Black hole collisions and gravitational waves

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Ole Miss Physics Seminar
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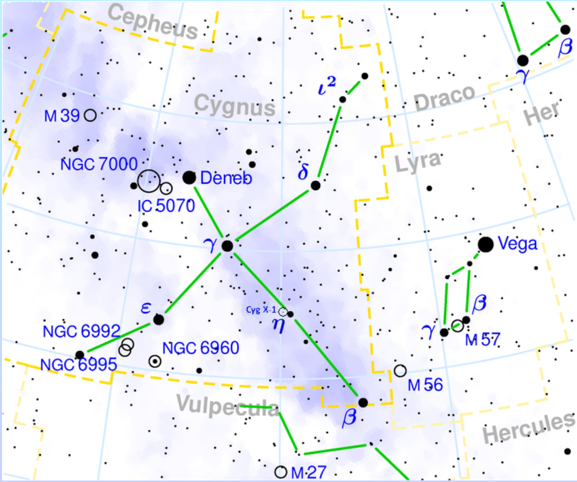
Overview

- Motivation
- Modeling black holes in GR
- Black holes in astrophysics
- Black holes in fundamental physics
  - Trans Planckian scattering
  - Non-assymptotically flat boundaries: AdS/CFT
- Other topics in $D \geq 5$
  - Instabilities of Myers-Perry BHs
  - Cosmic censorship in $D \geq 5$
- Summary

1. Motivation

Black holes are out there: Stellar BHs

- high-mass X-ray binaries: Cygnus X-1 (1964)

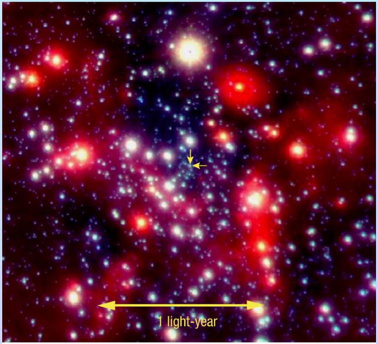


Black holes are out there: Stellar BHs

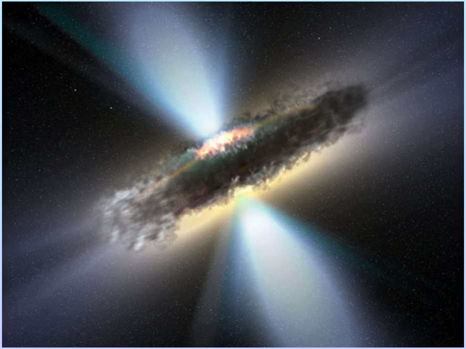
- One member is very compact and massive $\Rightarrow$ Black Hole

Black holes are out there: galactic BHs

- Supermassive BHs found at center of virtually all galaxies
- SMBHs conjectured to be responsible for quasars starting in the 1980s

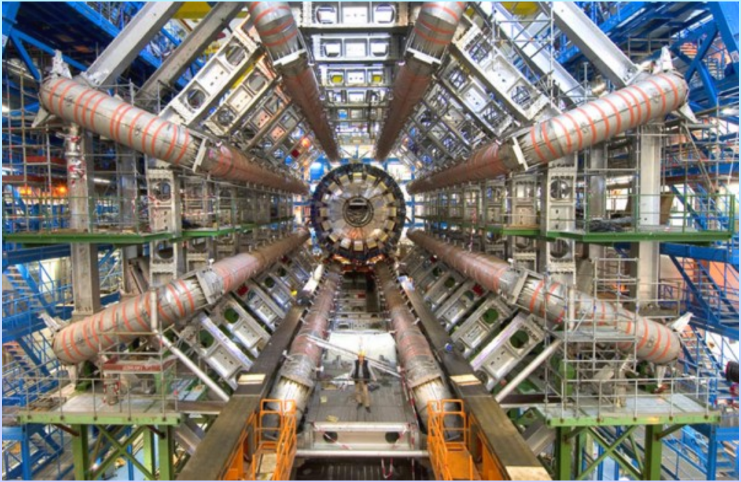


The Centre of the Milky Way
(VLT YEPUN + NACO)
ESO PR Photo 23a/02 (9 October 2002)



Black holes might be in here: LHC

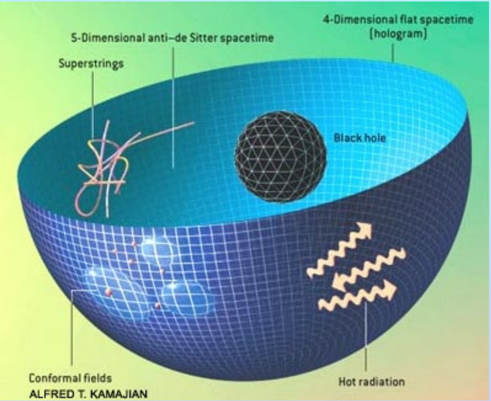
- LHC CERN



Motivation (AdS/CFT correspondence)

- BH spacetimes “know” about physics without BHs

  AdS/CFT correspondence Maldacena ’97



2. Modeling black holes in GR

How to get the metric?

Train cemetery
Uyuni, Bolivia

- Solve for the metric $g_{\alpha\beta}$

How to get the metric?

- The metric must obey the Einstein Equations

- Ricci-Tensor, Einstein Tensor, Matter Tensor

  $R_{\alpha\beta} \equiv R^{\mu}{}_{\alpha\mu\beta}$

  $G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R^{\mu}{}_{\mu}$ “Trace reversed” Ricci

  $T_{\alpha\beta}$ “Matter”

- Einstein Equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$

- Solutions: Easy! Take metric

  $\Rightarrow$ Calculate $G_{\alpha\beta}$

  $\Rightarrow$ Use that as matter tensor

- Physically meaningful solutions: Difficult! $\Rightarrow$ Numerics!

A list of tasks

- Target: Predict time evolution of BBH in GR
- Einstein equations: 1) Cast as evolution system
  2) Choose specific formulation
  3) Discretize for computer
- Choose coordinate conditions: Gauge
- Fix technical aspects: 1) Mesh refinement / spectral domains
  2) Singularity handling / excision
  3) Parallelization
- Construct realistic initial data
- Start evolution and waaaaiiiiit...
- Extract physics from the data

3. Black holes in astrophysics

Free parameters of BH binaries

- Total mass M

  Relevant for GW detection: Frequencies scale with M

  Not relevant for source modeling: trivial rescaling

- Mass ratio $q \equiv \frac{M_1}{M_2}$, $\eta \equiv \frac{M_1 M_2}{(M_1+M_2)^2}$

- Spin: $\vec{S}_1$, $\vec{S}_2$ (6 parameters)

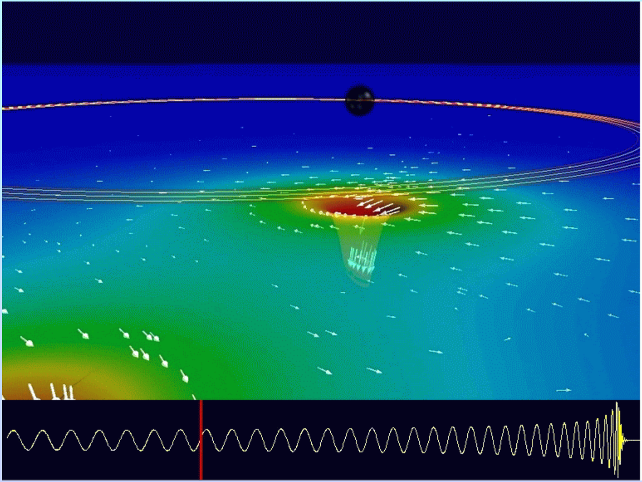
- Initial parameters

  Binding energy E_b | Separation

  Orbital ang. momentum L | Eccentricity

  Alternatively: frequency, eccentricity

Morphology of a BBH inspiral



Thanks to Caltech, CITA, Cornell

Gravitational recoil

- Anisotropic GW emission $\Rightarrow$ recoil of remnant BH

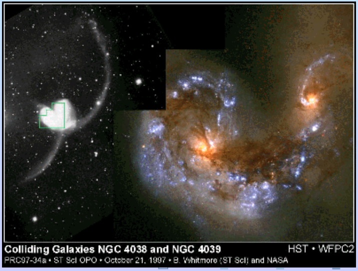
  Bonnor & Rotenburg '61,   Peres '62,   Bekenstein '73

- Escape velocities:

Globular clusters	30 $\mathrm{km/s}$
dSph	20 – 100 $\mathrm{km/s}$
dE	100 – 300 $\mathrm{km/s}$
Giant galaxies	$\sim$ 1000 $\mathrm{km/s}$

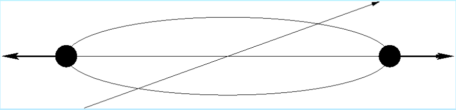
Ejection / displacement of BH $\Rightarrow$

- Growth history of SMBHs
- BH populations, IMBHs
- Structure of galaxies



Superkicks

- Kidder '95, UTB-RIT '07: maximum kick expected for



- Measured kicks $v \approx 2500\ \mathrm{km/s}$ for spin $a \approx 0.75$

  Extrapolated to maximal spins: $v_{\mathrm{max}} \approx 4000\ \mathrm{km/s}$

  González *et al.* '07,   Campanelli *et al.* '07

- Unlikely configuration!

  Bogdanović *et al.* '07,   Kesden, US & Berti '10, '10a

- Hyperbolic encounters: v up to $10000\ \mathrm{km/s}$

  Healy *et al.* '08

Spin precession and flip

- X-shaped radio sources

  Merrit & Ekers '07

- Jet along spin axis

- Spin re-alignment

  $\Rightarrow$ new + old jet

- Spin precession 98°

  Spin flip 71°

  UTB-RIT '06

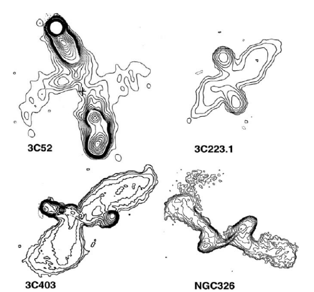
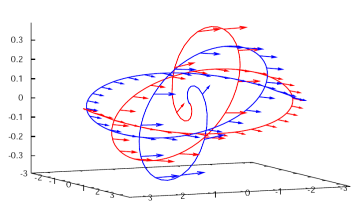


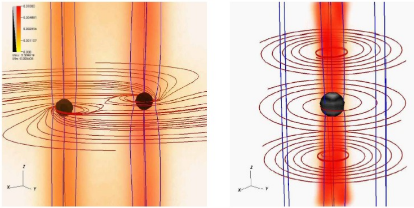
Fig. 2. A composite of four clear examples of the X-shape morphology. VLA radio observations of 3C52 (28), 3C223.1 (29), 3C403 (29), and NGC 326 (12).



Jets generated by binary BHs

Palenzuela, Lehner & Liebling ’10

- Non-spinning BH binary
- Einstein-Maxwell equtions with “force free” plasma
- Electromagnetic field extracts energy from **L** $\Rightarrow$ jets
- Optical signature: double jets

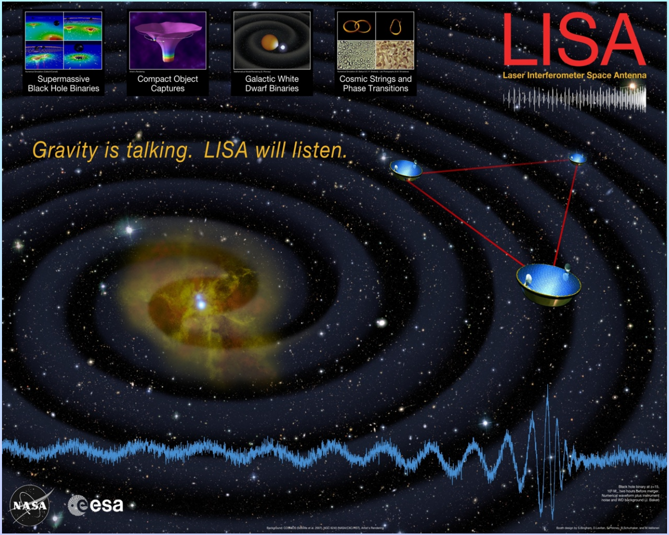


(a) $-8.2\,M_8$ hrs (b) $4.6\,M_8$ hrs

Gravitational Wave observations

- Accelerated masses generate GWs
- Interaction with matter very weak!
- Earth bound detectors: LIGO, VIRGO, GEO600, LCGT

Space interferometer LISA



Some targets of GW physics

- Confirmation of GR

  Hulse & Taylor 1993 Nobel Prize

- Parameter determination of BHs: M, $\vec{S}$

- Optical counter parts

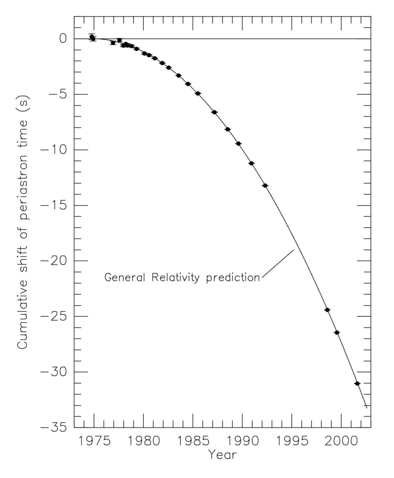
  Standard sirens (candles)

  Mass of graviton

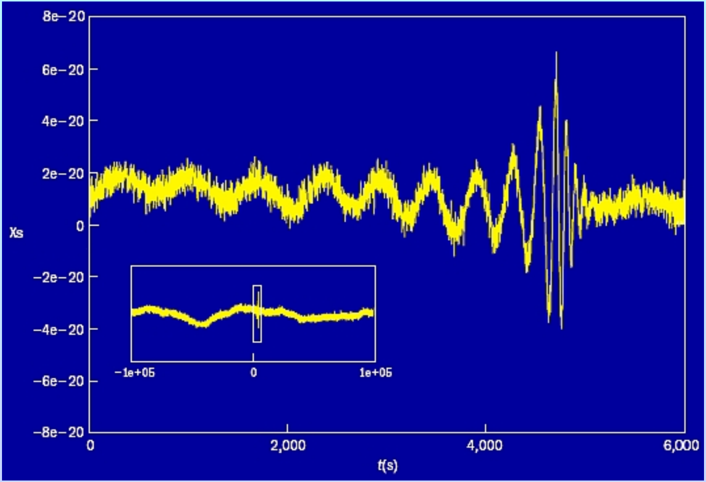
- Test Kerr Nature of BHs

- Cosmological sources

- Neutron stars: EOS



Matched filtering



Long, accurate waveforms required

$\Rightarrow$ combine NR with PN, perturbation theory

3. Black holes in fundamental physics

So what other interesting physics can we do with NR?

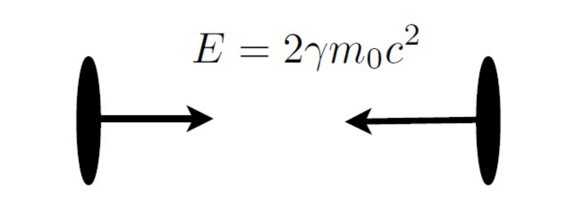
- High-energy physics
  - Trans-Planckian scattering
  - AdS/CFT duality
- Mathematical physics and theoretical physics
  - Cosmic censorship
  - Critical phenomena
  - BH instabilities (Myers-Perry)

3.1. Transplanckian scattering

BH formation and hoop conjecture

- Hoop conjecture

  Thorne '72



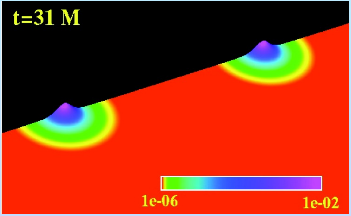
- de Broglie wavelength: $\lambda = \frac{hc}{E}$
- Schwarzschild radius: $r = \frac{2GE}{c^4}$
- BH will form if $\lambda < r \quad \Leftrightarrow \quad E \gtrsim \sqrt{\frac{hc^5}{G}} \equiv E_{\text{Planck}}$

BH formation in boson field collisions

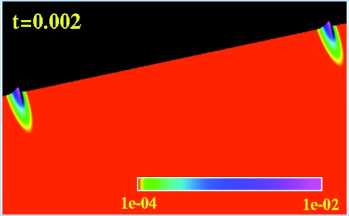
Pretorius & Choptuik '09

- Einstein plus minimally coupled, massive, complex scalar filed

  "Boson stars"



$\gamma = 1$



$\gamma = 4$

- BH formation threshold: $\gamma_{\rm thr} = 2.9 \pm 10~\%$
- About $1/3$ of hoop conjecture prediction

Motivation (High-energy physics)

- Matter does not matter at energies well above the Planck scale

  $\Rightarrow$ Model particle collisions by black-hole collisions

  Banks & Fischler '99; Giddings & Thomas '01

- TeV-gravity scenarios

  $\Rightarrow$ The Planck scale might be as low as TeVs due to extra dimensions

  Arkani-Hamed, Dimopulos & Dvali '98, Randall & Sundrum '99

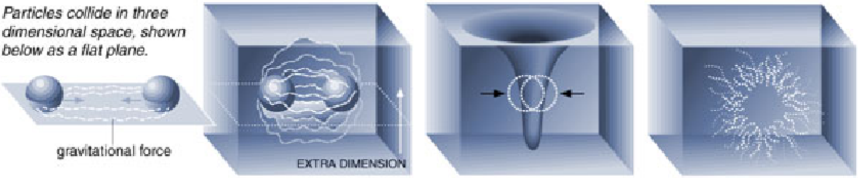
  $\Rightarrow$ Black holes could be produced in colliders

  Eardley & Giddings '02, Dimopoulos & Landsberg '01,...

Motivation (High-energy physics)

Black Holes on Demand

Scientists are exploring the possibility of producing miniature black holes on demand by smashing particles together. Their plans hinge on the theory that the universe contains more than the three dimensions of everyday life. Here's the idea:



As the particles approach in a particle accelerator, their gravitational attraction increases steadily.

When the particles are extremely close, they may enter space with more dimensions, shown above as a cube.

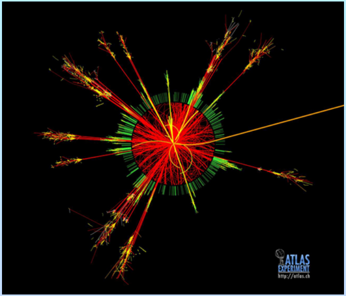
The extra dimensions would allow gravity to increase more rapidly so a black hole can form.

Such a black hole would immediately evaporate, sending out a unique pattern of radiation.

Experimental signature at the LHC

Black hole formation at the LHC could be detected by the properties of the jets resulting from Hawking radiation.

- Multiplicity of partons: Number of jets and leptons
- Large transverse energy
- Black-hole mass and spin are important for this!



ToDo:

- Exact cross section for BH formation
- Determine loss of energy in gravitational waves
- Determine spin of merged black hole

Black-hole collisions in $D = 4$

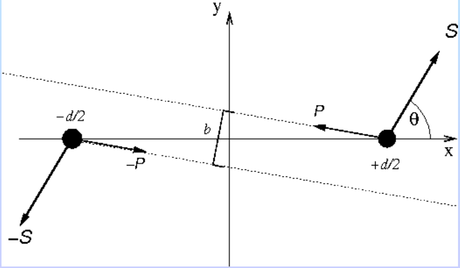
- Take two black holes

  Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

  Initial position: $\pm x_0$

  Linear momentum: $\mp P[\cos\alpha,\ \sin\alpha,\ 0]$

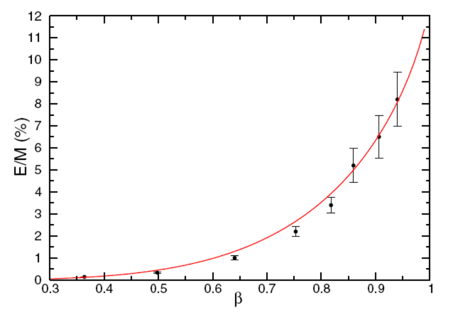
- Impact parameter: $b \equiv \frac{L}{P}$



Head-on collisions: $b = 0, \quad \vec{S} = 0$

- Total radiated energy: 14 ± 3 % for $v \to 1$ Sperhake *et al.* ’08

  About half of Penrose ’74

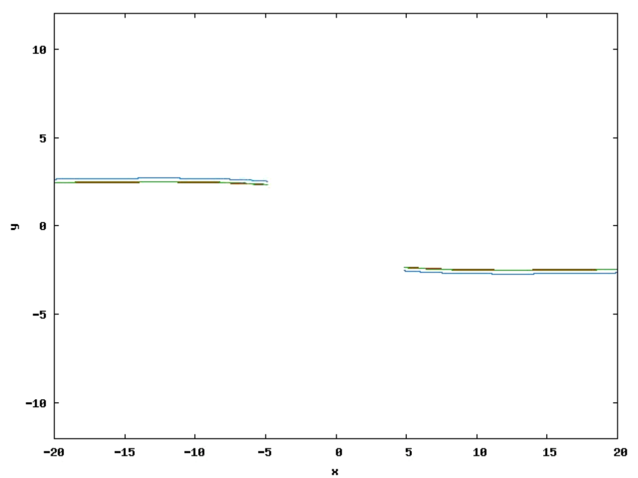


- Agreement with approximative methods

  Flat spectrum, multipolar GW structure Berti *et al.* ’10

Grazing collisions: $b \neq 0, \quad \vec{S} = 0, \quad \gamma = 1.52$

Immediate vs. Delayed vs. No merger   Sperhake *et al.* '09



Critical impact parameter

- $b < b_{\rm crit} \quad \Rightarrow \quad$ Merger

  $b > b_{\rm crit} \quad \Rightarrow \quad$ Scattering

- Numerical study: $\quad b_{\rm crit} = \frac{2.5 \pm 0.05}{v} M$

  Shibata *et al.* '08

- Independent study by Sperhake *et al.* '09

  $\gamma = 1.52$: $\quad 3.39 < b_{\rm crit}/M < 3.4$

  $\gamma = 2.93$: $\quad 2.3 < b_{\rm crit}/M < 2.4$

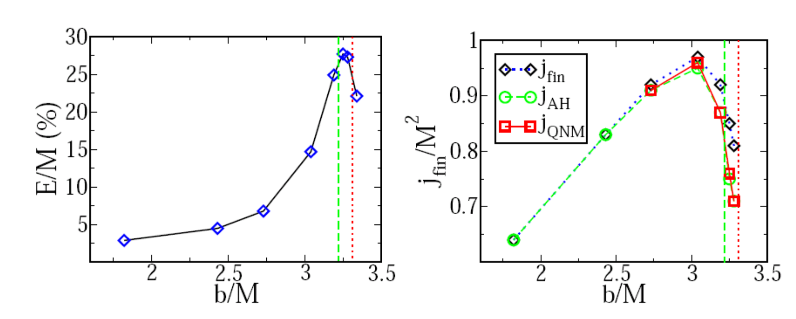
  $v \to 1$ limit still needs to be determined

- Limit from Penrose construction: $b_{\rm crit} = 1.685\ M$

  Yoshino & Rychkov '05

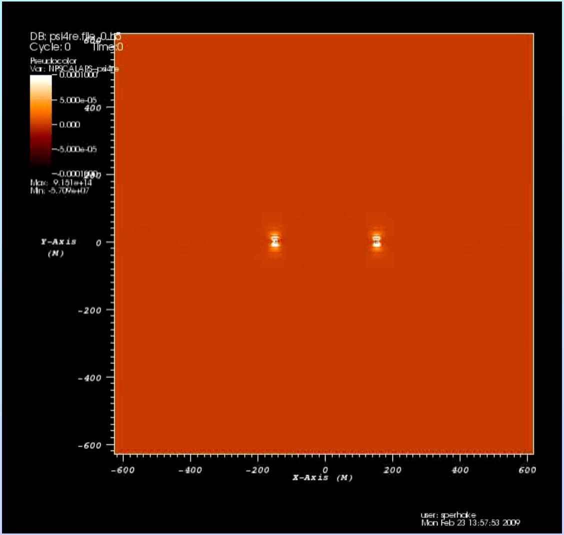
Radiated quantities

- $b-$sequence with $\gamma = 1.52$
- Final spin close to Kerr limit
- $E_{\rm rad} \sim 35$ % for $\gamma = 2.93$; about 10 % of Dyson luminosity

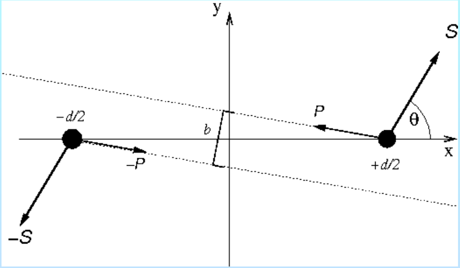


Sperhake *et al.* '09

Gravitational radiation: Delayed merger



Recoil in grazing collisions

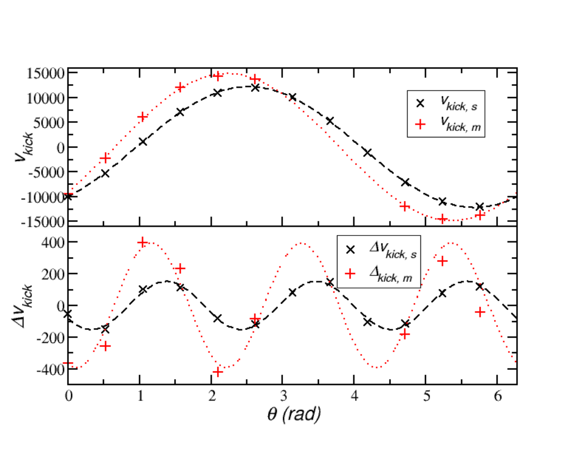


- equal-mass, superkick, $\chi = 0.621$
- $\gamma = 1.52$
- 2 sequences

  merging: $b = 3.34\ M$

  scattering: $b = 3.25\ M$

Recoil in grazing collisions



Expansion in θ according to   Boyle, Kesden & Nissanke '08

Recoil in grazing collisions

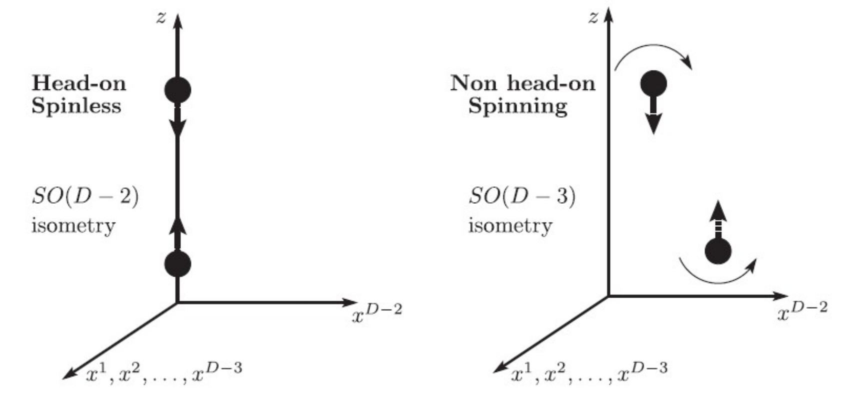
- $v_{\mathrm{max,s}} = 12\,200\ \mathrm{km/s}$
  $v_{\mathrm{max,m}} = 14\,900\ \mathrm{km/s}$
- Large recoils for merger and scattering!
- $v_{\mathrm{max}} \propto E_{\mathrm{rad}}$
- Antikicks can occur in both $\Rightarrow$ not a merger-only feature!
- Ultimate kick
  $v_{\mathrm{max}} \propto E_{\mathrm{rad}} \quad \Rightarrow \quad \sim 45\,000\ \mathrm{km/s}$
  spin insignificant for large $\gamma \quad \Rightarrow \quad \sim 25\,000\ \mathrm{km/s}$
  no simple picture $\quad \Rightarrow \quad$ more data needed...

Moving to $D > 4$

- Symmetries allow dimensional reduction

  Geroch '70

- Reduces to "3+1" plus quasi-matter terms: scalar field



BSSN formulation with quasi matter

$$\partial_t \tilde{\gamma}_{ij} = [\mathrm{BSSN}],$$

$$\partial_t \chi = [\mathrm{BSSN}],$$

$$\partial_t K = [\mathrm{BSSN}] + 4\pi\alpha(E + S),$$

$$\partial_t \tilde{A}_{ij} = [\mathrm{BSSN}] - 8\pi\alpha\left(\chi S_{ij} - \frac{1}{3} S \tilde{\gamma}_{ij}\right),$$

$$\partial_t \tilde{\Gamma}^i = [\mathrm{BSSN}] - 16\pi\alpha\chi^{-1} j^i,$$

$$\partial_t \zeta = -2\alpha K_\zeta + \beta^m \partial_m \zeta - \frac{2}{3}\zeta\partial_m \beta^m + 2\zeta\frac{\beta^y}{y},$$

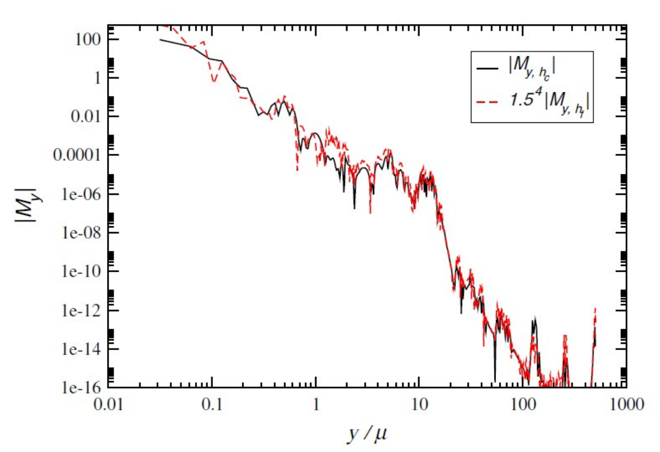
$$\partial_t K_\zeta = \ldots \quad ,$$

$$E,\ j^i,\ S_{ij} = f(\mathrm{BSSN},\ \zeta,\ K_\zeta).$$

Zilhão *et al.* '10

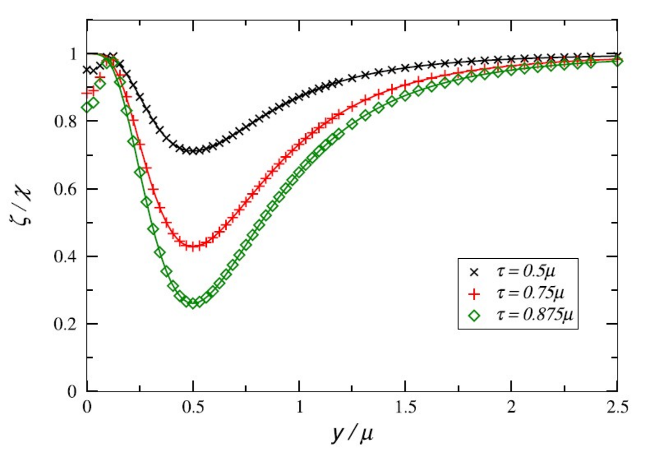
Single black hole in $D = 5$

Initial data: Tangherlini '63



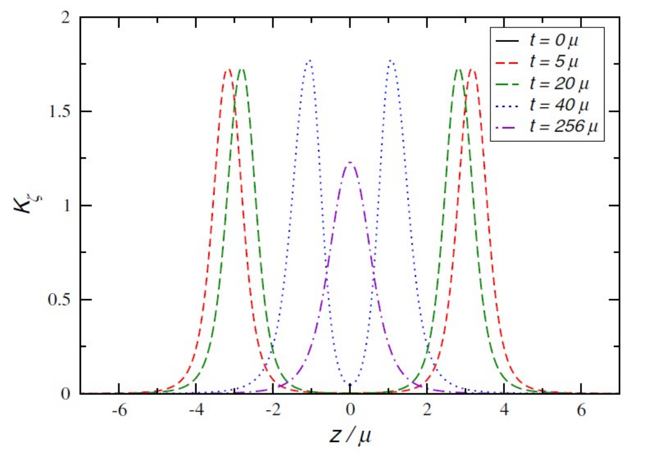
Single black hole in $D = 5$

In geodesic slicing

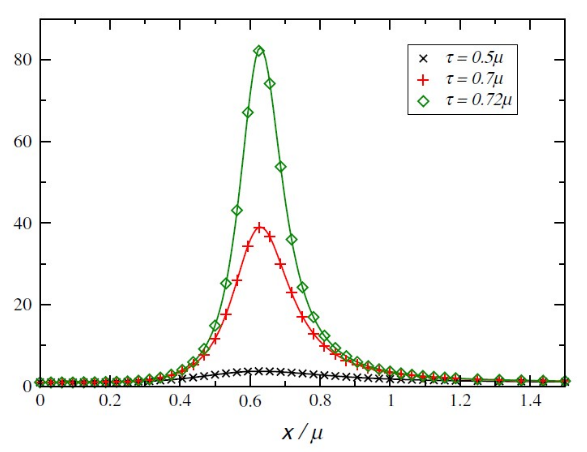


Head-on in $D = 5$

Initial data: $D = 5$ analogue of Brill-Lindquist data

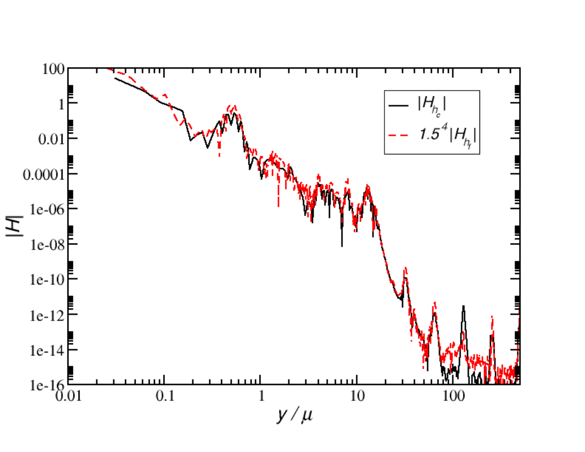


Single black hole in $D = 6$



Geoesic slicing, zero shift

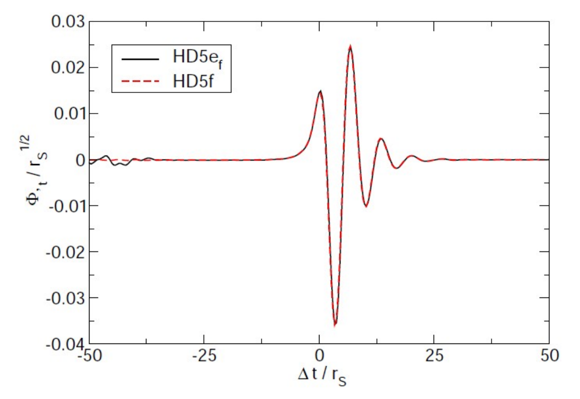
Single black hole in $D = 6$



ToDo: long term stable evolutions

GWs from head-on in $D = 5$

Wave extraction based on Kodama & Ishibashi '03

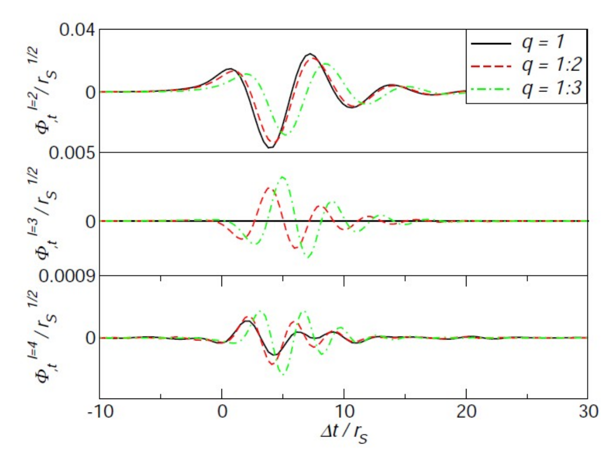


$E_{\rm rad} = 0.089\ \%M$ cf. $0.055\ \%M$ in $D = 4$

Witek *et al.* '10a

Unequal-mass head-on in $D = 5$

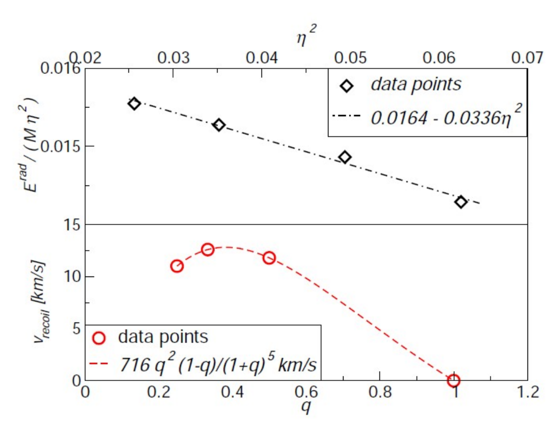
Kodama-Ishibashi multipoles



Witek *et al.* '10b

Unequal-mass head-on in $D = 5$

Radiated energy and momentum

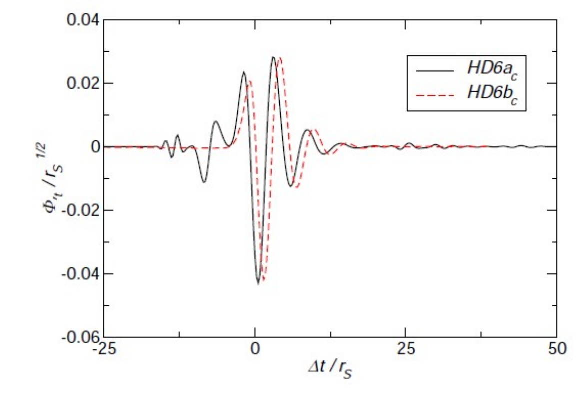


Agreement within < 5 % with extrapolated point particle calculations

Breaking news!

First black-hole collisions in $D = 6$

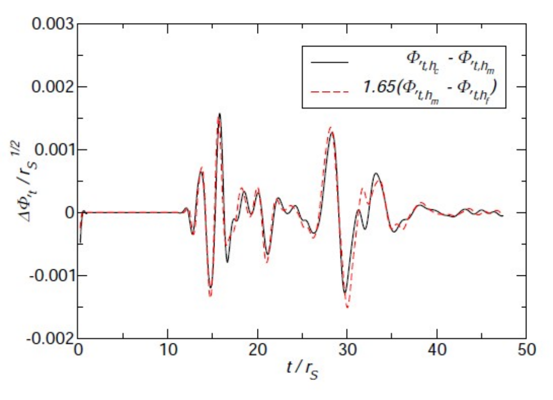
Witek *et al. '10*



- Adjust shift parameters
- Use LaSh system Witek, Hilditch & US '10

First black-hole collisions in $D = 6$

Witek *et al. '10*

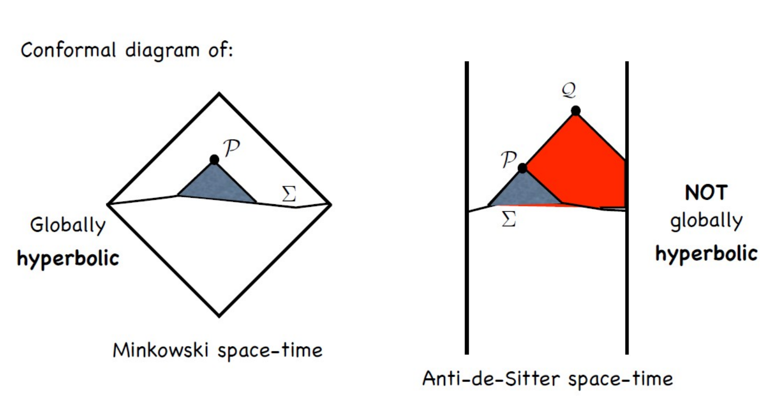


Second order convergence

3.2. Non-assymptotically flat boundaries: AdS/CFT

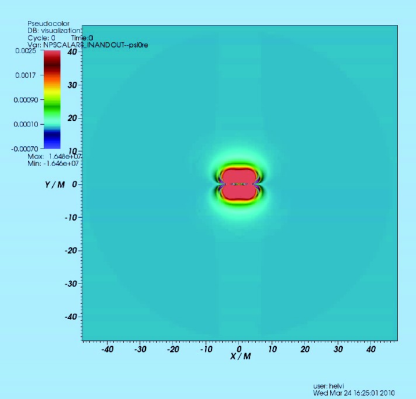
AdS/CFT correspondence

Challenge: Model the active role of the boundary!



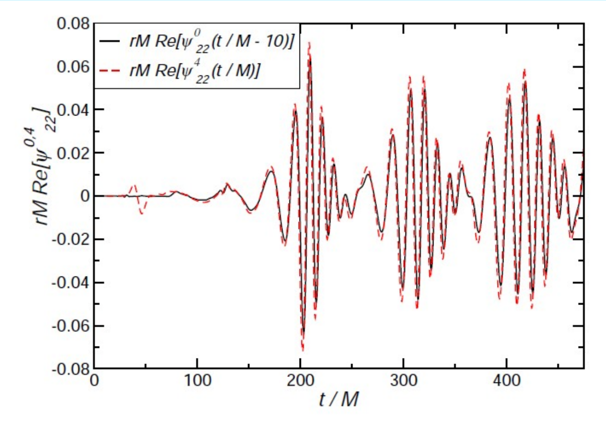
Toy model: Black hole inspiral in a lego sphere

- Lego sphere with reflective boundary
- Goddard R1 run   Baker *et al.* '06
- Calculate Ψ_4 and Ψ_0

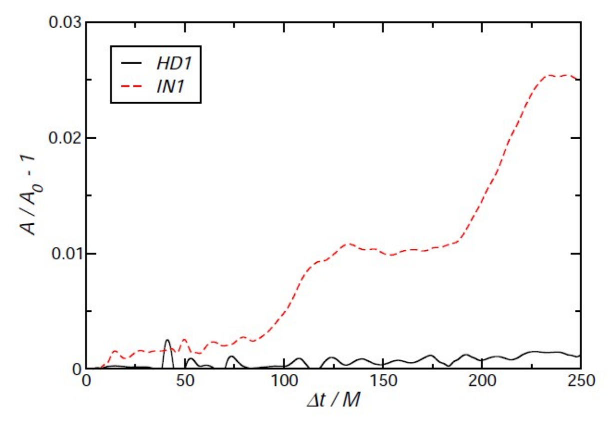


Quadrupole mode

Gravitational radiation (out going and ingoing)



Horizon area



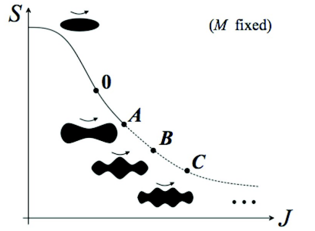
Superradiance: high frequency absorbed, low frequency amplified
No conclusive evidence yet...

4. Other topics in $D \geq 5$

Other topics: Instabilities of Myers-Perry

- Ultra-spinning Myers-Perry black holes (with single angular momentum parameter) should be unstable.
- Confirmed by linearized analysis of axisymmetric perturbations

Dias *et al.* ’09



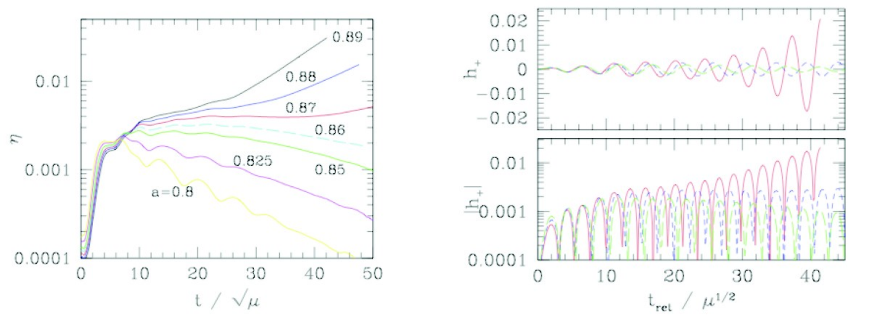
d	$(a/r_m)\|_{\ell=1}$	$(a/r_m)\|_{\ell=2}$	$(a/r_m)\|_{\ell=3}$
7	1.075	1.714	2.141
8	1.061	1.770	2.275
9	1.051	1.792	2.337

Other topics: Instabilities of Myers-Perry

- Numerical study of non-axisymmetric instabilities of $D = 5$ Myers-Perry BH with single ang. momentum parameter.

  Shibata & Yoshino '09

- Found onset of instabilities at spin $a/\sqrt{\mu} \approx 0.87$



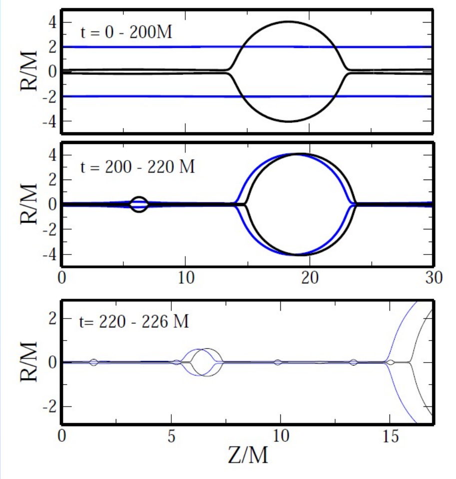
Other topics: Cosmic censorship in $D = 5$

Pretorius & Lehner '10

- Axisymmetric code
- Study evolution of black string...
- Gregory-Laflamme instability cascades down

  until string reaches zero radius

  $\Rightarrow$ naked singularity



5. Summary

Summary

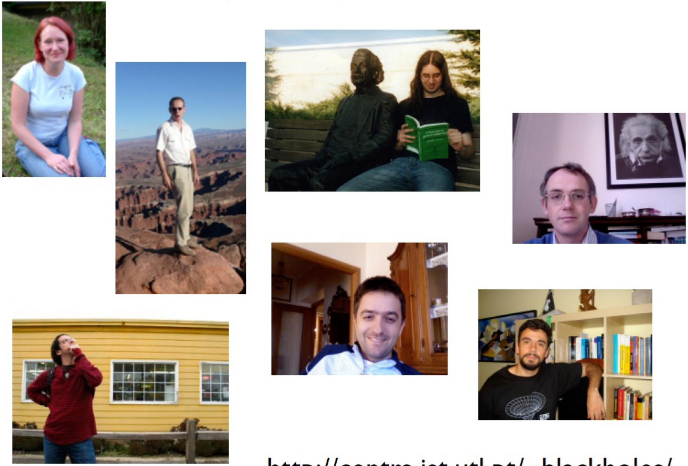
- Black holes are real objects in many areas of physics!
- Astrophysics: Recoil, Spin flips, jets
- Gravitational wave physics: template banks needed
- High-energy collisions in $D = 4$:

  largest kicks $\sim 15\,000$ km/s

  largest radiation ~ 30 %

  largest post-merger spin $a \lesssim 1$
- Formalism for arbitrary spatial dimension D
- Head-on collisions from rest
- Test non-assymptotically flat OBCs
- Signs of cosmic censorship violation in $D = 5$

The team



http://centra.ist.utl.pt/~blackholes/