Black holes in a box

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The evolution of black holes in "confining boxes" is interesting for a number of reasons, particularly because it mimics some aspects of anti-de Sitter spacetimes. We are here interested in the potential role that boundary conditions play in the evolution of such systems. Therefore, we imprison and study a black hole binary in a box, at which boundary we set mirror-like boundary conditions.

1. Introduction

Black holes (BHs) play a central role in fundamental physics, from gravitational wave astronomy to high energy physics. The gauge-gravity duality[1] has created a powerful framework for the study of strongly coupled gauge theories and found applications in connection with the experimental program on heavy ion collisions at Brookhaven's RHIC[2] and at the LHC at CERN,[3] among many others. A key feature of asymptotically AdS space is the "active role" played by its boundary. Null geodesics reach the boundary in finite coordinate time. One thus often refers to an asymptotically AdS space as a box.[4] We wish to explore BHs in AdS in the framework of Numerical Relativity. We model AdS backgrounds by a confining box with reflecting walls in which a BH binary evolves. This allows us to study a number of interesting phenomena. For example, since the final outcome of a generic BH binary merger is a rotating BH, one expects superradiant effects to play a role in the evolution of this system, when it is enclosed in a box.

2. Numerical Setup

The numerical simulations were carried out using Sperhake's LEAN code.[5] The evolution of Einstein equations is based on the χ-version of the Baumgarte-Shapiro-Shibata-Nakamura[6] system, with the moving puncture approach.[7] The punctures have initial parameter $m_i = 0.483$, $x_i = \pm 3.257$ and $p_{y_i} = 0.133$. We have used the resolution $h = 1/48M$ near the puncture. In order to mimic the numerical evolution of BHs in AdS spacetimes we impose a spherical box of radius r_B with reflecting boundary conditions around the BH binary.

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3. Results

First, we focus on a BH binary enclosed in a confining box with radius $r_B = 48M$. We have studied the gravitational radiation using the outgoing Newman-Penrose scalar Ψ_4 and the properties of the apparent horizon (AH) of the final BH using Thornburg's AHFinderDirect.[8] In the left panel of Fig. 1 we present the real part of the dominant $l = m = 2$ mode of Ψ_4, extracted at $r_{ex} = 40M$. The gravitational wave emitted during the inspiral and plunge of the BH binary is reflected back at the boundary, interacts with the final BH and travels outwards again which can be seen in the second and third cycle. Studying the waveform itself, we observe a broadening of the merger pulse upon each interaction with the BH. A possible explanation relies on superradiant amplification of the low-frequency part of the waveform, though more work is necessary to pinpoint the reason for this. We estimate the critical frequency for superradiance to be around $M\omega_c \sim 0.4$. The time evolution of AH

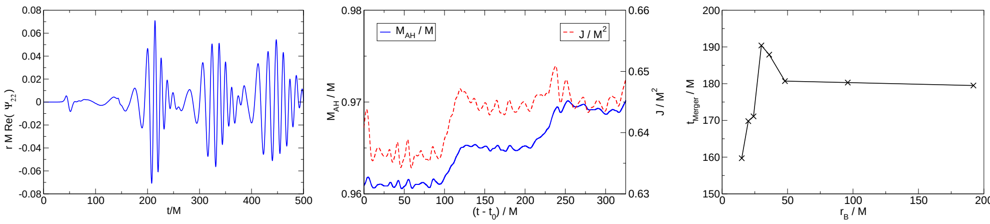

Fig. 1. Left: $l = m = 2$ mode of Ψ_4. Center: Time evolution of the mass and total spin of the final BH. Right: time of the merger of the BH binary depending on the box radius r_B.

mass and the total spin (assuming a Kerr BH as final state) is depicted in the mid panel of Fig. 1. The AH mass increases upon each interaction with the gravitational radiation pulse. We estimate that during the first and second interaction $\approx 15\%$ of the incident pulse is absorbed by the BH. In the course of the first interaction between the spinning BH and the ingoing gravitational wave angular momentum is absorbed. However, during the second interaction the spin of the central BH remains approximately constant. This process can be explained if we consider two competing phenomena, namely absorption of energy and angular momentum of high frequency modes of the radiation by the BH on the one hand and amplification of low frequency modes due to the superradiance effect on the other hand. This behaviour is in qualitative agreement with linearized studies,[9] but clearly deserves a more thorough investigation. We have also studied BH binaries in boxes with a different radii. We have studied only the pre-merger dynamics of those scenarios and observe a peculiar behaviour: The actual time of the merger seems to be driven by the interaction of the system with the reflected radiation in a non-monotonic way, as can be seen in the right panel of Fig. 1. We find three different phases: i) in case of small boxes the merger takes place earlier ii) in case of intermedium box radii the merger happens later than in the asymptotically flat scenario. iii) for large boxes in which the reflected signal does not interact with the BH binary before the merger, the plunge takes place at the same time as it would in the same setup but using outgoing boundary conditions. Why there is this non-monotonic pre-merger behaviour is not yet completely understood. A more detailed analysis is planned in future work.

The dynamics of BHs in generic spacetimes is a fascinating, yet complex problem. By considering a BH binary in a "boxed" spacetime we have accomplished the first steps towards a fully numerical treatment of BHs in AdS spacetimes. In future work we plan to investigate the presented issues in more detail, especially the numerical stability of the formulation and the pre-merger dynamics. Perhaps the most important conclusion of the present work is that these simulations *can* be done and represent the first successful step to a full numerical evolution of BHs in AdS spacetimes.

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