Statement of Research Interests

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August 27, 2013

1 Research interests

My main research interests lie in the field of computational astrophysics. I am strongly interested in computational science, especially numerical techniques applicable to general relativistic magnetohydrodynamics. My research interests include aspects of analytical relativity in particular black hole perturbation theory as well as semi-analytical aspects thereof.

I have worked on the gravitational and electromagnetic signatures of the disruption of white dwarfs by intermediate mass black holes [1, 2], the signature of gas present in the final merger phase of supermassive black holes [3, 4], extreme mass ratio inspirals [5, 6, 7, 8, 9], binary neutron star mergers [10], core collapse supernovae [11, 12], and developed multi-patch, adaptive mesh refinement codes for numerical hydrodynamics [13] and magnetohydrodynamics [14]. I am one of maintainers and core developers of the free, open-source Einstein Toolkit [15].

We are at the brink of being able to directly detect the gravitational waves predicted by General Relativity. With the advanced Laser Interferometer Gravitational-Wave Observatory (aLIGO) scheduled to come on-line in 2014 [16], there is great need to model the sources of gravitational waves that LIGO will detect. Only by comparing to model waveforms will we be able to extract the gravitational wave signal from LIGO's detector noise. These detections, together with electromagnetic observations of similar events, promise to offer answers to some of the most exciting questions in astrophysics. Are neutron star – neutron star mergers and black hole – neutron star mergers sources for short hard gamma ray bursts? How does the unknown equation of state of nuclear matter affect the merger and what microphysics governs matter at super-nuclear densities? Are nuclear accretion disks sites of r-process nucleosynthesis? All of these questions require that the system be accurately modelled and simulated using state of the art, highly parallel codes that included complex multi-physics combining a general relativistic description of gravity with a realistic description of matter and radiation.

1.1 Open source software development

The codes used for general relativistic magnetohydrodynamics simulations have grown beyond the point where a single researcher (or even research group) can develop them. Instead large collaborations bringing together physicists, computer scientists and applied mathematicians have formed to develop and maintain the codes [15, 17, 18].

Free, open source licensed software enables researchers to more easily share their work and combine efforts. This avoids duplication of effort, freeing physicists to explore new physics and facilitates collaboration with computer scientist to use state of the art methods to do so. Open source codes also improve reproducability of results. Several groups have in the past compared results of their vacuum and hydrodynamics codes for identical physical setups [19, 20]. Such comparisons, while useful, are hampered if codes are private and results cannot be reproduced independently. Finally open source software is in line with open science practices such as providing access to raw data from experiments and peer review of results.

From a practical point of view, open source software more easily allows for a separation of concerns, where computer scientists supply the computational infrastructure, and physicists the physics modules building on these. Different modules, developed by independent groups can benefit from using existing components [15]. Postdocs and graduate students that move from one group to another do not lose access to codes they helped develop and can continue to contribute to science without interruption. New groups can more easily join the field if there are basic software components available, leading to a richer science as more scenarios are explored using the code.

I am strongly involved in the Einstein Toolkit collaboration [15] which aims to provide an open source, stable, well maintained and documented code for general relativistic magnetohydrodynamics simulations. The Einstein Toolkit is based on the Cactus framework and used by groups at Caltech, LSU, AEI, GaTech, RIT and others for their numerical simulations. The toolkit currently has 93 registered users from 47 institutions around the world, with regular releases of updates and code fixes every 6 months. In the Einstein Toolkit we provide not only the code for users to download but also host weekly phone calls as well as an active mailing list and a bug tracking system to facilitate user feedback and to provide help to users.

Most recently we released a version of a our public general relativistic hydrodynamics code GRHydro [14]. GRHydro implements both constrained transport and divergence cleaning schemes to evolve the equations of ideal general relativistic magnetohydrodynamics. GRHydro supports simple analytic equation of states, as well as tabulated microphysical (hot) equations of state used in typical core-collapse supernova simulations.

Indeed the core-collapse simulations we performed at Caltech are based on the Einstein Toolkit and requirements of these simulations are a major driving factor in the development of the toolkit. I have been involved in these projects both as a scientist and providing computer support and infrastructure and physics code for these simulations.

Together with Christian Reisswig I worked on the publicly available multi-patch code Llama [21]. Llama provides a multi-patch framework for general relativistic magnetohydrodynamics which covers the simulation domain with multiple patches adapted to the symmetry of the problem. Typically a central Cartesian block capable of adaptive mesh refinement is surrounded by a set of inflated cube [22] patches with logarithmic spacing in the radial coordinate and constant angular resolution. Such a patch system is ideally suited to compact binary and core-collapse simulations where the central region benefits from mesh refinement and where spherical or cylindrical symmetry is not a good approximation. In the outer wave zone on the other hand, spherical symmetry is a good approximation to the system dynamics and the use of a coordinate system adapted to this symmetry greatly improves speed and accuracy of the simulation.

1.2 Binary neutron star mergers as sources of gravitational radiation

Double binary neutron star systems are among the expected sources of gravitational waves detectable by LIGO, VIRGO, and the Kamioka Gravitational Wave Detector (KAGRA) ([23] and references therein). Such a double binary neutron star system forms out of a binary star system that survives throughout the evolution and supernova explosion of both of its constituent stars [24]. Eventually gravitational wave emission by the orbiting system becomes the dominant source of orbital decay and drives the inspiral of the stars toward each other.. To date this has been observed only indirectly for example in the orbital decay of the Hulse-Taylor pulsar [25]. Finally the neutron stars collide and either form a temporary hypermassive neutron star or directly collapse to a black hole ([26] and references therein). During the final inspiral phase the system emits a chirp-like gravitational wave signal which increases in both frequency and amplitude as the stars approach each other. Once the stars come into contact the sinusoidal gravitational wave abruptly cuts off while the combined object settles into an almost spheroidal shape. The massive (neutron star) remnant may then continue to emit gravitational waves as hydrodynamic instabilities redistribute matter to form a non-vanishing quadrupole moment until sufficient angular momentum has been shed via gravitational wave emission to allow collapse to a black hole [27]. Finally the black hole settles down through quasinormal ringing.

The inspiral signal encodes information about the mass of the neutron star, while information about the neutron star equation of state is mostly accessible in the very late inspiral and early merger signal. Detecting the signal and in particular extracting system parameters using a matched filtering approach however requires an accurate model and numerical simulation of the inspiral and merger process [23]. This is usually done in a two step process, where a few accurate but expensive full numerical simulations are used to calibrate a secondary set of semi-analytic templates [28]. This is done as an efficiency measure because of the large cost associated with full simulations. Generally the secondary templates are more faithful if longer numerical waveforms are used to calibrate them.

Recent work [29, 30] showed that the equation of state of neutron star matter has measurable influence on the tidal deformability of the neutron stars and through this leaves an imprint in the gravitational wave signal. Any template used for parameter extraction thus has to take the equation of state of neutron star matter into account. Unfortunately many current simulations [28, 31, 32, 33, 34, 10] use a simplified Γ-law equation of state which is appropriate only during the inspiral phase. On the other hand those simulations that use a more realistic nuclear equation of state are very short [35], spanning only 5 – 7 orbits. It is expected [30] that longer numerical simulations will allow for a more accurate estimation of equation of state parameters from the detected signal.

In the simulating extreme spacetimes (SXS) collaboration [17] I am simulating the inspiral and merger of binary neutron stars. Using SpEC, we achieve high accuracy over long inspirals by employing a hybrid scheme that uses efficient spectral methods to solve for the smooth spacetime part of the Einstein-matter system and a finite volume method to solve for the matter variables.

We are currently capable of simulation $\gtrsim 22$ orbits which opens the window to infer information about the nuclear equation of state from the late inspiral signal.

I am implementing adaptive mesh refinement in the code, increasing the resolution in the crucial central region of the simulation domain where the merging neutron stars are located to further increase this accuracy.

1.3 Binary neutron star mergers as progenitors of short gamma ray burst central engines

The merger of double neutron star binaries and the accretion of a remnant disk on the central merger remnant is a leading candidate for the central engine in short gamma ray bursts [24]. During merger the neutron star material is heated and the cold equation of state used in most current simulations becomes an increasingly poorer approximation of the equation of state of the neutron stars. This has the possibility of affecting the predicted lifetime of a hypermassive merger remnant and the geometry of the surrounding disk. The effect of varying the equation of state was first studied in [36], [37] and [38]. [36] compared results of simulations using different hot equations of state, using the conformally flat approximation to General Relativity, while [37] employed full numerical relativity but only a single hot equation of state. Very recently [38] employed both general relativity, several (cold) equations of state and simulated inspirals lasting 10 orbits.

Once the stars have coalesced into a single object a jet of relativistic particles can be ejected along the spin axis. If the central object is a black hole, then strong magnetic fields driven by the magnetorotational instability in the disk extract energy from the spin of the black hole to launch the jet [39]. An alternative jet mechanism drives the jet by neutrino annihilation that deposits energy via pair production above the poles and provides energy in this manner. The strong neutrino flux facilitates the formation of r-process elements in the disk and may drive a neutrino-rich disk wind [40, 41]. In both the central black hole and central hypermassive neutron star case, the magnetic field will collimate outflowing material along the magnetic field lines.

To facilitate this research I and collaborators are currently adding a more realistic neutrino transport scheme, as well as support for magnetohydrodynamics to our existing adaptive mesh refined multi-patch code [42].

1.4 Core-collapse supernovae

When a massive main sequence star nears the end of its lifetime, it burns lighter elements to iron in a thermonuclear reaction and forms a growing iron core. Once the core reaches a critical mass, nuclear statistical equilibrium favours dissociation into α-particles and free nucleons and the core begins to collapse in a runaway process. As the core collapses its density rises to nuclear densities where the equation of state suddenly stiffens providing a new hydrostatic equilibrium. The core however overshoots this equilibrium position and bounces back into the still infalling material. This launches a shock into the infalling material that is ultimately responsible for the supernova explosion. As the shock travels outwards, it dissipates energy via dissociation of heavy nuclei as they fall through the shock front and via escaping neutrinos. Eventually the shock stalls and a successful explosion requires a means to revive the shock.

Currently there are two commonly accepted candidate mechanisms for shock revival: (a) the neutrino mechanism [43] where the shock is revived through the energy deposited by neutrinos escaping from the proto-neutron star core in a gain layer under the shock front and (b) the magnetohydrodynamic mechanism which operates by converting the rotational energy of the core into magnetic field energy which drives the explosion through magnetic pressure [44, 45].

Modern one-dimensional spherical symmetric studies [46] using full Boltzmann neutrino transport have shown that in spherical symmetry neutrino heating is not sufficient to revive the shock. A secondary mechanism is required to prolong the residency time of the infalling material in the gain region for it to absorb sufficient energy to revive the shock. Two possible mechanism have been observed in 2D axisymmetric simulations: the neutrino-driven convection [47] which creates a negative entropy gradient in the gain region, thus promoting convective overturn of the material, and the standing accretion shock instability (SASI) [48] which leads to a large scale, sloshing type motion along the axis of symmetry, also increasing the residence time of material in the gain region.

In a recent paper [11] we described a core-collapse simulation of collapse and protoneutron star formation, that for the first time employed full 3d numerics, high resolution, neutrino transport and a fully general relativistic treatment of gravity in a single simulation. This is in contrast to previous studies [49, 50, 51] that employed a combination of a Newtonian approximation to General Relativity, simplified neutrino treatment, lower resolution or were using axisymmetric codes. Our work improves tremendously on these aspects, we extract, for the first time, gravitational waves from full 3d general relativistic collapse and post-bounce core-collapse supernova simulations. We also verify the relative importance of the standing accretion shock instability and convective instabilities in 3d Cartesian simulations.

The simulation pushed the boundaries of what our simulation framework was able to handle both in size of the simulation as well as complexity of the physics involved, yielding output files of terabyte size that had to be postprocessed off-line to explore the dynamics of the core-collapse supernova engine.

We are now working on improving our neutrino transport scheme by implementing a 2 moment, energy dependent scheme as well as incorporating ideal magnetohydrodynamics into our code. These improvements will allow us to make quantitative predictions of the neutrino signal to be expected from the next galactic supernova.

1.5 Extreme mass ratio inspirals

Mass segregation effects concentrate stellar mass black holes and other heavy objects near the central massive black holes [52, 53] in galaxy cores. From there, scattering by other stars [54], tidal disruption events [55] and resonant relaxation [56] moves compact objects onto eccentric trajectories along which the objects enters the strong field region around the central black hole where gravitational wave emission dominates the orbital decay.

A possible future eLISA/NGO detector will detect such extreme mass ratio inspirals of a solar mass compact object into a supermassive black hole. During the inspiral eLISA/NGO will map the black hole spacetime in exquisite detail [57]. These mappings provide a test for the black hole uniqueness theorems of General Relativity as well as the chance to directly measure the mass of the supermassive black holes in the centre of galaxies [57] to very high accuracy. Just as the waveforms contain information about the central object so do they encode the orbital parameters, and detection of extreme mass ratio inspiral signals thus offers a unique opportunity to learn about the dynamics governing dense stellar clusters.

There are currently efforts underway to compute inspiral waveforms using either full 3d simulations or by interpolating the self-force driving the inspiral from a table of force values computed along geodesics [58, 59].

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