High-speed black hole collisions with application to trans-Planckian particle scattering

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Overview

- Motivation
- Black-hole collisions in 3+1 dimensions
- Black-hole collisions in higher dimensional spacetimes
- Further topics
- Conclusions and outlook

1. Motivation

The Hierarchy Problem of Physics

- Gravity $\approx 10^{-39} \times$ other forces
- Higgs field $\approx \mu_{obs} \approx 250\ GeV = \sqrt{\mu^2 - \Lambda^2}$

  where $\Lambda \approx 10^{16}\ GeV$ is the grand unification energy
- Requires enormous finetuning!!!
- Finetuning exist: $\frac{987654321}{123456789} = 8.0000000729$
- Or Planck mass is much lower?

  I.e. Gravity much stronger at small length scales?
- Gravity not measured below $0.16\ mm$! Diluted due to...
  - Large extra dimensions
  - Extra dimension with warp factor

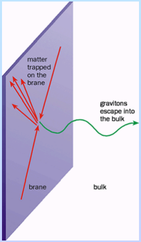
TeV Gravity

Large extra dimensions

Arkani-Hamed, Dimopoulos & Dvali '98

- SM confined to "3+1" brane
- Gravity lives in bulk

  $\Rightarrow$ Gravity diluted



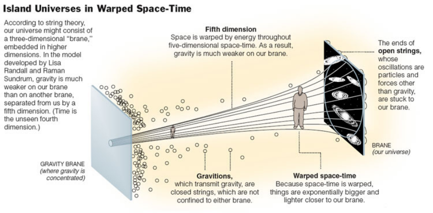
Warped geometry

Randall & Sundrum '99

- 5D AdS Universe with 2 branes:

  "our" 3+1 world, gravity brane
- 5^{th} dimension warped

  $\Rightarrow$ Gravity weakened

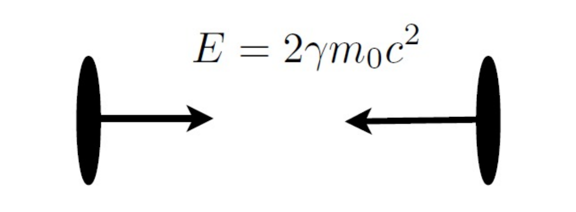


Either way: Gravity strong at $\gtrsim$ *TeV*

BH formation and hoop conjecture

- Hoop conjecture

  Thorne '72



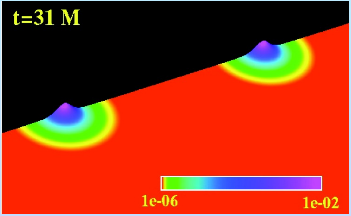
- de Broglie wavelength: $\lambda = \frac{hc}{E}$
- Schwarzschild radius: $r = \frac{2GE}{c^4}$
- BH will form if $\lambda < r \quad \Leftrightarrow \quad E \gtrsim \sqrt{\frac{hc^5}{G}} \equiv E_{\text{Planck}}$

BH formation in boson field collisions

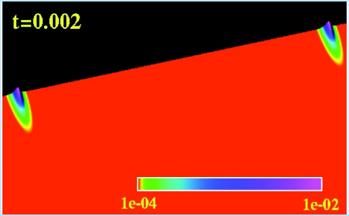
Pretorius & Choptuik '09

- Einstein plus minimally coupled, massive, complex scalar filed

  "Boson stars"



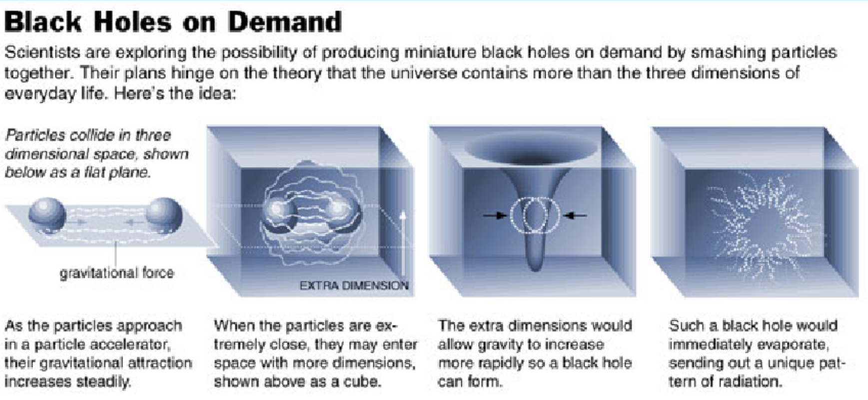
$\gamma = 1$



$\gamma = 4$

- BH formation threshold: $\gamma_{\text{thr}} = 2.9 \pm 10\ \%$
- About $1/3$ of hoop conjecture prediction

Motivation (High-energy physics)



- Matter does not matter at energies well above the Planck scale

$\Rightarrow$ Model particle collisions by black-hole collisions

Banks & Fischler '99; Giddings & Thomas '01

Black-hole formation in high-energy collisions

- Cosmic-rays hitting the earth's atmosphere
- Parton-parton collisions above TeV energies, LHC

$\rightarrow$ Talk by Colon, Sec. R9

Proton collisions at the LHC

Energy stored in a single beam:

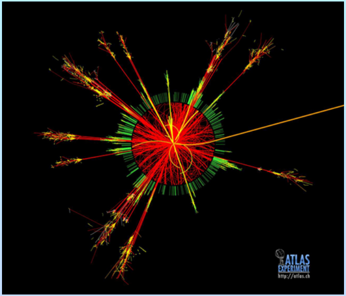
$$360\ MJ = 90\ kg \text{ of TNT}$$
$$= 15\ kg \text{ of chocolate}$$

Landsberg '11 talk at NRHEP Madeira

Experimental signature at the LHC

Black hole formation at the LHC could be detected by the properties of the jets resulting from Hawking radiation.

- Multiplicity of partons: Number of jets and leptons
- Large transverse energy
- Black-hole mass and spin are important for this!



ToDo:

- Exact cross section for BH formation
- Determine loss of energy in gravitational waves
- Determine spin of merged black hole

Further motivation

BH collisions and dynamics in general D of wide interest:

- Test Cosmic Censorship
- Study stability of black holes
- Probe GR in the most violent regime
- Zoom-whirl behaviour; “critical” phenomena
- Super-Planckian physics?
- AdS/CFT correspondence

2. BH collisions in 3+1 dimensions

Black-hole collisions in $D = 4$

- Numerical relativity breakthroughs carry over

  Pretorius '05, Goddard '05, Brownsville-RIT '05

- "Moving puncture" technique
- BSSN formulation; Shibata & Nakamura '95, Baumgarte & Shapiro '98
- $1 + \log$ slicing, Γ-driver shift condition
- Puncture ini-data; Bowen-York '80; Brandt & Brügmann '97; Ansorg et al. '04
- Mesh refinement `Cactus, Carpet`
- Wave extraction using Newman-Penrose scalar
- Apparent Horizon finder; e.g. Thornburg '96

Black-hole collisions in $D = 4$

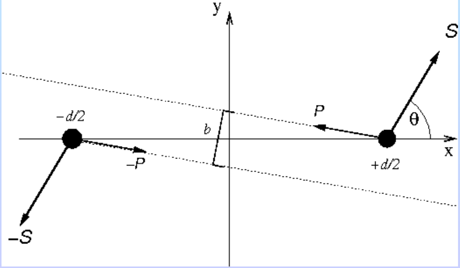
- Take two black holes

  Total rest mass: $M_0 = M_{A,\,0} + M_{B,\,0}$

  Initial position: $\pm x_0$

  Linear momentum: $\mp P[\cos\alpha,\ \sin\alpha,\ 0]$

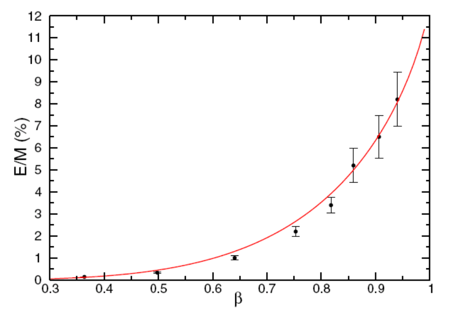
- Impact parameter: $b \equiv \frac{L}{P}$



Head-on collisions: $b = 0, \quad \vec{S} = 0$

- Total radiated energy: 14 ± 3 % for $v \to 1$ US *et al.* '08

  About half of Penrose '74



- Agreement with approximative methods

  Flat spectrum, multipolar GW structure Berti *et al.* '10

$b \neq 0$: Zoom whirl orbits

Pretorius & Khurana ’07

- 1-parameter family of initial data: impact parameter
  - Fine tune parameter

    $\Rightarrow$ “Threshold of

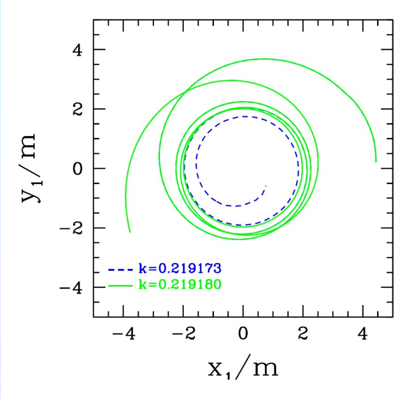
    immediate merger”
  - Analogue in geodesics
  - Remniscent of

    “Critical Phenomena”
  - Similar observations by

    Healy *et al.* ’09

    Zoom-whirl more likely for

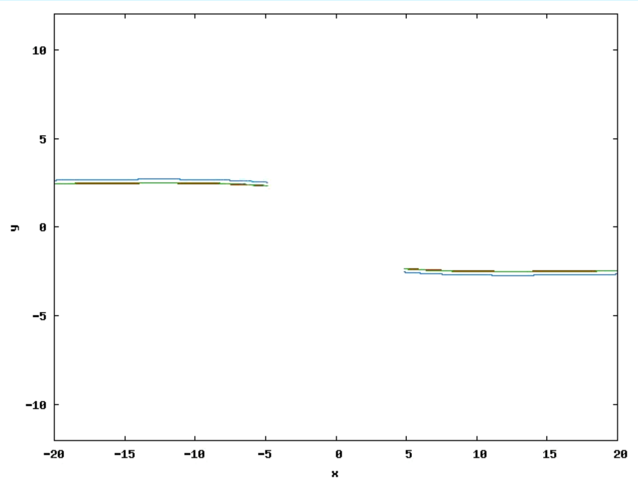
    larger q



Grazing collisions: $b \neq 0, \quad \vec{S} = 0, \quad \gamma = 1.52$

Immediate vs. Delayed vs. No merger

US, Cardoso, Pretorius, Berti, Hinderer & Yunes '09



Critical impact parameter

- $b < b_{\rm crit} \quad \Rightarrow \quad$ Merger

  $b > b_{\rm crit} \quad \Rightarrow \quad$ Scattering

- Numerical study: $\quad b_{\rm crit} = \frac{2.5 \pm 0.05}{v} M$

  Shibata, Okawa & Yamamoto '08

- Independent study by US, Pretorius, Cardoso, Berti *et al.* '09, '12

  $\gamma = 1.23 \ldots 2.93$:

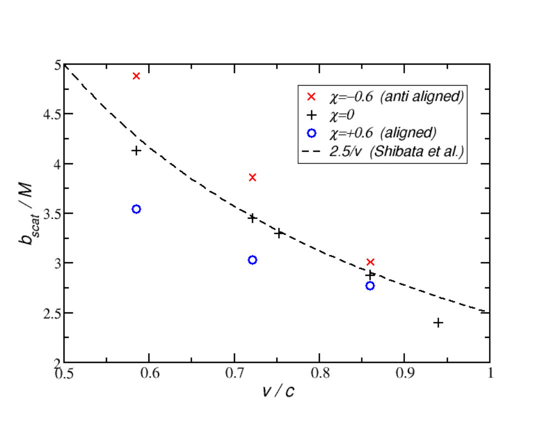
  $\chi = -0.6,\ 0,\ +0.6$ (anti-aligned, nonspinning, aligned)

- Limit from Penrose construction: $b_{\rm crit} = 1.685\ M$

  Yoshino & Rychkov '05

Critical impact parameter

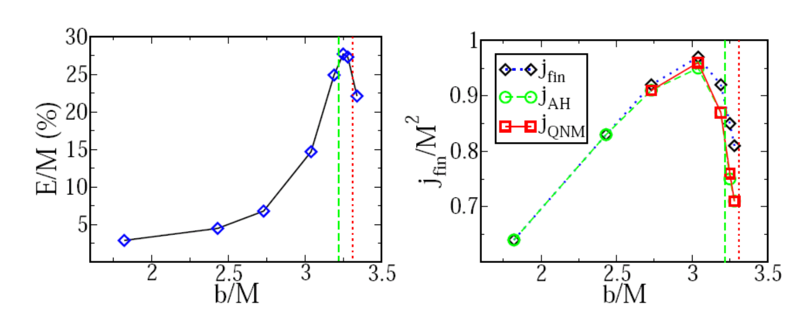
Preliminary results



- Effect of spin reduced for large γ
- $b_{\rm scat}$ for $v \to 1$ not quite certain

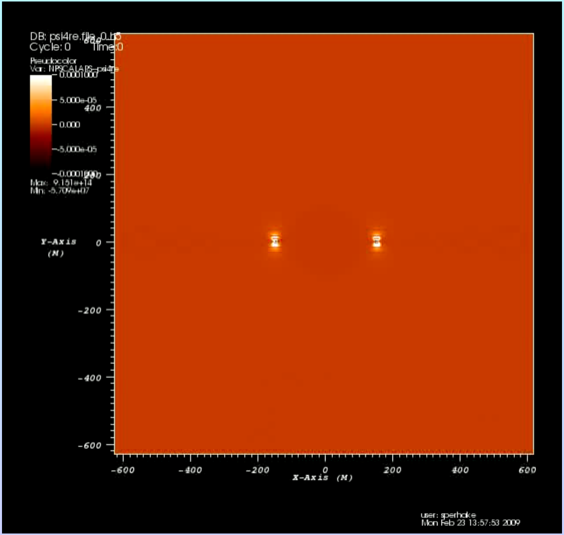
Radiated quantities

- $b-$sequence with $\gamma = 1.52$
- Final spin close to Kerr limit
- $E_{\rm rad} \sim 35$ % for $\gamma = 2.93$; about 10 % of Dyson luminosity

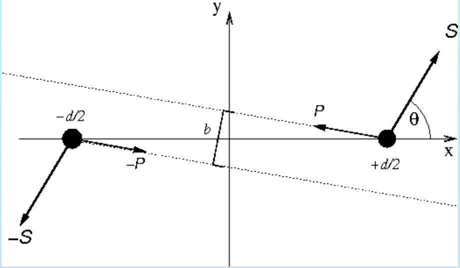


US, Cardoso, Pretorius, Berti, Hinderer & Yunes '09

Gravitational radiation: Delayed merger



Recoil in grazing collisions



- equal-mass, superkick, $\chi = 0.621$
- $\gamma = 1.52$
- 2 sequences

  merging: $b = 3.34\ M$

  scattering: $b = 3.25\ M$

Recoil in grazing collisions

- $v_{\mathrm{max,s}} = 12\,200\ \mathrm{km/s}$

  $v_{\mathrm{max,m}} = 14\,900\ \mathrm{km/s}$

- Large recoils for merger and scattering!

- $v_{\mathrm{max}} \propto E_{\mathrm{rad}}$

- Antikicks can occur in both $\Rightarrow$ not a merger-only feature!

- Ultimate kick

  $v_{\mathrm{max}} \propto E_{\mathrm{rad}} \quad \Rightarrow \quad \sim 45\,000\ \mathrm{km/s}$

  spin insignificant for large $\gamma \quad \Rightarrow \quad \sim 25\,000\ \mathrm{km/s}$

  no simple picture $\Rightarrow$ more data needed...

3. BH collisions in $D > 4$ dimensions

Moving to $D > 4$

- SACRA5D, SACRA-ND

  Shibata, Yoshino, Okawa, Nakao
- D-dim. vacuum Einstein Eqs.
- D-dim. vacuum BSSN Eqs.
- $SO(D-3)$ symmetry
- Modified CARTOON method
- D-dim. gauge conditions

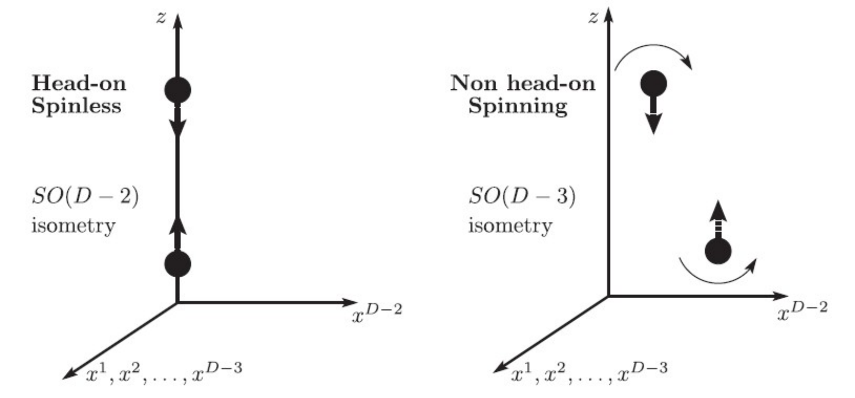
- LEAN

  Zilhão, Witek, US, Cardoso, Gualtieri & Nerozzi '10
- D-dim. vacuum Einstein Eqs.
- $SO(D-3)$ symmetry
- Dim. reduction; Geroch '70

  $\Rightarrow$ 4- dim. Einstein $+$ scalar
- $3+1$-dim. BSSN $+$ scalar
- Modified 4-dim. gauge

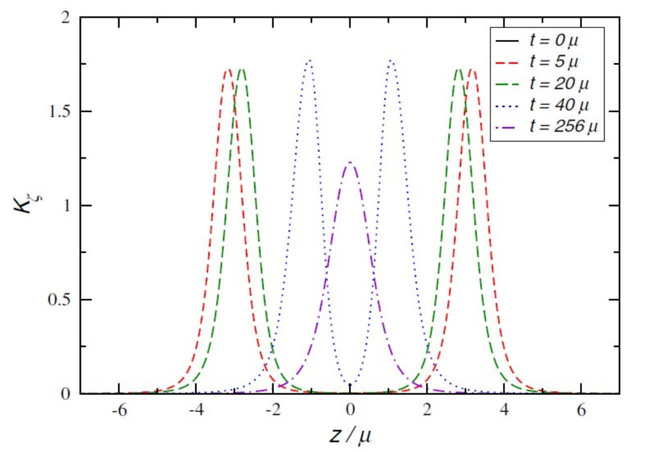
Moving to $D > 4$

- $SO(D-3)$ symmetry admits wide class of systems



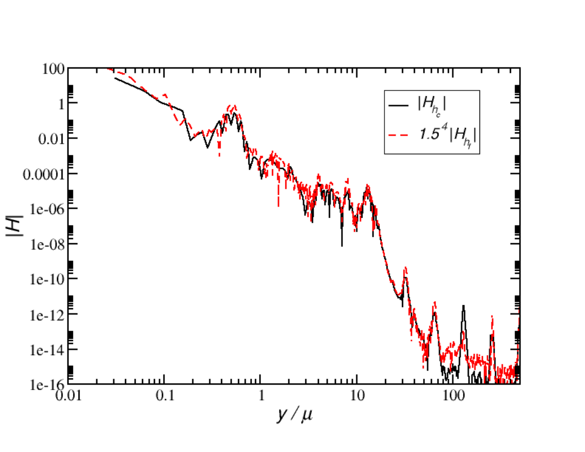
Head-on in $D = 5$

Initial data: $D = 5$ analogue of Brill-Lindquist data



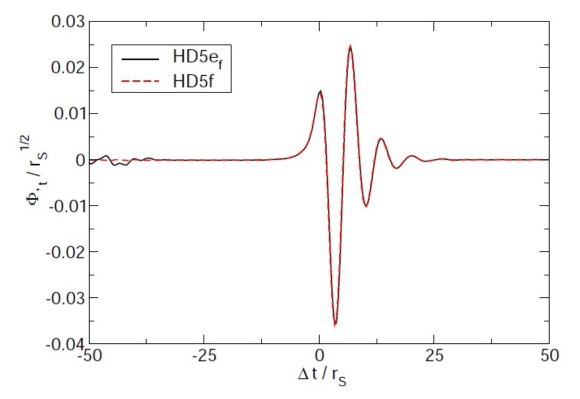
Single black hole in $D = 6$

Initial data: Tangherlini '63



GWs from head-on in $D = 5$

Wave extraction based on Kodama & Ishibashi '03

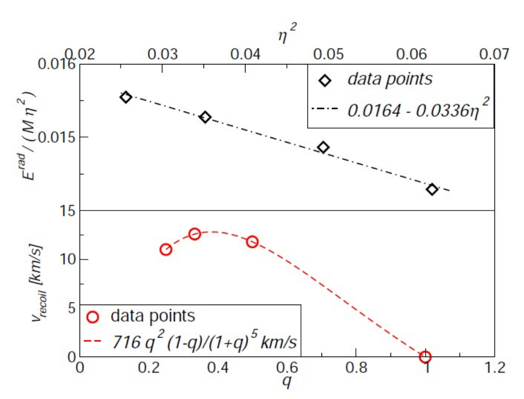


$E_{\rm rad} = 0.089\ \%M$ cf. $0.055\ \%M$ in $D = 4$

Witek *et al.* '10a

Unequal-mass head-on in $D = 5$

Radiated energy and momentum



Agreement within < 5 % with extrapolated point particle calculations

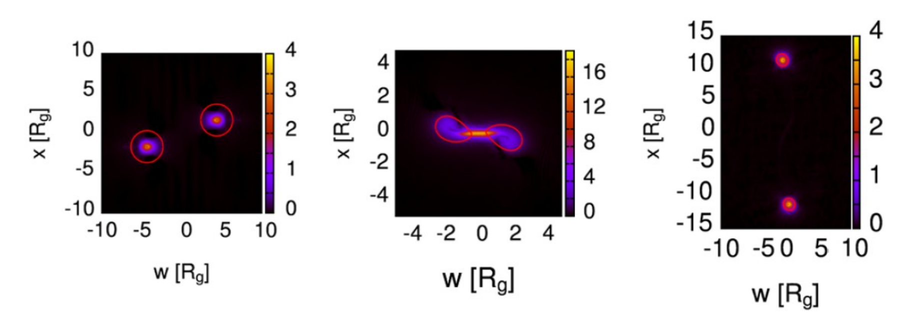
Witek *et al.* '10b

Boosted collisions in $D = 5$

Okawa, Nakao & Shibata '11

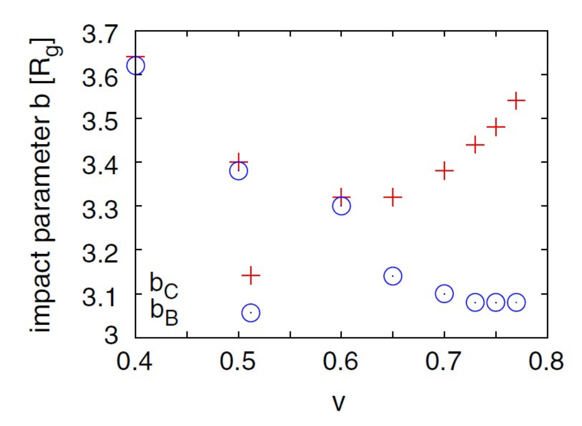
- Take Tangherlini metric; boost and translate
- Superpose two of those

$$\frac{\sqrt{R^{abcd}R_{abcd}}}{6\sqrt{2}E_P^2}$$



Scattering threshold in $D = 5$

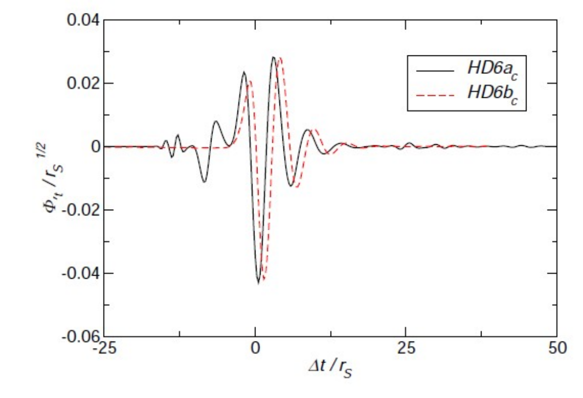
Okawa, Nakao & Shibata '11



Numerical stability still an issue...

First black-hole collisions in $D = 6$

Witek *et al.* '11

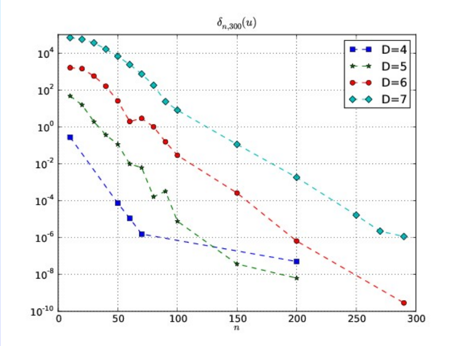


- Adjust shift parameters
- Use LaSh system Witek, Hilditch & US '10

Puncture initial data for boosted BHs in $D \geq 5$

- Generalize spectral code of Ansorg et al. '04
- Momentum constraint still solved analytically

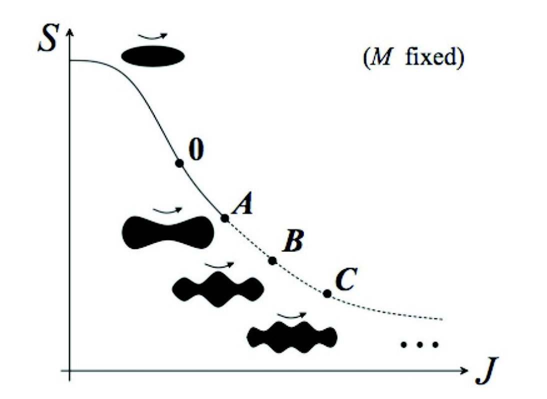
  Yoshino, Shiromizu & Shibata '06
- Spectral solver for Hamiltonian constraint; Zilhão et al. '11



4. More scenarios

Bar mode instability of Myers-Perry BH

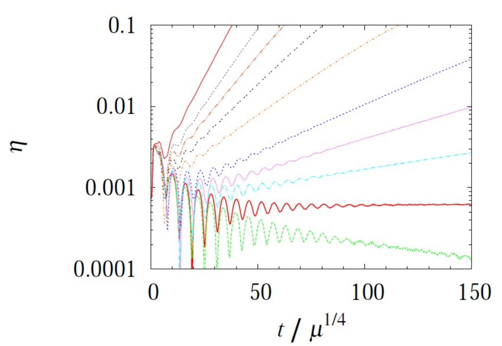
- MP BHs (with single ang.mom.) should be unstable.
- Linearized analysis Dias *et al.* '09



Non-linear analysis of MP instability

Shibata & Yoshino '10

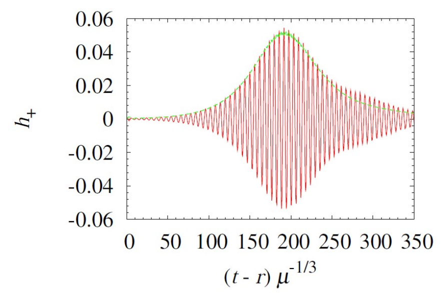
- Myers-Perry metric; transformed to Puncture like coordinate
- Add small bar-mode perturbation
- Deformation $\eta := \frac{2\sqrt{(l_0 - l_{\pi/2})^2 + (l_{\pi/4} - l_{3\pi/4})^2}}{l_0 + l_{\pi/2}}$



Non-linear analysis of MP instability

- Above dimension less q_{crit} instability
- GW emission; BH settles down to lower q configuration

d	5	6	7	8
$q_{\rm crit}$	0.87	0.74	0.73	0.77
a_*	1.76	0.91	0.83	0.86
C_p/C_e	0.38	0.65	0.68	0.67

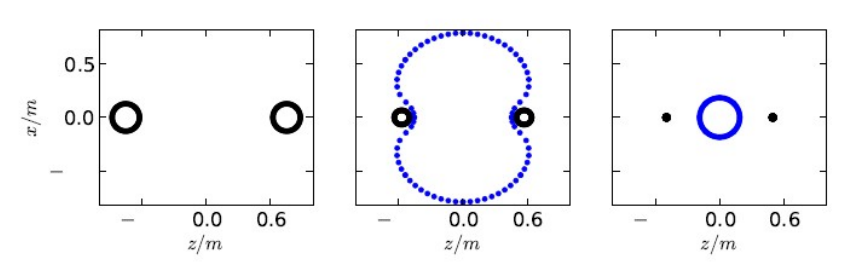


Black holes in de Sitter

Zilhão et al. ’12

- Two parameters: MH, d
- Initial data: McVittie type binaries McVittie ’33
- “Small BHs”: $d < d_{crit} \Rightarrow$ merger

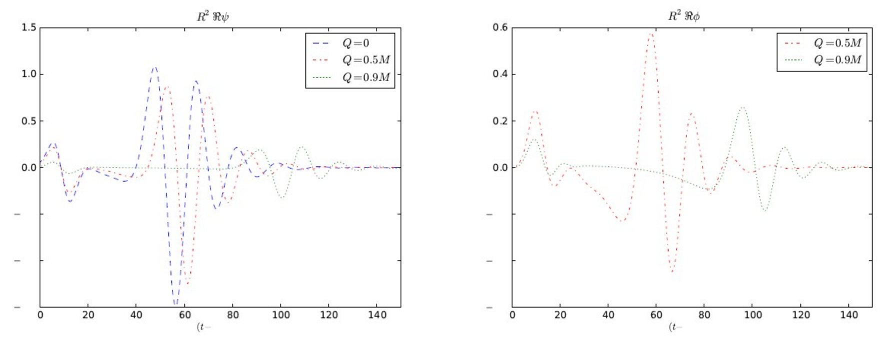
  $d > d_{crit} \Rightarrow$ no common AH
- “Large” holes at small d: Cosmic Censorship holds



Collisions of charged BHs in $D = 4$

Zilhão, Cardoso, Herdeiro, Lehner & US

- Electro-vacuum Einstein-Maxwell Eqs.; Moesta et al. '10
- Brill-Lindquist construction for equal mass, charge BHs
- Wave extraction $\Phi_2 := F_{\mu\nu}\bar{m}^{\mu}k^{\nu}$



5. Conclusions

Conlcusions

- "3+1" numerical framework can be modified for higher D
- Stability not yet as robust as in $D = 4$; gauge?
- Scattering threshold in 4D: $b_{\text{crit}} \approx \frac{2.5\ M}{v}$
- Cosmic Censorship holds
- Zoom-whirl behaviour in 4D
- BH collisions in 5D, 6D; b_{crit} still open
- Super-Planckian regime in 5D
- No zoom-whirl in 5D
- First charged BH collisions
- BH collisions in de Sitter: Censorship holds