Physics 236b assignment, Week 6:

(Feb 11, 2016. Due on Feb 18, 2016)

1. **Road rage as a gravitational wave source** [10 points]

   A Los Angeles motorist shakes his fist angrily at another motorist. In order of magnitude, what fraction of the expended energy goes into gravitational waves?

2. **Spinning rod** [15 points]

   A thin metal rod of mass M and length L spins with an angular frequency ω about its center, with the spin axis perpendicular to the rod.

   (a) Compute the power radiated in gravitational waves.

   (b) Centrifugal force will push electrons toward the ends of the rod, and this will induce a small electric quadrupole moment, and will lead to electromagnetic radiation. Assuming that the power in electric quadrupole radiation is ω^6 times the square of the electric quadrupole moment, estimate the power radiated in electromagnetic waves.

   (c) For a rod of density $\sim 10\text{g/cm}^3$ and $2\pi\omega \sim 1\text{kHz}$, what is the ratio of electromagnetic to gravitational radiation?

3. **Effect of GWs on particles in TT gauge** [15 points]

   Consider 2 particles A and B in the presence of a linearized gravitational wave. Choose a TT coordinate system in which, at time $t = 0$, both particles are at rest.

   (a) Write down the metric in this coordinate system in terms of h_{ij}.

   (b) Use the geodesic equation to show that both particles remain *at rest* in the TT coordinates, even while the wave is passing by. (Coordinates are only coordinates!)

(c) Let $x^i_{B(0)} = x^i_B - x^j_A$ be the coordinate separation between the particles, which you have just shown is constant in time. Transform into an *orthonormal* frame comoving with particle A, and show that

$$x^{\hat{i}}_B(\tau) = x^{\hat{k}}_{B(0)} \left(\delta^i_k + \frac{1}{2} h^j_k \right) \tag{1}$$

(in class, we derived this result a different way, using geodesic deviation).

4. **Circularization of binary orbit** [25 points]

Two point masses m_1 and m_2 are in *Newtonian* elliptical orbit with semimajor axis a and eccentricity e. Without loss of generality, assume that the orbit is in the xy plane and that pericenter is on the x axis.

We will show that gravitational radiation makes the orbit more circular as time goes on.

Recall that for a Keplerian orbit, if the distance between the particles is r, then

$$r = \frac{a(1-e^2)}{1 + e\cos\phi}, \tag{2}$$

where ϕ is the angle of the separation vector with respect to the x axis.

(a) Express the total energy E of the orbit and the total angular momentum L of the orbit in terms of a and e (only Newtonian formulas are needed).

(b) Compute the quadrupole tensor in terms of m_1, m_2, r, and ϕ.

(c) Write down the Newtonian equations of motion for the orbit, i.e. dr/dt and $d\phi/dt$.

(d) Compute the third time derivatives of the quadrupole tensor, using the equations of motion from part 4c to simplify your expressions.

(e) Compute the rate at which energy and angular momentum are carried off by gravitational waves. You will need to average over an orbit. Then trivially compute the rate of change of energy and angular momentum of the orbit, dE/dt and dL/dt, due to gravitational wave emission.

(f) Compute the rate of change of a and e, and show that the orbit becomes more circular as time goes on.

(g) Assume $e = 0$ in your expression for da/dt from part 4f, and use it to estimate the time t_{collide} at which the orbit shrinks to zero and the particles collide, if $a = a_0$ at $t = 0$. If $m_1 = m_2 = 1$ solar mass, what is the maximum that a_0 can be so that t_{collide} is less than the age of the universe (14 billion years)?