Black holes in higher dimensions

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Overview

- Motivation
- High-energy collisions of black holes
- AdS/CFT correspondence
- Black-hole Stability, Cosmic Censorship
- Conclusions and outlook

1. Motivation

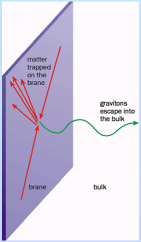
The Hierarchy proble in physics: TeV Gravity

Large extra dimensions

Arkani-Hamed, Dimopoulos & Dvali ’98

- SM confined to “3+1” brane
- Gravity lives in bulk

  $\Rightarrow$ Gravity diluted



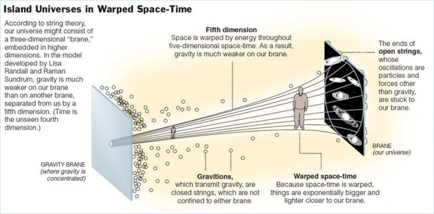
Warped geometry

Randall & Sundrum ’99

- 5D AdS Universe with 2 branes:

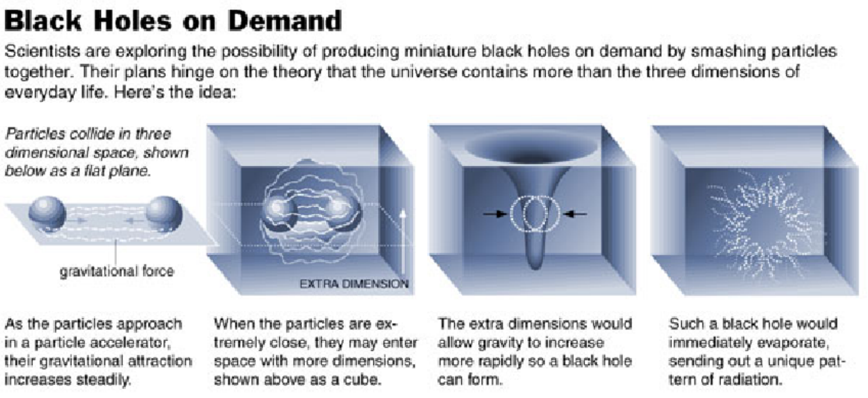
  “our” 3+1 world, gravity brane
- 5^{th} dimension warped

  $\Rightarrow$ Gravity weakened



Either way: Gravity strong at $\gtrsim TeV$

Motivation (High-energy physics)



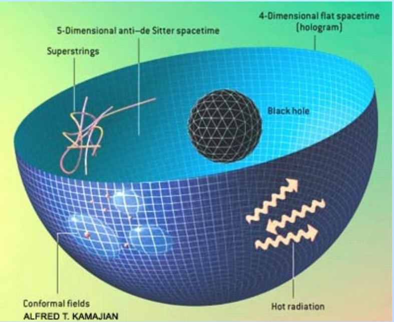
- Matter does not matter at energies well above the Planck scale

  $\Rightarrow$ Model particle collisions by black-hole collisions

  Banks & Fischler '99; Giddings & Thomas '01

AdS/CFT correspondence

- CFTs in $D = 4$ dual to asymptotically AdS BHs in $D = 5$
- Study cousins of QCD, e. g. $\mathcal{N} = 4$ SYM
- Applications
  - Quark-gluon plasma; heavy-ion collisions, RHIC
  - Condensed matter, superconductors
- Dictionary: Metric fall-off $\leftrightarrow$ $T_{\alpha\beta}$



Further motivation

BH collisions and dynamics in general *D* of wide interest:

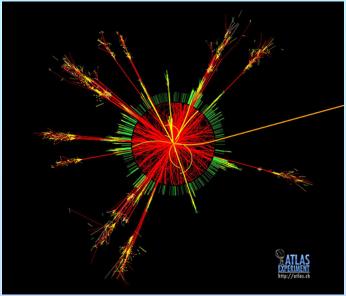
- Test Cosmic Censorship
- Study stability of black holes
- Probe GR in the most violent regime
- Zoom-whirl behaviour; “critical” phenomena
- Super-Planckian physics?

2. High-energy BH collisions

Experimental signature at the LHC

Black hole formation at the LHC could be detected by the properties of the jets resulting from Hawking radiation.

- Multiplicity of partons: Number of jets and leptons
- Large transverse energy
- Black-hole mass and spin are important for this!



ToDo:

- Exact cross section for BH formation
- Determine loss of energy in gravitational waves
- Determine spin of merged black hole

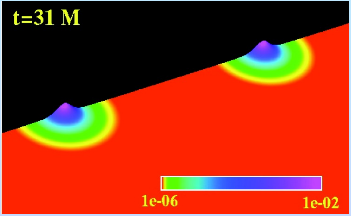
Does matter "matter"?

- Matter does not matter at energies $\ll E_{Planck}$

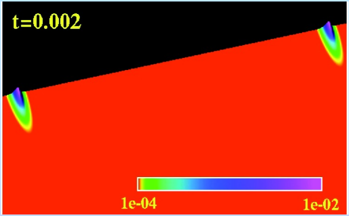
  Banks & Fischler '99;   Giddings & Thomas '01

- Einstein plus minimally coupled, massive, complex scalar filed

  "Boson stars"   Pretorius & Choptuik '09



$\gamma = 1$



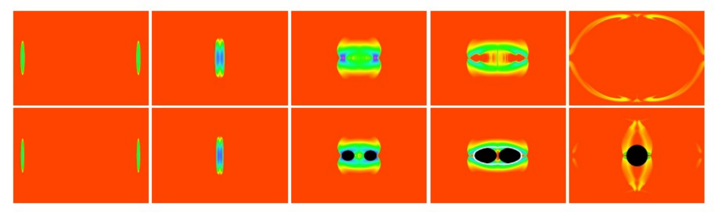
$\gamma = 4$

- BH formation threshold: $\gamma_{\rm thr} = 2.9 \pm 10\ \% \sim 1/3\ \gamma_{hoop}$
- Model particle collisions by BH collisions

Does matter "matter"?

- Perfect fluid "stars" model
- $\gamma = 8 \ldots 12$; BH formation below Hoop prediction

  East & Pretorius '12
- Gravitational focussing $\Rightarrow$ Formation of individual horizons



- Type-I critical behaviour
- Extrapolation by 60 orders would imply no BH formation at LHC

  Rezzolla & Tanaki '12

BH collisions: Computational framework

- Numerical relativity breakthroughs carry over

  Pretorius '05, Goddard '05, Brownsville-RIT '05
- "Moving puncture" technique
- BSSN formulation; Shibata & Nakamura '95, Baumgarte & Shapiro '98
- $1 + \log$ slicing, Γ-driver shift condition
- Puncture ini-data; Bowen-York '80; Brandt & Brügmann '97; Ansorg et al. '04
- Mesh refinement `Cactus, Carpet`
- Wave extraction using Newman-Penrose scalar
- Apparent Horizon finder; e.g. Thornburg '96

Initial setup

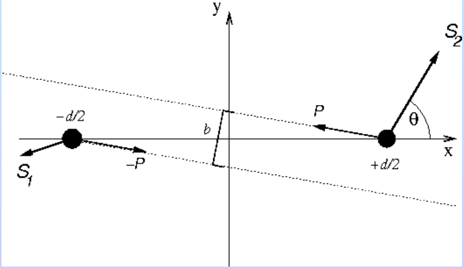
- Take two black holes

  Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

  Initial position: $\pm\frac{d}{2}$

  Linear momentum: $\mp P[\cos\alpha,\ \sin\alpha,\ 0]$

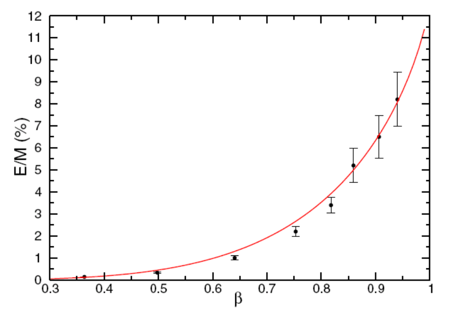
- Impact parameter: $b \equiv \frac{L}{P}$



Head-on: $D = 4, \quad b = 0, \quad \vec{S} = 0$

- Total radiated energy: 14 ± 3 % for $v \to 1$ US *et al.* '08

  About half of Penrose '74



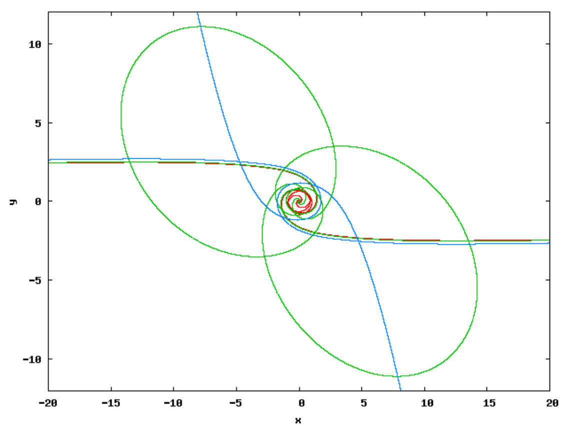
- Agreement with approximative methods

  Flat spectrum, multipolar GW structure Berti *et al.* '10

Grazing: $D = 4, \quad b \neq 0, \quad \gamma = 1.52$

- Zoom-whirl orbits   Pretorius & Khurana '07
- Immediate vs. Delayed vs. No merger

  US, Cardoso, Pretorius, Berti, Hinderer & Yunes '09



Scattering threshold b_{scat} in $D = 4$

- $b < b_{\rm scat}$ $\Rightarrow$ Merger

  $b > b_{\rm scat}$ $\Rightarrow$ Scattering

- Numerical study: $b_{\rm scat} = \frac{2.5 \pm 0.05}{v} M$

  Shibata, Okawa & Yamamoto '08

- Independent study by US, Pretorius, Cardoso, Berti *et al.* '09, '12

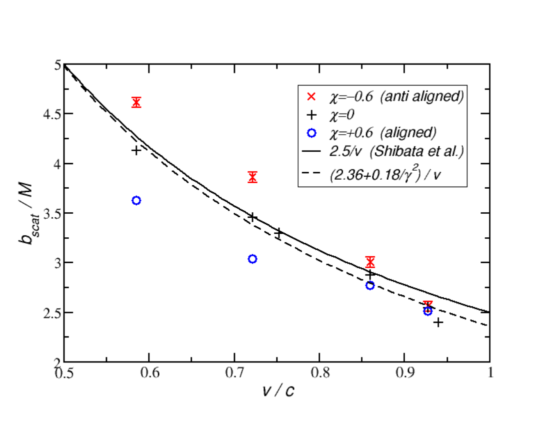
  $\gamma = 1.23 \ldots 2.93$:

  $\chi = -0.6,\ 0,\ +0.6$ (anti-aligned, nonspinning, aligned)

- Limit from Penrose construction: $b_{\rm crit} = 1.685\ M$

  Yoshino & Rychkov '05

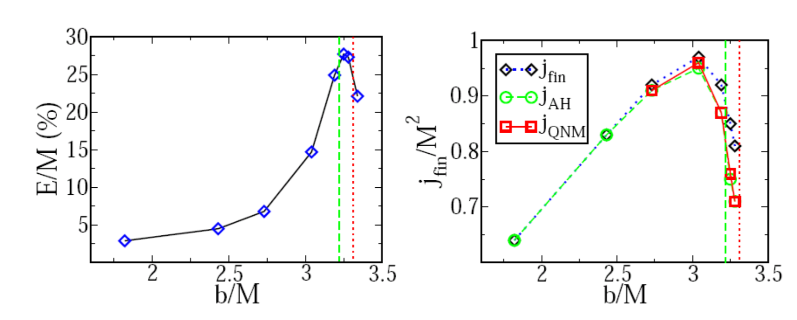
Diminishing impact of structure as $v \to 1$



- Effect of spin reduced for large γ
- $b_{\rm scat}$ for $v \to 1$ not quite certain

Radiated quantities: $b-$sequence with $\gamma = 1.52$

- Final spin close to Kerr limit
- $E_{\rm rad} \sim 35$ % for $\gamma = 2.93$; about 10 % of Dyson luminosity
- Diminishing "hang-up" effect as $v \to 1$

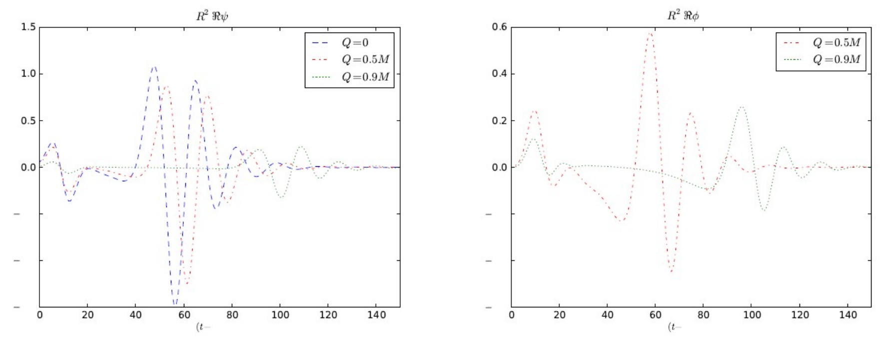


US, Cardoso, Pretorius, Berti, Hinderer & Yunes '09

Collisions of charged BHs in $D = 4$

Zilhão, Cardoso, Herdeiro, Lehner & US

- Electro-vacuum Einstein-Maxwell Eqs.; Moesta et al. '10
- Brill-Lindquist construction for equal mass, charge BHs
- Wave extraction $\Phi_2 := F_{\mu\nu}\bar{m}^\mu k^\nu$



Moving to $D > 4$

- SACRA5D, SACRA-ND

  Shibata, Yoshino, Okawa, Nakao
- D-dim. vacuum Einstein Eqs.
- D-dim. vacuum BSSN Eqs.
- $SO(D-3)$ symmetry
- Modified CARTOON method
- D-dim. gauge conditions

- LEAN

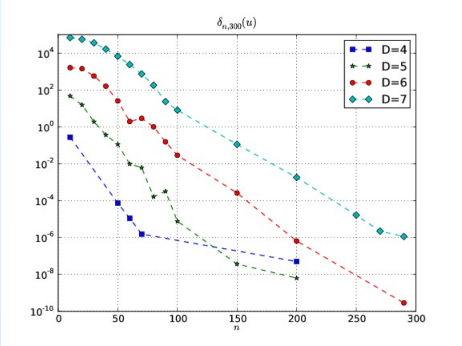
  Zilhão, Witek, US, Cardoso, Gualtieri & Nerozzi '10
- D-dim. vacuum Einstein Eqs.
- $SO(D-3)$ symmetry
- Dim. reduction; Geroch '70

  $\Rightarrow$ 4- dim. Einstein + scalar
- $3+1$-dim. BSSN + scalar
- Modified 4-dim. gauge

Puncture initial data for boosted BHs in $D \geq 5$

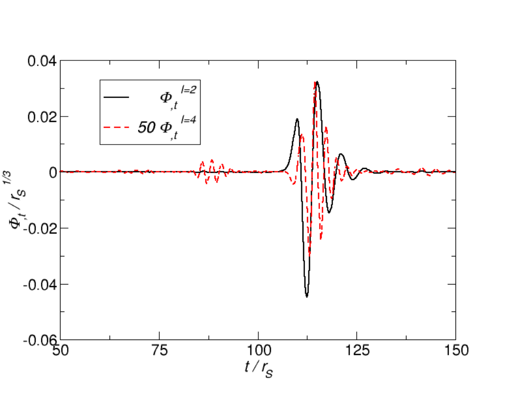
- Generalize spectral code of Ansorg et al. '04
- Momentum constraint still solved analytically

  Yoshino, Shiromizu & Shibata '06
- Spectral solver for Hamiltonian constraint; Zilhão et al. '11



Black-hole collisions in $D = 6$

Witek *et al.* *in prep.*



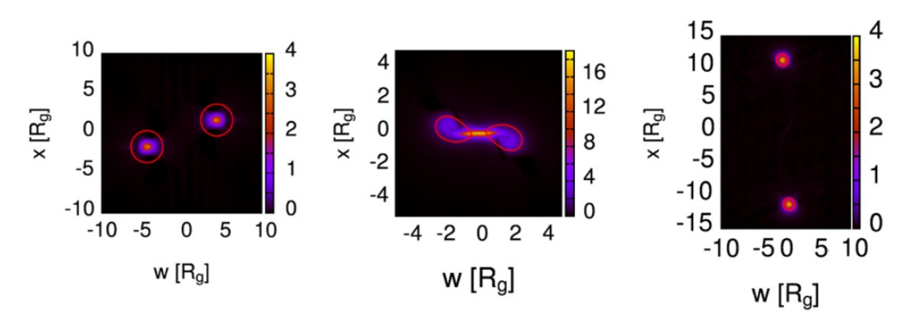
- $d/r_S = 6$
- QNM ringdown agrees with close-limit   Yoshino '05

Boosted collisions in $D = 5$

Okawa, Nakao & Shibata '11

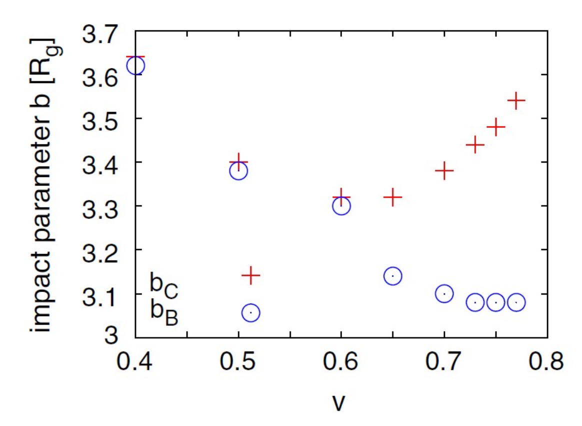
- Take Tangherlini metric; boost and translate
- Superpose two of those

$$\frac{\sqrt{R^{abcd}R_{abcd}}}{6\sqrt{2}E_P^2}$$



Scattering threshold in $D = 5$

Okawa, Nakao & Shibata ’11

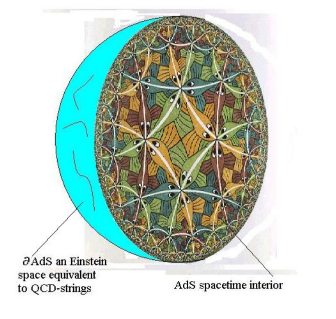


Numerical stability still an issue...

3. The AdS/CFT correspondence

Large N and holography

- Holography
  - BH entropy $\propto A_{Hor}$
  - For a Local Field Theory
    entropy $\propto V$
  - Gravity in D dims
    $\Leftrightarrow$ local FT in $D-1$ dims



- Large N limit
  - Perturbative expansion of gauge theory in g^2N
    $\sim$ loop expansion in string theory
  - N: # of "colors"
    g^2N: t'Hooft coupling

The AdS/CFT conjecture

Maldacena '98

- "strong form": Type IIb string theory on $AdS_5 \times S^5$

  $\Leftrightarrow \mathcal{N} = 4$ super Yang-Mills in $D = 4$

  Hard to prove; non-perturbative Type IIb String Theory?

- "weak form": low-energy limit of string-theory side

  $\Rightarrow$ Type IIb Supergravity on $AdS_5 \times S^5$

- Some assumptions, factor out S^5

  $\Rightarrow$ General Relativity on AdS_5

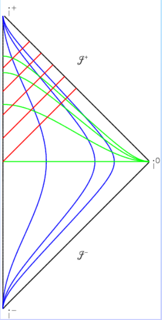
- Corresponds to limit of large N, $g^2 N$ in the field theory

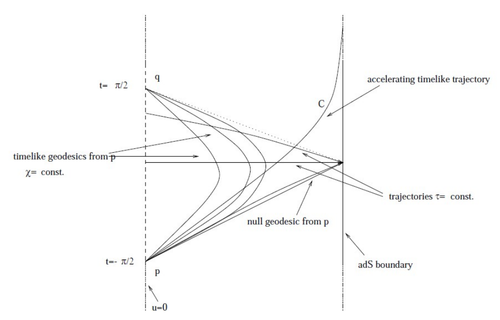
- E. g. Stationary AdS BH $\Leftrightarrow$ Thermal Equil. with T_{Haw} in dual FT

  Witten '98

The boundary in AdS

- Dictionary between metric properties and vacuum expectation values of CFT operators.
  E. g. $T_{\alpha\beta}$ operator of CFT $\leftrightarrow$ transverse metric on *AdS* boundary.
- The boundary plays an active role in *AdS*!   Metric singular!





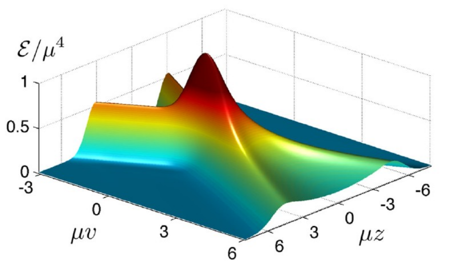
Collision of planar shockwaves in $\mathcal{N} = 4$ SYM

- Dual to colliding gravitational shock waves in AADS
- Characteristic study with translational invariance

  Chesler & Yaffe '10, '11

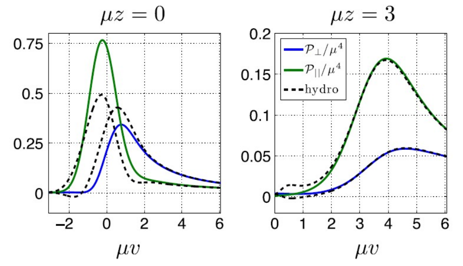
- Initial data: 2 superposed shockwaves

  $$ds^2 = r^2[-dx_+ dx_- + d\mathbf{x}_\perp] + \frac{1}{r^2}[dr^2 + h(x_\pm)dx_\pm^2]$$



Collision of planar shockwaves in $\mathcal{N} = 4$ SYM

- Initially system far from equilibrium
- Isotropization after $\Delta v \sim 4/\mu \sim 0.35\ fm/c$
- Confirms hydrodynamic simulations of QGP $\sim 1\ fm/c$ Heinz '04



- Non-linear vs. linear Einstein Eqs. agree within $\sim 20\ \%$

Heller et al. '12

Cauchy ("4+1") evolutions in asymptotically AdS

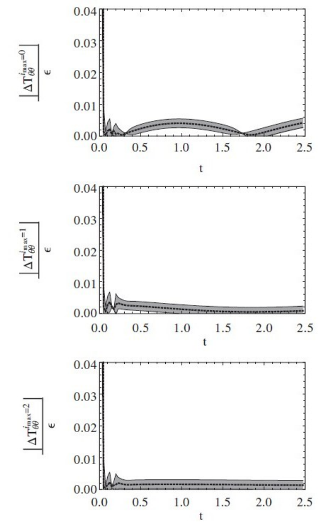
- Characteristic coordinates successful numerical tool in AdS/CFT
- But: restricted to symmetries, caustics problem...
- Cauchy evolution needed for general scenarios? Cf. BBH inspiral!!
- Cauchy scheme based on generalized harmonic formulation

  Bantilan & Pretorius '12

  - $SO(3)$ symmetry
  - Compactify "bulk radius"
  - Asymptotic symmetry of AdS_5: $SO(4, 2)$
  - Decompose metric into AdS_5 piece and deviation
  - Gauge must preserve asymptotic fall-off

Cauchy ("4+1") evolutions in asymptotically AdS

- Scalar field collapse
- BH formation and ringdown
- Low order QNMs $\sim$ perturbative studies, but mode coupling
- CFT stress-energy tensor consistent with thermalized $\mathcal{N} = 4$ SYM fluid
- Difference of CFT $T_{\theta\theta}$ and hydro ($+1^{st}$, 2^{nd} corrs.)



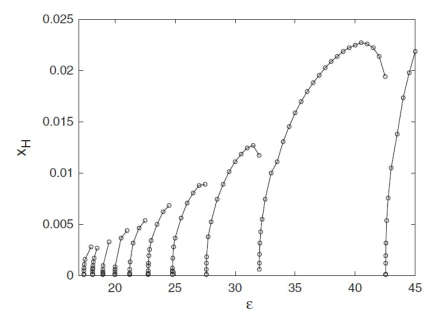
4. Stability, Cosmic Censorship

Stability of AdS

- $m = 0$ scalar field in as. flat spacetimes   Choptuik '93

  $p > p^* \Rightarrow$ BH, $p < p^* \Rightarrow$ flat

- $m = 0$ scalar field in as. AdS   Bizon & Rostworowski '11

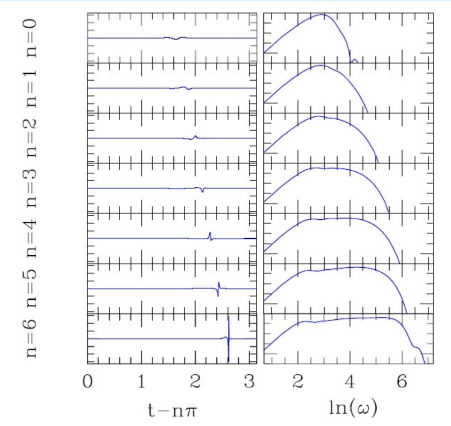


- Similar behaviour for "Geons"   Dias, Horowitz & Santos '11

Stability of AdS

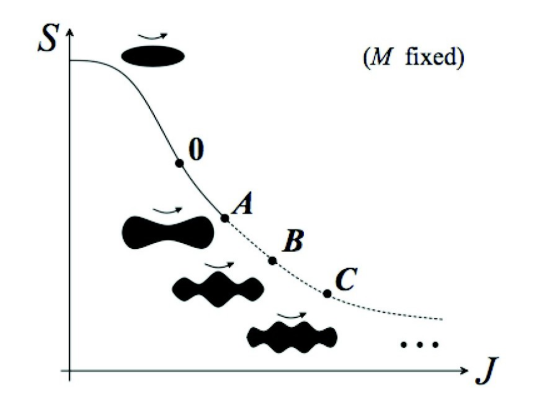
- Pulses narrow under successive reflections

  Buchel, Lehner & Liebling '12



Bar mode instability of Myers-Perry BH

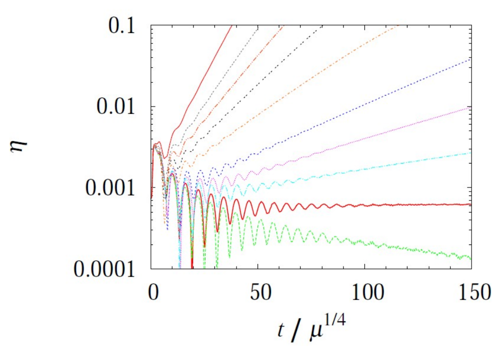
- MP BHs (with single ang.mom.) should be unstable.
- Linearized analysis Dias *et al.* '09



Non-linear analysis of MP instability

Shibata & Yoshino '10

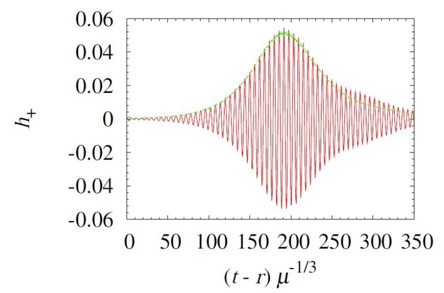
- Myers-Perry metric; transformed to Puncture like coordinate
- Add small bar-mode perturbation
- Deformation $\eta := \frac{2\sqrt{(l_0 - l_{\pi/2})^2 + (l_{\pi/4} - l_{3\pi/4})^2}}{l_0 + l_{\pi/2}}$



Non-linear analysis of MP instability

- Above dimensionless q_{crit} instability
- GW emission; BH settles down to lower q configuration

d	5	6	7	8
$q_{\rm crit}$	0.87	0.74	0.73	0.77
a_*	1.76	0.91	0.83	0.86
C_p/C_e	0.38	0.65	0.68	0.67



Cosmic Censorship in $D = 5$

Pretorius & Lehner '10

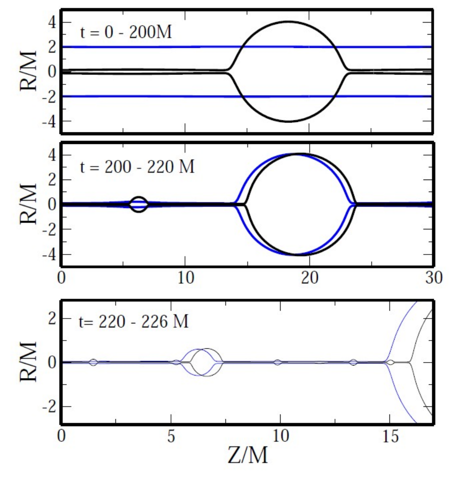
- Axisymmetric code
- Evolution of black string...
- Gregory-Laflamme instability

  cascades down

  in finite time

  until string has zero width

  $\Rightarrow$ naked singularity

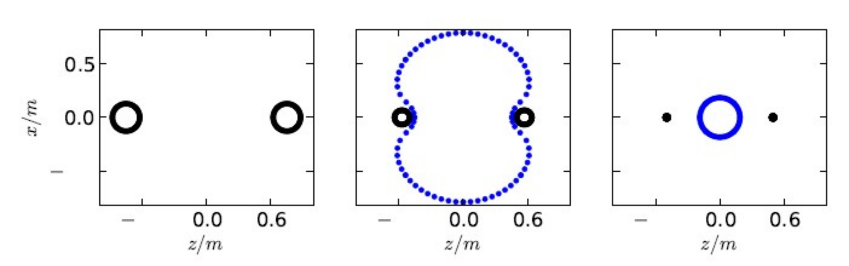


Cosmic Censorship in $D = 4$ de Sitter

Zilhão et al. '12

- Two parameters: MH, d
- Initial data: McVittie type binaries McVittie '33
- "Small BHs": $d < d_{crit} \Rightarrow$ merger

  $d > d_{crit} \Rightarrow$ no common AH
- "Large" holes at small d: Cosmic Censorship holds



5. Conclusions

Conlcusions

- "3+1" numerical framework can be modified for higher D
- High-energy collisions
  - In 4D b_{thresh} for $v \to 1$?
  - Zoom-whirl behaviour in 4D, but not 5D
  - For $v \to 1$ structure less important
- AdS/CFT correspondence
  - Numerical challenge; boundary
  - Results in characteristic framework; thermalization
  - First attempts in "3+1"
- AdS unstable against perturbations
- Myers Perry BH unstable above threshold spin
- Cosmic Censorship holds in 4D, but not 5D