Colliding black holes in 3+1 and higher dimensions

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CSIC-IEEC Barcelona

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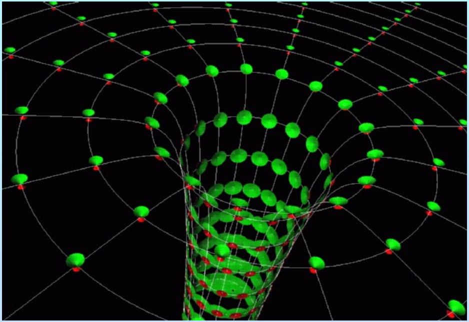
Overview

- Motivation
- Modeling black holes in GR
- Black holes in astrophysics
- Black holes in fundamental physics
  - Trans Planckian scattering
  - Non-assymptotically flat boundaries: AdS/CFT
- Other topics in $D \geq 5$
  - Instabilities of Myers-Perry BHs
  - Cosmic censorship in $D \geq 5$
- Summary

1. Motivation

What are black holes?



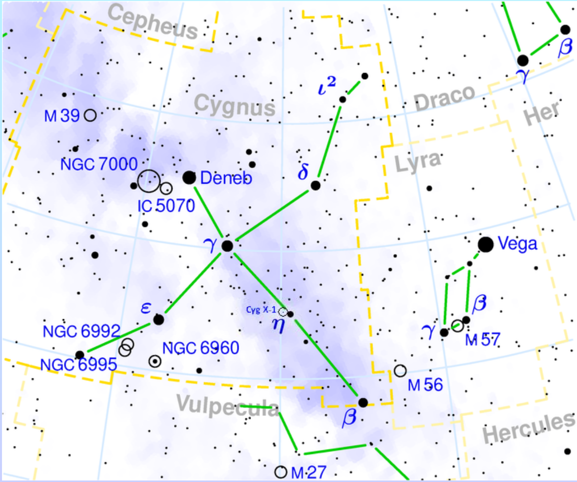
- Consider Lightcones
- In and outgoing light
- Calculate surface

  of outgoing light fronts
- Expansion $\equiv$ Rate of

  change of this surface
- Apparent Horizon $\equiv$ Outermost surface with zero expansion
- "Light cones tip over" due to curvature

Black holes are out there: Stellar BHs

- high-mass X-ray binaries: Cygnus X-1 (1964)

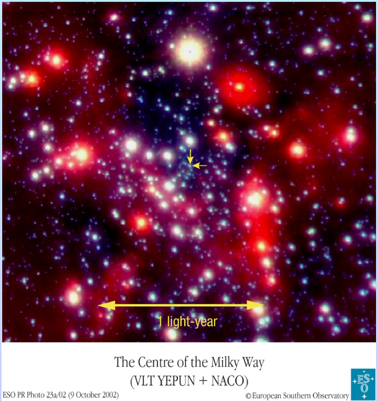


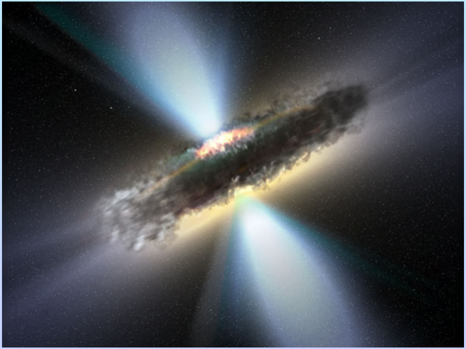
Black holes are out there: Stellar BHs

- One member is very compact and massive $\Rightarrow$ Black Hole

Black holes are out there: galactic BHs

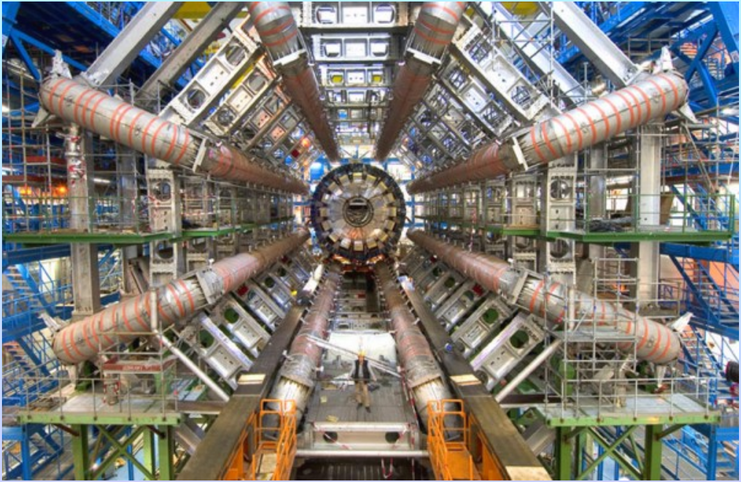
- Supermassive BHs found at center of virtually all galaxies
- SMBHs conjectured to be responsible for quasars starting in the 1980s





Black holes might be in here: LHC

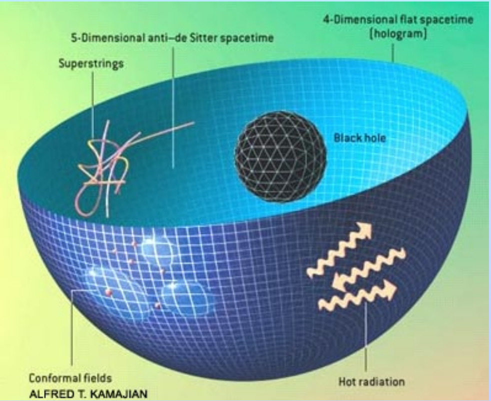
- LHC CERN



Motivation (AdS/CFT correspondence)

- BH spacetimes "know" about physics without BHs

  AdS/CFT correspondence   Maldacena '97



2. Modeling black holes in GR

General Relativity: Curvature

- Curvature generates acceleration

  "geodesic deviation"

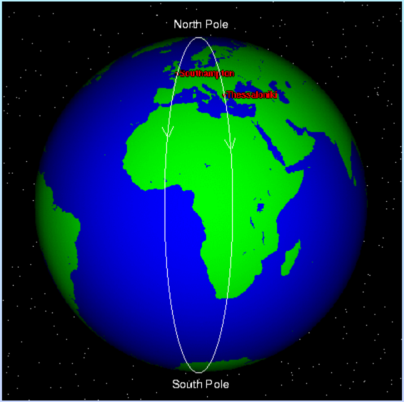
  No "force"!!

- Description of geometry

  Metric $g_{\alpha\beta}$

  Connection $\Gamma^{\alpha}_{\beta\gamma}$

  Riemann Tensor $R^{\alpha}{}_{\beta\gamma\delta}$



The metric defines everything

- Christoffel connection

  $$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} \left(\partial_\beta g_{\gamma\mu} + \partial_\gamma g_{\mu\beta} - \partial_\mu g_{\beta\gamma} \right)$$

- Covariant derivative

  $$\nabla_\alpha T^{\beta}{}_{\gamma} = \partial_\alpha T^{\beta}{}_{\gamma} + \Gamma^{\beta}_{\mu\alpha} T^{\mu}{}_{\gamma} - \Gamma^{\mu}_{\gamma\alpha} T^{\beta}{}_{\mu}$$

- Riemann Tensor

  $$R^{\alpha}{}_{\beta\gamma\delta} = \partial_\gamma \Gamma^{\alpha}_{\beta\delta} - \partial_\delta \Gamma^{\alpha}_{\beta\gamma} + \Gamma^{\alpha}_{\mu\gamma} \Gamma^{\mu}_{\beta\delta} - \Gamma^{\alpha}_{\mu\delta} \Gamma^{\mu}_{\beta\gamma}$$

- $\Rightarrow$ Geodesic deviation,

  Parallel transport,

  ...

How to get the metric?

Train cemetery
Uyuni, Bolivia

- Solve for the metric $g_{\alpha\beta}$

How to get the metric?

- The metric must obey the Einstein Equations

- Ricci-Tensor, Einstein Tensor, Matter Tensor

  $R_{\alpha\beta} \equiv R^{\mu}{}_{\alpha\mu\beta}$

  $G_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R^{\mu}{}_{\mu}$ “Trace reversed” Ricci

  $T_{\alpha\beta}$ “Matter”

- Einstein Equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$

- Solutions: Easy! Take metric

  $\Rightarrow$ Calculate $G_{\alpha\beta}$

  $\Rightarrow$ Use that as matter tensor

- Physically meaningful solutions: Difficult!

The Einstein Equations in vacuum

- "Spacetime tells matter how to move,
  matter tells spacetime how to curve"
- Field equations in vacuum: $R_{\alpha\beta} = 0$
  Second order PDEs for the metric components
  Invariant under coordinate (gauge) transformations
- System of equations extremely complex: Pile of paper!
  Analytic solutions: Minkowski, Schwarzschild, Kerr,
  Robertson-Walker, ...
- Numerical methods necessary for general scenarios!!!

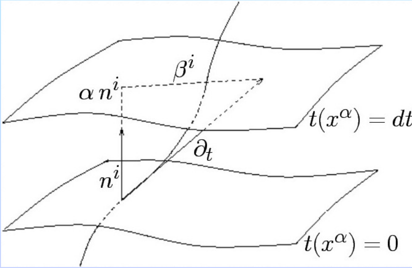
A list of tasks

- Target: Predict time evolution of BBH in GR
- Einstein equations: 1) Cast as evolution system
  2) Choose specific formulation
  3) Discretize for computer
- Choose coordinate conditions: Gauge
- Fix technical aspects: 1) Mesh refinement / spectral domains
  2) Singularity handling / excision
  3) Parallelization
- Construct realistic initial data
- Start evolution and waaaaiiiiit...
- Extract physics from the data

3+1 Decomposition

- GR: "Space and time exist as a unity: Spacetime"
- NR: ADM 3+1 split Arnowitt, Deser & Misner '62
  York '79, Choquet-Bruhat & York '80

$$g_{\alpha\beta} = \left(\begin{array}{c|c} -\alpha^2 + \beta_m \beta^m & \beta_j \\ \hline \beta_i & \gamma_{ij} \end{array} \right)$$



- 3-Metric γ_{ij}
  Lapse α
  Shift β^i
- lapse, shift $\Rightarrow$ Gauge

ADM Equations

The Einstein equations $R_{\alpha\beta} = 0$ become

- 6 Evolution equations

  $$(\partial_t - \mathcal{L}_\beta)\gamma_{ij} = -2\alpha K_{ij}$$

  $$(\partial_t - \mathcal{L}_\beta)K_{ij} = -D_i D_j \alpha + \alpha[R_{ij} - 2K_{im}K^m{}_j + K_{ij}K]$$

- 4 Constraints

  $$R + K^2 - K_{ij}K^{ij} = 0$$

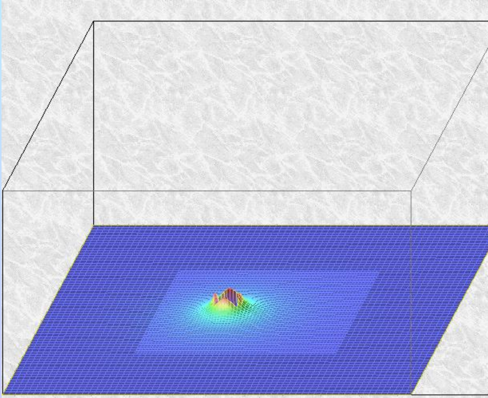
  $$-D_j K^{ij} + D^i K = 0$$

  preserved under evolution!

- Evolution

  1) Solve constraints

  2) Evolve data



Formulations I: BSSN

- One can easily change variables. E. g. wave equation

$$\partial_{tt} u - c\partial_{xx} u = 0 \qquad \Leftrightarrow \qquad \begin{aligned} \partial_t F - c\partial_x G &= 0 \\ \partial_x F - \partial_t G &= 0 \end{aligned}$$

- BSSN: rearrange degrees of freedom

$$\chi = (\det \gamma)^{-1/3} \qquad \tilde{\gamma}_{ij} = \chi \gamma_{ij}$$

$$K = \gamma_{ij} K^{ij} \qquad \tilde{A} = \chi \left(K_{ij} - \tfrac{1}{3} \gamma_{ij} K \right)$$

$$\tilde{\Gamma}^i = \tilde{\gamma}^{mn} \tilde{\Gamma}^i_{mn} = -\partial_m \tilde{\gamma}^{im}$$

Shibata & Nakamura '95, Baumgarte & Shapiro '98

Formulations I: BSSN

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\phi = \tfrac{1}{12}\ln\gamma \qquad \hat{\gamma}_{ij} = e^{-4\phi}\gamma_{ij}$$

$$K = \gamma_{ij}K^{ij} \qquad \hat{A}_{ij} = e^{-4\phi}\left(K_{ij} - \tfrac{1}{3}\gamma_{ij}K\right)$$

$$\hat{\Gamma}^i = \gamma^{ij}\hat{\Gamma}^i_{jk} = -\partial_j\hat{\gamma}^{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\hat{\gamma}_{ij} = -2\alpha\hat{A}_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\phi = -\tfrac{1}{6}\alpha K$$

$$(\partial_t - \mathcal{L}_\beta)\hat{A}_{ij} = e^{-4\phi}(-D_iD_j\alpha + \alpha R_{ij})^{\mathrm{TF}} + \alpha(K\hat{A}_{ij} - 2\hat{A}_{ik}\hat{A}^k{}_j)$$

$$(\partial_t - \mathcal{L}_\beta)K = -D^iD_i\alpha + \alpha\left(\hat{A}_{ij}\hat{A}^{ij} + \tfrac{1}{3}K^2\right)$$

$$\partial_t\hat{\Gamma}^i = 2\alpha\left(\hat{\Gamma}^i_{jk}\hat{A}^{jk} + 6\hat{A}^{ij}\partial_j\phi - \tfrac{2}{3}\hat{\gamma}^{ij}\partial_jK\right) - 2\hat{A}^{ij}\partial_j\alpha + \hat{\gamma}^{jk}\partial_j\partial_k\beta^i$$
$$+\tfrac{1}{3}\hat{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\hat{\Gamma}^i + \tfrac{2}{3}\hat{\Gamma}^i\partial_j\beta^j \underbrace{- \left(\chi + \tfrac{2}{3}\right)\left(\hat{\Gamma}^i - \hat{\gamma}^{jk}\hat{\Gamma}^i_{jk}\right)\partial_l\beta^l}_{\text{Yo et al. (2002)}}$$

Formulations II: Generalized harmonic (GHG)

- Harmonic gauge: choose coordinates such that

  $\nabla_\mu \nabla^\mu x^\alpha = 0$

- 4-dim. version of Einstein equations

  $R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \ldots$

  Principal part of wave equation

- Generalized harmonic gauge: $H_\alpha \equiv g_{\alpha\nu} \nabla_\mu \nabla^\mu x^\nu$

  $\Rightarrow R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \ldots - \frac{1}{2} \left(\partial_\alpha H_\beta + \partial_\beta H_\alpha \right)$

  Still principal part of wave equation !!!

The gauge in GHG

- Relation between H_α and lapse α and shift β^i:

  $$H_\mu n^\mu = -K - \frac{1}{\alpha^2}\left(\partial_0 \alpha - \beta^i \partial_i \alpha\right)$$

  $$\perp^i{}_\mu H^\mu = \frac{1}{\alpha}\gamma^{ik}\partial_k \alpha + \frac{1}{\alpha^2}\left(\partial_0 \beta^i - \beta^k \partial_k \beta^i\right) - \gamma^{mn}\Gamma^i_{mn}$$

- Auxiliary constraint

  $$C_\gamma \equiv H_\gamma - \Gamma^\mu_{\mu\gamma} + g^{\mu\nu}\partial_\mu g_{\nu\gamma}$$

  Requires constraint damping

  Gundlach *et al.* '05

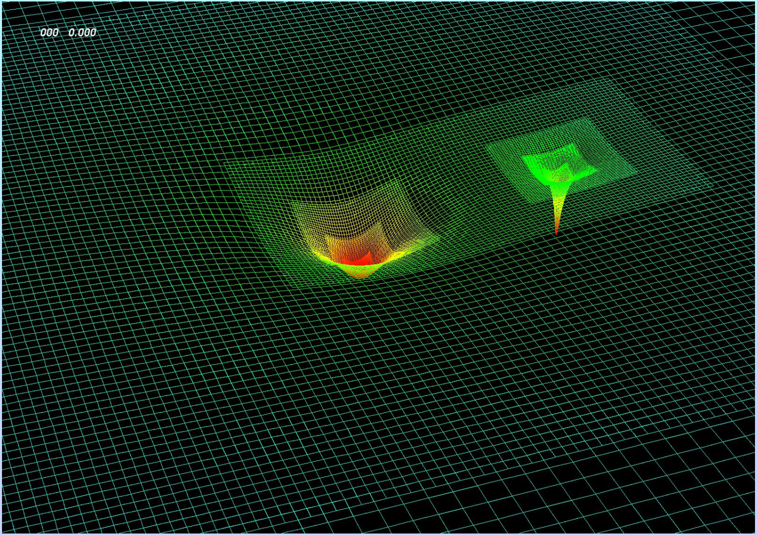
The gauge freedom

- Remember: Einstein equations say nothing about $\alpha, \quad \beta^i$
- Any choice of lapse and shift gives a solution
- This represents the coordinate freedom of GR
- Physics do not depend on $\alpha, \quad \beta^i$

  So why bother?
- The performance of the numerics DO depend strongly on the gauge!
- How do we get good gauge?

  Singularity avoidance, avoid coordinate stretching, well posedness

What goes wrong with bad gauge?



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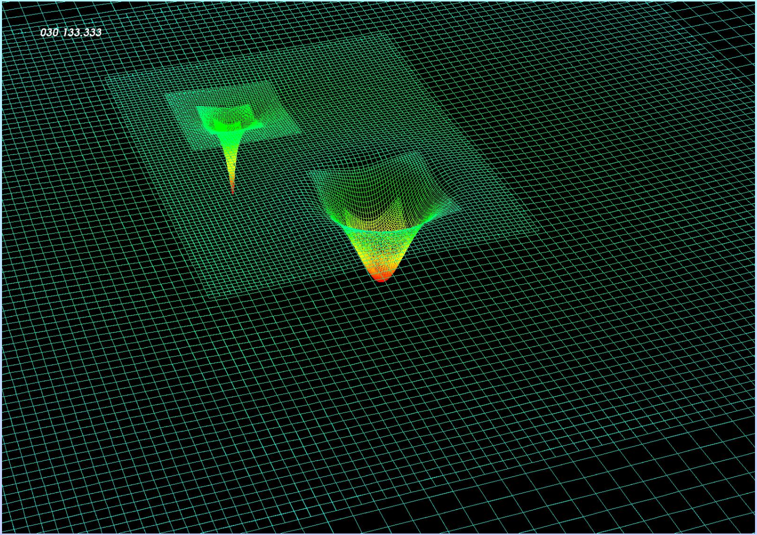

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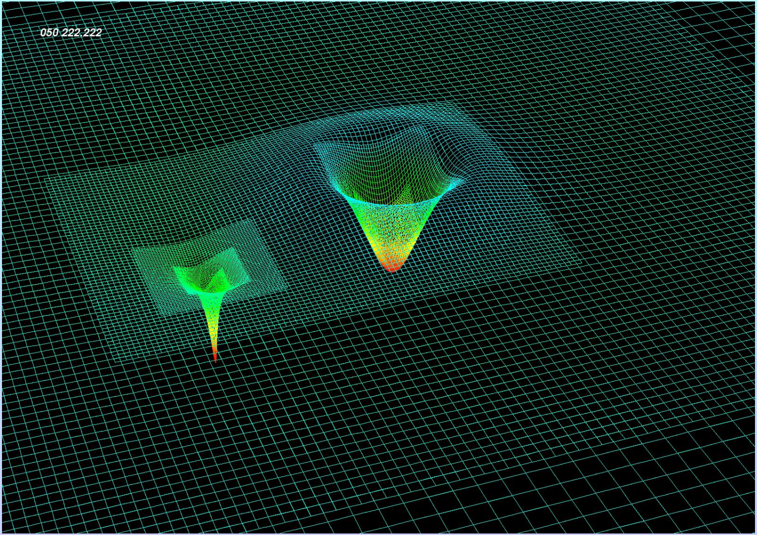

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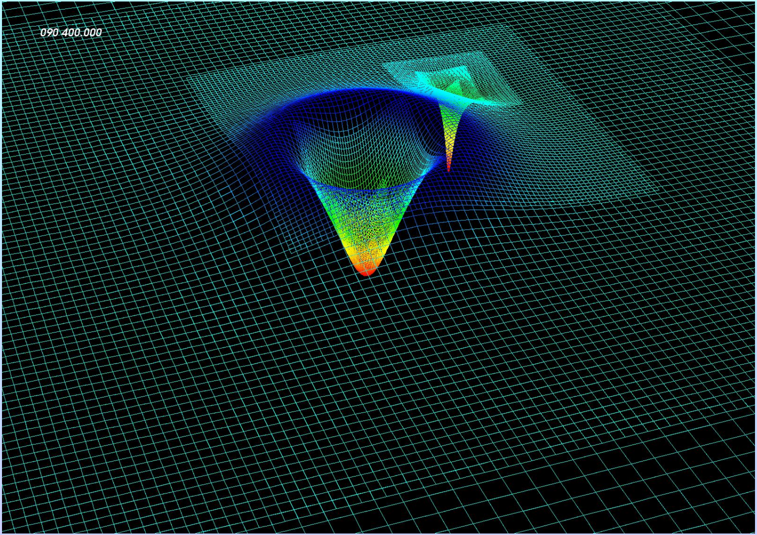


Initial data

Two problems: Constraints, realistic data

- Rearrange degrees of freedom

  York-Lichnerowicz split: $\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$

  $$K_{ij} = A_{ij} + \frac{1}{3}\gamma_{ij} K$$

  York & Lichnerozwicz, O'Murchadha & York,

  Wilson & Mathews, York

- Make simplifying assumptions

  Conformal flatness: $\tilde{\gamma}_{ij} = \delta_{ij}$

- Find good elliptic solvers

Two families of initial data

- Generalized analytic solutions:

  Isotropic Schwarzschild $ds^2 = \frac{M-2r}{M+2r} dt^2 + \left(1 + \frac{M}{2r}\right)^4 \left(dr^2 + r^2 d\Omega\right)$

  $\Rightarrow$ Time-symmetric N holes — Brill & Lindquist, Misner '60s

  $\Rightarrow$ Spin, Momenta — Bowen & York '80

  $\Rightarrow$ Punctures — Brandt & Brügmann '97

- Excision data: horizon boundary conditions

  Meudon Group, Pfeiffer, Ansorg

- Remaining problems: 1) junk radiation

  2) We often want zero eccentricity

Mesh refinement

3 Length scales :	BH	$\sim 1\ M$
	Wavelength	$\sim 10...100\ M$
	Wave zone	$\sim 100...1000\ M$

- Critical phenomena

  Choptuik '93

- First used for BBHs

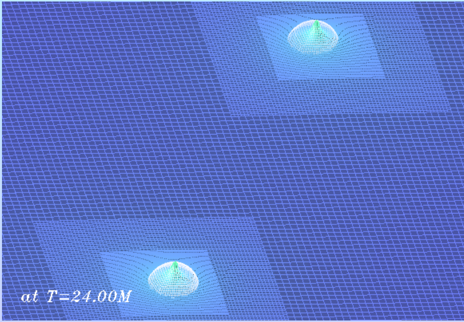
  Brügmann '96

- Available Packages:

  Paramesh MacNeice *et al.* '00

  Carpet Schnetter *et al.* '03

  SAMRAI MacNeice *et al.* '00



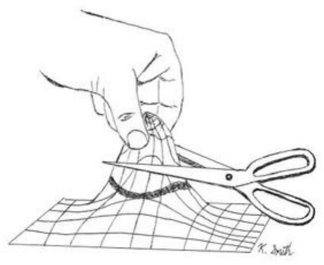
Singularity treatment

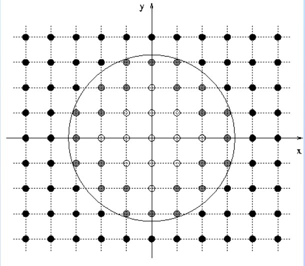
- Cosmic censorship ⇒ horizon protects outside
- We get away with it...

  Moving Punctures

  UTB, NASA Goddard '05
- Excision: Cut out region around singularity

  Caltech-Cornell, Pretorius





Extracting physics I: Global quantities

- ADM mass: Total energy of the spacetime

  $M_{\rm ADM} = \frac{1}{16\pi} \lim_{r\to\infty} \int_{S_r} \sqrt{\gamma}\gamma^{ij}\gamma^{kl} \left(\partial_j \gamma_{ik} - \partial_k \gamma_{ij}\right) dS_l$

- Total angular momentum of the spacetime

  $P_i = \frac{1}{8\pi} \lim_{r\to\infty} \int_{S_r} \sqrt{\gamma} \left(K^m{}_i - \delta^m{}_i K\right) dS_m$

  $J_i = \frac{1}{8\pi} \epsilon_{il}{}^m \lim_{r\to\infty} \int_{S_r} \sqrt{\gamma} x^l \left(K^n{}_m - \delta^n{}_m K\right) dS_n$

By construction all of these are time independent !!

Extracting physics II: Local quantities

- Often impossible to define!!
- Isolated horizon framework Ashtekar *et al.*

  $\rightarrow$ Calculate apparent horizon $\rightarrow$ Irreducible mass, momenta associated with horizon

  $M_{\rm irr} = \sqrt{\frac{A_{\rm AH}}{16\pi}}$
- Total BH mass Christodoulou

  $M^2 = M_{\rm irr}^2 + \frac{S^2}{4M_{\rm irr}^2} + P^2$
- Binding energy of a binary: $E_{\rm b} = M_{\rm ADM} - M_1 - M_2$

Extracting physics III: Gravitational Waves

- Most important diagnostic: Emitted GWs
- Newman-Penrose scalar

  $\Psi_4 = C_{\alpha\beta\gamma\delta} n^\alpha \bar{m}^\beta n^\gamma \bar{m}^\delta$

  Complex $\Rightarrow$ 2 free functions
- GWs allow us to measure

  $\rightarrow$ Radiated energy $E_{\rm rad}$

  $\rightarrow$ Radiated momenta $P_{\rm rad}$, $J_{\rm rad}$

  $\rightarrow$ Angular dependence of radiation

  $\rightarrow$ Gravitational wave strain h_+, $h_\times$

Angular dependence of GWs

- Waves are normally extracted at fixed radius $r_{\rm ex}$

  $\Rightarrow \Psi_4 = \Psi_4(t, \theta, \phi)$

  θ, ϕ are viewed from the source frame!

- Decompose angular dependence using spherical harmonics

  $\Psi_4 = \sum_{\ell,m} \psi_{\ell m}(t) Y^{-2} \ell m(\theta, \phi)$

  Modes $\psi_{\ell m}(t) = A_{\ell m}(t) \times e^{i\phi(t)}$

  Spin-weighted spherical harmonics $Y^{-2}_{\ell m}$

A brief history of BH simulations

- Pioneers: Hahn & Lindquist '60s, Eppley, Smarr *et al.* '70s
- Grand Challenge: First $3D$ Code Anninos *et al.* '90s
- Further attempts: Bona & Massó, Pitt-PSU-Texas

  AEI-Potsdam, Alcubierre *et al.*

  PSU: first orbit Brügmann *et al.* '04

Codes unstable!

- Breakthrough: Pretorius '05 GHG

  UTB, Goddard'05 Moving Punctures
- Currently about 10 codes world wide

3. Black holes in astrophysics

Free parameters of BH binaries

- Total mass M

  Relevant for GW detection: Frequencies scale with M

  Not relevant for source modeling: trivial rescaling

- Mass ratio $q \equiv \frac{M_1}{M_2}$, $\qquad \eta \equiv \frac{M_1 M_2}{(M_1+M_2)^2}$

- Spin: $\vec{S}_1$, $\vec{S}_2$ (6 parameters)

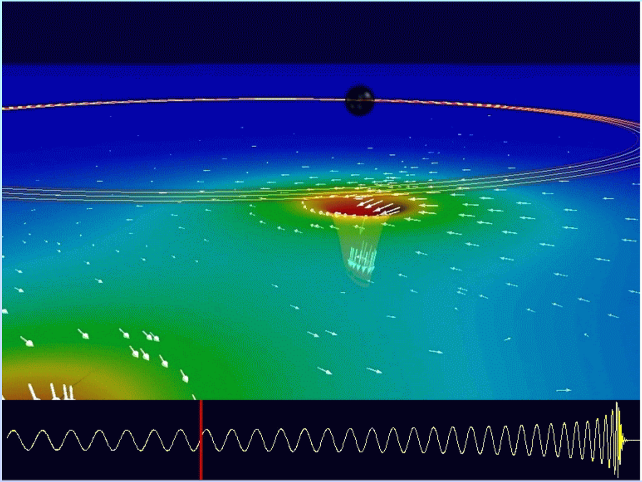
- Initial parameters

  Binding energy E_{b} | Separation

  Orbital ang. momentum L | Eccentricity

  Alternatively: frequency, eccentricity

Morphology of a BBH inspiral



Thanks to Caltech, CITA, Cornell

Gravitational recoil

- Anisotropic GW emission $\Rightarrow$ recoil of remnant BH

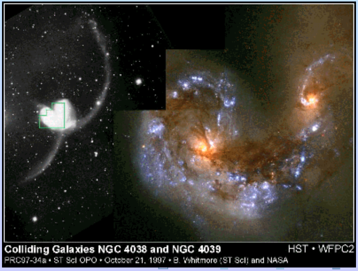
  Bonnor & Rotenburg '61, Peres '62, Bekenstein '73

- Escape velocities:

Globular clusters	30 $\mathrm{km/s}$
dSph	$20 - 100$ $\mathrm{km/s}$
dE	$100 - 300$ $\mathrm{km/s}$
Giant galaxies	~ 1000 $\mathrm{km/s}$

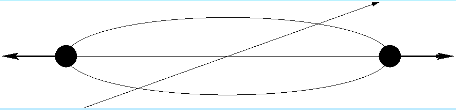
Ejection / displacement of BH $\Rightarrow$

- Growth history of SMBHs
- BH populations, IMBHs
- Structure of galaxies



Superkicks

- Kidder '95, UTB-RIT '07: maximum kick expected for



- Measured kicks $v \approx 2500\ \mathrm{km/s}$ for spin $a \approx 0.75$

  Extrapolated to maximal spins: $v_{\max} \approx 4000\ \mathrm{km/s}$

  González *et al.* '07, Campanelli *et al.* '07

- Unlikely configuration!

  Bogdanović *et al.* '07, Kesden, US & Berti '10, '10a

- Hyperbolic encounters: v up to $10000\ \mathrm{km/s}$

  Healy *et al.* '08

Spin precession and flip

- X-shaped radio sources

  Merrit & Ekers '07

- Jet along spin axis

- Spin re-alignment

  $\Rightarrow$ new + old jet

- Spin precession 98°

  Spin flip 71°

  UTB-RIT '06

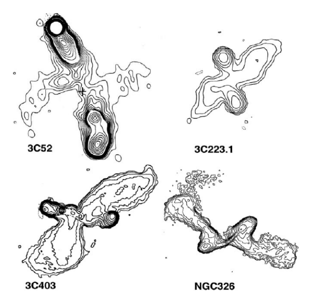
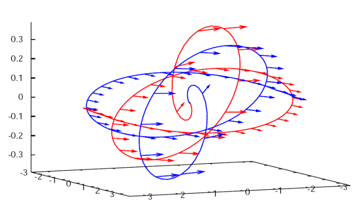


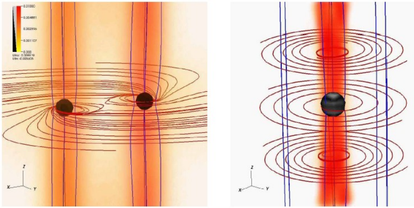
Fig. 2. A composite of four clear examples of the X-shape morphology. VLA radio observations of 3C52 (28), 3C223.1 (29), 3C403 (29), and NGC 326 (12).



Jets generated by binary BHs

Palenzuela, Lehner & Liebling '10

- Non-spinning BH binary
- Einstein-Maxwell equtions with "force free" plasma
- Electromagnetic field extracts energy from **L** $\Rightarrow$ jets
- Optical signature: double jets

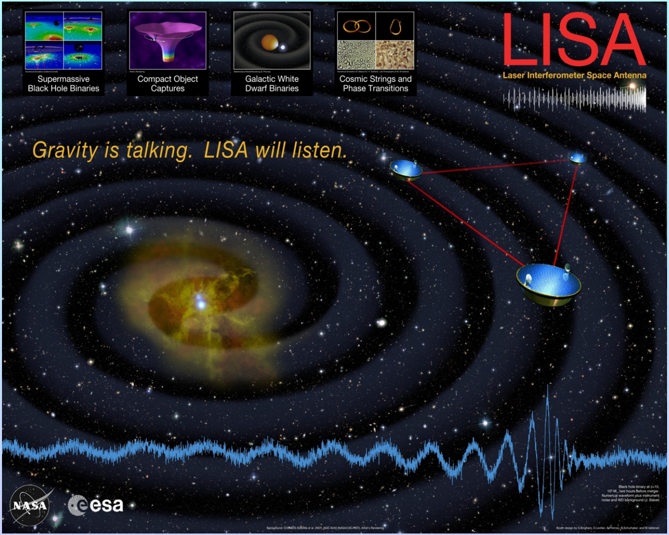


(a) $-8.2\, M_8$ hrs (b) $4.6\, M_8$ hrs

Gravitational Wave observations

- Accelerated masses generate GWs
- Interaction with matter very weak!
- Earth bound detectors: GEO600, LIGO, TAMA, VIRGO

Space interferometer LISA



Some targets of GW physics

- Confirmation of GR

  Hulse & Taylor 1993 Nobel Prize

- Parameter determination

  of BHs: $M, \ \vec{S}$

- Optical counter parts

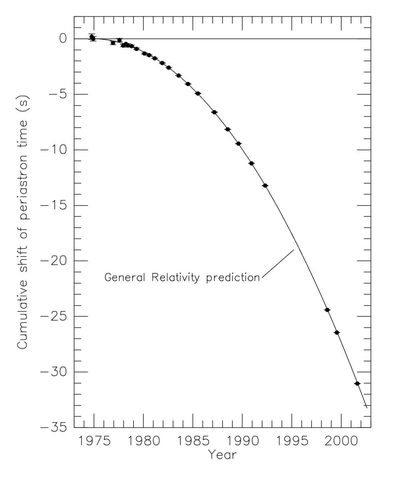
  Standard sirens (candles)

  Mass of graviton

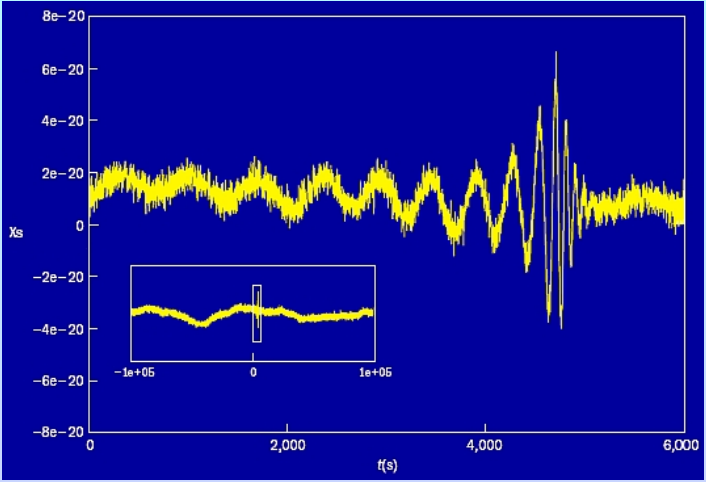
- Test Kerr Nature of BHs

- Cosmological sources

- Neutron stars: EOS



Matched filtering



Long, accurate waveforms required

⇒ combine NR with PN, perturbation theory

3. Black holes in fundamental physics

So what other interesting physics can we do with NR?

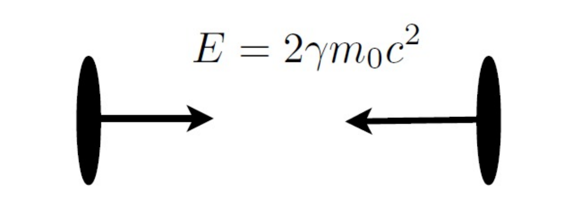
- High-energy physics
  - Trans-Planckian scattering
  - AdS/CFT duality
- Mathematical physics and theoretical physics
  - Cosmic censorship
  - Critical phenomena
  - BH instabilities (Myers-Perry)

3.1. Transplanckian scattering

BH formation and hoop conjecture

- Hoop conjecture

  Thorne '72



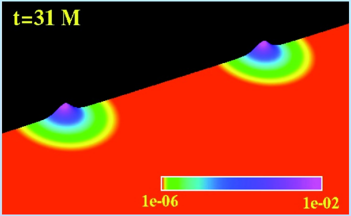
- de Broglie wavelength: $\lambda = \frac{hc}{E}$
- Schwarzschild radius: $r = \frac{2GE}{c^4}$
- BH will form if $\lambda < r \quad \Leftrightarrow \quad E \gtrsim \sqrt{\frac{hc^5}{G}} \equiv E_{\mathrm{Planck}}$

BH formation in boson field collisions

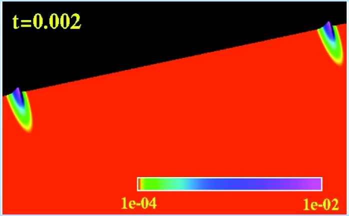
Pretorius & Choptuik ’09

- Einstein plus minimally coupled, massive, complex scalar filed

  “Boson stars”



$\gamma = 1$



$\gamma = 4$

- BH formation threshold: $\gamma_{\rm thr} = 2.9 \pm 10$ %
- About $1/3$ of hoop conjecture prediction

Motivation (High-energy physics)

- Matter does not matter at energies well above the Planck scale

  $\Rightarrow$ Model particle collisions by black-hole collisions

  Banks & Fischler '99; Giddings & Thomas '01

- TeV-gravity scenarios

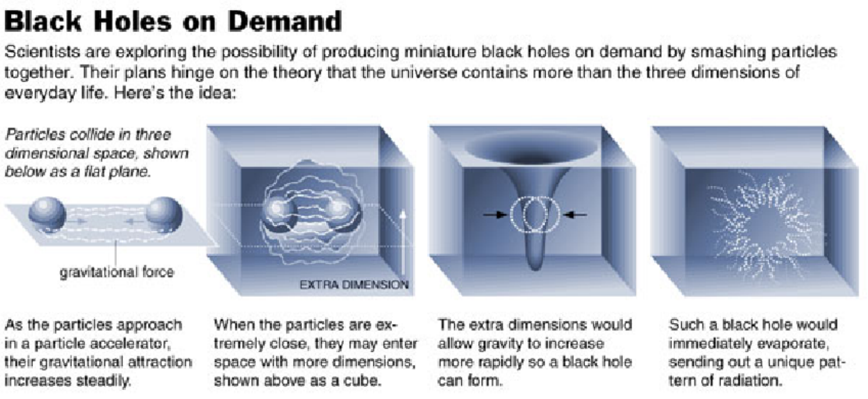
  $\Rightarrow$ The Planck scale might be as low as TeVs due to extra dimensions

  Arkani-Hamed, Dimopulos & Dvali '98, Randall & Sundrum '99

  $\Rightarrow$ Black holes could be produced in colliders

  Eardley & Giddings '02, Dimopoulos & Landsberg '01,...

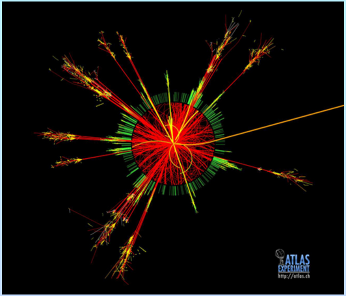
Motivation (High-energy physics)



Experimental signature at the LHC

Black hole formation at the LHC could be detected by the properties of the jets resulting from Hawking radiation.

- Multiplicity of partons: Number of jets and leptons
- Large transverse energy
- Black-hole mass and spin are important for this!



ToDo:

- Exact cross section for BH formation
- Determine loss of energy in gravitational waves
- Determine spin of merged black hole

Black-hole collisions in $D = 4$

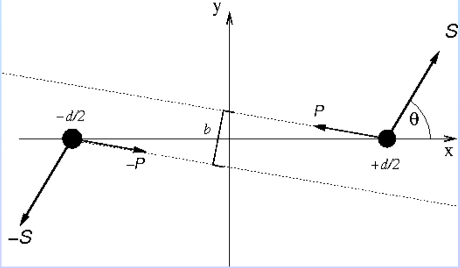
- Take two black holes

  Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

  Initial position: $\pm x_0$

  Linear momentum: $\mp P[\cos\alpha,\ \sin\alpha,\ 0]$

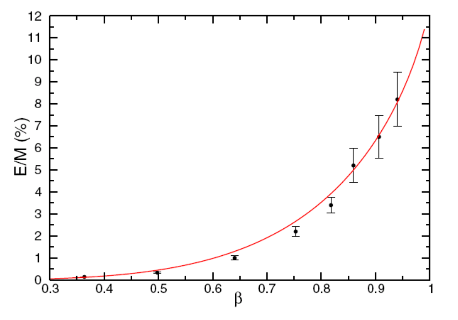
- Impact parameter: $b \equiv \frac{L}{P}$



Head-on collisions: $b = 0$, $\vec{S} = 0$

- Total radiated energy: 14 ± 3 % for $v \to 1$ Sperhake *et al.* '08

  About half of Penrose '74

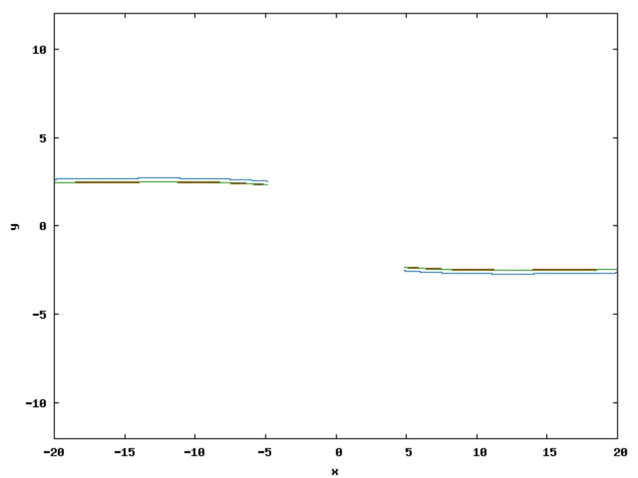


- Agreement with approximative methods

  Flat spectrum, multipolar GW structure Berti *et al.* '10

Grazing collisions: $b \neq 0, \quad \vec{S} = 0, \quad \gamma = 1.52$

Immediate vs. Delayed vs. No merger   Sperhake *et al.* '09



Critical impact parameter

- $b < b_{\rm crit} \quad \Rightarrow \quad$ Merger

  $b > b_{\rm crit} \quad \Rightarrow \quad$ Scattering

- Numerical study: $\quad b_{\rm crit} = \frac{2.5 \pm 0.05}{v} M$

  Shibata *et al.* '08

- Independent study by Sperhake *et al.* '09

  $\gamma = 1.52$: $\quad 3.39 < b_{\rm crit}/M < 3.4$

  $\gamma = 2.93$: $\quad 2.3 < b_{\rm crit}/M < 2.4$

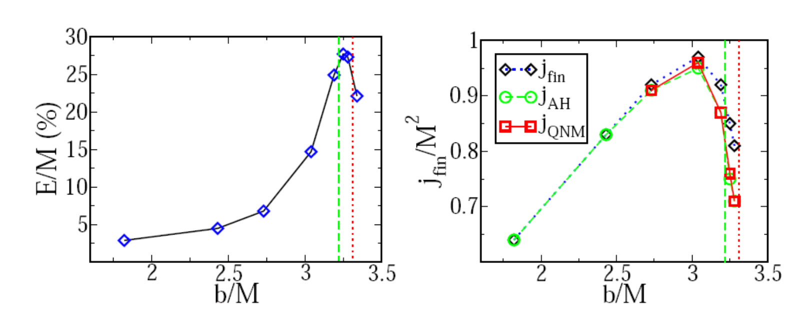
  $v \to 1$ limit still needs to be determined

- Limit from Penrose construction: $b_{\rm crit} = 1.685\, M$

  Yoshino & Rychkov '05

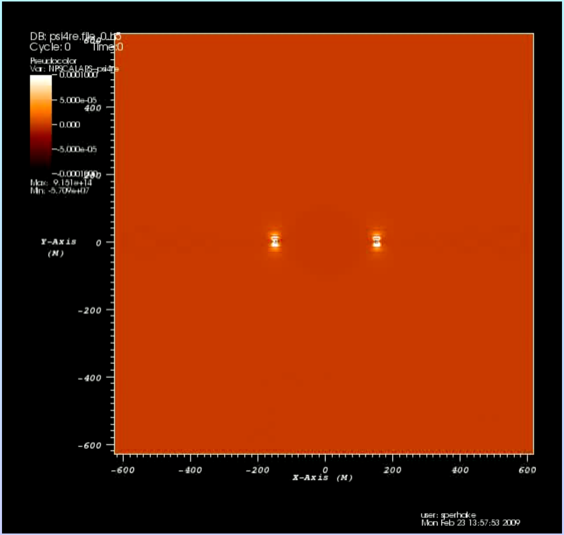
Radiated quantities

- $b-$sequence with $\gamma = 1.52$
- Final spin close to Kerr limit
- $E_{\rm rad} \sim 35$ % for $\gamma = 2.93$; about 10 % of Dyson luminosity

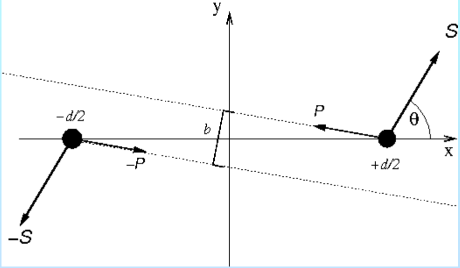


Sperhake *et al.* '09

Gravitational radiation: Delayed merger



Recoil in grazing collisions

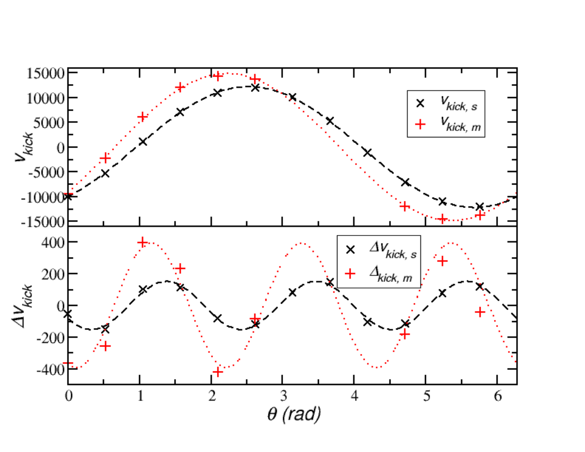


- equal-mass, superkick, $\chi = 0.621$
- $\gamma = 1.52$
- 2 sequences

  merging: $b = 3.34\ M$

  scattering: $b = 3.25\ M$

Recoil in grazing collisions



Expansion in θ according to Boyle, Kesden & Nissanke '08

Recoil in grazing collisions

- $v_{\mathrm{max,s}} = 12\,200~\mathrm{km/s}$

  $v_{\mathrm{max,m}} = 14\,900~\mathrm{km/s}$

- Large recoils for merger and scattering!

- $v_{\mathrm{max}} \propto E_{\mathrm{rad}}$

- Antikicks can occur in both $\Rightarrow$ not a merger-only feature!

- Ultimate kick

  $v_{\mathrm{max}} \propto E_{\mathrm{rad}} \quad \Rightarrow \quad \sim 45\,000~\mathrm{km/s}$

  spin insignificant for large $\gamma \quad \Rightarrow \quad \sim 25\,000~\mathrm{km/s}$

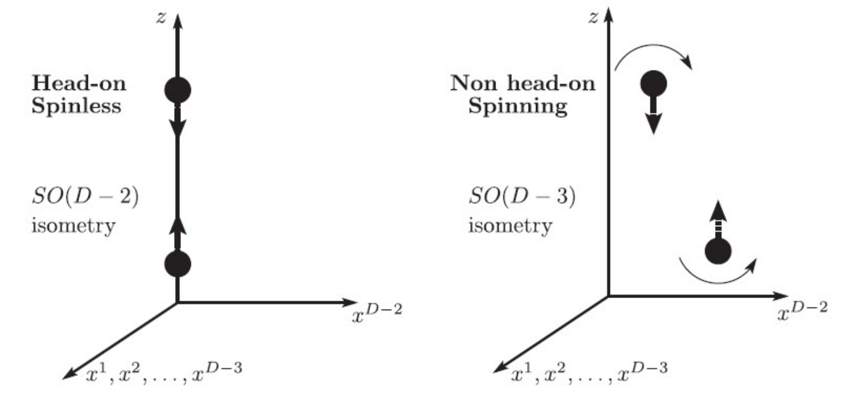
  no simple picture $\quad \Rightarrow \quad$ more data needed...

Moving to $D > 4$

- Symmetries allow dimensional reduction

  Geroch '70

- Reduces to "3+1" plus quasi-matter terms: scalar field



BSSN formulation with quasi matter

$$\partial_t \tilde{\gamma}_{ij} = [\mathrm{BSSN}],$$

$$\partial_t \chi = [\mathrm{BSSN}],$$

$$\partial_t K = [\mathrm{BSSN}] + 4\pi\alpha(E + S),$$

$$\partial_t \tilde{A}_{ij} = [\mathrm{BSSN}] - 8\pi\alpha\left(\chi S_{ij} - \frac{1}{3} S \tilde{\gamma}_{ij}\right),$$

$$\partial_t \tilde{\Gamma}^i = [\mathrm{BSSN}] - 16\pi\alpha\chi^{-1} j^i,$$

$$\partial_t \zeta = -2\alpha K_\zeta + \beta^m \partial_m \zeta - \frac{2}{3}\zeta \partial_m \beta^m + 2\zeta \frac{\beta^y}{y},$$

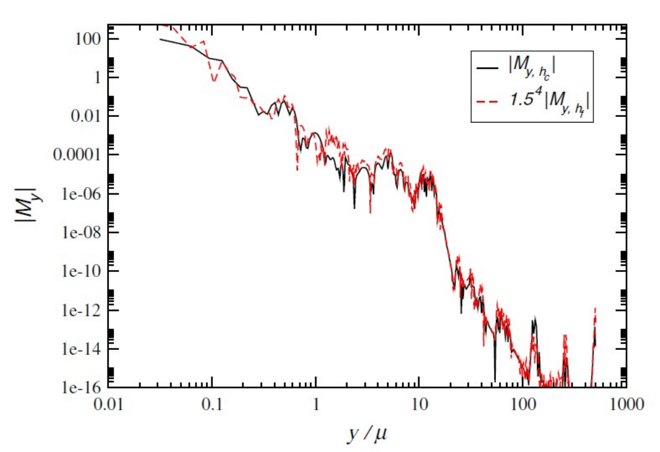
$$\partial_t K_\zeta = \ldots \quad ,$$

$$E,\ j^i,\ S_{ij} = f(\mathrm{BSSN},\ \zeta,\ K_\zeta).$$

Zilhão *et al.* '10

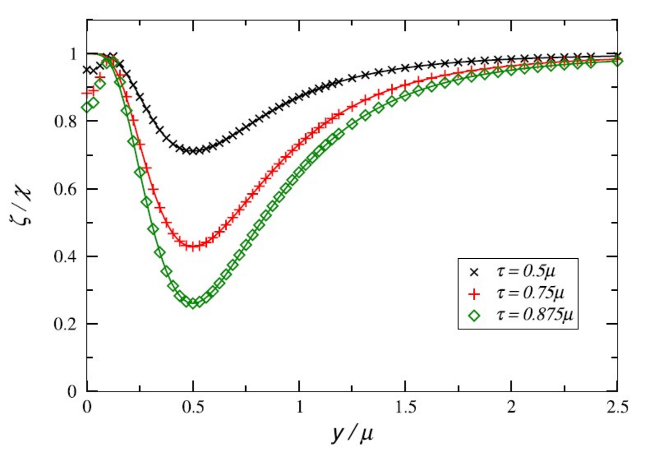
Single black hole in $D = 5$

Initial data: Tangherlini '63



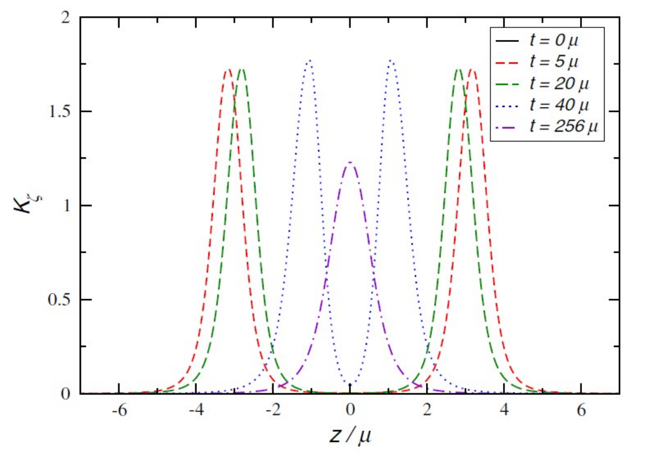
Single black hole in $D = 5$

In geodesic slicing

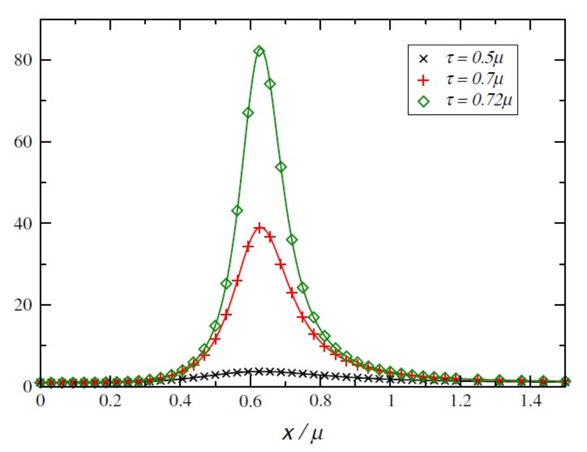


Head-on in $D = 5$

Initial data: $D = 5$ analogue of Brill-Lindquist data

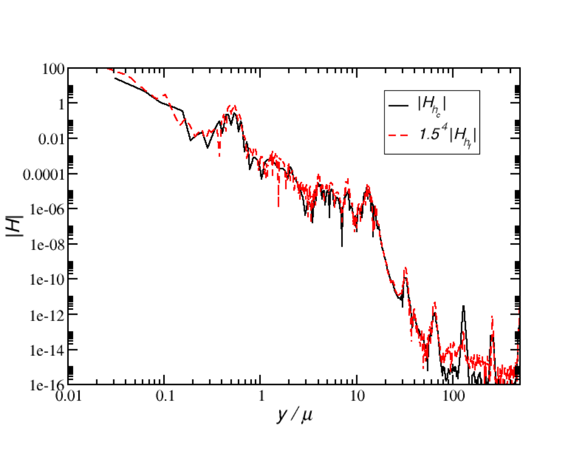


Single black hole in $D = 6$



Geoesic slicing, zero shift

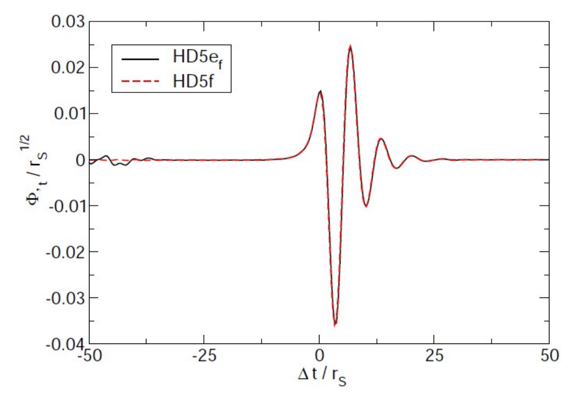
Single black hole in $D = 6$



ToDo: long term stable evolutions

GWs from head-on in $D = 5$

Wave extraction based on Kodama & Ishibashi '03

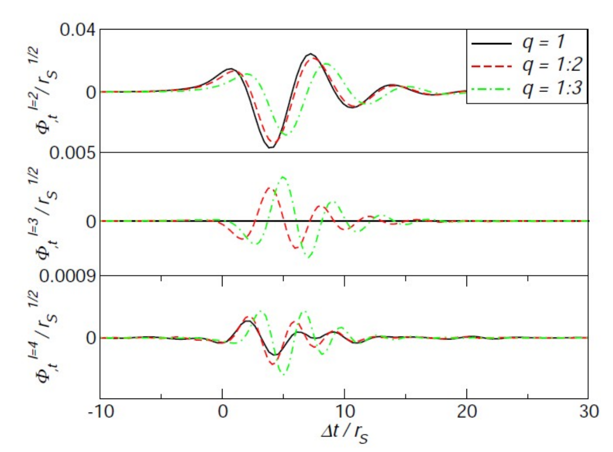


$E_{\rm rad} = 0.089\,\%M$ cf. $0.055\,\%M$ in $D = 4$

Witek *et al.* '10a

Unequal-mass head-on in $D = 5$

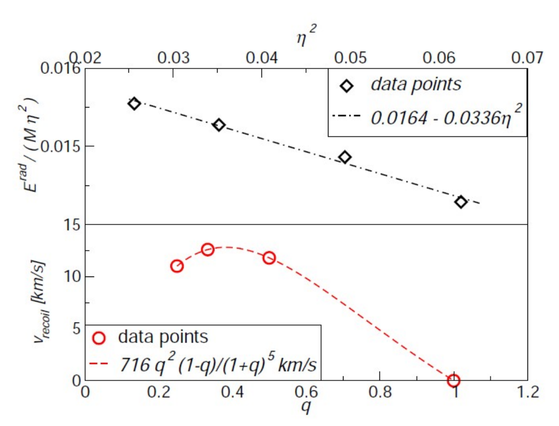
Kodama-Ishibashi multipoles



Witek *et al.* '10b

Unequal-mass head-on in $D = 5$

Radiated energy and momentum

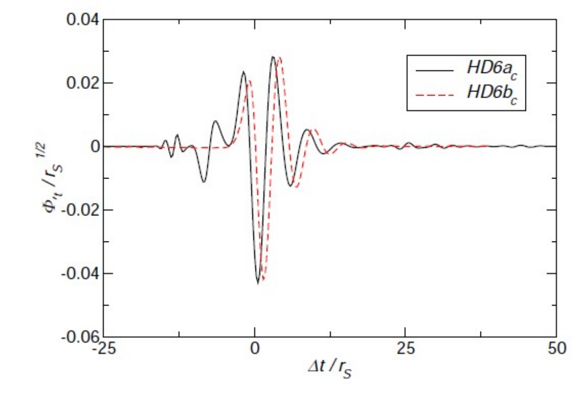


Agreement within < 5 % with extrapolated point particle calculations

Breaking news!

First black-hole collisions in $D = 6$

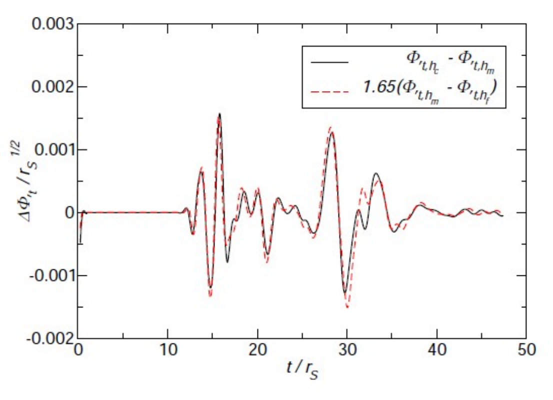
Witek *et al. '10*



- Adjust shift parameters
- Use LaSh system Witek, Hilditch & US '10

First black-hole collisions in $D = 6$

Witek *et al. '10*

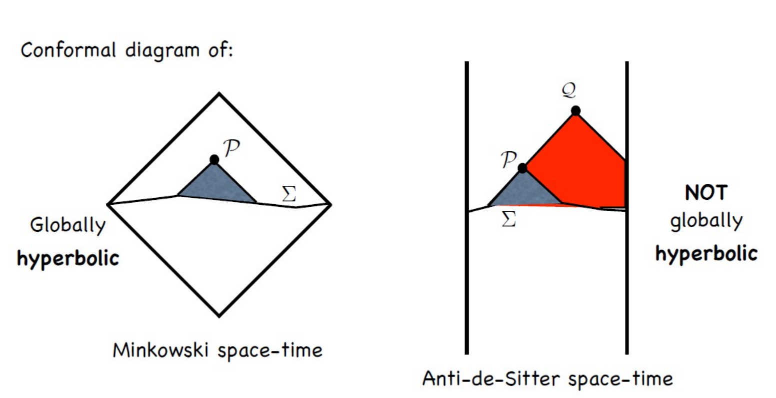


Second order convergence

3.2. Non-assymptotically flat boundaries: AdS/CFT

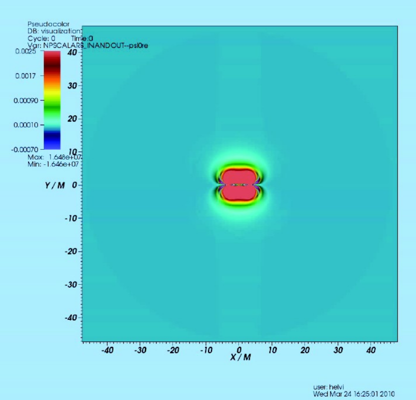
AdS/CFT correspondence

Challenge: Model the active role of the boundary!



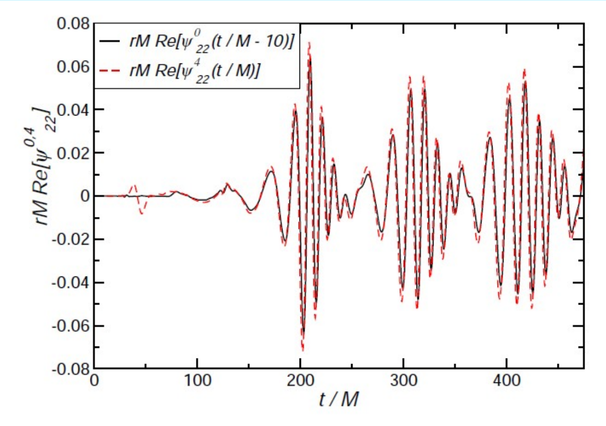
Toy model: Black hole inspiral in a lego sphere

- Lego sphere with reflective boundary
- Goddard R1 run   Baker *et al.* '06
- Calculate Ψ_4 and Ψ_0

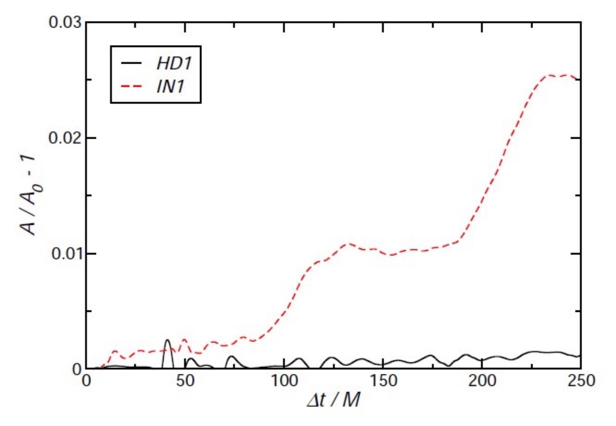


Quadrupole mode

Gravitational radiation (out going and ingoing)



Horizon area



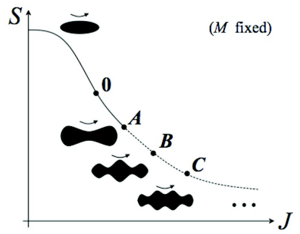
Superradiance: high frequency absorbed, low frequency amplified
No conclusive evidence yet...

4. Other topics in $D \geq 5$

Other topics: Instabilities of Myers-Perry

- Ultra-spinning Myers-Perry black holes (with single angular momentum parameter) should be unstable.
- Confirmed by linearized analysis of axisymmetric perturbations

Dias *et al.* '09



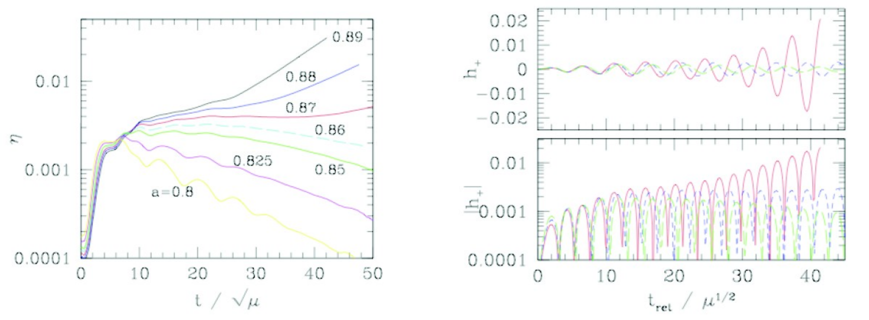
d	$(a/r_m)\|_{\ell=1}$	$(a/r_m)\|_{\ell=2}$	$(a/r_m)\|_{\ell=3}$
7	1.075	1.714	2.141
8	1.061	1.770	2.275
9	1.051	1.792	2.337

Other topics: Instabilities of Myers-Perry

- Numerical study of non-axisymmetric instabilities of $D = 5$ Myers-Perry BH with single ang. momentum parameter.

  Shibata & Yoshino '09

- Found onset of instabilities at spin $a/\sqrt{\mu} \approx 0.87$



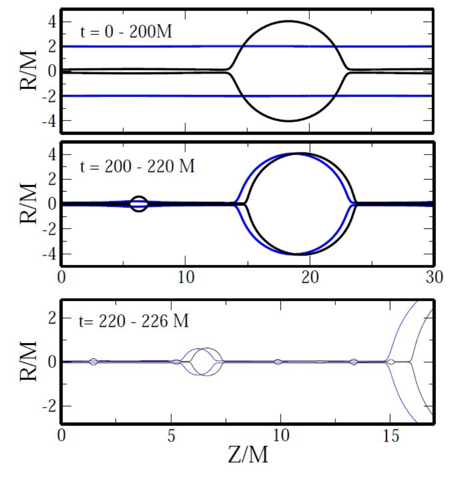
Other topics: Cosmic censorship in $D = 5$

Pretorius & Lehner '10

- Axisymmetric code
- Study evolution of black string...
- Gregory-Laflamme instability cascades down

  until string reaches zero radius

  $\Rightarrow$ naked singularity



5. Summary

Summary

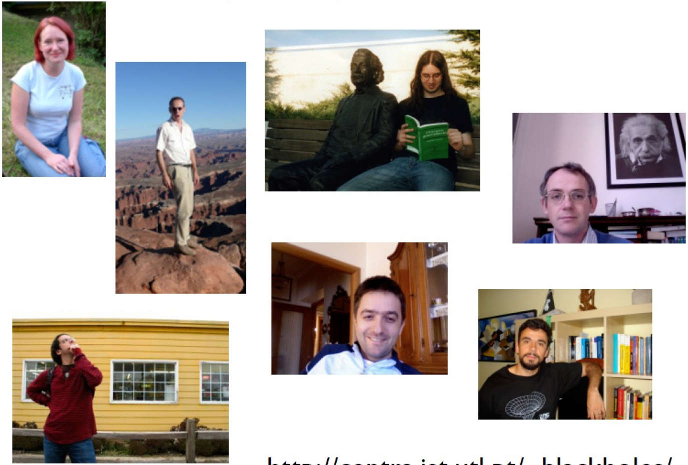
- Black holes are real objects in many areas of physics!
- Astrophysics: Recoil, Spin flips, jets
- Gravitational wave physics: template banks needed
- High-energy collisions in $D = 4$:

  largest kicks $\sim 15\,000\ \mathrm{km/s}$

  largest radiation $\sim 30\ \%$

  largest post-merger spin $a \lesssim 1$
- Formalism for arbitrary spatial dimension D
- Head-on collisions from rest
- Test non-assymptotically flat OBCs
- Signs of cosmic censorship violation in $D = 5$

The team



http://centra.ist.utl.pt/~blackholes/