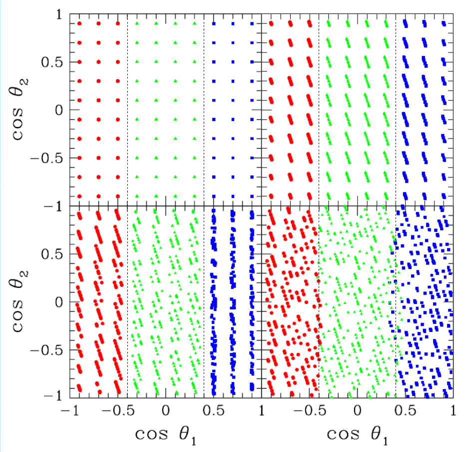
3. Final spins

Resonance capturing in practice: $q = 9/11$



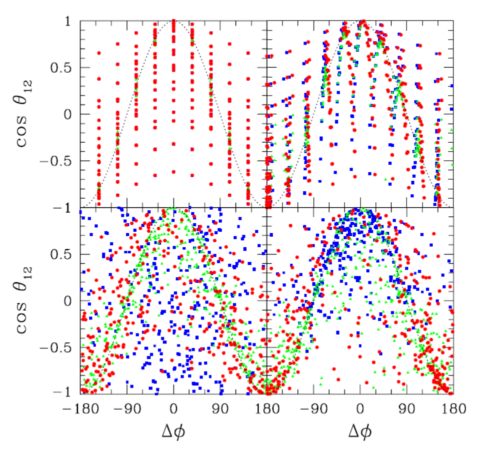
- Isotropic $10 \times 10 \times 10$ grid of configurations
- At $R = 1000\ M + \epsilon,\ 1000\ M,\ 100\ M,\ 10\ M$

Resonance capturing in practice: $q = 1/3$



- Isotropic $10 \times 10 \times 10$ grid of configurations
- At $R = 1000\ M + \epsilon,\ 1000\ M,\ 100\ M,\ 10\ M$

Resonance capturing in practice: $q = 9/11$

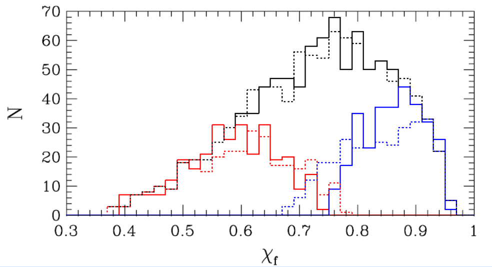


- Isotropic $10 \times 10 \times 10$ grid of configurations
- At $R = 1000\,M + \epsilon,\ 1000\,M,\ 100\,M,\ 10\,M$

Final spin of merged BBH

Numerical relativity $\Rightarrow$ fitting formula $(q,\ \mathbf{S}_1,\ \mathbf{S}_2)\ \rightarrow\ \mathbf{S}_f$

Here: Barausse & Rezzolla '09, but similar results for others



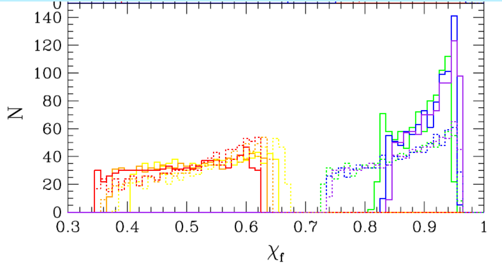
- $\theta_1(t_0)$, $\theta_2(t_0)$, $\Delta\phi(t_0)$ isotropic $10 \times 10 \times 10$
- large θ_1, all 1000 binaries, small θ_1
- Initially isotropic stays isotropic

  cf. Bogdanović, Reynolds & Miller '07

Final spin of merged BBH

Numerical relativity $\Rightarrow$ fitting formula $(q,\ \mathbf{S}_1,\ \mathbf{S}_2)\ \rightarrow\ \mathbf{S}_f$

Here: Barausse & Rezzolla '09, but similar results for others



- $\theta_1(t_0) = 170^\circ,\ 160^\circ,\ 150^\circ,\ 30^\circ,\ 20^\circ,\ 10^\circ$
- $\theta_2(t_0),\ \Delta\phi(t_0)$: 30×30 isotropic
- dotted: switching off precession

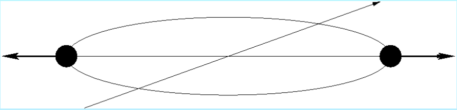
  solid: with precession

Summary: Final spins

- Resonances act as attractor for random binaries
- This is a statistical effect!
- Initially isotropic ensembles stay isotropic; cancelation
- $\Delta\phi = 0^\circ$ resonances increase final spin
  (alignment of $\mathbf{S}_1$, $\mathbf{S}_2$)
- $\Delta\phi = 180^\circ$ resonances mildly decrease final spin

4. Suppression of superkicks

Superkicks



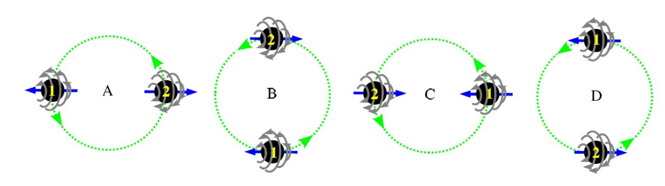
$$\theta_1 = \theta_2 = 90^\circ,\ \Delta\phi = 180^\circ$$

- Superkicks: up to several 1000 km/s

  González, Hannam, Sperhake, Brügmann & Husa, PRL 98, 231101 (2007)

  Campanelli, Lousto, Zlochower & Merritt, ApJ 659, L5 (2007)

- $>$ escape velocities from giant galaxies!



Setup

- BBHs inspiral from 1000 M to 10 M
- Ensemble 1: $10 \times 10 \times 10$ isotropic
- Ensemble 2: 30×30 isotropic in θ_2, $\Delta\phi$

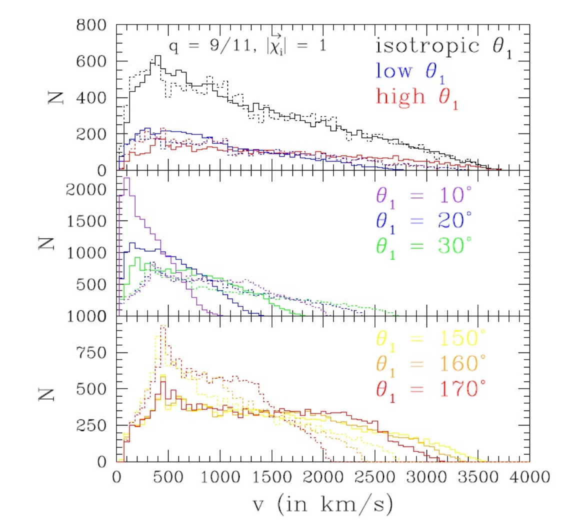
  fix $\theta_1(t_0) = 170^\circ, 160^\circ, 150^\circ, 30^\circ, 20^\circ, 10^\circ$
- Map $\mathbf{S}_1$, $\mathbf{S}_2$, q to $v_{\rm kick}$

  $\vec{v}(q, \chi_1, \chi_2) = v_m \hat{\mathbf{e}}_1 + v_\perp(\cos\xi \hat{\mathbf{e}}_1 + \sin\xi \hat{\mathbf{e}}_2) + v_{||}\hat{\mathbf{e}}_z$

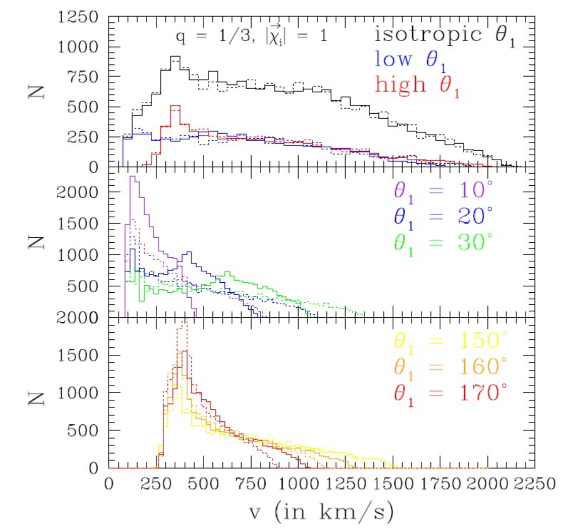
  $v_{||} \sim |\mathbf{\Delta}^\perp|, \qquad \mathbf{\Delta} = \frac{q\chi_2 - \chi_1}{1+q}$

  Campanelli, Lousto, Zlochower & Merritt ’07

Kick distributions with and without PN inspiral $q = \frac{9}{11}$



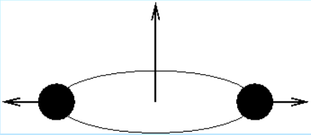
Kick distributions with and without PN inspiral $q = \frac{1}{3}$



Even larger kicks: superkick and hang-up

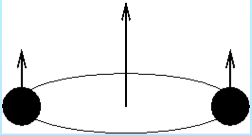
Lousto & Zlochower, arXiv:1108.2009 [gr-qc]

Superkicks

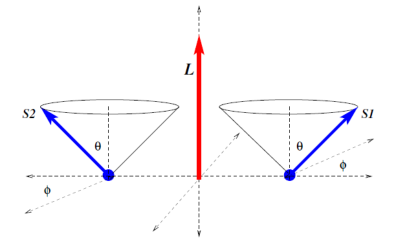


- Moderate GW generation
- Large kicks

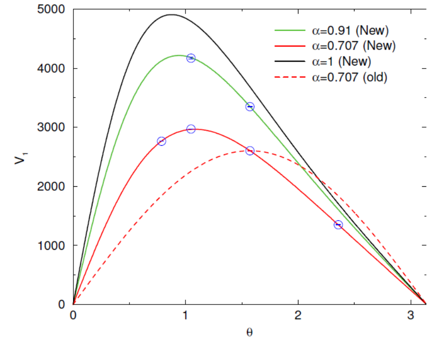
Hangup



- Strong GW generation
- No kicks

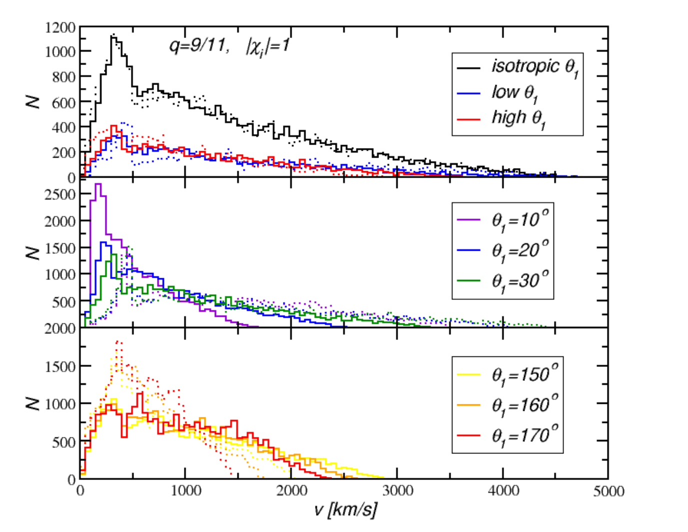


Superkicks and orbital hang-up



- Maximum kick about 25 % larger: $v_{\max} \approx 5\,000$ km/s
- Distribution asymmetric in θ
- Largest recoil for partial alignment

Kick distributions with and without PN inspiral $q = \frac{9}{11}$



Summary: Kick suppression

- Resonances attract aligned (anti aligned) configurations towards $\Delta\phi = 0^\circ$ (180°)
- Superkicks suppressed (enhanced) for $\Delta\phi = 0^\circ$ $(\Delta\phi = 180^\circ)$ resonances
- If accretion torque partially aligns $\vec{S}_1$ with $\vec{L}_N$ $\Rightarrow \Delta\phi = 0^\circ$ resonances dominate and suppress kicks
- Kick suppression still effective for hang-up kicks
- Why? Because the key angle is $\Delta\phi$

5. Stellar-mass BH binary formation

A simplified scenario for stellar-mass BBH formation

- Stellar binary: $M'_{Si},\ M''_{Si} = 35,\ 16.75\ M_\odot$ or $30,\ 24\ M_\odot$
- Primary expands to fill Roche lobe
- 50% M transfer to Secondary until core remant $M'_C = 8.5$ or $8M_\odot$
- Primary explodes as SN $\rightarrow$ BH with $M'_{BH} = 7.5$ or $6M_\odot$
- SN kick tilts $\boldsymbol{L}$
- Tides may align $\boldsymbol{S}''$ and circularize orbit
- Secondary expands to fill Roche lobe $\Rightarrow$ Common envelope
- Secondary becomes helium core with $M''_C = 8$ or $8.5M_\odot$
- Secondary explodes as SN $\rightarrow$ BH with $M'_{BH} = 6$ or $7.5M_\odot$
- SN kick again tilts orbital plane

Comments: Initial separation

- a_0 drawn from logarithmic distribution $[a_{\rm min},\ a_{\rm max}]$

  $a_{\rm max}$: Primary fills Roche lobe

  $a_{\rm min}$: Secondary does not fill Roche lobe at transfer

- $a_0 > a_{\rm max} \Rightarrow$ binary unbound by SN kick

  $a_0 < a_{\rm min} \Rightarrow$ merger in CE phase

Comments: Mass transfer

- Star fills Roche lobe $\Rightarrow$ stable transfer or CE
- Our q $\Rightarrow$ SN1 $\rightarrow$ stable transfer, SN2 $\rightarrow$ CE

  Clausen, Wade, Kopparapu & O'Shaughnessy '12
- Accretion by secondary: $M''_{Sf} = M''_{Si} + f_a(M'_{Si} - M'_C)$

  We choose semi-conservative: $f_a = 0.5$
- f_a tied to fraction of RMR vs. SMR

  $\Rightarrow$ potentially measurable via GWs

Comments: SN kicks

- Calibrate kick using observed motion of young pulsars

  $v_{\rm pNS} \in$ Maxwellian with $\sigma = 265~{\rm km/s}$

- Fallback $\Rightarrow v_{\rm BH} = (1 - f_{\rm fb}) v_{\rm pNS}$

  For our q, simulations suggest $f_{\rm fb} = 0.8$

  Fryer '99, Fryer & Kalogera '01

- Kicks $\in$ cone with θ_b about $\boldsymbol{S}$

  We consider: isotropic $\theta_b = 90^\circ$, polar $\theta_b = 10^\circ$

Comments: Kick effect on orbit

- SN $\Rightarrow$ mass reduction, tilt of orbit
- SN equally likely anywhere in orbit $\Rightarrow$ true anomaly
- At SN1: assume $\boldsymbol{S}_{1,2}$ aligned with $\boldsymbol{L}$
- a_f, e_f from conservation of energy, ang. mom.
- $e_f > 1 \Rightarrow$ Binary unbound
- Overall: isotropic kicks less likely to unbind binary

  $\Rightarrow$ wider ranges of tilt angles

Comments: Tidal alignment

- Tidal dissipation $\Rightarrow$ circularize orbit; align $\boldsymbol{S}_2$ with $\boldsymbol{L}$
- We consider two extremes: i) fully efficient tides, ii) no tidal effects
- Tidal effects on BH can be safely ignored
- Tidal effects operate when secondary fills Roche lobe
- Change in separation due to tides negligible compared with CE phase

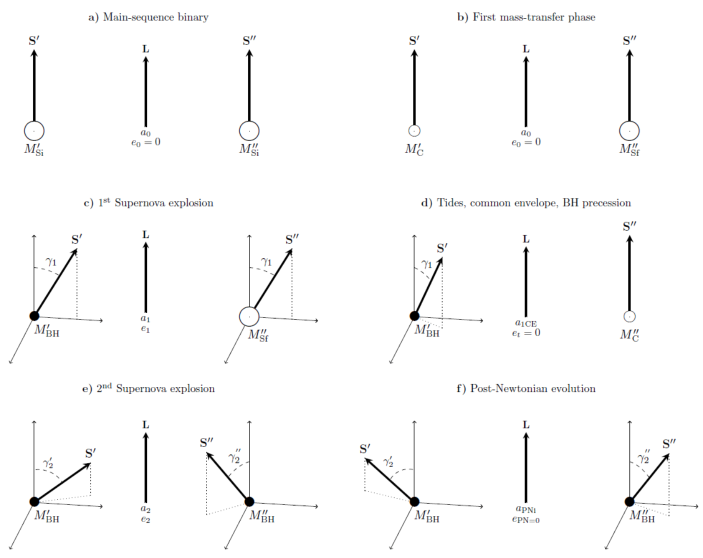
Comments: Common envelope phase

- If a_1 after SN1 too large $\Rightarrow$ no CE phase; game over
- Otherwise: CE has $E_b = -\frac{GM''_{Sf}(M''_{Sf} - M''_C)}{\lambda \mathcal{R}_L}$

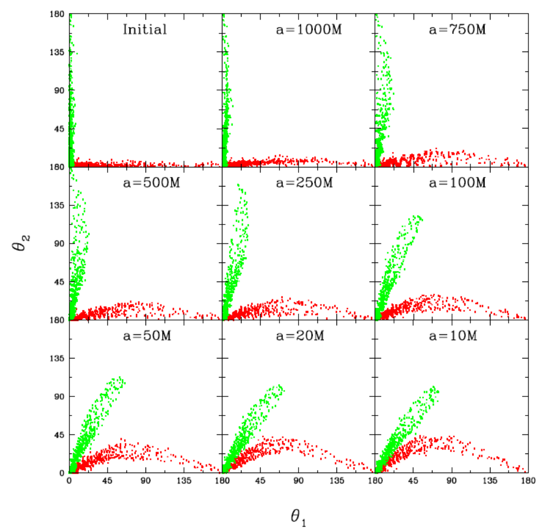
  We use λ from analytic fit of Dominik et al. '12
- Energy, momentum conservation $\Rightarrow a_{1\mathrm{CE}}$
- $a_{1\mathrm{CE}}$ too small

  $\Rightarrow$ helium core fills Roche lobe, prompt merger; game over
- We neglect accretion onto BH

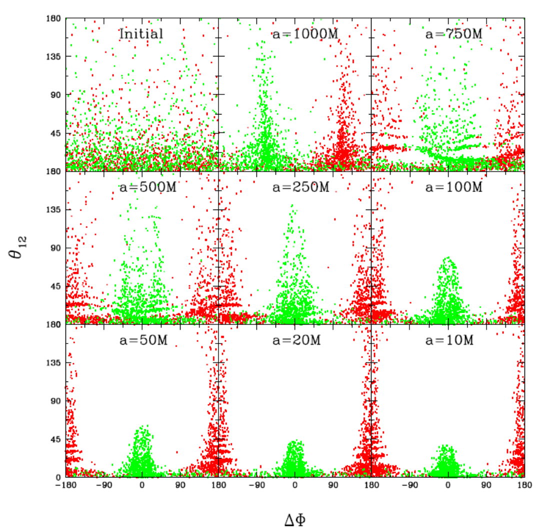
Summary: Evolution sequence



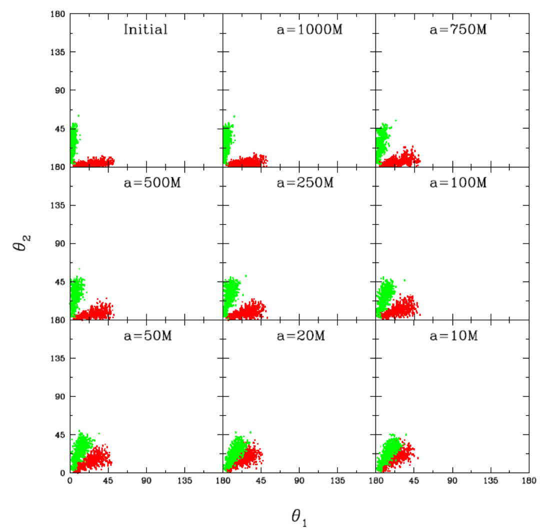
Spin evolution θ_1, θ_2, tides, iso-kick: SMR, RMR



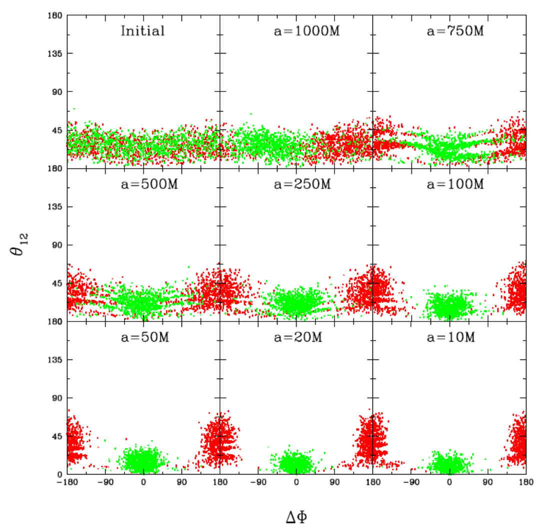
Spin evolution $\Delta\phi$, θ_{12}, tides, iso-kick: SMR, RMR



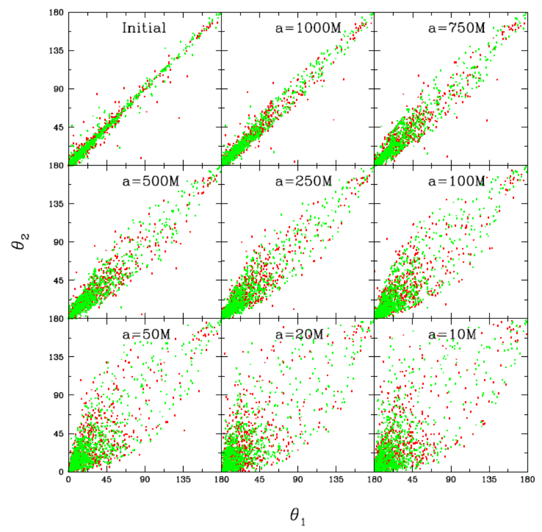
Spin evolution θ_1, θ_2, tides, pol-kick: SMR, RMR



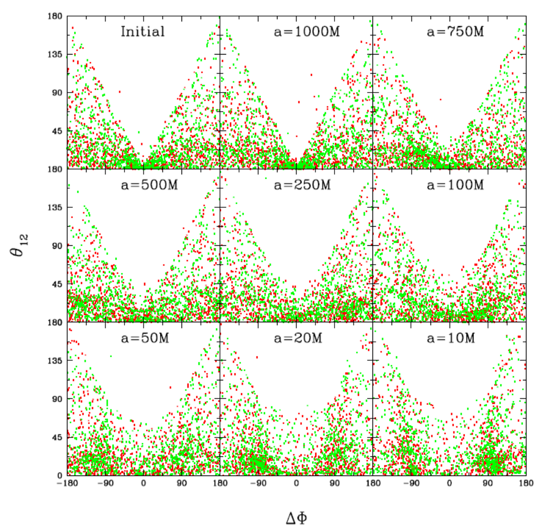
Spin evolution $\Delta\phi$, θ_{12}, tides, pol-kick: SMR, RMR



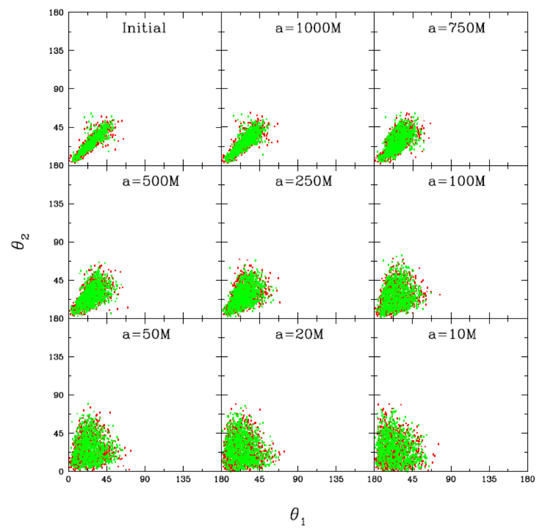
Spin evolution θ_1, θ_2, no tides, iso-kick: SMR, RMR



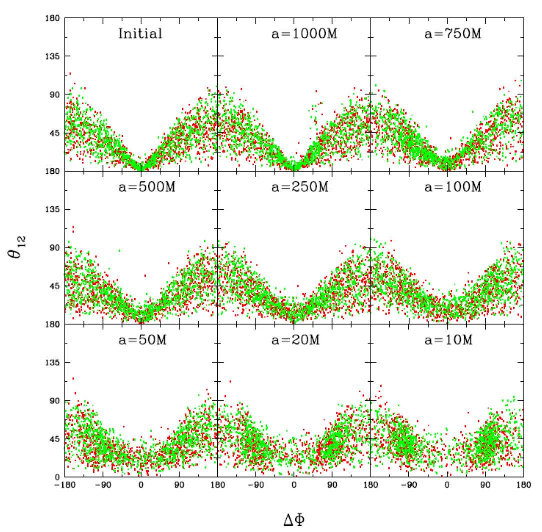
Spin evolution $\Delta\phi$, θ_{12}, no tides, iso-kick: SMR, RMR



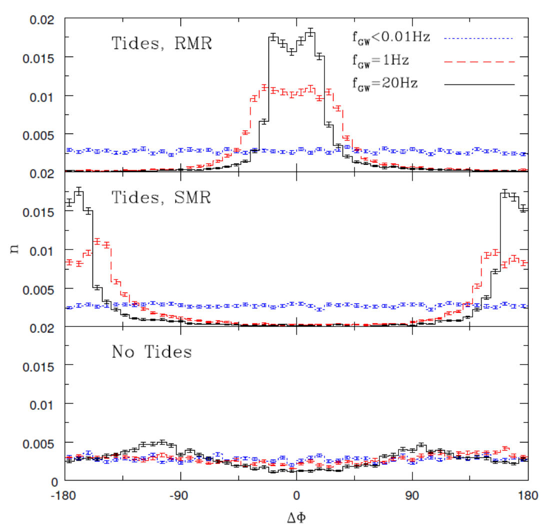
Spin evolution θ_1, θ_2, no tides, pol-kick: SMR, RMR



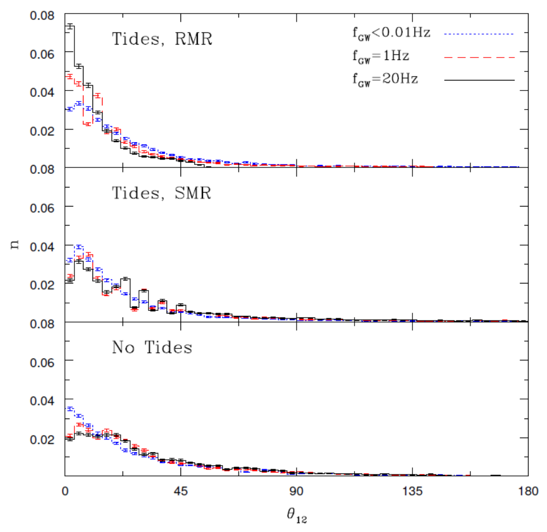
Spin evolution $\Delta\phi$, θ_{12}, no tides, pol-kick: SMR, RMR



Spin distribution at GW frequencies: $\Delta\phi$

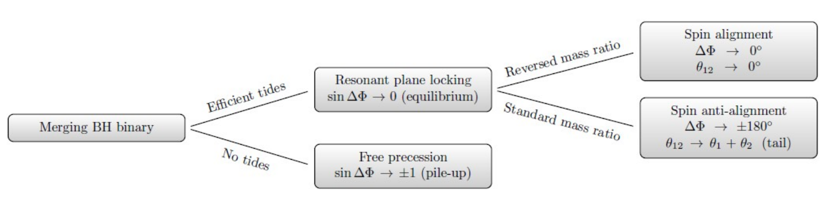


Spin distribution at GW frequencies: θ_{12}



Summary: BH binary formation

- Simplified model for stellar mass BHB formation
- Key ingredients: mass reversal, tides



Conclusions

- Spin orbit resonances attract inspiraling binaries
- 2 classes of resonances: $\Delta\phi = 0^\circ, \quad 180^\circ$
- Isotropic ensembles remain isotropic
- Non-isotropic ensembles can be drastically affected
- Superkicks suppressed if heavy BH's $\boldsymbol{S}$ more aligned with $\boldsymbol{L}$
- Stellar-mass BH binary formation affected by resonances depending on mass transfer, tides