Black-hole binary simulations on supercomputers

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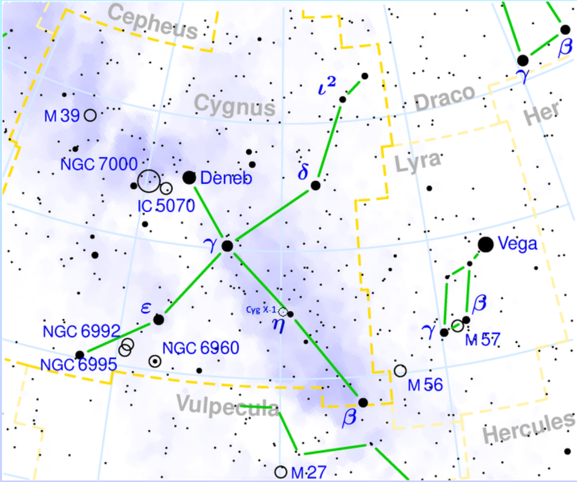
Overview

- Motivation
- Modeling black holes in GR
- Black holes in astrophysics
- Black holes in GW physics
- Trans-Planckian scattering
- AdS/CFT, Cosmic Censorship, BH instabilities
- Summary

1. Motivation

Black holes are out there: Stellar BHs

- high-mass X-ray binaries: Cygnus X-1 (1964)

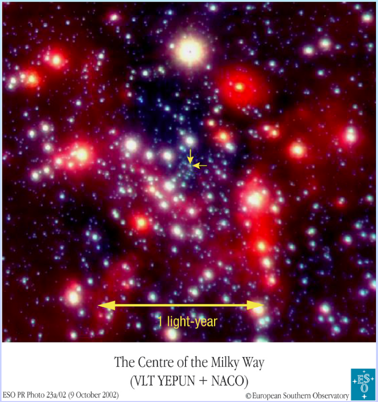


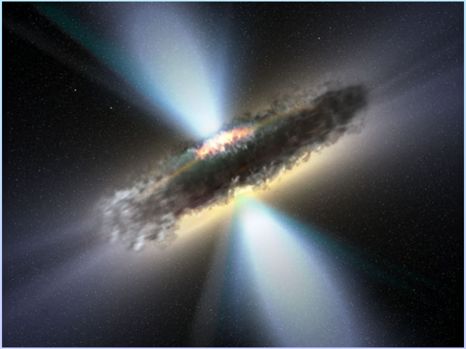
Black holes are out there: Stellar BHs

- One member is very compact and massive ⇒ Black Hole

Black holes are out there: galactic BHs

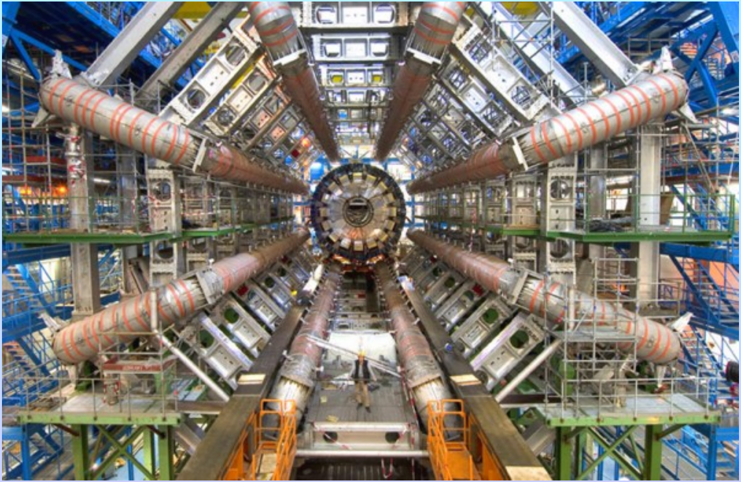
- Supermassive BHs found at center of virtually all galaxies
- SMBHs conjectured to be responsible for quasars starting in the 1980s





Black holes might be in here: LHC

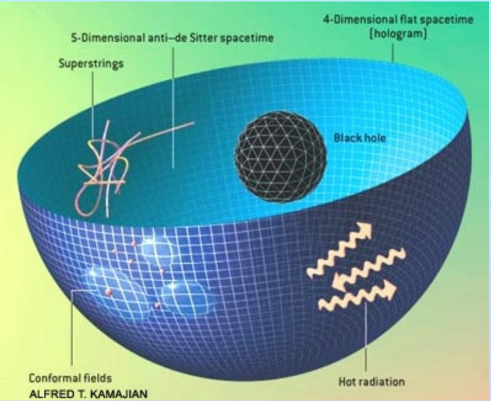
- LHC CERN



Motivation (AdS/CFT correspondence)

- BH spacetimes “know” about physics without BHs

  AdS/CFT correspondence   Maldacena ’97



2. Modeling black holes in GR

General Relativity: Curvature

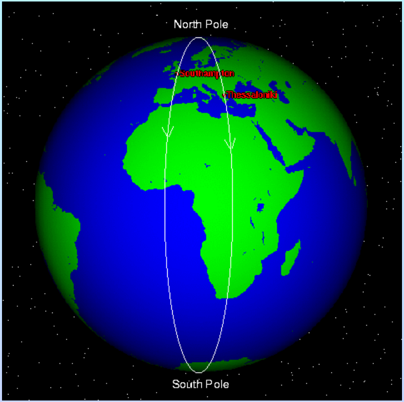
- Curvature generates acceleration

  "geodesic deviation"

  No "force"!!

- Description of geometry

Metric	$g_{\alpha\beta}$
Connection	$\Gamma^{\alpha}_{\beta\gamma}$
Riemann Tensor	$R^{\alpha}{}_{\beta\gamma\delta}$



How to get the metric?

Train cemetery
Uyuni, Bolivia

- Solve for the metric $g_{\alpha\beta}$

How to get the metric?

- The metric must obey the Einstein Equations
- Ricci-Tensor, Einstein Tensor, Matter Tensor

  $R_{\alpha\beta} \equiv R^{\mu}{}_{\alpha\mu\beta}$

  $G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R^{\mu}{}_{\mu}$ "Trace reversed" Ricci

  $T_{\alpha\beta}$ "Matter"
- Einstein Equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$
- Solutions: Easy! Take metric

  $\Rightarrow$ Calculate $G_{\alpha\beta}$

  $\Rightarrow$ Use that as matter tensor
- Physically meaningful solutions: Difficult! $\Rightarrow$ Numerics!

A set of tasks

- To get a time evolution pf BBHs in GR
- Einstein equations: 1) Canonical ADM "3+1" split
  2) Formulation: BSSN, GHG
  3) Discretization: differencing, spectral
- Gauge: moving punctures, generalize harmonic gauge
- 1) Mesh refinement: Carpet, Paramesh, SAMRAI,...
  2) Singularities: moving puncturs, excision
  3) Parallelization: MPI, OpenMP,...
- Initial data: York-Lichnerowicz conformal split, Bowen-York
- Run duration: days, weeks, months
- Diagnostics: Newman-Penrose, Pert.Theory, Horizons, ADM

Free parameters of BH binaries

- Total mass M

  Relevant for GW detection: Frequencies scale with M

  Not relevant for source modeling: trivial rescaling

- Mass ratio $q \equiv \frac{M_1}{M_2}$, $\eta \equiv \frac{M_1 M_2}{(M_1+M_2)^2}$

- Spin: $\vec{S}_1$, $\vec{S}_2$ (6 parameters)

- Initial parameters

  Binding energy E_b Separation

  Orbital ang. momentum L Eccentricity

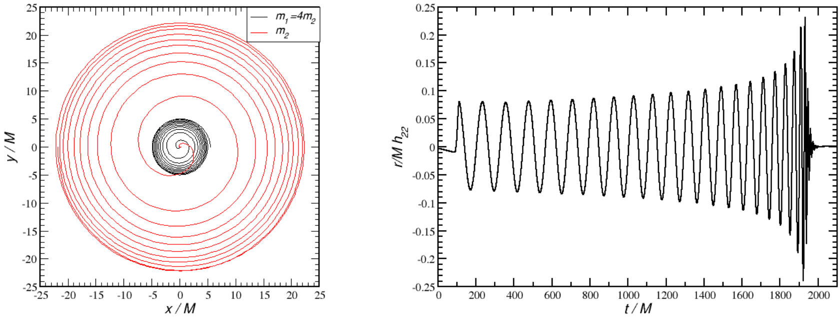
  Alternatively: frequency, eccentricity

BBH trajectory and waveform

- $q = 4$, non-spinning binary; ~ 11 orbits

  US, Brügmann, Müller & Sopuerta '11

Trajectory | Quadrupole mode



3. Black holes in astrophysics

Gravitational recoil

- Anisotropic GW emission $\Rightarrow$ recoil of remnant BH

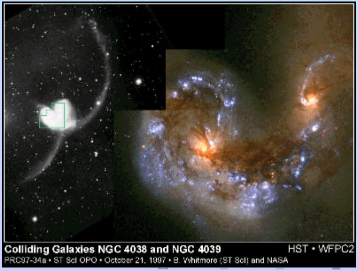
  Bonnor & Rotenburg '61, Peres '62, Bekenstein '73

- Escape velocities:

Globular clusters	30 $\mathrm{km/s}$
dSph	$20-100$ $\mathrm{km/s}$
dE	$100-300$ $\mathrm{km/s}$
Giant galaxies	~ 1000 $\mathrm{km/s}$

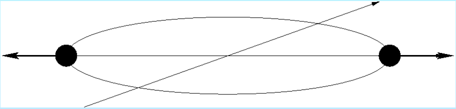
Ejection / displacement of BH $\Rightarrow$

- Growth history of SMBHs
- BH populations, IMBHs
- Structure of galaxies



Superkicks

- Kidder '95, UTB-RIT '07: maximum kick expected for



- Measured kicks $v \approx 2500\ \mathrm{km/s}$ for spin $a \approx 0.75$

  Extrapolated to maximal spins: $v_{\mathrm{max}} \approx 4000\ \mathrm{km/s}$

  González *et al.* '07, Campanelli *et al.* '07

- Unlikely configuration! Kick suppression **S** **L** alignment

  Bogdanović *et al.* '07, Kesden, US & Berti '10, '10a

- "Hang-up" kicks: v up to $5\,000\ \mathrm{km/s}$; Suppressed?

  Lousto & Zlochower '11

Spin precession and flip

- X-shaped radio sources

  Merrit & Ekers ’07

- Jet along spin axis

- Spin re-alignment

  $\Rightarrow$ new + old jet

- Spin precession 98°

  Spin flip 71°

  UTB-RIT ’06

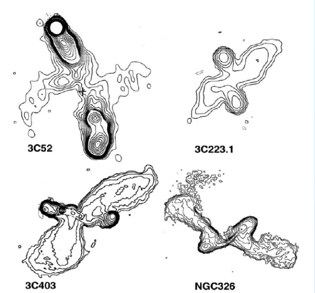
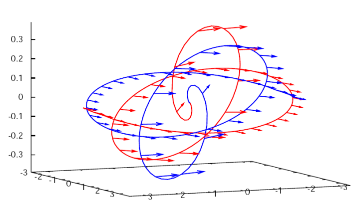


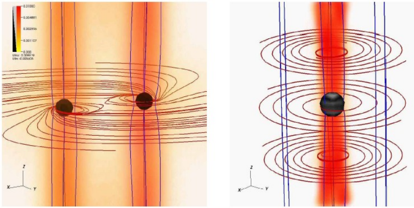
Fig. 2. A composite of four clear examples of the X-shape morphology. VLA radio observations of 3C52 (28), 3C223.1 (29), 3C403 (29), and NGC 326 (12).



Jets generated by binary BHs

Palenzuela, Lehner & Liebling ’10

- Blanford-Znajek for non-spinning BH binary
- Einstein-Maxwell equtions with “force free” plasma
- Electromagnetic field extracts energy from **L** $\Rightarrow$ jets
- Optical signature: double jets



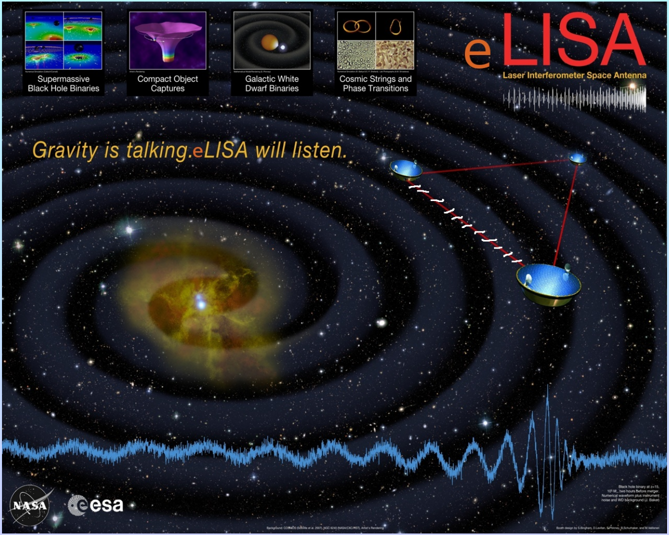
(a) $-8.2\, M_8$ hrs (b) $4.6\, M_8$ hrs

4. Black holes in GW physics

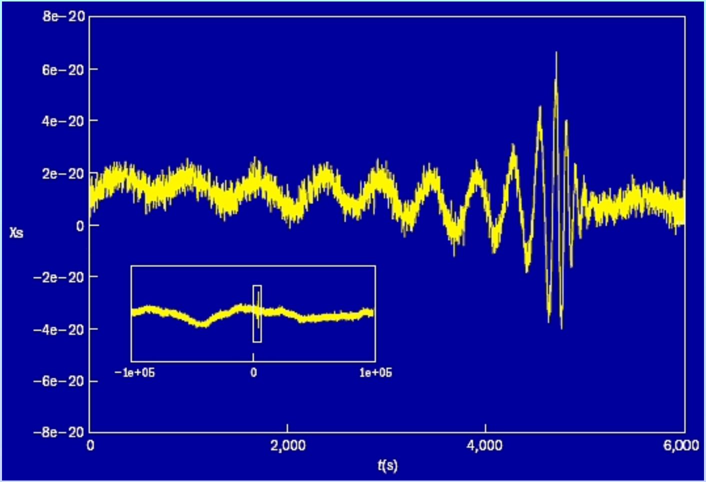
Gravitational Wave observations

- Accelerated masses generate GWs
- Interaction with matter very weak!
- Earth bound detectors: LIGO, VIRGO, GEO600, LCGT

Space interferometer LISA



Matched filtering



Long, accurate waveforms required

⇒ combine NR with PN, perturbation theory

GW data analysis

- Wave strain $h \equiv h_+ - ih_\times = \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' \Psi_4$

  Reisswig & Pollney '11

- Inner product $\langle h, g \rangle \equiv 4 Re \int_0^\infty \frac{\bar{h}(f) \bar{g}^*(f)}{S_N(f)} df$

  Finn & Chernoff '93, Cutler & Flanagan '94

- SNR $\rho_m = \frac{\langle h_e, h_m \rangle}{||h_m||}$

- Mismatch $\rho_m = (1 - \mathcal{M}) \frac{\langle h_e, h_e \rangle}{||h_e||}$

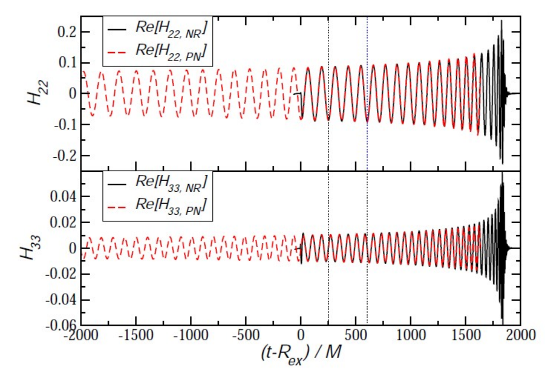
- Loss of sources $\sim 3\mathcal{M}$ %

- Accuracy requirements $\frac{||\delta h||}{||h||} < \begin{cases} 1/\rho & \text{for parameter estimation,} \\ \sqrt{2\mathcal{M}_{\max}} & \text{for detection.} \end{cases}$

  Lindblom *et al.* '10

Template construction

- Stitch together PN and NR waveforms
- EOB or phenomenological templates for $\geq$ 7-dim. par. space



- Community wide Ninja2 and NRAR projects; cf. talk by Husa

Accuracy requirements on numerical simulations

- Errors dominated by PN contributions $\Leftrightarrow$ Too few NR orbits

  Hannam *et al.* '11

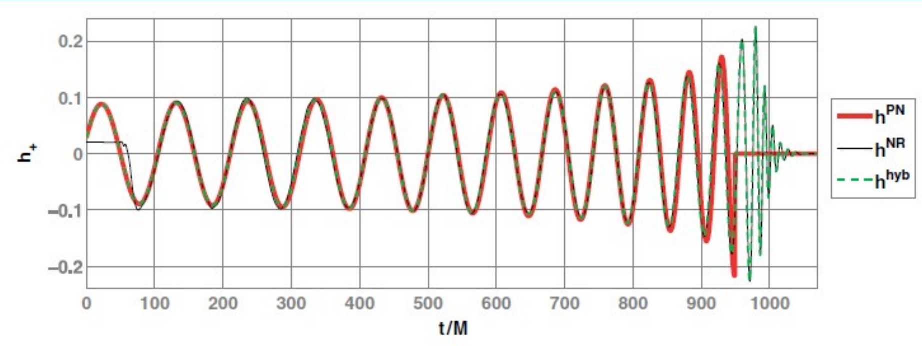
- Details depend on
  - Acceptable $\mathcal{M}$
  - Binary parameters
  - Purpose (detection parameter estimation)
  - Detector

- Predicted range   several to > 30 orbits

  Hannam *et al.*'10, Macdonald *et al.*'11, Ohme *et al.*'11, Lovelace *et al.* '12

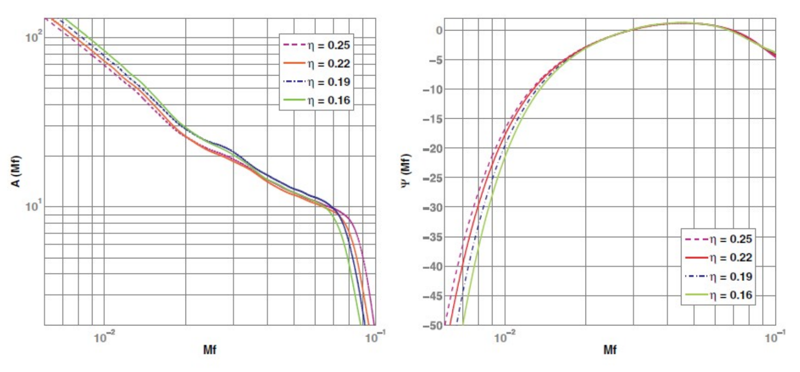
Phenomenological waveform templates

Non-spinning BHBs from Ajith *et al.* ’07



Waveform in the Fourier domain

$$h(f) = \mathcal{A}(f)e^{i\Psi(f)}$$



The template bank

$$\mathcal{A}_{eff}(f) = \begin{cases} (f/f_{mer})^{-7/6} & \textit{if} \quad f < f_{merg} \\ (f/f_{mer})^{-2/3} & \textit{if} \quad f_{merg} \le f < f_{ring} \\ (f \times \mathcal{L}(f, f_{ring}, \sigma) & \textit{if} \quad f_{ring} \le f < f_{cut} \end{cases}$$

$$\mathcal{L}(f, f_{ring}, \sigma) = \left(\frac{1}{2\pi}\right) \frac{\sigma}{(f - f_{ring})^2 + \sigma^2/4}$$

$$\Psi_{eff}(f) = 2\pi f t_0 + \phi_0 + \psi_0 f^{-5/3} + \psi_2 f^{-1} + \psi_3 f^{-2/3} + \psi_4 f^{-1/3} + \psi_6 f^{1/3}$$

Free parameters: $\{f_{merg}, f_{ring}, f_{cut}, \sigma\}, \quad \{\psi_0, \psi_2, \psi_3, \psi_4, \psi_6\}$

Create map with physical parameters $\{M, \eta\}$

- Non-spinning binaries:

  Ajith *et al.* '07, Ajith '08, Ajith *et al.* '08

- Subsets of spinning binaries:

  Ajith *et al.* '09, Santamaria *et al.* '10, Sturani *et al.* '10

Effective One Body templates

EOB method Buonanno & Damour ’99, ’00

- Map GR two body problem into particle motion in effective metric
- Components of effective metric calculated to 3PN order
- Improve model by adding pseudo PN terms of higher order
  (to be derived from NR)
- Further improvements: resum PN, model non-adiabatic effects
  e.g. Damour ’10
- Match inspiral-plunge waveform to merger-ringdown

EOB construction and comparison with NR

- Non-spinning binaries

  Buonanno *et al.*'07, '09, Damour *et al.*'07a, '07b, 08,

- Non-precessing, spinning binaries

  Pan *et al.*'09, '11, Taracchini *et al.*'12

- Comparison between EOB and phenom. models

  Damour, Trias & Nagar '11

  - Use EOB as reference, phenom. as model
  - OK for detection with initial detectors
  - Problems for advanced detectors, parameter estimation
  - Phenom. models do not use exact PN in early inspiral

- Improved models under construction

“Extreme” binary configurations

- Mass ratio 1 : 100

  Lousto & Zlochower ’10, Nakano *et al.*’11

  Calculate perturbative waveforms from NR trajectories

- Nearly extremal spins

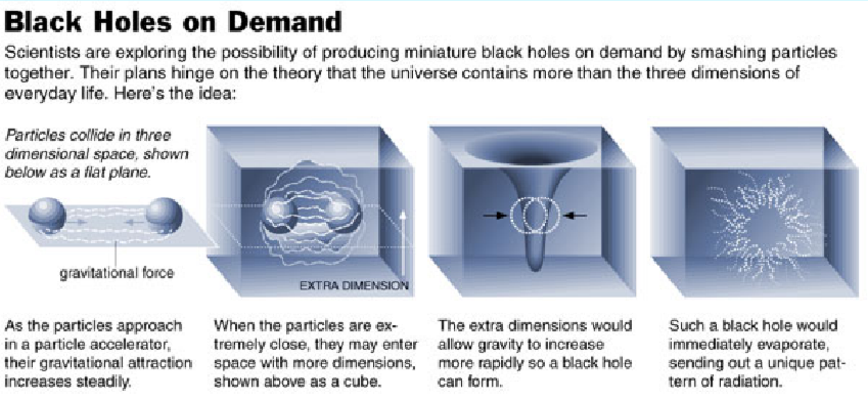
  Lovelac *et al.*’08, ’11, ’12

  - $E_{rad} = 10.952\ \%\ M$
  - Spin evolution, AH area agree well with Alvi ’01
  - 25.5 orbits insufficient for par. estimation in low-mass binaries

5. Transplanckian scattering

Motivation (High-energy physics)

TeV gravity: Arkani-Hamed, Dimopoulos & Dvali '98; Randal & Sundrum '99



- Identify jet multiplicity, transverse energy
- Requires BH mass, spin, cross section

Black-hole collisions in $D = 4$

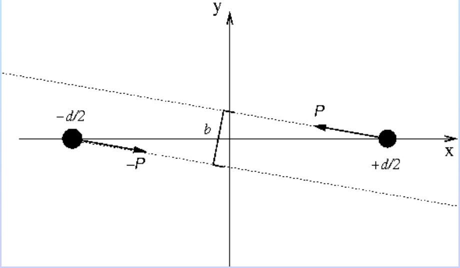
- Take two black holes

  Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

  Initial position: $\pm x_0$

  Linear momentum: $\mp P[\cos\alpha,\ \sin\alpha,\ 0]$

- Impact parameter: $b \equiv \frac{L}{P}$



$b \neq 0$: Critical impact parameter

- $b < b_{\rm crit} \quad \Rightarrow \quad$ Merger

  $b > b_{\rm crit} \quad \Rightarrow \quad$ Scattering

- Numerical study: $b_{\rm crit} = \frac{2.5 \pm 0.05}{v} M$

  Shibata *et al.* '08

- Independent study by Sperhake *et al.* '09

  $\gamma = 1.52$: $3.39 < b_{\rm crit}/M < 3.4$

  $\gamma = 2.93$: $2.3 < b_{\rm crit}/M < 2.4$

  $v \to 1$ limit still needs to be determined

- Enormous GW energie: $\sim 35\%$ M
- Go to $D \geq 5$: Dimensional reduction

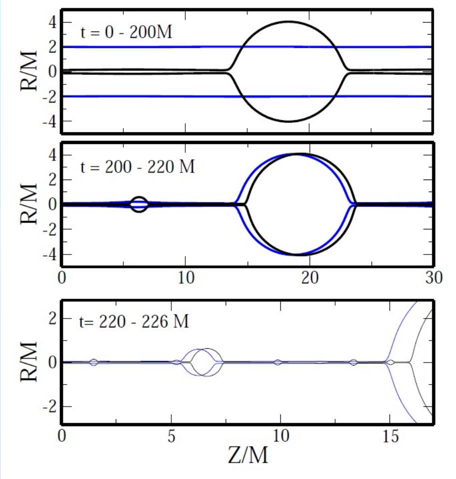
6. AdS/CFT, BH stability, Censorship

Cosmic censorship

Pretorius & Lehner '10

- $D = 5$ Axisymmetric code
- Study evolution of black string...
- Gregory-Laflamme instability cascades down

  until string reaches zero radius

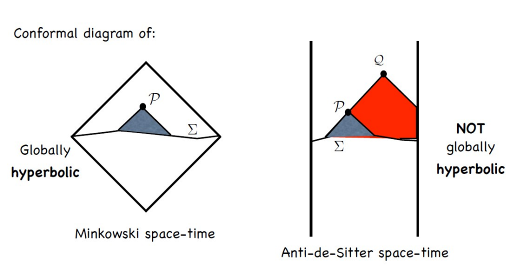


Gauge-gravity duality: AdS/CFT

- Model strongly coupled gauge theories via $D+1$ gravity

  E. g. quark-gluon plasma, isotropization, hydrodynamics.

- Challenge: Model active role of boundary



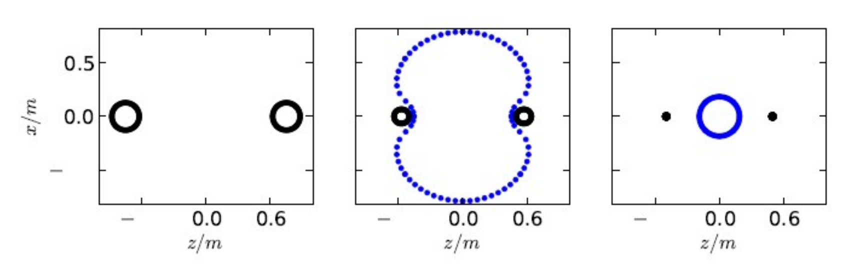
- First numerical studies

  Chesler & Jaffe ’09, ’11, Bantilan *et al.* *’12*

Black holes in de Sitter

- Two parameters: MH, d
- Initial data: McVittie type binaries McVittie '33
- "Small BHs": $d < d_{crit} \Rightarrow$ merger

  $d > d_{crit} \Rightarrow$ no common AH
- "Large" holes at small d: Cosmic Censorship holds



Summary

- Black holes are real objects in many areas of physics!
- Astrophysics: Recoil, Spin flips, jets
- GW physics:
  - Template banks: phenom.models, EOB
  - Accuracy requirements may be high
  - High spins, mass ratios explored
- Further applications of NR:
  - TeV gravity scenarios
  - Cosmic censorship
  - AdS/CFT correspondence