Numerical relativity: Triumphs and challenges of binary black hole simulations

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SXS Collaboration: www.black-holes.org

ROM-GR, Jun 06 2013

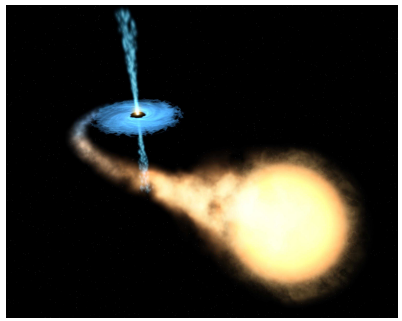
Outline

1. Introduction: Gravitational-wave sources and numerical relativity
2. Triumphs: Current capabilities of simulations
3. Challenges

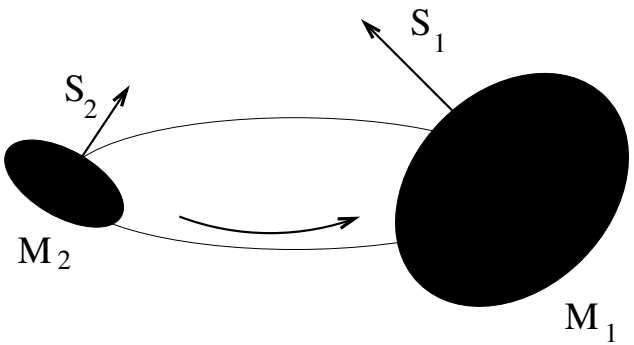
1. Introduction

Black Holes

- **Black holes**: most strongly gravitating objects in the Universe.
  - Obey Einstein's general relativistic field equations.
  - Form from collapse of matter (stars, gas, . . .)
  - Energy source for many astrophysical phenomena.



- **Black-hole binaries** expected to occur in the Universe.



- Orbit decays as energy lost to **gravitational radiation**.
- Eventually black holes collide, merge, and form a final black hole.

Neutron Stars

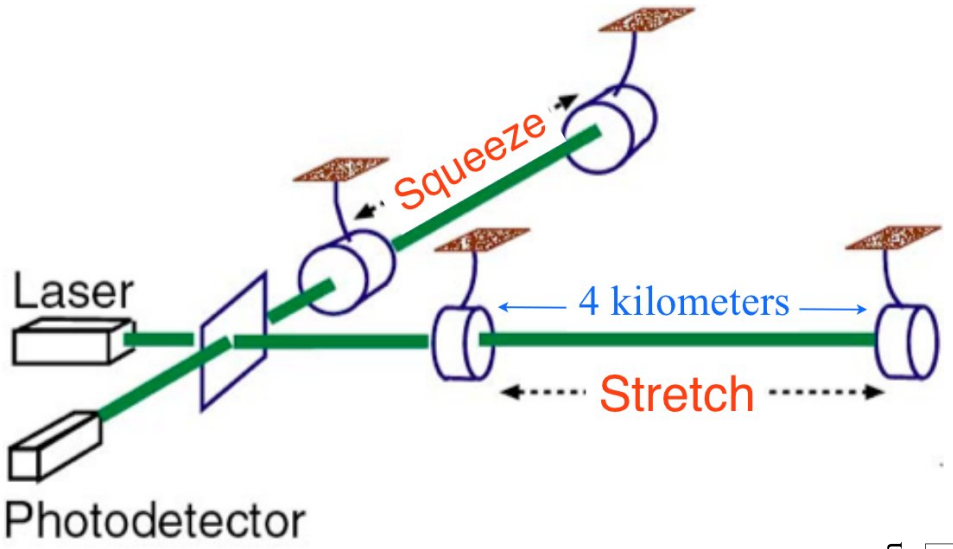
Neutron star (NS): made of degenerate matter at nuclear density. Formed in supernovae.

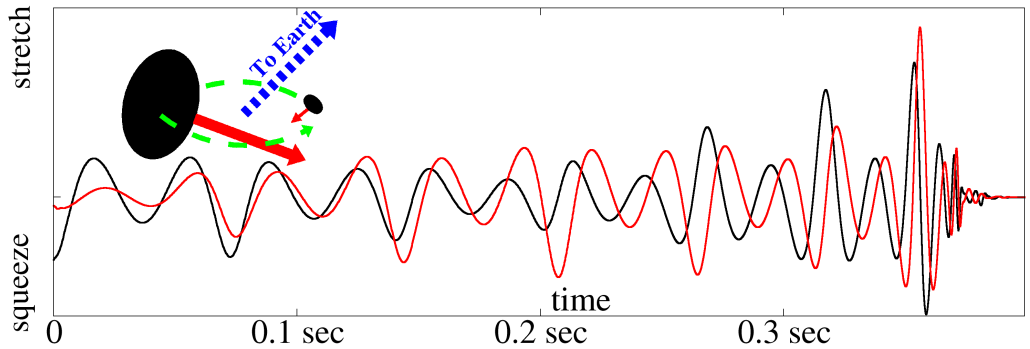
NS/NS & BH/NS binaries produce gravitational radiation.

- Need additional physics besides general relativity
  - Hydrodynamics (shocks, . . .)
  - Microphysics (finite temperature, composition, . . .)
  - Magnetic fields
  - Neutrino transport
- Many more parameters (but some less important for LIGO)
- Remainder of talk: concentrate on BH/BH

LIGO

- LIGO: Laser Interferometer Gravitational-Wave Observatory



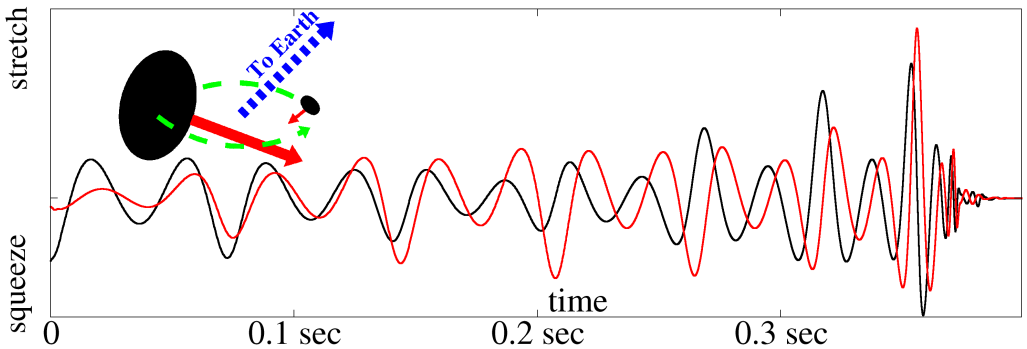


- LIGO planned in 2 phases:
  - Initial, 2005-2010.
  - Advanced, $\sim$ 2015.
- Advanced LIGO should detect waves from compact binaries.
- Similar detectors in Europe (Virgo), Japan (KAGRA)

Source modeling and LIGO

Detailed models of gravitational waves sources will help:

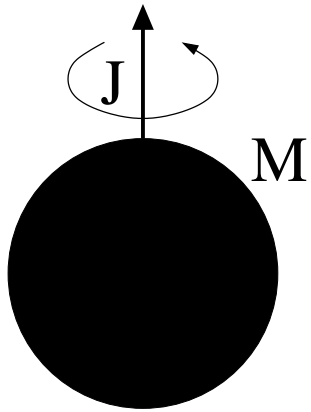
- Detection of signals
  - Matched filtering technique requires waveform templates, greatly improves detection rate.



- Parameter estimation
  - Compare measured signal with model to learn about sources
    - Test general relativity
    - Measure populations/distributions/properties of BHs, NSs
    - Learn about microphysics of dense matter
    - Multimessenger astronomy
    - ...

What do we know about black holes?

- A handful of exact solutions
  - Schwarzschild (1916): Static black hole
  - Kerr (1963): Stationary, rotating black hole



$\chi = J/M^2; |\chi| \leq 1$

(units: $G = c = 1$)

- Global theorems (e.g. Hawking area theorem)
- Perturbations about exact solutions

- No way to solve dynamical strong-gravity problems until recently.

Numerical relativity (NR)

- Write Einstein's field equations as an initial value problem for $g_{\mu\nu}$.

$$\boxed{G_{\mu\nu} = 8\pi T_{\mu\nu}} \Rightarrow \begin{cases} \text{Constraints} & (\text{like } \nabla \cdot B = 0) \\ \text{Evolution eqs.} & (\text{like } \partial_t B = -\nabla \times E) \end{cases}$$

- Choose unconstrained data on an initial time slice
- Choose gauge (=coordinate) conditions

Get yourself a computer cluster, and

- Solve constraints at $t = 0$

  (For us: 4 (+1) coupled nonlinear 2nd-order elliptic PDEs)
- Use evolution eqs. to advance in time

  (For us: 50 coupled nonlinear 1st-order hyperbolic PDEs)

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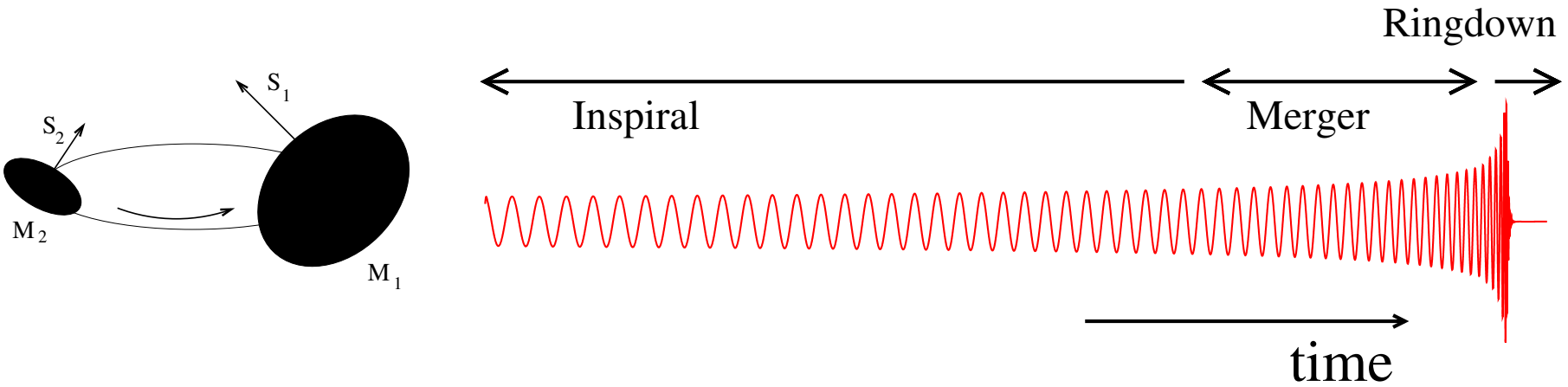
- First successful black-hole binary computation: Pretorius 2005
- Today several research groups worldwide have NR codes.

Binary black hole waveforms



Waveform divided into 3 parts:

- Inspiral: BHs far apart, described by post-Newtonian (PN) theory.
- Merger: Nonlinear, need NR.
- Ringdown: Single BH, described by pert. theory or NR.

PN: perturbative expansion in powers of v/c

Idea: Match NR simulation to PN, just before PN becomes inaccurate.

2. Capabilities of BBH Simulations

NR Codes

About a dozen in existence

Formulation of Equations:	BSSN	Generalized Harmonic
Treatment of Singularities:	Moving Punctures	Excision
Numerical Methods:	Finite Differencing	Spectral

NR Codes

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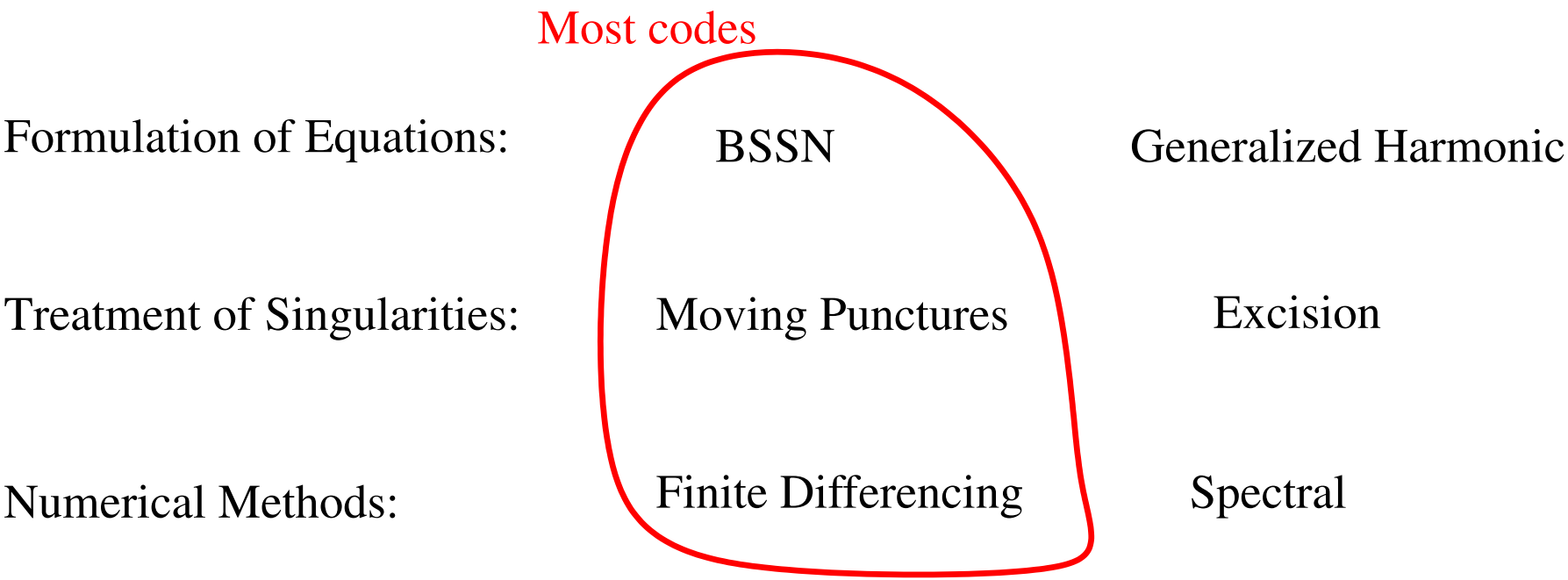

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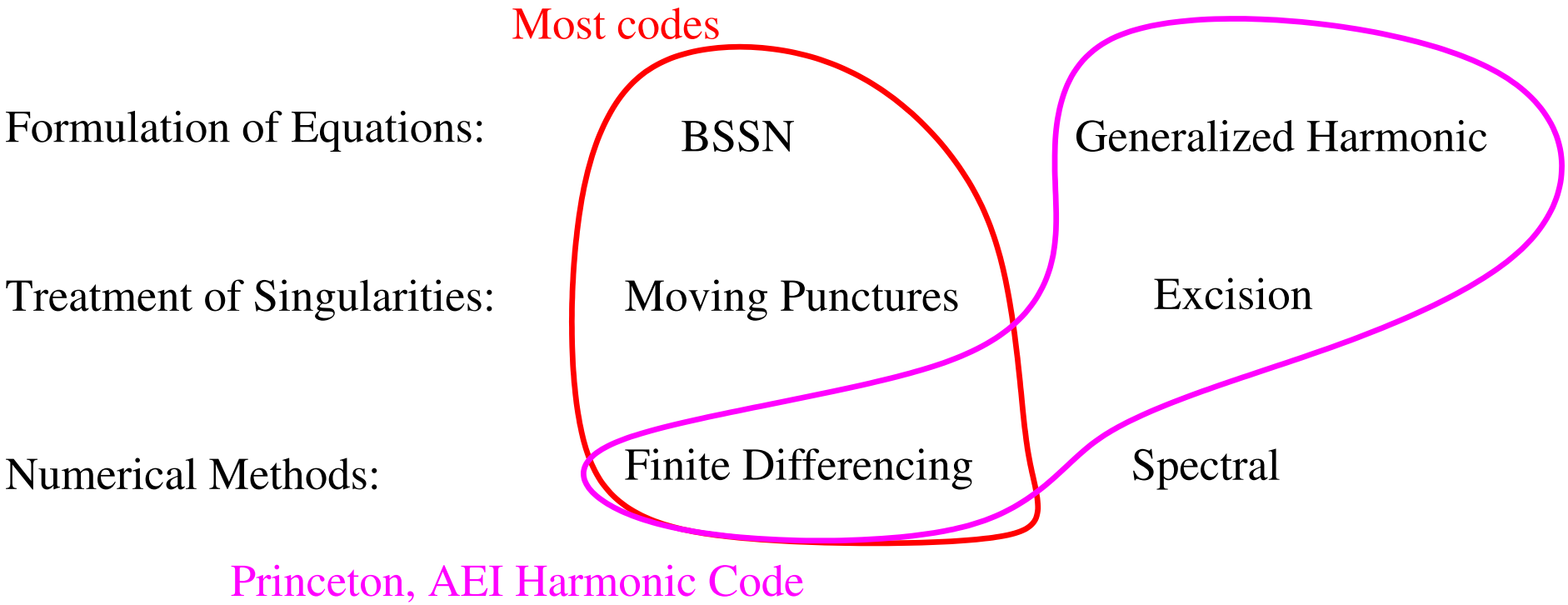

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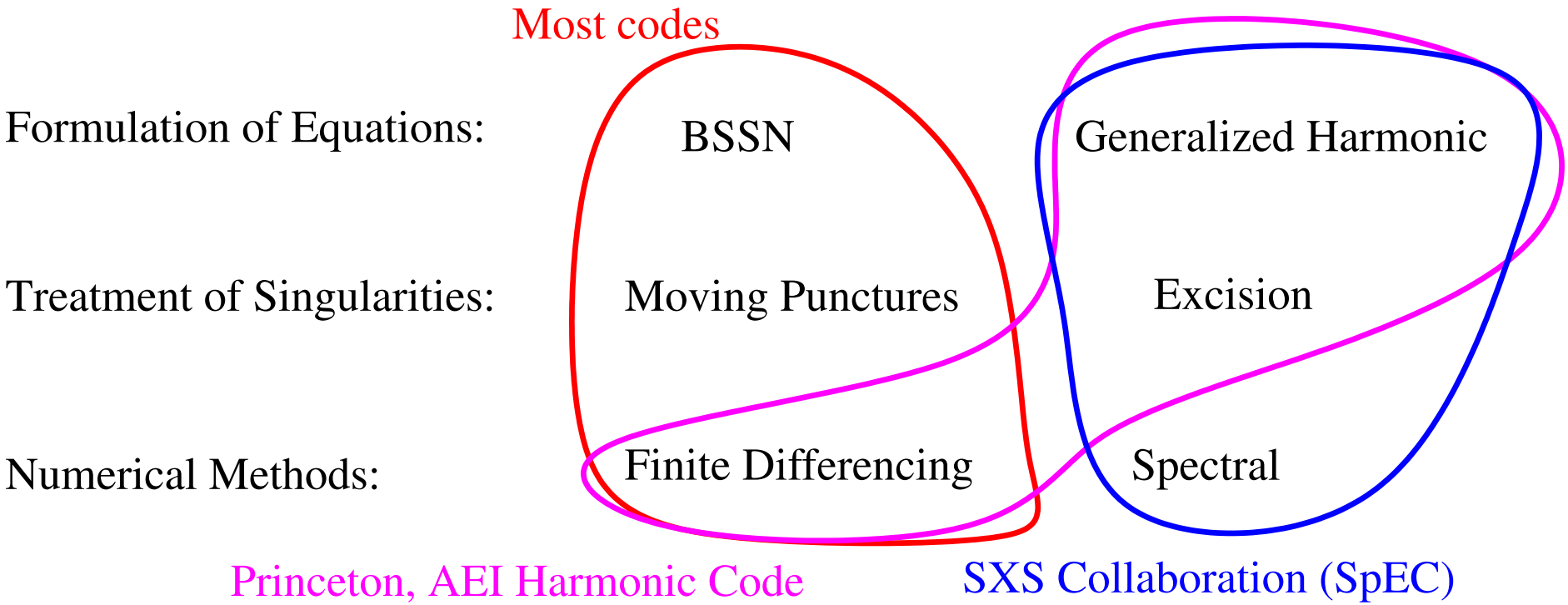

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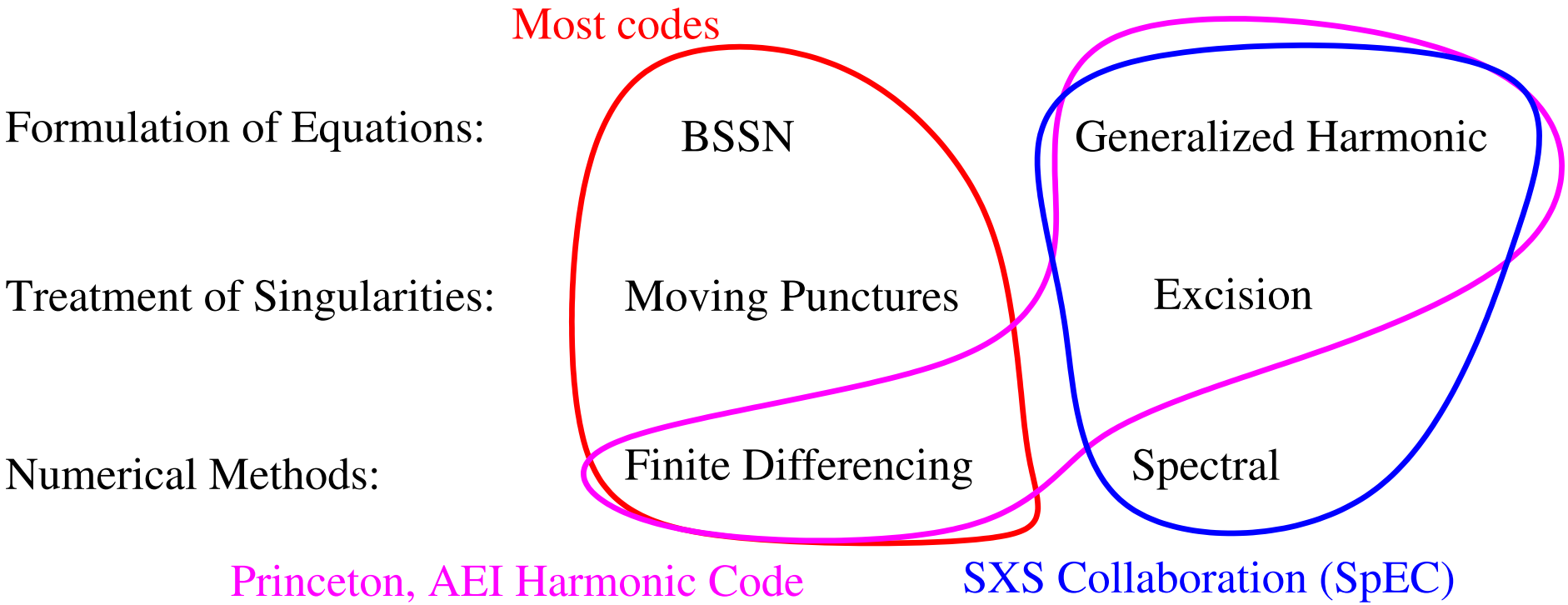


- Comparing different codes improves confidence in results.
- Most examples I will show will be from SpEC.

SpEC - Spectral Einstein Code

http://www.black-holes.org/SpEC.html

Parallel computer code developed at Caltech, Cornell, CITA (Toronto), Washington State, UC Fullerton, plus several contributors at other institutions.

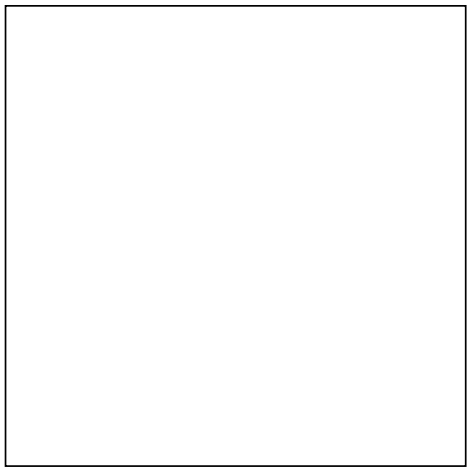
- Solves nonlinear Einstein equations in 3+1 dimensions.
- Handles dynamical black holes.
- Relativistic Hydrodynamics.

Over 50 researchers have contributed to SpEC.

Brief Description of SpEC GR Module

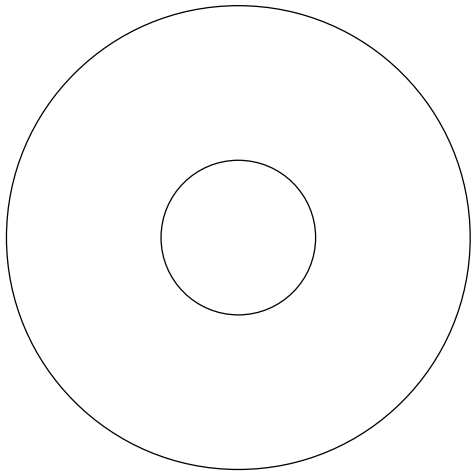
Spectral methods

Solve equations **on finite spatial regions** called **subdomains**.



$$f(x,y,z) = \sum_{\ell mn}^{LMN} f_{\ell mn} T_\ell(x) T_m(y) T_n(z)$$

$$f(r,\theta,\phi) = \sum_{\ell mn}^{LMN} f_{\ell mn} T_n(r) Y_{\ell m}(\theta,\phi)$$



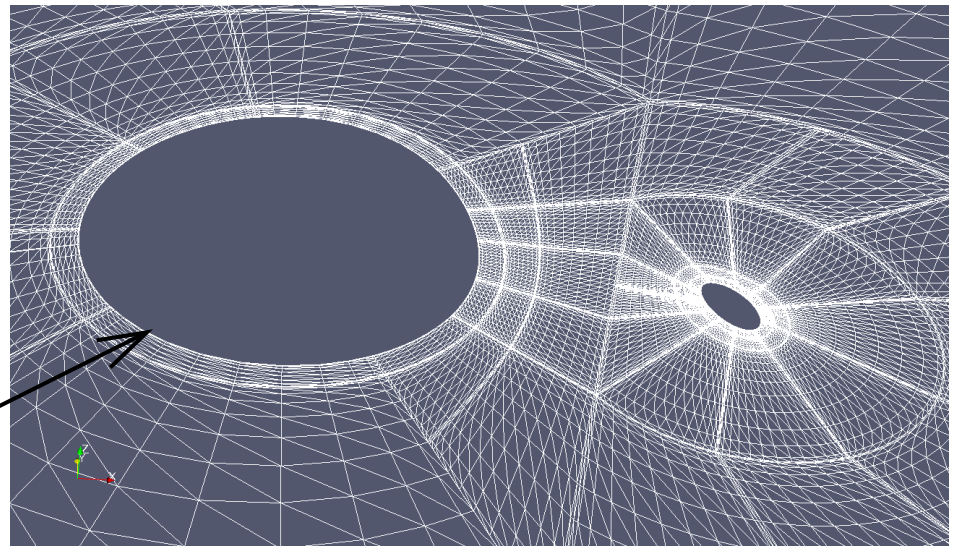
- Choose **spectral basis functions** based on subdomain shapes.
- Exponential convergence for smooth problems. **High accuracy**.

(This differs from widely used finite-difference methods)

Brief Description of SpEC GR Module

Excision

To avoid singularities, we **excise** the interiors of BHs.

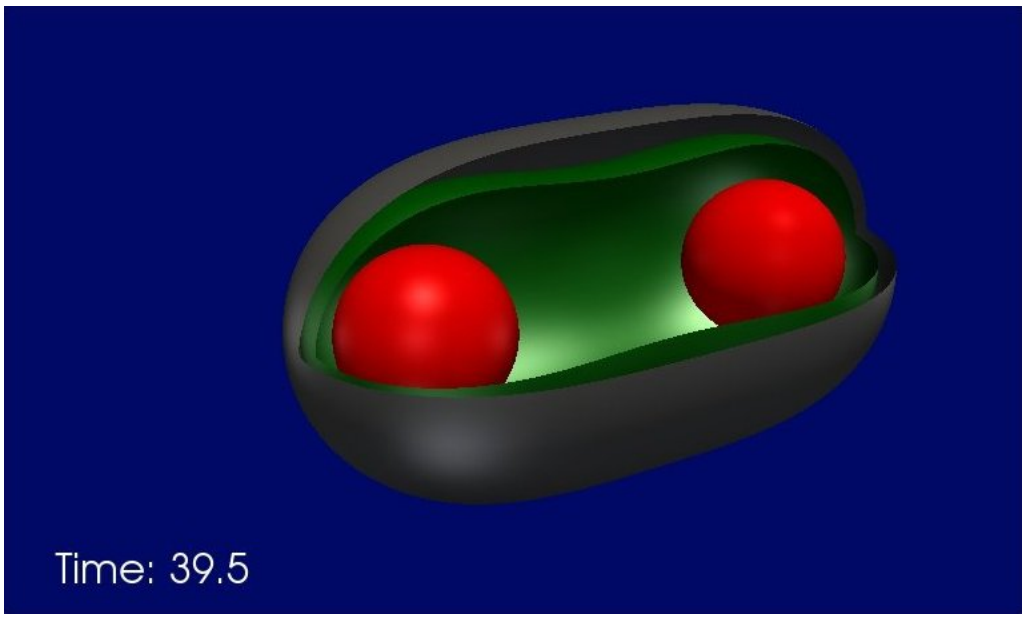


excision boundary
(slightly inside horizon)

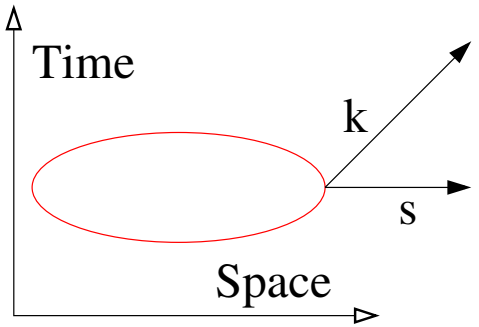
(This differs from another widely used approach called “moving punctures”)

Aside: Apparent horizons

- Event horizon (EH)
  - Boundary of region where photons can escape to infinity.
  - **Nonlocal**
- Apparent horizon (AH)
  - Smooth closed surface of zero null expansion. $\nabla^\mu k_\mu = 0$
  - **Local**



grey=EH; red,green=AH

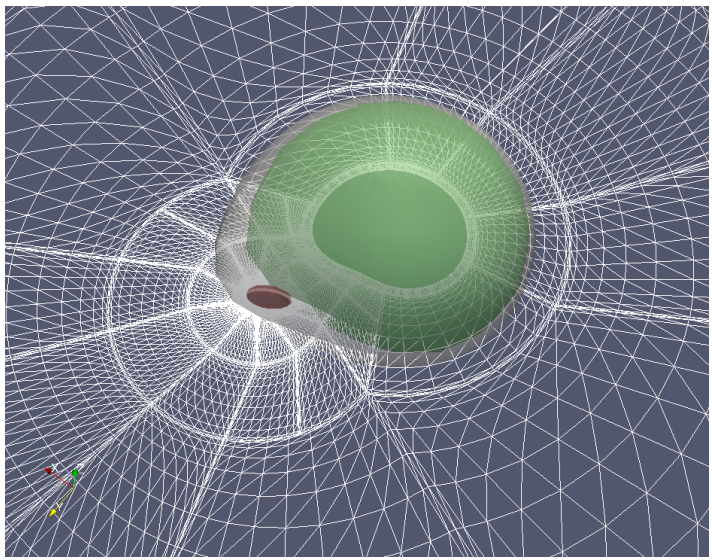


Theorem: if an AH exists, it cannot be outside an EH.

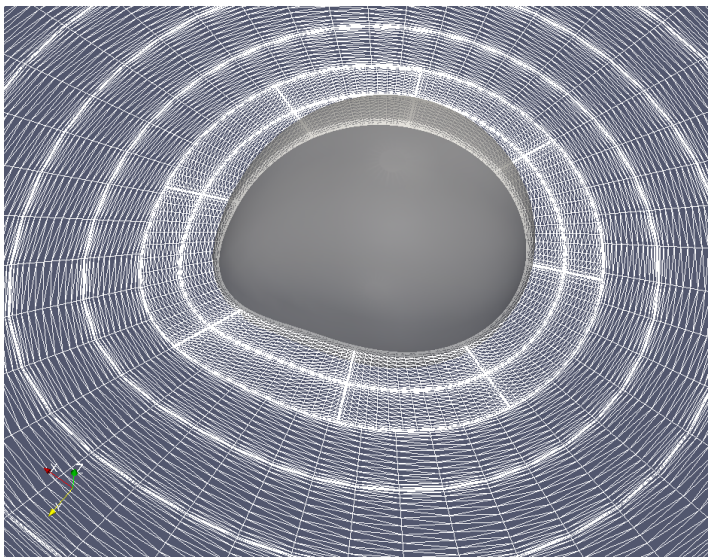
Brief Description of SpEC GR Module

Handling Merger

- Eventually, common horizon forms around both BHs.
- **Regrid** onto new grid with only one excised region.
- Continue evolution on new grid, until final BH settles down.



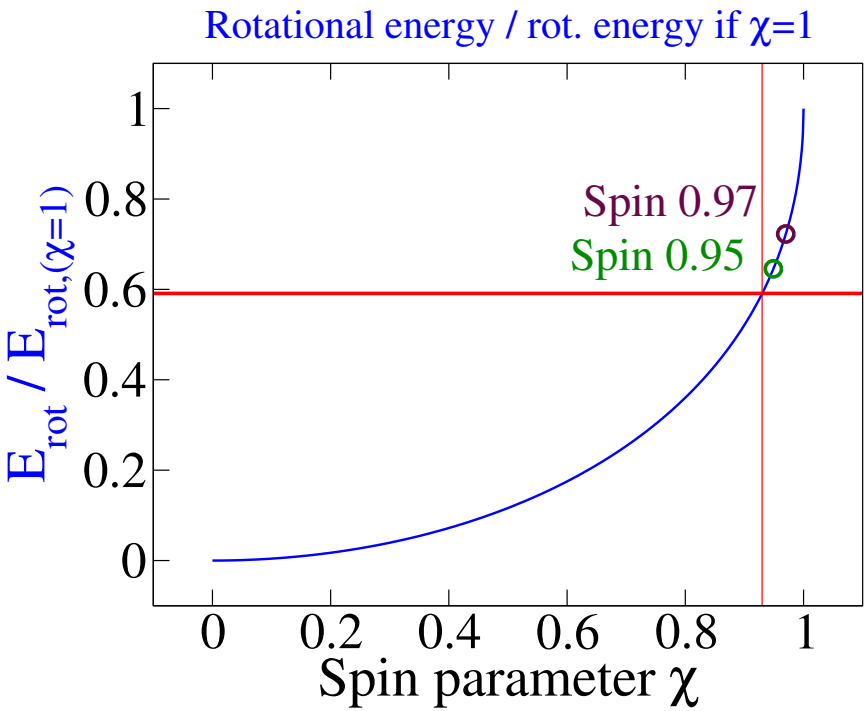
$\Rightarrow$



Current BBH capabilities: spins

Spin parameter: $\chi = J/M^2$, $|\chi| \leq 1$

- Straightforward initial data construction limited to spins $\chi <\sim 0.93$.
- Even $\chi = 0.93$ is only 60% of possible $E_{\rm rot}$



Adapted from Lovelace, MAS, Szilagyi PRD83:024010,2011

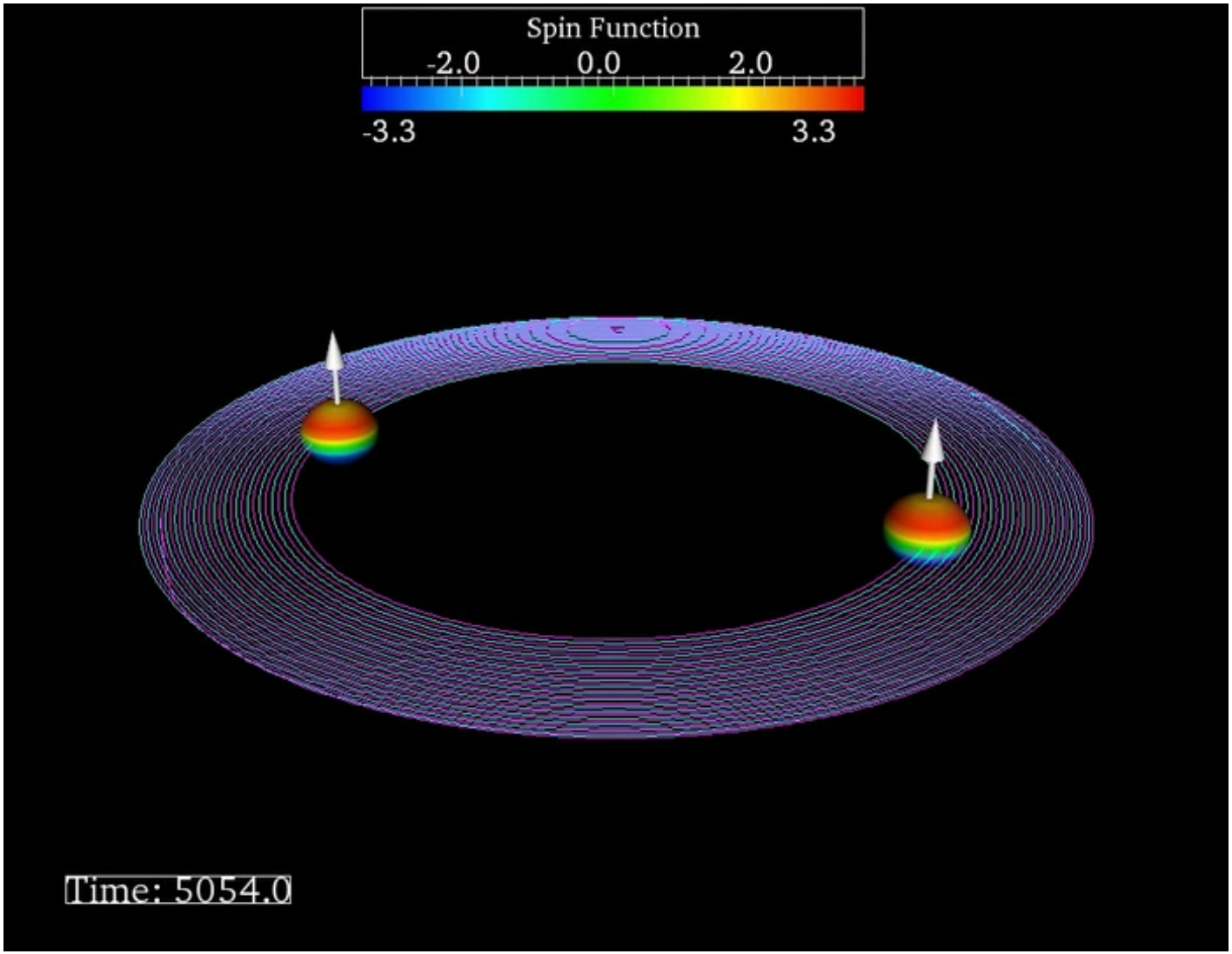
What are spins of real black holes?

- Highly uncertain
- Accretion models: $\chi \sim 0.95$
- Some observations suggest $\chi > 0.98$

- Want to explore large spins.

Current BBH capabilities: spins

Lovelace, Boyle, MAS, Szilágyi, CQG 29:045003 (2012)



Spins ~ 0.97. Movie: Geoffrey Lovelace

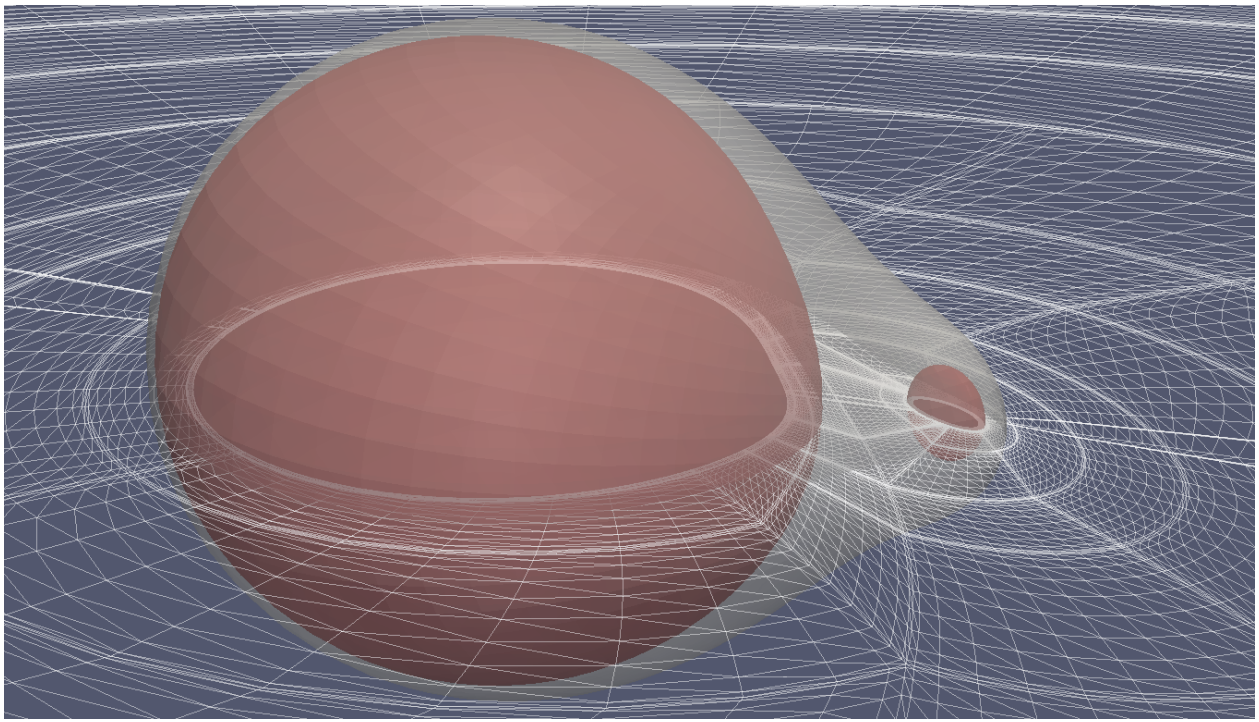
Current BBH capabilities: mass ratios

Want to explore all mass ratios.
Extreme mass ratio: pert. theory.

Large mass ratios are difficult:

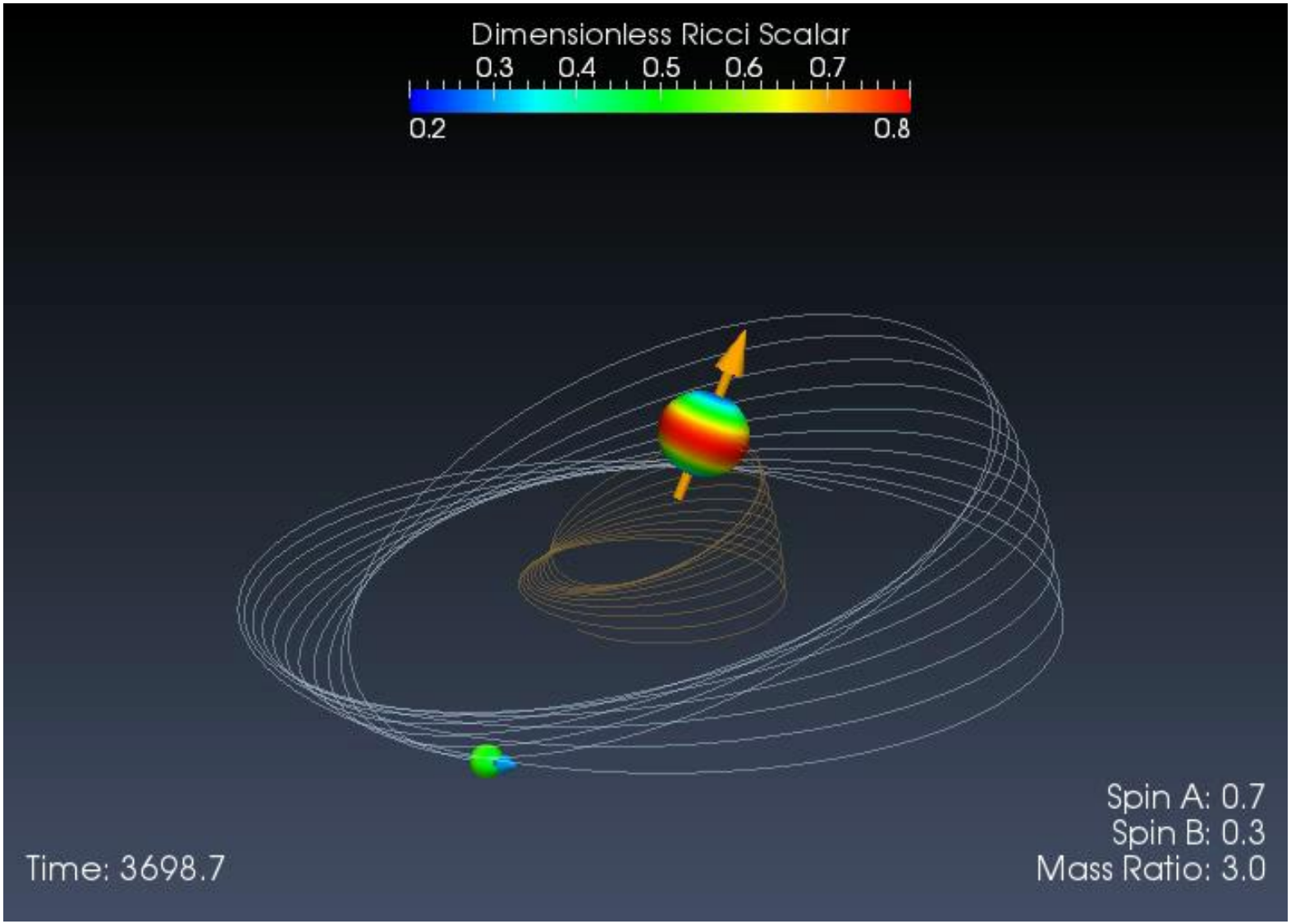
- Time scale of orbit $\sim M_1 + M_2$
- Size of time step $\sim M_{\rm small}$

- Largest to date is 100:1 (2 orbits), Lousto & Zlochower 2011



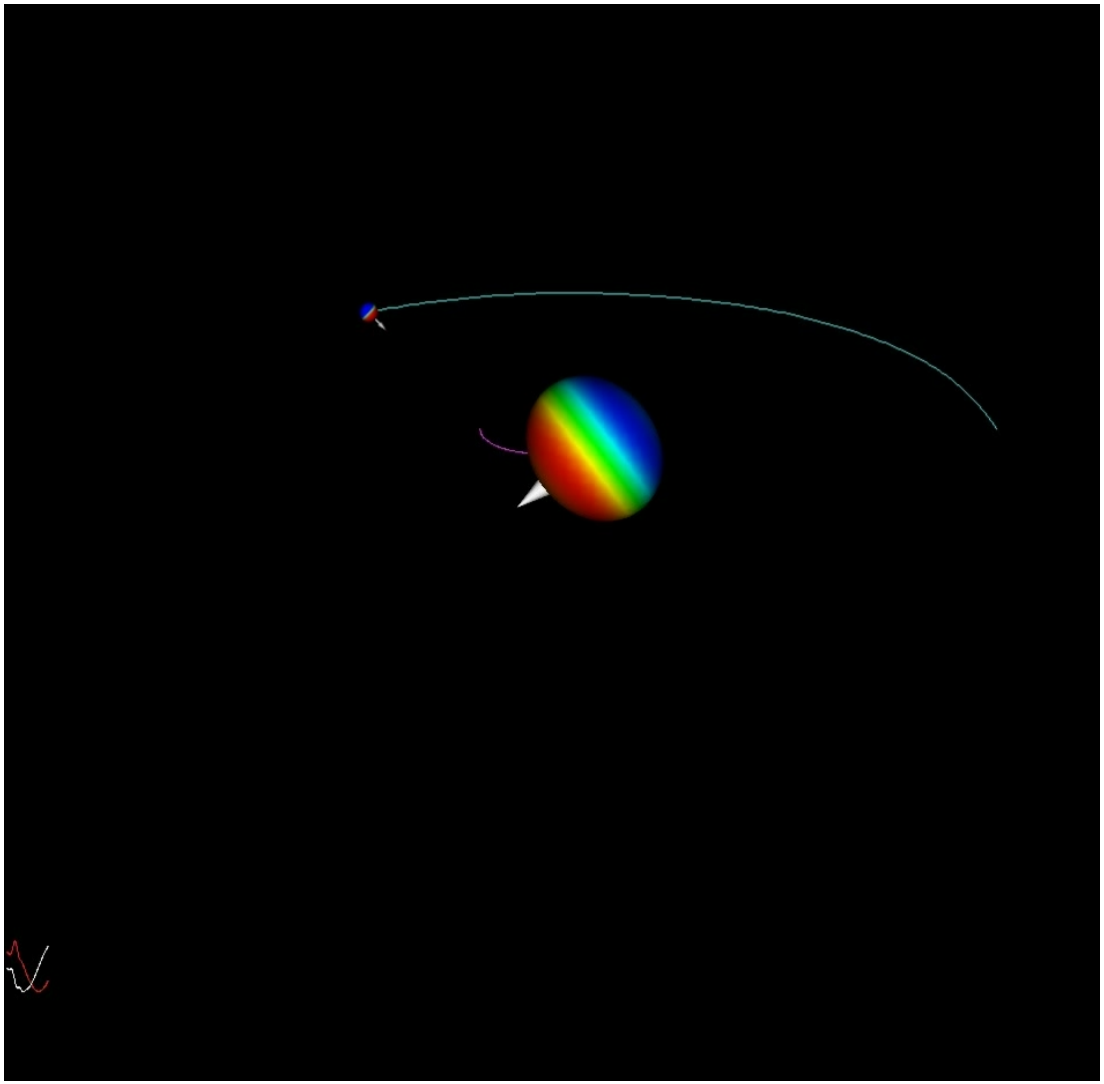
SXS Collaboration: Mass ratio 8:1

Current BBH capabilities: Precession



Simulation and Movie: Nick Taylor, SXS Collaboration

Current BBH capabilities: Precession



Color = Vorticity
(a measure of spin)

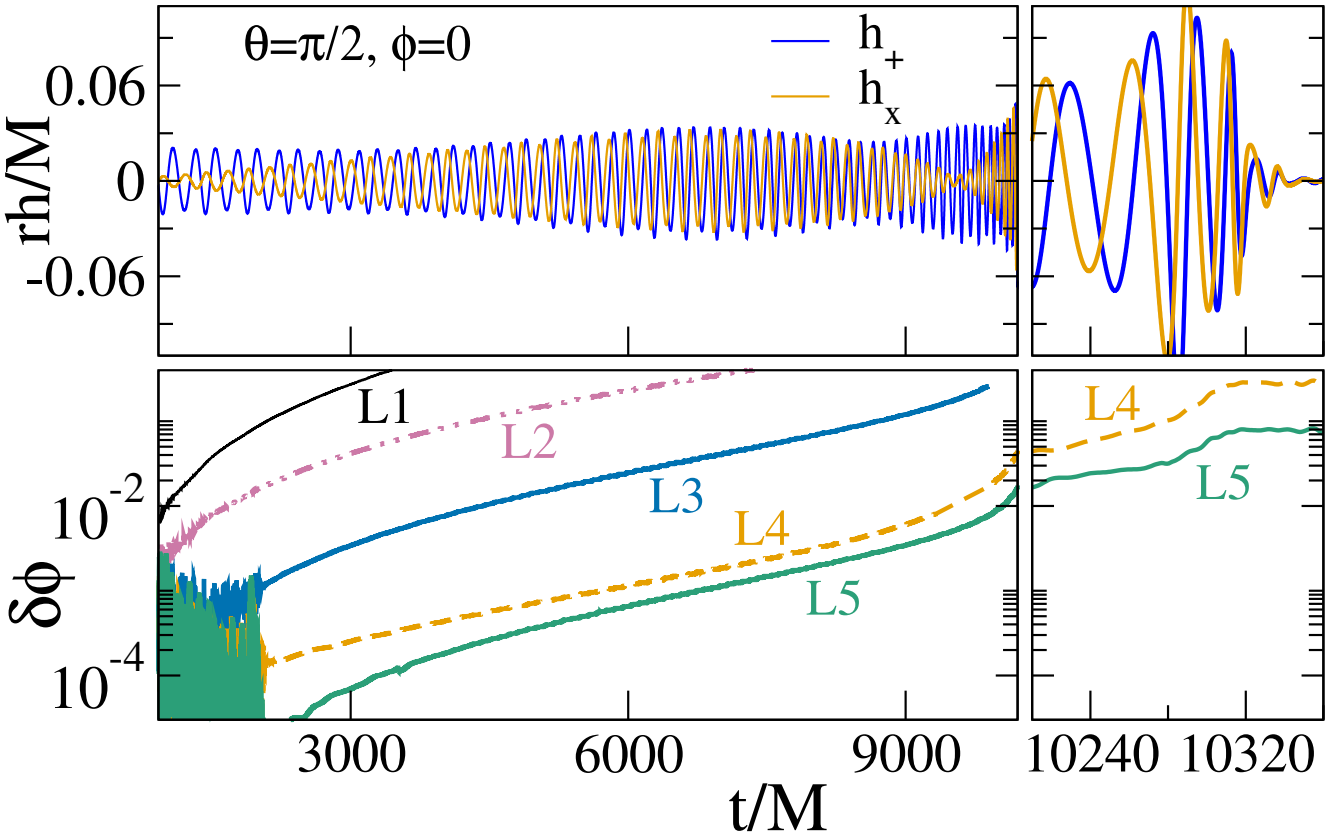
Mass ratio 6

Large hole spin $\sim$ 0.91

Small hole spin $\sim$ 0.3

Movie: Robert McGehee and Alex Streicher, SXS Collaboration

Current BBH capabilities: Accuracy



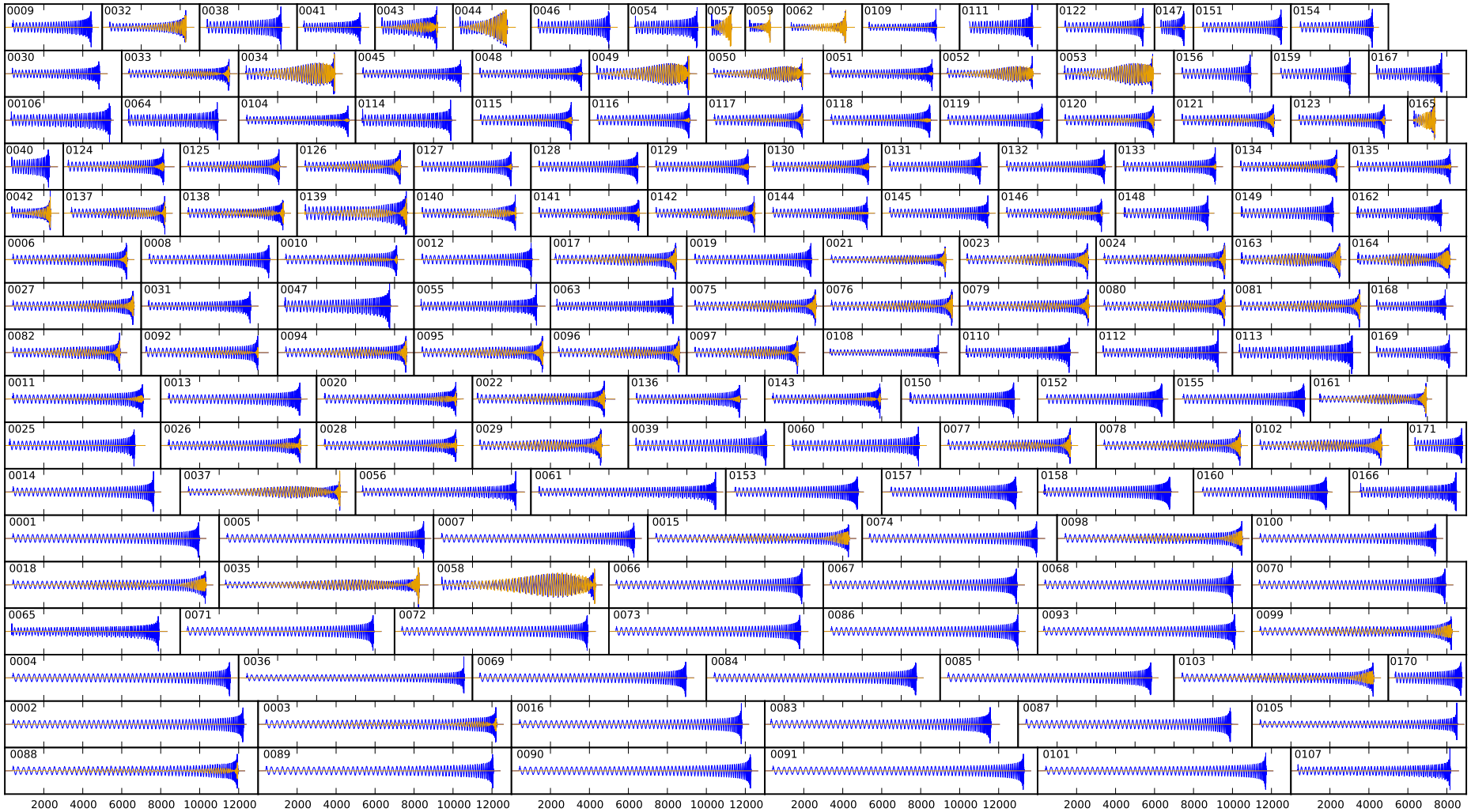
Mass ratio=3
Large hole spin=0.5
Small hole spin=0

31 orbits
Precession

Mroue et. al., SXS Collaboration, arXiv:1304.6077

New BBH capabilities: simulation catalogs

171 simulations, all with inspiral, merger, and ringdown



Mroue, MAS, Szilagyi, Pfeiffer, Boyle, Hemberger, Kidder, Lovelace, Ossokine, Taylor, Zenginoglu, Buchman, Chu, Giesler, Owen, Teukolsky, arXiv:1304.6077

New BBH capabilities: simulation catalogs

Other catalogs:

- NRAR (Numerical Relativity/Analytical Relativity) project
  - Goal: Improve analytic waveform models using NR simulations.
  - 9 NR codes.
  - 25 simulations in first round; in preparation.
- NINJA (Numerical INJection Analysis) collaboration
  - Goal:
    - Add numerical waveforms into LIGO/Virgo detector noise
    - Test how well detection pipelines can detect/identify them
  - 8 NR groups.
  - 56 hybridized waveforms, CQG 29, 124001 (2012)
- Georgia Tech
  - 191 simulations (Pekowsky et al, arXiv:1304.3176)

Applications of BBH simulations

Gravitational-wave studies

- Calibrate and test analytic waveform models
- Inject into LIGO data analysis pipelines
- Construct NR-only template banks
- ...

Applications of BBH simulations

Astrophysics

- Construct formulae for remnant properties
- Study gravitational recoil
- Examine precession effects
- Examine effects of eccentricity
- ...

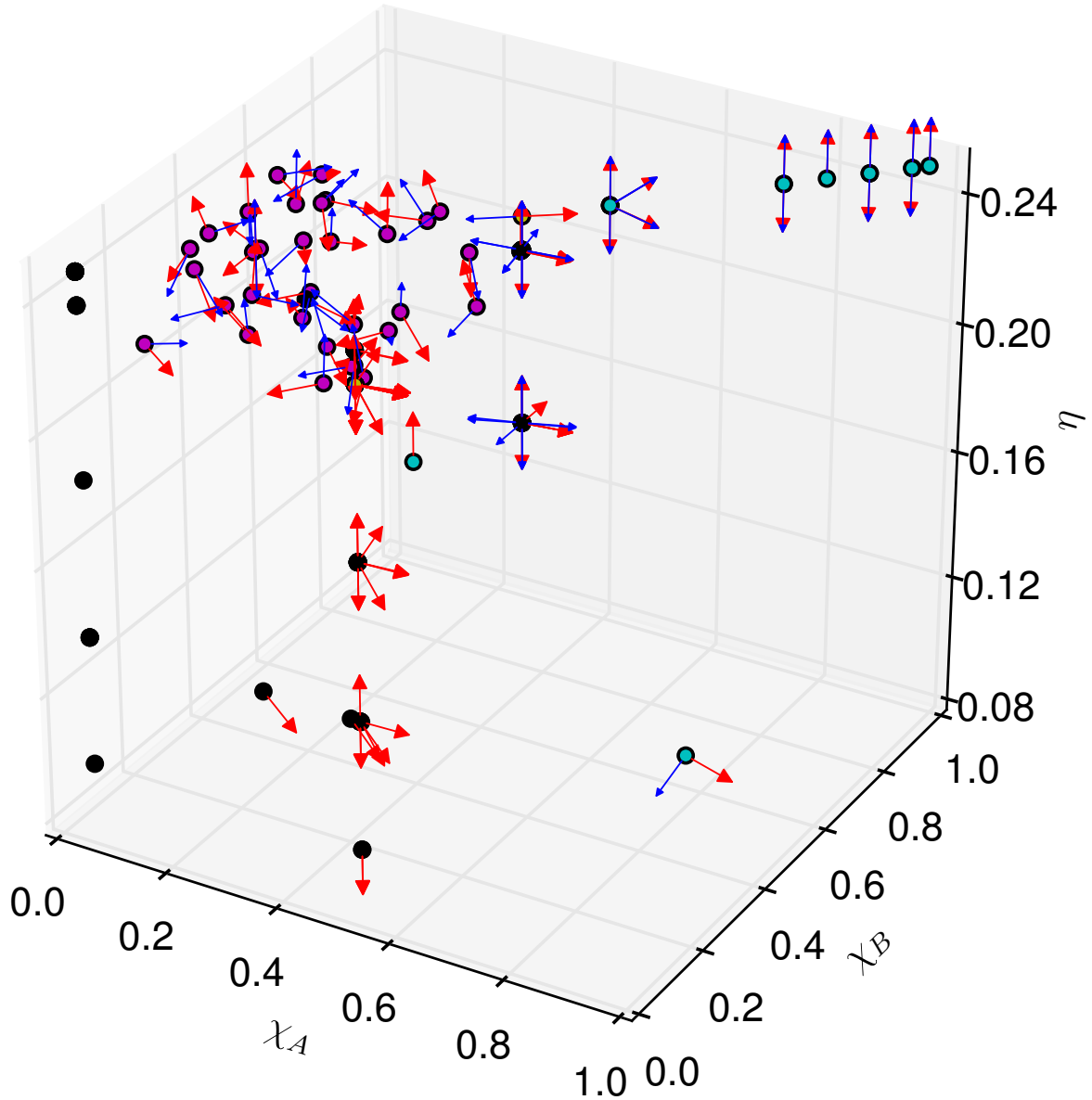
Applications of BBH simulations

Fundamental relativity

- Study critical phenomena
- Study black holes in higher dimensions
- Examine relativistic head-on collisions
- Study topology and behavior of event horizon during merger
- Understand dynamics of strong gravity and wave generation
- ...

3. Challenges

Challenge: parameter space coverage



$$\eta = m_A m_B/(m_A + m_B)^2$$

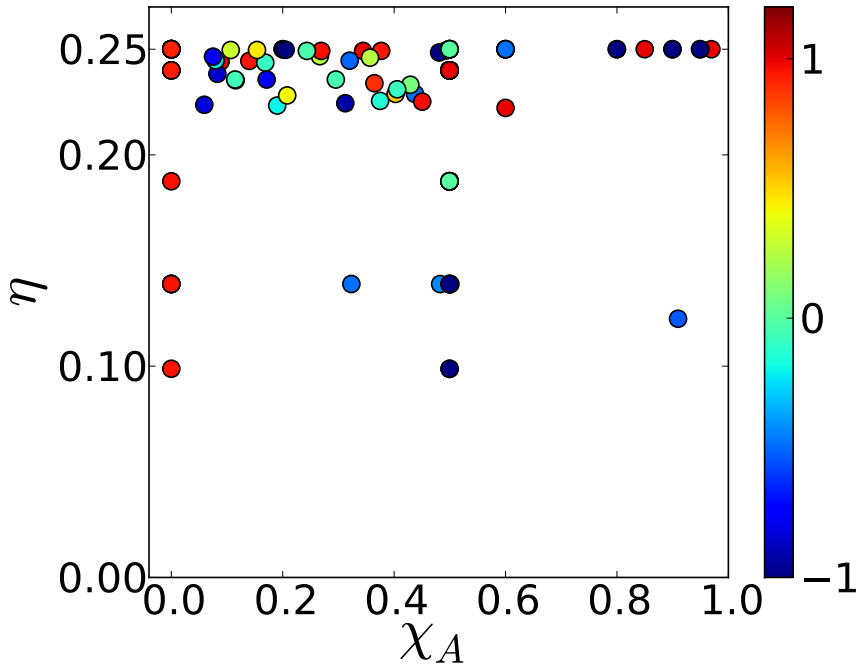
- Red/blue arrows = Initial spin directions
- Spins up to 0.97
- Mass ratios up to 8

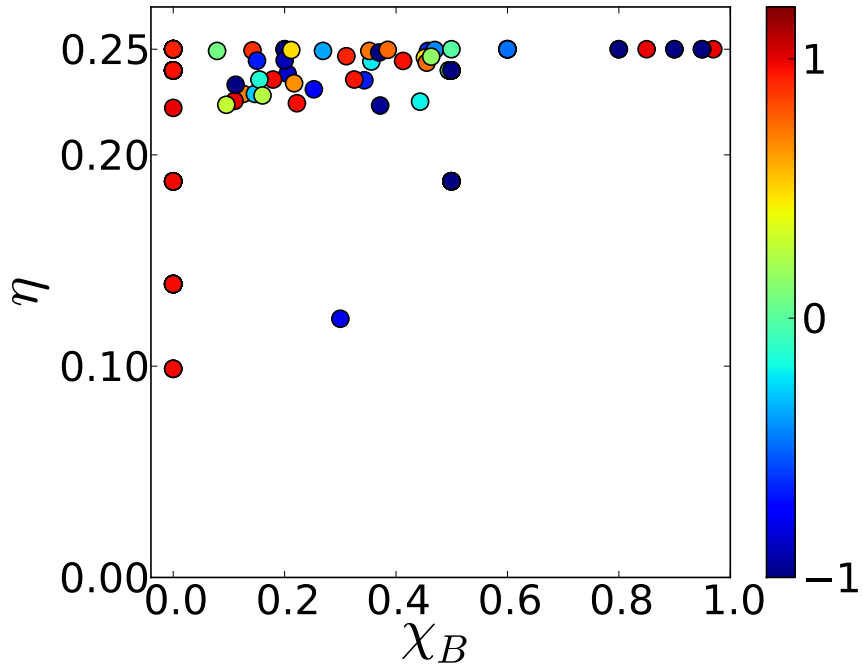
- Very sparse coverage!

SXS Collaboration catalog, Mroue et. al. arXiv:1304.6077

Challenge: parameter space coverage

$\eta = m_A m_B/(m_A + m_B)^2$





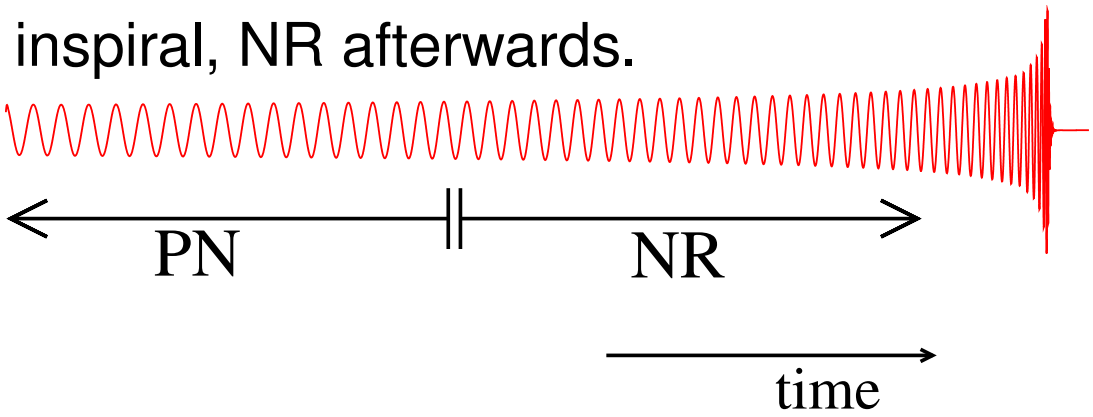
SXS Collaboration catalog, Mroue et. al. arXiv:1304.6077

Dual challenge:

- Parameter space is large, 7D.
- Extreme parameter values are difficult.

Challenge: many orbits

- Waveform visible to LIGO includes hundreds of binary orbits.
- NR simulates many fewer orbits (most to date is 34).
- Solution: Use PN for inspiral, NR afterwards.



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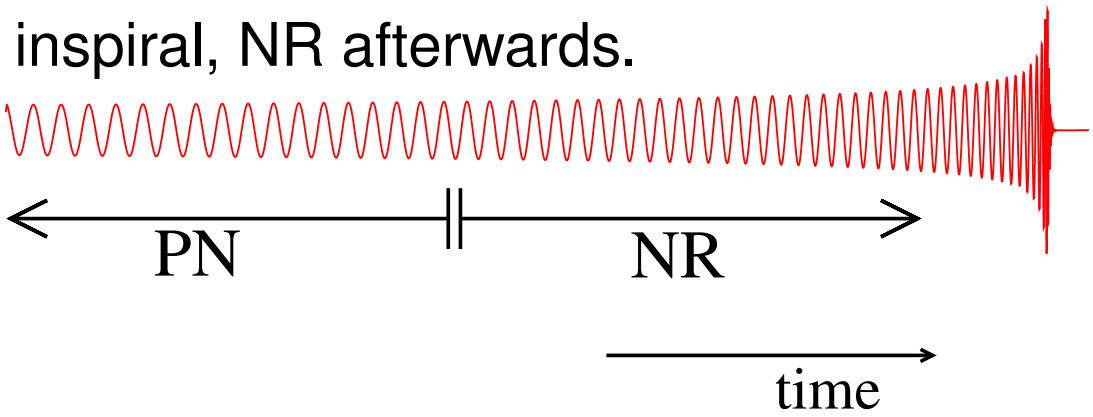


- Problem: Where is the matching point?
  - Equal mass, no spin: can match 10 orbits before merger.
  - 'BH/NS' parameters: (mass ratio ~ 7, moderate spins), PN is still inaccurate dozens of orbits before merger.
- Dual challenge:
  - NR needs to simulate more orbits.
  - We don't know how many orbits are needed! (but are testing this)

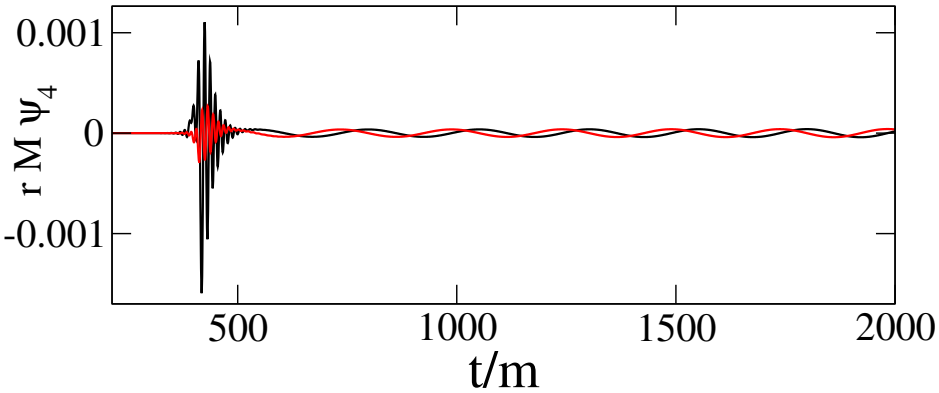
Challenge: Initial data

Astrophysical initial data

- Initial data should be 'snapshot' of inspiral from $t = -\infty$
- Tidal distortion, initial gravitational radiation are not correct.

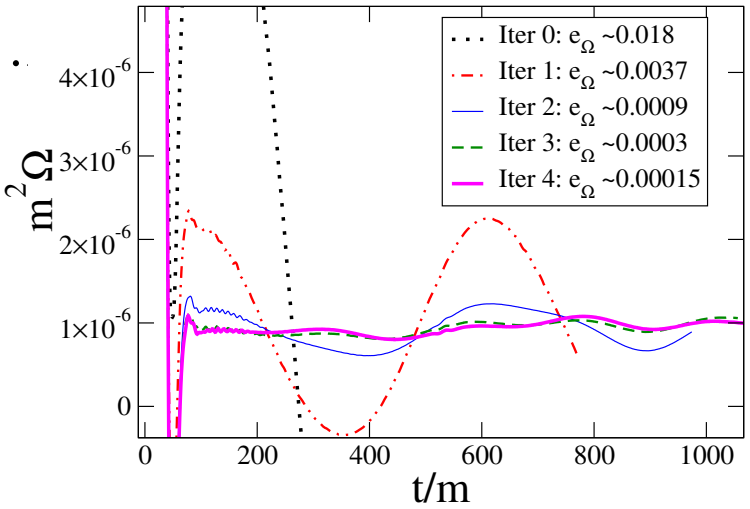
$\Rightarrow$ "Junk radiation" spoils beginning of simulation.

$\Rightarrow$ Masses & spins relax during junk epoch.



Eccentricity

- Cannot *a priori* choose initial data to get desired eccentricity.
- Can produce small eccentricity via iterative scheme: expensive.



Mroue & Pfeiffer, arXiv:1210.2958

Challenge: Computational expense

Example runs from SpEC code (SXS collaboration):

Mass Ratio	Spin A	Spin B	N orbits	Run time (CPU-h)
1	0	0	16	8k
1	0.95	0.95	25	200k
3	0.7	0.3	26	34k
6	0.91	0.3	6.5	38k

- Best efficiency: few (~ 50) cores, run many simulations at once.
- Days to months wallclock time, depending on parameters.
- Recent improvement by a factor of ~ 5 for SpEC (Bela Szilagyi).

Challenge: Computational expense

- NR too computationally expensive for
  - Covering 7D parameter space by random sampling.
  - 'Live' NR during data analysis.
- Parameter space, expense worse when including neutron stars.

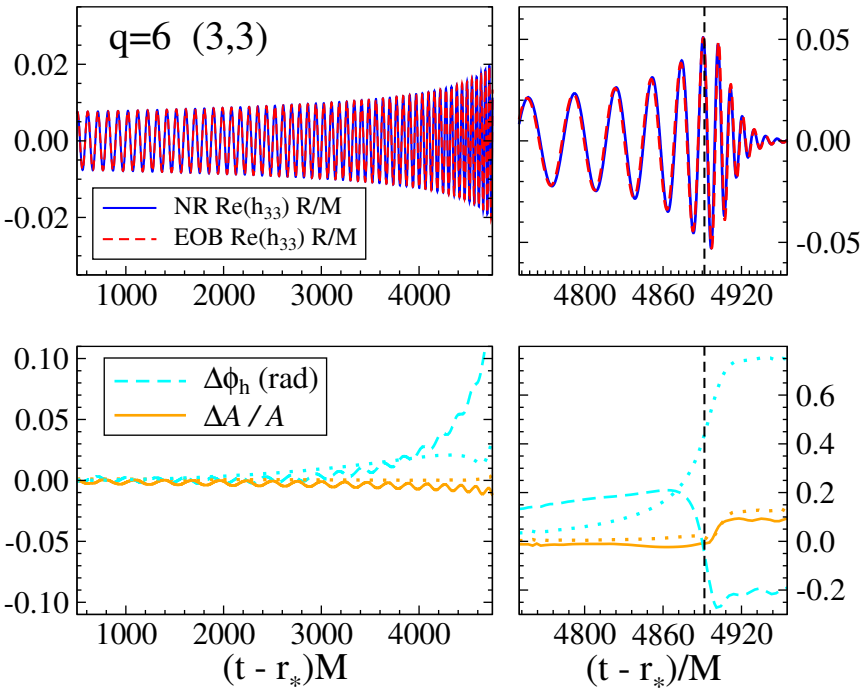
What to do?

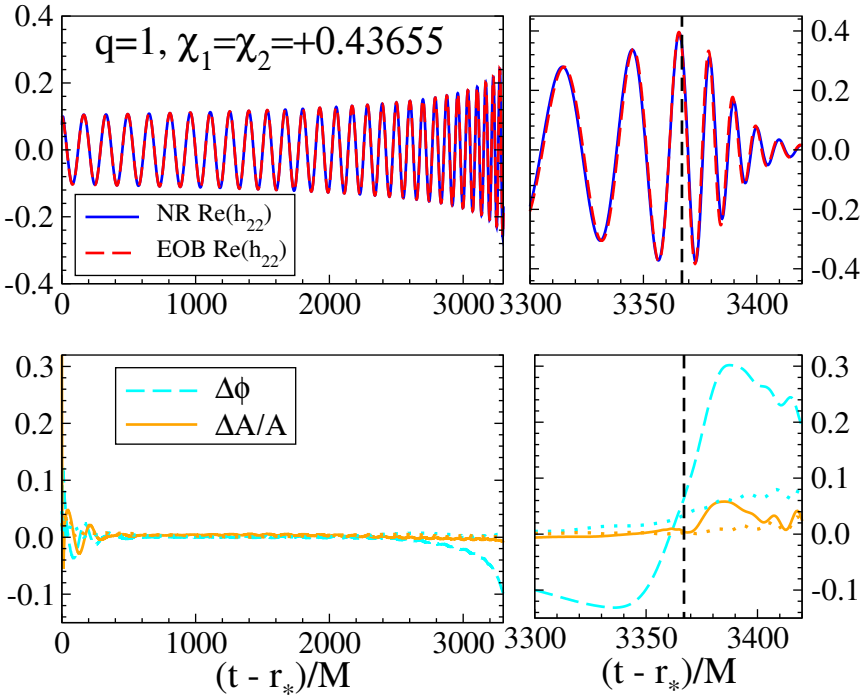
- Build **analytical** models ("EOB", "PhenomC") w/ unknown coefs.
  $\Rightarrow$ Determine coefficients by fitting to numerical simulations.
- **Reduced order modeling**.

Analytical waveform models

"Effective One Body" model fitted to SpEC waveforms.

- No spin: "EOBNRv2"
  Pan et. al. PRD 84:124052 (2011)





- Spins, no precession:"SEOBNRv1"
  Taracchini ea., PRD 86:024011 (2012)

Summary

- Numerical Relativity now becoming mature, especially for BBH
- 7D parameter space for BBH, more for NS/NS, NS/BH
- Challenges:
  - Simulating enough binary orbits.
  - Difficult corners of parameter space.
  - How to choose 'important' points in parameter space.
  - Computational expense.
- In the future
  - Improvements in efficiency, accuracy, parameter coverage.
  - Include more physics for simulations with matter.
  - Reduced Order Modeling