

# Colliding black holes in 3+1 and higher dimensions

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Pizza Lunch Seminar, CSIC-IEEC  
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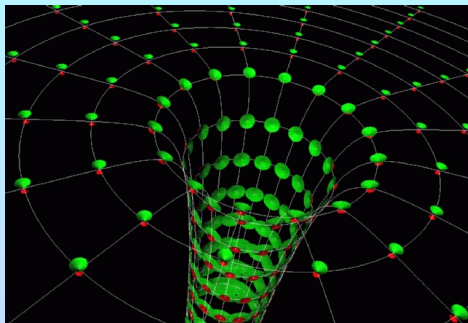
# Overview

- Motivation
- Modeling black holes in GR
- Black holes in astrophysics
- Black holes in fundamental physics
  - Trans Planckian scattering
  - Non-asymptotically flat boundaries: AdS/CFT
- Other topics in  $D \geq 5$ 
  - Instabilities of Myers-Perry BHs
  - Cosmic censorship in  $D \geq 5$
- Summary

# 1. Motivation

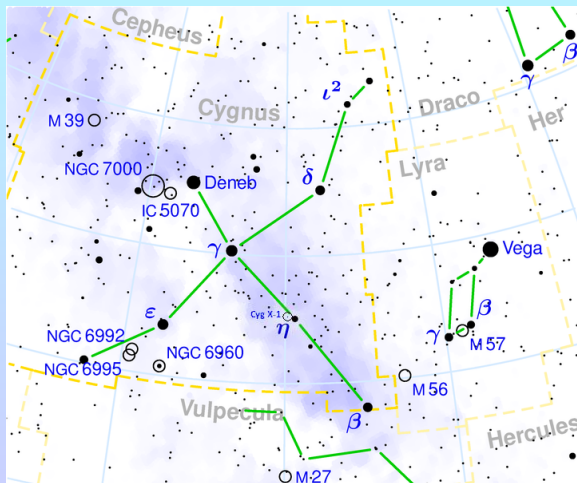
# What are black holes?

- Consider Lightcones
- In and outgoing light
- Calculate surface of outgoing light fronts
- Expansion  $\equiv$  Rate of change of this surface
- Apparent Horizon  $\equiv$  Outermost surface with zero expansion
- “Light cones tip over” due to curvature



# Black holes are out there: Stellar BHs

- high-mass X-ray binaries: Cygnus X-1 (1964)



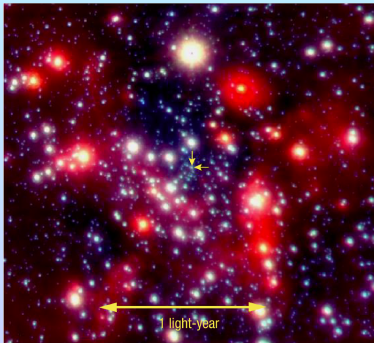
# Black holes are out there: Stellar BHs

- One member is very compact and massive  $\Rightarrow$  Black Hole



# Black holes are out there: galactic BHs

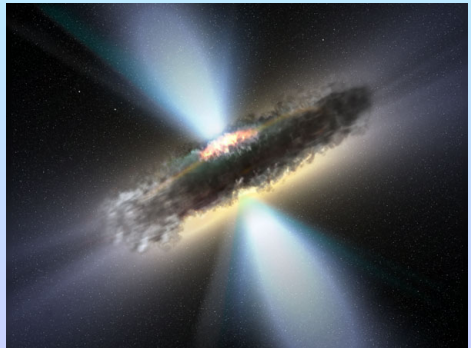
- Supermassive BHs found at center of virtually all galaxies
- SMBHs conjectured to be responsible for quasars starting in the 1980s



The Centre of the Milky Way  
(VLT YEPUN + NACO)

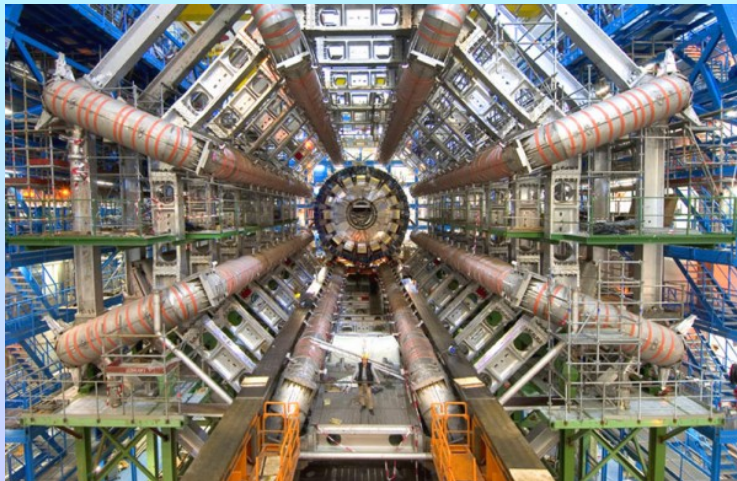
ESO PR Photo 23a/02 (9 October 2002)

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# Black holes might be in here: LHC

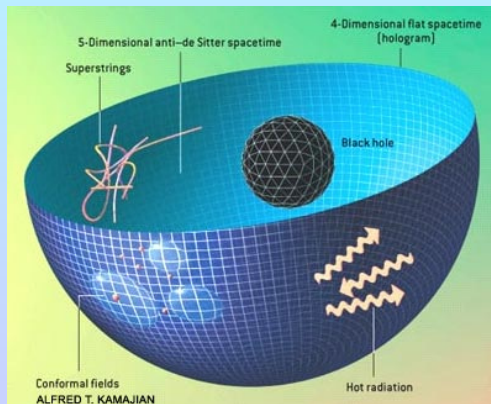
- LHC CERN





# Motivation (AdS/CFT correspondence)

- BH spacetimes “know” about physics without BHs  
AdS/CFT correspondence Maldacena '97



## 2. Modeling black holes in GR

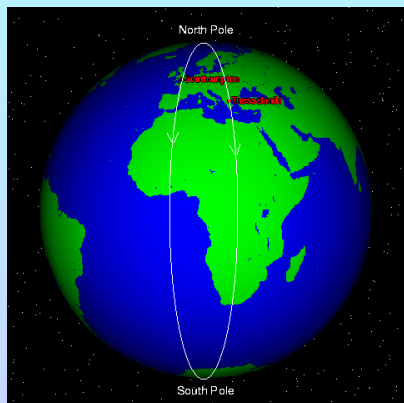
# General Relativity: Curvature

- Curvature generates acceleration  
“geodesic deviation”  
No “force”!!
- Description of geometry

Metric  $g_{\alpha\beta}$

Connection  $\Gamma_{\beta\gamma}^{\alpha}$

Riemann Tensor  $R^{\alpha}{}_{\beta\gamma\delta}$



# How to get the metric?



Train cemetery  
Uyuni, Bolivia

- Solve for the metric  $g_{\alpha\beta}$

# How to get the metric?

- The metric must obey the Einstein Equations
- Ricci-Tensor, Einstein Tensor, Matter Tensor

$$R_{\alpha\beta} \equiv R^{\mu}{}_{\alpha\mu\beta}$$

$$G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R^{\mu}{}_{\mu} \quad \text{“Trace reversed” Ricci}$$

$$T_{\alpha\beta} \quad \text{“Matter”}$$

- Einstein Equations  $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$

- Solutions: Easy!  $\Rightarrow$  Calculate  $G_{\alpha\beta}$

$\Rightarrow$  Use that as matter tensor

- Physically meaningful solutions: Difficult!

# The Einstein Equations in vacuum

- “Spacetime tells matter how to move, matter tells spacetime how to curve”
- Field equations in vacuum:  $R_{\alpha\beta} = 0$   
Second order PDEs for the metric components  
Invariant under coordinate (gauge) transformations
- System of equations extremely complex: **Pile of paper!**  
Analytic solutions: Minkowski, Schwarzschild, Kerr,  
Robertson-Walker, ...
- **Numerical methods** necessary for general scenarios!!!

# A list of tasks

- Target: Predict time evolution of BBH in GR
- Einstein equations:
  - 1) Cast as evolution system
  - 2) Choose specific formulation
  - 3) Discretize for computer
- Choose coordinate conditions: Gauge
- Fix technical aspects:
  - 1) Mesh refinement / spectral domains
  - 2) Singularity handling / excision
  - 3) Parallelization
- Construct realistic initial data
- Start evolution and waaaaiiiit...
- Extract physics from the data

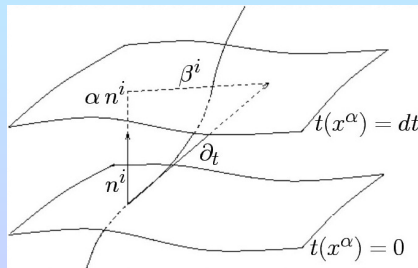
# 3+1 Decomposition

- GR: “Space and time exist as a unity: Spacetime”
- NR: ADM 3+1 split    Arnowitt, Deser & Misner '62  
York '79, Choquet-Bruhat & York '80

$$g_{\alpha\beta} = \left( \begin{array}{c|c} -\alpha^2 + \beta_m \beta^m & \beta_j \\ \hline \beta_i & \gamma_{ij} \end{array} \right)$$

- 3-Metric     $\gamma_{ij}$
- Lapse         $\alpha$
- Shift         $\beta^i$

- lapse, shift  $\Rightarrow$  Gauge





# ADM Equations

The Einstein equations  $R_{\alpha\beta} = 0$  become

- 6 Evolution equations

$$(\partial_t - \mathcal{L}_\beta)\gamma_{ij} = -2\alpha K_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)K_{ij} = -D_i D_j \alpha + \alpha [R_{ij} - 2K_{im} K^m_j + K_{ij} K]$$

- 4 Constraints

$$R + K^2 - K_{ij} K^{ij} = 0$$

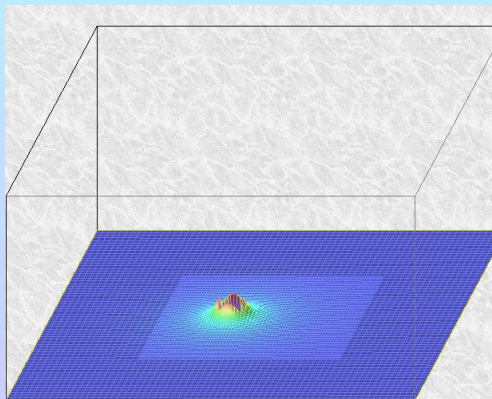
$$-D_j K^{ij} + D^i K = 0$$

preserved under evolution!

- Evolution

1) Solve constraints

2) Evolve data



# Formulations I: BSSN

- One can easily change variables. E. g. wave equation

$$\begin{aligned} \partial_{tt}u - c\partial_{xx}u = 0 & \quad \Leftrightarrow \quad \partial_t F - c\partial_x G = 0 \\ & \quad \quad \quad \partial_x F - \partial_t G = 0 \end{aligned}$$

- **BSSN**: rearrange degrees of freedom

$$\begin{aligned} \chi &= (\det \gamma)^{-1/3} & \tilde{\gamma}_{ij} &= \chi \gamma_{ij} \\ K &= \gamma_{ij} K^{ij} & \tilde{A} &= \chi (K_{ij} - \frac{1}{3} \gamma_{ij} K) \\ \tilde{\Gamma}^i &= \tilde{\gamma}^{mn} \tilde{\Gamma}_{mn}^i = -\partial_m \tilde{\gamma}^{im} \end{aligned}$$

Shibata & Nakamura '95,

Baumgarte & Shapiro '98

# Formulations I: BSSN

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\begin{aligned} \phi &= \frac{1}{12} \ln \gamma & \hat{\gamma}_{ij} &= e^{-4\phi} \gamma_{ij} \\ K &= \gamma_{ij} K^{ij} & \hat{A}_{ij} &= e^{-4\phi} \left( K_{ij} - \frac{1}{3} \gamma_{ij} K \right) \\ \hat{\Gamma}^i &= \gamma^{ij} \hat{\Gamma}_{jk}^i = -\partial_j \hat{\gamma}^{ij} \end{aligned}$$

$$(\partial_t - \mathcal{L}_\beta) \hat{\gamma}_{ij} = -2\alpha \hat{A}_{ij}$$

$$(\partial_t - \mathcal{L}_\beta) \phi = -\frac{1}{6} \alpha K$$

$$(\partial_t - \mathcal{L}_\beta) \hat{A}_{ij} = e^{-4\phi} (-D_i D_j \alpha + \alpha R_{ij})^{\text{TF}} + \alpha (K \hat{A}_{ij} - 2 \hat{A}_{ik} \hat{A}^k{}_j)$$

$$(\partial_t - \mathcal{L}_\beta) K = -D^i D_i \alpha + \alpha (\hat{A}_{ij} \hat{A}^{ij} + \frac{1}{3} K^2)$$

$$\begin{aligned} \partial_t \hat{\Gamma}^i &= 2\alpha \left( \hat{\Gamma}_{jk}^i \hat{A}^{jk} + 6 \hat{A}^{ij} \partial_j \phi - \frac{2}{3} \hat{\gamma}^{ij} \partial_j K \right) - 2 \hat{A}^{ij} \partial_j \alpha + \hat{\gamma}^{jk} \partial_j \partial_k \beta^i \\ &\quad + \frac{1}{3} \hat{\gamma}^{ij} \partial_j \partial_k \beta^k + \beta^j \partial_j \hat{\Gamma}^i + \frac{2}{3} \hat{\Gamma}^i \partial_j \beta^j \end{aligned}$$

Yo et al. (2002)

## Formulations II: Generalized harmonic (GHG)

- **Harmonic gauge:** choose coordinates such that

$$\nabla_{\mu} \nabla^{\mu} x^{\alpha} = 0$$

- **4-dim. version of Einstein equations**

$$R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_{\mu} \partial_{\nu} g_{\alpha\beta} + \dots$$

Principal part of wave equation

- **Generalized harmonic gauge:**  $H_{\alpha} \equiv g_{\alpha\nu} \nabla_{\mu} \nabla^{\mu} x^{\nu}$

$$\Rightarrow R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_{\mu} \partial_{\nu} g_{\alpha\beta} + \dots - \frac{1}{2} (\partial_{\alpha} H_{\beta} + \partial_{\beta} H_{\alpha})$$

Still principal part of wave equation !!!

# The gauge in GHG

- Relation between  $H_\alpha$  and lapse  $\alpha$  and shift  $\beta^i$ :

$$H_\mu n^\mu = -K - \frac{1}{\alpha^2} (\partial_0 \alpha - \beta^i \partial_i \alpha)$$

$$\perp^i{}_\mu H^\mu = \frac{1}{\alpha} \gamma^{ik} \partial_k \alpha + \frac{1}{\alpha^2} (\partial_0 \beta^i - \beta^k \partial_k \beta^i) - \gamma^{mn} \Gamma_{mn}^i$$

- Auxiliary constraint

$$C_\gamma \equiv H_\gamma - \Gamma_{\mu\gamma}^\mu + g^{\mu\nu} \partial_\mu g_{\nu\gamma}$$

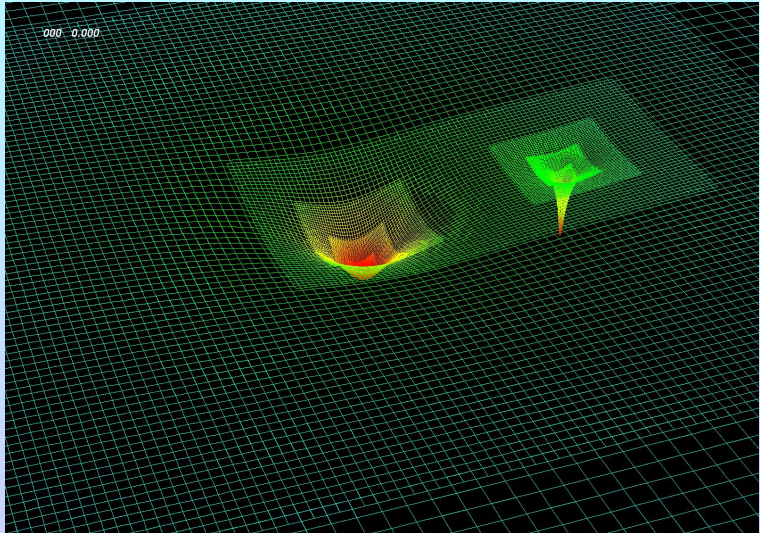
Requires constraint damping

Gundlach *et al.* '05

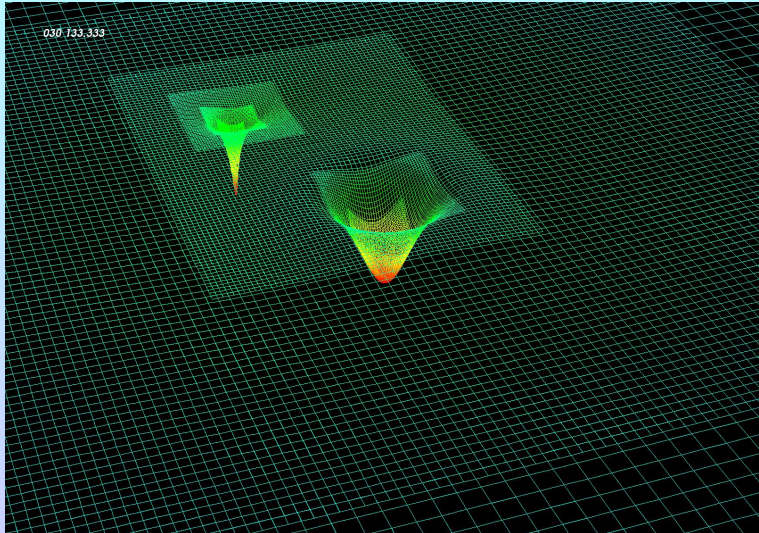
# The gauge freedom

- Remember: Einstein equations say nothing about  $\alpha$ ,  $\beta^i$
- Any choice of lapse and shift gives a solution
- This represents the coordinate freedom of GR
- Physics do not depend on  $\alpha$ ,  $\beta^i$   
So why bother?
- The performance of the numerics DO depend strongly on the gauge!
- How do we get good gauge?  
Singularity avoidance, avoid coordinate stretching, well posedness

# What goes wrong with bad gauge?

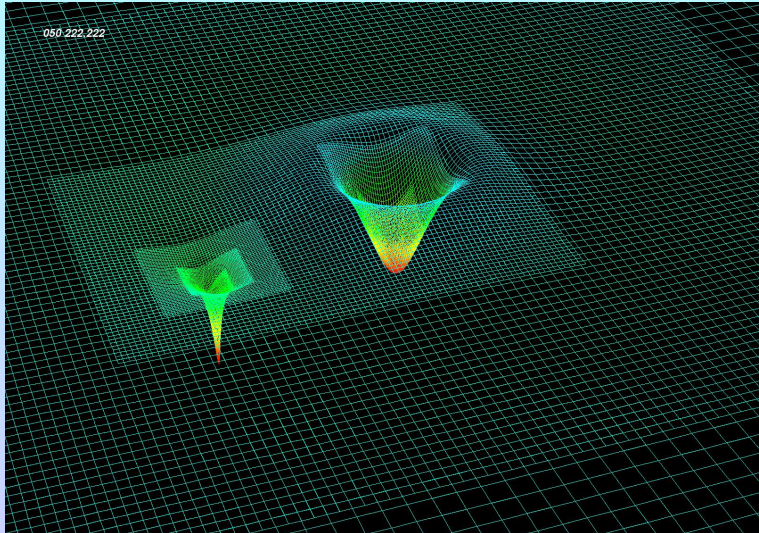


# What goes wrong with bad gauge?

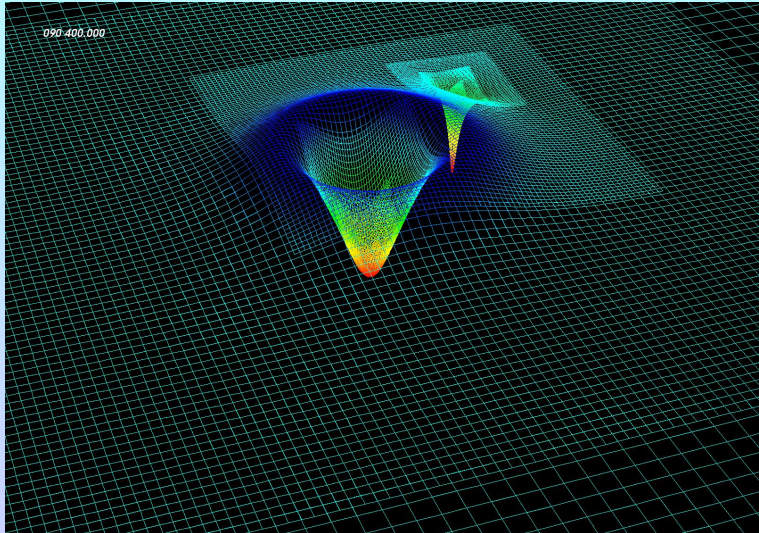




# What goes wrong with bad gauge?



# What goes wrong with bad gauge?



# Initial data

Two problems: Constraints, realistic data

- Rearrange degrees of freedom

York-Lichnerowicz split:  $\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$

$$K_{ij} = A_{ij} + \frac{1}{3} \gamma_{ij} K$$

York & Lichnerowicz, O'Murchadha & York,

Wilson & Mathews, York

- Make simplifying assumptions

Conformal flatness:  $\tilde{\gamma}_{ij} = \delta_{ij}$

- Find good elliptic solvers

# Two families of initial data

- Generalized analytic solutions:

Isotropic Schwarzschild  $ds^2 = \frac{M-2r}{M+2r} dt^2 + \left(1 + \frac{M}{2r}\right)^4 (dr^2 + r^2 d\Omega)$

⇒ Time-symmetric N holes      Brill & Lindquist, Misner '60s

⇒ Spin, Momenta      Bowen & York '80

⇒ Punctures      Brandt & Brügmann '97

- Excision data: horizon boundary conditions

Meudon Group, Pfeiffer, Ansorg

- Remaining problems: 1) junk radiation

2) We often want zero eccentricity

# Mesh refinement

3 Length scales :	BH	$\sim 1 M$
	Wavelength	$\sim 10 \dots 100 M$
	Wave zone	$\sim 100 \dots 1000 M$

- Critical phenomena

Choptuik '93

- First used for BBHs

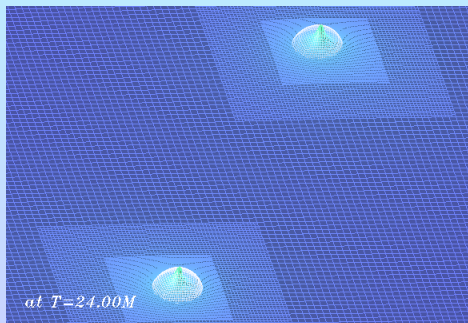
Brügmann '96

- Available Packages:

Paramesh MacNeice *et al.* '00

Carpet Schnetter *et al.* '03

SAMRAI MacNeice *et al.* '00



# Singularity treatment

- Cosmic censorship  $\Rightarrow$  horizon protects outside

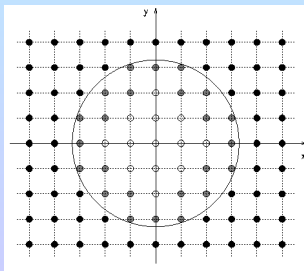
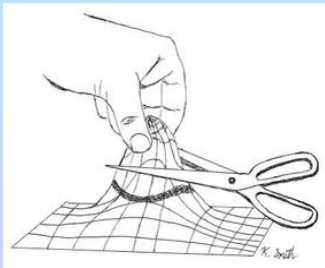
- We get away with it...

Moving Punctures

UTB, NASA Goddard '05

- Excision: Cut out region around singularity

Caltech-Cornell, Pretorius



# Extracting physics I: Global quantities

- ADM mass: Total energy of the spacetime

$$M_{\text{ADM}} = \frac{1}{16\pi} \lim_{r \rightarrow \infty} \int_{S_r} \sqrt{\gamma} \gamma^{ij} \gamma^{kl} (\partial_j \gamma_{ik} - \partial_k \gamma_{ij}) dS_l$$

- Total angular momentum of the spacetime

$$P_i = \frac{1}{8\pi} \lim_{r \rightarrow \infty} \int_{S_r} \sqrt{\gamma} (K^m{}_i - \delta^m{}_i K) dS_m$$

$$J_i = \frac{1}{8\pi} \epsilon_{il}{}^m \lim_{r \rightarrow \infty} \int_{S_r} \sqrt{\gamma} x^l (K^n{}_m - \delta^n{}_m K) dS_n$$

By construction all of these are time independent !!

# Extracting physics II: Local quantities

- Often impossible to define!!
- Isolated horizon framework Ashtekar *et al.*
  - Calculate **apparent horizon** → Irreducible mass, momenta associated with horizon

$$M_{\text{irr}} = \sqrt{\frac{A_{\text{AH}}}{16\pi}}$$

- Total BH mass Christodoulou

$$M^2 = M_{\text{irr}}^2 + \frac{S^2}{4M_{\text{irr}}^2} + P^2$$

- Binding energy of a binary:  $E_b = M_{\text{ADM}} - M_1 - M_2$



# Extracting physics III: Gravitational Waves

- Most important diagnostic: Emitted GWs

- Newman-Penrose scalar

$$\Psi_4 = C_{\alpha\beta\gamma\delta} n^\alpha \bar{m}^\beta n^\gamma \bar{m}^\delta$$

Complex  $\Rightarrow$  2 free functions

- GWs allow us to measure

$\rightarrow$  Radiated energy  $E_{\text{rad}}$

$\rightarrow$  Radiated momenta  $P_{\text{rad}}, J_{\text{rad}}$

$\rightarrow$  Angular dependence of radiation

$\rightarrow$  Gravitational wave strain  $h_+, h_\times$

# Angular dependence of GWs

- Waves are normally extracted at fixed radius  $r_{\text{ex}}$

$$\Rightarrow \Psi_4 = \Psi_4(t, \theta, \phi)$$

$\theta, \phi$  are viewed from the source frame!

- Decompose angular dependence using spherical harmonics

$$\Psi_4 = \sum_{\ell, m} \psi_{\ell m}(t) Y^{-2\ell m}(\theta, \phi)$$

Modes  $\psi_{\ell m}(t) = A_{\ell m}(t) \times e^{i\phi(t)}$

Spin-weighted spherical harmonics  $Y_{\ell m}^{-2}$

# A brief history of BH simulations

- Pioneers: Hahn & Lindquist '60s, Eppley, Smarr *et al.* '70s
- Grand Challenge: First 3D Code Anninos *et al.* '90s
- Further attempts: Bona & Massó, Pitt-PSU-Texas  
AEI-Potsdam, Alcubierre *et al.*  
PSU: first orbit Brügmann *et al.* '04

Codes unstable!

- 
- Breakthrough: Pretorius '05 GHG  
UTB, Goddard'05 Moving Punctures
  - Currently about 10 codes world wide

# 3. Black holes in astrophysics

# Free parameters of BH binaries

- Total mass  $M$

Relevant for GW detection: Frequencies scale with  $M$

Not relevant for source modeling: trivial rescaling

- Mass ratio  $q \equiv \frac{M_1}{M_2}$ ,  $\eta \equiv \frac{M_1 M_2}{(M_1 + M_2)^2}$

- Spin:  $\vec{S}_1, \vec{S}_2$  (6 parameters)

- Initial parameters

Binding energy  $E_b$

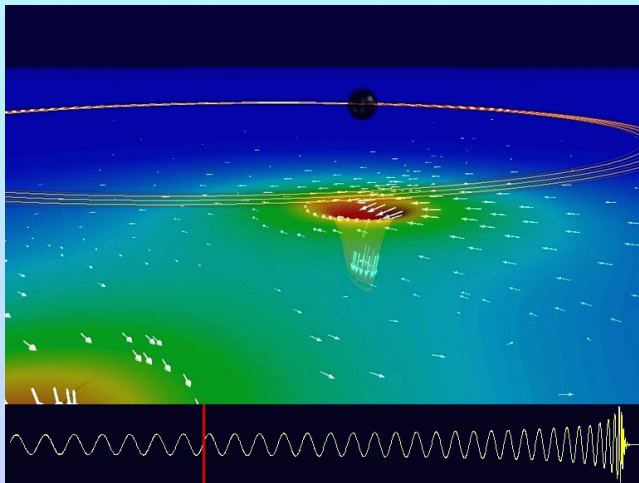
Separation

Orbital ang. momentum  $L$

Eccentricity

Alternatively: frequency, eccentricity

# Morphology of a BBH inspiral



Thanks to Caltech, CITA, Cornell

# Gravitational recoil

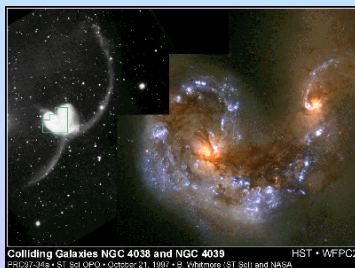
- Anisotropic GW emission  $\Rightarrow$  recoil of remnant BH

Bonnor & Rotenberg '61, Peres '62, Bekenstein '73

- Escape velocities:    Globular clusters    30 km/s  
                                  dSph                            20 – 100 km/s  
                                  dE                                100 – 300 km/s  
                                  Giant galaxies             $\sim$  1000 km/s

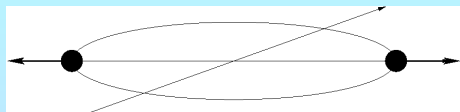
Ejection / displacement of BH  $\Rightarrow$

- Growth history of SMBHs
- BH populations, IMBHs
- Structure of galaxies



# Superkicks

- Kidder '95, UTB-RIT '07: maximum kick expected for



- Measured kicks  $v \approx 2500$  km/s for spin  $a \approx 0.75$   
Extrapolated to maximal spins:  $v_{\max} \approx 4000$  km/s  
González *et al.* '07, Campanelli *et al.* '07
- Unlikely configuration!  
Bogdanović *et al.* '07, Kesden, US & Berti '10, '10a
- Hyperbolic encounters:  $v$  up to 10000 km/s  
Healy *et al.* '08



# Spin precession and flip

- X-shaped radio sources

Merrit & Ekers '07

- Jet along spin axis

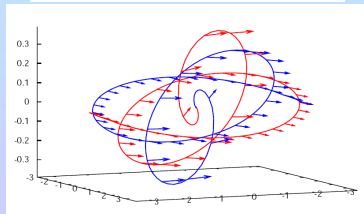
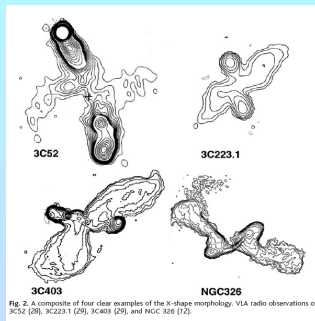
- Spin re-alignment

⇒ new + old jet

- Spin precession  $98^\circ$

Spin flip  $71^\circ$

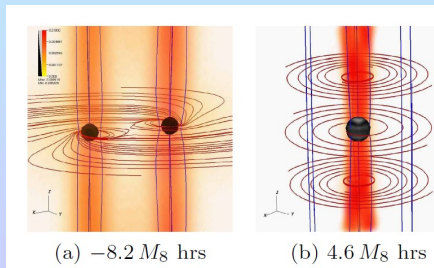
UTB-RIT '06



# Jets generated by binary BHs

Palenzuela, Lehner & Liebling '10

- Non-spinning BH binary
- Einstein-Maxwell equations with “force free” plasma
- Electromagnetic field extracts energy from  $\mathbf{L} \Rightarrow$  jets
- Optical signature: **double jets**

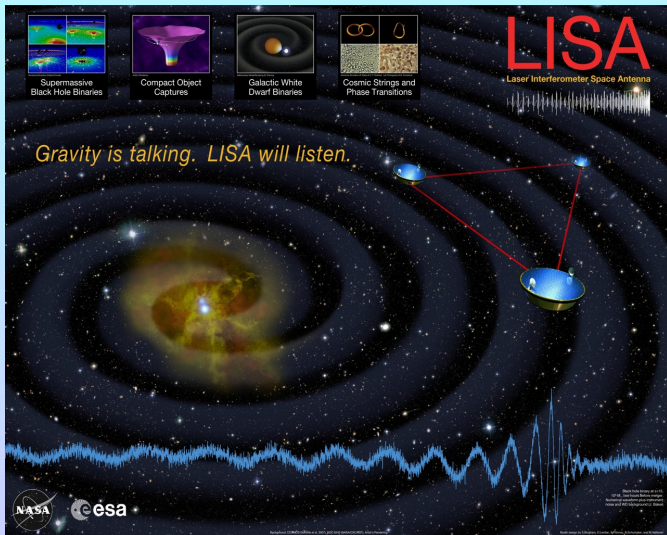


# Gravitational Wave observations

- Accelerated masses generate GWs
- Interaction with matter *very weak!*
- Earth bound detectors: GEO600, LIGO, TAMA, VIRGO

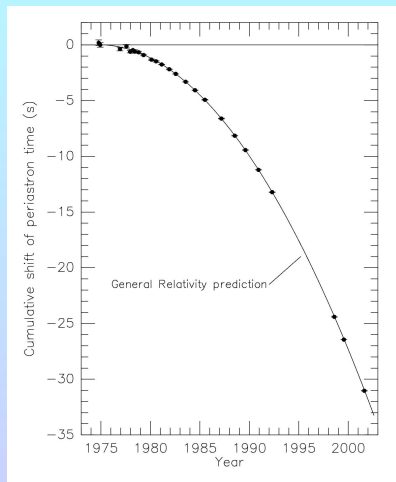


# Space interferometer LISA

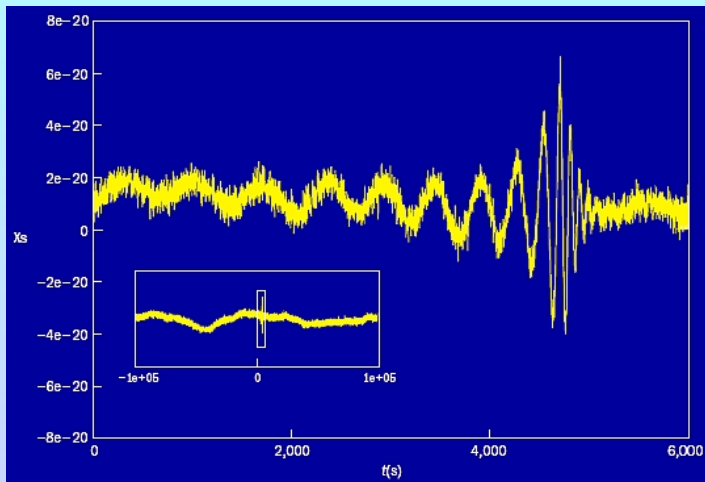


# Some targets of GW physics

- Confirmation of GR
  - Hulse & Taylor 1993 Nobel Prize
- Parameter determination of BHs:  $M$ ,  $\vec{S}$
- Optical counter parts
  - Standard sirens (candles)
  - Mass of graviton
- Test Kerr Nature of BHs
- Cosmological sources
- Neutron stars: EOS



# Matched filtering



Long, accurate waveforms required

⇒ combine NR with PN, perturbation theory

# 3. Black holes in fundamental physics

# So what other interesting physics can we do with NR?

- High-energy physics
  - Trans-Planckian scattering
  - AdS/CFT duality
- Mathematical physics and theoretical physics
  - Cosmic censorship
  - Critical phenomena
  - BH instabilities (Myers-Perry)

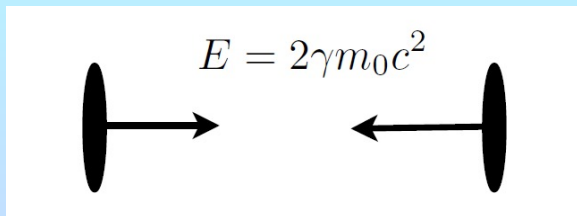


# 3.1. Transplanckian scattering

# BH formation and hoop conjecture

- Hoop conjecture

Thorne '72



- de Broglie wavelength:  $\lambda = \frac{hc}{E}$

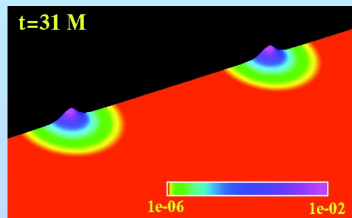
- Schwarzschild radius:  $r = \frac{2GE}{c^4}$

- BH will form if  $\lambda < r \Leftrightarrow E \gtrsim \sqrt{\frac{hc^5}{G}} \equiv E_{\text{Planck}}$

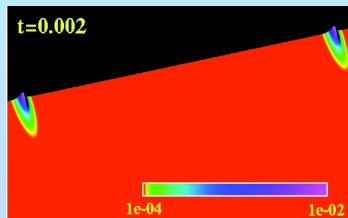
# BH formation in boson field collisions

Pretorius & Choptuik '09

- Einstein plus minimally coupled, massive, complex scalar field  
“Boson stars”



$$\gamma = 1$$



$$\gamma = 4$$

- BH formation threshold:  $\gamma_{\text{thr}} = 2.9 \pm 10 \%$
- About 1/3 of hoop conjecture prediction

# Motivation (High-energy physics)

- **Matter does not matter** at energies well above the Planck scale  
⇒ Model particle collisions by black-hole collisions

Banks & Fischler '99; Giddings & Thomas '01

- **TeV-gravity** scenarios

⇒ The Planck scale might be as low as TeVs due to extra dimensions

Arkani-Hamed, Dimopoulos & Dvali '98, Randall & Sundrum '99

⇒ **Black holes could be produced in colliders**

Eardley & Giddings '02, Dimopoulos & Landsberg '01,...

# Motivation (High-energy physics)

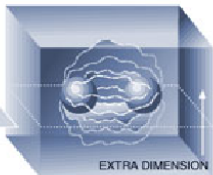
## Black Holes on Demand

Scientists are exploring the possibility of producing miniature black holes on demand by smashing particles together. Their plans hinge on the theory that the universe contains more than the three dimensions of everyday life. Here's the idea:

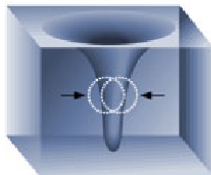
*Particles collide in three dimensional space, shown below as a flat plane.*



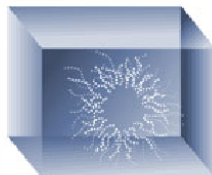
As the particles approach in a particle accelerator, their gravitational attraction increases steadily.



When the particles are extremely close, they may enter space with more dimensions, shown above as a cube.



The extra dimensions would allow gravity to increase more rapidly so a black hole can form.

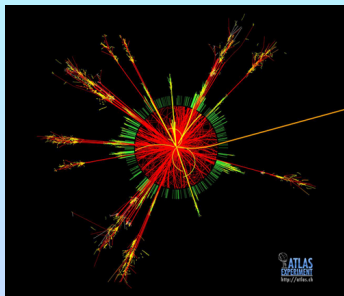


Such a black hole would immediately evaporate, sending out a unique pattern of radiation.

# Experimental signature at the LHC

Black hole formation at the LHC could be detected by the properties of the jets resulting from Hawking radiation.

- Multiplicity of partons: Number of jets and leptons
- Large transverse energy
- Black-hole mass and spin are important for this!



ToDo:

- Exact cross section for BH formation
- Determine loss of energy in gravitational waves
- Determine spin of merged black hole

# Black-hole collisions in $D = 4$

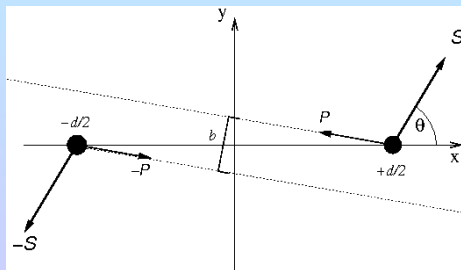
- Take two black holes

Total rest mass:  $M_0 = M_{A,0} + M_{B,0}$

Initial position:  $\pm x_0$

Linear momentum:  $\mp P[\cos \alpha, \sin \alpha, 0]$

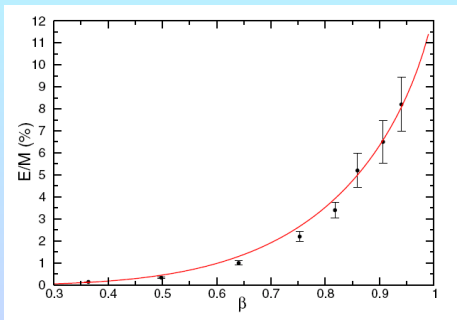
- Impact parameter:  $b \equiv \frac{L}{P}$



# Head-on collisions: $b = 0$ , $\vec{S} = 0$

- Total radiated energy:  $14 \pm 3 \%$  for  $\nu \rightarrow 1$  Sperhake *et al.* '08

About half of Penrose '74



- Agreement with approximative methods

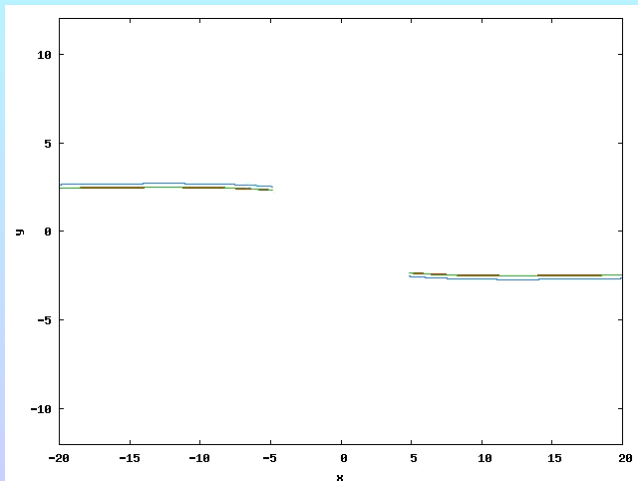
Flat spectrum, multipolar GW structure

Berti *et al.* '10



# Grazing collisions: $b \neq 0$ , $\vec{S} = 0$ , $\gamma = 1.52$

Immediate vs. Delayed vs. No merger      Sperhake *et al.* '09

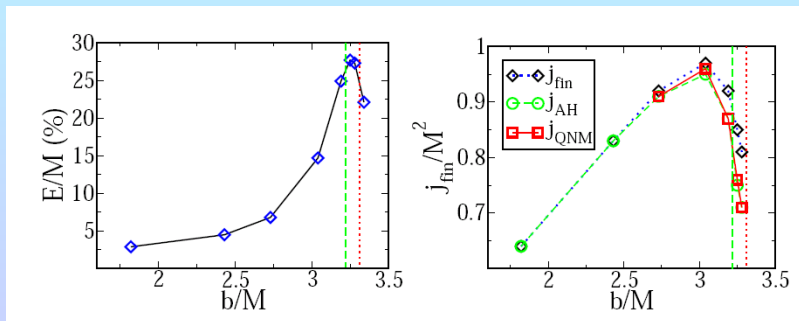


# Critical impact parameter

- $b < b_{\text{crit}} \Rightarrow$  Merger
- $b > b_{\text{crit}} \Rightarrow$  Scattering
- Numerical study:  $b_{\text{crit}} = \frac{2.5 \pm 0.05}{v} M$   
Shibata *et al.* '08
- Independent study by Sperhake *et al.* '09
  - $\gamma = 1.52: \quad 3.39 < b_{\text{crit}}/M < 3.4$
  - $\gamma = 2.93: \quad 2.3 < b_{\text{crit}}/M < 2.4$
  - $v \rightarrow 1$  limit still needs to be determined
- Limit from Penrose construction:  $b_{\text{crit}} = 1.685 M$   
Yoshino & Rychkov '05

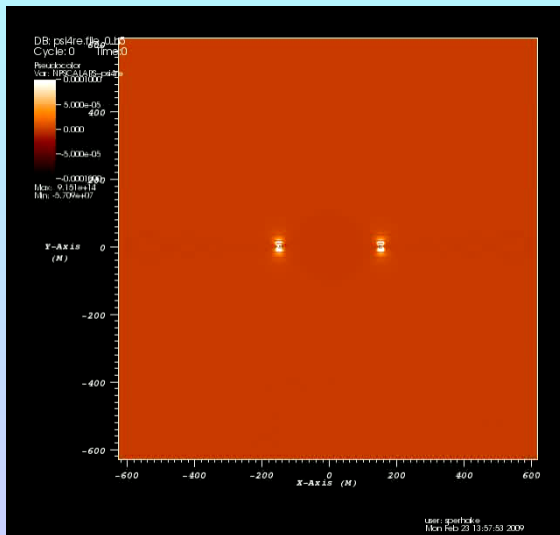
# Radiated quantities

- $b$ -sequence with  $\gamma = 1.52$
- Final spin close to Kerr limit
- $E_{\text{rad}} \sim 35\%$  for  $\gamma = 2.93$ ; about 10% of Dyson luminosity

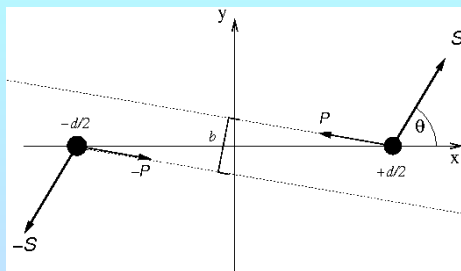


Sperhake *et al.* '09

# Gravitational radiation: Delayed merger

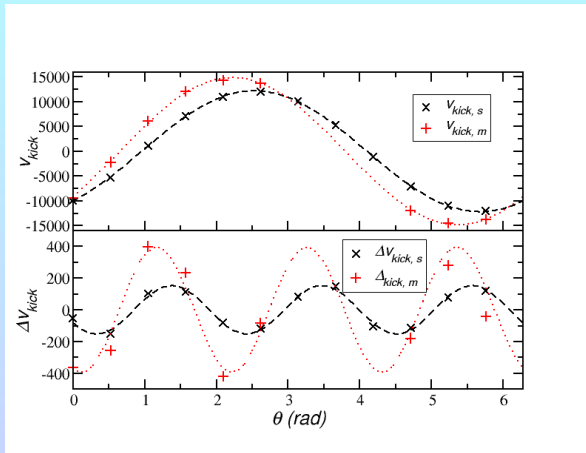


# Recoil in grazing collisions



- equal-mass, **superkick**,  $\chi = 0.621$
- $\gamma = 1.52$
- 2 sequences
  - merging:  $b = 3.34 M$
  - scattering:  $b = 3.25 M$

# Recoil in grazing collisions



Expansion in  $\theta$  according to Boyle, Kesden & Nissanke '08

# Recoil in grazing collisions

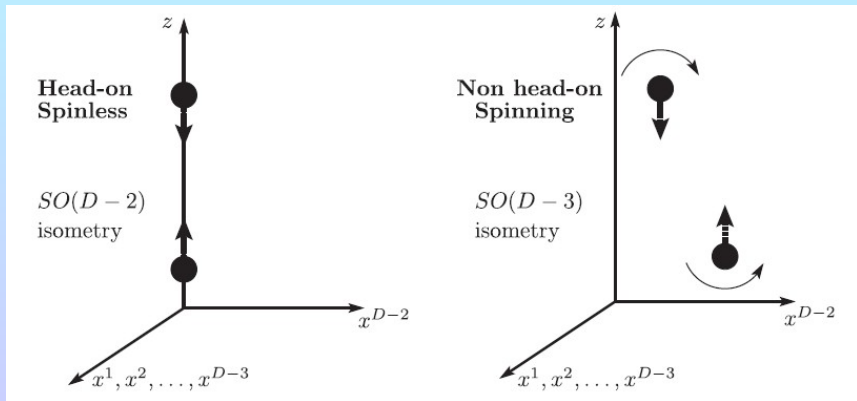
- $v_{\max,s} = 12\,200 \text{ km/s}$   
 $v_{\max,m} = 14\,900 \text{ km/s}$
- Large recoils for merger and scattering!
- $v_{\max} \propto E_{\text{rad}}$
- Antikicks can occur in both  $\Rightarrow$  not a merger-only feature!
- Ultimate kick  
 $v_{\max} \propto E_{\text{rad}} \Rightarrow \sim 45\,000 \text{ km/s}$   
spin insignificant for large  $\gamma \Rightarrow \sim 25\,000 \text{ km/s}$   
no simple picture  $\Rightarrow$  more data needed...

# Moving to $D > 4$

- Symmetries allow dimensional reduction

Gerch '70

- Reduces to “3+1” plus quasi-matter terms: scalar field





# BSSN formulation with quasi matter

$$\partial_t \tilde{\gamma}_{ij} = [\text{BSSN}],$$

$$\partial_t \chi = [\text{BSSN}],$$

$$\partial_t K = [\text{BSSN}] + 4\pi\alpha(E + S),$$

$$\partial_t \tilde{A}_{ij} = [\text{BSSN}] - 8\pi\alpha(\chi S_{ij} - \frac{1}{3}S\tilde{\gamma}_{ij}),$$

$$\partial_t \tilde{\Gamma}^i = [\text{BSSN}] - 16\pi\alpha\chi^{-1}j^i,$$

$$\partial_t \zeta = -2\alpha K_\zeta + \beta^m \partial_m \zeta - \frac{2}{3}\zeta \partial_m \beta^m + 2\zeta \frac{\beta^y}{y},$$

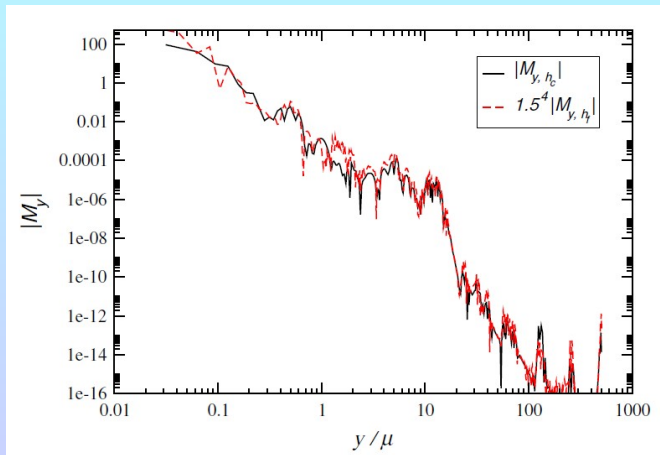
$$\partial_t K_\zeta = \dots ,$$

$$E, j^i, S_{ij} = f(\text{BSSN}, \zeta, K_\zeta).$$

Zilhão *et al.* '10

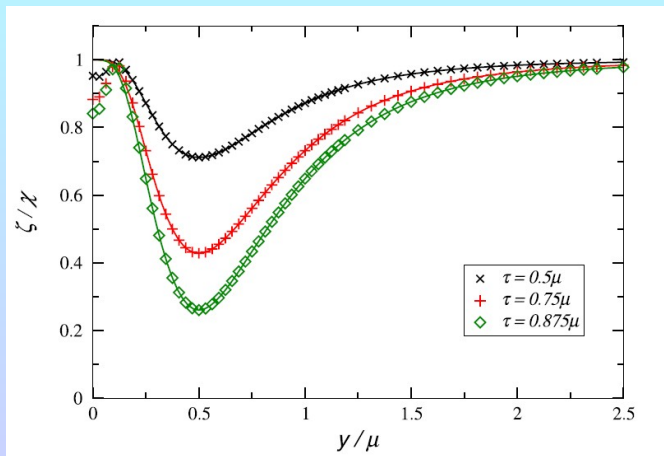
# Single black hole in $D = 5$

Initial data: Tangherlini '63



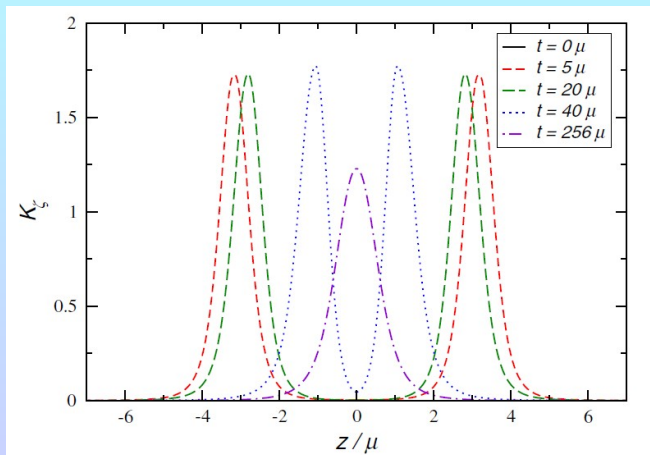
# Single black hole in $D = 5$

In geodesic slicing

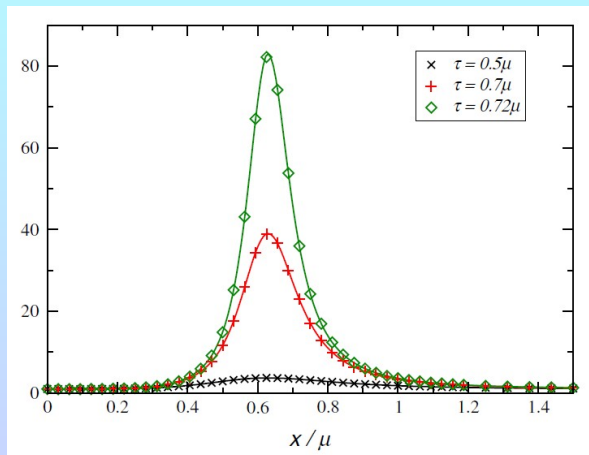


# Head-on in $D = 5$

Initial data:  $D = 5$  analogue of Brill-Lindquist data

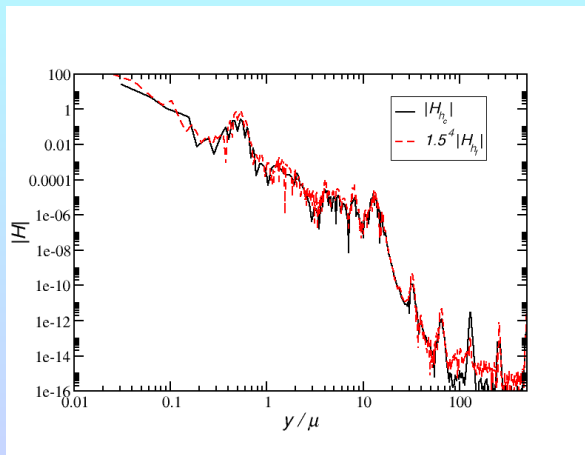


# Single black hole in $D = 6$



Geodesic slicing, zero shift

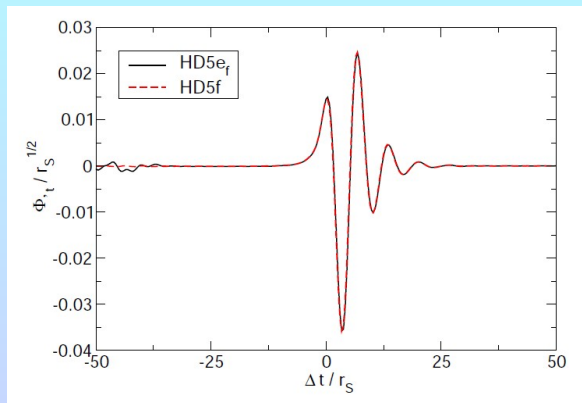
# Single black hole in $D = 6$



ToDo: long term stable evolutions

# GWs from head-on in $D = 5$

Wave extraction based on Kodama & Ishibashi '03

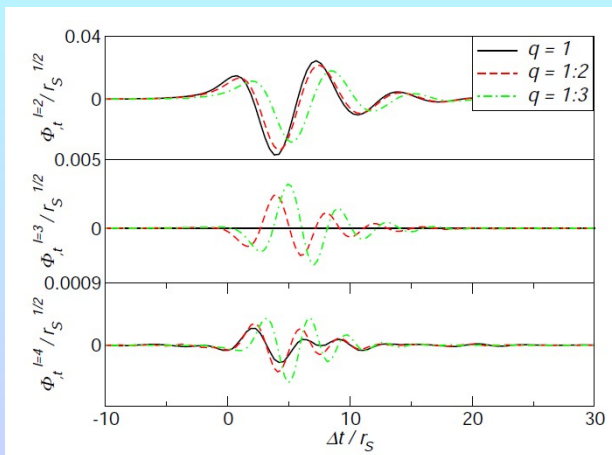


$E_{\text{rad}} = 0.089 \% M$  cf.  $0.055 \% M$  in  $D = 4$

Witek *et al.* '10a

# Unequal-mass head-on in $D = 5$

## Kodama-Ishibashi multipoles

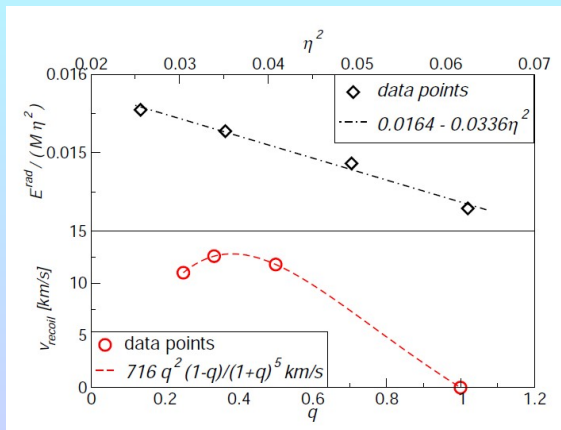


Witek *et al.* '10b



# Unequal-mass head-on in $D = 5$

## Radiated energy and momentum

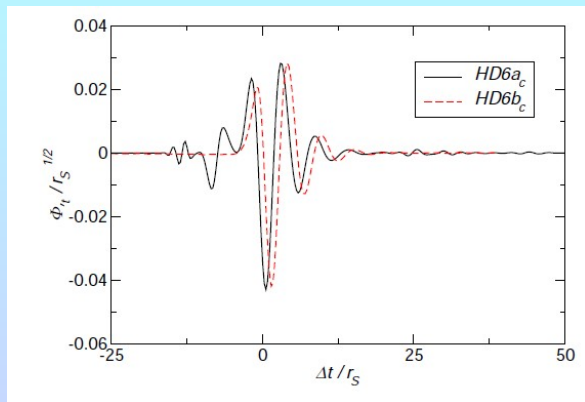


Agreement within  $< 5\%$  with extrapolated point particle calculations

# Breaking news!

# First black-hole collisions in $D = 6$

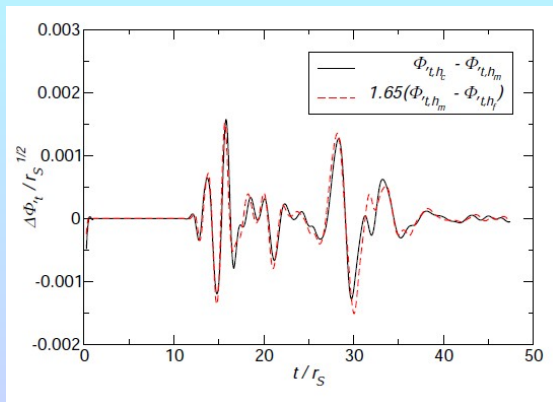
Witek *et al.* '10



- Adjust shift parameters
- Use LaSh system Witek, Hilditch & US '10

# First black-hole collisions in $D = 6$

Witek *et al.* '10



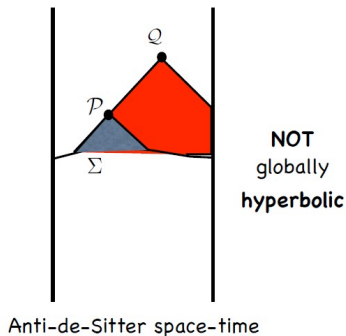
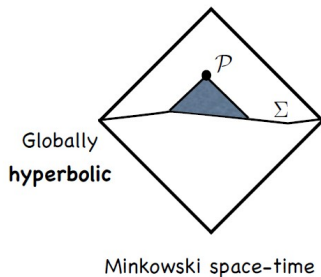
Second order convergence

## 3.2. Non-asymptotically flat boundaries: AdS/CFT

# AdS/CFT correspondence

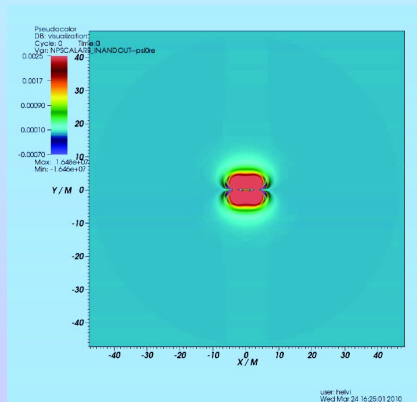
Challenge: Model the active role of the boundary!

Conformal diagram of:



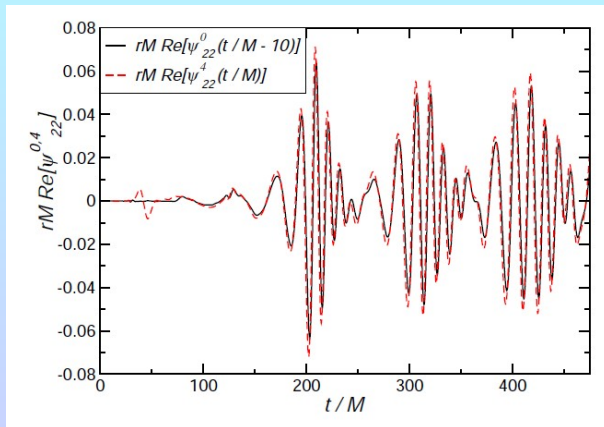
# Toy model: Black hole inspiral in a lego sphere

- Lego sphere with reflective boundary
- Goddard R1 run Baker *et al.* '06
- Calculate  $\Psi_4$  and  $\Psi_0$



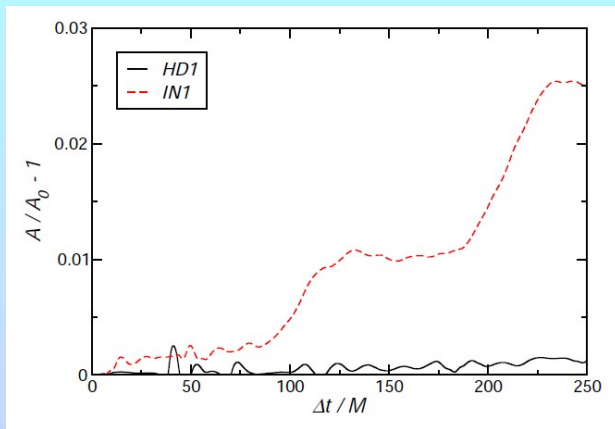
# Quadrupole mode

Gravitational radiation (out going and ingoing)





# Horizon area



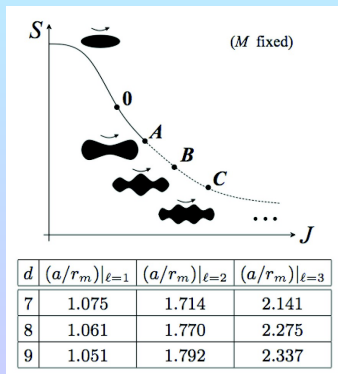
Superradiance: high frequency absorbed, low frequency amplified  
No conclusive evidence yet...

# 4. Other topics in $D \geq 5$

# Other topics: Instabilities of Myers-Perry

- Ultra-spinning Myers-Perry black holes (with single angular momentum parameter) should be unstable.
- Confirmed by linearized analysis of axisymmetric perturbations

Dias *et al.* '09

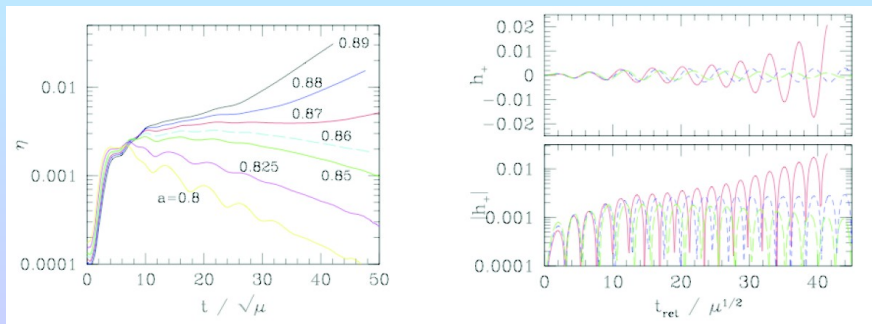


## Other topics: Instabilities of Myers-Perry

- Numerical study of non-axisymmetric instabilities of  $D = 5$  Myers-Perry BH with single ang. momentum parameter.

Shibata & Yoshino '09

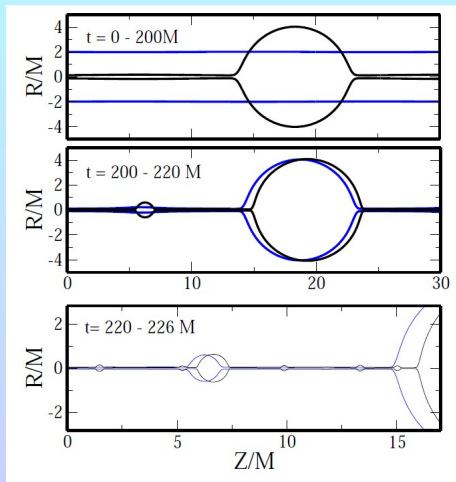
- Found onset of instabilities at spin  $a/\sqrt{\mu} \approx 0.87$



# Other topics: Cosmic censorship in $D = 5$

Pretorius & Lehner '10

- Axisymmetric code
- Study evolution of black string...
- Gregory-Laflamme instability cascades down until string reaches zero radius  
 $\Rightarrow$  naked singularity

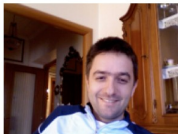


# 5. Summary

# Summary

- Black holes are real objects in many areas of physics!
- Astrophysics: Recoil, Spin flips, jets
- Gravitational wave physics: template banks needed
- High-energy collisions in  $D = 4$ :
  - largest kicks  $\sim 15\,000$  km/s
  - largest radiation  $\sim 30\%$
  - largest post-merger spin  $a \lesssim 1$
- Formalism for arbitrary spatial dimension  $D$
- Head-on collisions from rest
- Test non-asymptotically flat OBCs
- Signs of cosmic censorship violation in  $D = 5$

# The team



<http://centra.ist.utl.pt/~blackholes/>