

How do you get GWs in the first place?

Moving mass-energy.

EM analogy: most ^{dominant} radiation from electric dipole

$$\frac{d\text{Energy}}{dt} \sim |\ddot{\vec{d}}|^2$$

$$\text{where } \vec{d} = \sum_A q_A \vec{x}_A$$

Next is magnetic dipole

$$\frac{d\text{Energy}}{dt} \sim |\ddot{\vec{\mu}}|^2$$

$$\vec{\mu} = \sum_A \vec{r}_A \times q \vec{v}_A \quad , \text{ current}$$

Gravitation: "dipole" = $\vec{d} = \sum_A m_A \vec{x}_A$

$$\dot{\vec{d}} = \sum_A m_A \vec{v}_A = \vec{P} \quad \text{conserved}$$

$$\text{"m dipole"} \quad \vec{\mu} = \sum_A \vec{r}_A \times m \vec{v}_A = \vec{J} \quad \text{conserved}$$

No grav. dipole radiation

Next is electric quadrupole ¹brings.

Will show, for weakly gravitating sources, $\left(v \lesssim \left(\frac{M}{R} \right)^{1/2} \ll 1 \right)$
 $R \ll \lambda$

$$h_{ij}^{\text{TT}}(t-r) = \frac{2}{r} \ddot{I}_{ij}^{\text{TT}}(t-r)$$

$$\circ = \frac{\partial}{\partial t}$$

where $I_{ij} \equiv \int T^{00} (x_i x_j - \frac{1}{3} \delta_{ij} r^2) d^3x$

reduced quadrupole
moment

and $I_{ij}^{\text{TT}} \equiv P_{il} P_{jm} I^{lm} - \frac{1}{2} P_{ij} P_{lm} I^{lm}$

$$P_{ij} \equiv \delta_{ij} - \frac{x_i x_j}{r^2}$$

Also, will show

$$\left. \frac{dE}{dt} \right|_{\text{GW}} = \frac{1}{5} \langle \ddot{I}_{ij} \ddot{I}^{ij} \rangle$$

$$\left. \frac{dL_i}{dt} \right|_{\text{GW}} = \frac{2}{5} \epsilon_{ijk} \langle \ddot{I}^{jl} \ddot{I}_l^k \rangle$$