Physics 236b assignment, Week 2:

(Jan 14, 2016. Due on Jan 21, 2016)

1. **Ideal neutron gas** [50 points]

   The equation of state for a *nonrelativistic, extremely degenerate* ideal neutron gas is

$$P = K\rho_0^\Gamma, \tag{1}$$

   where the constants Γ and K are given by

$$\Gamma = 5/3, \tag{2}$$

$$K = \frac{3^{2/3}\pi^{4/3}}{5}\frac{\hbar^2}{m_n^{8/3}}. \tag{3}$$

   Here ρ_0 is the rest mass density, and m_n is the rest mass of a neutron. By nonrelativistic, we mean that the energy of a single neutron is dominated by its rest mass. Thus, for parts 1a and 1b below, assume that $\rho = \rho_0$.

   (a) Write down the equation of state in the form of two equations, $P(n)$ and $\rho(n)$, where n is the number density of neutrons, assuming $\rho = \rho_0$.

   (b) Numerically integrate the *Newtonian* equations of hydrostatic equilibrium (i.e. the equations for dm/dr and dP/dr) to construct the structure of a star with a given central density ρ_c. Use your favorite numerical integration scheme (this is possible to do by hand using a calculator, but feel free to use other code or tools such as Numerical Recipes, Mathematica, Scipy, etc.). Note: when doing numerical integrations, it is strongly recommended to define and work with dimensionless variables to avoid large and awkward numbers; for example, one might work with the variable $\tilde{m}(r) = m(r)/M_\odot$ instead of $m(r)$.

i. Plot M (mass of the star) vs ρ_c for $10^{11} < \rho_c(\mathrm{g\ cm}^{-3}) < 10^{17}$.

ii. How does the mass of the star scale with the radius of the star R?

iii. Which configurations are stable and which are unstable to radial perturbations?

iv. What is the maximum possible mass of a stable neutron star in this model? What is the maximum central density?

(c) From now on we will allow ρ to include non-rest energy, so that $\rho \neq \rho_0$. But we will still use the same equation of state (1), which is expressed in terms of ρ_0. Assuming adiabatic changes, use the first law of thermodynamics and Eq. (1) to compute ρ in terms of ρ_0.

(d) Numerically integrate the *general relativistic* structure equations, using ρ from part 1c and P from Eq. (1).

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ii. Which configurations are stable and which are unstable to radial perturbations?

iii. What is the maximum possible mass of a stable neutron star in this model? What is the maximum central density?

iv. What is the maximum redshift of a photon emitted from the surface of a stable neutron star (in this model) and observed at infinity?

2. **Holding a particle above a black hole** [10 points]

A small particle of mass m is held stationary at some radius $r > 2M$ near a Schwarzshild black hole of mass M by a (massless, stretchless) string. The other end of the string is held by a stationary observer at large radius r_{obs}.

(a) What is the force (tension) F exerted by the string on the particle?

(b) What is the force F exerted on the string by the observer at r_{obs}?

(c) What is the force from part 2b if $r_{\mathrm{obs}} \to \infty$?