Numerical Relativity simulations of black holes: Methodology and Computational Framework

U. Sperhake

CSIC-IEEC Barcelona

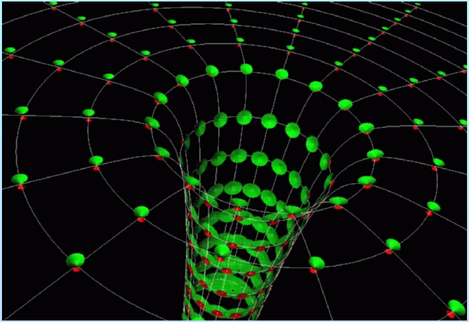
Numerical Cosmology 2012
19$^{\text{th}}$ July 2012

Overview

- Motivation
- Modeling black holes in GR
- Black holes in astrophysics
- High-energy collisions of black holes
- The AdS/CFT correspondence
- Stability, Cosmic Censorship
- Conclusions

1. Motivation

What are black holes?



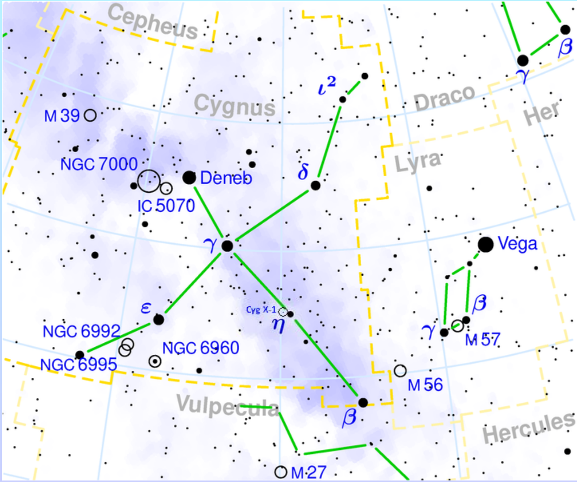
- Consider Lightcones
- In and outgoing light
- Calculate surface

  of outgoing light fronts
- Expansion $\equiv$ Rate of

  change of this surface
- Apparent Horizon $\equiv$ Outermost surface with zero expansion
- "Light cones tip over" due to curvature

Black holes are out there: Stellar BHs

- high-mass X-ray binaries: Cygnus X-1 (1964)

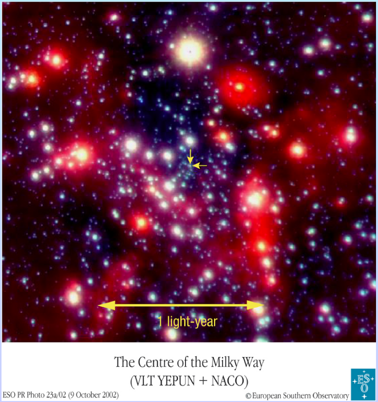


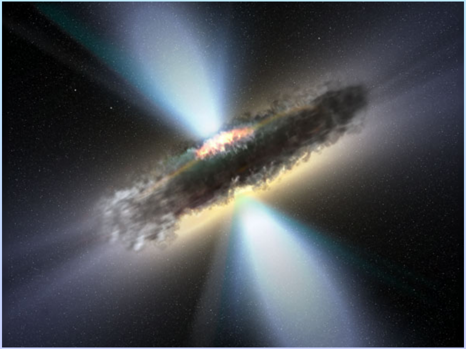
Black holes are out there: Stellar BHs

- One member is very compact and massive ⇒ Black Hole

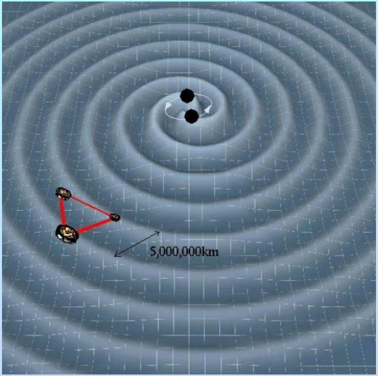
Black holes are out there: galactic BHs

- Supermassive BHs found at center of virtually all galaxies
- SMBHs conjectured to be responsible for quasars starting in the 1980s





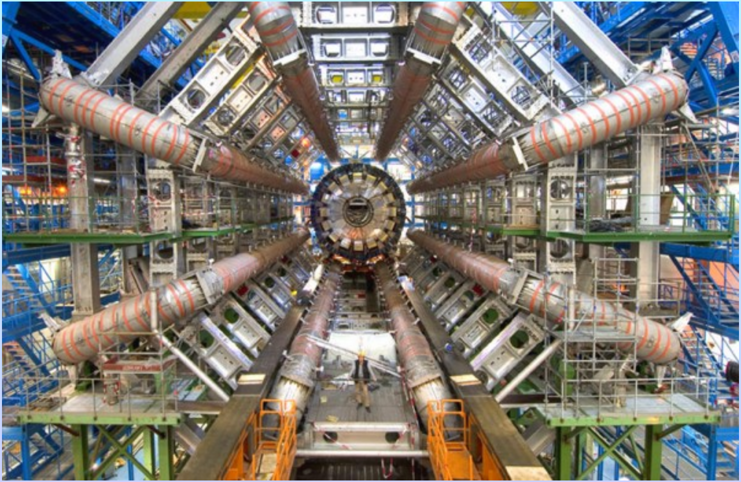
BHs are strong sources of gravitational waves



- BH binaries source of GWs for LIGO, VIRGO, GEO600, “LISA”
- Cross corellate model waveforms with data stream

Black holes might be in here: LHC

- LHC CERN

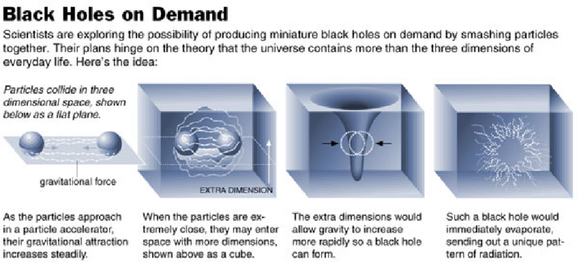


BH generation in *TeV*-gravity scenarios

- Extra dimensions can explain hierarchy problem

  Arkani-Hamed, Dimopoulos & Dvali '98   Randall & Sundrum '98

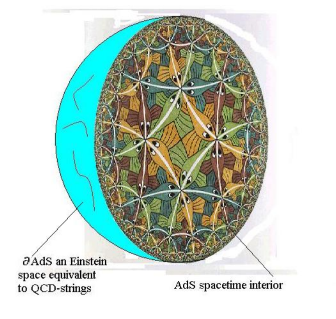
- Gravity dominant at $\sim$ *TeV* $\Rightarrow$ BH formation in LHC collisions



- Signature: # jets, leptons, transverse energy
- TODO: determine Cross section, GW loss, BH spin

AdS/CFT correspondence

- CFTs in $D = 4$ dual to asymptotically AdS BHs in $D = 5$
- Study cousins of QCD, e. g. $\mathcal{N} = 4$ SYM
- Applications
  - Quark-gluon plasma; heavy-ion collisions, RHIC
  - Condensed matter, superconductors
- Dictionary: Metric fall-off $\leftrightarrow$ $T_{\alpha\beta}$



2. Modeling black holes in GR

General Relativity: Curvature

- Curvature generates acceleration

  "geodesic deviation"

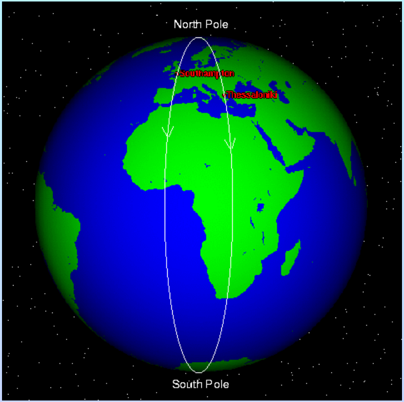
  No "force"!!

- Description of geometry

  Metric $g_{\alpha\beta}$

  Connection $\Gamma^{\alpha}_{\beta\gamma}$

  Riemann Tensor $R^{\alpha}{}_{\beta\gamma\delta}$



How to get the metric?

Train cemetery
Uyuni, Bolivia

- Solve for the metric $g_{\alpha\beta}$

How to get the metric?

- The metric must obey the Einstein Equations
- Ricci-Tensor, Einstein Tensor, Matter Tensor

  $R_{\alpha\beta} \equiv R^{\mu}{}_{\alpha\mu\beta}$

  $G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R^{\mu}{}_{\mu}$ "Trace reversed" Ricci

  $T_{\alpha\beta}$ "Matter"

- Einstein Equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$
- Solutions: Easy! Take metric

  $\Rightarrow$ Calculate $G_{\alpha\beta}$

  $\Rightarrow$ Use that as matter tensor

- Physically meaningful solutions: Difficult! $\Rightarrow$ Numerics

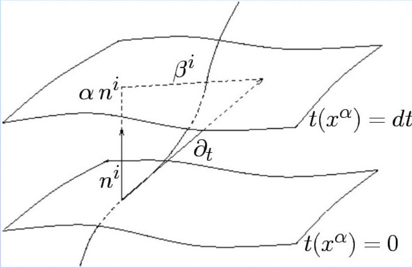
A list of tasks

- Target: Predict time evolution of BBH in GR
- Einstein equations: 1) Cast as evolution system
  2) Choose specific formulation
  3) Discretize for computer
- Choose coordinate conditions: Gauge
- Fix technical aspects: 1) Mesh refinement / spectral domains
  2) Singularity handling / excision
  3) Parallelization
- Construct realistic initial data
- Start evolution and waaaaiiiiit...
- Extract physics from the data

3+1 Decomposition

- GR: "Space and time exist as a unity: Spacetime"
- NR: ADM 3+1 split   Arnowitt, Deser & Misner '62
  York '79, Choquet-Bruhat & York '80

$$g_{\alpha\beta} = \left(\begin{array}{c|c} -\alpha^2 + \beta_m \beta^m & \beta_j \\ \hline \beta_i & \gamma_{ij} \end{array} \right)$$



- 3-Metric γ_{ij}
  Lapse α
  Shift β^i
- lapse, shift $\Rightarrow$ Gauge

ADM Equations

The Einstein equations $R_{\alpha\beta} = 0$ become

- 6 Evolution equations

  $$(\partial_t - \mathcal{L}_\beta)\gamma_{ij} = -2\alpha K_{ij}$$

  $$(\partial_t - \mathcal{L}_\beta)K_{ij} = -D_i D_j \alpha + \alpha[R_{ij} - 2K_{im}K^m{}_j + K_{ij}K]$$

- 4 Constraints

  $$R + K^2 - K_{ij}K^{ij} = 0$$

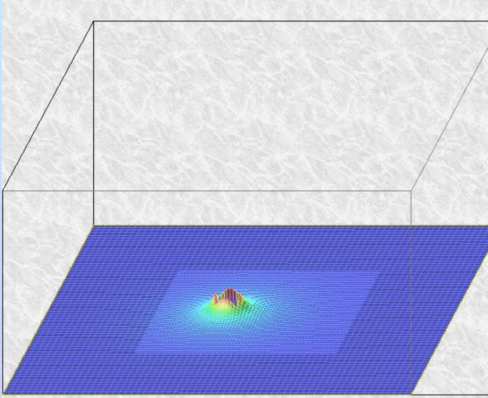
  $$-D_j K^{ij} + D^i K = 0$$

  preserved under evolution!

- Evolution

  1) Solve constraints

  2) Evolve data



Formulations I: BSSN

- One can easily change variables. E. g. wave equation

$$\partial_{tt} u - c\partial_{xx} u = 0 \qquad \Leftrightarrow \qquad \begin{aligned} \partial_t F - c\partial_x G &= 0 \\ \partial_x F - \partial_t G &= 0 \end{aligned}$$

- BSSN: rearrange degrees of freedom

$$\chi = (\det \gamma)^{-1/3} \qquad \tilde{\gamma}_{ij} = \chi \gamma_{ij}$$

$$K = \gamma_{ij} K^{ij} \qquad \tilde{A}_{ij} = \chi \left(K_{ij} - \tfrac{1}{3} \gamma_{ij} K \right)$$

$$\tilde{\Gamma}^i = \tilde{\gamma}^{mn} \tilde{\Gamma}^i_{mn} = -\partial_m \tilde{\gamma}^{im}$$

Shibata & Nakamura '95, Baumgarte & Shapiro '98

- BSSN strongly hyperbolic, but depends on details...

Sarbach *et al.*'02, Gundlach & Martín-García '06

Formulations I: BSSN

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\phi = \tfrac{1}{12}\ln\gamma \qquad \hat{\gamma}_{ij} = e^{-4\phi}\gamma_{ij}$$

$$K = \gamma_{ij}K^{ij} \qquad \hat{A}_{ij} = e^{-4\phi}\left(K_{ij} - \tfrac{1}{3}\gamma_{ij}K\right)$$

$$\hat{\Gamma}^i = \gamma^{ij}\hat{\Gamma}^i_{jk} = -\partial_j\hat{\gamma}^{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\hat{\gamma}_{ij} = -2\alpha\hat{A}_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\phi = -\tfrac{1}{6}\alpha K$$

$$(\partial_t - \mathcal{L}_\beta)\hat{A}_{ij} = e^{-4\phi}(-D_iD_j\alpha + \alpha R_{ij})^{\mathrm{TF}} + \alpha(K\hat{A}_{ij} - 2\hat{A}_{ik}\hat{A}^k{}_j)$$

$$(\partial_t - \mathcal{L}_\beta)K = -D^iD_i\alpha + \alpha\left(\hat{A}_{ij}\hat{A}^{ij} + \tfrac{1}{3}K^2\right)$$

$$\partial_t\hat{\Gamma}^i = 2\alpha\left(\hat{\Gamma}^i_{jk}\hat{A}^{jk} + 6\hat{A}^{ij}\partial_j\phi - \tfrac{2}{3}\hat{\gamma}^{ij}\partial_j K\right) - 2\hat{A}^{ij}\partial_j\alpha + \hat{\gamma}^{jk}\partial_j\partial_k\beta^i$$

$$+\tfrac{1}{3}\hat{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\hat{\Gamma}^i + \tfrac{2}{3}\hat{\Gamma}^i\partial_j\beta^j \underbrace{- \left(\chi + \tfrac{2}{3}\right)\left(\hat{\Gamma}^i - \hat{\gamma}^{jk}\hat{\Gamma}^i_{jk}\right)\partial_l\beta^l}_{\text{Yo et al. (2002)}}$$

Formulations II: Generalized harmonic (GHG)

- Harmonic gauge: choose coordinates such that

  $\nabla_\mu \nabla^\mu x^\alpha = 0$

- 4-dim. version of Einstein equations

  $R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \ldots$

  Principal part of wave equation

- Generalized harmonic gauge: $H_\alpha \equiv g_{\alpha\nu} \nabla_\mu \nabla^\mu x^\nu$

  $\Rightarrow R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \ldots - \frac{1}{2} \left(\partial_\alpha H_\beta + \partial_\beta H_\alpha \right)$

  Still principal part of wave equation !!! Manifestly hyperbolic

  Friedrich '85, Garfinkle '02, Pretorius '05

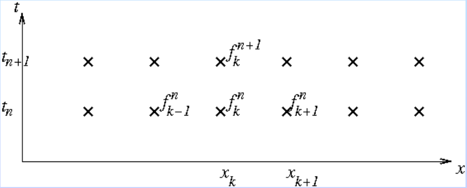
- Constraint preservation; constraint satisfying BCs

  Gundlach et al. '05, Lindblom et al. '06

Discretization of the time evolution

- Finite differencing (FD)

  Pretorius, RIT, Goddard, Georgia Tech, LEAN, BAM, UIUC,...
- Spectral   Caltech-Cornell-CITA
- Parallelization with MPI, $\sim$ 128 cores, $\sim$ 256 *Gb* RAM
- Example: advection equation $\partial_t f = \partial_x f$, FD
- Array f^n_k for fixed n



$$f_k^{n+1} = f_k^n + \Delta t \frac{f_{k+1}^n - f_{k-1}^n}{2\Delta x}$$

Initial data

Two problems: Constraints, realistic data

- Rearrange degrees of freedom

  York-Lichnerowicz split: $\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$

  $$K_{ij} = A_{ij} + \frac{1}{3}\gamma_{ij} K$$

  York & Lichnerozwicz, O'Murchadha & York,

  Wilson & Mathews, York

- Make simplifying assumptions

  Conformal flatness: $\tilde{\gamma}_{ij} = \delta_{ij}$

- Find good elliptic solvers, e. g. Ansorg et al. '04

Mesh refinement

3 Length scales : BH $\sim 1\ M$

Wavelength $\sim 10 \ldots 100\ M$

Wave zone $\sim 100 \ldots 1000\ M$

- Critical phenomena

  Choptuik '93

- First used for BBHs

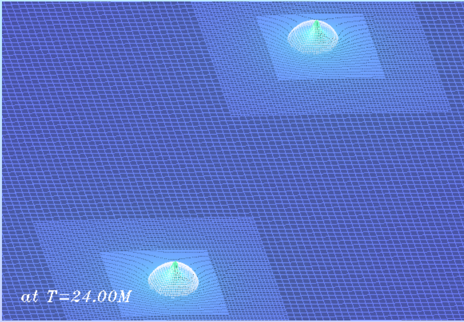
  Brügmann '96

- Available Packages:

  Paramesh MacNeice *et al.* '00

  Carpet Schnetter *et al.* '03

  SAMRAI MacNeice *et al.* '00



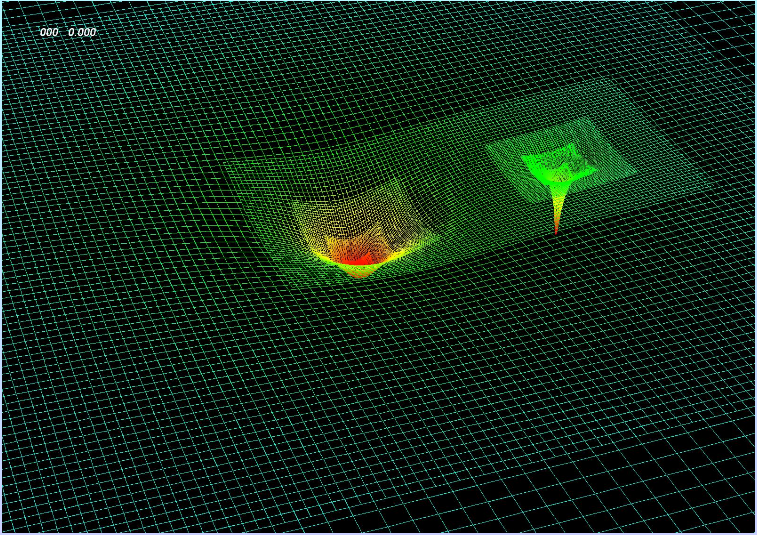
The gauge freedom

- Remember: Einstein equations say nothing about $\alpha, \quad \beta^i$
- Any choice of lapse and shift gives a solution
- This represents the coordinate freedom of GR
- Physics do not depend on $\alpha, \quad \beta^i$

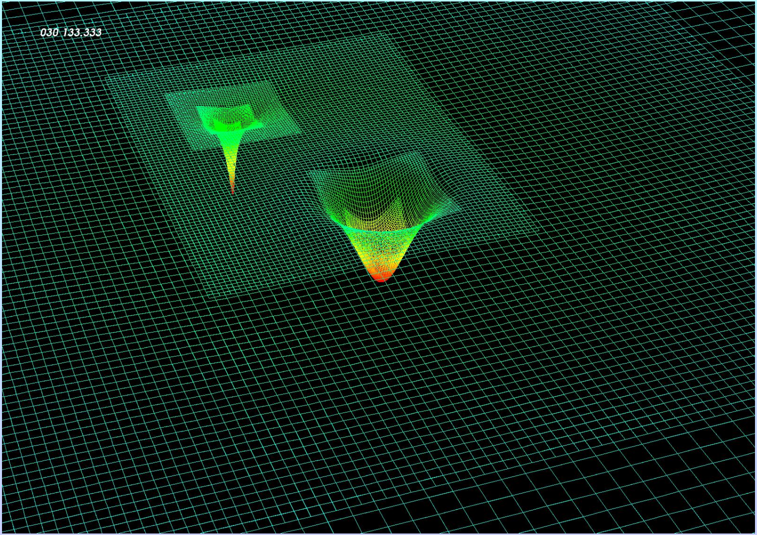
  So why bother?
- The performance of the numerics DO depend strongly on the gauge!
- How do we get good gauge?

  Singularity avoidance, avoid coordinate stretching, well posedness

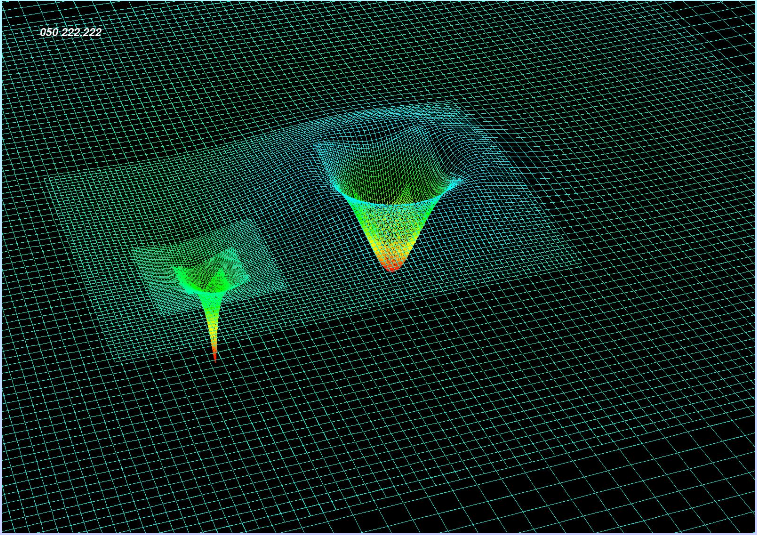
What goes wrong with bad gauge?



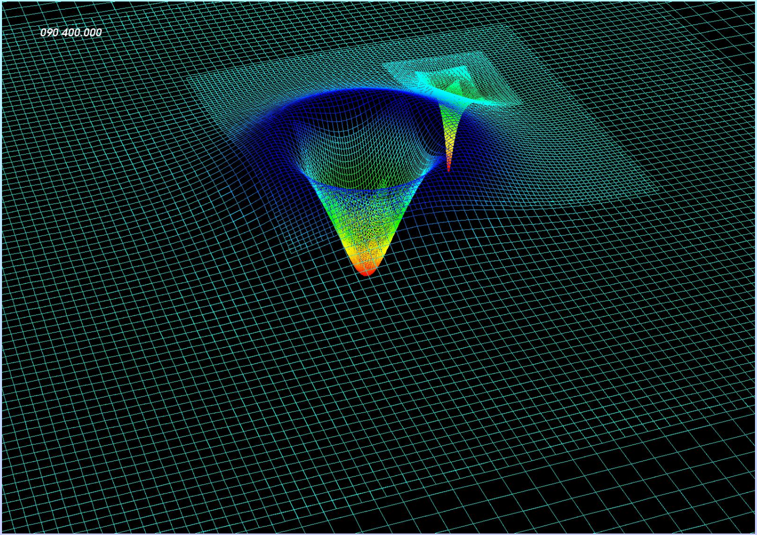
What goes wrong with bad gauge?



What goes wrong with bad gauge?



What goes wrong with bad gauge?



A brief history of BH simulations

- Pioneers: Hahn & Lindquist '60s, Eppley, Smarr *et al.* '70s
- Grand Challenge: First $3D$ Code Anninos *et al.* '90s
- Further attempts: Bona & Massó, Pitt-PSU-Texas

  AEI-Potsdam, Alcubierre *et al.*

  PSU: first orbit Brügmann *et al.* '04

Codes unstable!

- Breakthrough: Pretorius '05 GHG

  UTB, Goddard'05 Moving Punctures
- Currently about 10 codes world wide

3. BHs in GW and astrophysics

Free parameters of BH binaries

- Total mass M

  Relevant for GW detection: Frequencies scale with M

  Not relevant for source modeling: trivial rescaling

- Mass ratio $q \equiv \frac{M_1}{M_2}$, $\eta \equiv \frac{M_1 M_2}{(M_1+M_2)^2}$

- Spin: $\vec{S}_1$, $\vec{S}_2$ (6 parameters)

- Initial parameters

  Binding energy $E_{\rm b}$ Separation

  Orbital ang. momentum L Eccentricity

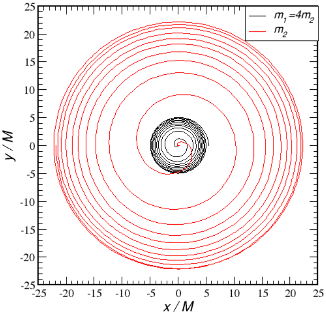
  Alternatively: frequency, eccentricity

BBH trajectory and waveform

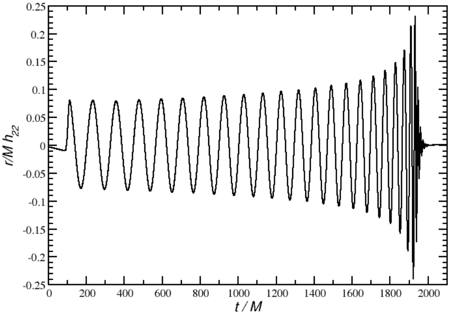
- $q = 4$, non-spinning binary; $\sim$ 11 orbits

  US, Brügmann, Müller & Sopuerta '11

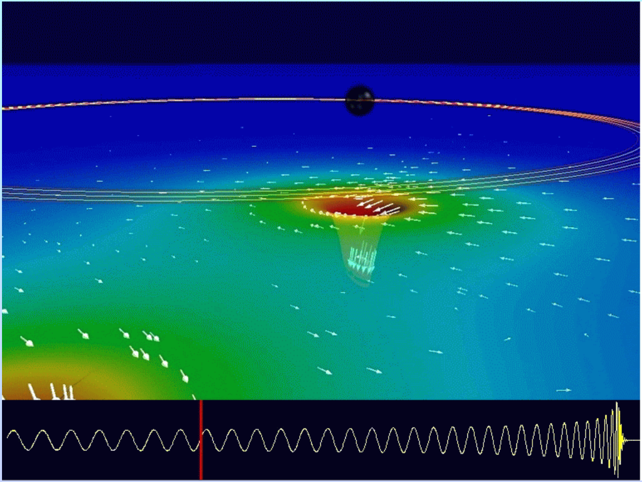
Trajectory



Quadrupole mode



Morphology of a BBH inspiral



Thanks to Caltech, Cornell, CITA

Gravitational recoil

- Anisotropic GW emission ⇒ recoil of remnant BH

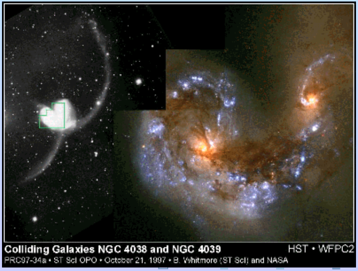
  Bonnor & Rotenburg '61, Peres '62, Bekenstein '73

- Escape velocities:

Globular clusters	30 $\mathrm{km/s}$
dSph	20 – 100 $\mathrm{km/s}$
dE	100 – 300 $\mathrm{km/s}$
Giant galaxies	$\sim$ 1000 $\mathrm{km/s}$

Ejection / displacement of BH ⇒

- Growth history of SMBHs
- BH populations, IMBHs
- Structure of galaxies

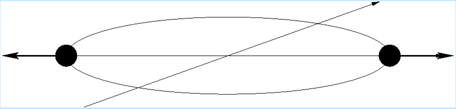


Superkicks

- Kicks from non-spinning BHs up to $\sim 180\ km/s$

  González et al. '06

- Kidder '95, UTB-RIT '07: maximum kick expected for



- Kicks up to $v_{\rm max} \approx 4\,000\ \mathrm{km/s}$

  González *et al.* '07, Campanelli *et al.* '07

- "Hang-up kicks" of up to $5\,000\ km/s$ Lousto & Zlochower '12

- Suppression via spin alignment and Resonance effects in inspiral

  Schnittman '04, Bogdanovicź et al. '07, Kesden, US & Berti '10, '10a, '12

Gravitational Wave observations

- Accelerated masses generate GWs
- Interaction with matter very weak!
- Earth bound detectors: GEO600, LIGO, TAMA, VIRGO

Some targets of GW physics

- Confirmation of GR

  Hulse & Taylor 1993 Nobel Prize

- Parameter determination

  of BHs: M, $\vec{S}$

- Optical counter parts

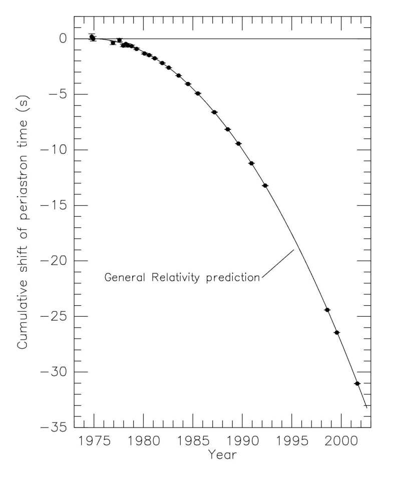
  Standard sirens (candles)

  Mass of graviton

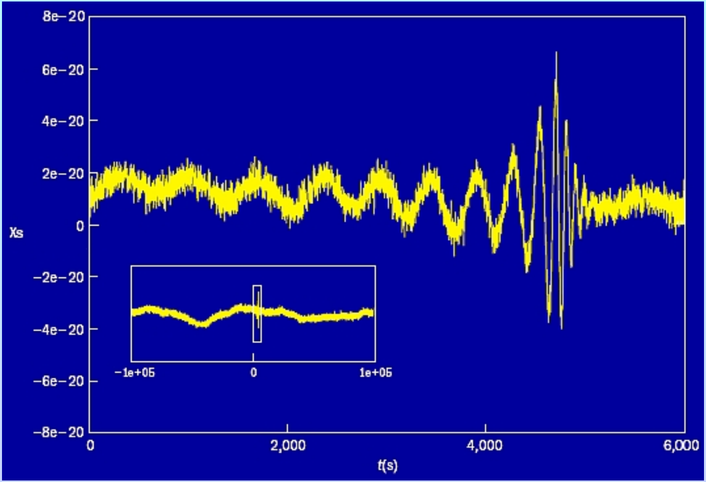
- Test Kerr Nature of BHs

- Cosmological sources

- Neutron stars: EOS



Matched filtering

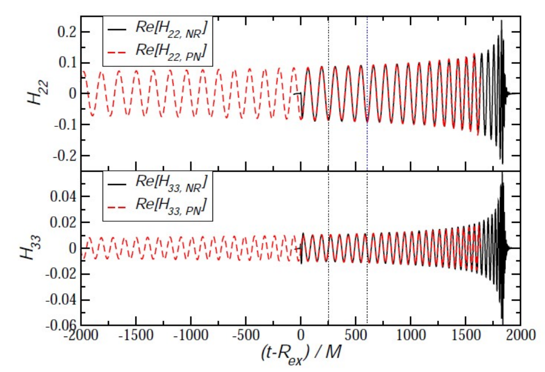


Long, accurate waveforms required

⇒ combine NR with PN, perturbation theory

Template construction

- Stitch together PN and NR waveforms
- EOB or phenomenological templates for $\geq$ 7-dim. par. space



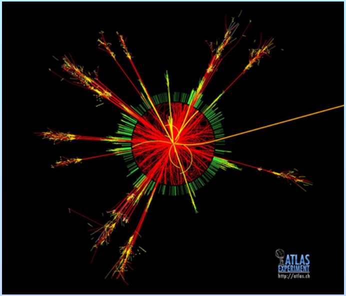
- Community wide Ninja2 and NRAR projects

4. High-energy collisions of black holes

Experimental signature at the LHC

Black hole formation at the LHC could be detected by the properties of the jets resulting from Hawking radiation.

- Multiplicity of partons: Number of jets and leptons
- Large transverse energy
- Black-hole mass and spin are important for this!



ToDo:

- Exact cross section for BH formation
- Determine loss of energy in gravitational waves
- Determine spin of merged black hole

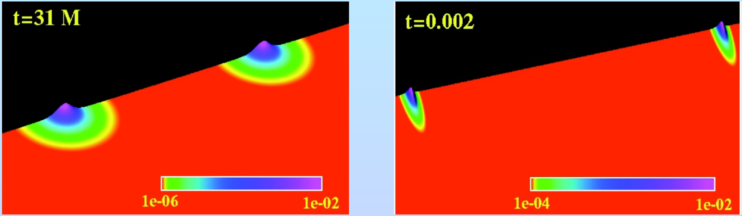
Does matter “matter”?

- Matter does not matter at energies $\ll E_{Planck}$

  Banks & Fischler ’99; Giddings & Thomas ’01

- Einstein plus minimally coupled, massive, complex scalar filed

  “Boson stars” Pretorius & Choptuik ’09



$\gamma = 1$ $\gamma = 4$

- BH formation threshold: $\gamma_{\rm thr} = 2.9 \pm 10\ \% \sim 1/3\ \gamma_{hoop}$
- Model particle collisions by BH collisions

Initial setup

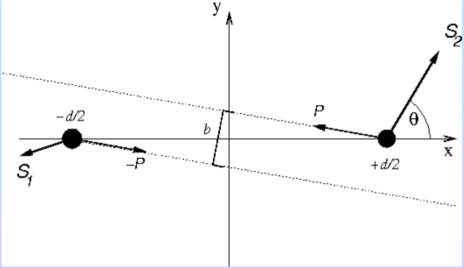
- Take two black holes

  Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

  Initial position: $\pm\frac{d}{2}$

  Linear momentum: $\mp P[\cos\alpha,\ \sin\alpha,\ 0]$

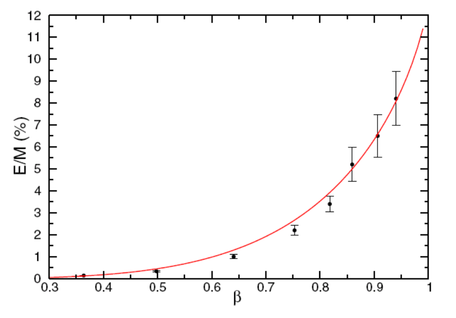
- Impact parameter: $b \equiv \frac{L}{P}$



Head-on: $D = 4, \quad b = 0, \quad \vec{S} = 0$

- Total radiated energy: 14 ± 3 % for $v \to 1$ US *et al.* '08

  About half of Penrose '74



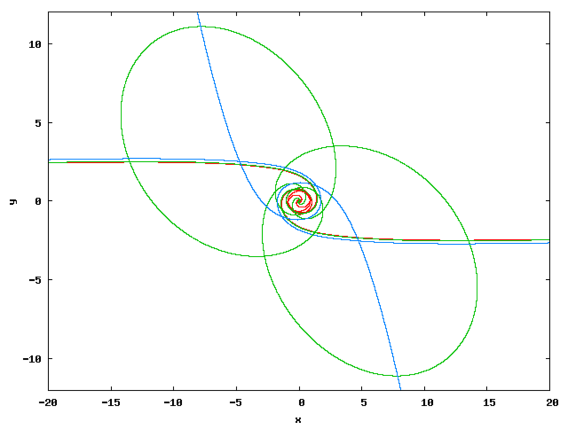
- Agreement with approximative methods

  Flat spectrum, multipolar GW structure Berti *et al.* '10

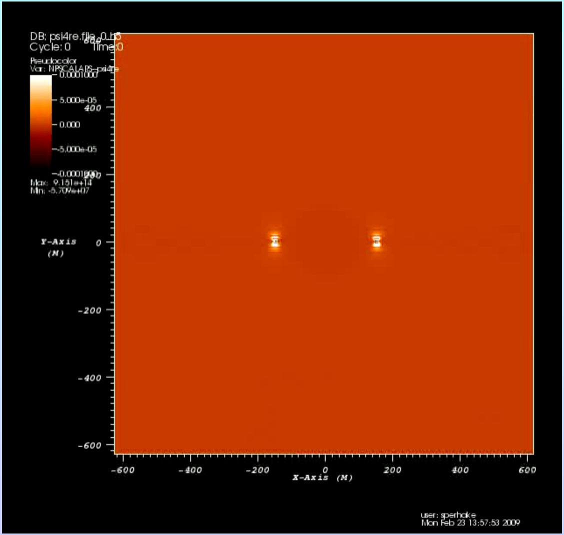
Grazing: $D = 4, \quad b \neq 0, \quad \gamma = 1.52$

- Zoom-whirl orbits   Pretorius & Khurana '07
- Immediate vs. Delayed vs. No merger

  US, Cardoso, Pretorius, Berti, Hinderer & Yunes '09



Gravitational radiation: Delayed merger



Scattering threshold b_{scat} in $D = 4$

- $b < b_{\mathrm{scat}} \quad \Rightarrow \quad$ Merger

  $b > b_{\mathrm{scat}} \quad \Rightarrow \quad$ Scattering

- Numerical study: $b_{\mathrm{scat}} = \frac{2.5 \pm 0.05}{v} M$

  Shibata, Okawa & Yamamoto '08

- Independent study by US, Pretorius, Cardoso, Berti *et al.* '09, '12

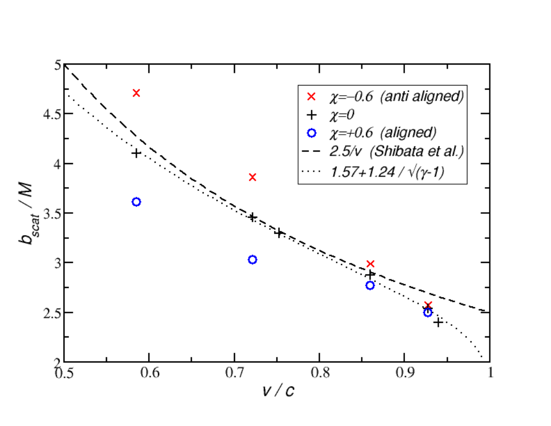
  $\gamma = 1.23 \ldots 2.93$:

  $\chi = -0.6,\ 0,\ +0.6$ (anti-aligned, nonspinning, aligned)

- Limit from Penrose construction: $b_{\mathrm{crit}} = 1.685\ M$

  Yoshino & Rychkov '05

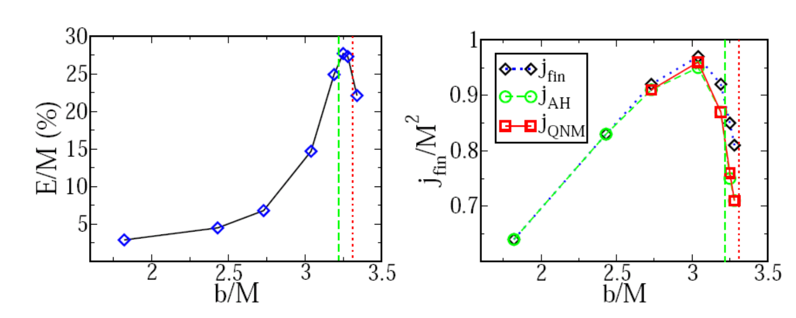
Diminishing impact of structure as $v \to 1$



- Effect of spin reduced for large γ
- $b_{\rm scat}$ for $v \to 1$ not quite certain

Radiated quantities: $b-$sequence with $\gamma = 1.52$

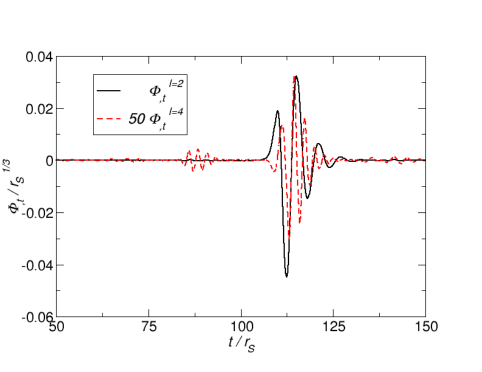
- Final spin close to Kerr limit
- $E_{\rm rad} \sim 35$ % for $\gamma = 2.93$; about 10 % of Dyson luminosity
- Diminishing "hang-up" effect as $v \to 1$



US, Cardoso, Pretorius, Berti, Hinderer & Yunes '09

Black-hole head-on collisions in $D = 6$

Witek *et al.* *in prep.*



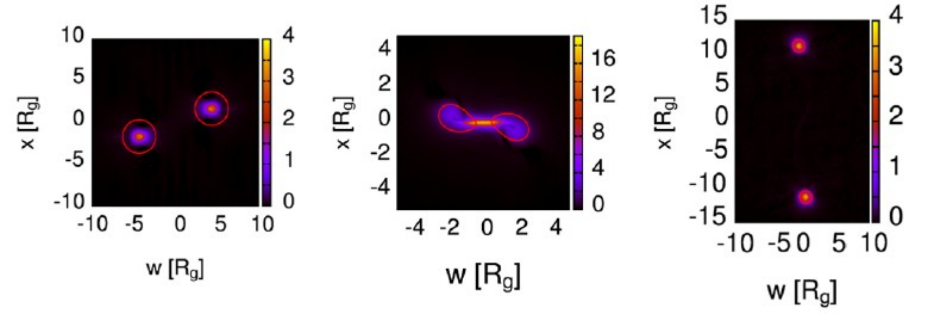
- Dimensional reduction, $SO(D-3)$ symmetry
- $d/r_S = 6$
- QNM ringdown agrees with close-limit Yoshino '05

Boosted collisions in $D = 5$

Okawa, Nakao & Shibata '11

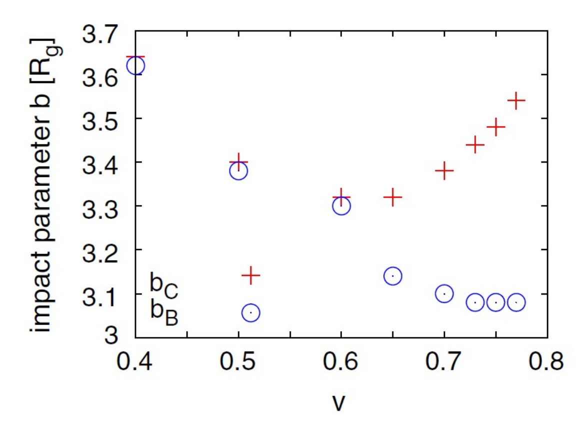
- Take Tangherlini metric; boost, translate, superpose
- Use $SO(D-3)$ symmetry via CARTOON

$$\frac{\sqrt{R^{abcd}R_{abcd}}}{6\sqrt{2}E_P^2}$$



Scattering threshold in $D = 5$

Okawa, Nakao & Shibata '11



Numerical stability still an issue...

5. The AdS/CFT correspondence

The AdS/CFT conjecture

Maldacena '98

- "strong form": Type IIb string theory on $AdS_5 \times S^5$

  $\Leftrightarrow \mathcal{N} = 4$ super Yang-Mills in $D = 4$

  Hard to prove; non-perturbative Type IIb String Theory?

- "weak form": low-energy limit of string-theory side

  $\Rightarrow$ Type IIb Supergravity on $AdS_5 \times S^5$

- Some assumptions, factor out S^5

  $\Rightarrow$ General Relativity on AdS_5

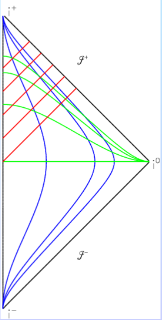
- Corresponds to limit of large N, $g^2 N$ in the field theory

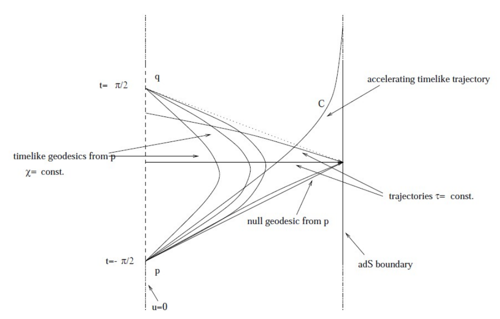
- E. g. Stationary AdS BH $\Leftrightarrow$ Thermal Equil. with T_{Haw} in dual FT

  Witten '98

The boundary in AdS

- Dictionary between metric properties and vacuum expectation values of CFT operators.
  E. g. $T_{\alpha\beta}$ operator of CFT $\leftrightarrow$ transverse metric on *AdS* boundary.
- The boundary plays an active role in *AdS*! Metric singular!



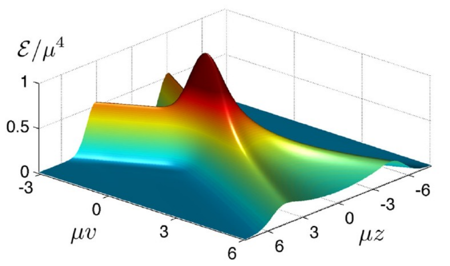


Collision of planar shockwaves in $\mathcal{N} = 4$ SYM

- Dual to colliding gravitational shock waves in AADS
- Characteristic study with translational invariance

  Chesler & Yaffe ’10, ’11

- Initial data: 2 superposed shockwaves
- Isotropization after $\Delta v \sim 4/\mu \sim 0.35$ fm/c



Cauchy ("4+1") evolutions in asymptotically AdS

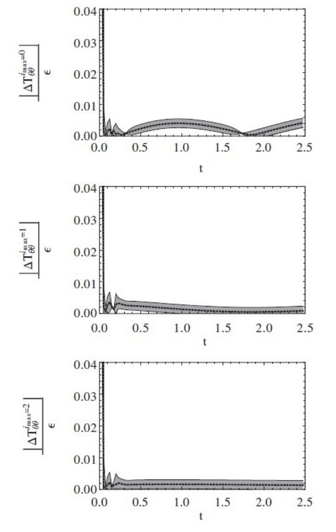
- Characteristic coordinates successful numerical tool in AdS/CFT
- But: restricted to symmetries, caustics problem...
- Cauchy evolution needed for general scenarios? Cf. BBH inspiral!!
- Cauchy scheme based on generalized harmonic formulation

  Bantilan & Pretorius '12

  - $SO(3)$ symmetry
  - Compactify "bulk radius"
  - Asymptotic symmetry of AdS_5: $SO(4,2)$
  - Decompose metric into AdS_5 piece and deviation
  - Gauge must preserve asymptotic fall-off

Cauchy ("4+1") evolutions in asymptotically AdS

- Scalar field collapse
- BH formation and ringdown
- Low order QNMs $\sim$ perturbative studies, but mode coupling
- CFT stress-energy tensor consistent with thermalized $\mathcal{N} = 4$ SYM fluid
- Difference of CFT $T_{\theta\theta}$ and hydro ($+1^{st}$, 2^{nd} corrs.)



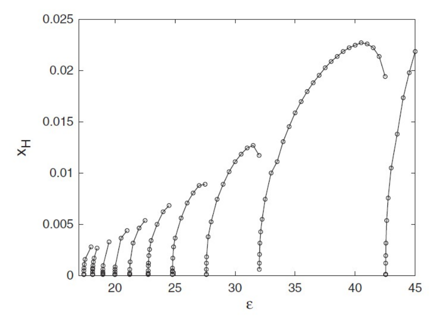
6. Stability, Cosmic Censorship

Stability of AdS

- $m = 0$ scalar field in as. flat spacetimes   Choptuik '93

  $p > p^* \Rightarrow$ BH, $p < p^* \Rightarrow$ flat

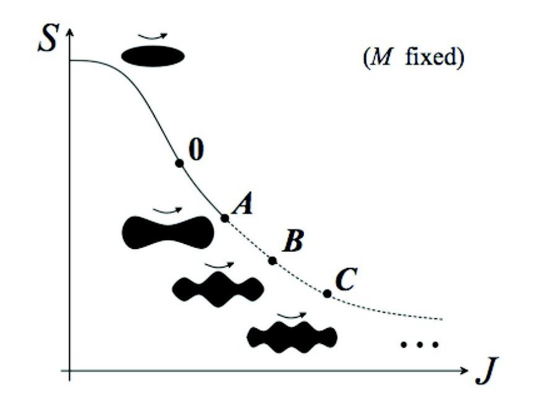
- $m = 0$ scalar field in as. AdS   Bizon & Rostworowski '11



- Similar behaviour for “Geons”   Dias, Horowitz & Santos '11

Bar mode instability of Myers-Perry BH

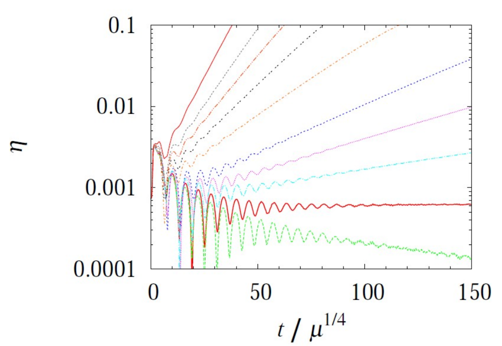
- MP BHs (with single ang.mom.) should be unstable.
- Linearized analysis Dias *et al.* '09



Non-linear analysis of MP instability

Shibata & Yoshino '10

- Myers-Perry metric; transformed to Puncture like coordinate
- Add small bar-mode perturbation
- Unstable for rotation parameter $q \gtrsim 0.75$



Cosmic Censorship in $D = 5$

Pretorius & Lehner '10

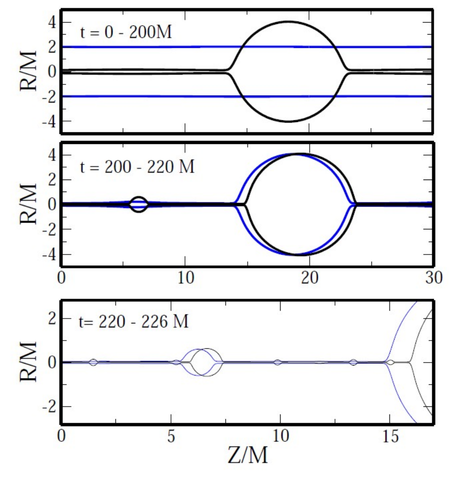
- Axisymmetric code
- Evolution of black string...
- Gregory-Laflamme instability

  cascades down

  in finite time

  until string has zero width

  $\Rightarrow$ naked singularity

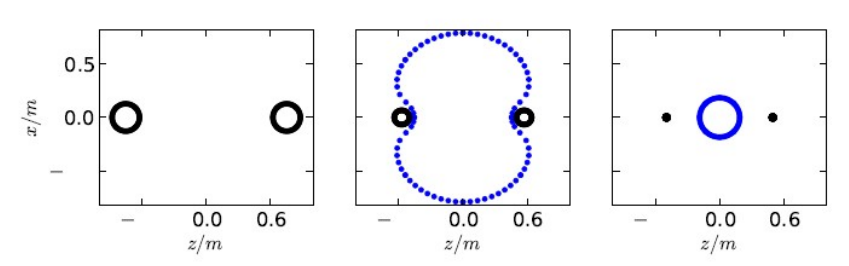


Cosmic Censorship in $D = 4$ de Sitter

Zilhão et al. '12

- Two parameters: MH, d
- Initial data: McVittie type binaries McVittie '33
- "Small BHs": $d < d_{crit} \Rightarrow$ merger

  $d > d_{crit} \Rightarrow$ no common AH
- "Large" holes at small d: Cosmic Censorship holds



7. Conclusions

Conclusions

- NR breakthroughs in 2005
- Typical simulations: 128 cores, 256 Gb RAM, $\sim$ weeks
- Explicit discretization, MPI parallelized, OpenMP
- Astrophysics, GW physics
- High-energy collisions of black holes
- AdS/CFT correspondence
- BH Stability, Cosmic Censorship
- ... ?