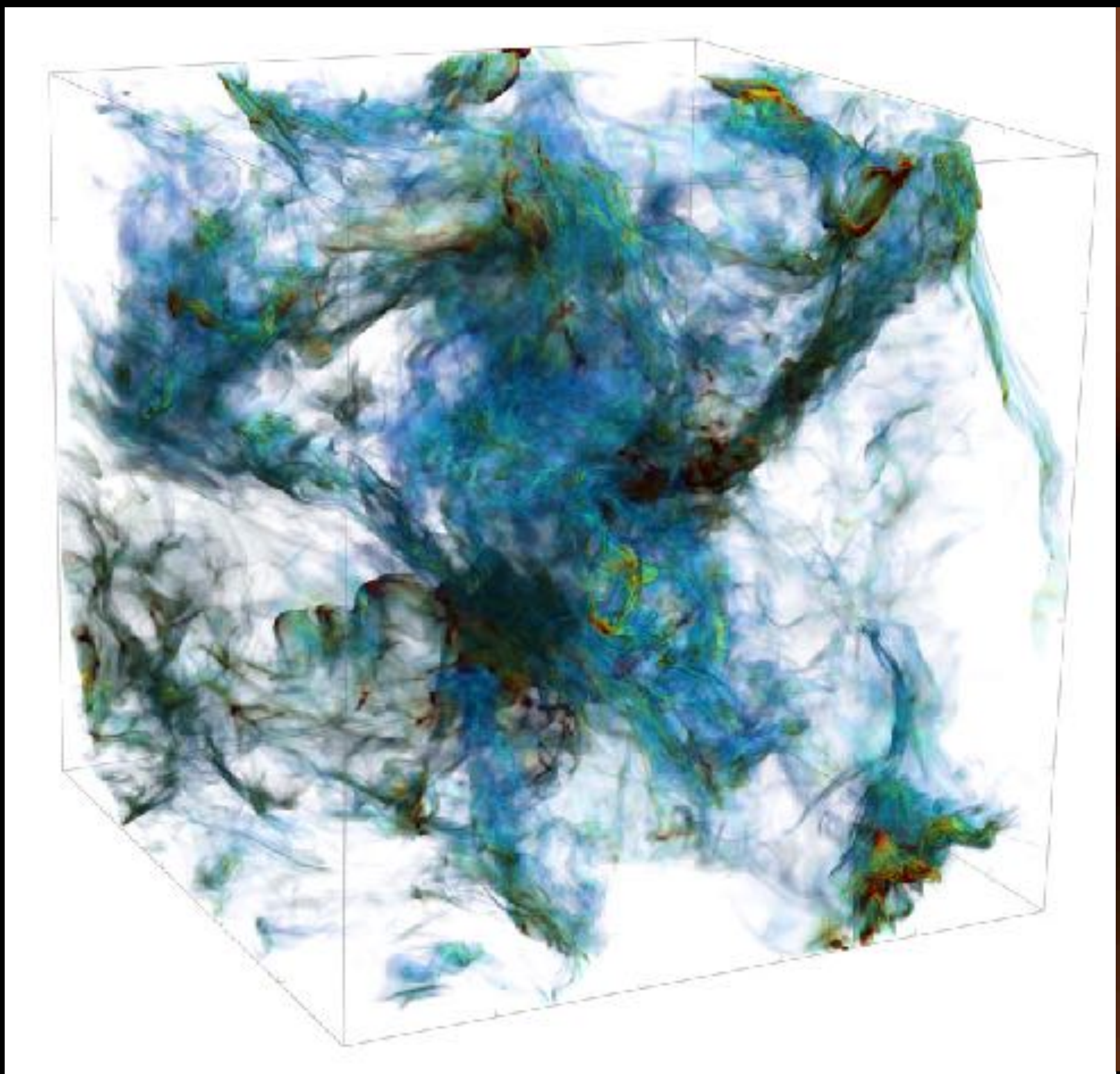
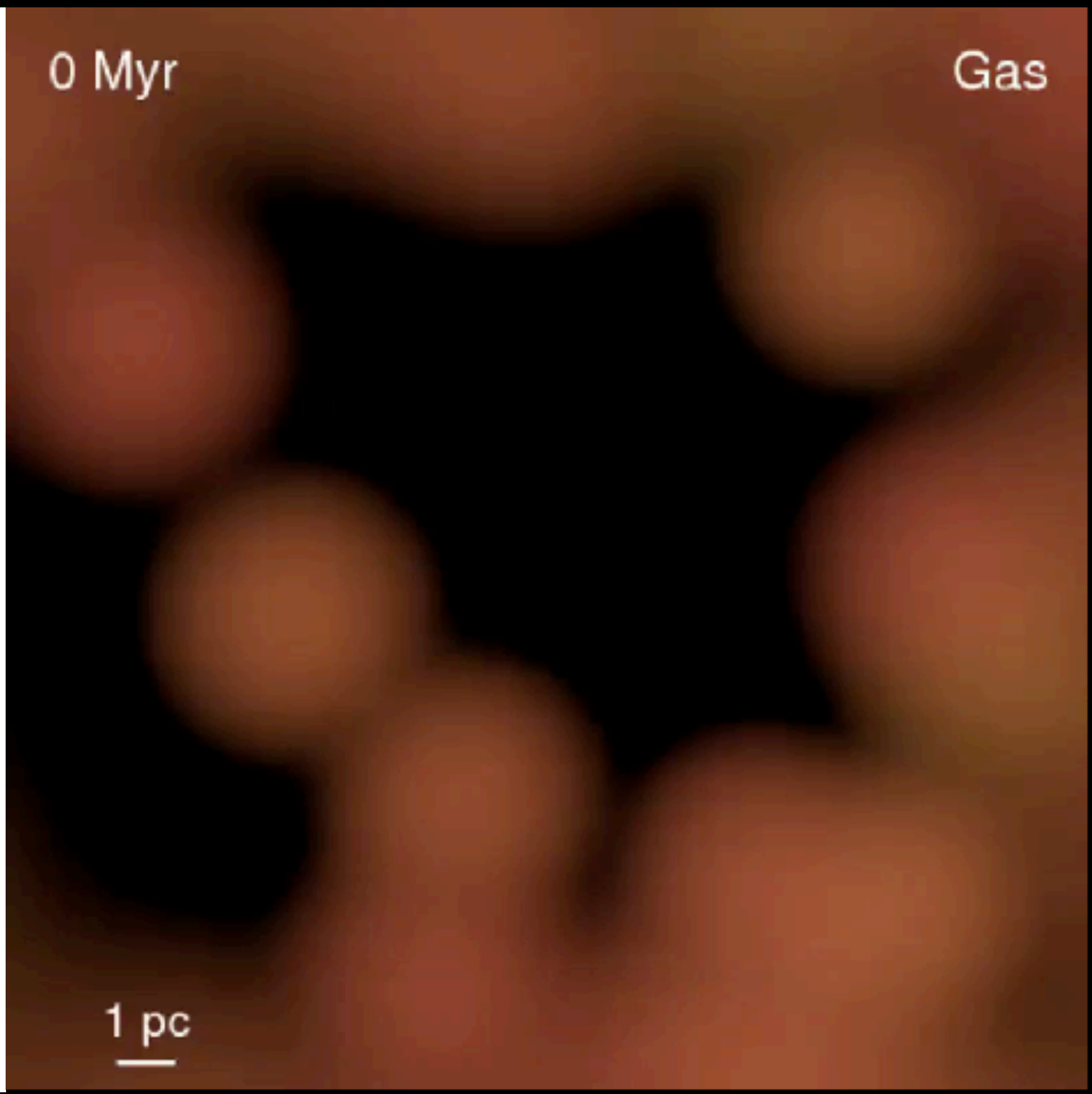
# A New Approach to Turbulence:

## Origins of ISM Structure, Stellar Clustering & the IMF, and (perhaps?) Planet Formation



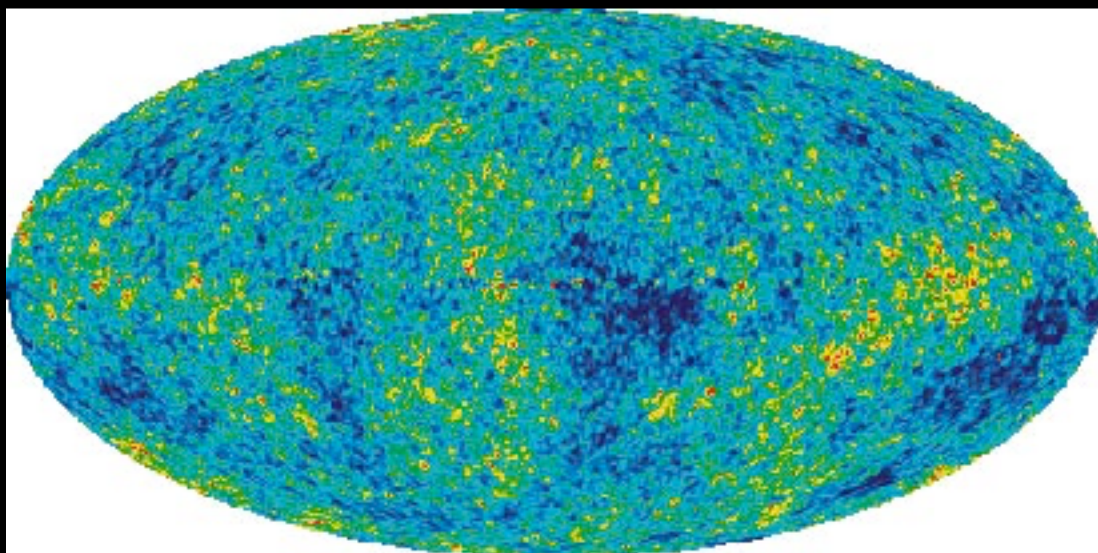


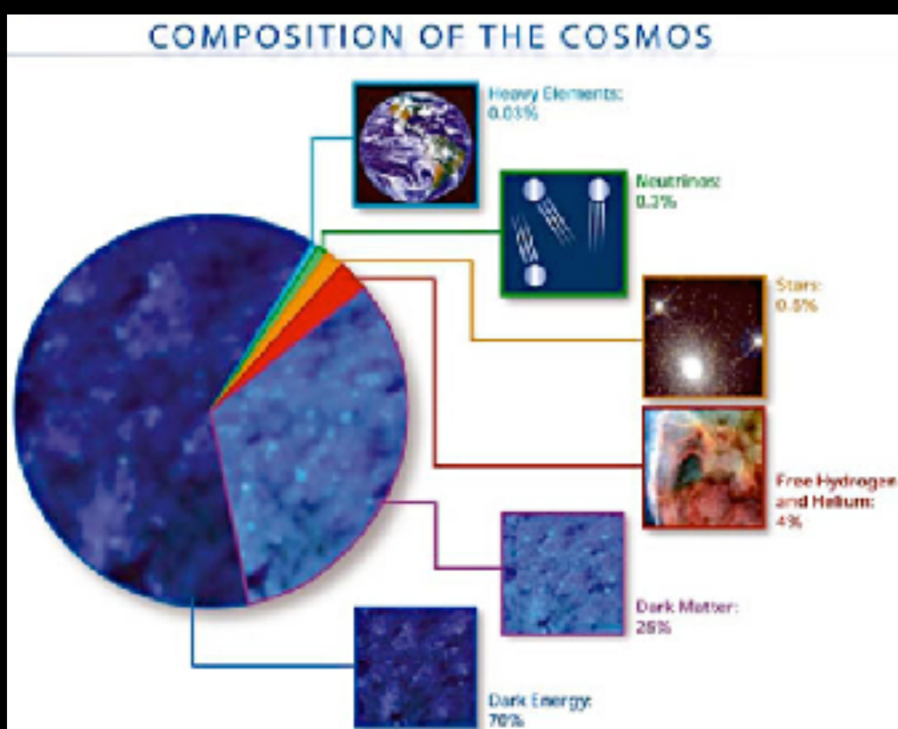

Philip Hopkins

(and many, many others who did all the hard bits)

# What’s the Big Picture?

# The Big Question:
## HOW DO WE GO FROM BIG BANG TO MILKY WAY?






z~1090
(t~400,000 yr)

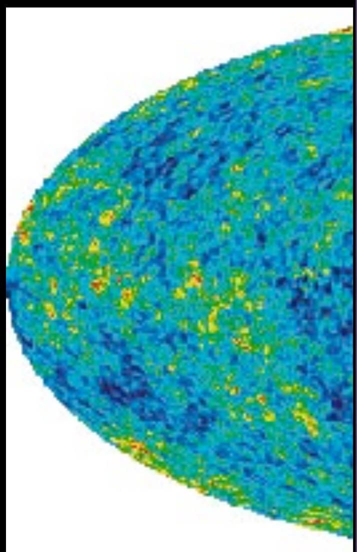
?

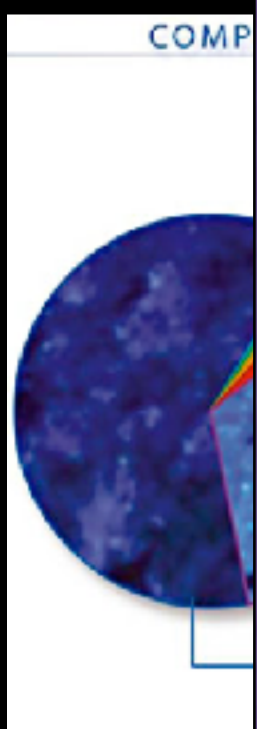
Today

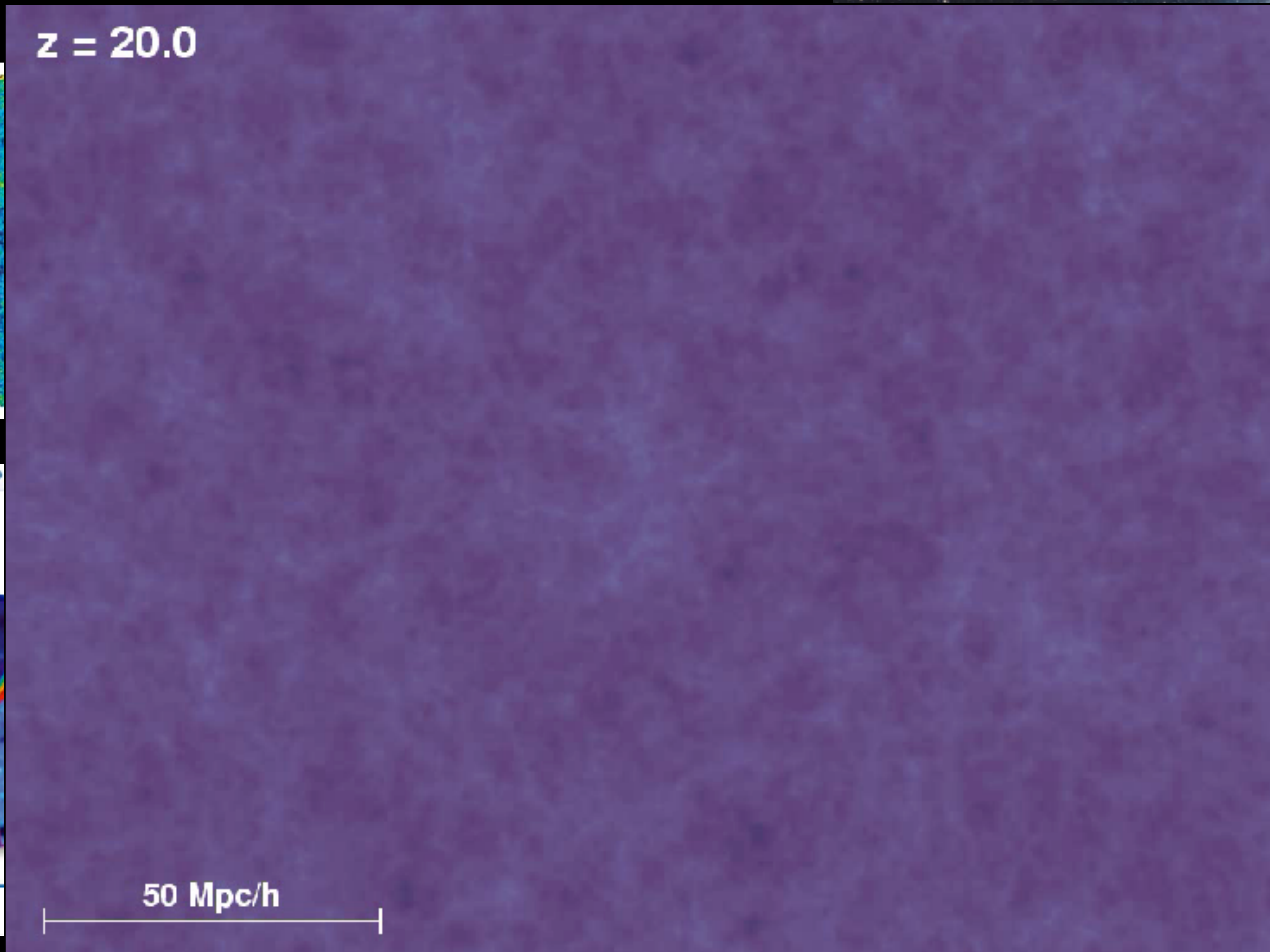
# The Big Question:

HOW DO WE GO FROM BIG BANG TO MILKY WAY?

Today



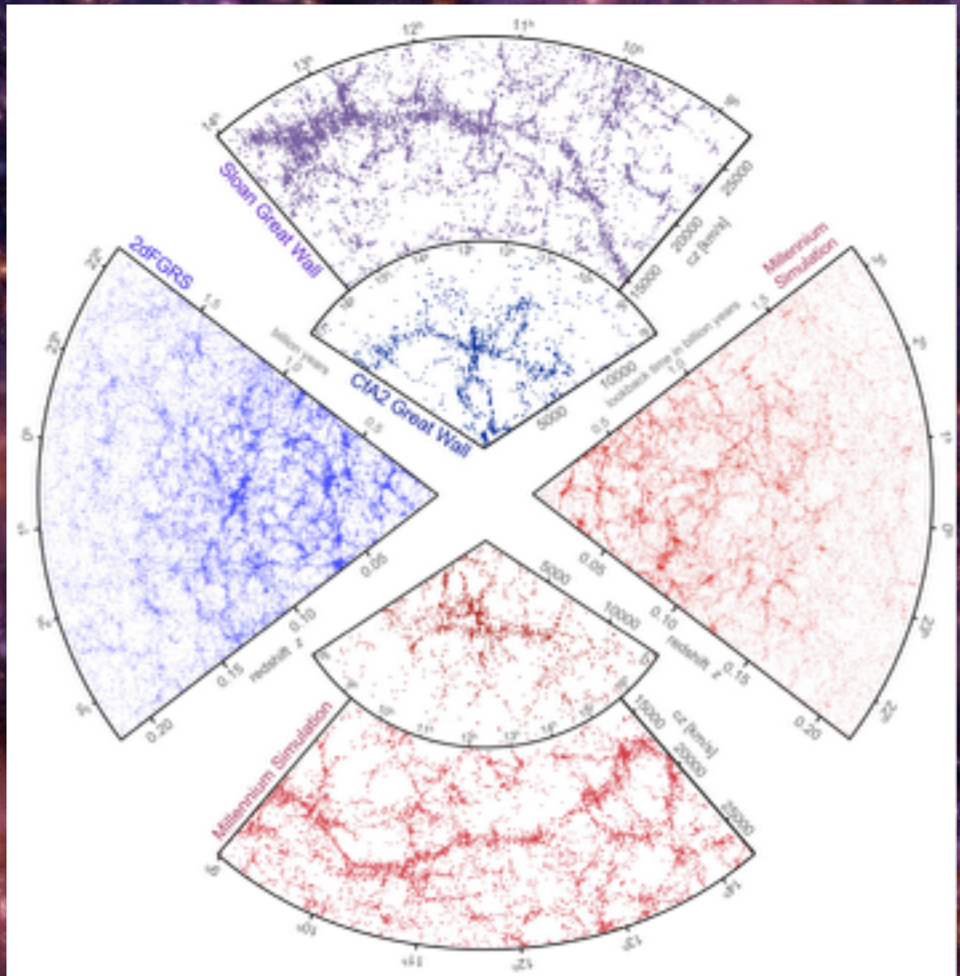


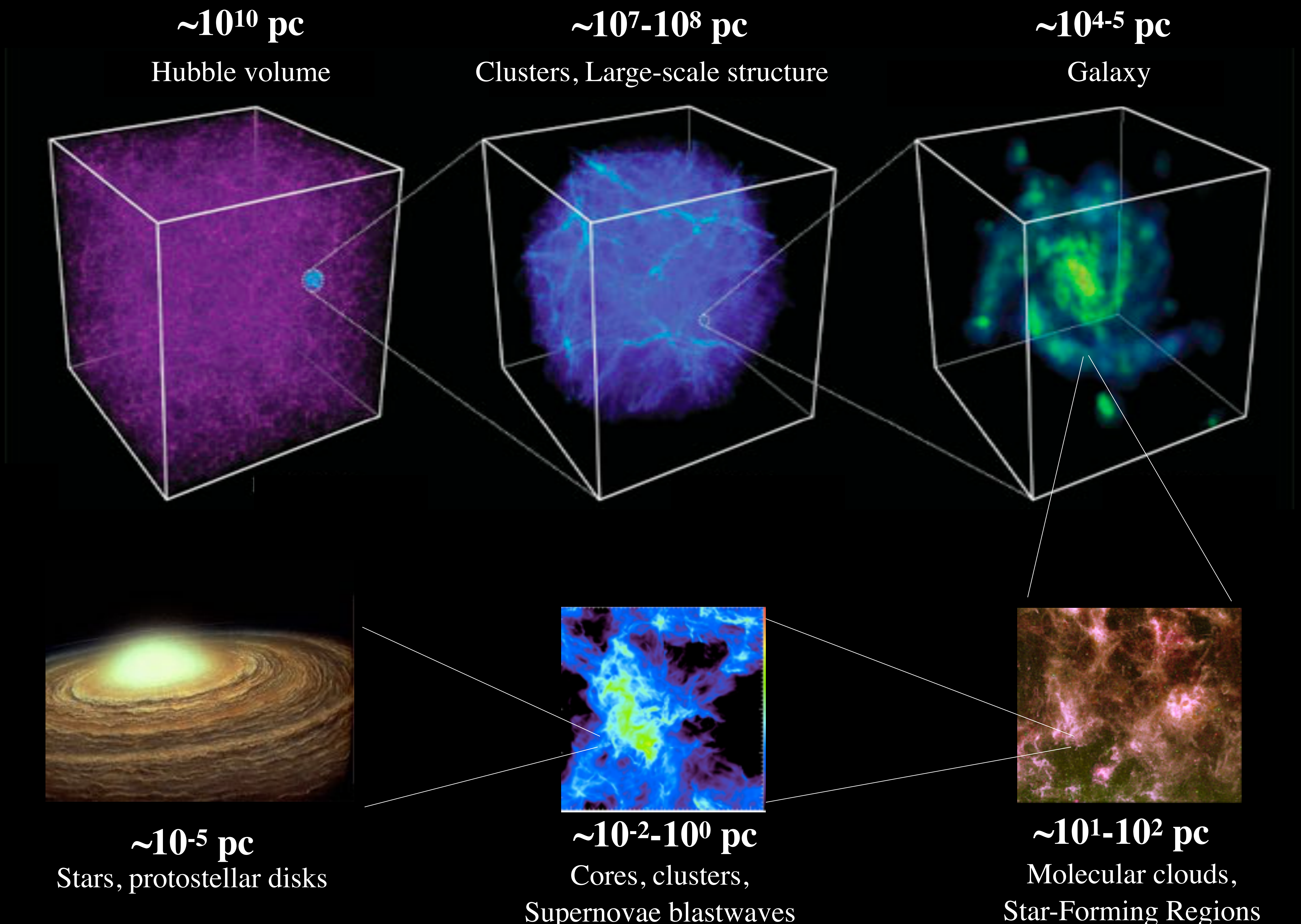

z~1090
(t~400,000 yr)

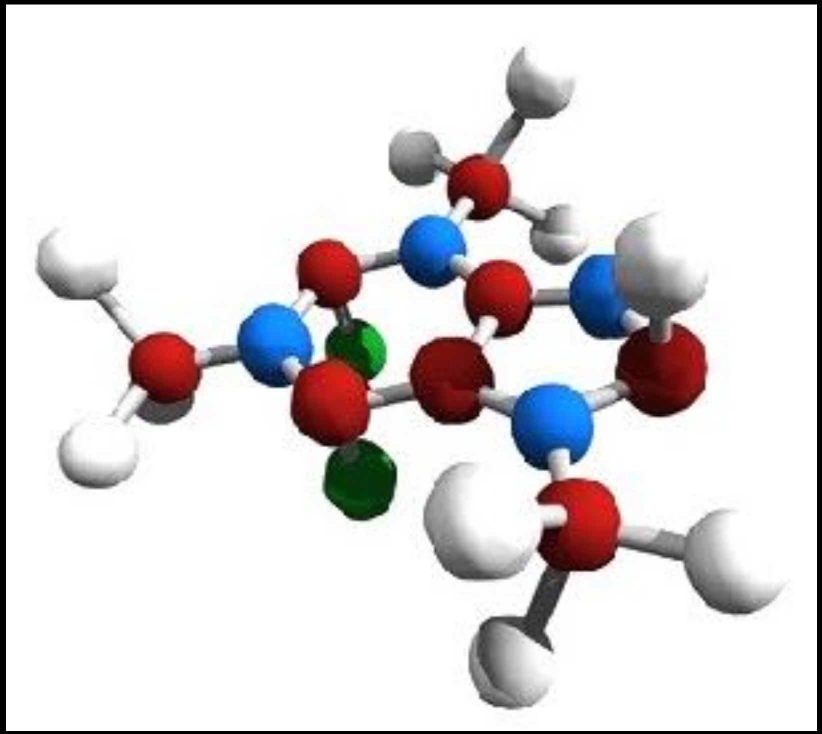
# Large scales: Gravity + Dark Matter/Energy Works!

Observations vs Theory
(SDSS vs Millennium Simulation)



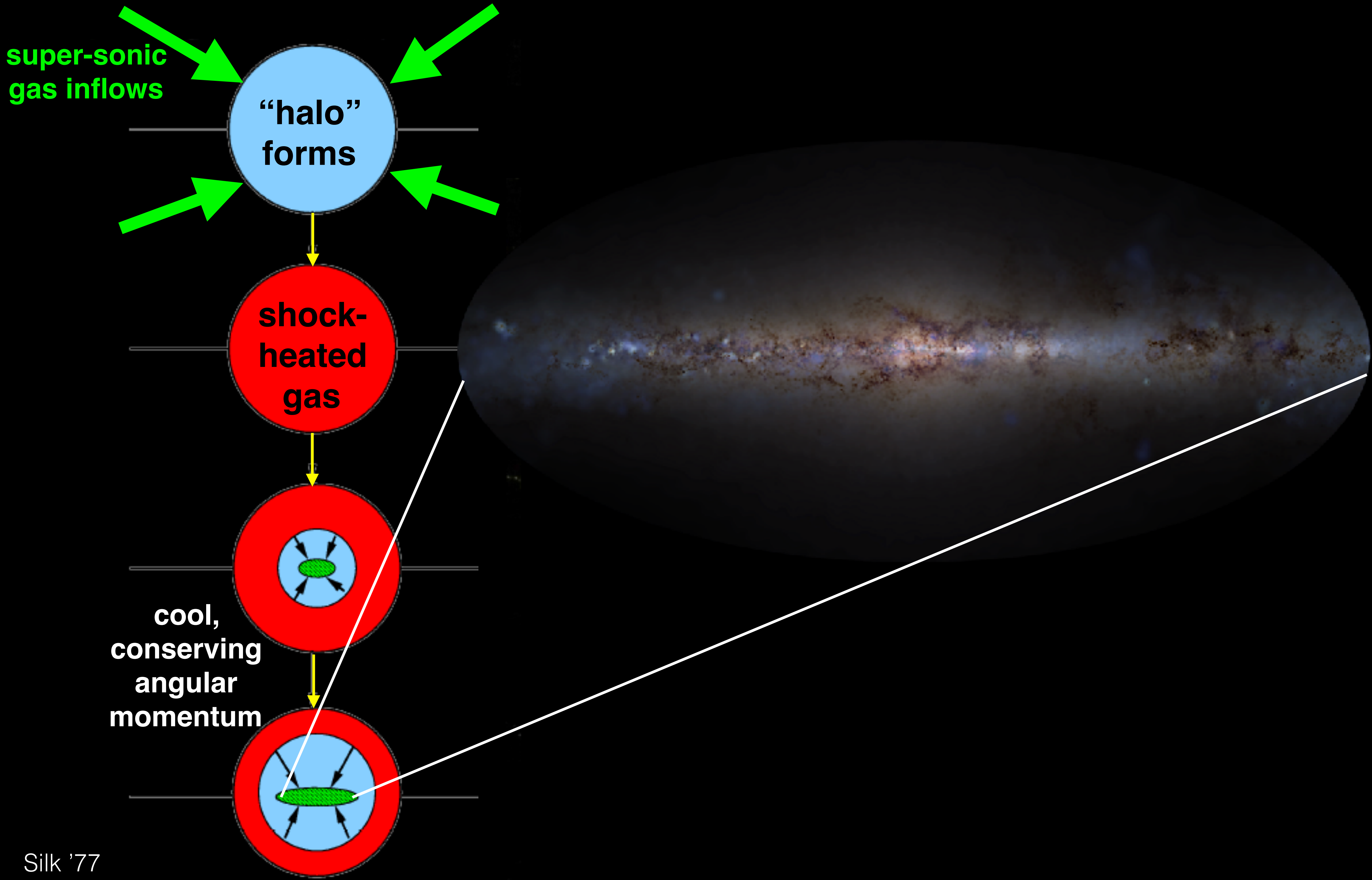
# Our work:


$\sim 10^{10}$ pc
Hubble volume
$\sim 10^{7}$-$10^{8}$ pc
Clusters, Large-scale structure
$\sim 10^{4-5}$ pc
Galaxy
$\sim 10^{-5}$ pc
Stars, protostellar disks
$\sim 10^{-2}$-$10^{0}$ pc
Cores, clusters,
Supernovae blastwaves
$\sim 10^{1}$-$10^{2}$ pc
Molecular clouds,
Star-Forming Regions



Add some fluid dynamics
and chemistry, and go!

# The Basic Picture:

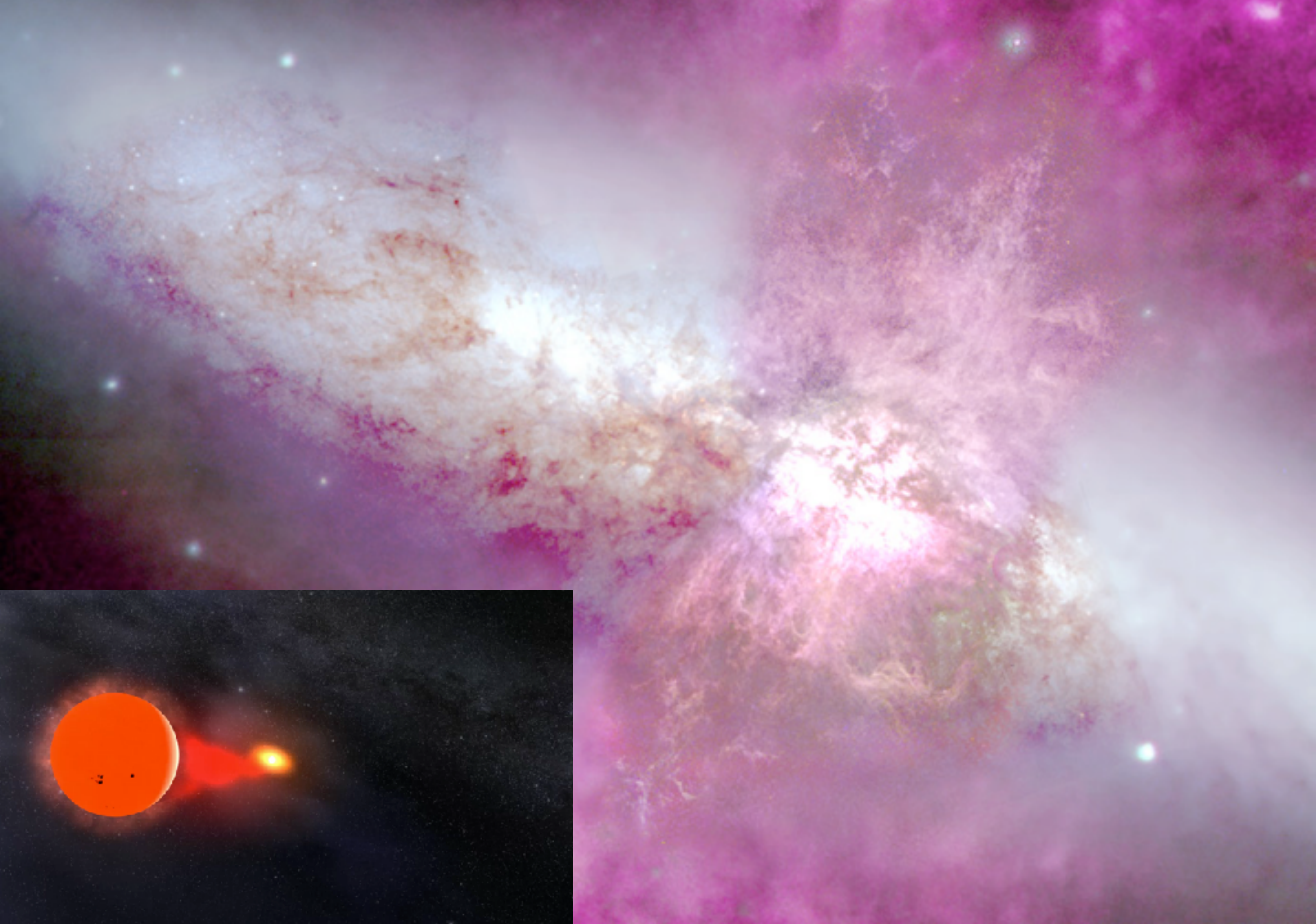



Silk '77
Binney '77
Rees & Ostriker '77

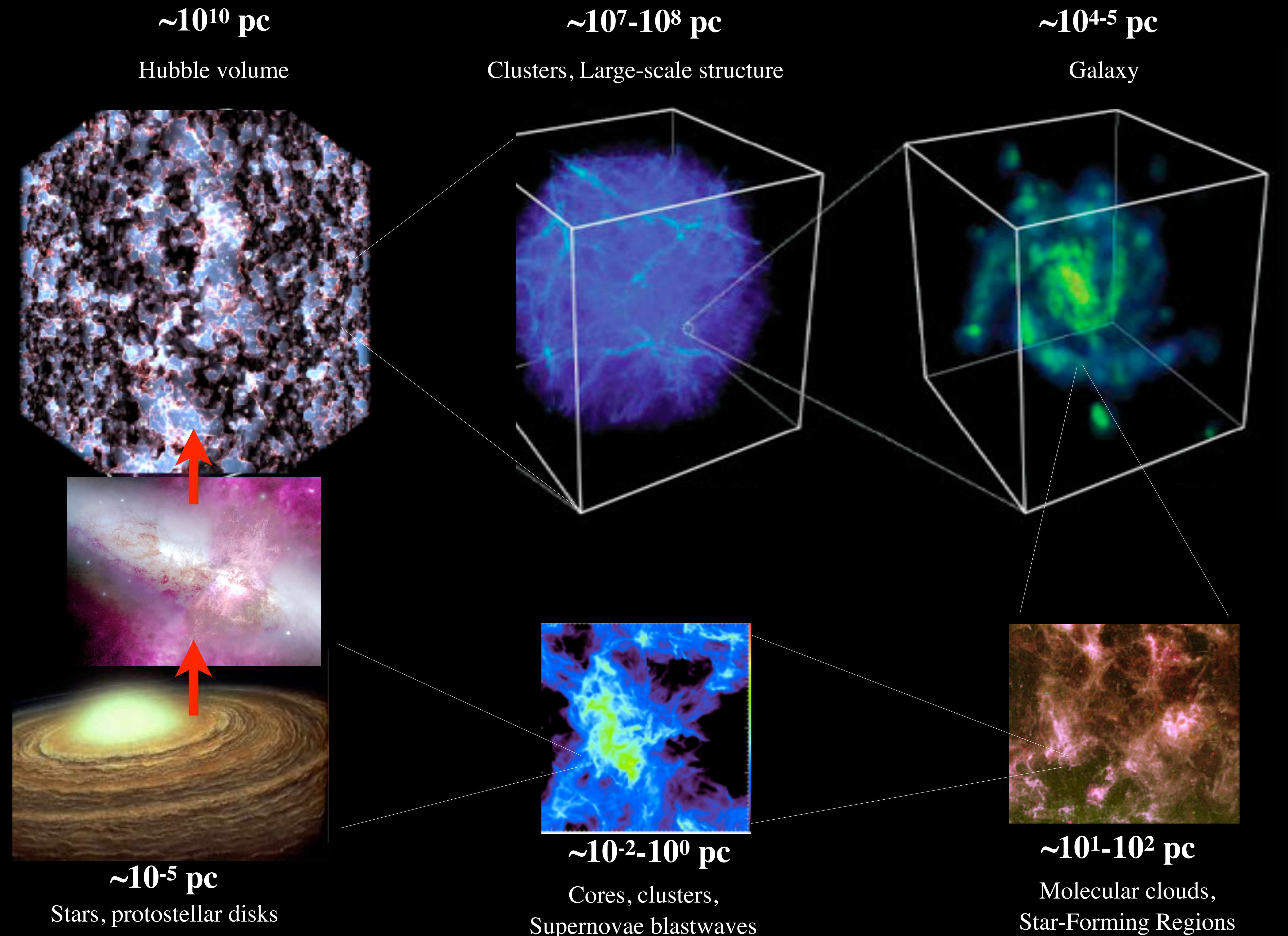
Done!

Not so fast…

# Stars Matter

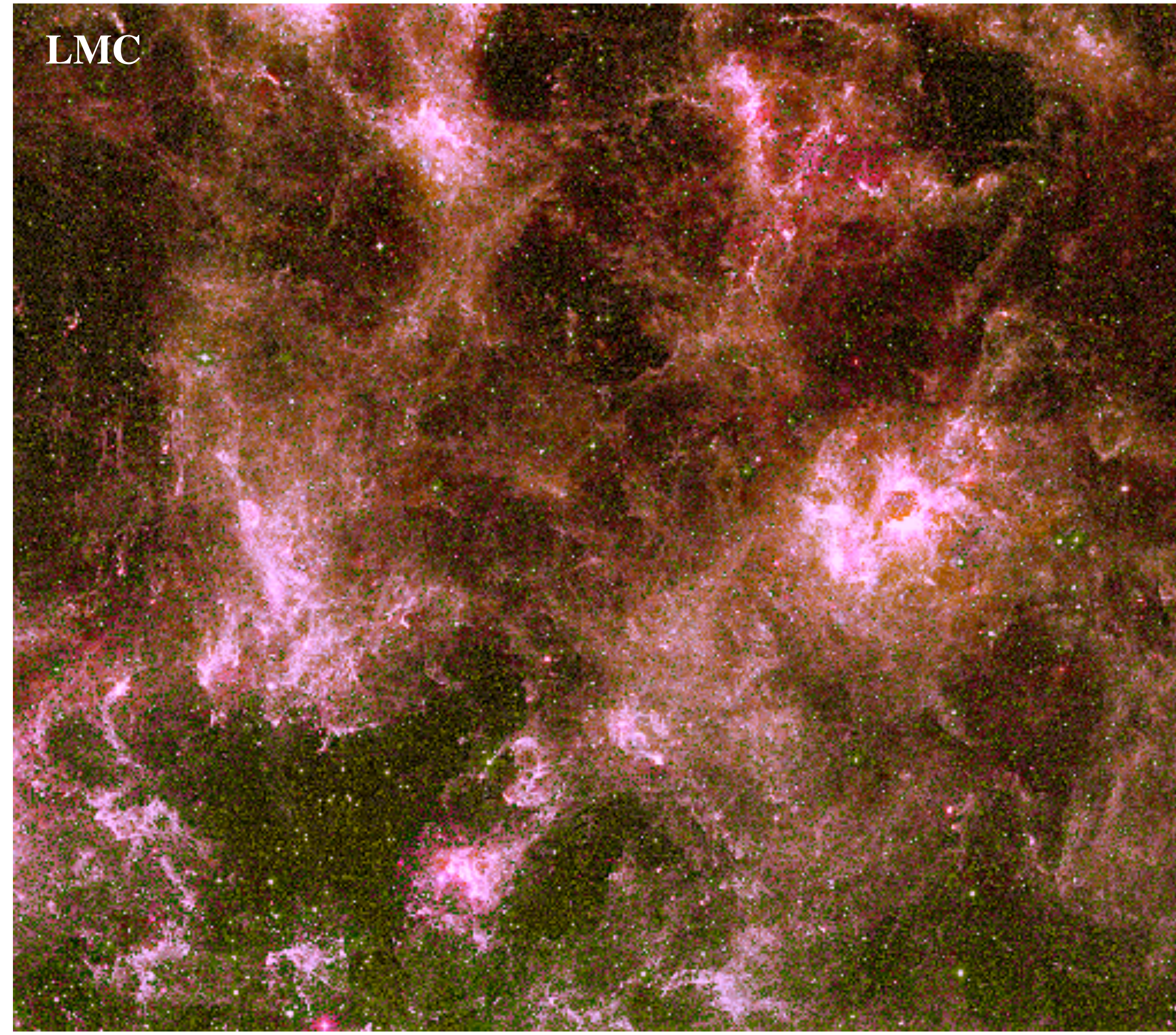


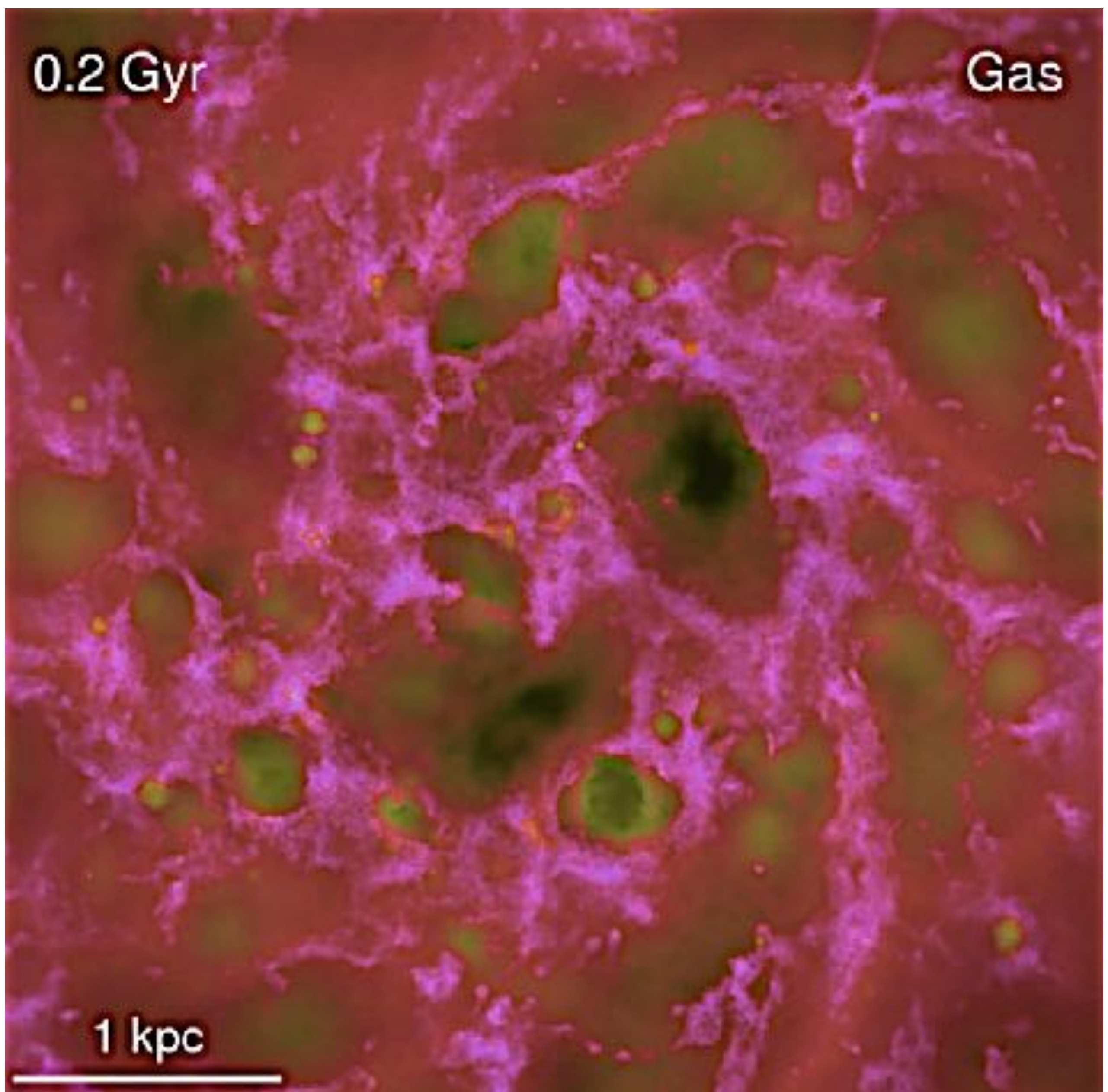
# … Nature hates theorists



# The Turbulent, Multi-Physics ISM

IMPORTANT ON
(ALMOST) ALL SCALES



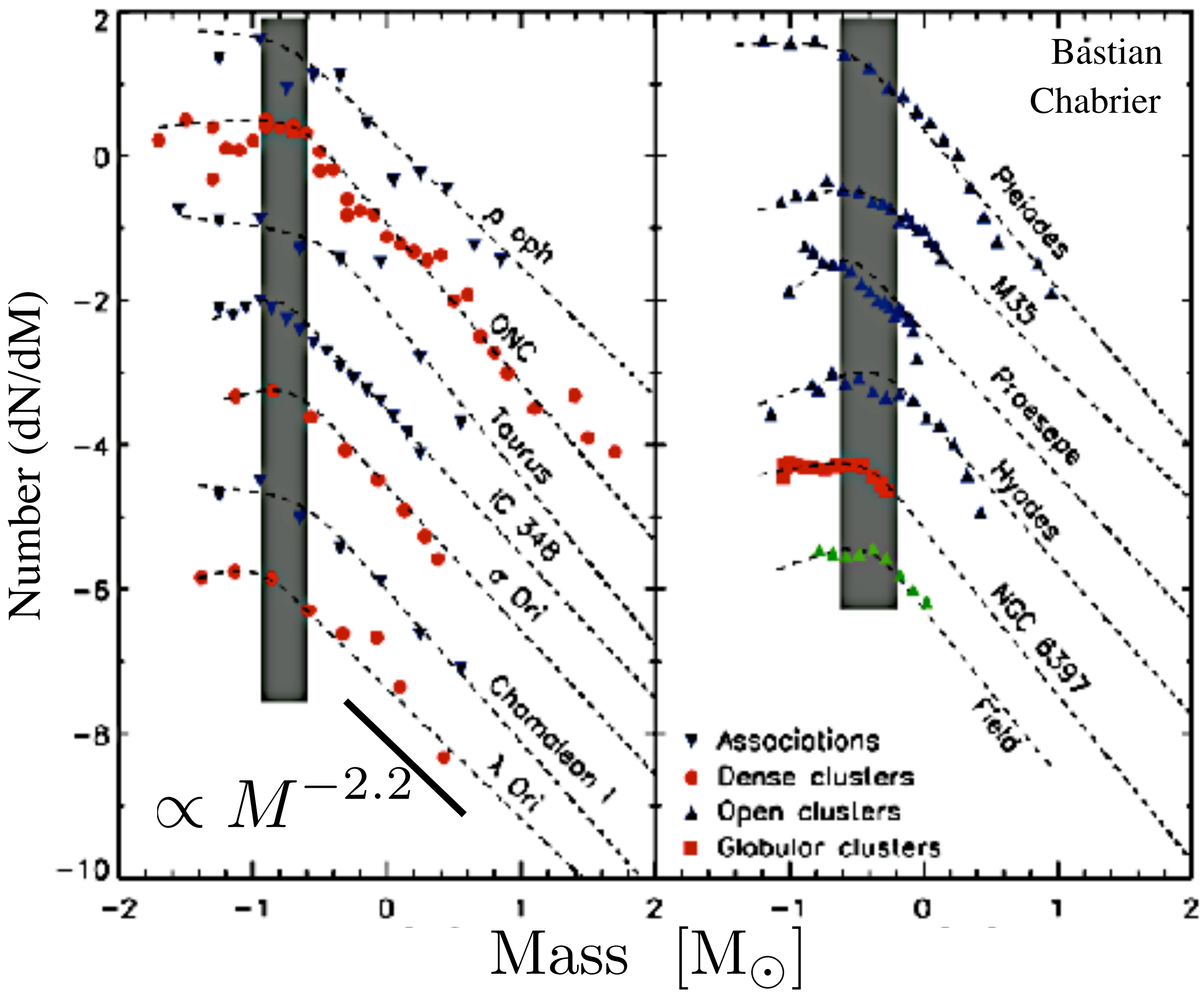




- Gravity
- Turbulence
- Magnetic, Thermal, Cosmic Ray, Radiation Pressure
- Cooling (atomic, molecular, metal-line, free-free)
- Star & BH Formation/Growth
- "Feedback": Massive stars, SNe, BHs,
  external galaxies, etc.

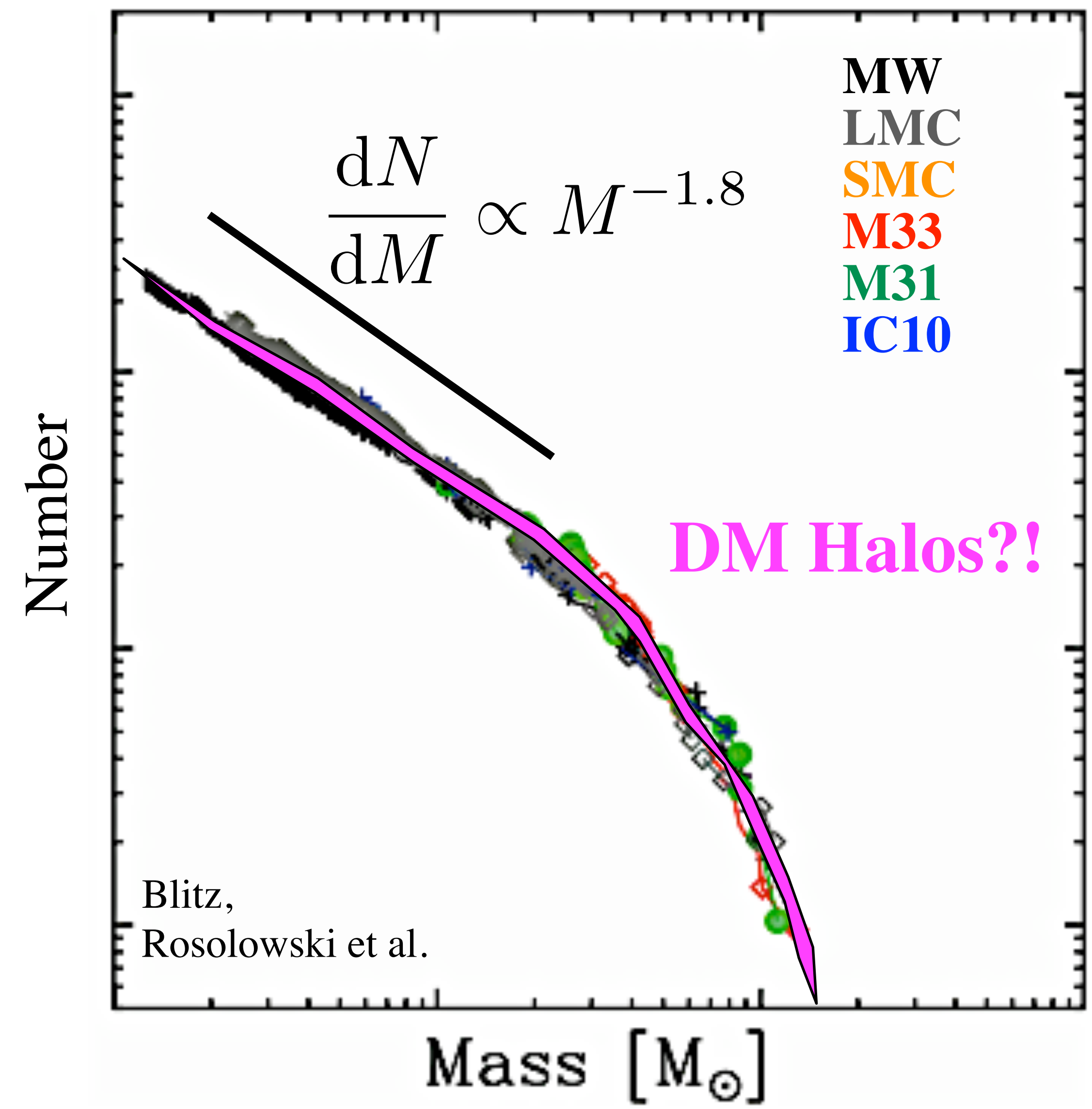
# The ISM is Messy...

## YET THERE IS SURPRISING REGULARITY

Stars & Pre-Stellar Gas Cores:



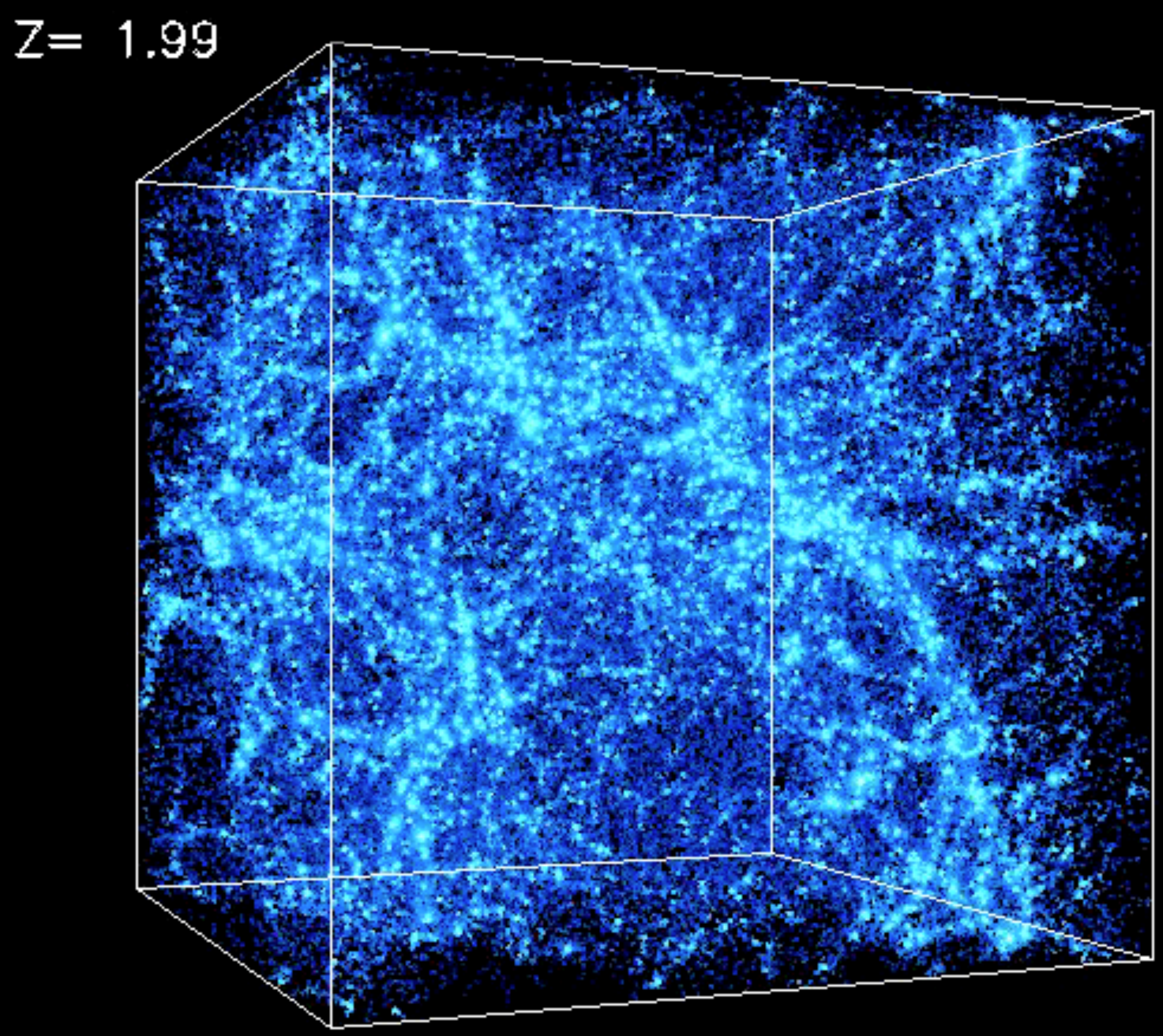

Giant Molecular Clouds:




universality between IMF, CMF MF, halo MF, etc.

# Is this an accident?

STRUCTURE FORMATION

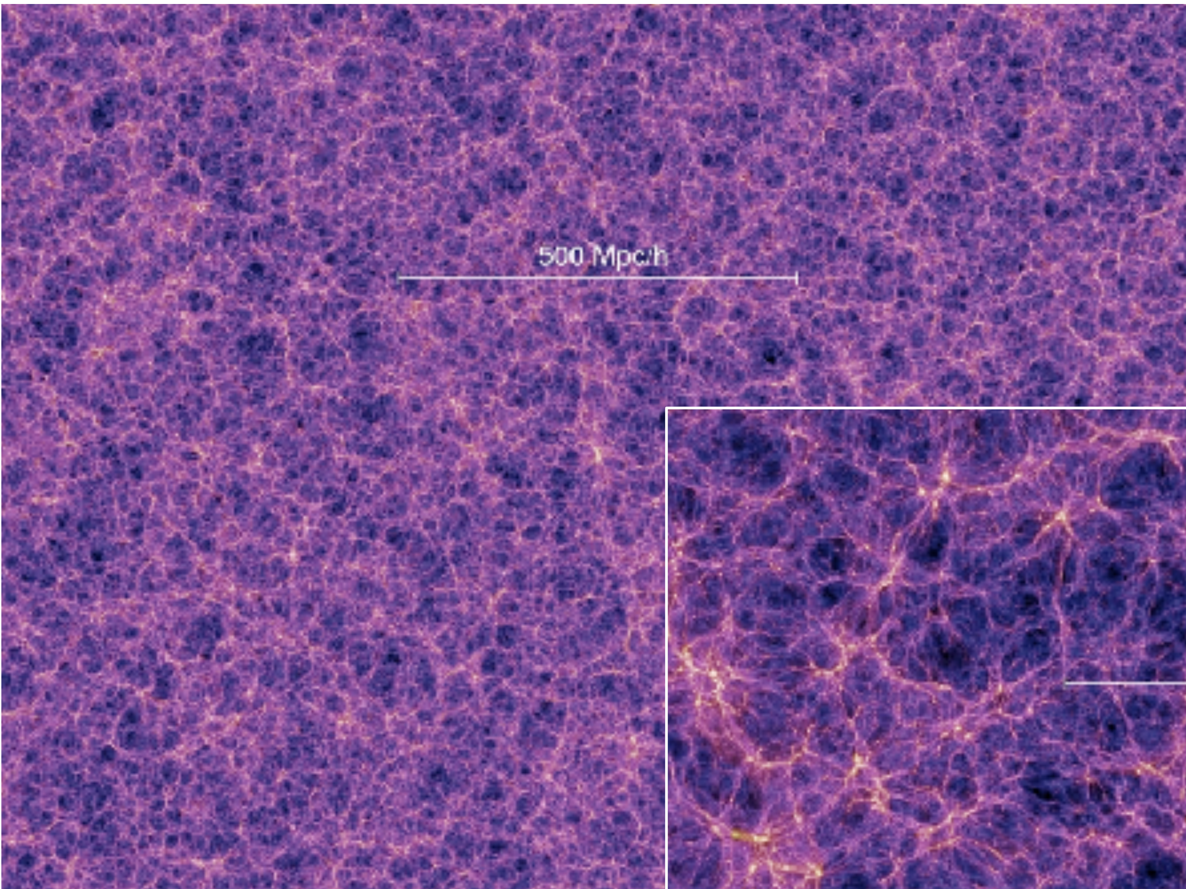


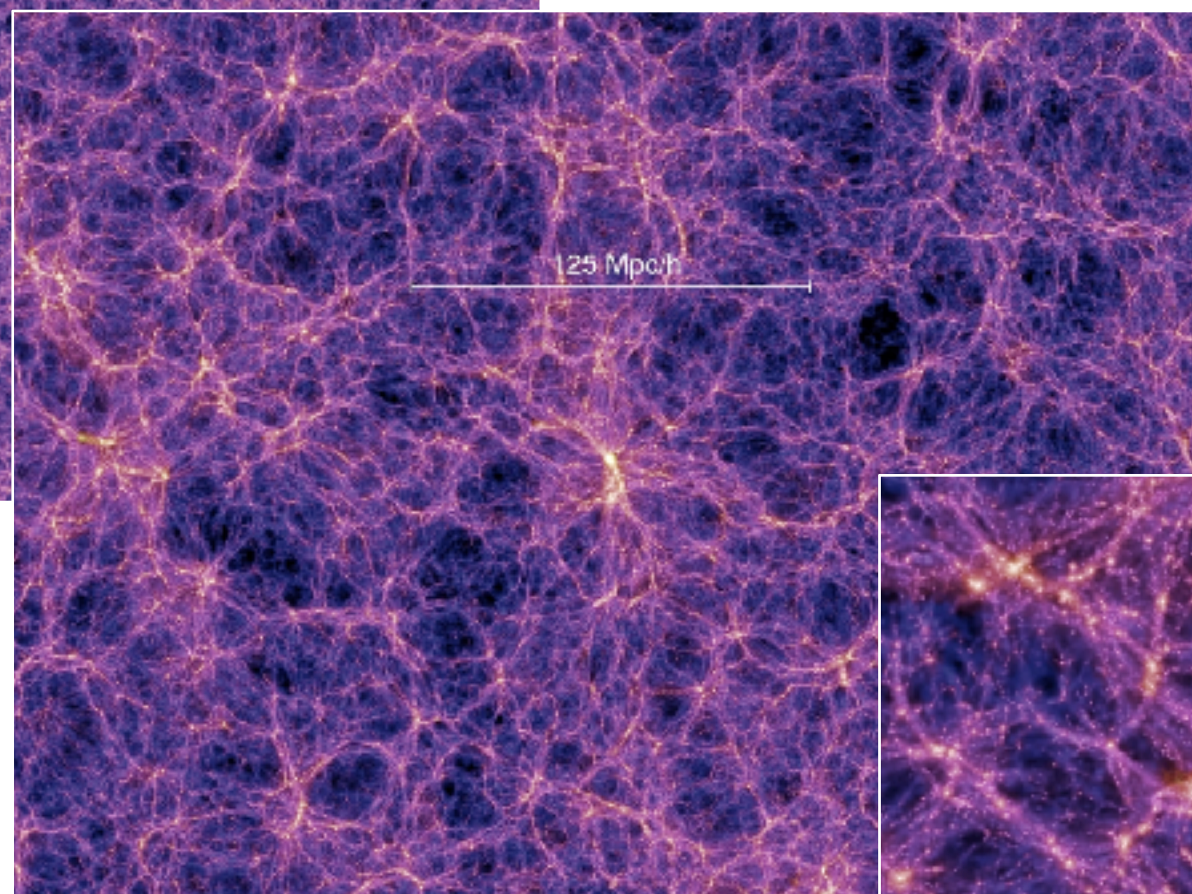

STAR FORMATION

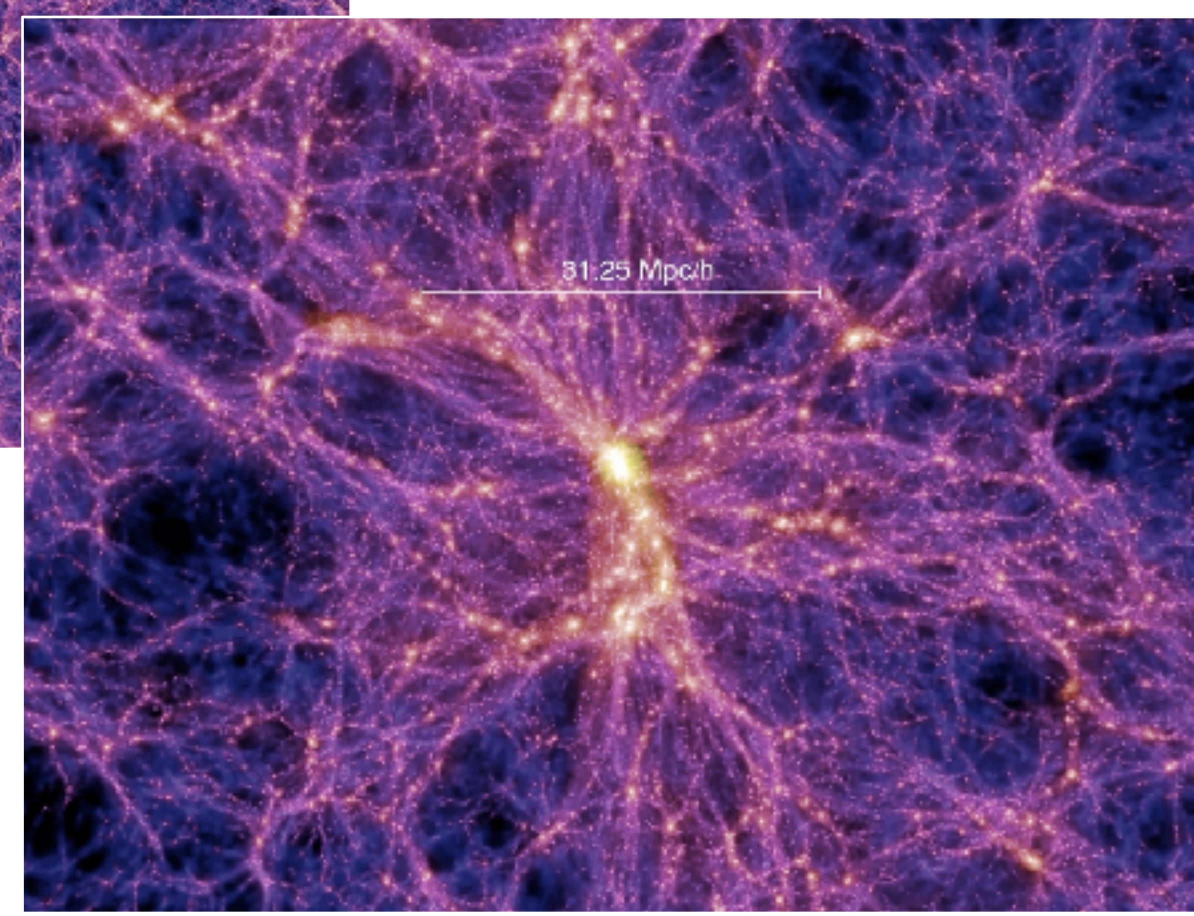

Guszejnov 15,16
PFH 10,12

→ Planet formation (PFH & Christiansen)

# A solution from cosmology?




- Press & Schechter '74:
  - $\rho$ Fluctuations a Gaussian random field
  - Power spectrum P(k~1/r): variance
- "Count" mass above critical fluctuation:
  - Turnaround & gravitational collapse

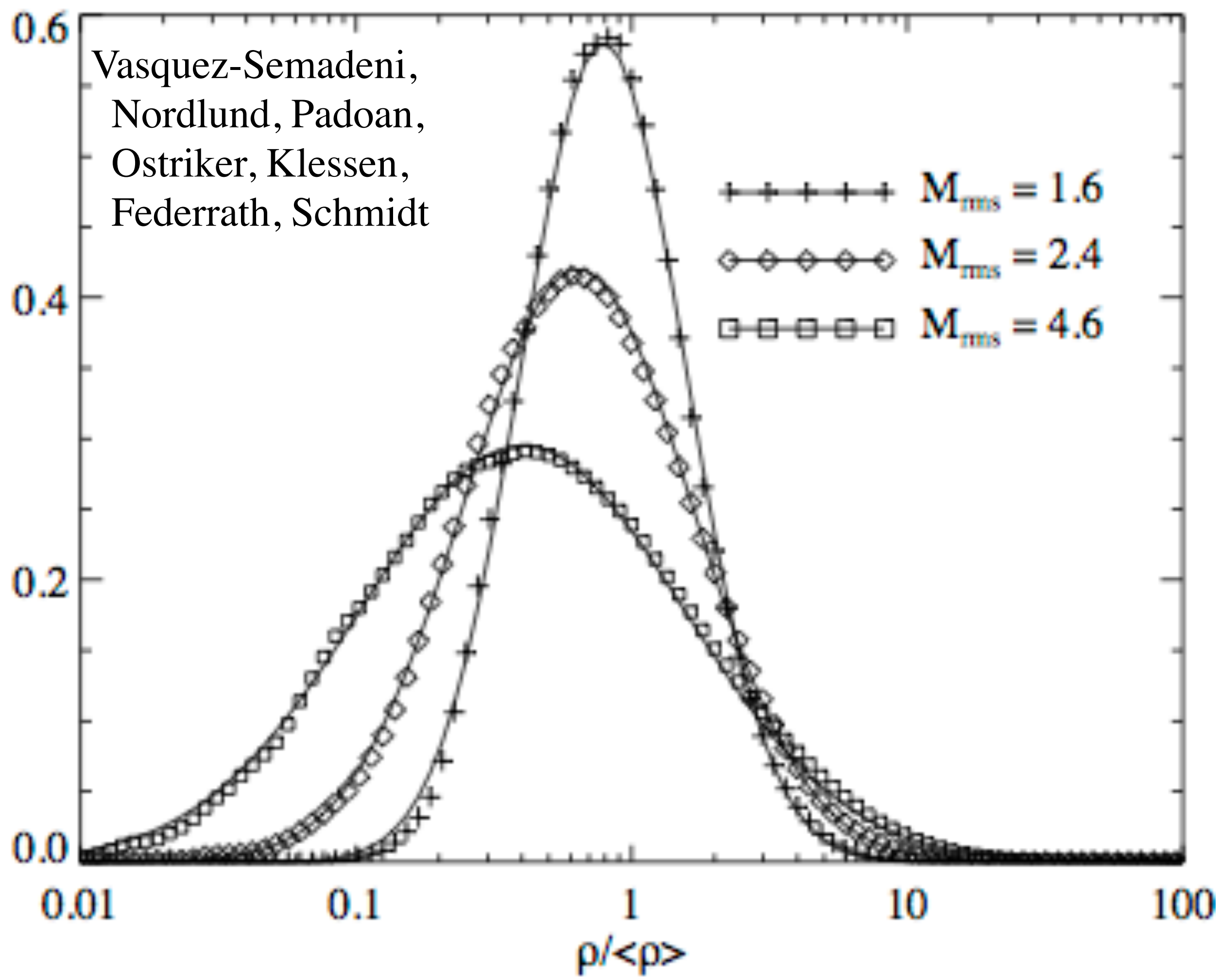
$$\bar{\rho}(< R \sim 1/k) > \rho_{\rm crit}$$

# Turbulence

## BASIC EXPECTATIONS: FROM THE SIMULATIONS

Velocity: $v^2_{\rm turb}(R) \propto R^{1/2} \qquad (E(k) \propto k^{-p})$

Density:




$$\frac{{\rm d}p}{{\rm d}\ln\rho} \propto \exp\left[-\frac{\ln^2(\rho)}{2\,S(R)}\right]$$

$$S(R) \approx \ln\left[1 + \frac{v^2_{\rm turb}(R)}{c_s^2}\right]$$

# What Defines a Fluctuation of Interest?

## DISPERSION RELATION:

$$\omega^2 = \kappa^2 + c_s^2\, k^2 + u_t(k)^2\, k^2 - \frac{4\pi\, G\, \rho\, |k| h}{1 + |k| h}$$

Angular Momentum

$$\kappa \sim \frac{V_{\rm disk}}{R_{\rm disk}}$$

Thermal Pressure

$$\propto r^{-2}$$

Turbulence

$$\propto r^{p-3} \sim r^{-1}$$

$$r > r_{\rm sonic}: \quad u_t^2 > c_s^2$$

Gravity

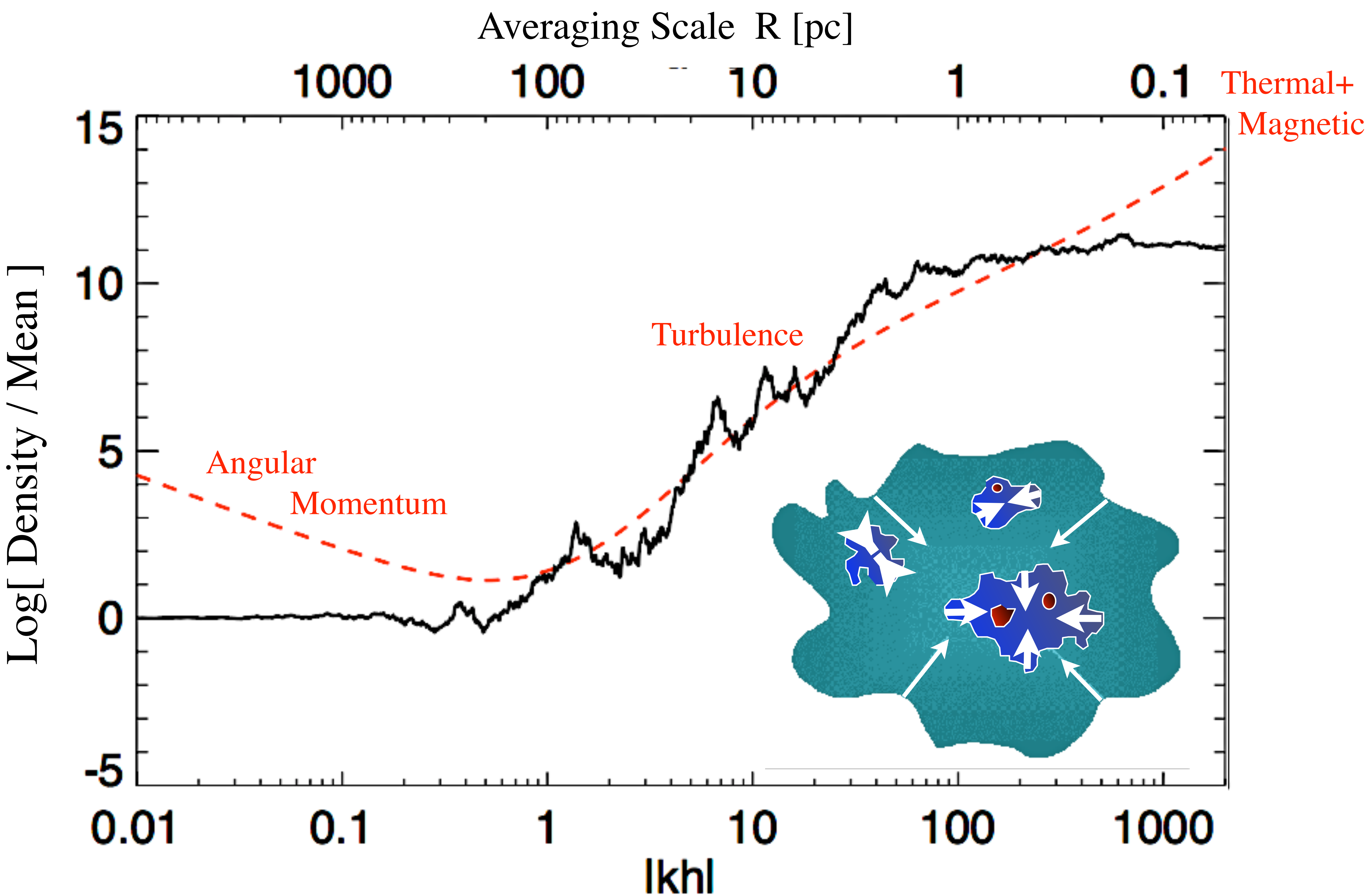
Mode Grows (Collapses) when $\omega < 0$: $\rho > \rho_{\chi\rho\iota\tau}$

Chandrasekhar ‘51, Vandervoort ‘70, Toomre ‘77

# Turbulent Fragmentation: From GMCs to Stars

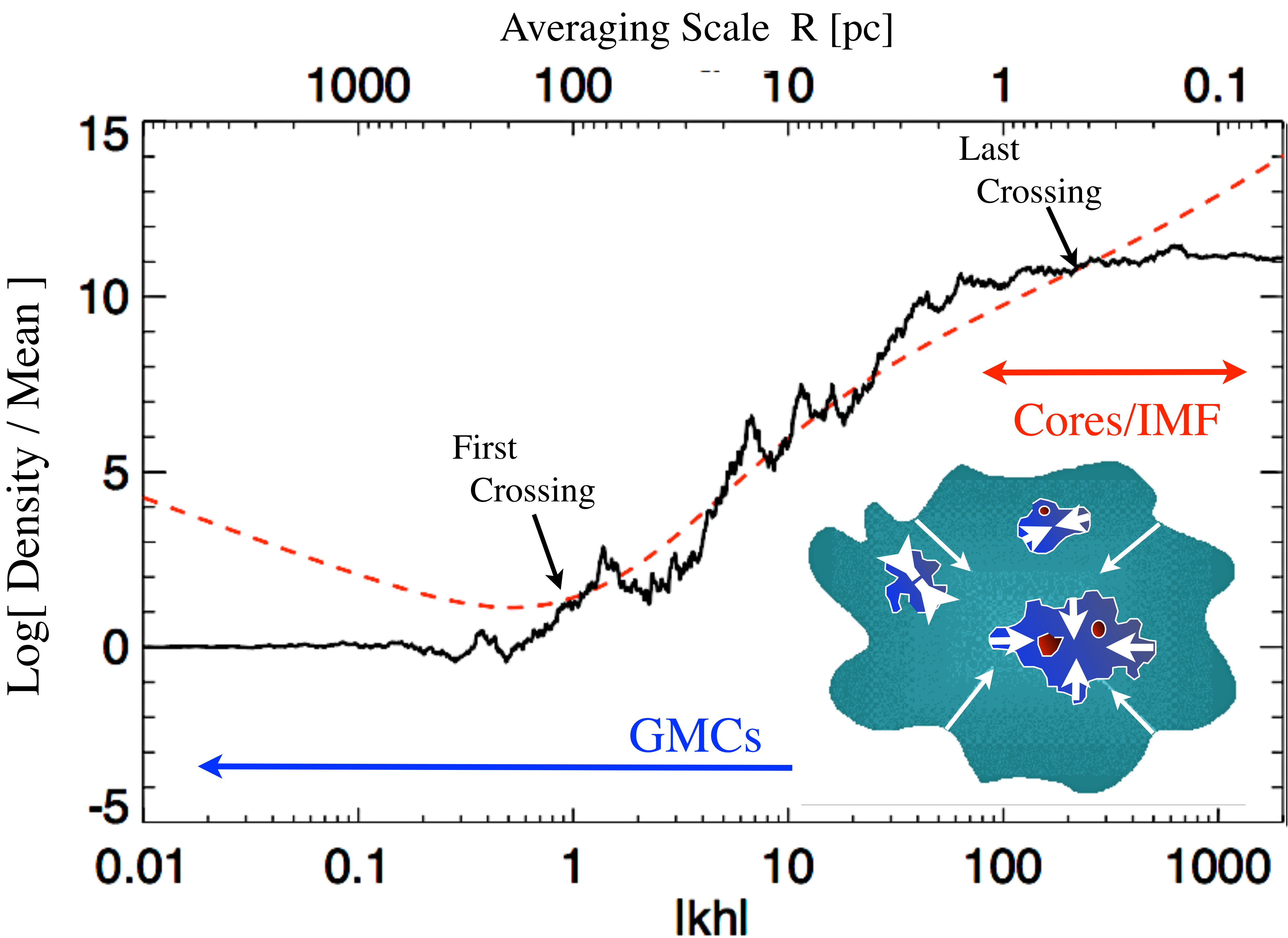


JUST COUNTING "CLOUDS IN CLOUDS"



# "Counting" Collapsing Objects

## EVALUATE DENSITY FIELD vs. "BARRIER"



# What Does This Mean for ISM Structure and Star Formation?

# The "First Crossing" Mass Function

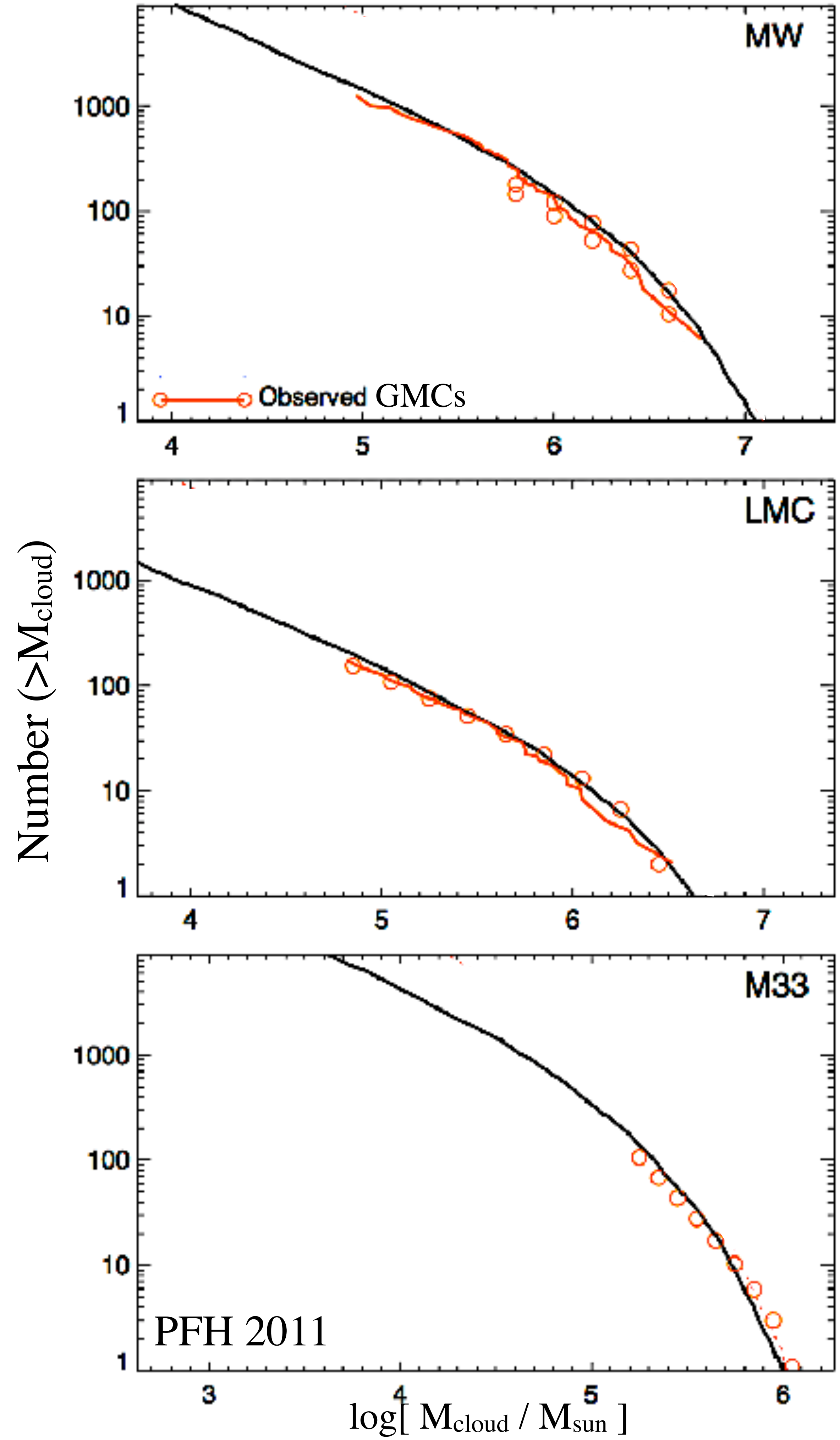
## VS GIANT MOLECULAR CLOUDS

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto M^{-\alpha}\, e^{-(M/M_J)^{\beta}}$$

(Toomre mass)

$$\alpha \approx 2 - \epsilon(M)$$

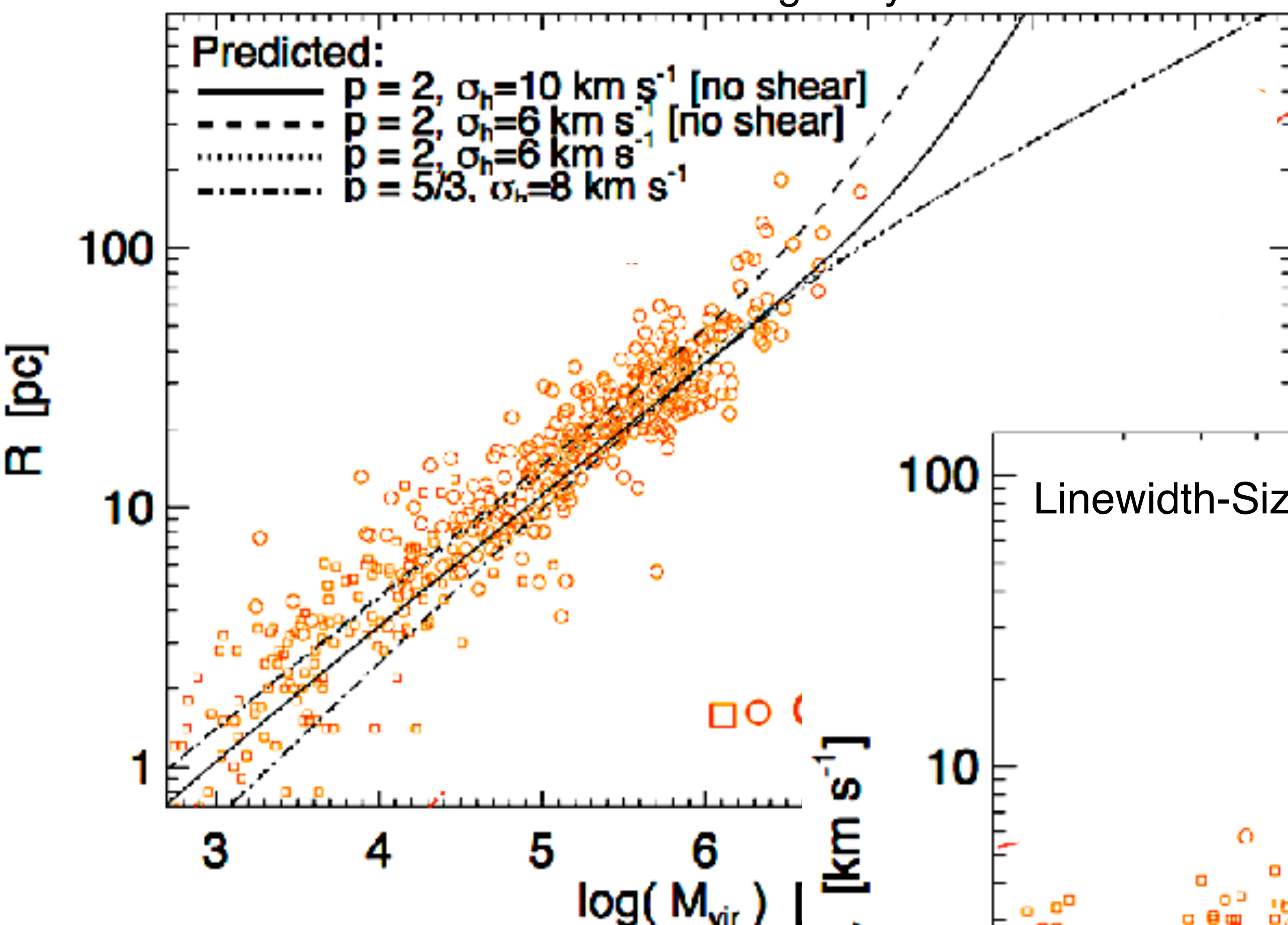
(scale-free)



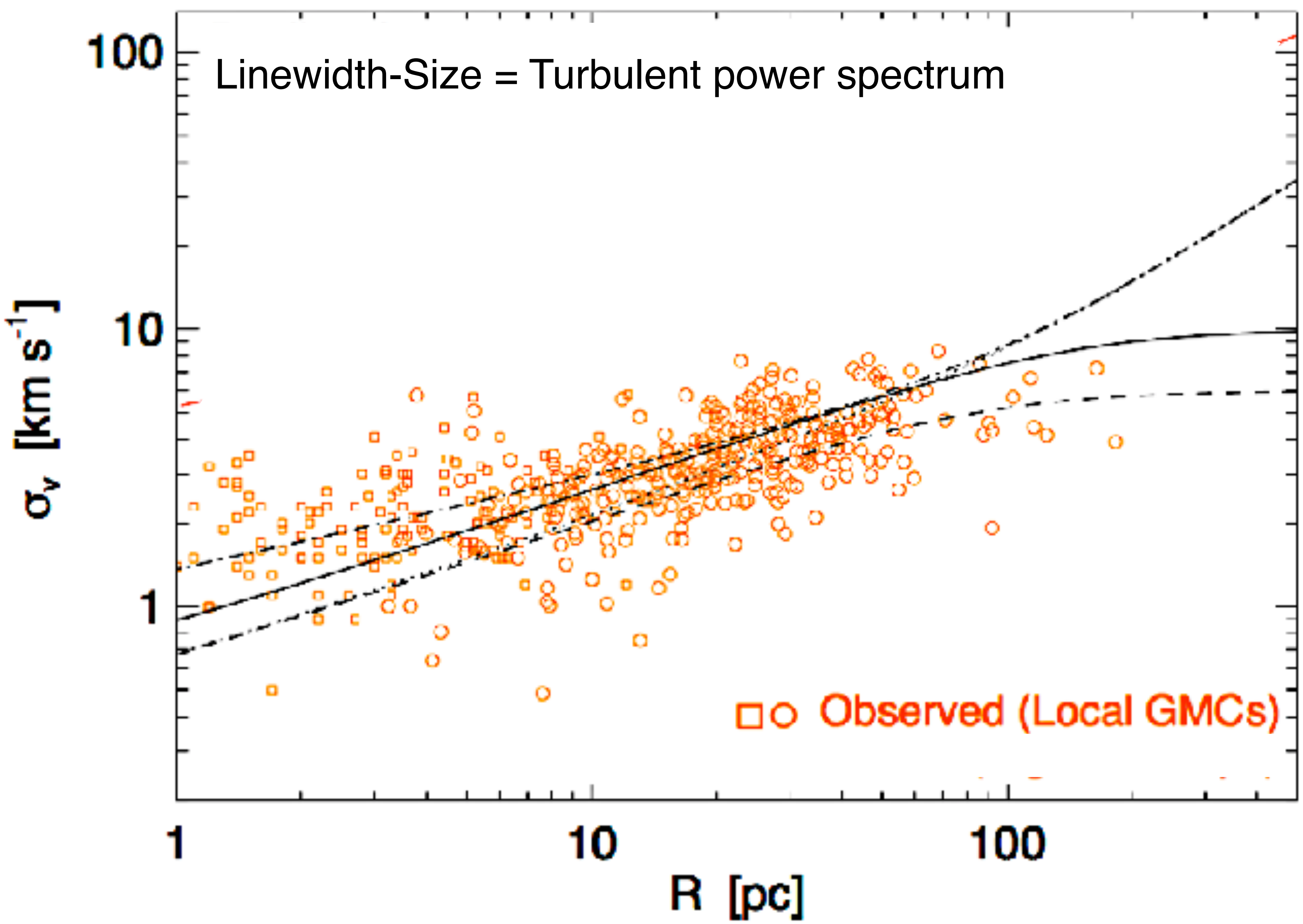
# Structural Properties of "Clouds"

## LARSON'S LAWS EMERGE NATURALLY

Size-Mass = Self-gravity criteria



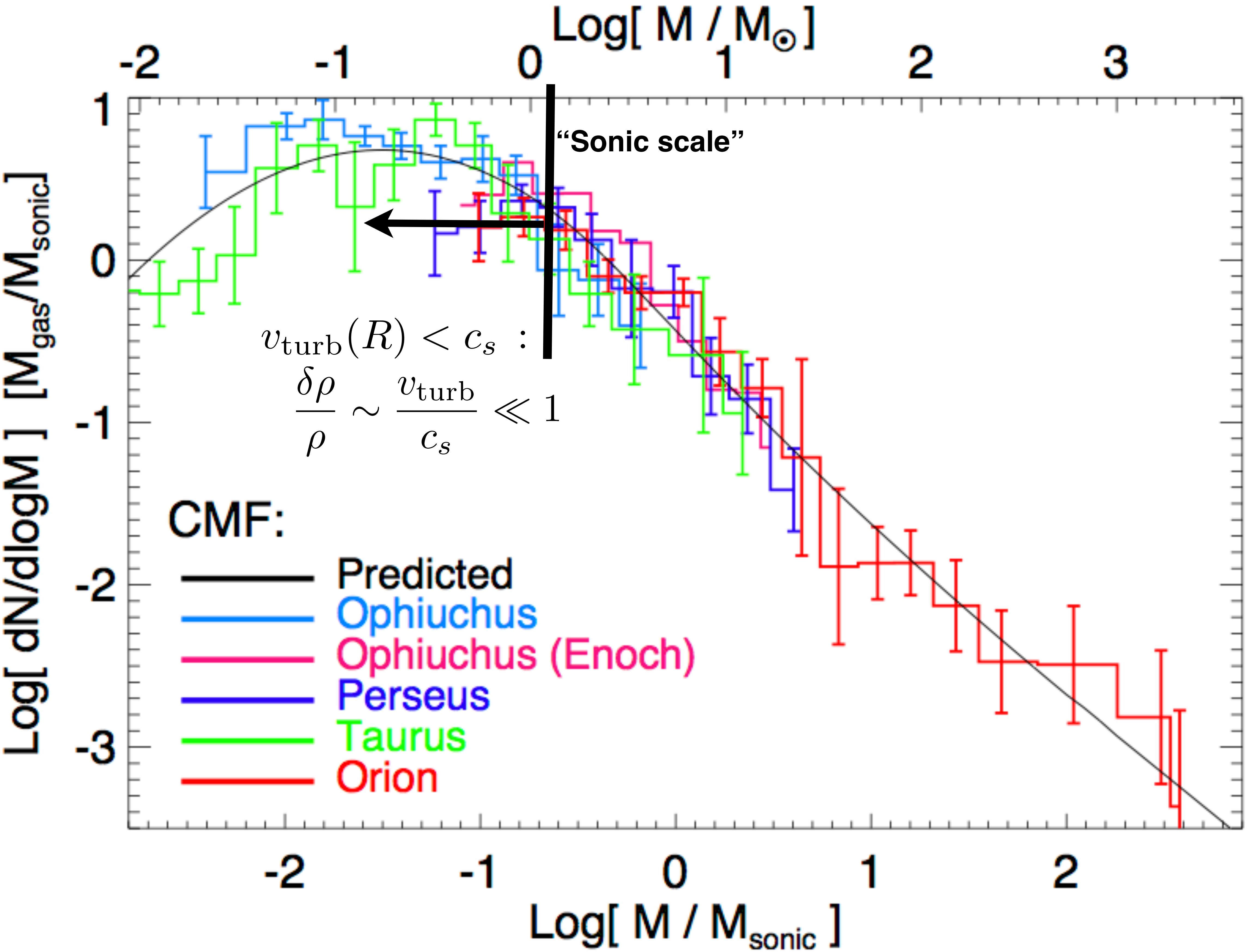

Linewidth-Size = Turbulent power spectrum



## The “Last Crossing” Mass Function

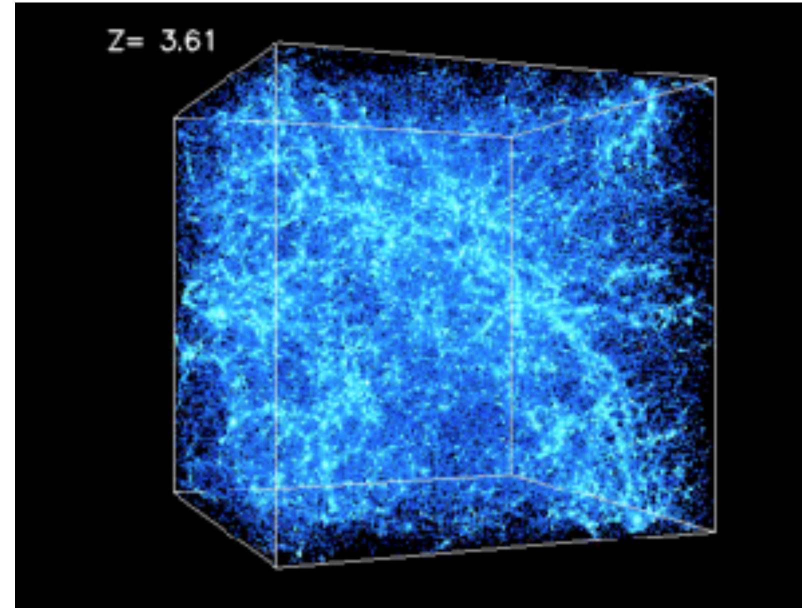
VS PROTOSTELLAR CORES

(Hennebelle & Chabrier, Padoan & Nordlund, PFH 2012)



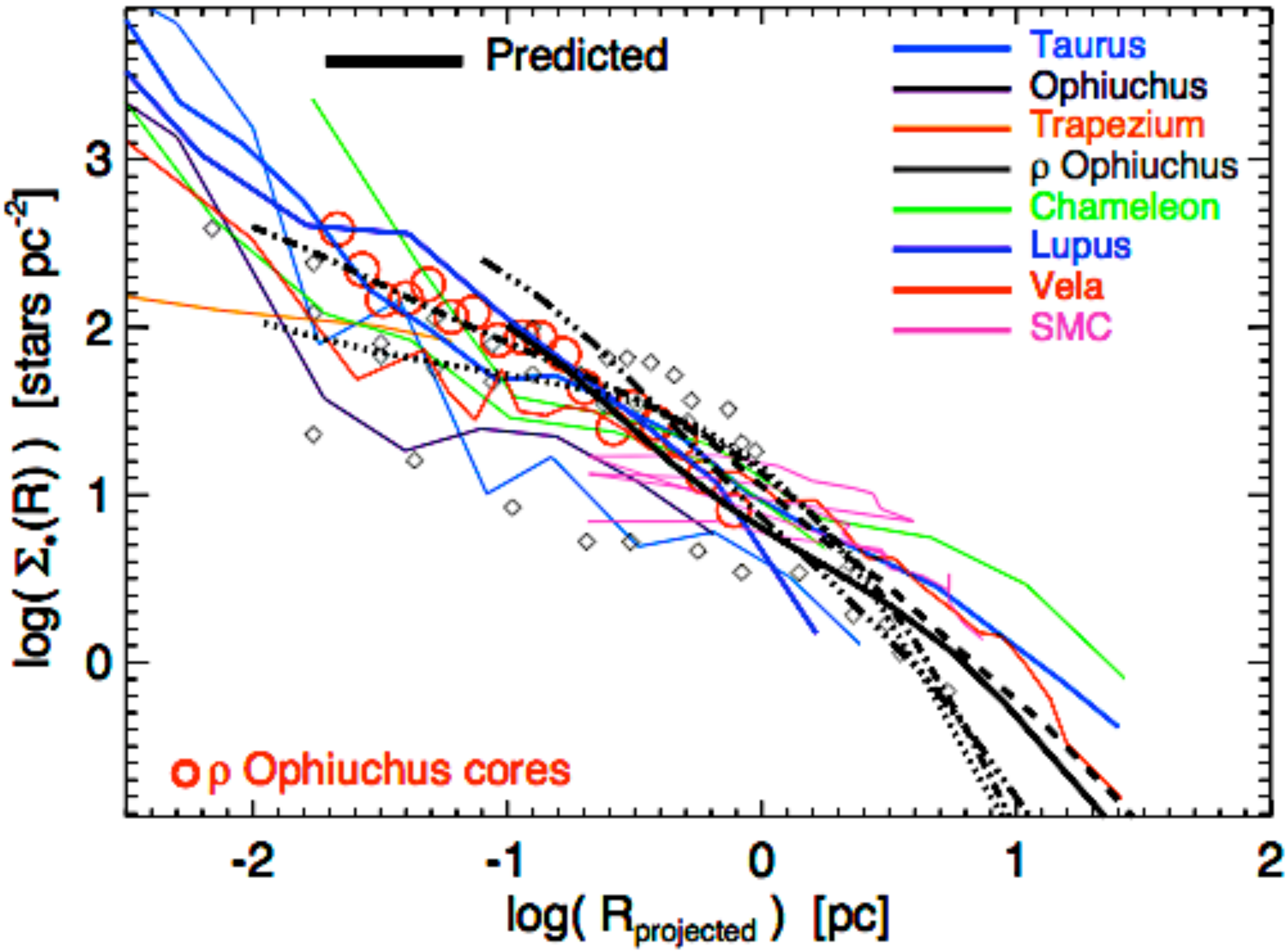
# Why Is Star Formation Clustered?

## CLUSTERING IS *INEVITABLE*





PFH 2012



# Open Questions:

## 1. What Maintains the Turbulence?

Efficient Cooling: $$\dot{P}_{\rm diss} \sim \frac{M_{\rm gas}\, v_{\rm turb}}{t_{\rm crossing}}$$

## 2. Why Doesn't Everything Collapse?

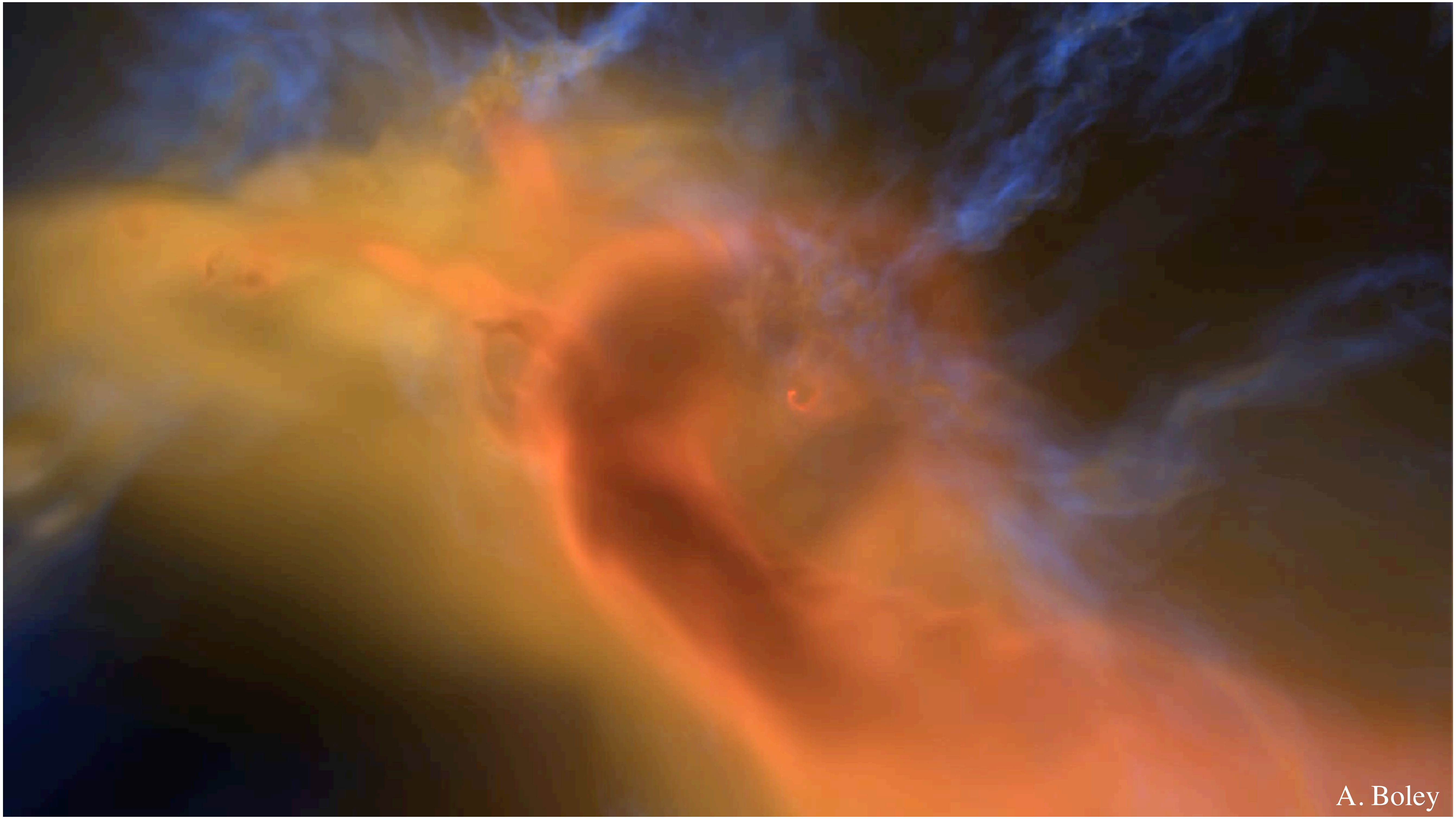
"Top-down" turbulence can't stop collapse once self-gravitating

Fast Cooling: $$\dot{M}_* \sim \frac{M_{\rm gas}}{t_{\rm freefall}}$$

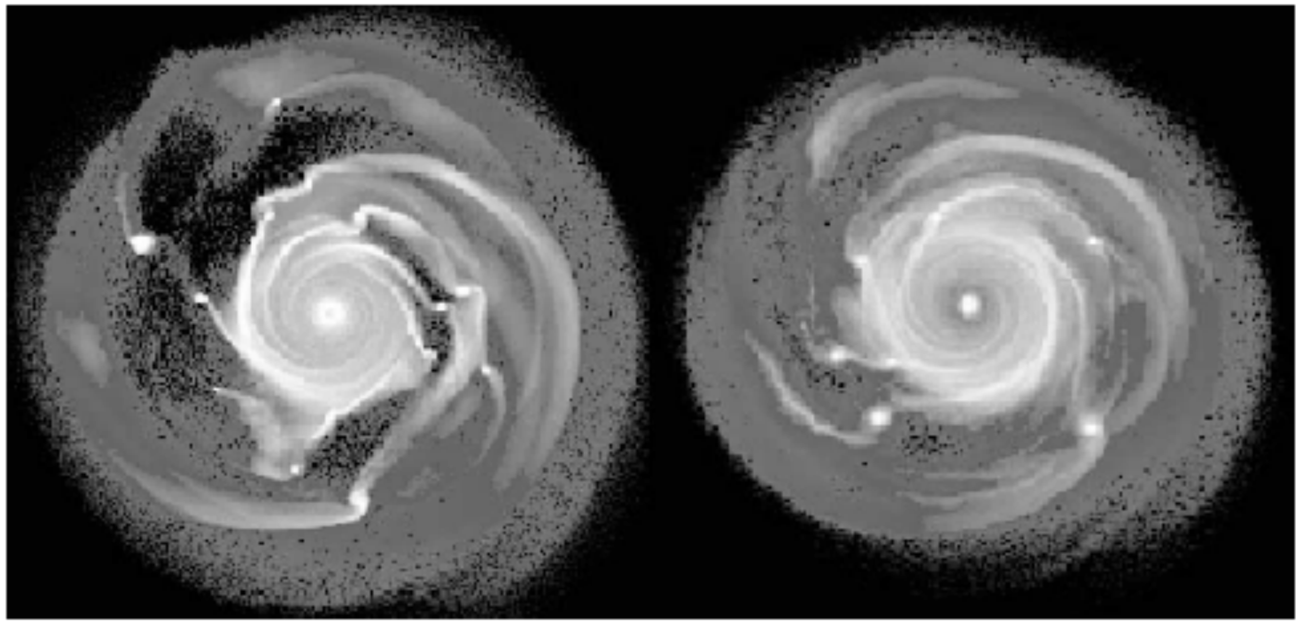
# What About Planets?

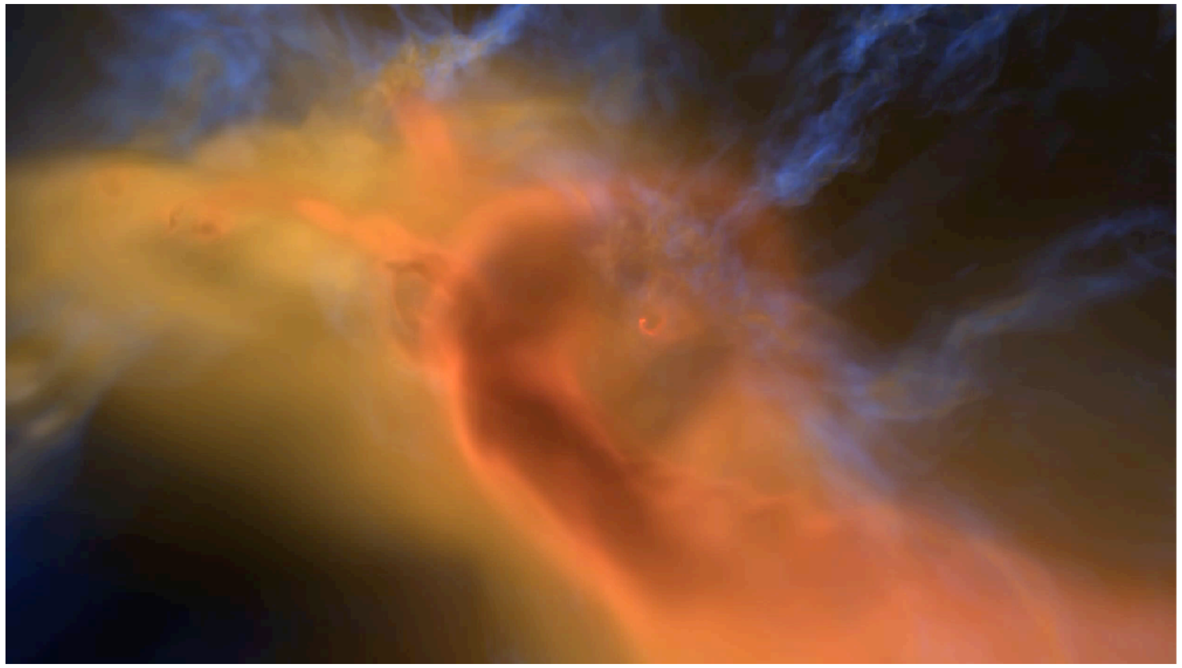
# Planet Formation?

- Two channels:
  - (1) "Core accretion"
  - (2) "Direct Collapse"



# Standard (Toomre) Criterion for Direct Collapse:





$$Q = \frac{c_s\,\Omega}{\pi\,G\,\Sigma_{\rm gas}} \sim \frac{1}{\rho}\,\frac{M_*}{r_*^3}$$

$$Q \sim 100\left(\frac{\Sigma_{\rm gas}}{\Sigma_{\rm MMSN}}\right)^{-1} r_{*,\,{\rm AU}}^{-1/4}$$

# But, What if the Disks Are Turbulent?

- Need density fluctuation: $$\frac{\rho}{\langle\rho\rangle} \gtrsim \frac{1}{\langle\rho\rangle}\,\frac{M_*}{r_*^3} \sim Q$$

- Turbulence:: stochastic fluctuations with $$\sigma_{\ln\rho} \approx \sqrt{\ln\left(1+\mathcal{M}^2\right)} \sim \mathcal{M}$$

- So, at any instant, in a given region: $$P_\rho \sim \mathrm{erfc}\left[\frac{\ln Q}{\sqrt{2}\,\sigma_{\ln\rho}}\right]$$

- Q~100, *M*~0.1 :: $P_\rho \sim 10^{-7}$ is small!

# But, What if the Disks Are Turbulent?

- Most unstable wavelength ("size" of regions) : $\sim h$
- So have $N_{\rm volumes} \sim \left(\frac{r_*}{h}\right)^2$ independent "samples" (at a given time)
- Turbulence evolves stochastically with coherence time ~ eddy turnover time:

$$t_{\rm ``reset"} \approx t_{\rm cross}({\rm turb}) \approx t_{\rm dyn} = \Omega^{-1} \sim {\rm yr}$$

- And disks have a long lifetime $t_{\rm disk} \sim {\rm Myr}$

so "resample" it $\dfrac{t_{\rm disk}}{t_{\rm dyn}}$ independent times

$$P_{\rm tot} \sim \left(\frac{t_{\rm disk}}{t_{\rm dyn}}\right)\left(\frac{r_*}{h}\right)^2 {\rm erfc}\left[\frac{\ln Q}{\sqrt{2}\,\sigma_{\ln\rho}}\right]$$

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$$P_{\rm tot} \sim \left(\frac{t_{\rm disk}}{t_{\rm dyn}}\right)\left(\frac{r_*}{h}\right)^2 {\rm erfc}\left[\frac{\ln Q}{\sqrt{2}\,\sigma_{\ln\rho}}\right] \gtrsim 1 \quad \text{for} \quad \begin{matrix} Q \sim 100 \\ \mathcal{M} \gtrsim 0.1 \end{matrix}$$

# What About Core Accretion?

# Core Accretion: The “Meter Barrier”

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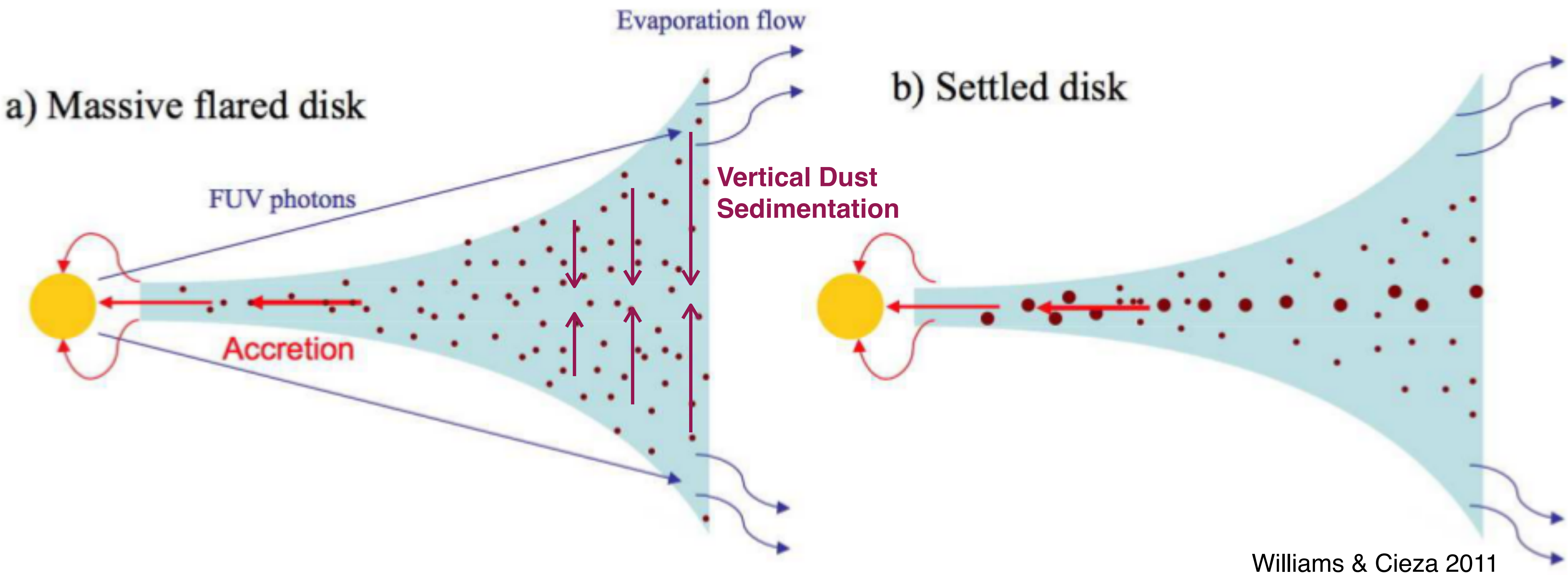


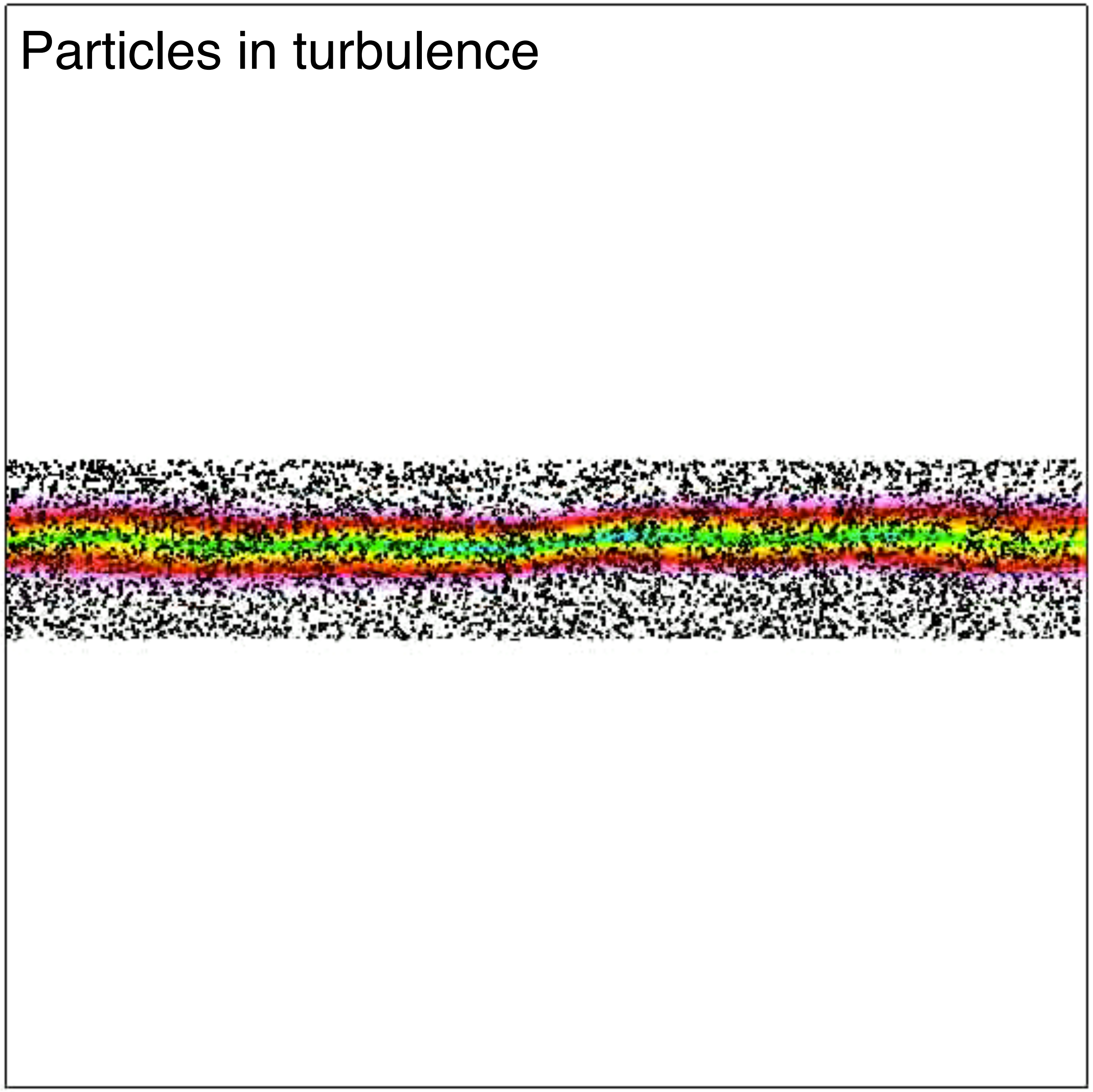

Williams & Cieza 2011

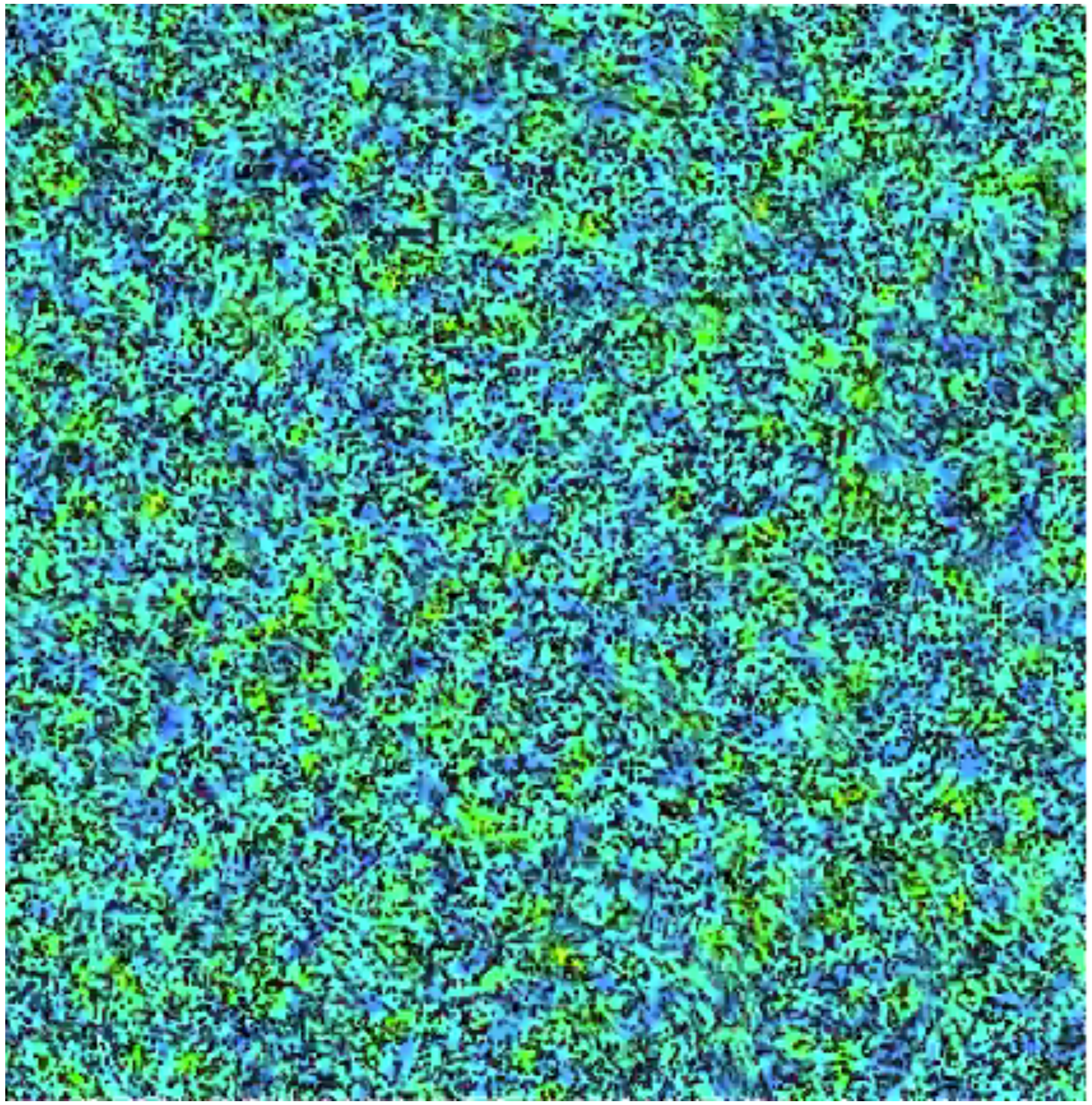
- Goldreich & Ward ‘73:
  concentrate (small) grains until they collapse to km-size!

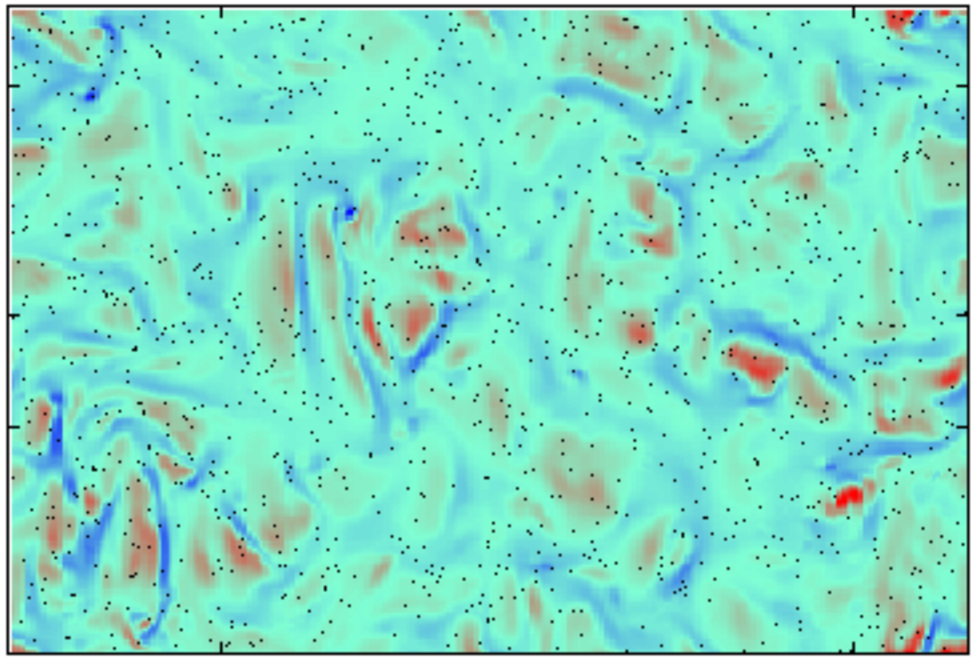
- Problem: *turbulence*

# But Is Turbulence So Bad?

## WHAT ELSE DOES IT DO?





# What is Happening?

VORTICITY = PREFERENTIAL CONCENTRATION OF GRAINS

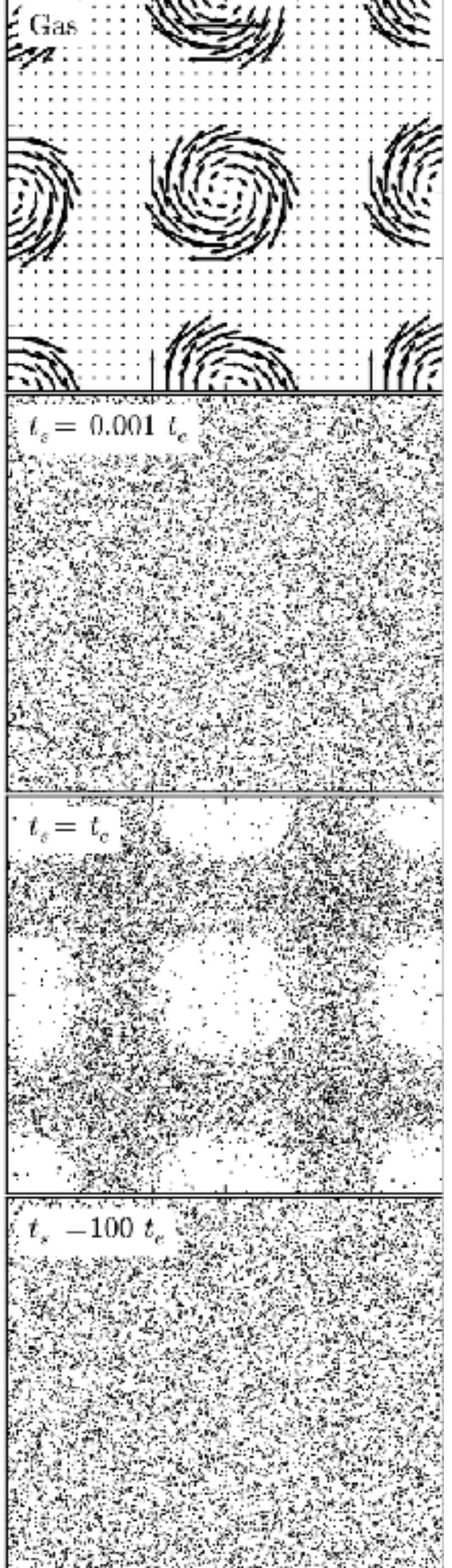
- **Dust is not Gas**
  - Instabilities segregate gas and dust

$$\frac{D\mathbf{v}_{\rm grain}}{Dt} = -\frac{(\mathbf{v}_{\rm grain} - \mathbf{u}_{\rm gas})}{t_{\rm stop}} + \mathbf{F}_{\rm ext}$$

$$t_{\rm stop} = \frac{\bar{\rho}_{\rm grain}\, R_{\rm grain}}{\rho_{\rm gas}\, c_{s,\,{\rm gas}}}$$

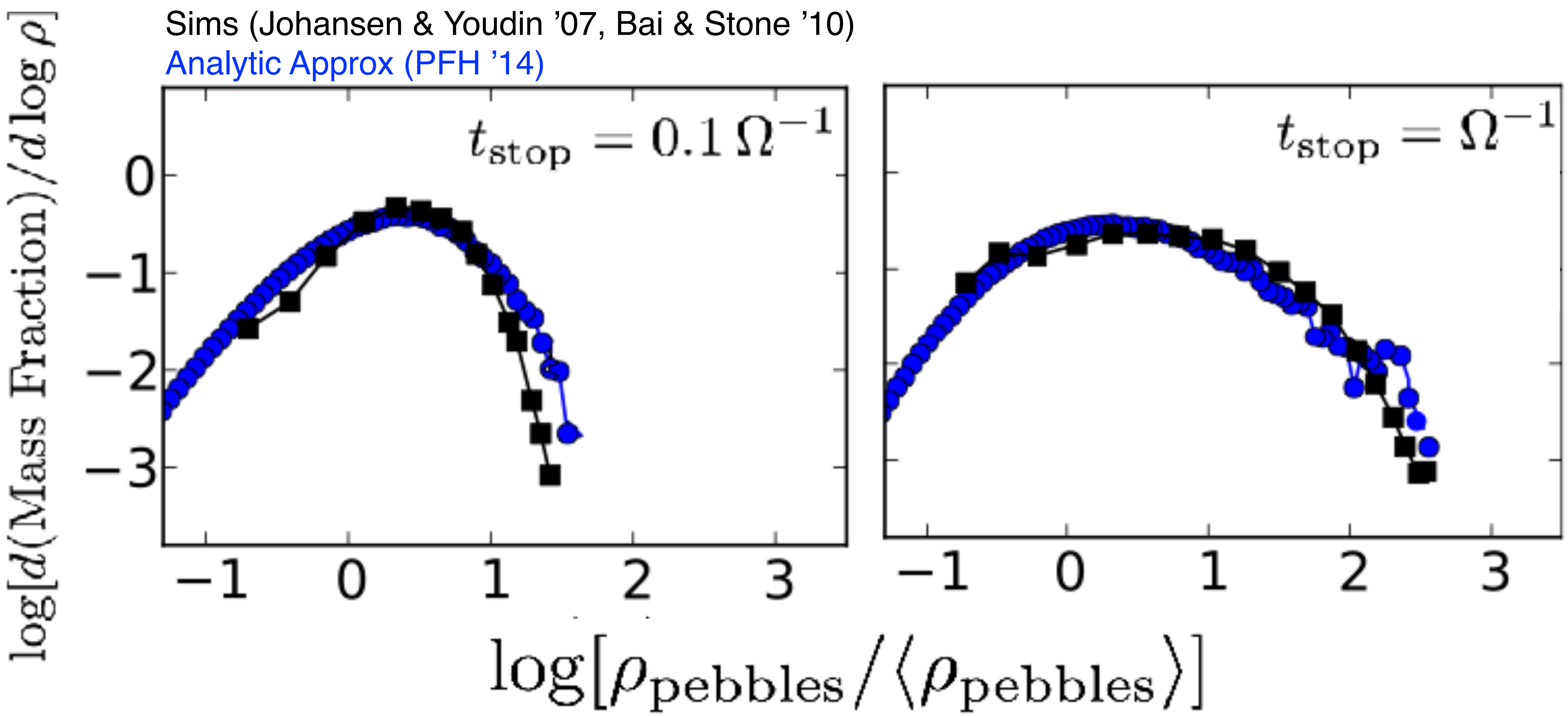
  - e.g. vortex traps, preferential concentration, streaming instability, zonal flows

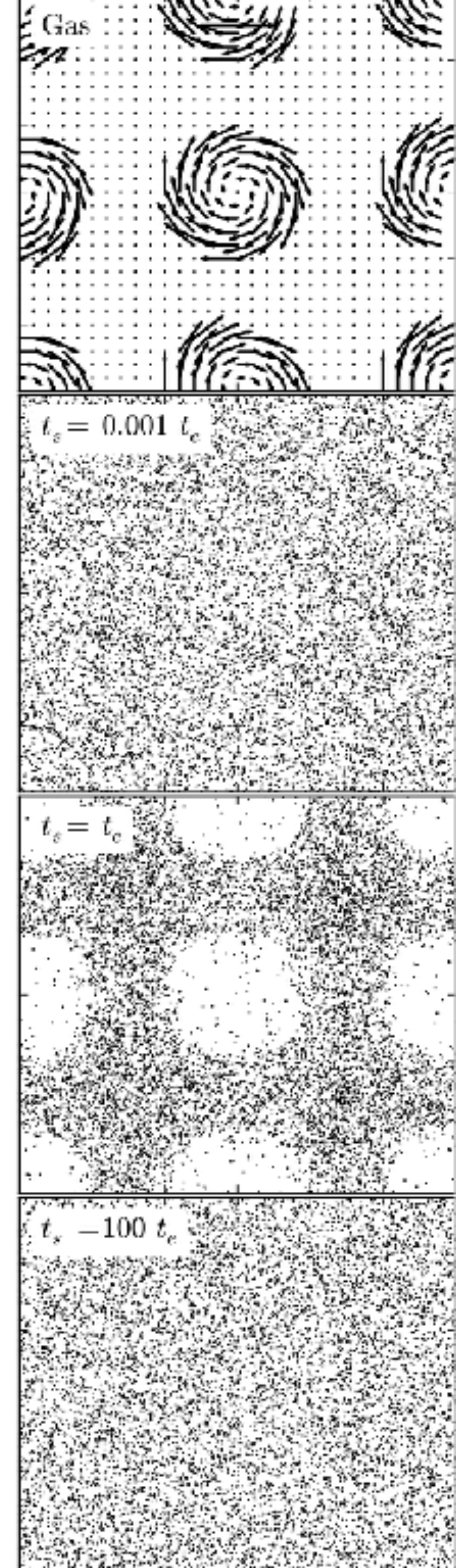
(see Barge & Sommeria '95, Bracco '99, Cuzzi '01, Johansen & Youdin '07, Carballido '08, Lyra '08, Bai & Stone '10, Pan '11, Zhu '14 and others)



# Multiply over a whole turbulent cascade and…

## GET A LOG-NORMAL-LIKE PDF



# Numerical Methods

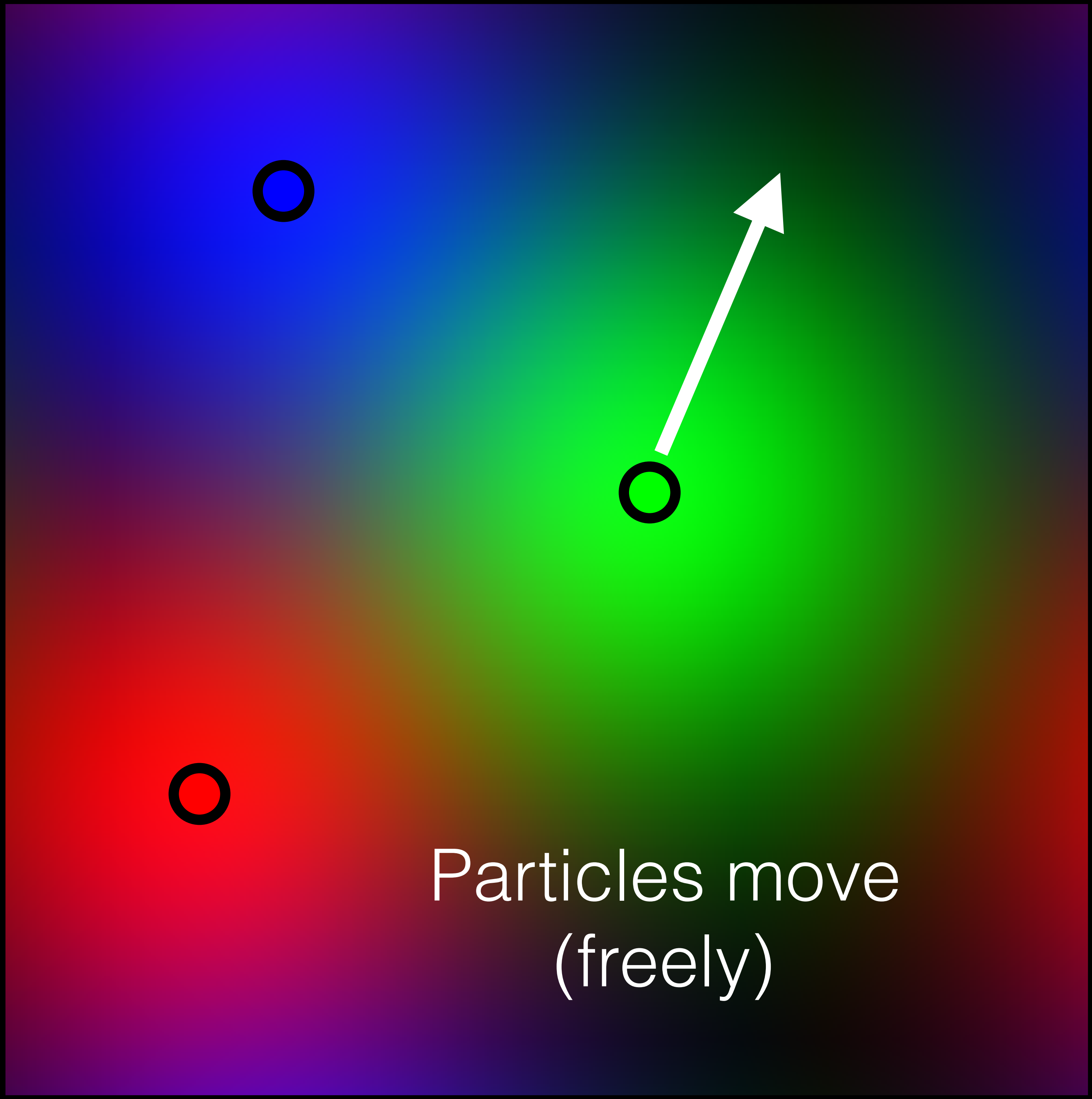
(aka: why did we switch from SPH?)

Challenge:
POPULAR METHODS FOR
HYDRODYNAMICS HAVE PROBLEMS

Lucy 77, Gingold & Monaghan 77
Reviews by: Springel 11, Price 12

## Smoothed-Particle Hydrodynamics

- Lagrangian, adaptive, simple, conservative



Challenge:
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Lucy 77, Gingold & Monaghan 77
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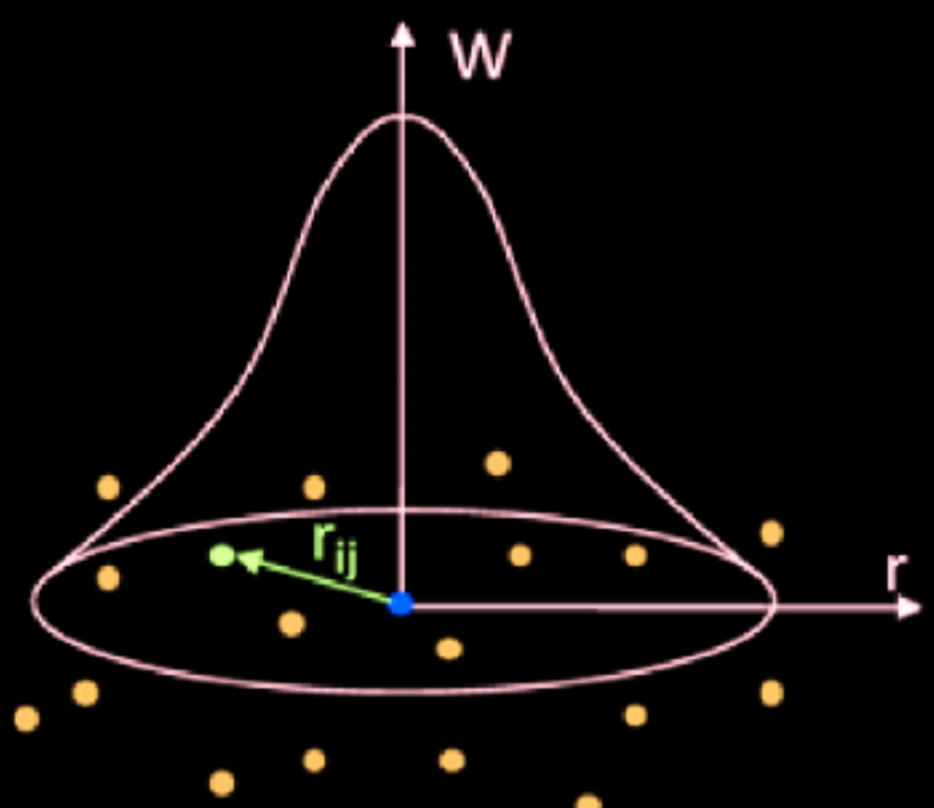
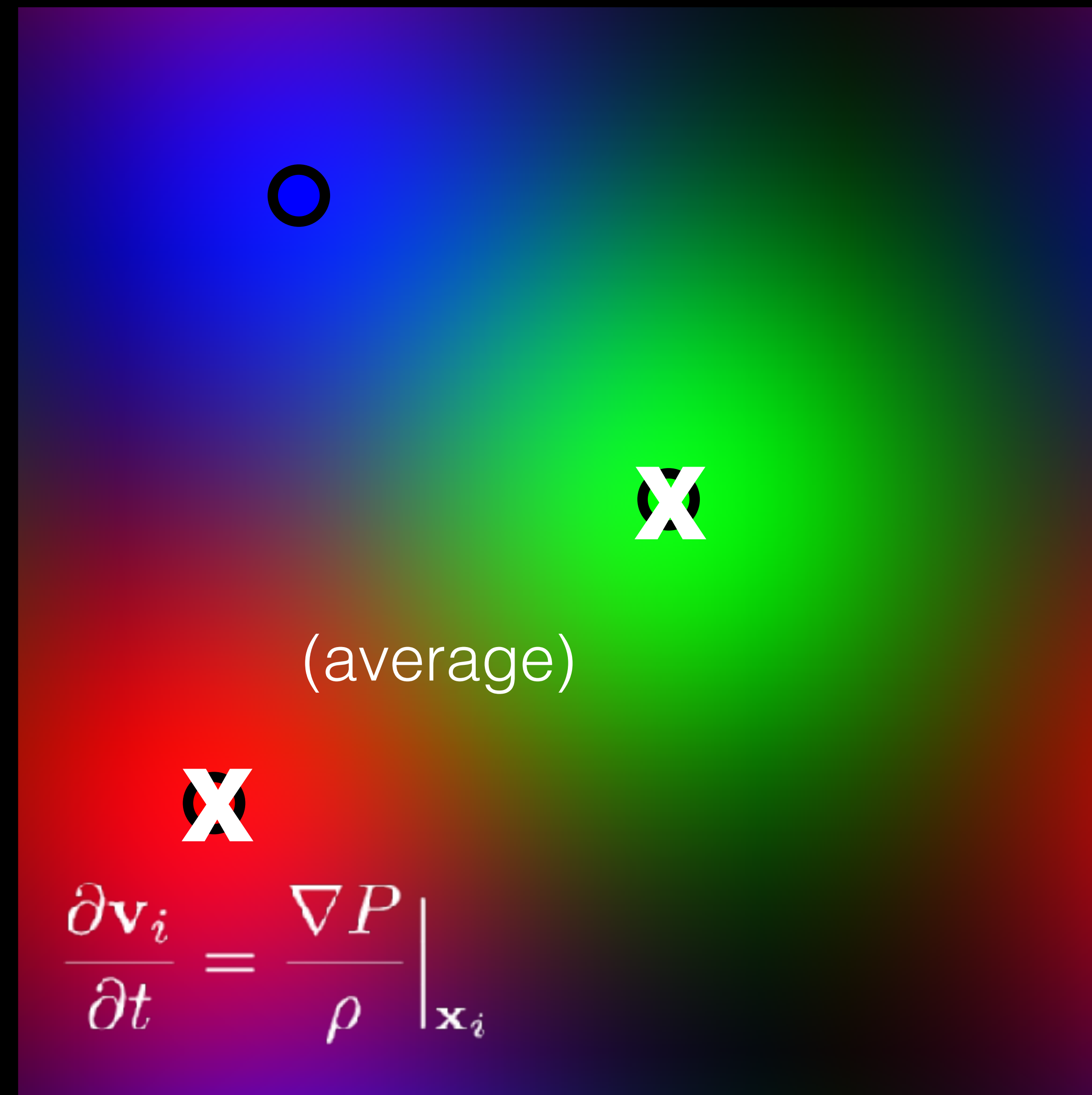
# Smoothed-Particle Hydrodynamics

- No volume partition: point-like particles smoothed into fields [ok in "continuum limit"]



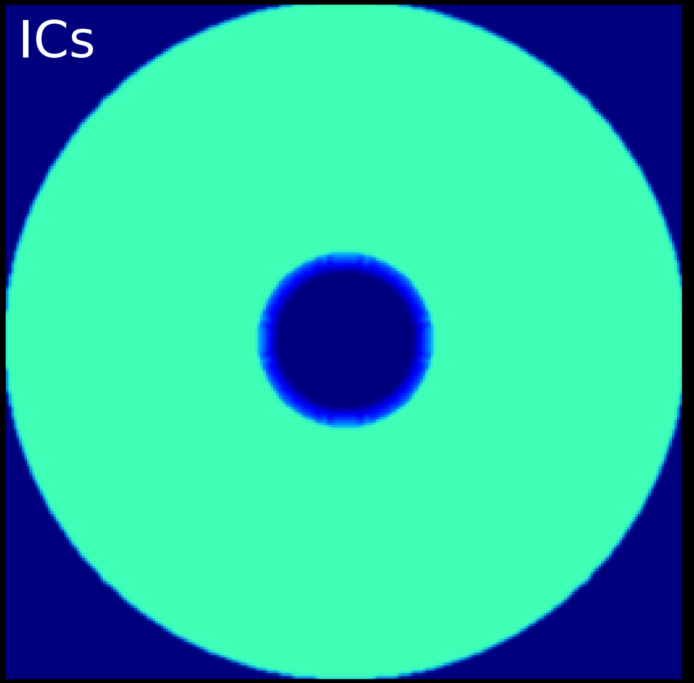

- Solve EOM at particle locations (stabilize with artificial diffusion)

Challenge:
POPULAR METHODS FOR
HYDRODYNAMICS HAVE PROBLEMS

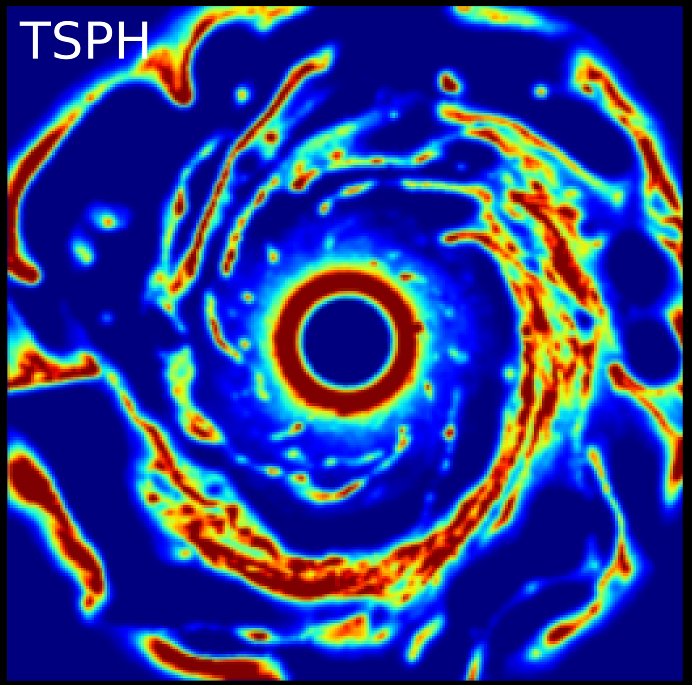
# Smoothed-Particle Hydrodynamics

- Lagrangian, adaptive, simple, conservative
- Artificial diffusion terms:
  - excess diffusion, viscosity

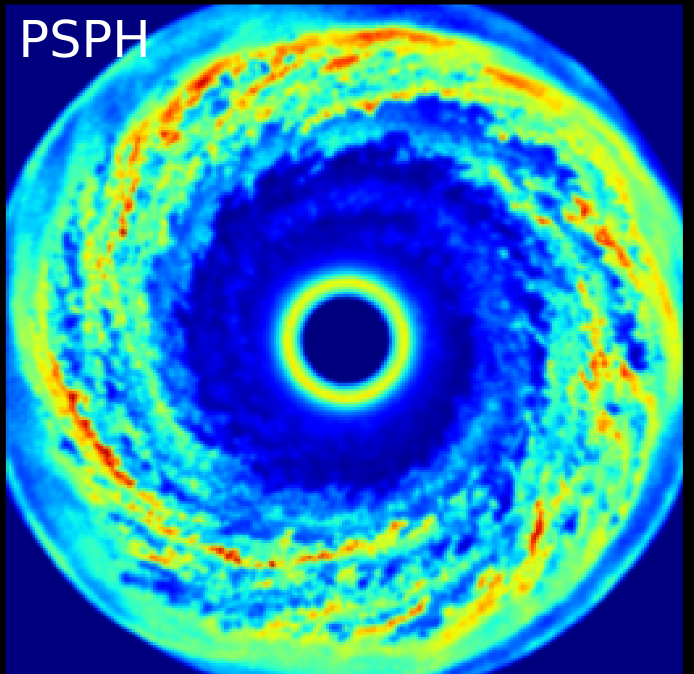
Keplerian disk/ring
(should conserve ICs)




"old" SPH
(Springel 02)
(after 20 orbits)




"new" SPH
(Hopkins 13)




Morris 97, Okamoto 03,
Cullen & Dehnen 10, Bauer & Springel 12

Ritchie & Thomas 01, Agertz 07, Price 12, Read 12

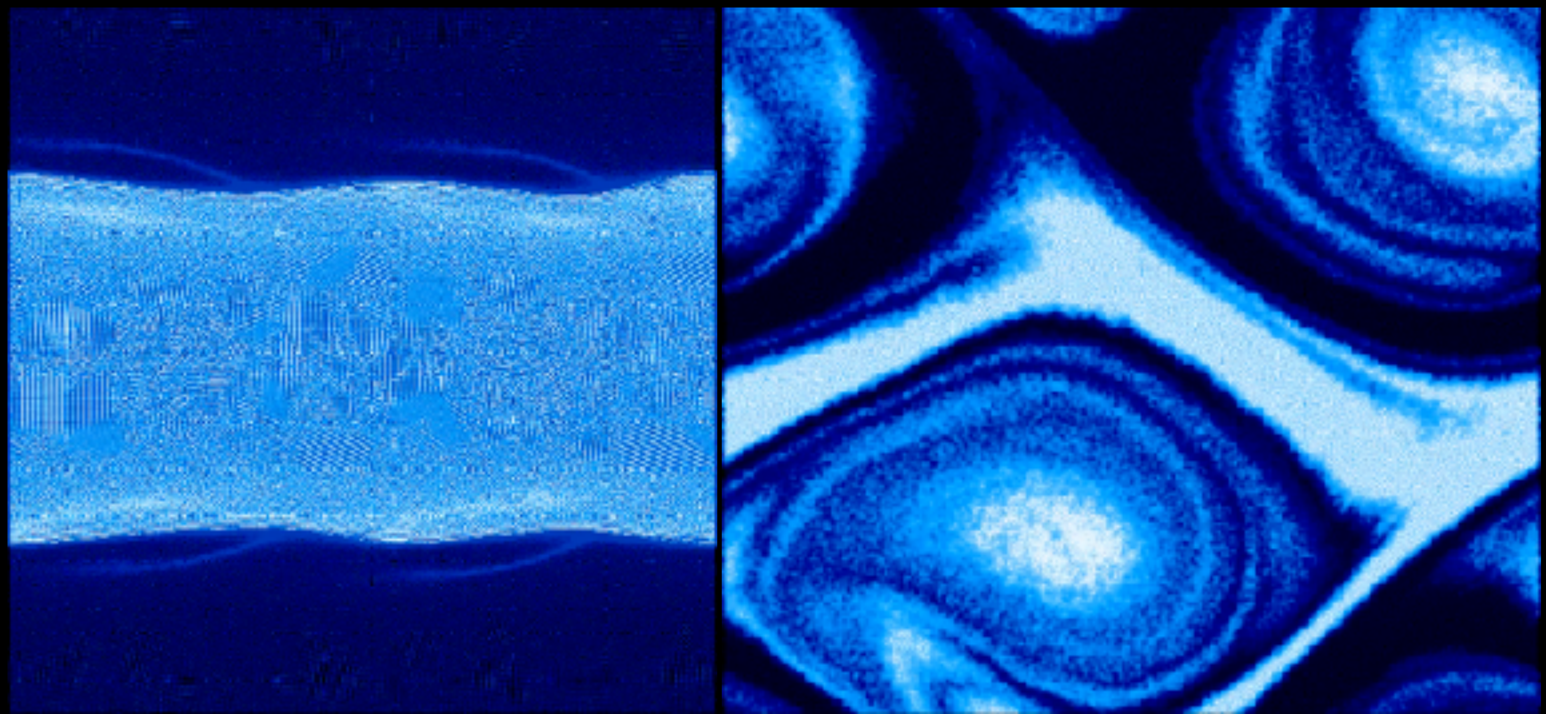
# Challenge:

## POPULAR METHODS FOR HYDRODYNAMICS HAVE PROBLEMS

- **“Traditional SPH”**
  - GADGET/(old)GASOLINE
  - ~32 neighbors (cubic spline)
  - constant artificial viscosity
  - “density” formulation

- **“Modern SPH”**
  - P-SPH/SPHS/PHANTOM
  - ~128-500 neighbors (alt. kernels) (many people: Read, Dehnen)
  - high-order switches (Cullen+Dehnen)
  - “pressure” formulation (Hopkins, Saitoh+Makino)
  - artificial diffusion for entropy (Price, Wadsley)

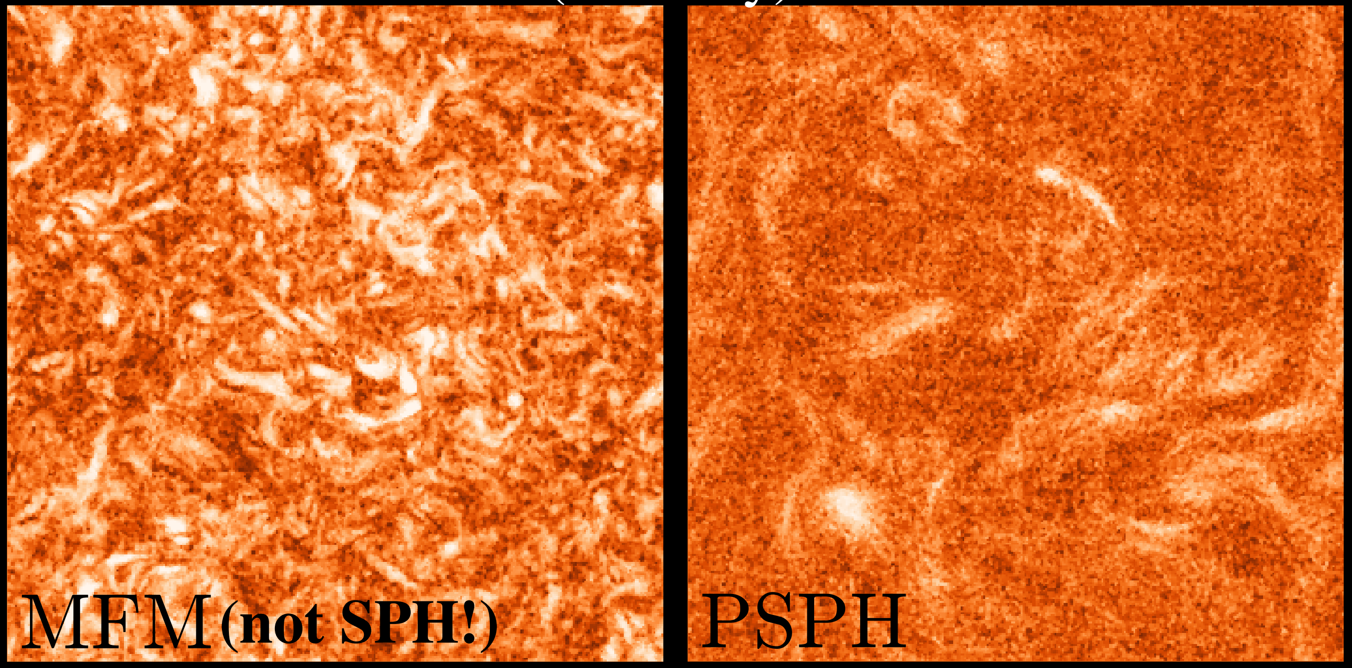
Kelvin-Helmholtz Instabilities



“old” SPH (Springel 02)

“new” SPH (PSPH) (Hopkins ’13): >>100 neighbors

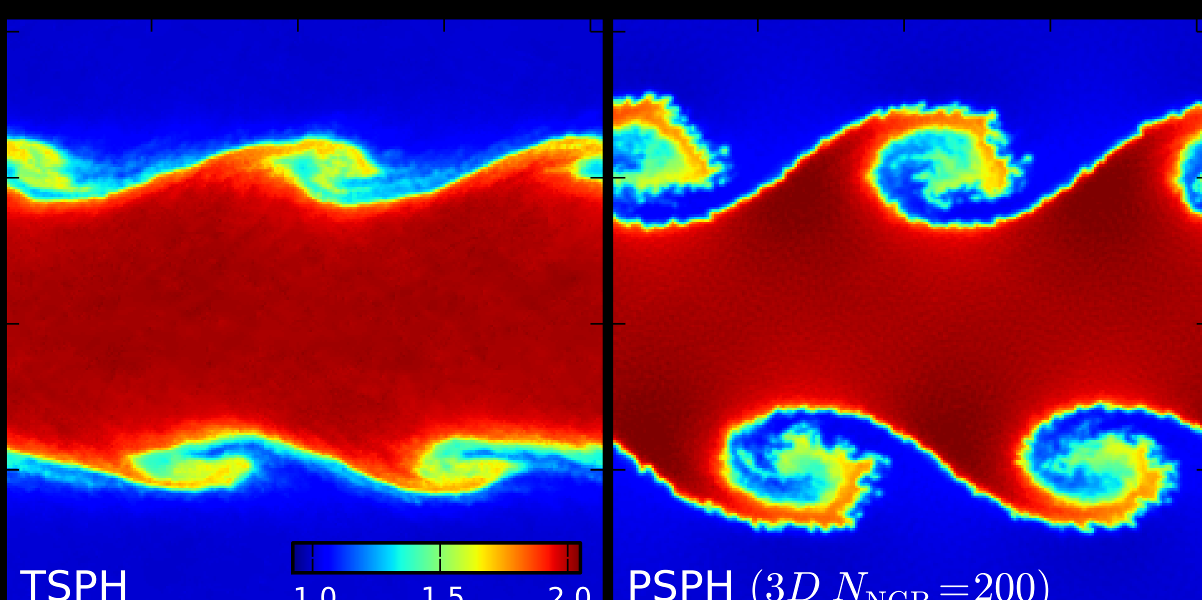
Sub-sonic turbulence (vorticity)



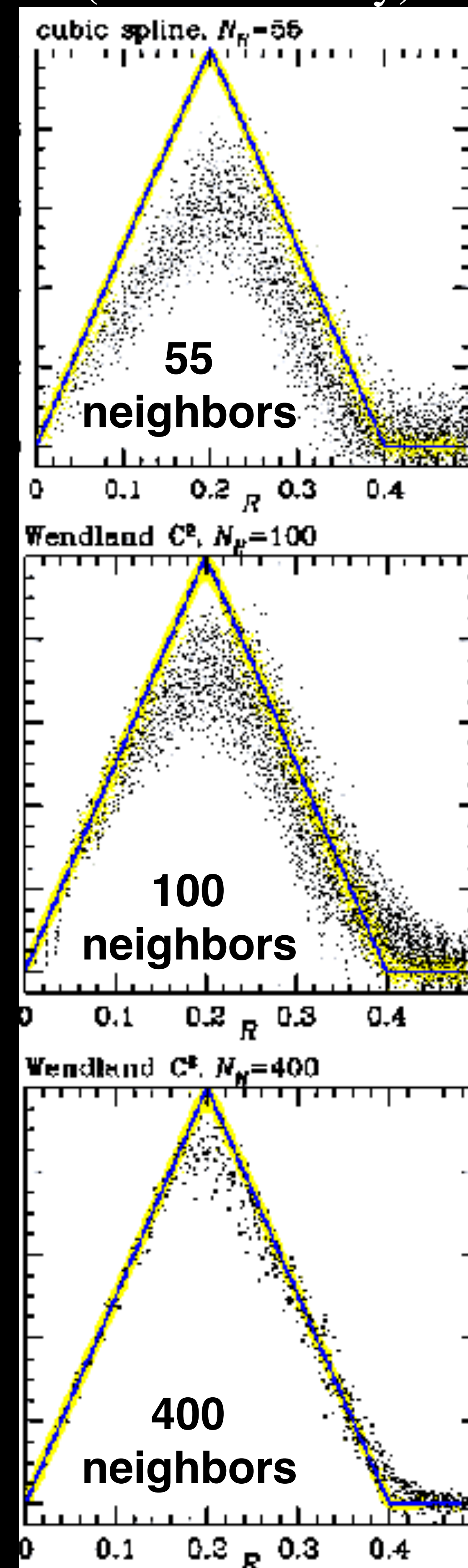
# Challenge:
## POPULAR METHODS FOR HYDRODYNAMICS HAVE PROBLEMS

- Fundamental low-order errors:
  - converge slowly:
    "beat down" by
    increasing kernel size,
    but this is *not efficient!*

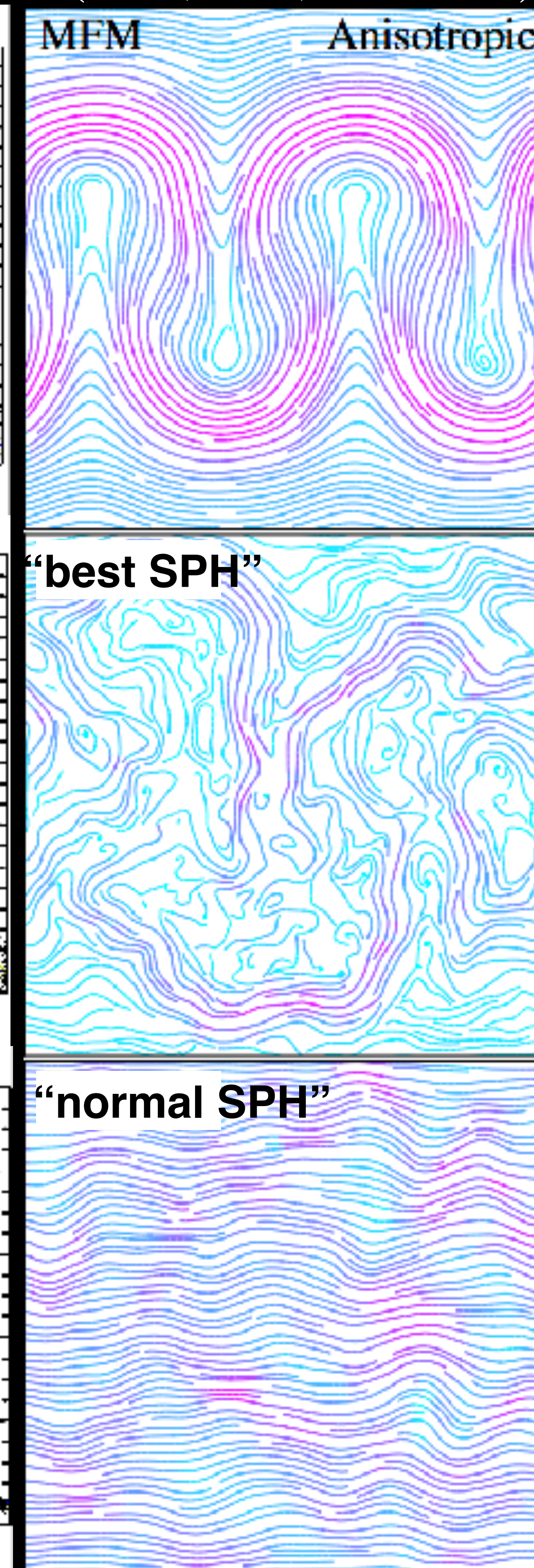
- MHD & anisotropic
  diffusion operators ill-posed




Gresho vortex (Dehnen & Aly)




Anisotropic Conduction (MTI, HBI, Hall MRI)

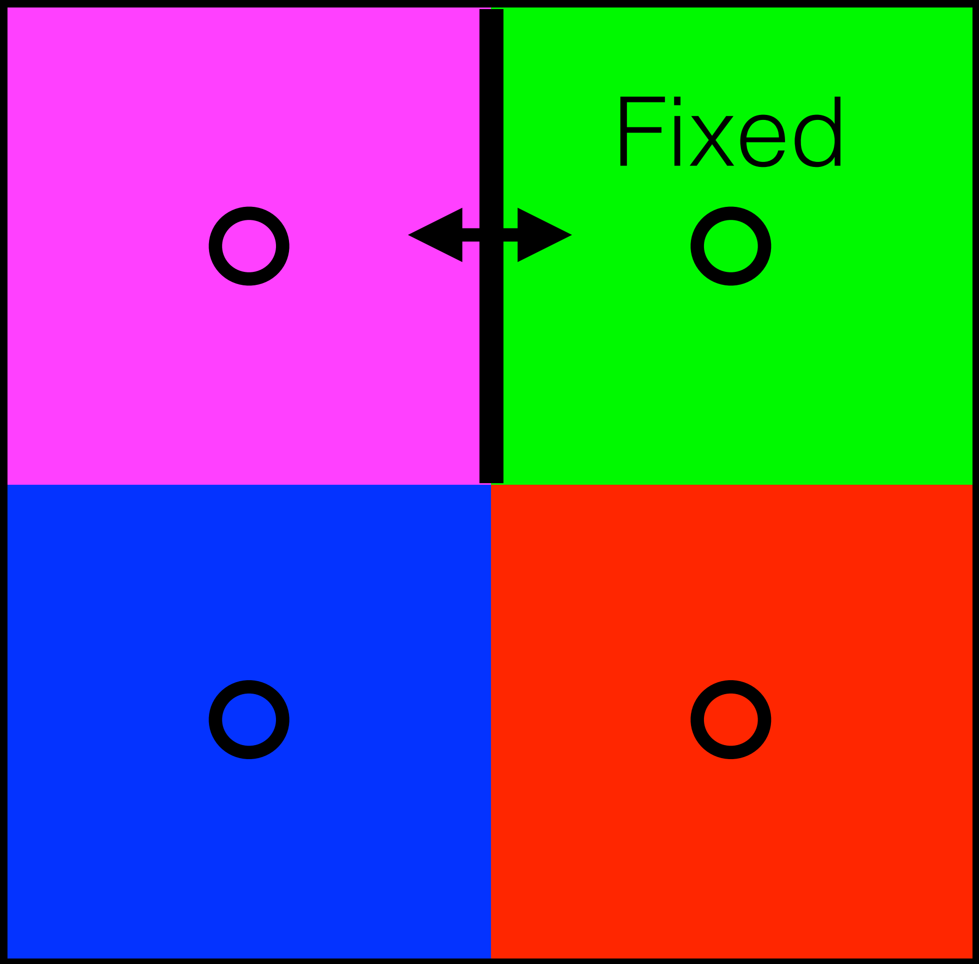



Dehnen & Aly, Rosswog, Hopkins, Tricco & Price, Read

Challenge:

POPULAR METHODS FOR HYDRODYNAMICS HAVE PROBLEMS

Berger & Colella 89 (& others)
Reviews by: Teyssier 14




$$\Delta m_i = \int_{\mathrm{cell}} \frac{\partial \rho}{\partial t} d^3\mathbf{x} = -\int_{\mathrm{cell}} \nabla \cdot (\rho\,\mathbf{v})\, d^3\mathbf{x}$$

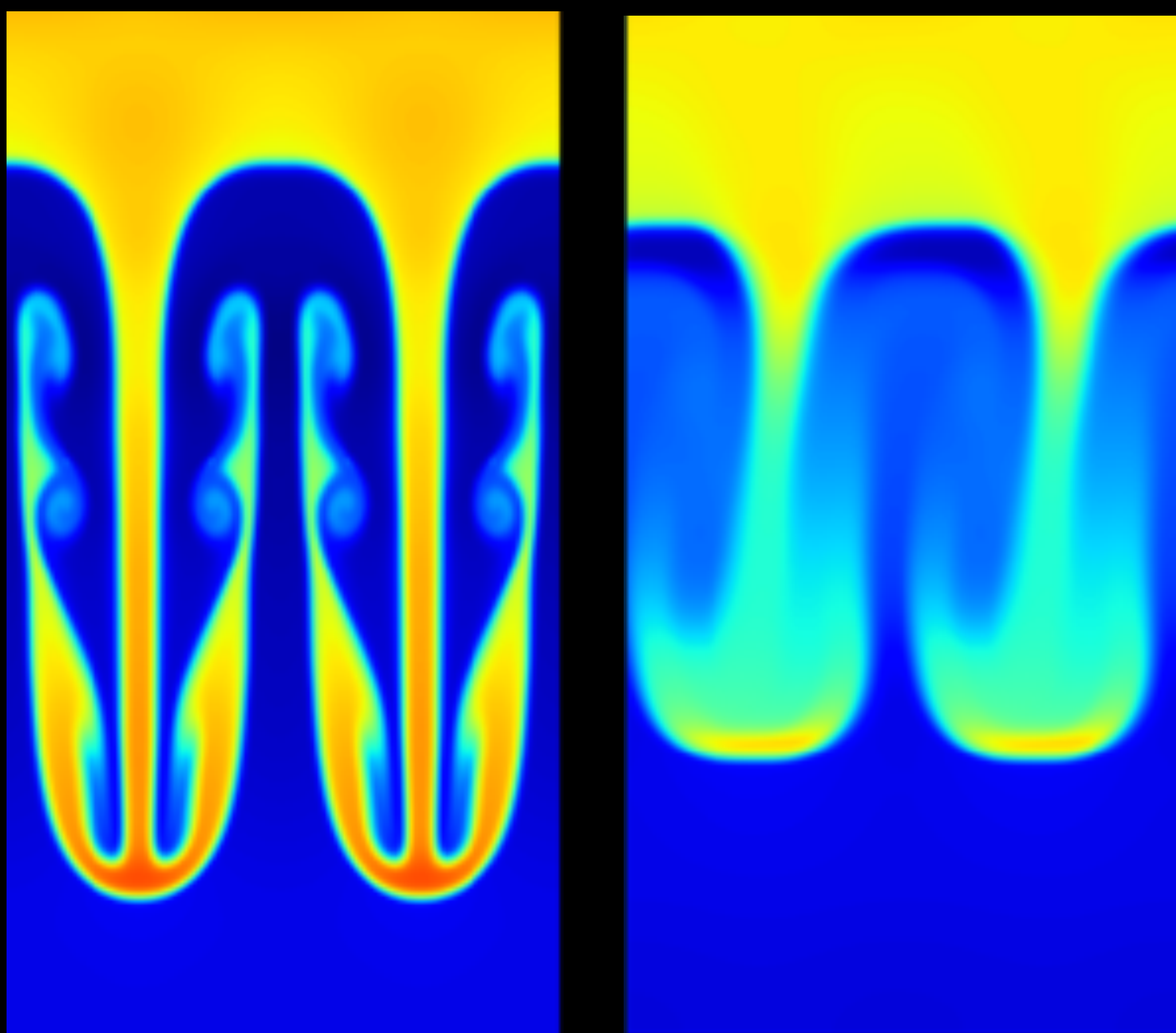
## Adaptive Mesh Refinement

- Eulerian, well-studied, high-order
- Each cell carries conserved quantities inside volume $V_i$
- Solve Reimann problem between geometric faces

# Adaptive Mesh Refinement (AMR)

## CHALLENGE: POPULAR METHODS HAVE PROBLEMS

Rayleigh-Taylor instability
(AMR, $256^2$)



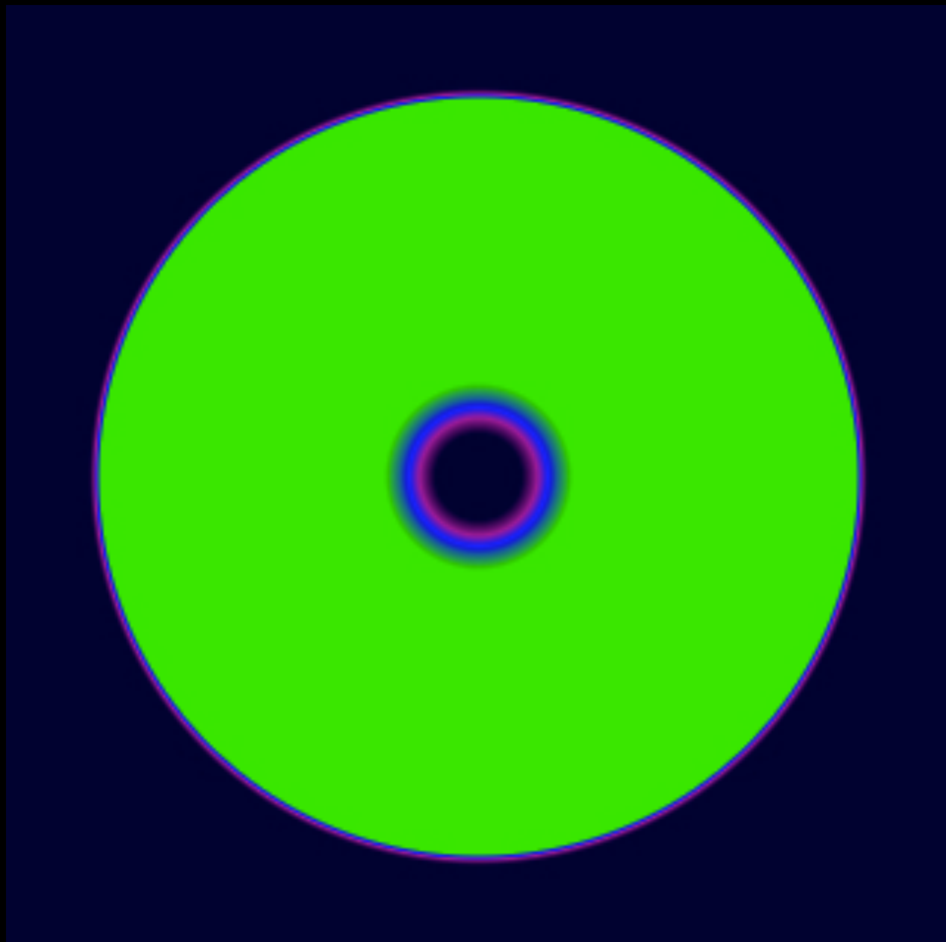
(no bulk motion) Mach 5 boost

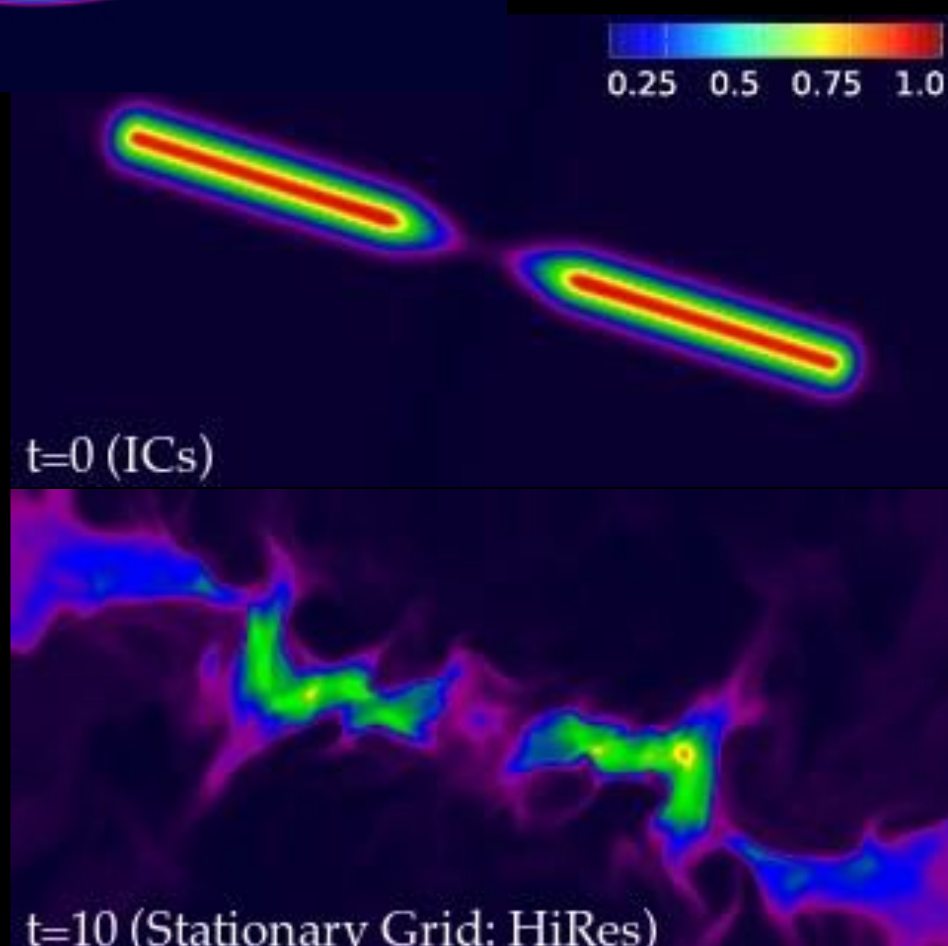
- Eulerian, well-studied, high-order
- Excessive mixing/diffusion
  when fluid moves over cells

Bryan 95, Wadsley 08,
Tasker & Bryan 08, Springel 10

# Adaptive Mesh Refinement (AMR)

## CHALLENGE: POPULAR METHODS HAVE PROBLEMS



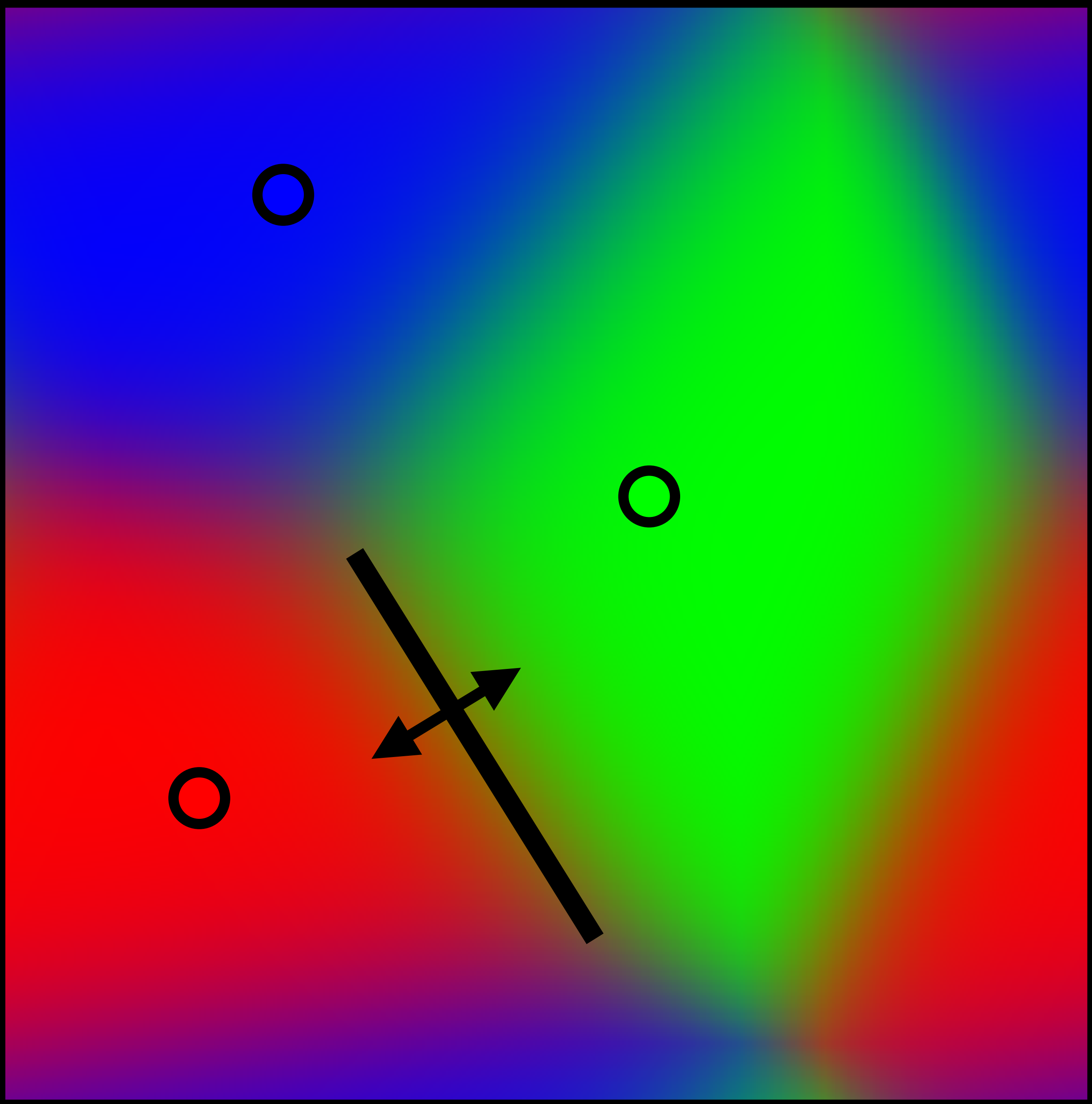



- Eulerian, well-studied, high-order
- Excessive mixing/diffusion when fluid moves over cells
- Geometric effects:
  - carbuncle instability (shocks)
  - loss of angular momentum
  - grid-alignment (disks)
- Also “beaten down” with resolution, but *expensive*
  - Hahn ’10: >>$512^2$ resolution to avoid grid-alignment

Peery & Imlay 88,
Mueller & Steinmetz 95, Hahn 10

Challenge:

POPULAR METHODS FOR
HYDRODYNAMICS HAVE PROBLEMS



## New Methods Combine (some) Advantages of Both

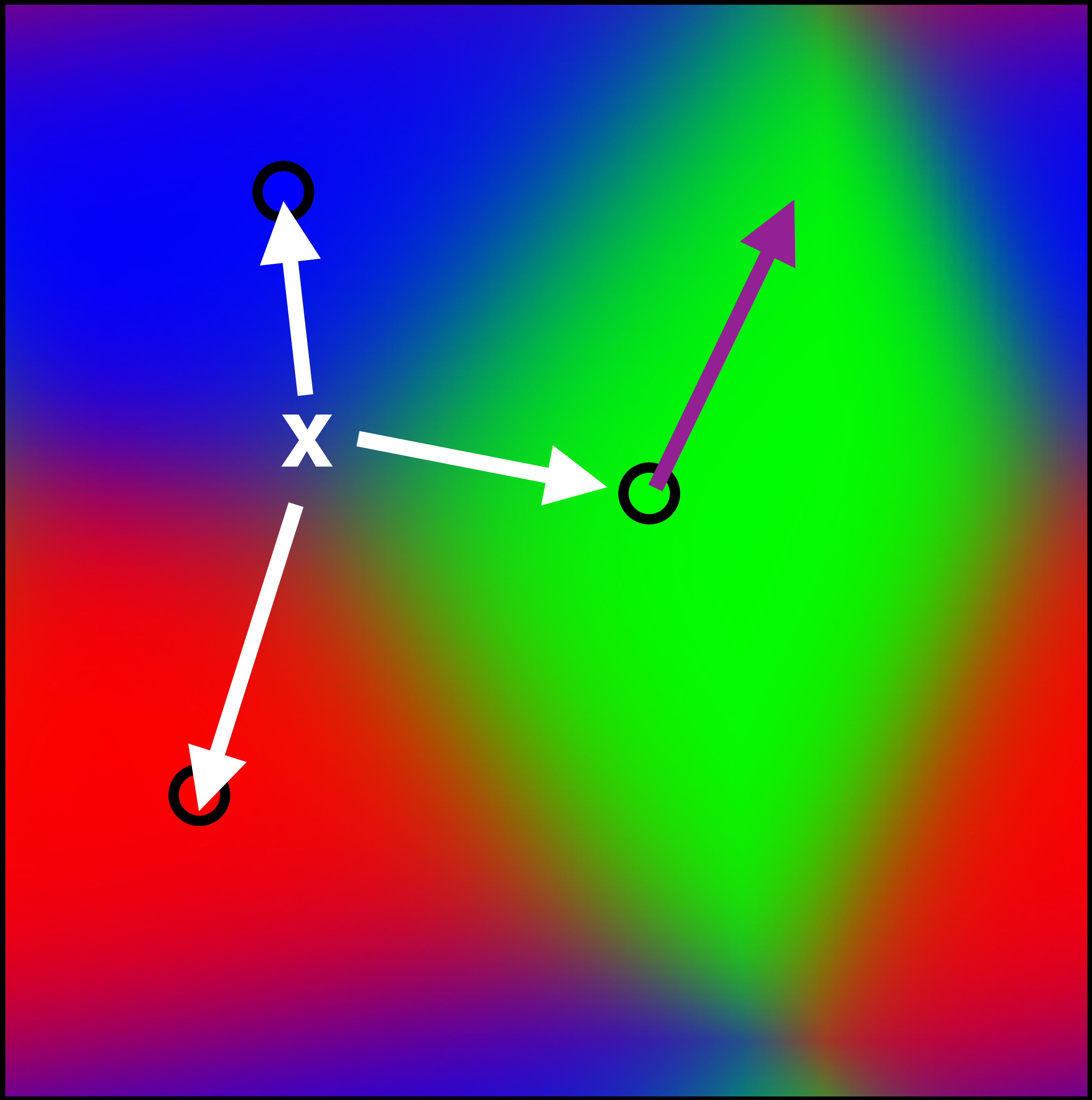
- Moving-meshes (AREPO), meshless finite-volume (GIZMO), high-order ALE methods
- Move with flow, no preferred geometry, but also accurate, high-order, and shock-capturing
- Less well-tested **!**

**AREPO**: Springel 2010
**TESS/DISCO**: Duffel 2011
**FVMHD3D**: Gaburov 2012
**GIZMO**: Hopkins 2015 (arXiv:1409.7395)

Challenge:

POPULAR METHODS FOR HYDRODYNAMICS HAVE PROBLEMS

Lanson & Vila 2008
Gaburov & Nitadori 2011
PFH 2014, 2015, 2016



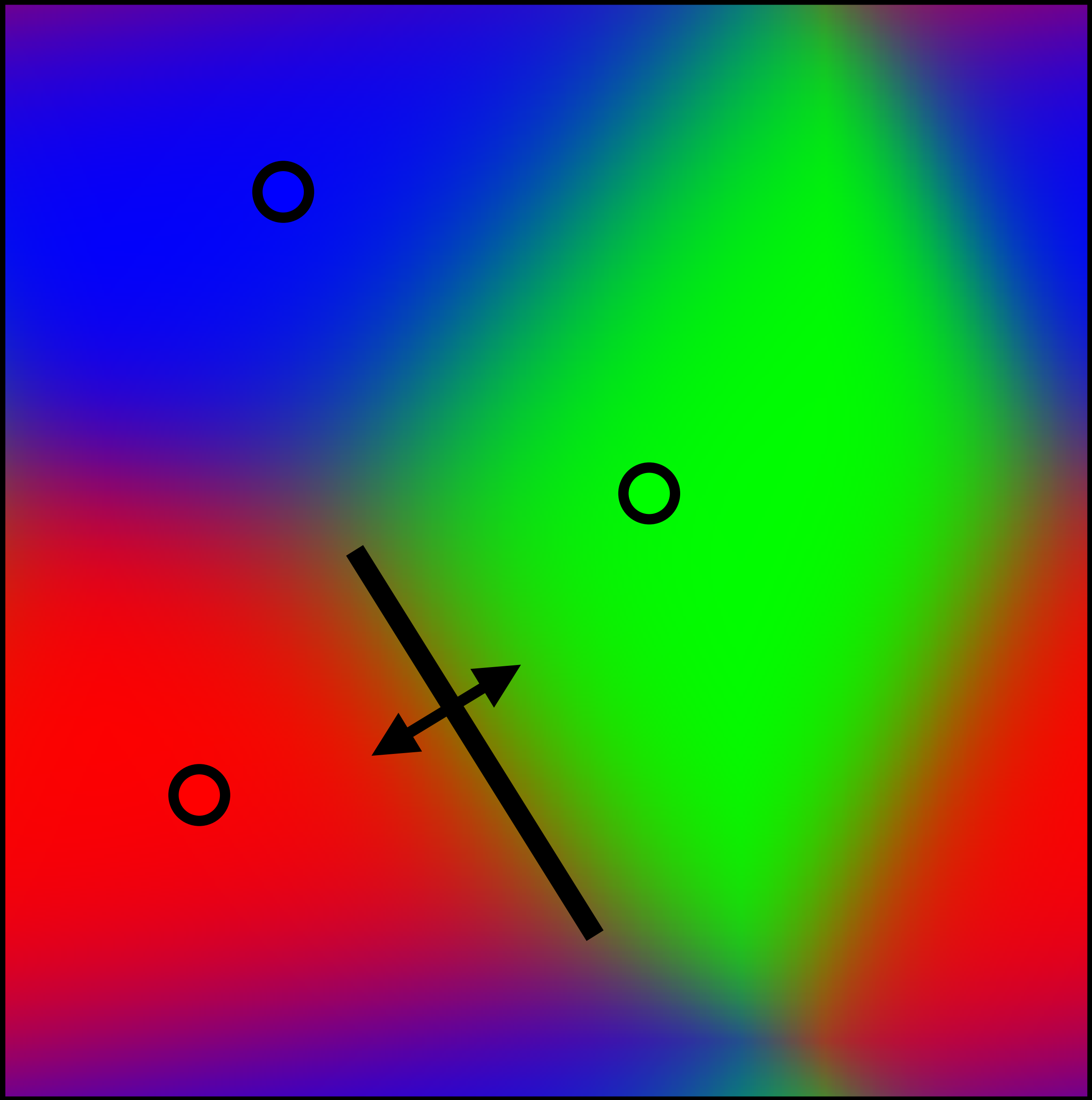

- Mesh-generating points move (if desired)
- Volume is "partitioned" with a continuous kernel (MFM/MFV) or step function (moving-mesh)

$$d\mathrm{Vol}_{i,j,k} = d^3\mathbf{x}\,\frac{W(\mathbf{x}-\mathbf{x}_{i,j,k})}{\sum W_{i,j,k}}$$

Lanson & Vila 2008
Gaburov & Nitadori 2011
PFH 2014, 2015, 2016

# Challenge:

## POPULAR METHODS FOR HYDRODYNAMICS HAVE PROBLEMS



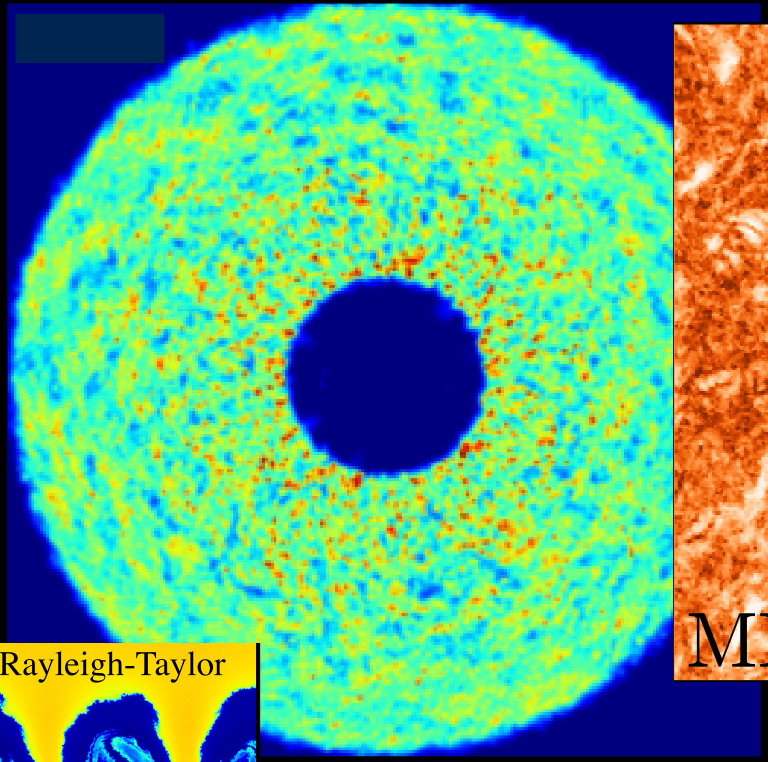
- Integrate EOM over volume: equivalent to Reimann problem at “effective face” (quadrature)

$$\Delta m_i = \int_{\rm vol} \frac{\partial \rho}{\partial t} d^3\mathbf{x} = -\int_{\rm vol} \nabla \cdot (\rho\,\mathbf{v})\, d^3\mathbf{x}$$

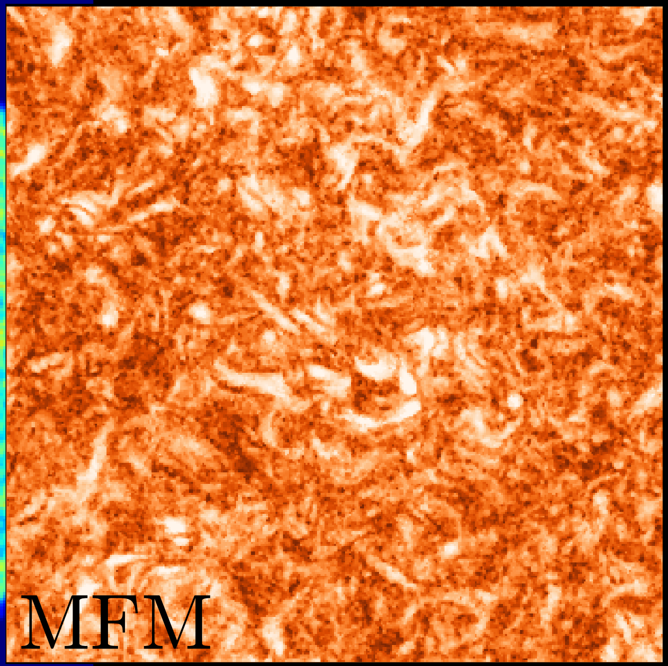
# New Methods Combine (some) Advantages of Both:

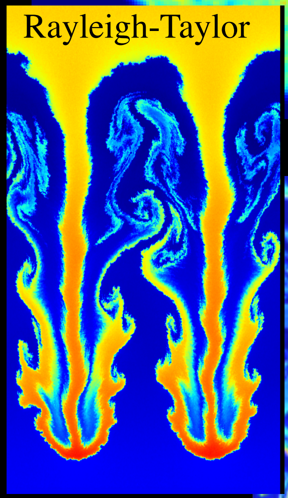
(BUT REMAIN LESS WELL-TESTED)

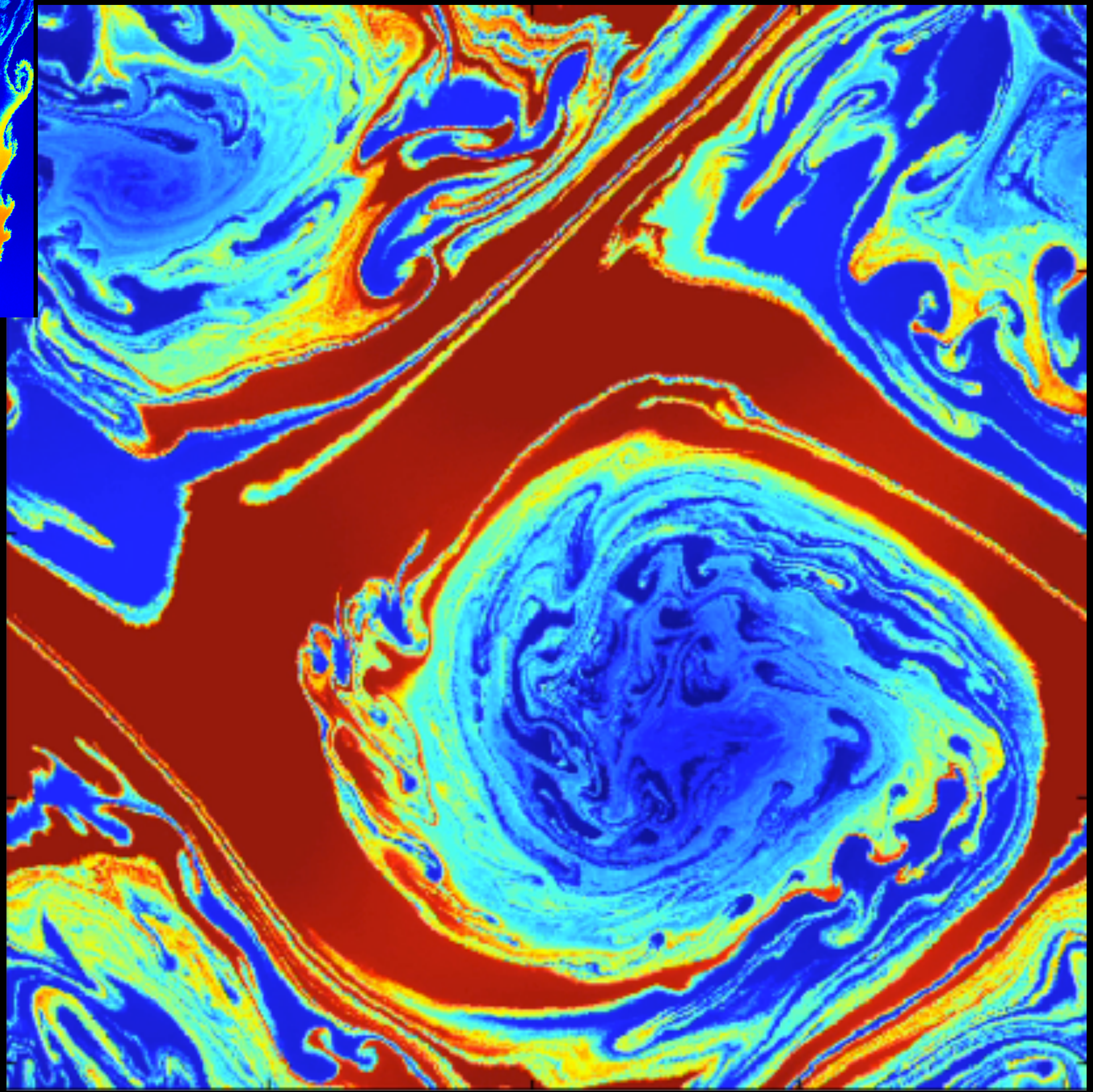
GIZMO: disk after 100 orbits



sub-sonic turbulence



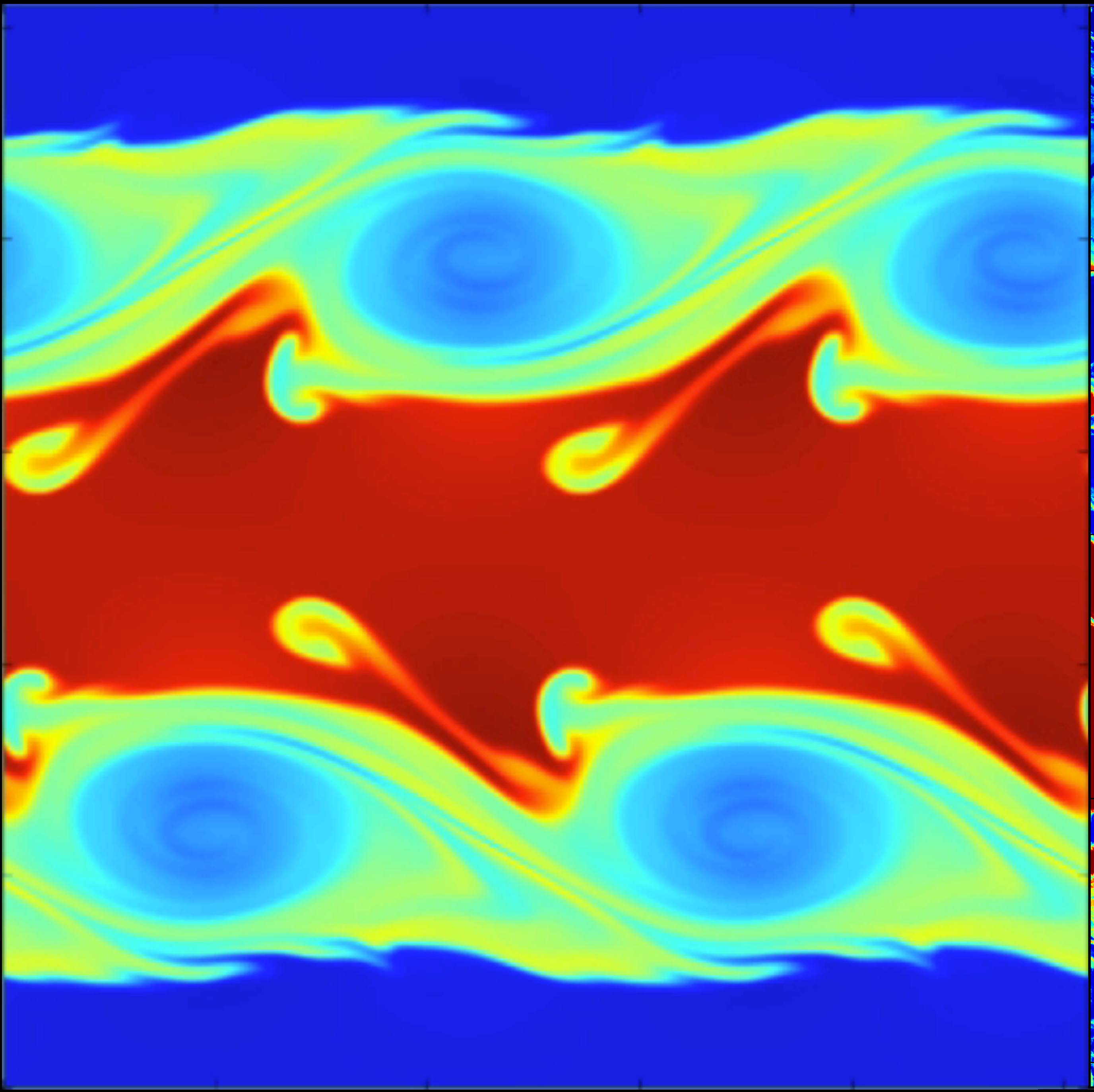



- Moving-meshes (AREPO), meshless finite-volume (GIZMO), high-order ALE methods
- Move with flow, no preferred geometry, but also accurate, high-order, and shock-capturing
- Grid noise is more severe

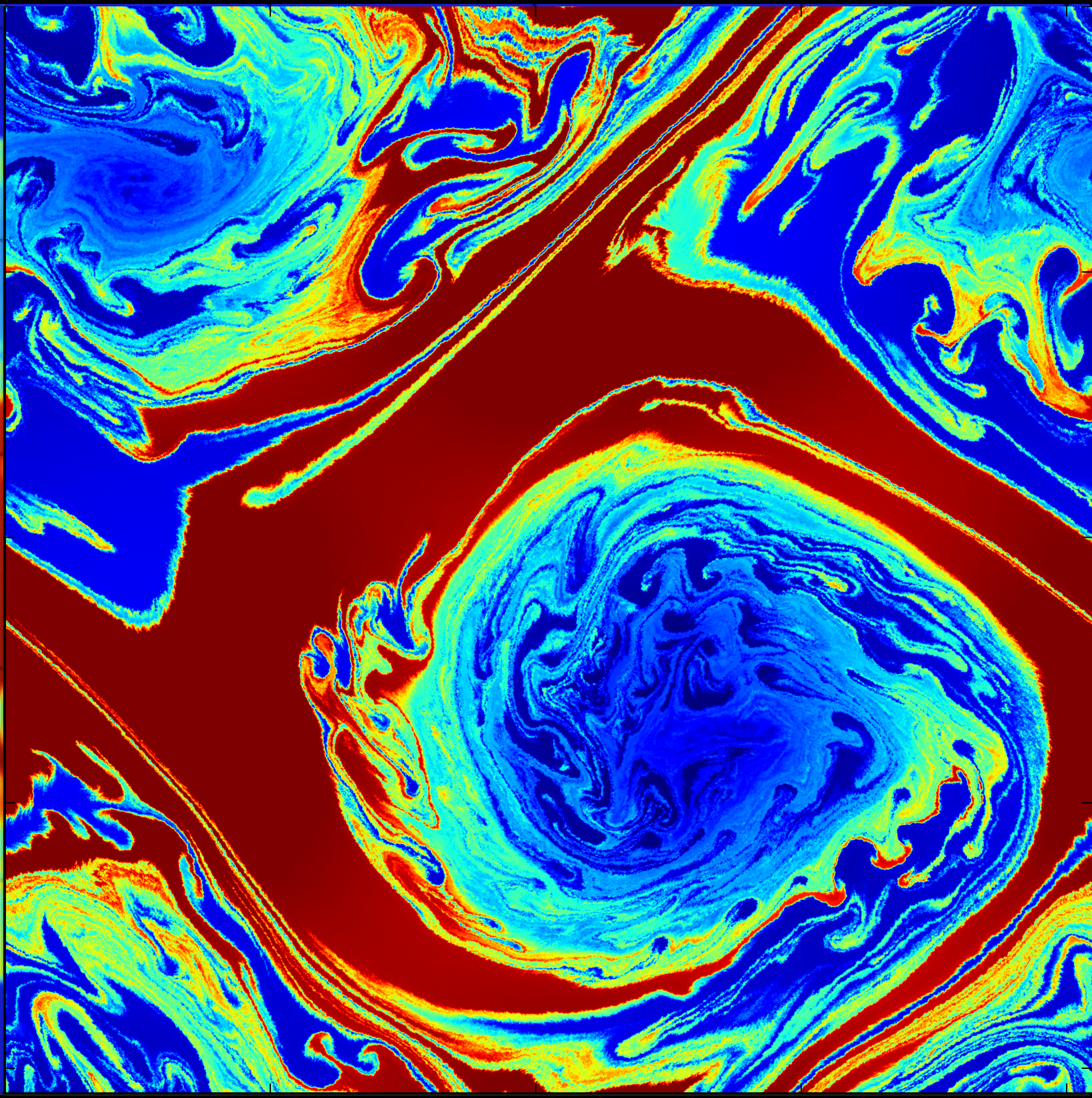
**AREPO**: Springel 2010
**TESS/DISCO**: Duffel 2011
**FVMHD3D**: Gaburov 2012
**GIZMO**: Hopkins 2015

# GIZMO: New Meshless Methods & Fluid Mixing

*(www.tapir.caltech.edu/~phopkins)*



Cartesian Grid

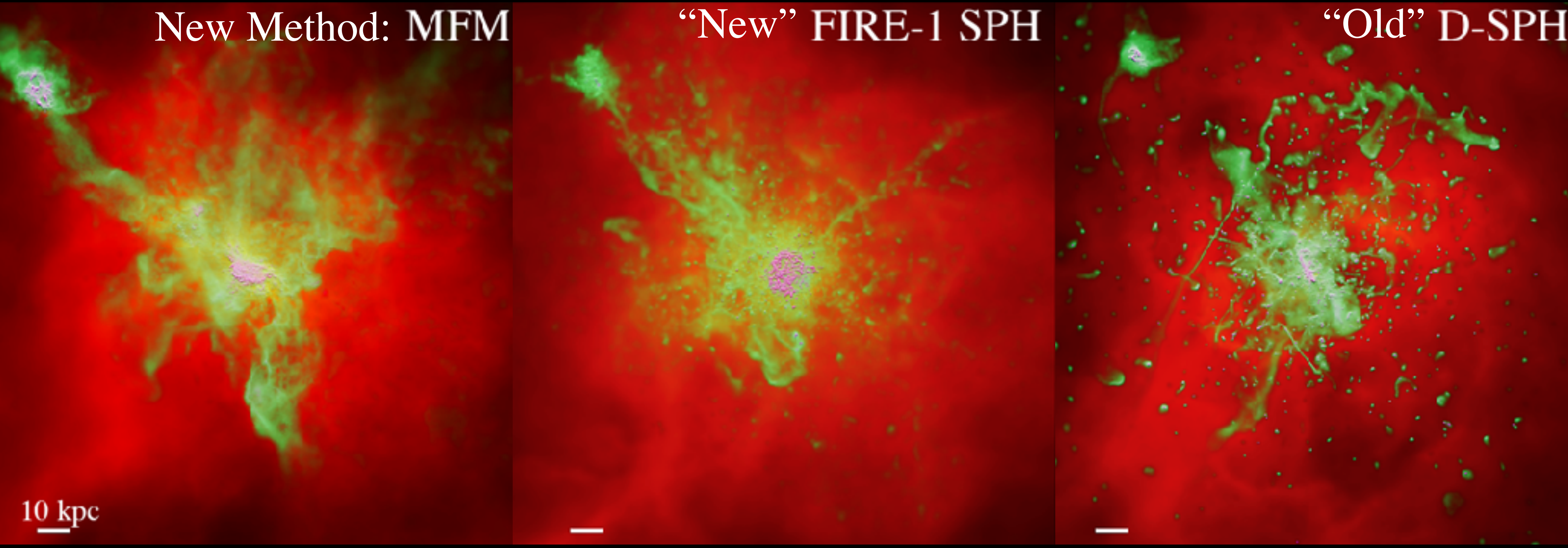


Meshless Finite Volume

Hopkins 2015 (arXiv:1409.7395)

# Getting the Hydro Right Can Matter

## BUT IT DEPENDS ON WHAT YOU CARE ABOUT




Agertz 07 & many others

*BUT* only factor ~1.5 difference in mass!

# Summary:

*** ISM *statistics* are far more fundamental than we typically assume ***

- **Turbulence + Gravity:** ISM structure follows
  - **GMC Mass Function & Structure** (“first crossing”)
  - **Core MF, IMF** (“last crossing”) & **Linewidth-Size-Mass**
  - **Clustering** of Stars (correlation functions)
  - Predictions for **IMF Variation** in ultra-high Mach numbers

- **Planet Formation:**
  - **Direct Collapse:** modest turbulence (Mach >0.3) could induce
  - **“Pebble Piles”:** could form beyond the ice line

- **Numerical Methods:**
  - SPH: Problems that may not converge
  - New Lagrangian Finite-Volume Methods: promising but poorly-understood?