Black-hole collisions and gravitational waves

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Overview

- Motivation
- Introduction
- Ingredients of numerical relativity
- Results
  - Precambrium: before the 2005 explosion
  - Gravitational wave observations
  - Black holes in astrophysics
  - Black holes in fundamental physics
- Summary

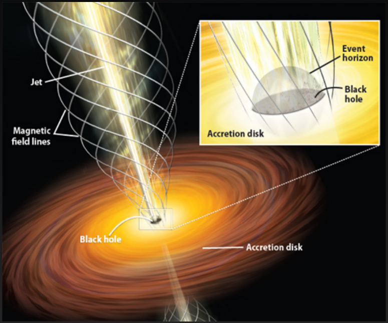
1. Motivation

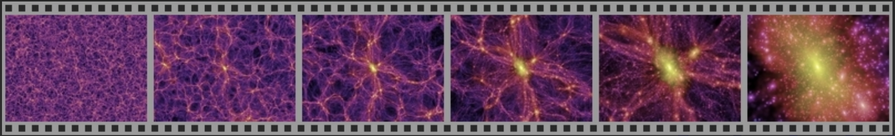
Black holes in Astrophysics

- Black holes are important in many astrophysical processes
  - Galaxies host BHs
  - Important sources of

    electromagnetic radiation
  - Structure formation

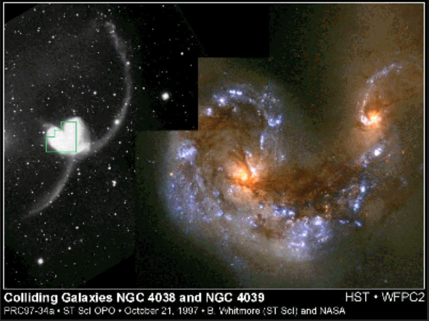
    in the Universe





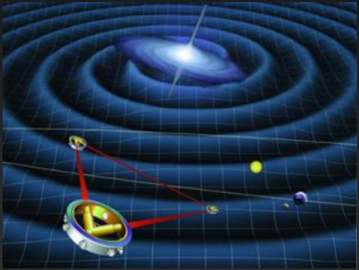
Black holes in Astrophysics

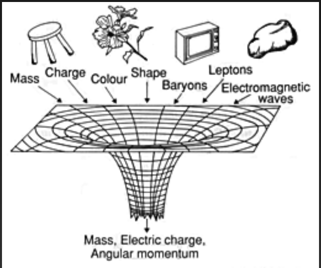
- Black holes are important in many astrophysical processes
  - Structure of galaxies
  - Cosmic projectiles



Black holes in Fundamental Physics

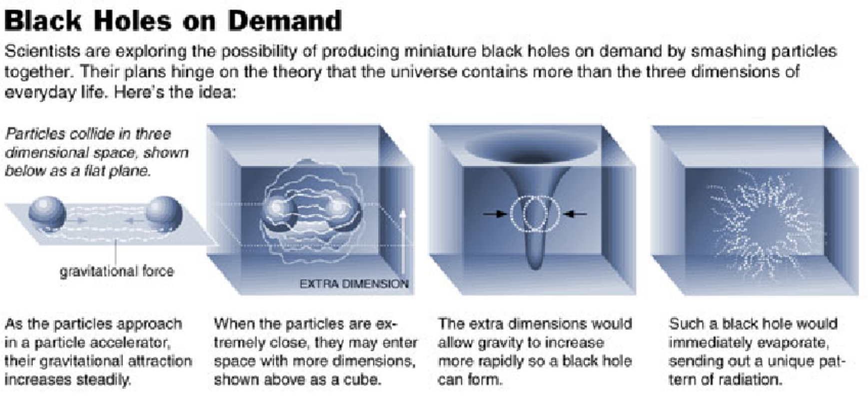
- Black holes allow new tests of fundamental physics
  - Strongest sources of Gravitational Waves (GWs)
  - Test alternative theories of Gravity
  - No-hair theorem of GR





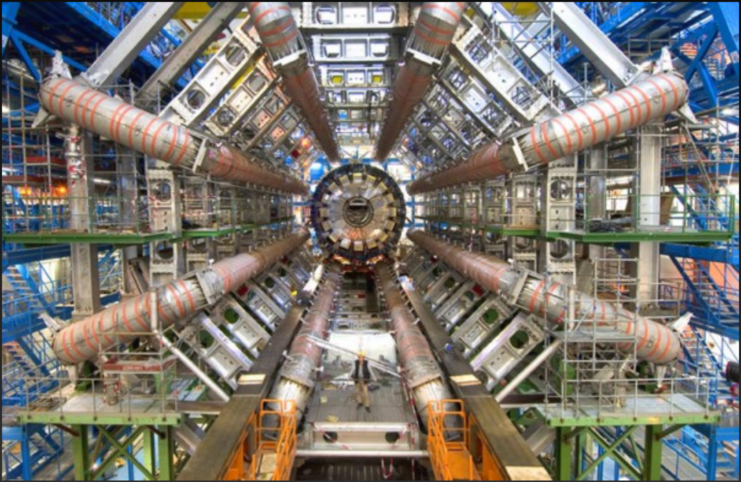
Black holes in Fundamental Physics

- Black holes allow new tests of fundamental physics
  - Production in particle accelerators



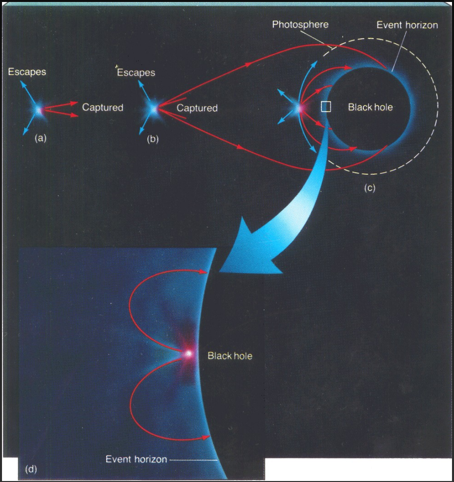
Black holes in Fundamental Physics

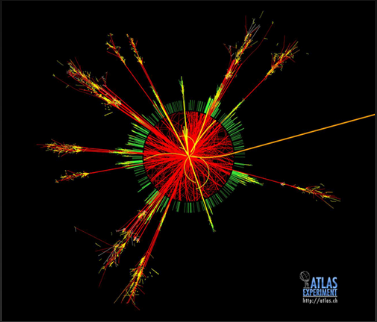
- LHC CERN



Black holes in Fundamental Physics

- BH evaporation via Hawking radiation





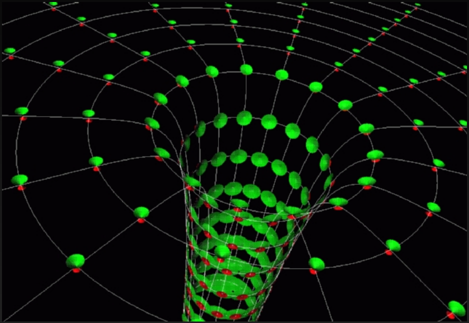
Black holes in Fundamental Physics

- BH spacetimes “know” about physics without BHs
  AdS-CFT Correspondence

2. What are Black Holes?

How to characterize Black HoleS?

- Consider Lightcones
- In and outgoing light
- Calculate surface of outgoing light fronts
- Expansion ≡ Rate of change of this surface
- Apparent Horizon ≡ Outermost surface with zero expansion
- "Light cones tip over" due to curvature



Schwarzschild metric

Schwarzschild–Metrik

$$\mathbf{g}^{\mu\nu} = \begin{pmatrix} g^{tt} & 0 & 0 & 0 \\ 0 & g^{rr} & 0 & 0 \\ 0 & 0 & g^{\theta\theta} & 0 \\ 0 & 0 & 0 & g^{\phi\phi} \end{pmatrix} = \begin{pmatrix} -1/(1-2\,M/r) & 0 & 0 & 0 \\ 0 & (1-2\,M/r) & 0 & 0 \\ 0 & 0 & 1/r^2 & 0 \\ 0 & 0 & 0 & 1/(r\sin\theta)^2 \end{pmatrix}$$

$$\mathbf{g}_{\mu\nu} = \begin{pmatrix} g_{tt} & 0 & 0 & 0 \\ 0 & g_{rr} & 0 & 0 \\ 0 & 0 & g_{\theta\theta} & 0 \\ 0 & 0 & 0 & g_{\phi\phi} \end{pmatrix} = \begin{pmatrix} -(1-2\,M/r) & 0 & 0 & 0 \\ 0 & 1/(1-2\,M/r) & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}$$

- Unfortunate coordinates
- Singularity at $r = 2\,M$
- What does this mean?

Kruskal coordinates

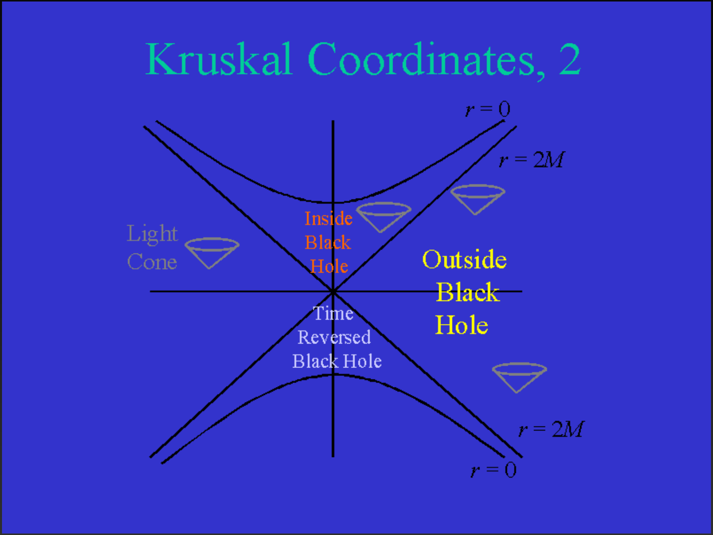
Kruskal Coordinates, 1

With $u = \pm\left|\frac{r}{2M} - 1\right|^{1/2} e^{r/4M} \cosh\left(\frac{ct}{4M}\right)$

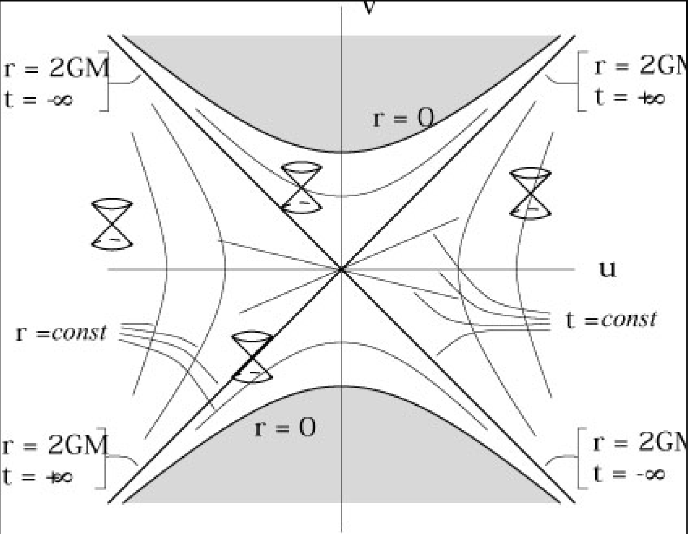
and $v = \pm\left|\frac{r}{2M} - 1\right|^{1/2} e^{r/4M} \sinh\left(\frac{ct}{4M}\right)$

The Schwarzschild solution is represented by the following:

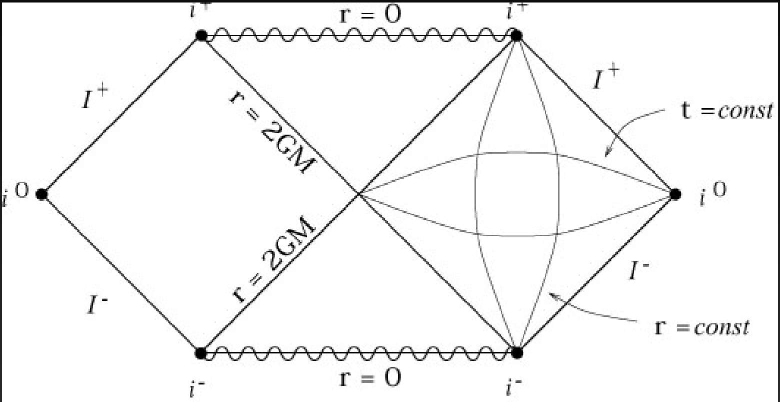
Kruskal coordinates



Kruskal coordinates



Penrose diagram

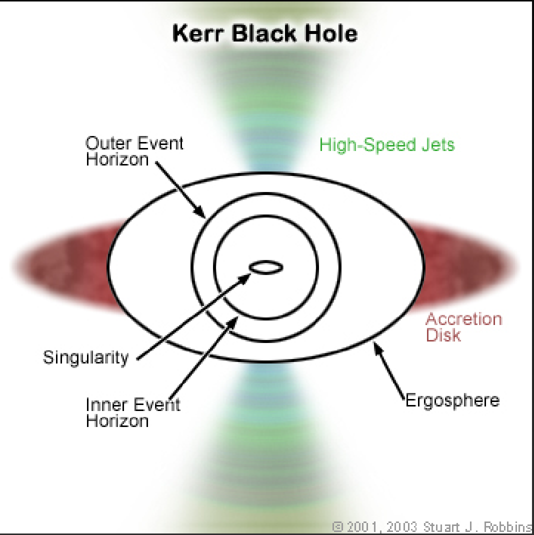


Rotating BHs: Kerr metric

$$
\begin{aligned}
ds^2 &= -\frac{\Delta_r}{\chi^2\,\rho^2}\,\left(dt - a\,\sin^2\theta\,d\phi\right)^2 + \frac{\Delta_\theta\,\sin^2\theta}{\chi^2\,\rho^2} \\
&\quad \times\left[a\,dt - (r^2+a^2)\,d\phi\right]^2 + \rho^2\,\left(\frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta}\right)
\end{aligned}
$$

$$
\begin{aligned}
\rho^2 &= r^2 + \cos^2\theta \\
\Delta_r &= (r^2+a^2)\,(1-\frac{1}{3}\,\Lambda r^2) - 2Mr + Q^2 \\
\Delta_\theta &= 1 + \frac{1}{3}\,\Lambda a^2\cos^2\theta \\
\chi &= 1 + \frac{1}{3}\,\Lambda a^2
\end{aligned}
$$

Rotating BHs: Kerr BH



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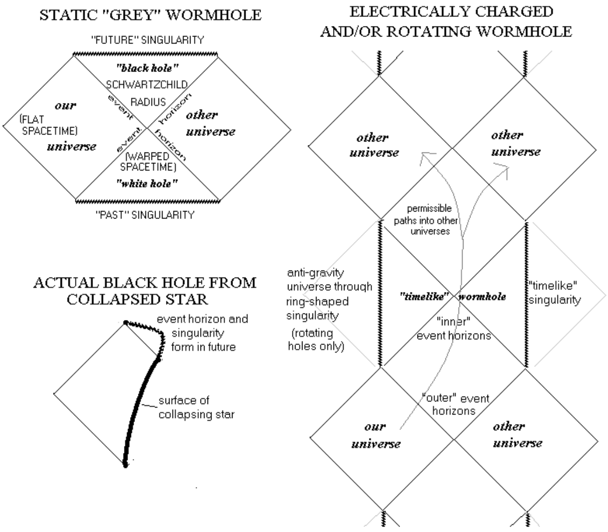
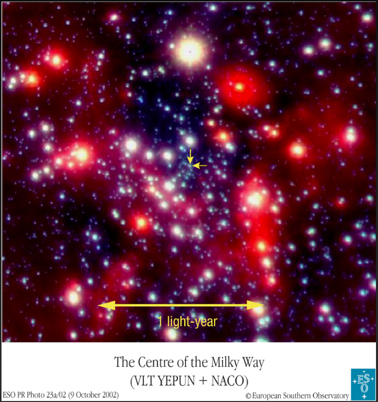
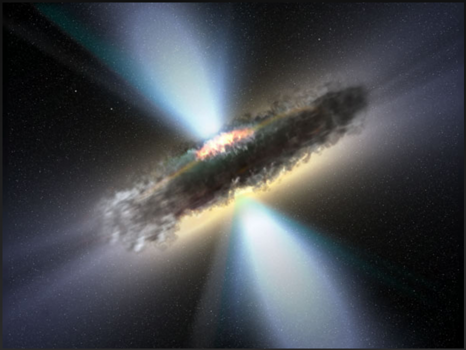


BHs for astrophysicists

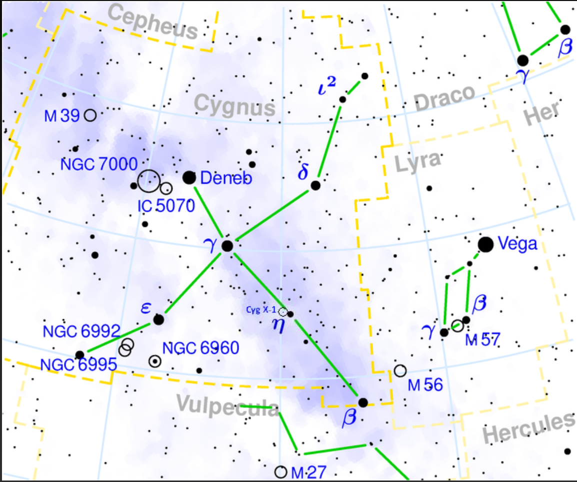
- Supermassive BHs found at center of virtually all galaxies





Stellar BHs

- In stellar binary systems: Cygnus XR-1



Stellar BHs

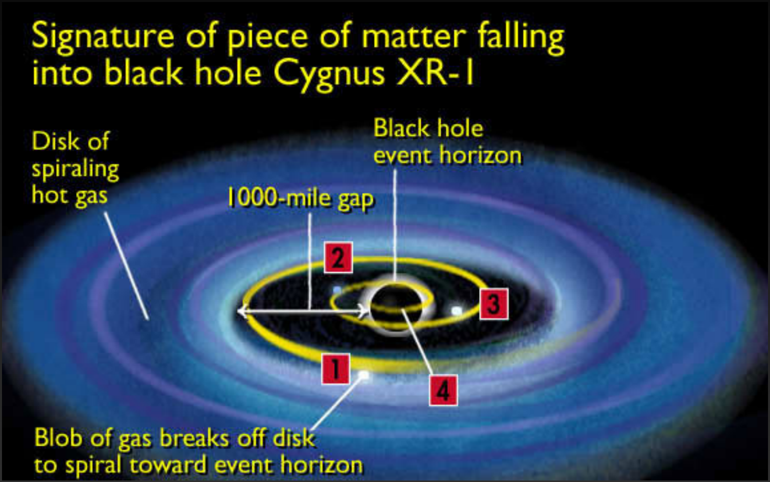
- X ray source!

Stellar BHs

- One member is very compact and massive $\Rightarrow$ Black Hole!

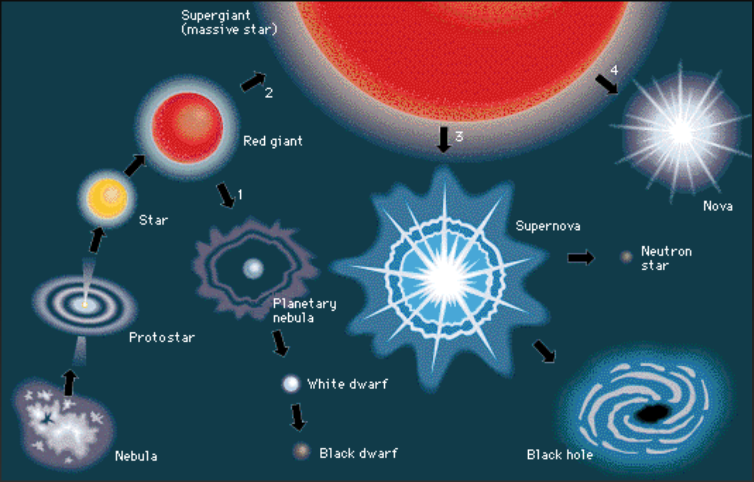
Stellar BHs

- Mass transfer, accretion



How are Black Holes formed?

- Stellar BHs: Supernovae



3. Gravitational Wave observations

Gravitational Waves

- Einstein's equations have wave like solutions:

  Gravitational Waves: $h_{ij} = h_{ij}(r - t)$

- Effect on test particles

Gravitational Wave detectors

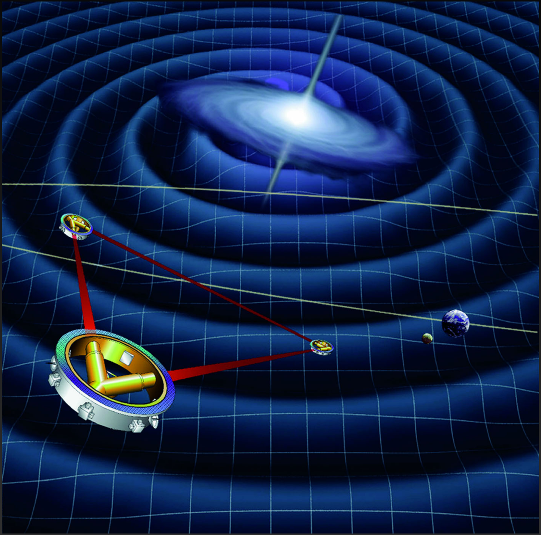
- Accelerated masses generate GWs
- Interaction with matter very weak!
- Earth bound detectors: GEO600, LIGO, TAMA, VIRGO

Detection principle

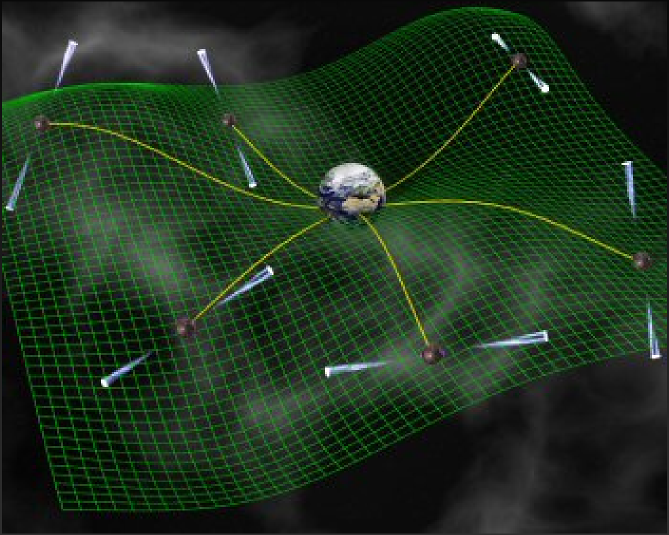
- Principle of measurement: Michelson-Morley interferometer

  but muuuuuuch more accurate: fraction of nucleus per km

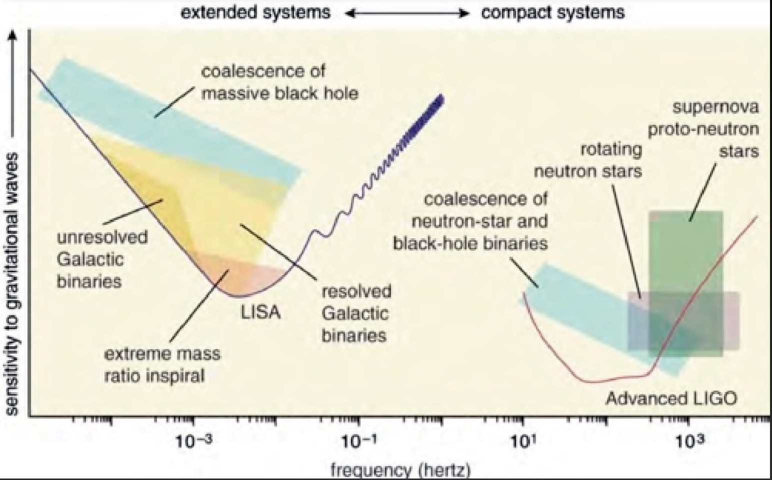
Space interferometer LISA



Pulsar timing arrays



Expected sources



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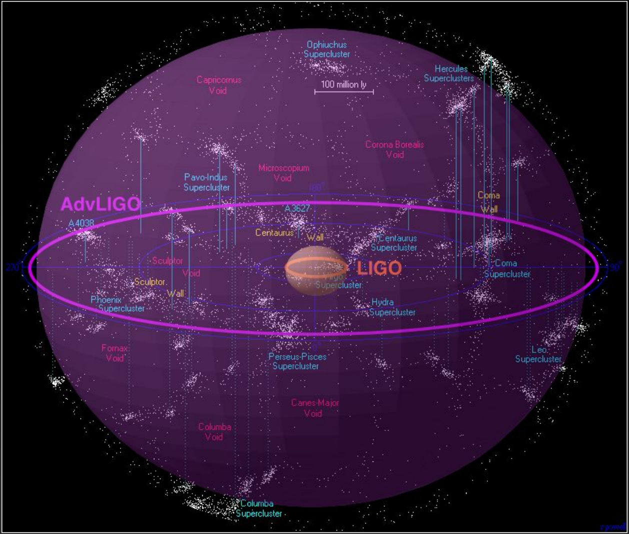


Some targets of GW physics

- Confirmation of GR

  Hulse & Taylor 1993 Nobel Prize

- Parameter determination of BHs: M, $\vec{S}$

- Optical counter parts

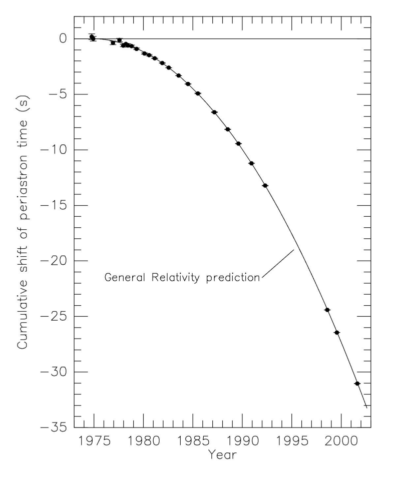
  Standard sirens (candles)

  Mass of graviton

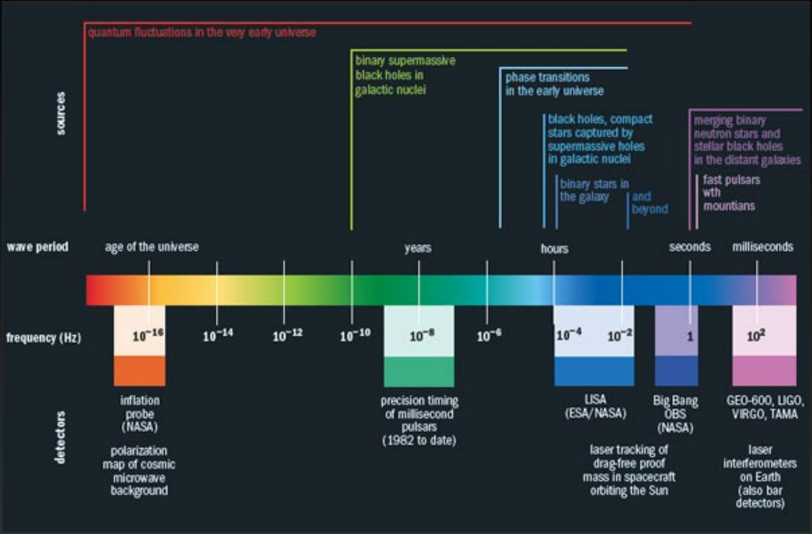
- Test Kerr Nature of BHs

- Cosmological sources

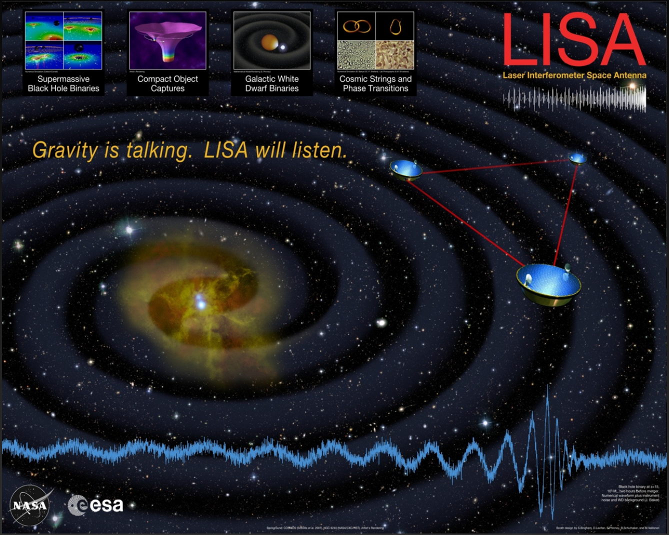
- Neutron stars: EOS



Some targets of GW physics



GW physics with LISA



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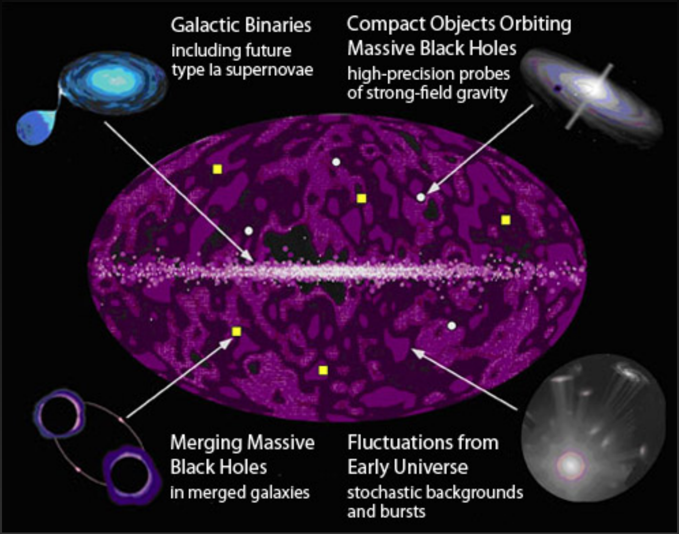

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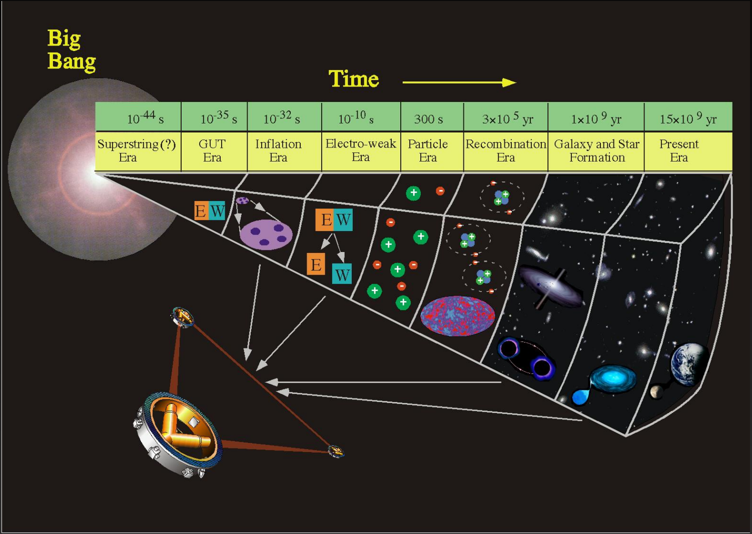
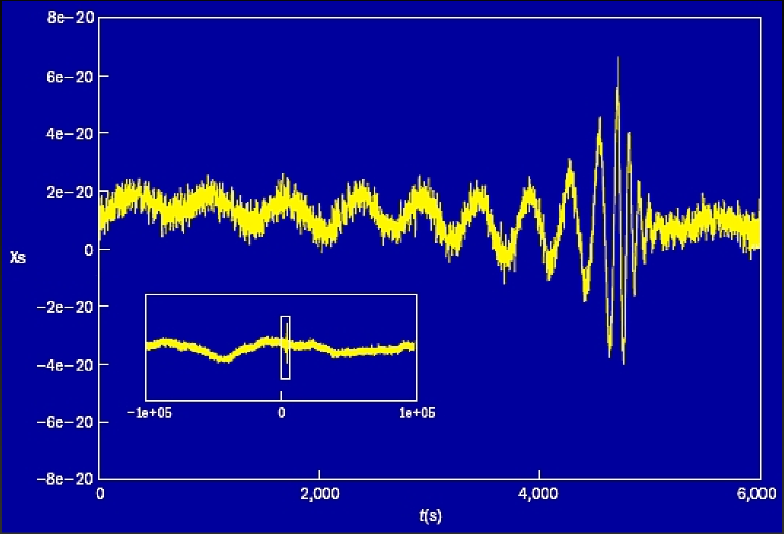


Matched filtering



3. Numerical Framework

General Relativity: Curvature

- Curvature generates acceleration

  "geodesic deviation"

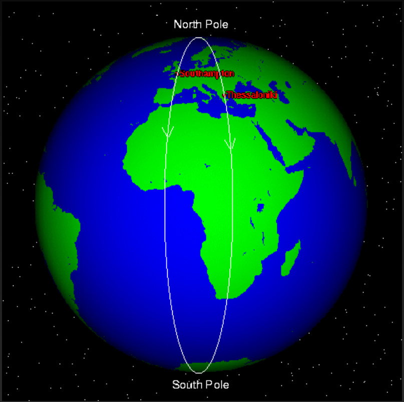
  No "force"!!

- Description of geometry

  Metric $g_{\alpha\beta}$

  Connection $\Gamma^{\alpha}_{\beta\gamma}$

  Riemann Tensor $R^{\alpha}{}_{\beta\gamma\delta}$



The metric defines everything

- Christoffel connection

  $$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} \left(\partial_{\beta} g_{\gamma\mu} + \partial_{\gamma} g_{\mu\beta} - \partial_{\mu} g_{\beta\gamma} \right)$$

- Covariant derivative

  $$\nabla_{\alpha} T^{\beta}{}_{\gamma} = \partial_{\alpha} T^{\beta}{}_{\gamma} + \Gamma^{\beta}_{\mu\alpha} T^{\mu}{}_{\gamma} - \Gamma^{\mu}_{\gamma\alpha} T^{\beta}{}_{\mu}$$

- Riemann Tensor

  $$R^{\alpha}{}_{\beta\gamma\delta} = \partial_{\gamma} \Gamma^{\alpha}_{\beta\delta} - \partial_{\delta} \Gamma^{\alpha}_{\beta\gamma} + \Gamma^{\alpha}_{\mu\gamma} \Gamma^{\mu}_{\beta\delta} - \Gamma^{\alpha}_{\mu\delta} \Gamma^{\mu}_{\beta\gamma}$$

- $\Rightarrow$ Geodesic deviation,

  Parallel transport,

  ...

How to get the metric?

- The metric must obey the Einstein Equations
- Ricci-Tensor, Einstein Tensor, Matter Tensor

  $R_{\alpha\beta} \equiv R^{\mu}{}_{\alpha\mu\beta}$

  $G_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R^{\mu}{}_{\mu}$ "Trace reversed" Ricci

  $T_{\alpha\beta}$ "Matter"

- Einstein Equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$
- Solutions: Easy! Take metric

  $\Rightarrow$ Calculate $G_{\alpha\beta}$

  $\Rightarrow$ Use that as matter tensor

- Physically meaningful solutions: Difficult!

The Einstein Equations in vacuum

- “Spacetime tells matter how to move,
  matter tells spacetime how to curve”
- Field equations in vacuum: $R_{\alpha\beta} = 0$
  Second order PDEs for the metric components
  Invariant under coordinate (gauge) transformations
- System of equations extremely complex: Pile of paper!
  Analytic solutions: Minkowski, Schwarzschild, Kerr,
  Robertson-Walker, ...
- Numerical methods necessary for general scenarios!!!

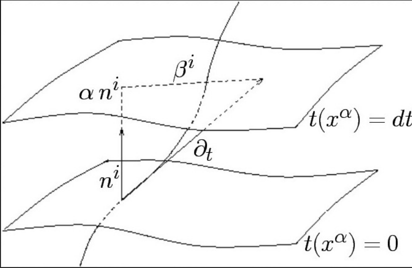
A list of tasks

- Target: Predict time evolution of BBH in GR
- Einstein equations: 1) Cast as evolution system
  2) Choose specific formulation
  3) Discretize for computer
- Choose coordinate conditions: Gauge
- Fix technical aspects: 1) Mesh refinement / spectral domains
  2) Singularity handling / excision
  3) Parallelization
- Construct realistic initial data
- Start evolution and waaaaiiiiit...
- Extract physics from the data

3+1 Decomposition

- GR: “Space and time exist as a unity: Spacetime”
- NR: ADM 3+1 split   Arnowitt, Deser & Misner ’62
  York ’79, Choquet-Bruhat & York ’80

$$g_{\alpha\beta} = \left(\begin{array}{c|c} -\alpha^2 + \beta_m \beta^m & \beta_j \\ \hline \beta_i & \gamma_{ij} \end{array} \right)$$



- 3-Metric γ_{ij}
  Lapse α
  Shift β^i
- lapse, shift $\Rightarrow$ Gauge

ADM Equations

The Einstein equations $R_{\alpha\beta} = 0$ become

- 6 Evolution equations

  $$(\partial_t - \mathcal{L}_\beta)\gamma_{ij} = -2\alpha K_{ij}$$

  $$(\partial_t - \mathcal{L}_\beta)K_{ij} = -D_i D_j \alpha + \alpha[R_{ij} - 2K_{im}K^m{}_j + K_{ij}K]$$

- 4 Constraints

  $$R + K^2 - K_{ij}K^{ij} = 0$$

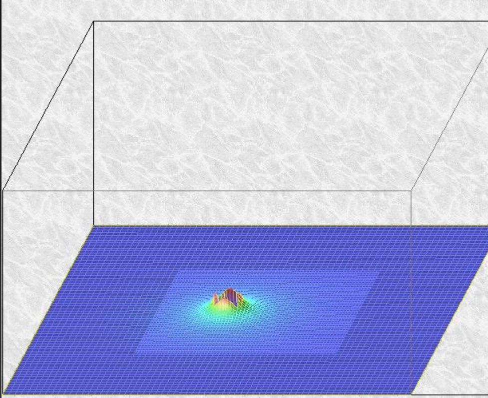
  $$-D_j K^{ij} + D^i K = 0$$

  preserved under evolution!

- Evolution

  1) Solve constraints

  2) Evolve data



GR specific problems

- Initial data must satisfy constraints

  $\Rightarrow$ Numerical solution of elliptic PDEs

  E. g. Puncture data Brandt & Brügmann '97
- Formulation of the Einstein equations
- Coordinates are constructed $\Rightarrow$ Gauge conditions
- Different length scales $\Rightarrow$ Mesh refinement
- Extremely long equations $\Rightarrow$ Turnover time
- Interpretation of the results? What is "Energy", "Mass"?

Gourgoulhon gr-qc/0703035, Alcubierre '07

Formulations I: BSSN

- One can easily change variables. E. g. wave equation

$$\partial_{tt} u - c\partial_{xx} u = 0 \qquad \Leftrightarrow \qquad \begin{aligned} \partial_t F - c\partial_x G &= 0 \\ \partial_x F - \quad \partial_t G &= 0 \end{aligned}$$

- BSSN: rearrange degrees of freedom

$$\chi = (\det \gamma)^{-1/3} \qquad \tilde{\gamma}_{ij} = \chi \gamma_{ij}$$

$$K = \gamma_{ij} K^{ij} \qquad \tilde{A} = \chi \left(K_{ij} - \tfrac{1}{3}\gamma_{ij} K \right)$$

$$\tilde{\Gamma}^i = \tilde{\gamma}^{mn} \tilde{\Gamma}^i_{mn} = -\partial_m \tilde{\gamma}^{im}$$

Shibata & Nakamura '95, Baumgarte & Shapiro '98

Formulations I: BSSN

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\phi = \tfrac{1}{12}\ln\gamma \qquad \hat{\gamma}_{ij} = e^{-4\phi}\gamma_{ij}$$

$$K = \gamma_{ij}K^{ij} \qquad \hat{A}_{ij} = e^{-4\phi}\left(K_{ij} - \tfrac{1}{3}\gamma_{ij}K\right)$$

$$\hat{\Gamma}^i = \gamma^{ij}\hat{\Gamma}^i_{jk} = -\partial_j\hat{\gamma}^{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\hat{\gamma}_{ij} = -2\alpha\hat{A}_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\phi = -\tfrac{1}{6}\alpha K$$

$$(\partial_t - \mathcal{L}_\beta)\hat{A}_{ij} = e^{-4\phi}(-D_iD_j\alpha + \alpha R_{ij})^{\mathrm{TF}} + \alpha(K\hat{A}_{ij} - 2\hat{A}_{ik}\hat{A}^k{}_j)$$

$$(\partial_t - \mathcal{L}_\beta)K = -D^iD_i\alpha + \alpha\left(\hat{A}_{ij}\hat{A}^{ij} + \tfrac{1}{3}K^2\right)$$

$$\partial_t\hat{\Gamma}^i = 2\alpha\left(\hat{\Gamma}^i_{jk}\hat{A}^{jk} + 6\hat{A}^{ij}\partial_j\phi - \tfrac{2}{3}\hat{\gamma}^{ij}\partial_jK\right) - 2\hat{A}^{ij}\partial_j\alpha + \hat{\gamma}^{jk}\partial_j\partial_k\beta^i$$
$$+\tfrac{1}{3}\hat{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\hat{\Gamma}^i + \tfrac{2}{3}\hat{\Gamma}^i\partial_j\beta^j \underbrace{- \left(\chi + \tfrac{2}{3}\right)\left(\hat{\Gamma}^i - \hat{\gamma}^{jk}\hat{\Gamma}^i_{jk}\right)\partial_l\beta^l}_{\text{Yo et al. (2002)}}$$

Formulations II: Generalized harmonic (GHG)

- Harmonic gauge: choose coordinates such that

  $\nabla_\mu \nabla^\mu x^\alpha = 0$

- 4-dim. version of Einstein equations

  $R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \ldots$

  Principal part of wave equation

- Generalized harmonic gauge: $H_\alpha \equiv g_{\alpha\nu} \nabla_\mu \nabla^\mu x^\nu$

  $\Rightarrow R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \ldots - \frac{1}{2} (\partial_\alpha H_\beta + \partial_\beta H_\alpha)$

  Still principal part of wave equation !!!

The gauge in GHG

- Relation between H_α and lapse α and shift β^i:

  $$H_\mu n^\mu = -K - \frac{1}{\alpha^2}\left(\partial_0\alpha - \beta^i\partial_i\alpha\right)$$

  $$\perp^i{}_\mu H^\mu = \frac{1}{\alpha}\gamma^{ik}\partial_k\alpha + \frac{1}{\alpha^2}\left(\partial_0\beta^i - \beta^k\partial_k\beta^i\right) - \gamma^{mn}\Gamma^i_{mn}$$

- Auxiliary constraint

  $$C_\gamma \equiv H_\gamma - \Gamma^\mu_{\mu\gamma} + g^{\mu\nu}\partial_\mu g_{\nu\gamma}$$

  Requires constraint damping

  Gundlach *et al.* '05

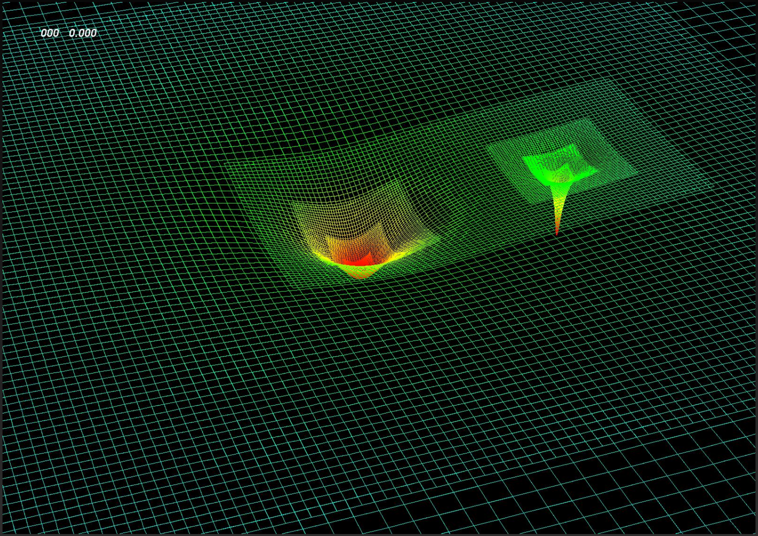
The gauge freedom

- Remember: Einstein equations say nothing about $\alpha, \quad \beta^i$
- Any choice of lapse and shift gives a solution
- This represents the coordinate freedom of GR
- Physics do not depend on $\alpha, \quad \beta^i$

  So why bother?
- The performance of the numerics DO depend strongly on the gauge!
- How do we get good gauge?

  Singularity avoidance, avoid coordinate stretching, well posedness

What goes wrong with bad gauge?



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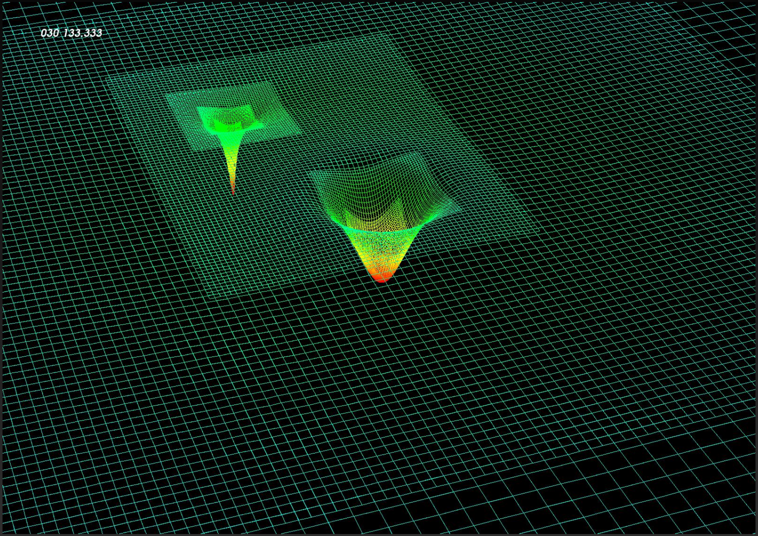

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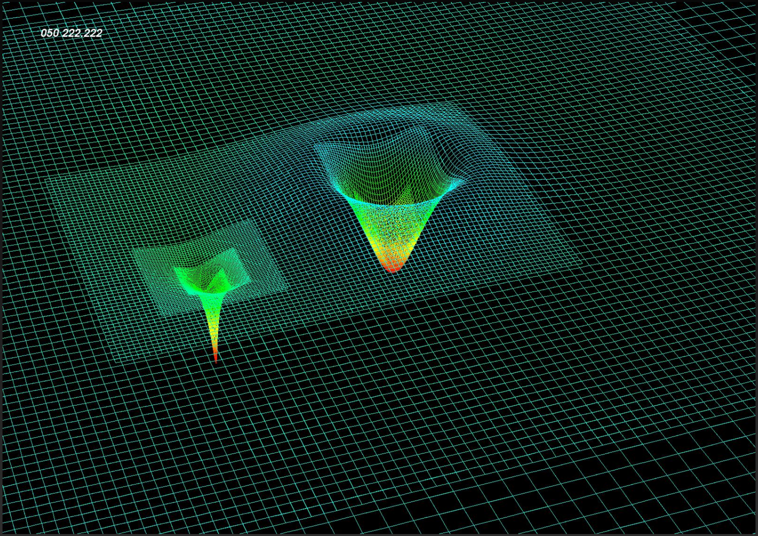

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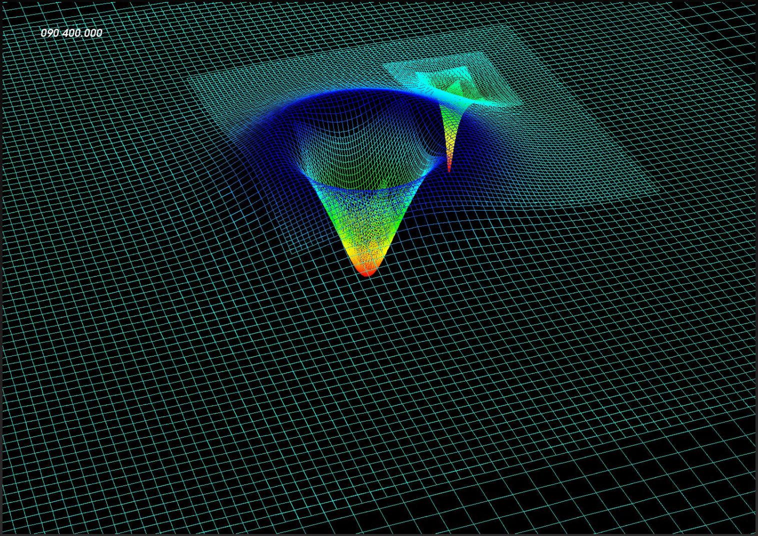


Ingredients for good gauge

- Singularity avoidance
- Avoid slice stretching
- Aim at stationarity in comoving frame
- Well posedness of system
- Generalize “good” gauge, e .g. harmonic
- Lots of good luck!

Bona & Massó ’95,

AEI: Alcubierre *et al.* 00s,

Alcubierre ’03,

Garfinkle ’04, Pretorius ’05

Initial data

Two problems: Constraints, realistic data

- Rearrange degrees of freedom

  York-Lichnerowicz split: $\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$

  $$K_{ij} = A_{ij} + \frac{1}{3}\gamma_{ij} K$$

  York & Lichnerozwicz, O'Murchadha & York,

  Wilson & Mathews, York

- Make simplifying assumptions

  Conformal flatness: $\tilde{\gamma}_{ij} = \delta_{ij}$

- Find good elliptic solvers

Two families of initial data

- Generalized analytic solutions:

  Isotropic Schwarzschild $ds^2 = \frac{M-2r}{M+2r}dt^2 + \left(1 + \frac{M}{2r}\right)^4 \left(dr^2 + r^2 d\Omega\right)$

  $\Rightarrow$ Time-symmetric N holes — Brill & Lindquist, Misner '60s

  $\Rightarrow$ Spin, Momenta — Bowen & York '80

  $\Rightarrow$ Punctures — Brandt & Brügmann '97

- Excision data: horizon boundary conditions

  Meudon Group, Pfeiffer, Ansorg

- Remaining problems: 1) junk radiation

  2) We often want zero eccentricity

Mesh refinement

3 Length scales :	BH	$\sim 1\ M$
	Wavelength	$\sim 10...100\ M$
	Wave zone	$\sim 100...1000\ M$

- Critical phenomena

  Choptuik '93

- First used for BBHs

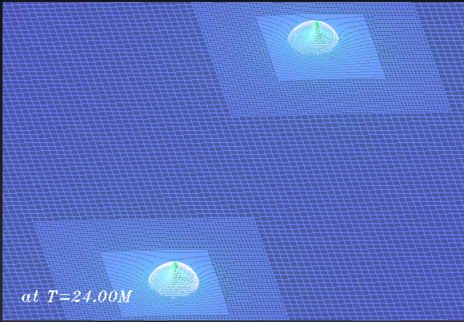
  Brügmann '96

- Available Packages:

  Paramesh MacNeice *et al.* '00

  Carpet Schnetter *et al.* '03

  SAMRAI MacNeice *et al.* '00



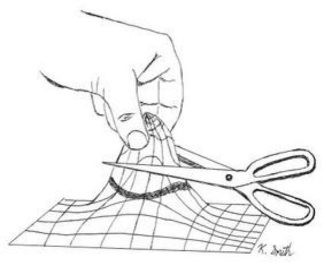
Singularity treatment

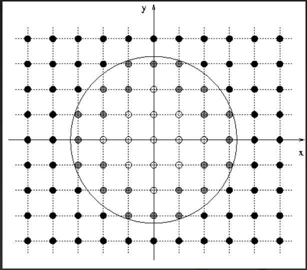
- Cosmic censorship ⇒ horizon protects outside
- We get away with it...

  Moving Punctures

  UTB, NASA Goddard '05
- Excision: Cut out region around singularity

  Caltech-Cornell, Pretorius





Extracting physics I: Global quantities

- ADM mass: Total energy of the spacetime

  $M_{\mathrm{ADM}} = \frac{1}{16\pi} \lim_{r\to\infty} \int_{S_r} \sqrt{\gamma}\gamma^{ij}\gamma^{kl}\left(\partial_j\gamma_{ik} - \partial_k\gamma_{ij}\right) dS_l$

- Total angular momentum of the spacetime

  $P_i = \frac{1}{8\pi} \lim_{r\to\infty} \int_{S_r} \sqrt{\gamma}\,(K^m{}_i - \delta^m{}_i K)\, dS_m$

  $J_i = \frac{1}{8\pi} \epsilon_{il}{}^m \lim_{r\to\infty} \int_{S_r} \sqrt{\gamma} x^l\,(K^n{}_m - \delta^n{}_m K)\, dS_n$

By construction all of these are time independent !!

Extracting physics II: Local quantities

- Often impossible to define!!
- Isolated horizon framework Ashtekar *et al.*

  $\to$ Calculate apparent horizon $\to$ Irreducible mass, momenta associated with horizon

  $$M_{\rm irr} = \sqrt{\frac{A_{\rm AH}}{16\pi}}$$

- Total BH mass Christodoulou

  $$M^2 = M_{\rm irr}^2 + \frac{S^2}{4M_{\rm irr}^2} + P^2$$

- Binding energy of a binary: $E_{\rm b} = M_{\rm ADM} - M_1 - M2$

Extracting physics III: Gravitational Waves

- Most important diagnostic: Emitted GWs
- Newman-Penrose scalar

  $\Psi_4 = C_{\alpha\beta\gamma\delta} n^\alpha \bar{m}^\beta n^\gamma \bar{m}^\delta$

  Complex $\Rightarrow$ 2 free functions
- GWs allow us to measure

  $\rightarrow$ Radiated energy $E_{\rm rad}$

  $\rightarrow$ Radiated momenta $P_{\rm rad}$, $J_{\rm rad}$

  $\rightarrow$ Angular dependence of radiation

  $\rightarrow$ Gravitational wave strain h_+, $h_\times$

Angular dependence of GWs

- Waves are normally extracted at fixed radius $r_{\rm ex}$

  $\Rightarrow \Psi_4 = \Psi_4(t, \theta, \phi)$

  θ, ϕ are viewed from the source frame!

- Decompose angular dependence using spherical harmonics

  $\Psi_4 = \sum_{\ell,m} \psi_{\ell m}(t) Y^{-2}\ell m(\theta, \phi)$

  Modes $\psi_{\ell m}(t) = A_{\ell m}(t) \times e^{i\phi(t)}$

  Spin-weighted spherical harmonics $Y^{-2}_{\ell m}$

4. Results

4.1. A brief history

A brief history of BH simulations

- Pioneers: Hahn & Lindquist '60s, Eppley, Smarr *et al.* '70s
- Grand Challenge: First $3D$ Code Anninos *et al.* '90s
- Further attempts: Bona & Massó, Pitt-PSU-Texas

  AEI-Potsdam, Alcubierre *et al.*

  PSU: first orbit Brügmann *et al.* '04

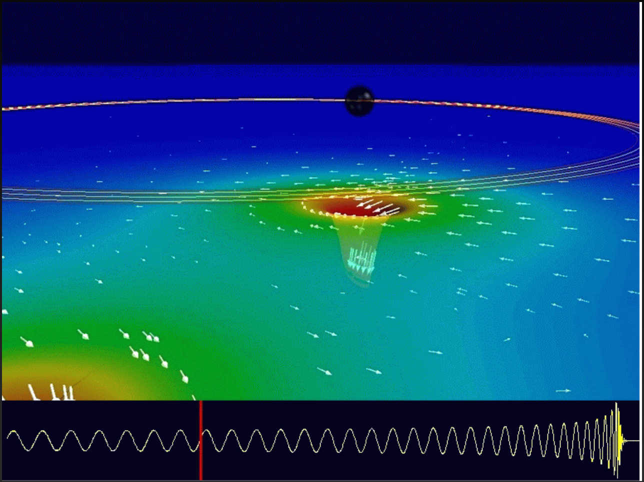
  Codes unstable!

- Breakthrough: Pretorius '05 — GHG

  UTB, Goddard'05 — Moving Punctures

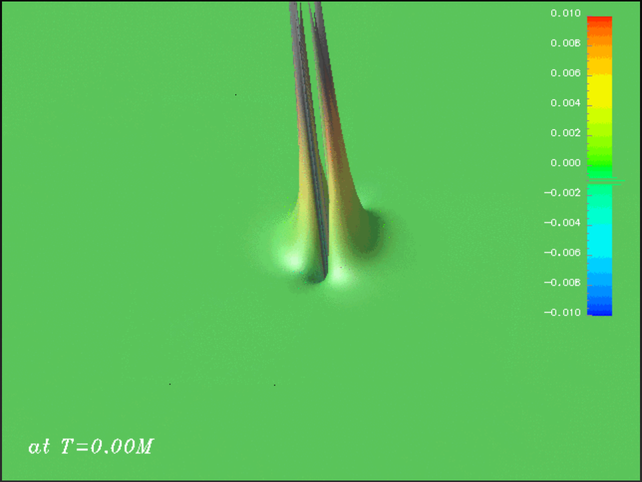
- Currently about 10 codes world wide

Animations: BBH inspiral

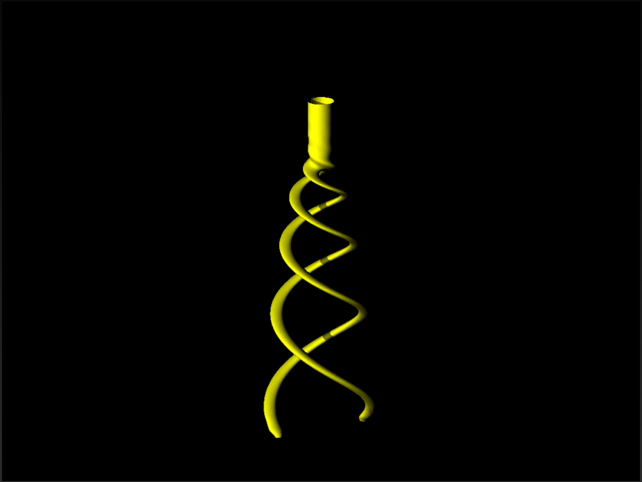


Thanks to Caltech, CITA, Cornell

Animations: The GW signal



Animations: The event horizon



Thanks to Marcus Thierfelder, Jena