

Black holes in D dimensions

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Overview

- Motivation
- High-energy collisions of black holes
- AdS/CFT correspondence
- Black-hole Stability, Cosmic Censorship
- Conclusions and outlook

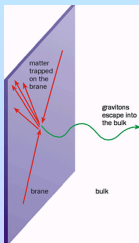
1. Motivation

The Hierarchy problem in physics: TeV Gravity

Large extra dimensions

Arkani-Hamed, Dimopoulos & Dvali '98

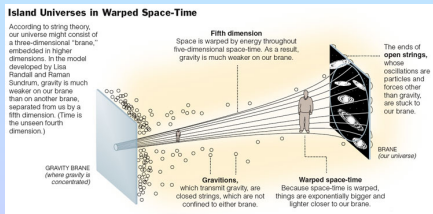
- SM confined to “3+1” brane
 - Gravity lives in bulk
- ⇒ Gravity diluted



Warped geometry

Randall & Sundrum '99

- 5D AdS Universe with 2 branes:
- “our” 3+1 world, gravity brane
- 5th dimension warped
- ⇒ Gravity weakened



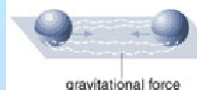
Either way: Gravity strong at $\gtrsim TeV$

Motivation (High-energy physics)

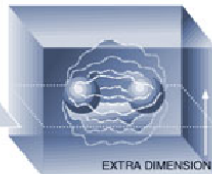
Black Holes on Demand

Scientists are exploring the possibility of producing miniature black holes on demand by smashing particles together. Their plans hinge on the theory that the universe contains more than the three dimensions of everyday life. Here's the idea:

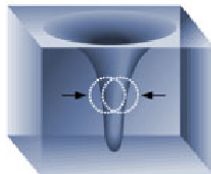
Particles collide in three dimensional space, shown below as a flat plane.



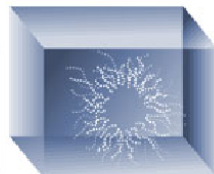
As the particles approach in a particle accelerator, their gravitational attraction increases steadily.



When the particles are extremely close, they may enter space with more dimensions, shown above as a cube.



The extra dimensions would allow gravity to increase more rapidly so a black hole can form.



Such a black hole would immediately evaporate, sending out a unique pattern of radiation.

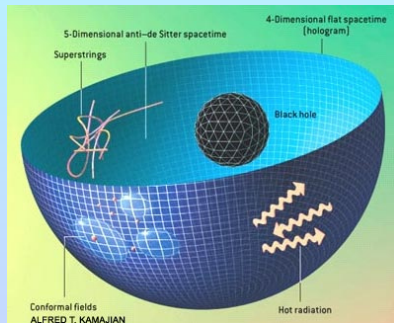
- Matter does not matter at energies well above the Planck scale
⇒ Model particle collisions by black-hole collisions

Banks & Fischler '99;

Giddings & Thomas '01

AdS/CFT correspondence

- CFTs in $D = 4$ dual to asymptotically AdS BHs in $D = 5$
- Study cousins of QCD, e. g. $\mathcal{N} = 4$ SYM
- Applications
 - Quark-gluon plasma; heavy-ion collisions, RHIC
 - Condensed matter, superconductors
- Dictionary: Metric fall-off $\leftrightarrow T_{\alpha\beta}$



Further motivation

BH collisions and dynamics in general D of wide interest:

- Test **Cosmic Censorship**
- Study **stability** of black holes
- Probe GR in the most violent regime
- **Zoom-whirl** behaviour; “critical” phenomena
- Super-Planckian physics?

2. High-energy BH collisions

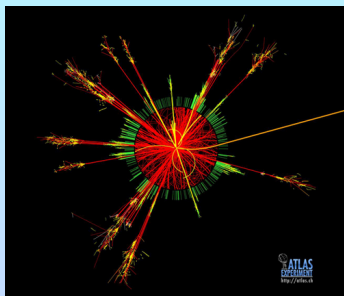
Experimental signature at the LHC

Black hole formation at the LHC could be detected by the properties of the jets resulting from Hawking radiation.

- Multiplicity of partons: Number of jets and leptons
- Large transverse energy
- Black-hole mass and spin are important for this!

ToDo:

- Exact cross section for BH formation
- Determine loss of energy in gravitational waves
- Determine spin of merged black hole



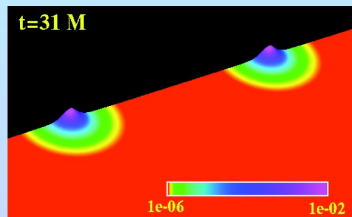
Does matter “matter”?

- Matter does not matter at energies $\ll E_{Planck}$

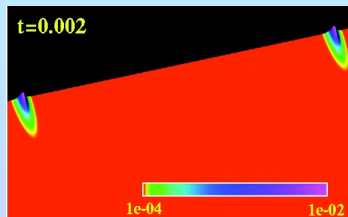
Banks & Fischler '99; Giddings & Thomas '01

- Einstein plus minimally coupled, massive, complex scalar field

“Boson stars” Pretorius & Choptuik '09



$\gamma = 1$



$\gamma = 4$

- BH formation threshold: $\gamma_{thr} = 2.9 \pm 10 \% \sim 1/3 \gamma_{hoop}$
- Model particle collisions by BH collisions

Computational framework

- Numerical relativity breakthroughs carry over
Pretorius '05, Goddard '05, Brownsville-RIT '05
- “Moving puncture” technique
- **BSSN** formulation; Shibata & Nakamura '95, Baumgarte & Shapiro '98
- $1 + \log$ slicing, Γ -driver shift condition
- Puncture ini-data; Bowen-York '80; Brandt & Brügmann '97; Ansorg et al. '04
- Mesh refinement *Cactus*, *Carpet*
- Wave extraction using **Newman-Penrose scalar**
- **Apparent Horizon** finder; e.g. Thornburg '96

Initial setup

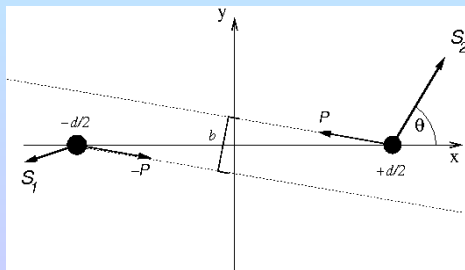
- Take two black holes

Total rest mass: $M_0 = M_{A,0} + M_{B,0}$

Initial position: $\pm \frac{d}{2}$

Linear momentum: $\mp P[\cos \alpha, \sin \alpha, 0]$

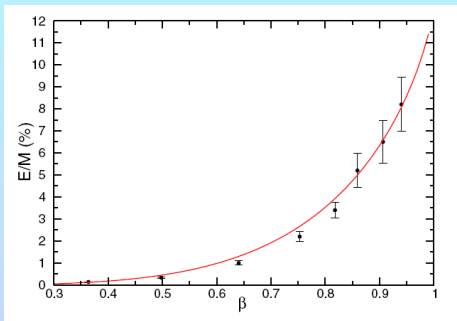
- Impact parameter: $b \equiv \frac{L}{P}$



Head-on: $D = 4$, $b = 0$, $\vec{S} = 0$

- Total radiated energy: $14 \pm 3 \%$ for $\nu \rightarrow 1$ US *et al.* '08

About half of Penrose '74



- Agreement with approximative methods

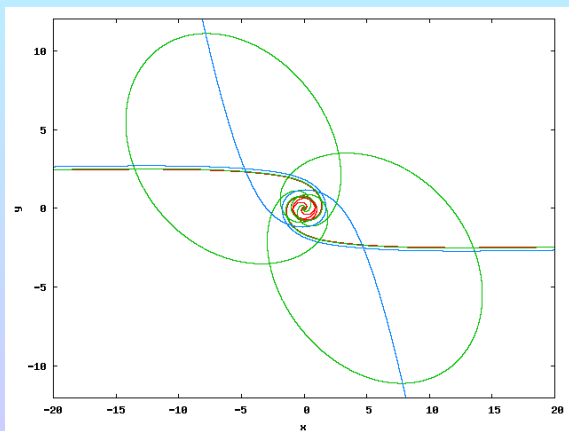
Flat spectrum, multipolar GW structure

Berti *et al.* '10

Grazing: $D = 4$, $b \neq 0$, $\gamma = 1.52$

- Zoom-whirl orbits Pretorius & Khurana '07
- Immediate vs. Delayed vs. No merger

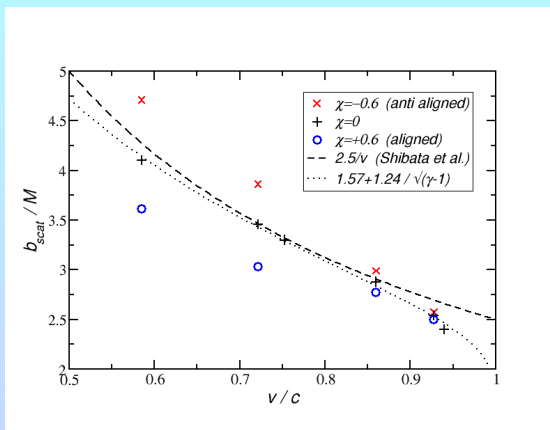
US, Cardoso, Pretorius, Berti, Hinderer & Yunes '09



Scattering threshold b_{scat} in $D = 4$

- $b < b_{\text{scat}} \Rightarrow$ Merger
- $b > b_{\text{scat}} \Rightarrow$ Scattering
- Numerical study: $b_{\text{scat}} = \frac{2.5 \pm 0.05}{v} M$
Shibata, Okawa & Yamamoto '08
- Independent study by US, Pretorius, Cardoso, Berti *et al.* '09, '12
 $\gamma = 1.23 \dots 2.93$:
 $\chi = -0.6, 0, +0.6$ (anti-aligned, nonspinning, aligned)
- Limit from Penrose construction: $b_{\text{crit}} = 1.685 M$
Yoshino & Rychkov '05

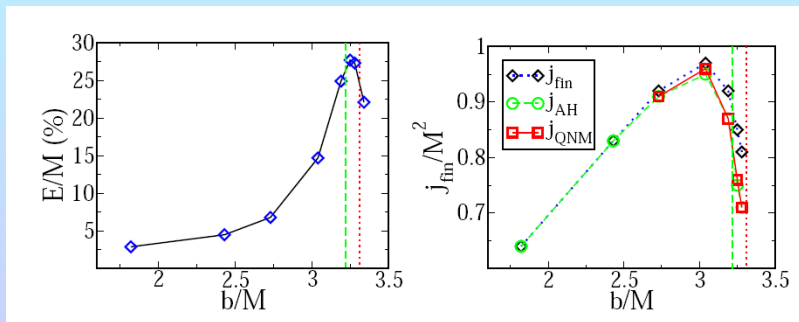
Diminishing impact of structure as $v \rightarrow 1$



- Effect of spin reduced for large γ
- b_{scat} for $v \rightarrow 1$ not quite certain

Radiated quantities: b -sequence with $\gamma = 1.52$

- Final spin close to Kerr limit
- $E_{\text{rad}} \sim 35\%$ for $\gamma = 2.93$; about 10% of Dyson luminosity
- Diminishing “hang-up” effect as $\nu \rightarrow 1$

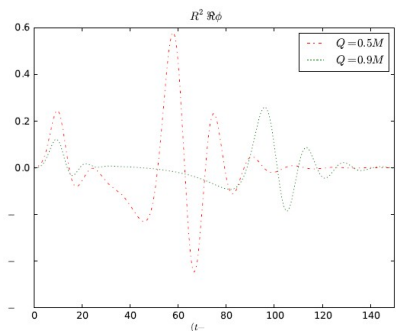
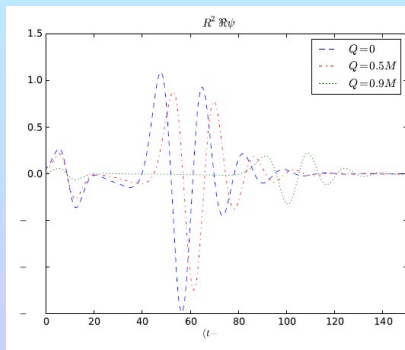


US, Cardoso, Pretorius, Berti, Hinderer & Yunes '09

Collisions of charged BHs in $D = 4$

Zilhão, Cardoso, Herdeiro, Lehner & US

- Electro-vacuum Einstein-Maxwell Eqs.; Moesta et al. '10
- Brill-Lindquist construction for equal mass, charge BHs
- Wave extraction $\Phi_2 := F_{\mu\nu} \bar{m}^\mu k^\nu$



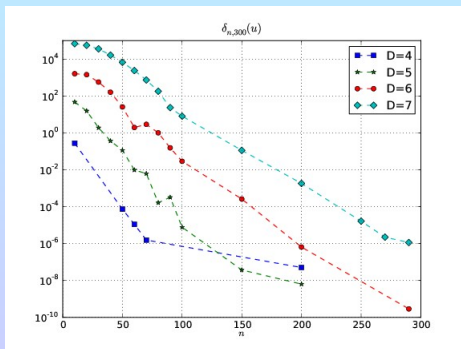
Moving to $D > 4$

- SACRA5D, SACRA-ND
Shibata, Yoshino, Okawa, Nakao
- D -dim. vacuum Einstein Eqs.
- D -dim. vacuum BSSN Eqs.
- $SO(D - 3)$ symmetry
- Modified CARTOON method
- D -dim. gauge conditions

- LEAN
Zilhão, Witek, US, Cardoso, Gualtieri & Nerozzi '10
- D -dim. vacuum Einstein Eqs.
- $SO(D - 3)$ symmetry
- Dim. reduction; Geroch '70
 \Rightarrow 4- dim. Einstein + scalar
- 3 + 1-dim. BSSN + scalar
- Modified 4-dim. gauge

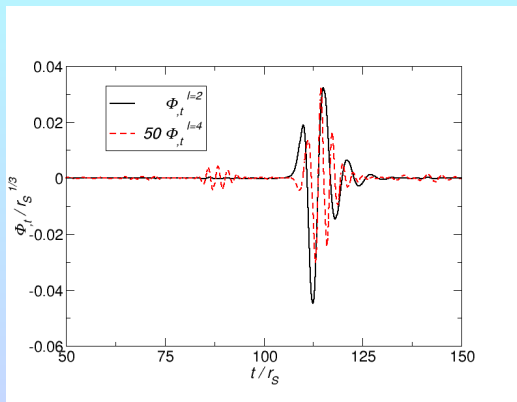
Puncture initial data for boosted BHs in $D \geq 5$

- Generalize spectral code of Ansorg et al. '04
- Momentum constraint still solved analytically
Yoshino, Shiromizu & Shibata '06
- Spectral solver for Hamiltonian constraint; Zilhão et al. '11



Black-hole collisions in $D = 6$

Witek *et al.* *in prep.*



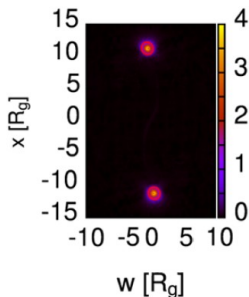
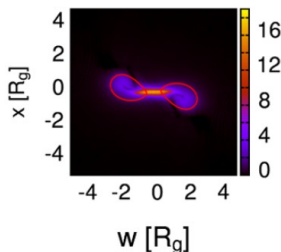
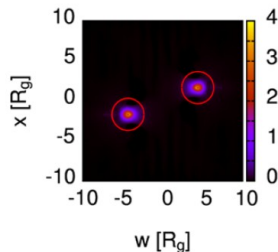
- $d/r_S = 6$
- QNM ringdown agrees with close-limit Yoshino '05

Boosted collisions in $D = 5$

Okawa, Nakao & Shibata '11

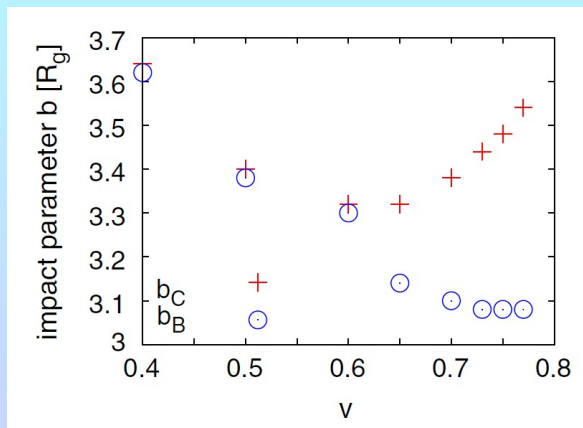
- Take Tangherlini metric; **boost** and translate
- Superpose two of those

$$\frac{\sqrt{R^{abcd}R_{abcd}}}{6\sqrt{2}E_P^2}$$



Scattering threshold in $D = 5$

Okawa, Nakao & Shibata '11



Numerical stability still an issue...

3. The AdS/CFT correspondence

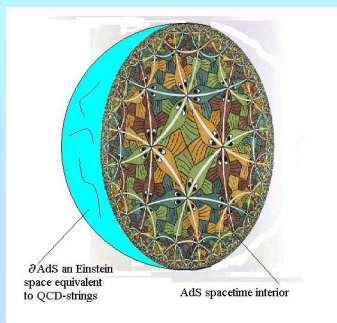
Large N and holography

- Holography

- BH entropy $\propto A_{Hor}$
- For a Local Field Theory entropy $\propto V$
- Gravity in D dims
 \Leftrightarrow local FT in $D - 1$ dims

- Large N limit

- Perturbative expansion of gauge theory in $1/N$ and $g^2 N$
 \sim loop expansion in string theory
- N : # of “colors”
 $g^2 N$: t’Hooft coupling



The AdS/CFT conjecture

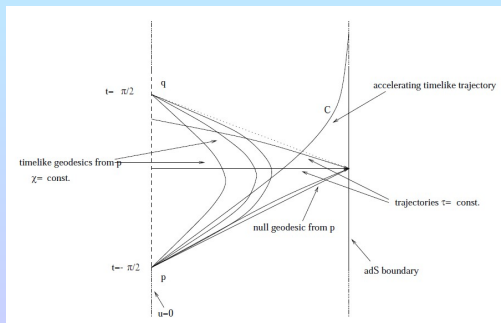
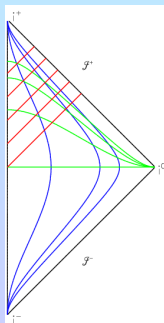
Maldacena '98

- “strong form”: Type IIb string theory on $AdS_5 \times S^5$
 $\Leftrightarrow \mathcal{N} = 4$ super Yang-Mills in $D = 4$
Hard to prove; non-perturbative Type IIb String Theory?
- “weak form”: low-energy limit of string-theory side
 \Rightarrow Type IIb Supergravity on $AdS_5 \times S^5$
- Some assumptions, factor out S^5
 \Rightarrow General Relativity on AdS_5
- Corresponds to limit of large N , $g^2 N$ in the field theory
- E. g. Stationary AdS BH \Leftrightarrow Thermal Equil. with T_{Haw} in dual FT

Witten '98

The boundary in AdS

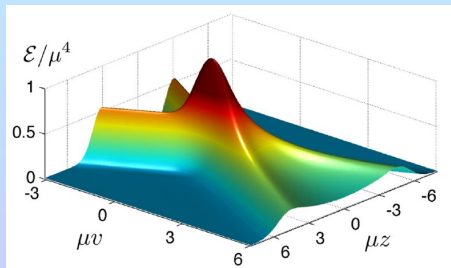
- Dictionary between metric properties and vacuum expectation values of CFT operators.
E. g. $T_{\alpha\beta}$ operator of CFT \leftrightarrow transverse metric on *AdS* boundary.
- The boundary plays an active role in *AdS*! Metric singular!



Collision of planar shockwaves in $\mathcal{N} = 4$ SYM

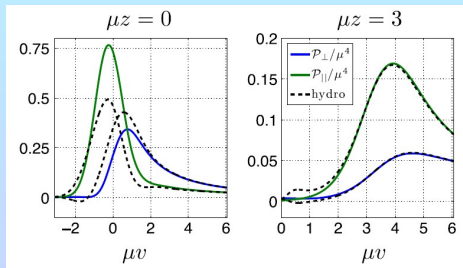
- Dual to colliding gravitational shock waves in AADS
- Characteristic study with translational invariance
Chesler & Yaffe '10, '11
- Initial data: 2 superposed shockwaves

$$ds^2 = r^2[-dx_+ dx_- + d\mathbf{x}_\perp] + \frac{1}{r^2}[dr^2 + h(x_\pm) dx_\pm^2]$$



Collision of planar shockwaves in $\mathcal{N} = 4$ SYM

- Initially system far from equilibrium
- Isotropization after $\Delta v \sim 4/\mu \sim 0.35 \text{ fm}/c$
- Confirms hydrodynamic simulations of QGP $\sim 1 \text{ fm}/c$ Heinz '04



- Non-linear vs. linear Einstein Eqs. agree within $\sim 20 \%$

Heller et al. '12

Cauchy (“4+1”) evolutions in asymptotically AdS

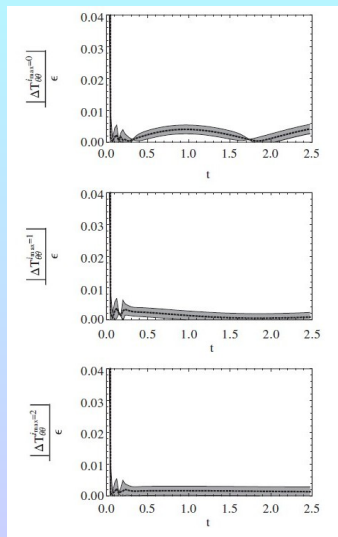
- Characteristic coordinates successful numerical tool in AdS/CFT
- But: restricted to symmetries, caustics problem...
- Cauchy evolution needed for general scenarios? Cf. BBH inspiral!!
- Cauchy scheme based on generalized harmonic formulation

Bantilan & Pretorius '12

- $SO(3)$ symmetry
- Compactify “bulk radius”
- Asymptotic symmetry of AdS_5 : $SO(4, 2)$
- Decompose metric into AdS_5 piece and deviation
- Gauge must preserve asymptotic fall-off

Cauchy (“4+1”) evolutions in asymptotically AdS

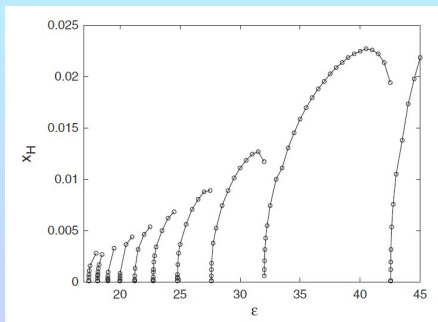
- Scalar field collapse
- BH formation and ringdown
- Low order QNMs \sim perturbative studies, but mode coupling
- CFT stress-energy tensor consistent with thermalized $\mathcal{N} = 4$ SYM fluid
- Difference of CFT $T_{\theta\theta}$ and hydro (+1st, 2nd corr.)



4. Stability, Cosmic Censorship

Stability of AdS

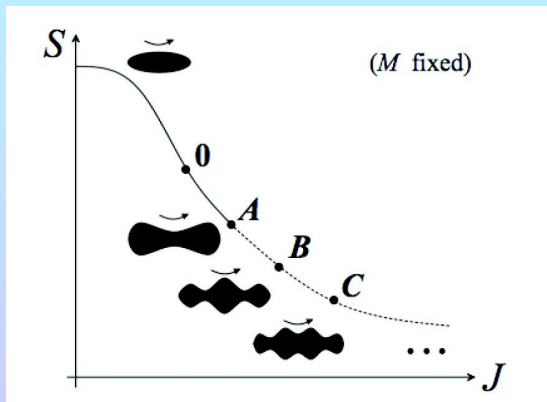
- $m = 0$ scalar field in as. flat spacetimes Choptuik '93
 $p > p^* \Rightarrow \text{BH}, \quad p < p^* \Rightarrow \text{flat}$
- $m = 0$ scalar field in as. AdS Bizon & Rostworowski '11



- Similar behaviour for “Geons” Dias, Horowitz & Santos '11

Bar mode instability of Myers-Perry BH

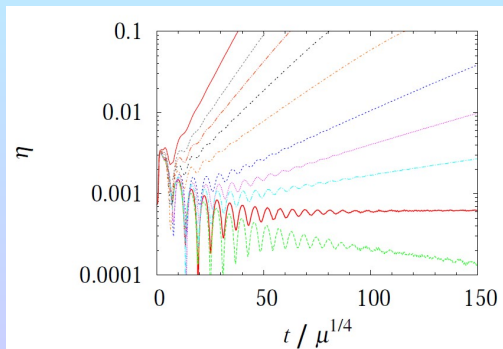
- MP BHs (with single ang.mom.) should be unstable.
- Linearized analysis Dias *et al.* '09



Non-linear analysis of MP instability

Shibata & Yoshino '10

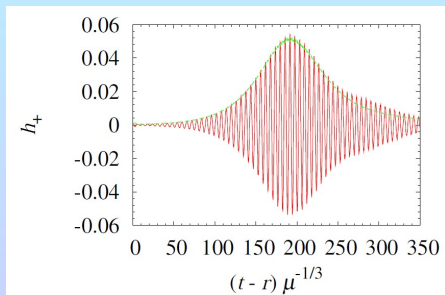
- Myers-Perry metric; transformed to Puncture like coordinate
- Add small bar-mode perturbation
- Deformation $\eta := \frac{2\sqrt{(l_0 - l_{\pi/2})^2 + (l_{\pi/4} - l_{3\pi/4})^2}}{l_0 + l_{\pi/2}}$



Non-linear analysis of MP instability

- Above dimensionless q_{crit} instability
- GW emission; BH settles down to lower q configuration

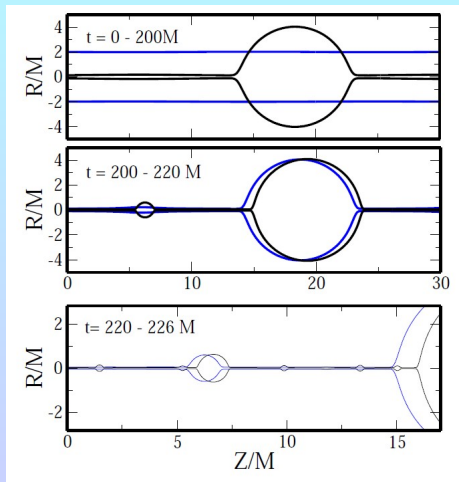
d	5	6	7	8
q_{crit}	0.87	0.74	0.73	0.77
a_*	1.76	0.91	0.83	0.86
C_p/C_e	0.38	0.65	0.68	0.67



Cosmic Censorship in $D = 5$

Pretorius & Lehner '10

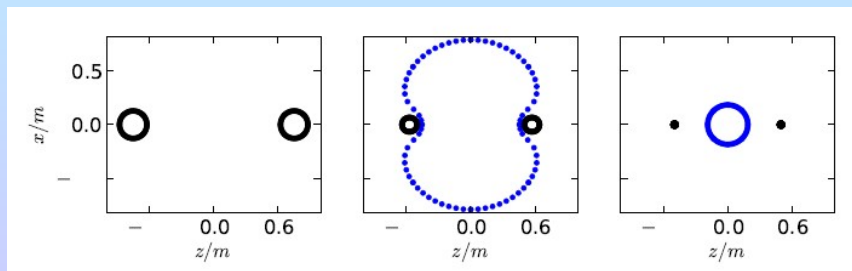
- Axisymmetric code
- Evolution of black string...
- Gregory-Laflamme instability cascades down in finite time until string has zero width \Rightarrow naked singularity



Cosmic Censorship in $D = 4$ de Sitter

Zilhão et al. '12

- Two parameters: MH , d
- Initial data: McVittie type binaries McVittie '33
- “Small BHs”: $d < d_{crit} \Rightarrow$ merger
 $d > d_{crit} \Rightarrow$ no common AH
- “Large” holes at small d : Cosmic Censorship holds



5. Conclusions

Conclusions

- “3+1” numerical framework can be modified for higher D
- High-energy collisions
 - In 4D b_{thresh} for $v \rightarrow 1$?
 - Zoom-whirl behaviour in 4D, but not 5D
 - For $v \rightarrow 1$ structure less important
- AdS/CFT correspondence
 - Numerical challenge; boundary
 - Results in characteristic framework; thermalization
 - First attempts in “3+1”
- AdS unstable against perturbations
- Myers Perry BH unstable above threshold spin
- Cosmic Censorship holds in 4D, but not 5D