

# Numerics, Physics, Resolution

Towards Predictive Galaxy Formation Simulations

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Caltech

# Physics

(a question of philosophy)

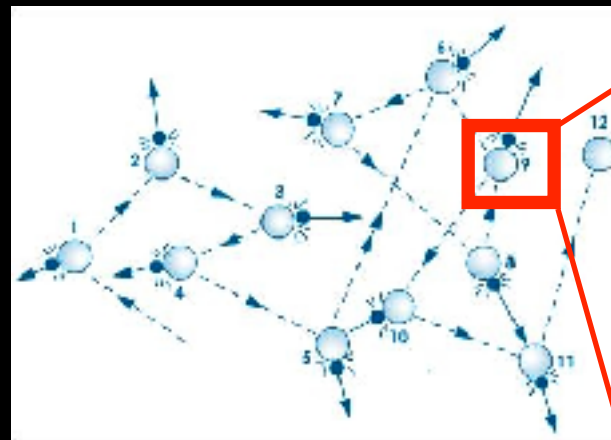
# Everything is sub-grid

Hydrodynamics



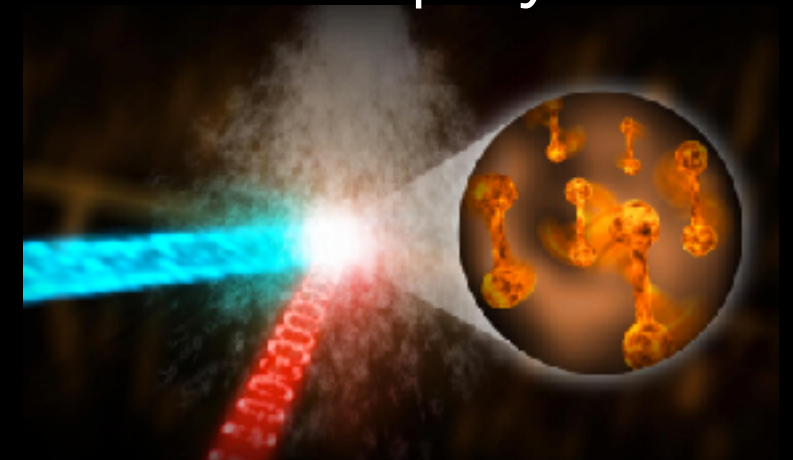
$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{v}$$

Statistical mechanics



$$\begin{aligned} Q(t) &= f[t; g[Q]] - f^{(0)}[t; g^{(0)}[Q]] \\ &+ \epsilon \left( \int ds \frac{\delta f^{(0)}(t)}{\delta g^{(0)}(s)} g^{(1)}[s; Q] + f^{(1)}[t; g^{(0)}[Q]] \right) \\ &+ \epsilon^2 \left( \int ds \frac{\delta f^{(0)}(t)}{\delta g^{(0)}(s)} g^{(2)}[s; Q] \right. \\ &\quad \left. + \frac{1}{2} \int ds ds' \frac{\delta^2 f^{(0)}(t)}{\delta g^{(0)}(s) \delta g^{(0)}(s')} g^{(1)}[s; Q] g^{(1)}[s'; Q] \right. \\ &\quad \left. + \int ds \frac{\delta f^{(1)}(t)}{\delta g^{(0)}(s)} g^{(1)}[s; Q] + f^{(2)}[t; g^{(0)}[Q]] \right) + O(\epsilon^3) \end{aligned}$$

Particle physics



$$\begin{aligned} \mathcal{L}_{GWS} &= \sum_f (\bar{\psi}_f (i \gamma^\mu \partial_\mu - m_f) \psi_f - g Q_f \bar{\psi}_f \gamma^\mu \psi_f A_\mu) - \\ &+ \frac{g}{\sqrt{2}} \sum_i (\bar{e}_L \gamma^\mu \nu_L W_\mu^+ + \bar{\nu}_L \gamma^\mu e_L W_\mu^-) + \frac{g}{2c_W} \sum_f \bar{\psi}_f \gamma^\mu (T_f^3 - 2Q_f \sin^2 \theta_W) \psi_f Z_\mu + \\ &- \frac{1}{4} |\partial_\mu A_\nu - \partial_\nu A_\mu - ie(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+)|^2 - \frac{1}{2} |\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + \\ &\quad - ie(W_\mu^+ A_\nu - W_\nu^+ A_\mu) + ig' c_W (W_\mu^+ Z_\nu - W_\nu^+ Z_\mu)|^2 + \\ &- \frac{1}{4} |\partial_\mu Z_\nu - \partial_\nu Z_\mu + ig' c_W (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+)|^2 \\ &- \frac{1}{2} M_\eta^2 \eta^2 - \frac{g M_\eta^2}{8 M_W} \eta^3 - \frac{g'^2 M_\eta^2}{32 M_W} \eta^4 + [M_W W_\mu^+ + \frac{g}{2} \eta W_\mu^+ + \\ &\quad + \frac{1}{2} (D_\mu \eta + i M_Z Z_\mu + \frac{ig}{2c_W} \eta Z_\mu)]^2 - \sum_f \frac{g}{2} \frac{m_f}{M_W} \bar{\psi}_f \psi_f \eta \end{aligned}$$

# 2 philosophies of sub-grid:

- 1. Parameterize unknowns, marginalize over them (fit to observations)
  - bias in BAO/LSS cosmology
  - MCMC SAMs / Illustris/Eagle philosophy
- 2. Derive from theory/observations on small scales, after “smoothing”
  - (magneto) hydrodynamics
  - FIRE philosophy:  $M_{\text{wind}} = (\text{whatever the input physics predicts})$



# Example: Supernovae

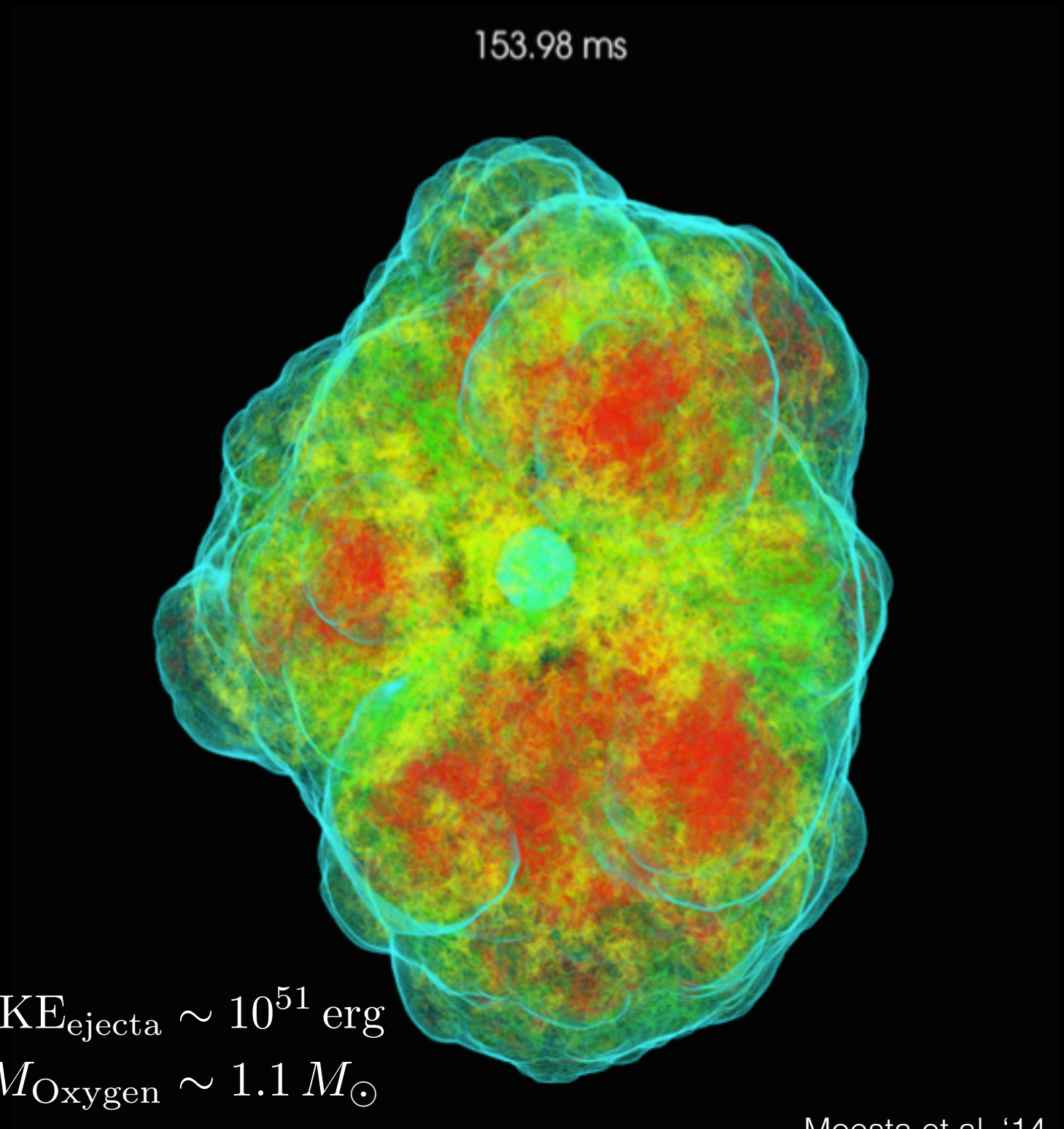
(building up a sub-grid model)

# Example: SNe

Resolution:

$$m_i < 10^{-6} M_{\odot}$$

Predict: Explosion



Sub-grid physics:

- (magneto) hydrodynamics
- nuclear Rx rates
- neutrino transfer

$$\text{KE}_{\text{ejecta}} \sim 10^{51} \text{ erg}$$

$$M_{\text{Oxygen}} \sim 1.1 M_{\odot}$$

...

Moesta et al. '14

# Example: SNe

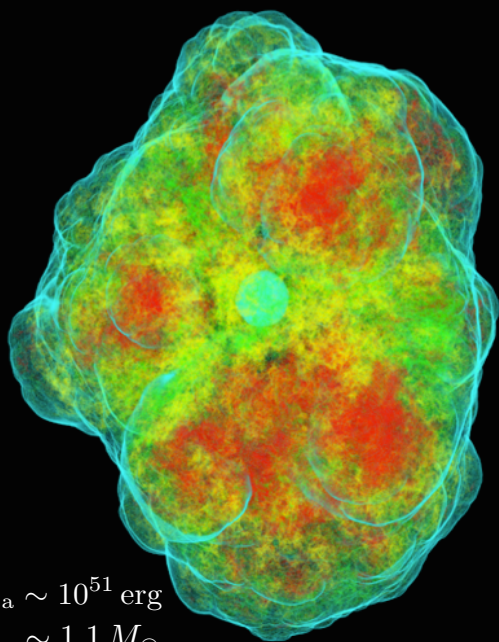
Resolution:

$$m_i \sim 1 - 100 M_{\odot}$$

Sub-grid physics:

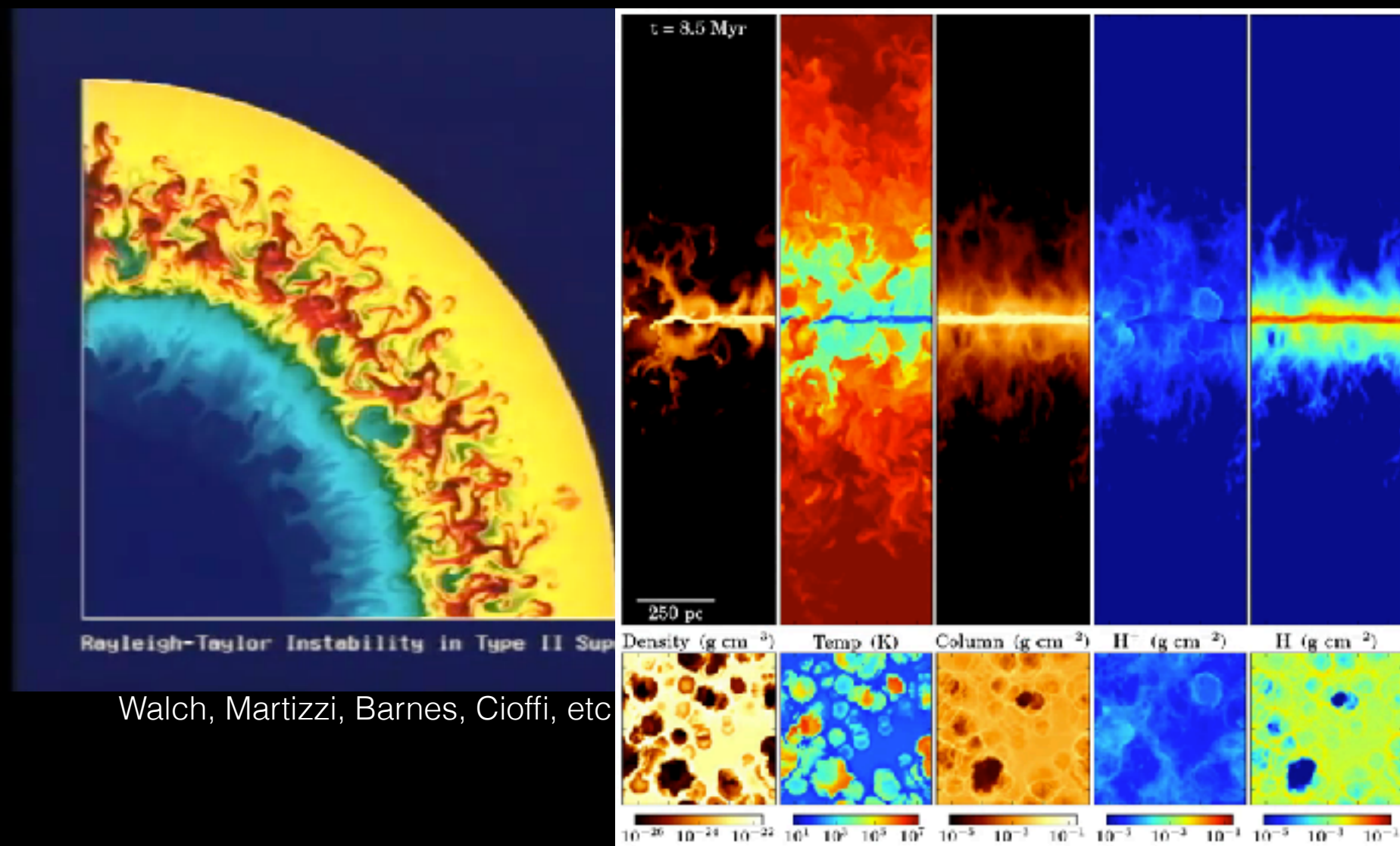
- SNe explosion
- ejecta energy, yields

153.98 ms



$$\begin{aligned} KE_{\text{ejecta}} &\sim 10^{51} \text{ erg} \\ M_{\text{Oxygen}} &\sim 1.1 M_{\odot} \\ &\dots \end{aligned}$$

Predict: Blastwave Evolution/ISM Interaction



End of energy-to-momentum (single SNe):

$$M_{\text{snowplow, final}} \sim 3000 M_{\odot}$$

Final momentum:

$$\langle M_s v_s \rangle_{\text{final, SNr}} \sim 10^{5.5} M_{\odot} \frac{\text{km}}{\text{s}}$$





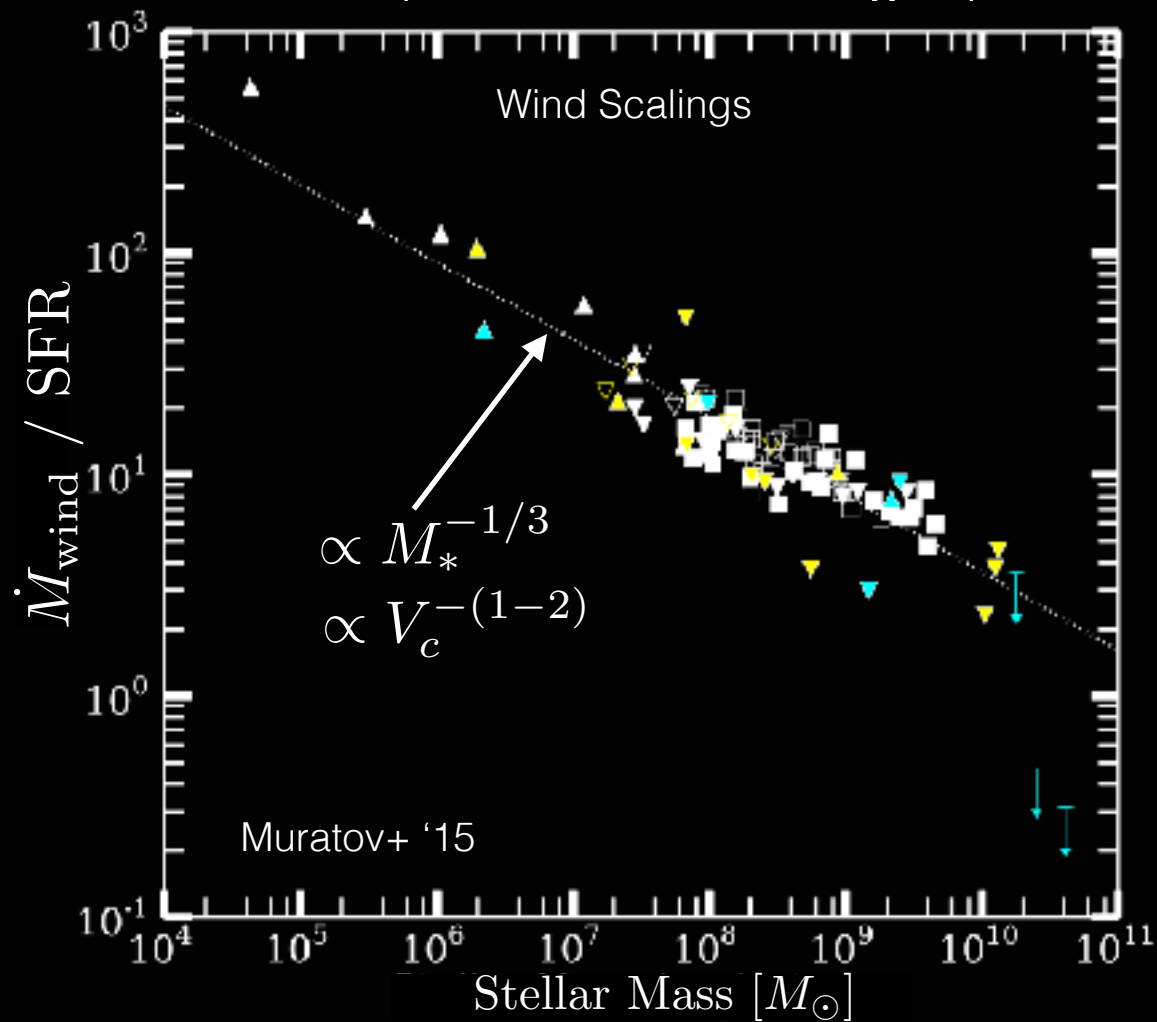
# Example: SNe

Resolution:

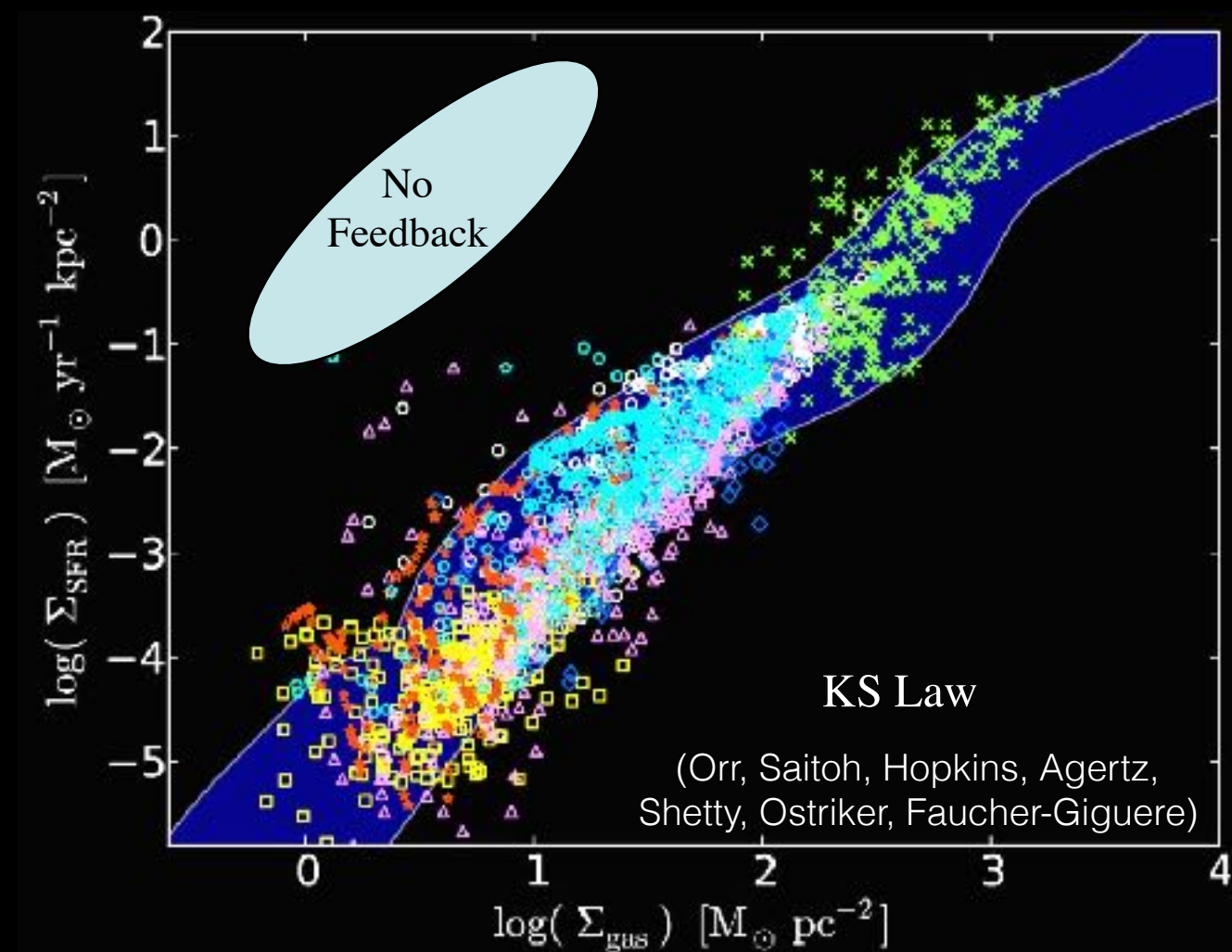
$$m_i \sim 10^{2-4} M_{\odot}$$

Sub-grid physics:

- single SNr evolution
- stellar evolution (rates)
- ~~SFR (dense molecular gas)~~



Predict: Overlap: super-bubbles & winds



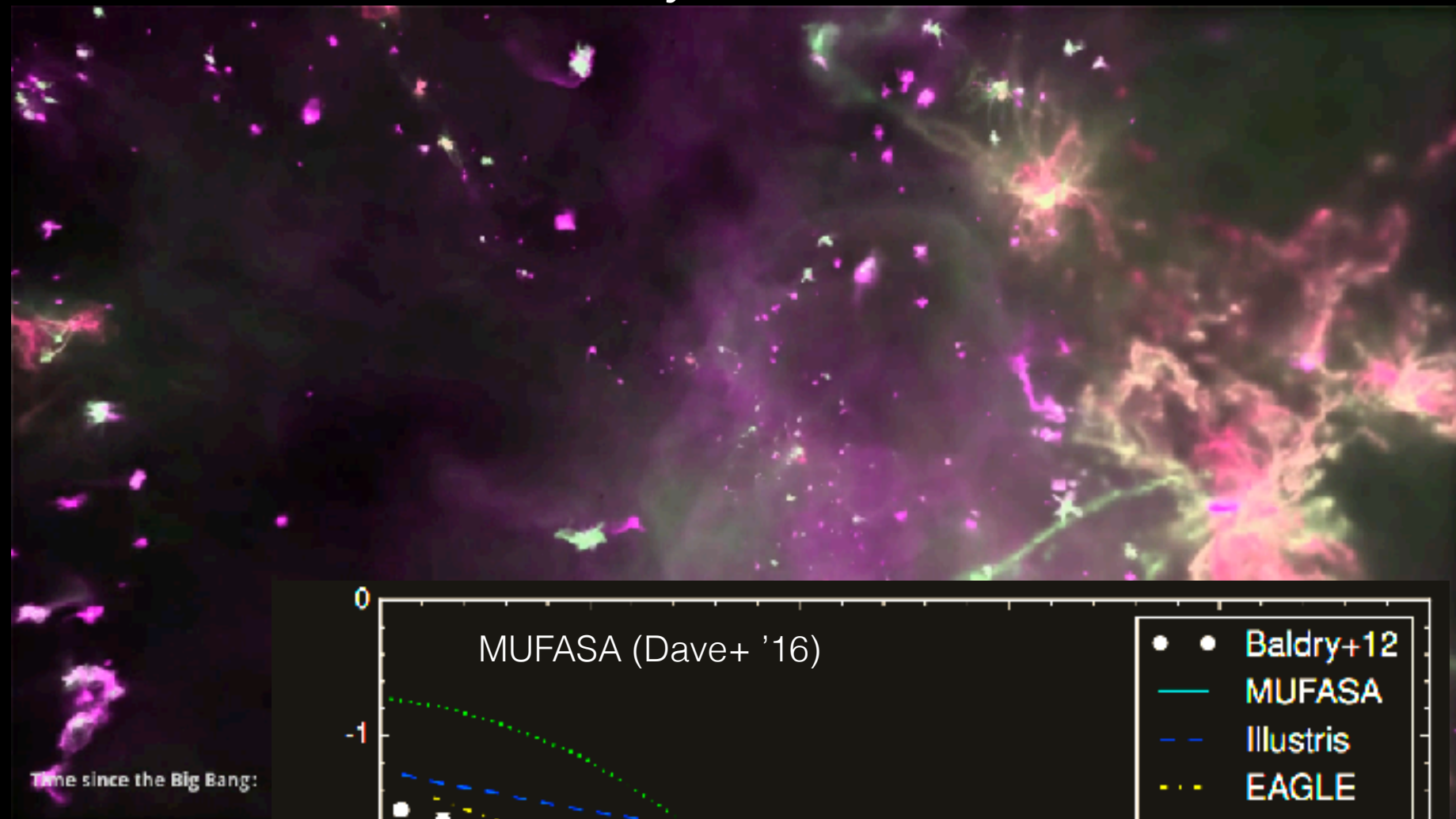
FIRE

# Example: SNe

Predict: Galaxy SFHs, IGM enrichment

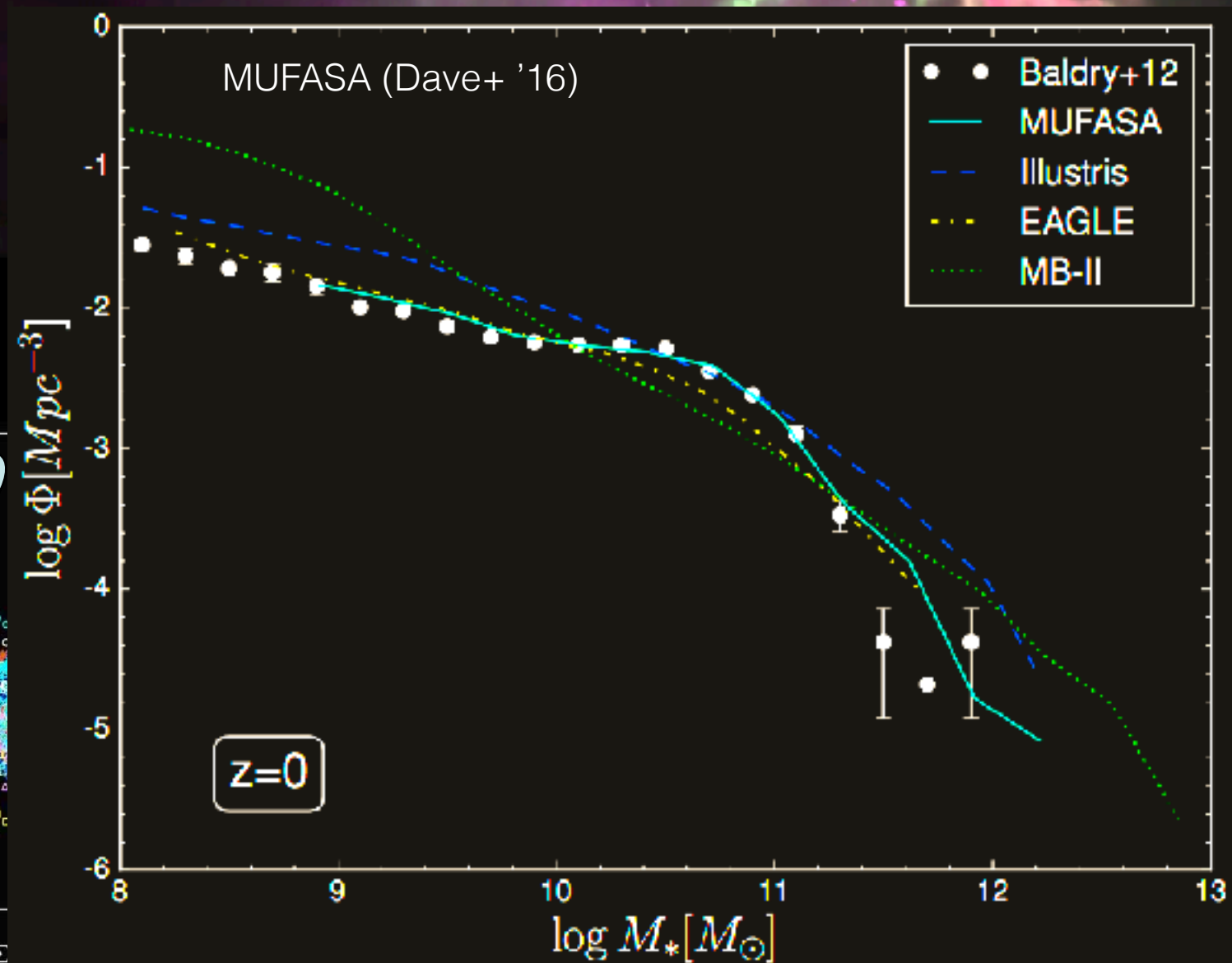
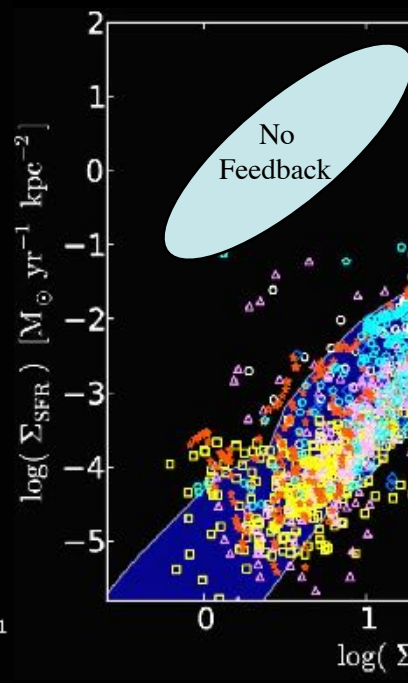
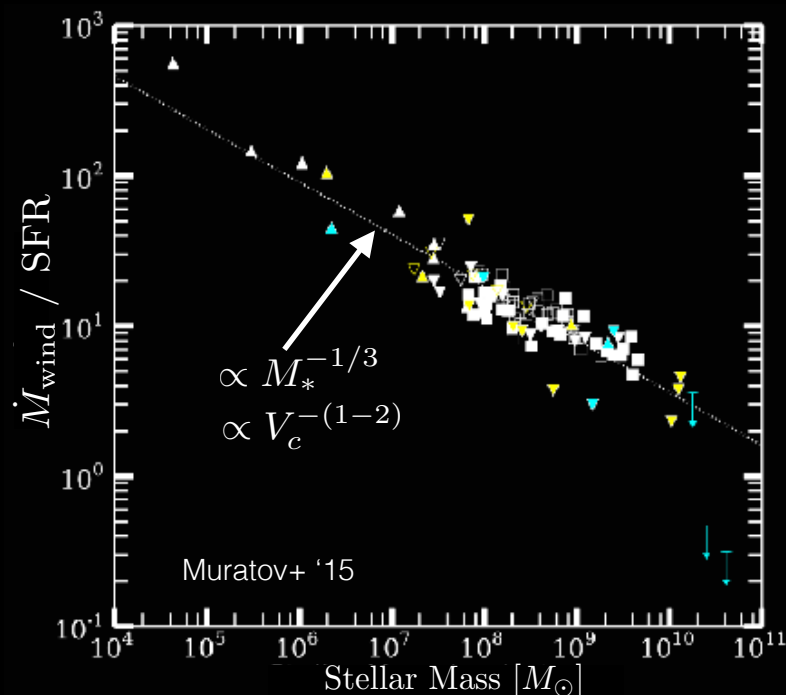
Resolution:

$$m_i \gtrsim 10^6 M_\odot$$



Sub-grid physics:

- SFR (kpc/low density gas)
- wind scalings (galaxy-scale)

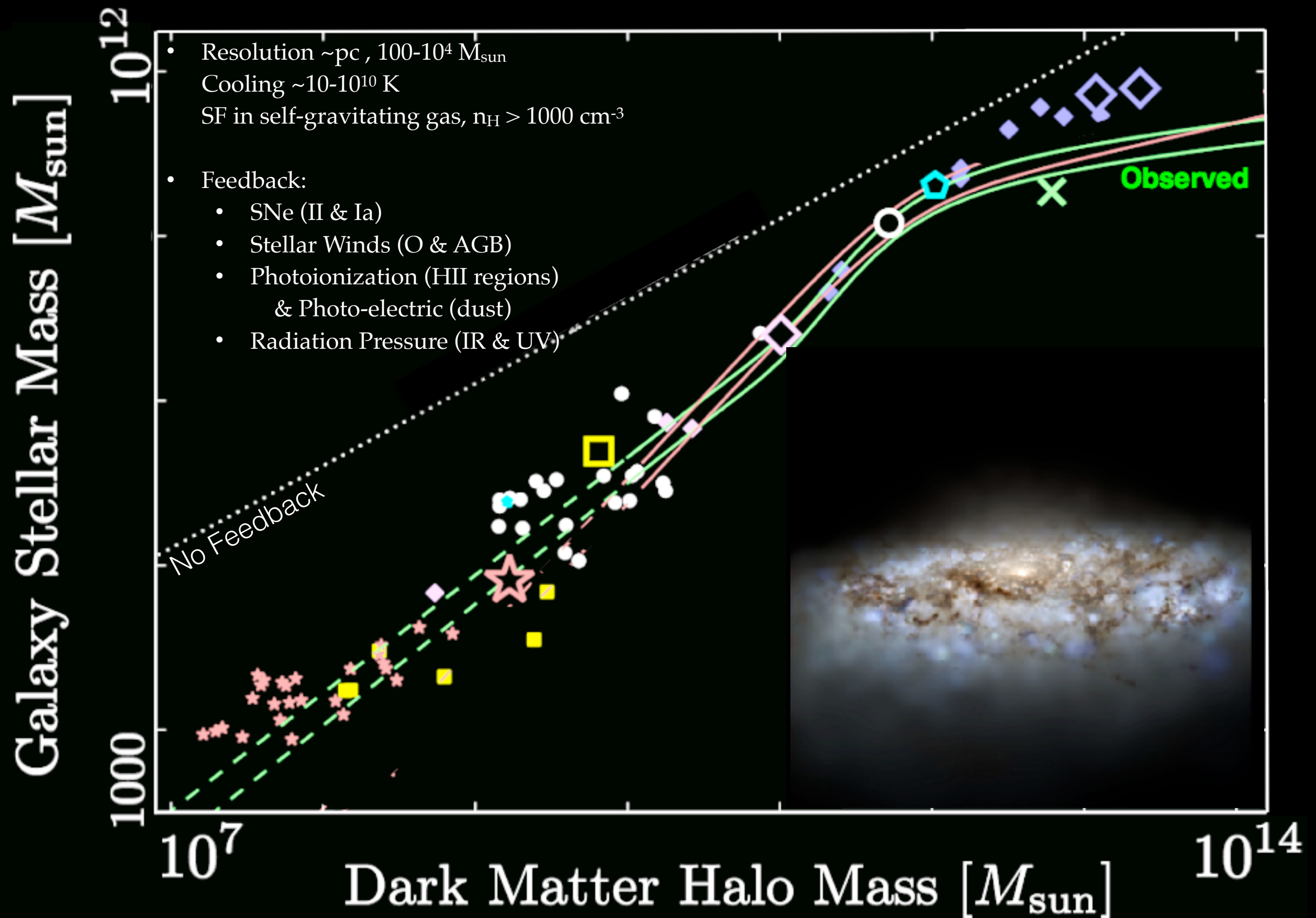


It Works!

PFH et al.

(arXiv:1311.2073)

THIS APPROACH IS PRODUCING REALISTIC GALAXIES







It Works!

- Resolution  $\sim \text{pc}$  ,  $100\text{-}10^4 M_{\text{sun}}$   
Cooling  $\sim 10\text{-}10^{10} \text{ K}$   
SF in self-gravitating gas,  $n_{\text{H}} > 1000$
- Feedback:
  - SNe (II & Ia)
  - Stellar Winds (O & AGB)
  - Photoionization (HII regions)  
& Photo-electric (dust)
  - Radiation Pressure (IR & UV)



# What Matters?

(depends 100% on *what you care about predicting*)

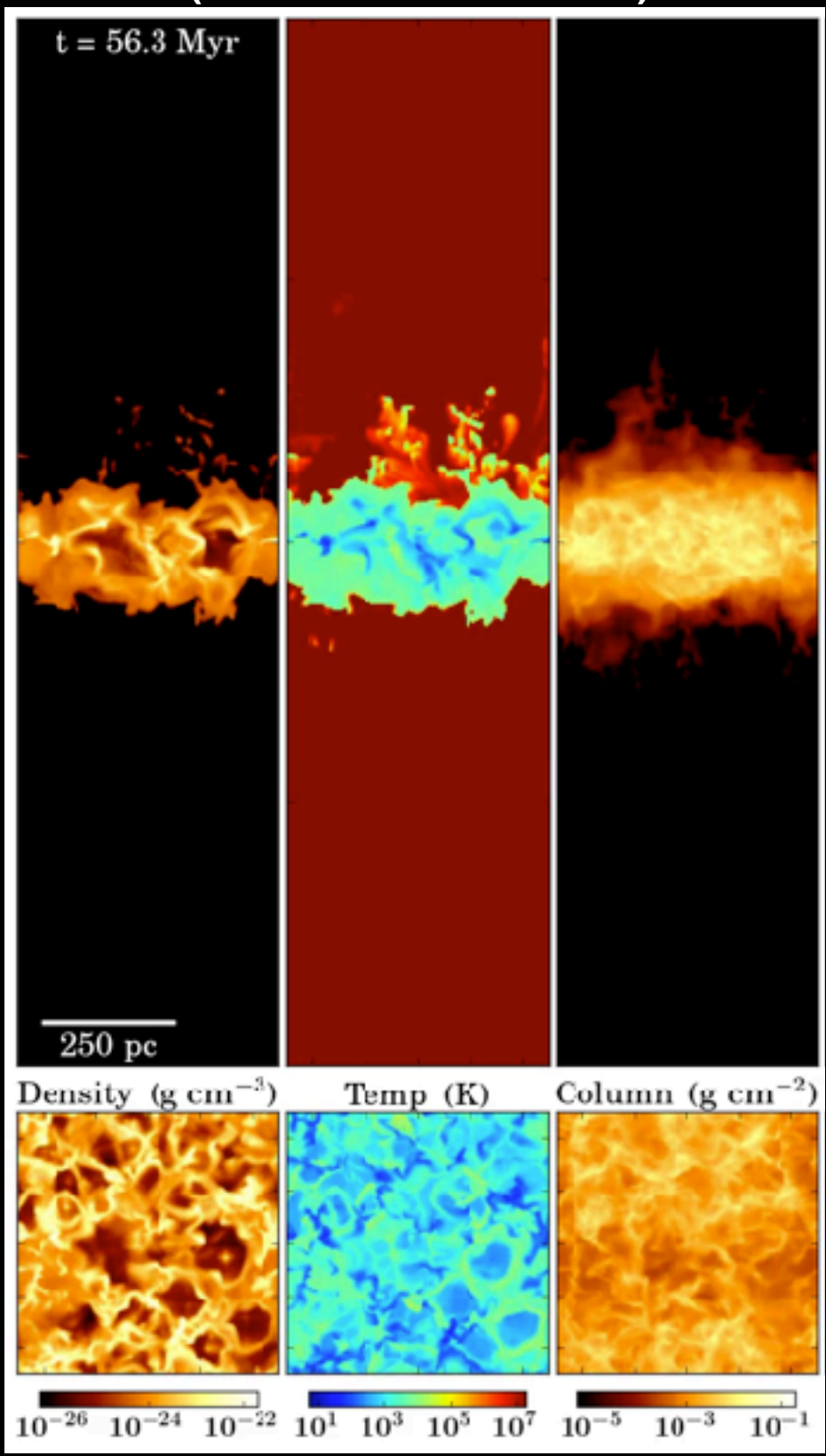


# Doing the “sub-grid” right can matter

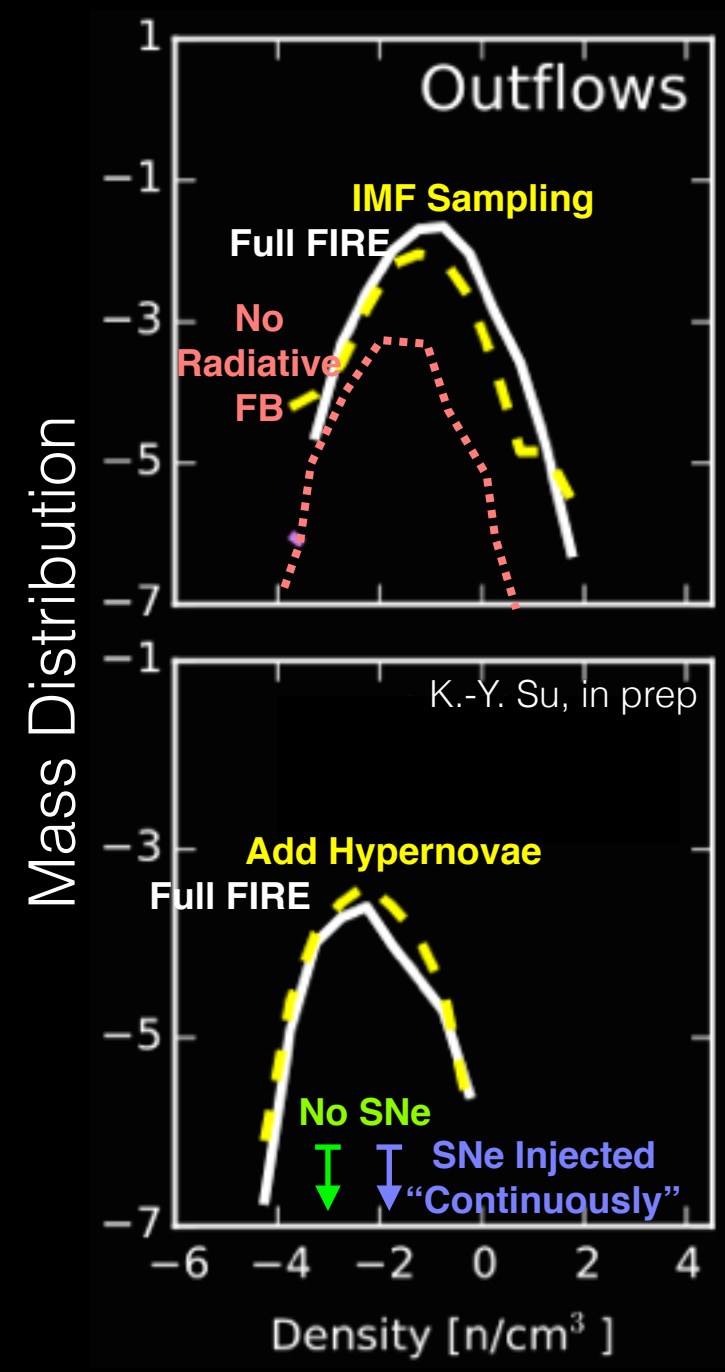
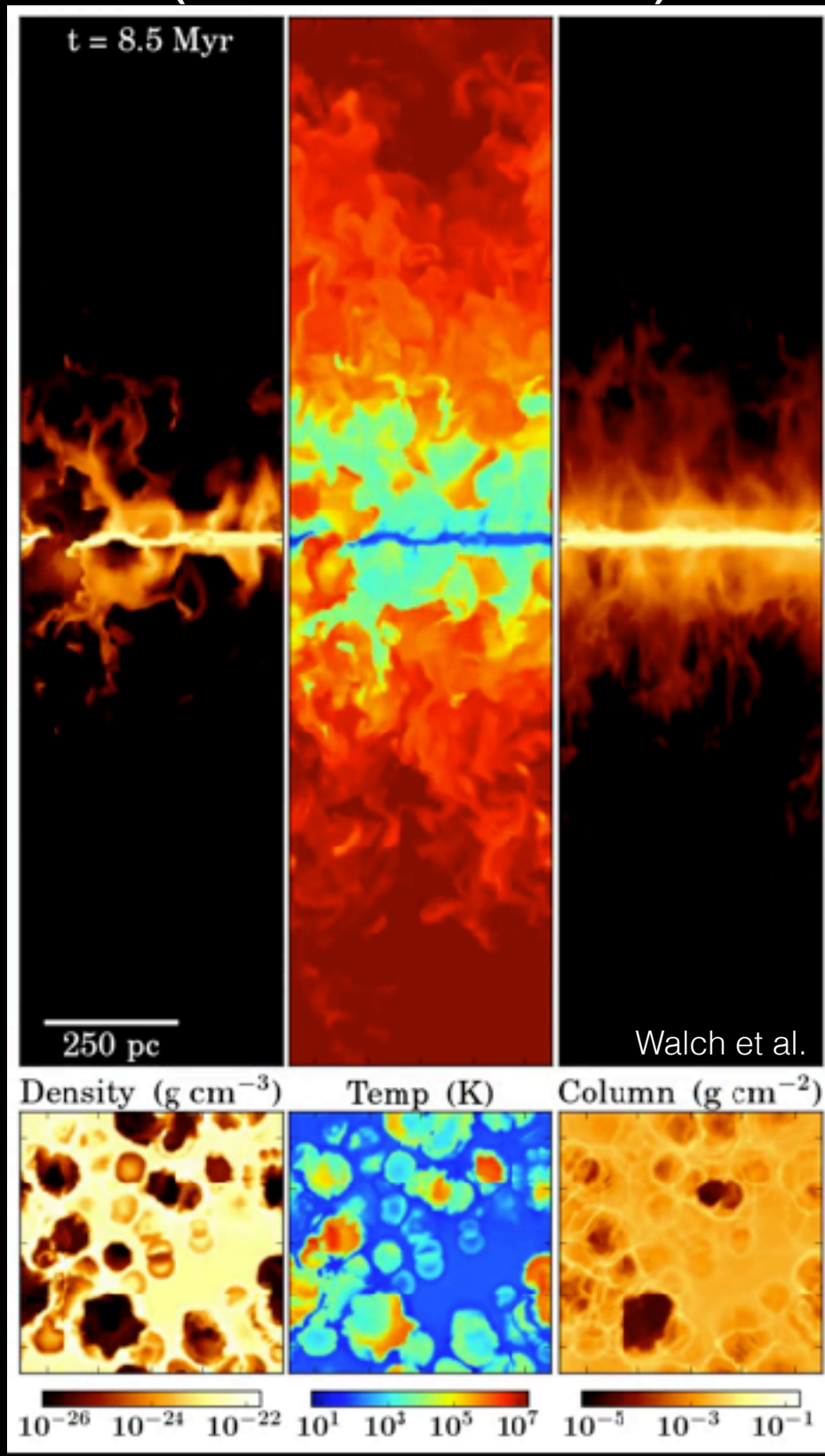
IF RESOLVE DENSE GAS, NEED PHYSICS FOR IT!

Murray+, Martizzi+,  
Walch+, Barnes+  
Hopkins+, Hayward+,  
Shetty+, Hennebelle+

SNe Explode in Density Peaks  
(no radiative feedback)



SNe Clustered & Off-Peak  
(with radiative feedback)

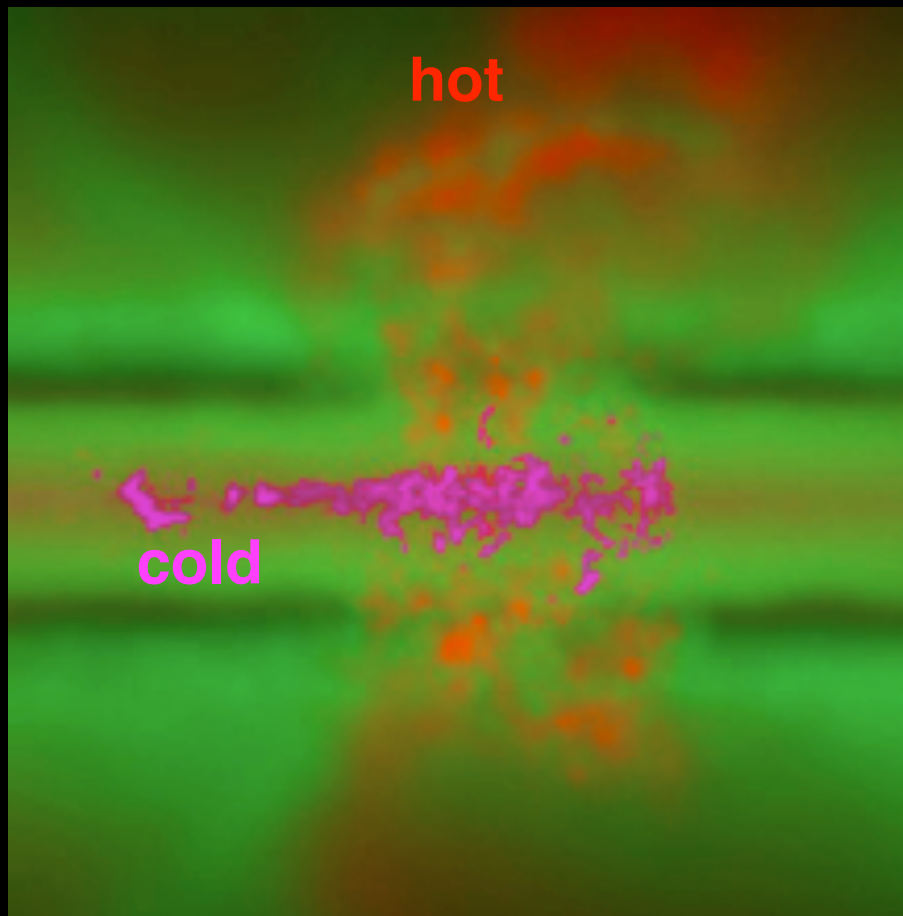


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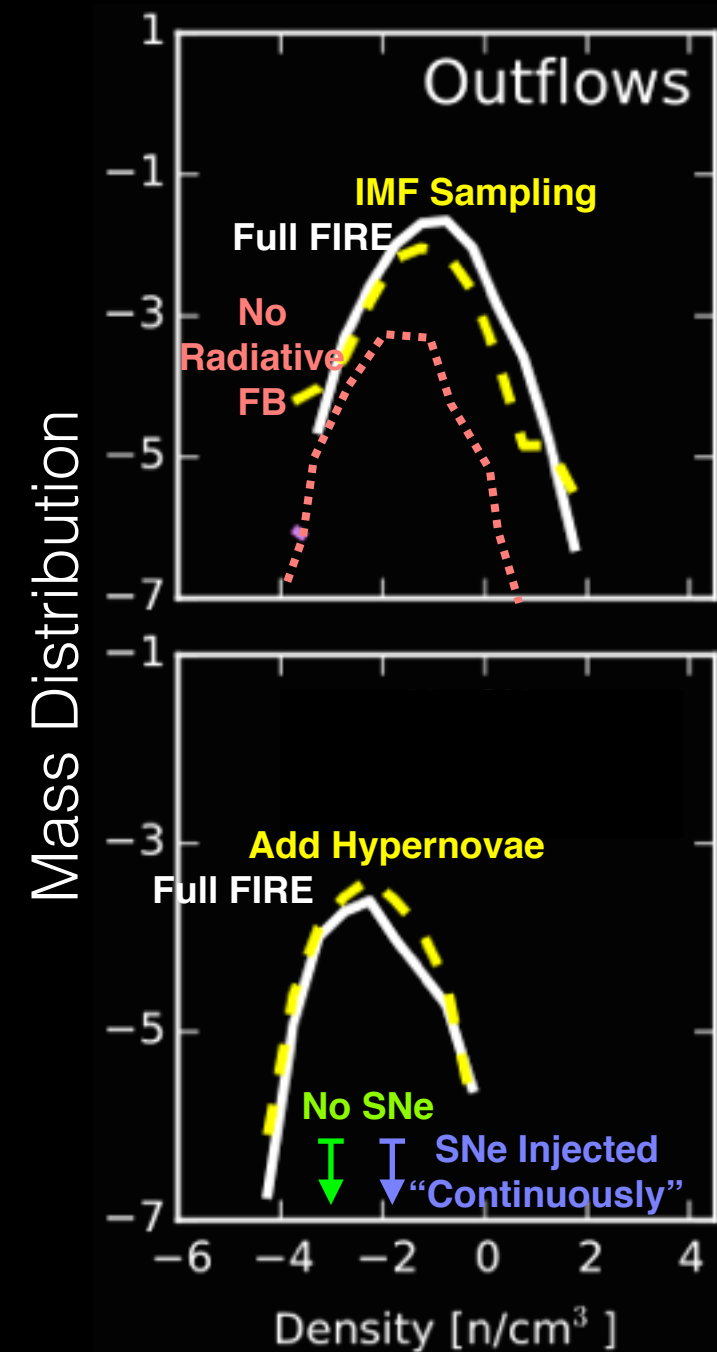
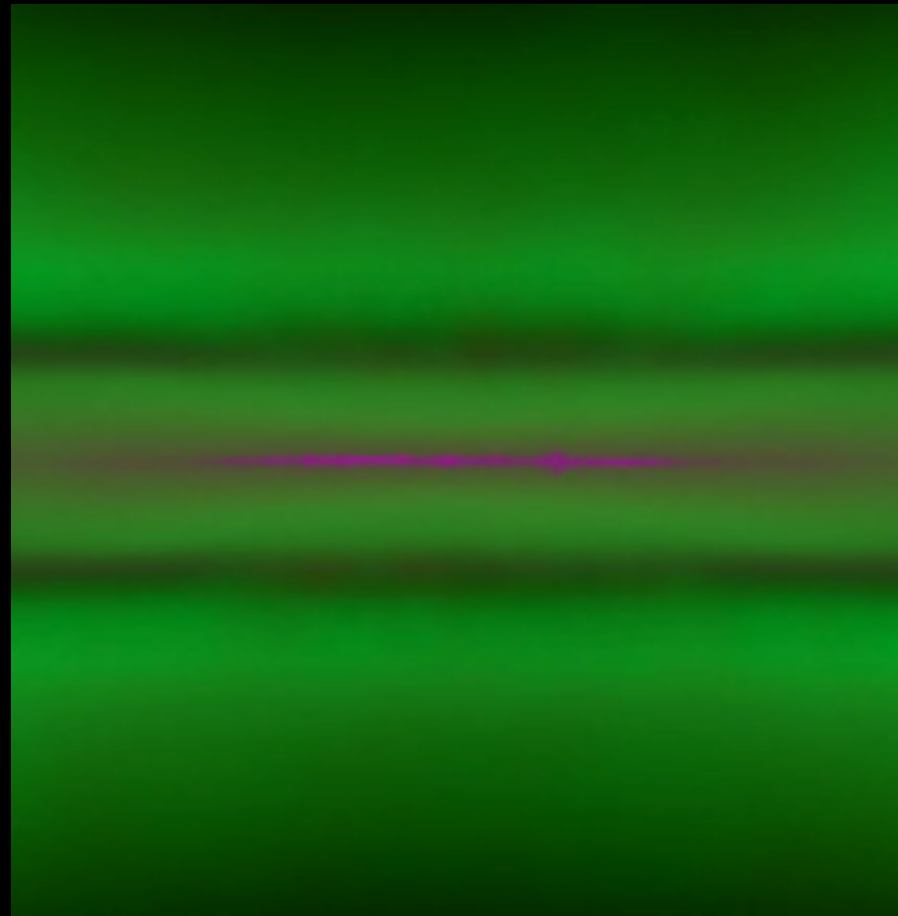
IF RESOLVE BUBBLES, NEED PHYSICS FOR IT!

Klessen+, Ostriker+  
Hopkins+  
K.-Y. Su, in prep

Treat each SNe explicitly  
following resolved explosion



Continuously dump  
thermal energy  $\sim$  SFR





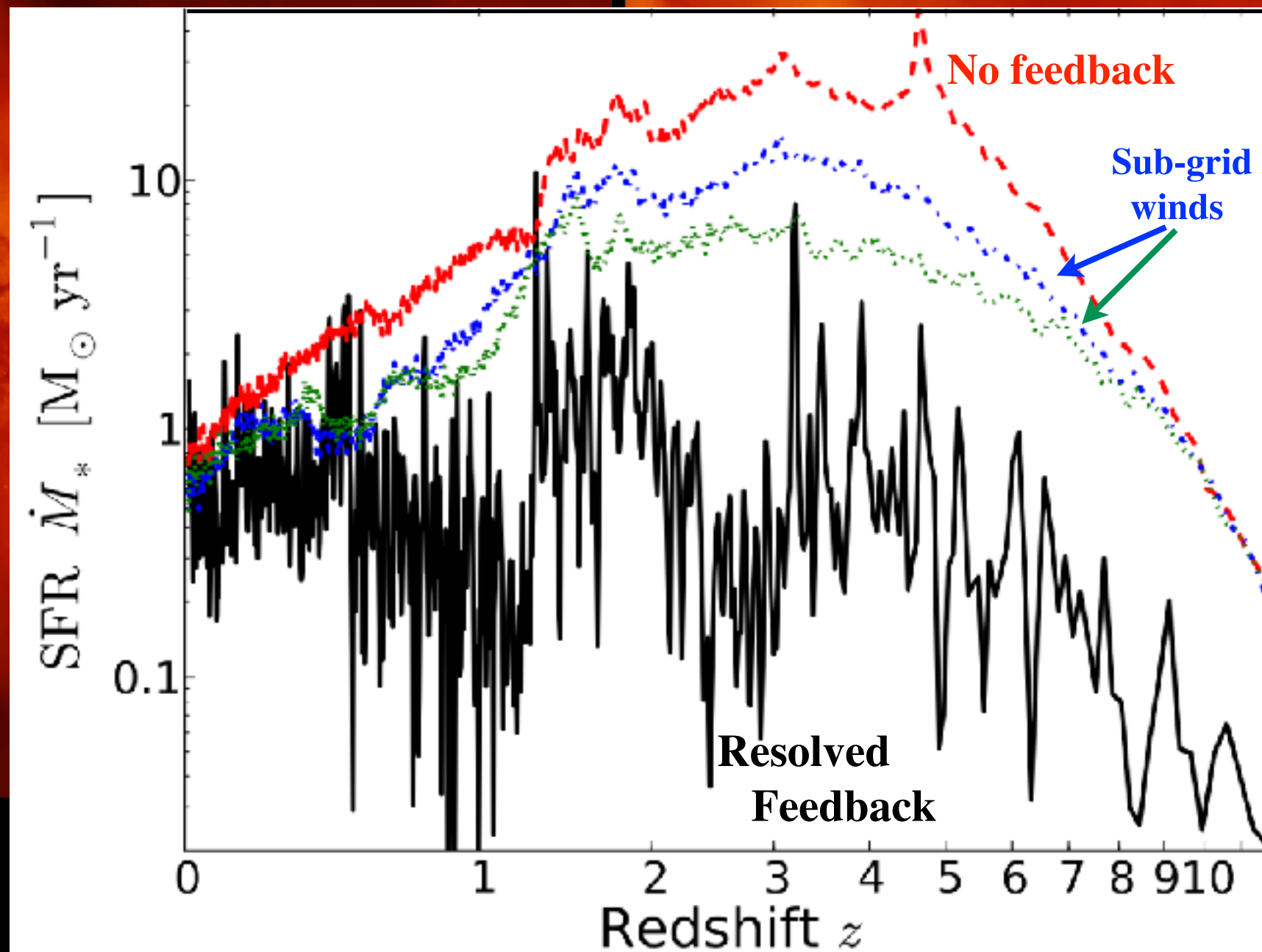
# Doing the “sub-grid” right can matter

## DANGERS OF ONLY FITTING MASSES

### Proto-Milky Way: Gas Temperature:

Simple Sub-Grid ( $\dot{M}_{\text{wind}} = \eta \dot{M}_*$ )

Following Full Feedback





# Resolution: Needs to Match Your Physics!

DIFFERENT PREDICTIONS REQUIRE DIFFERENT RESOLUTION

Fragmentation / GMCs / Dense Gas:

$$m_i \lesssim 10^5 M_\odot \ll M_{\text{Toomre}}$$

$$\epsilon_{\text{grav}}^{\text{min}} \ll 100 \text{ pc [guaranteed if adaptive]}$$

Super-bubbles / overlaps / chimneys:

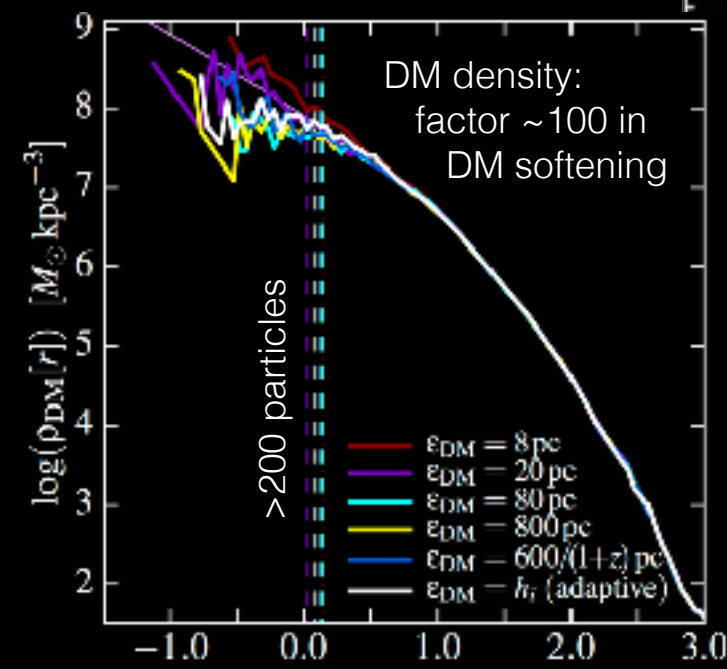
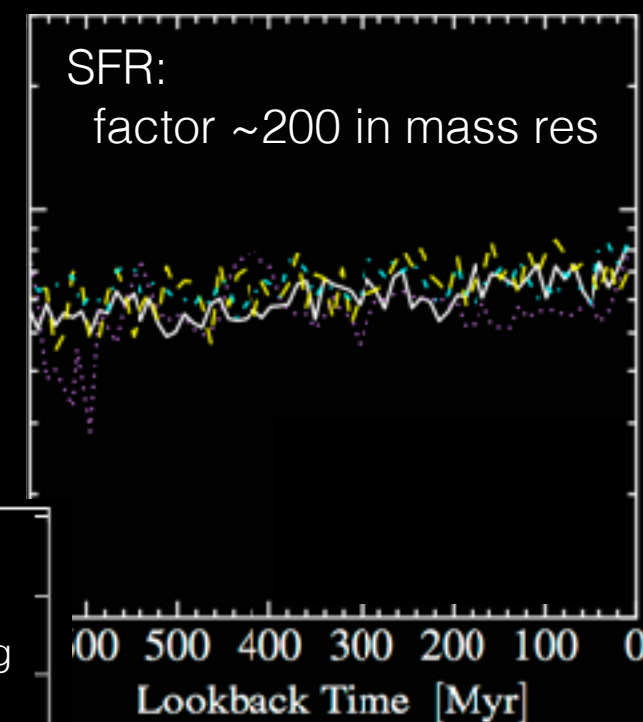
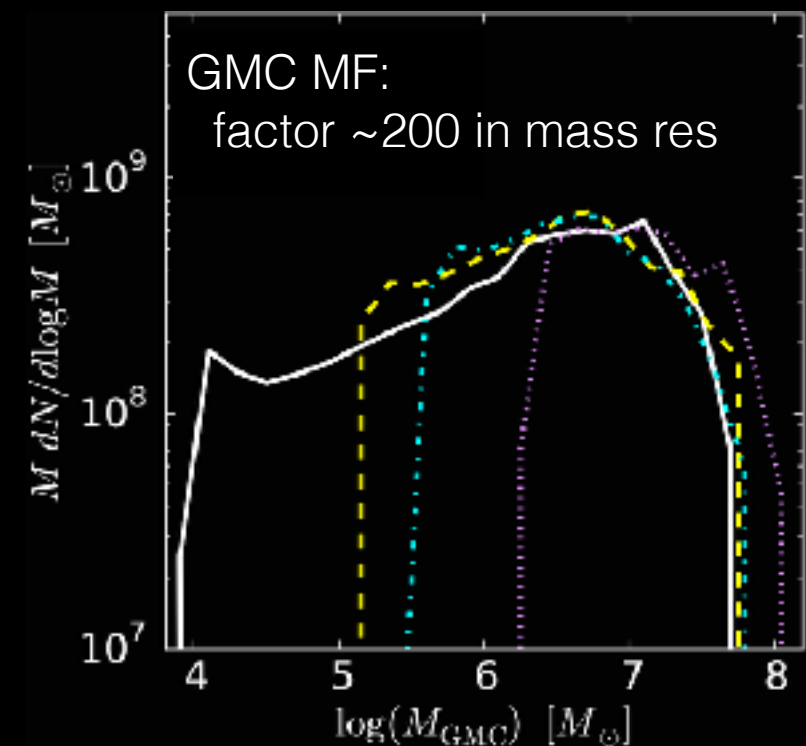
$$m_i \lesssim 10^5 M_\odot \ll M_{\text{Bubble}}$$

Individual SNe (no sub-grid SNe momentum):

$$m_i \lesssim 10^3 M_\odot \ll M_{\text{Cooling}}$$

Dwarf galaxy “bursty-ness”:

$$m_i \lesssim 10^{-6} M_{\text{halo}}$$

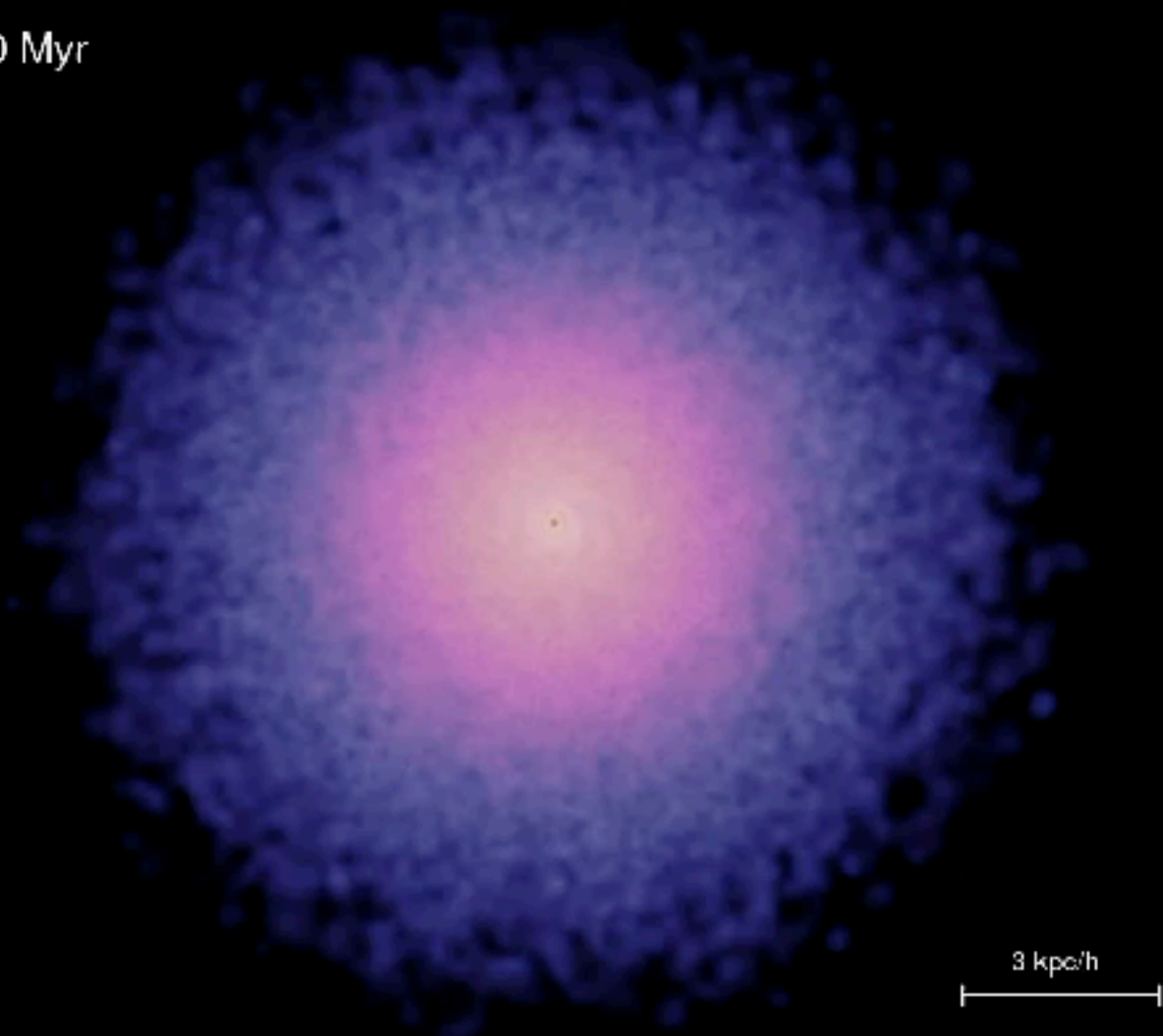


# Doing the “sub-grid” right can matter

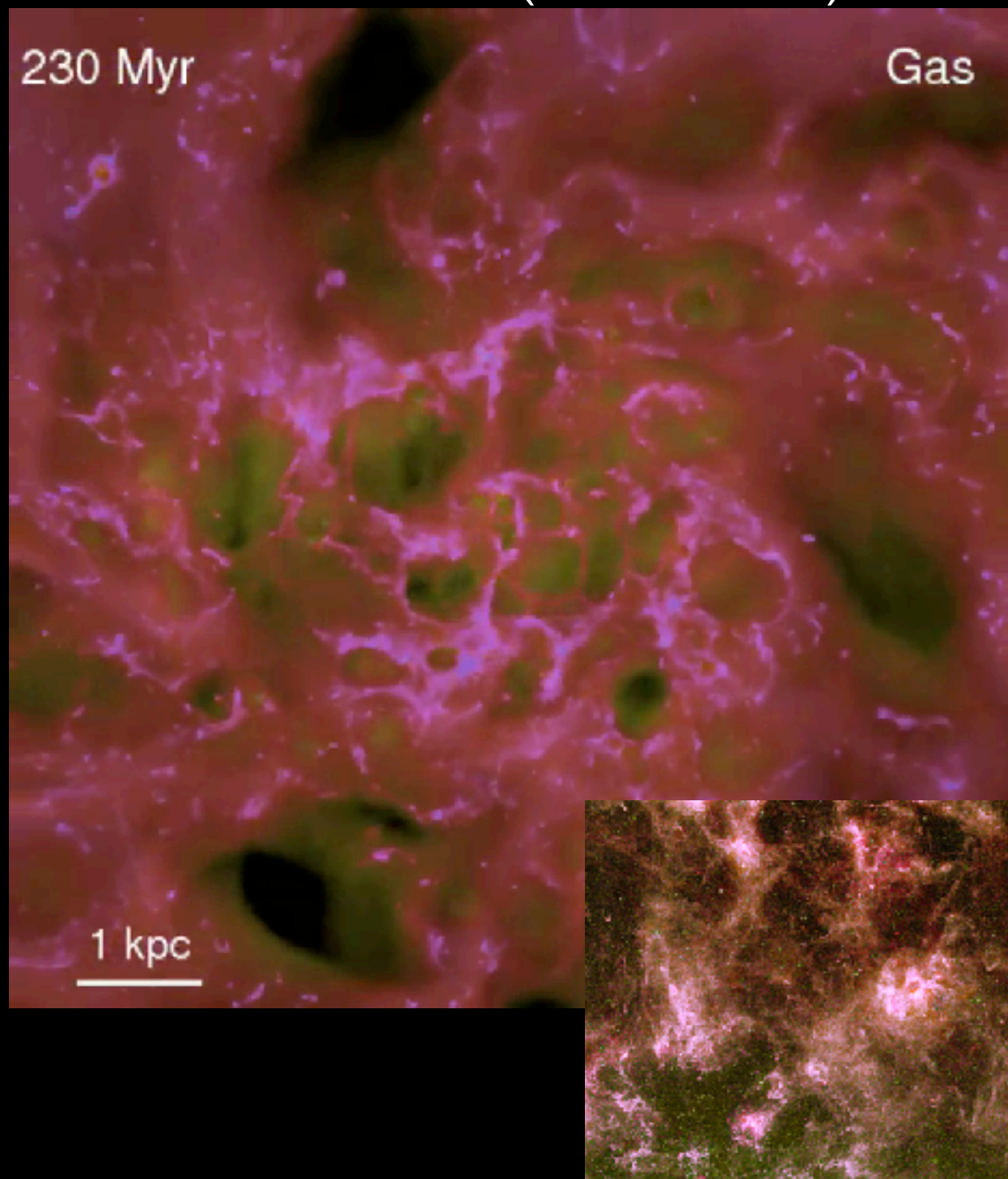
## NEED PHYSICS TO PUSH BEYOND YOUR SUB-GRID SCALE

### Sub-Grid ISM (Illustris, Eagle)

$T = 0$  Myr



### Resolved ISM (FIRE, SILCC)



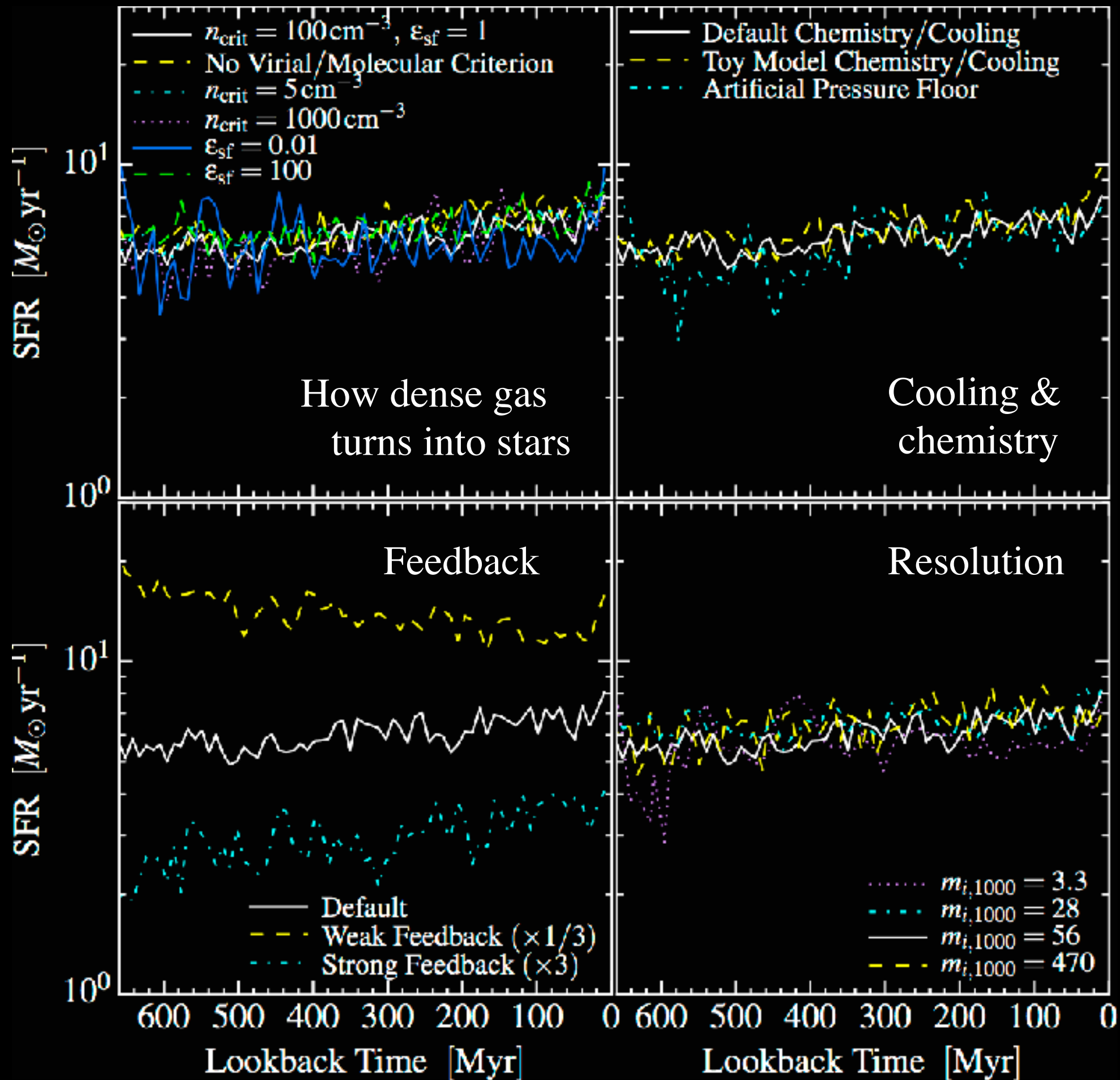
# What Doesn't Matter?

(depends 100% on *what you care about predicting*)

# (Galactic) Star Formation Rates are *INDEPENDENT* of how stars form!



Matt Orr (in prep)  
Saitoh+ 11  
Hopkins+ 11,12,14  
Agertz+14

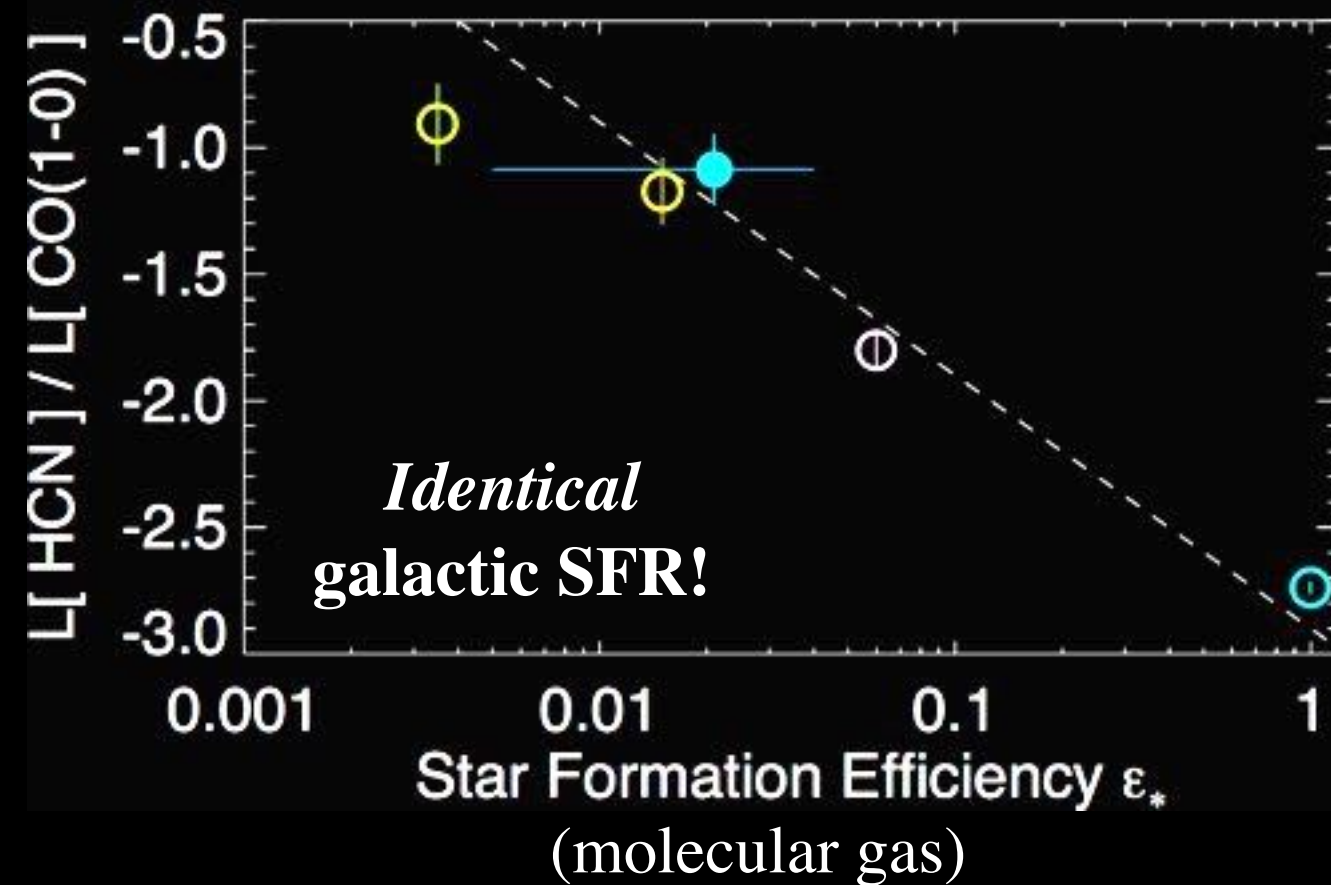
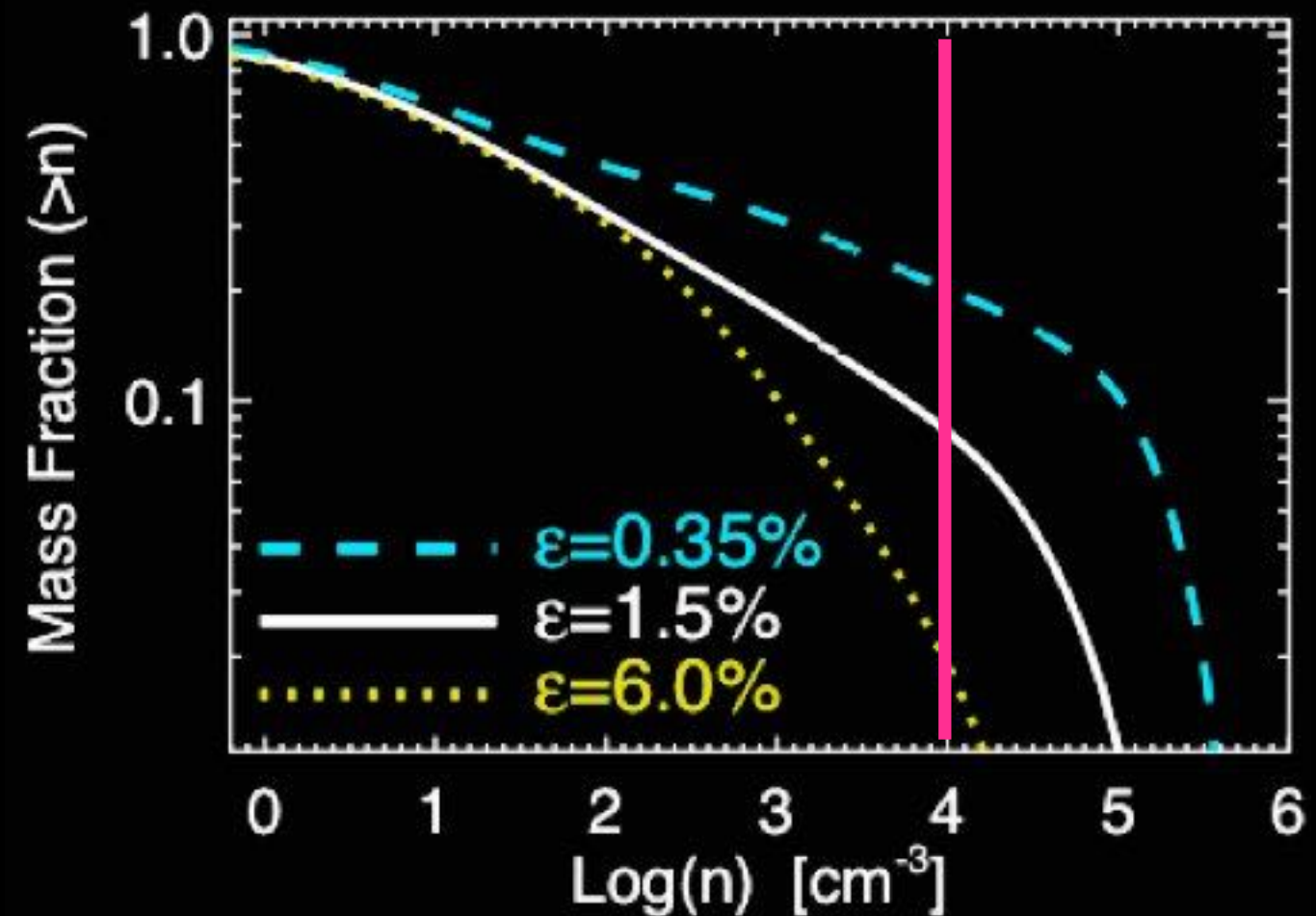




# Dense Gas *Does* Change

## SELF-REGULATES TO “NEEDED” SFR LEVEL

Efficiency (SF per  $t_{\text{dyn}}$ ) in *dense* gas



Matt Orr (in prep)  
Hopkins+ 11,12,14  
Shetty+ 14  
Narayanan+ 13

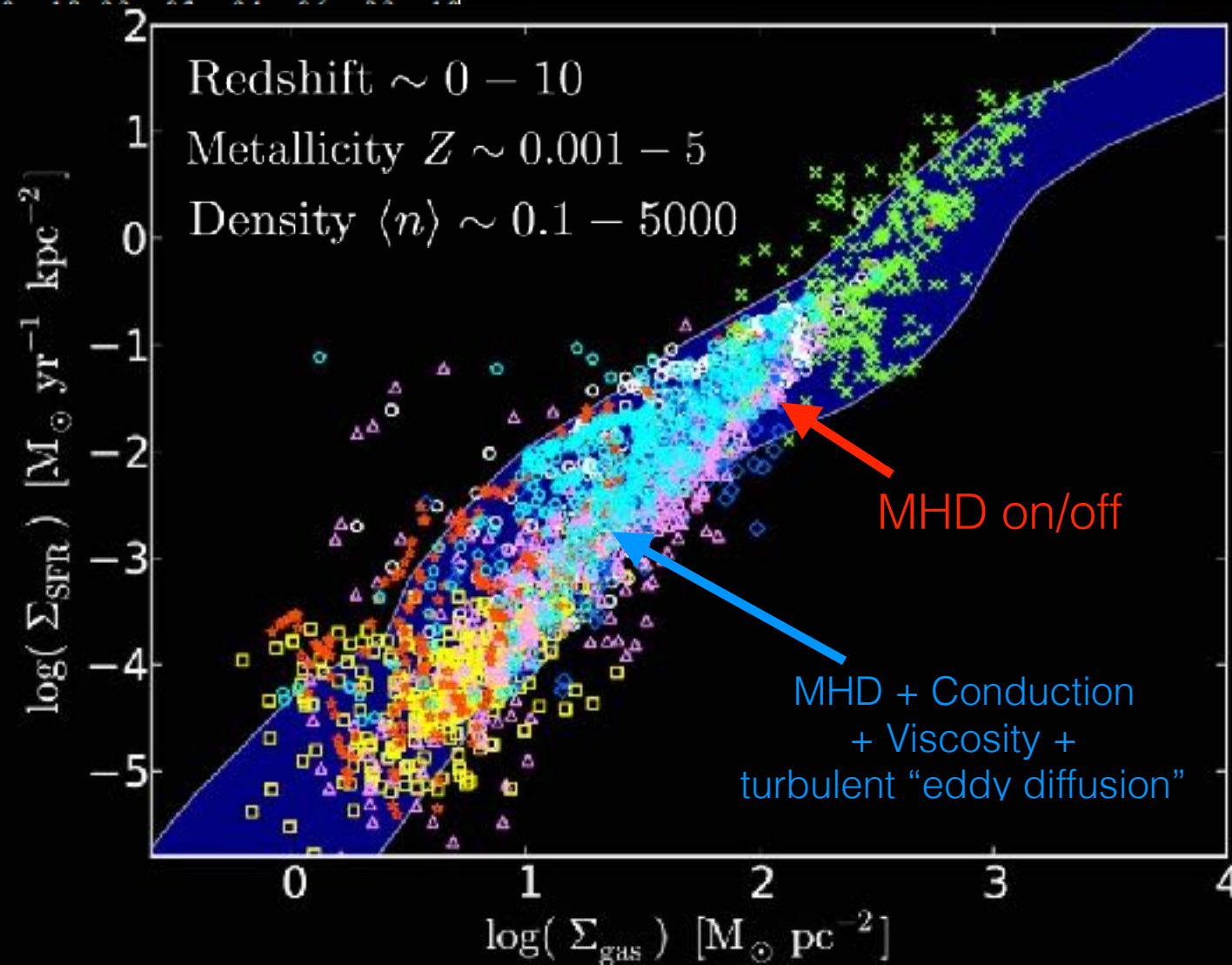
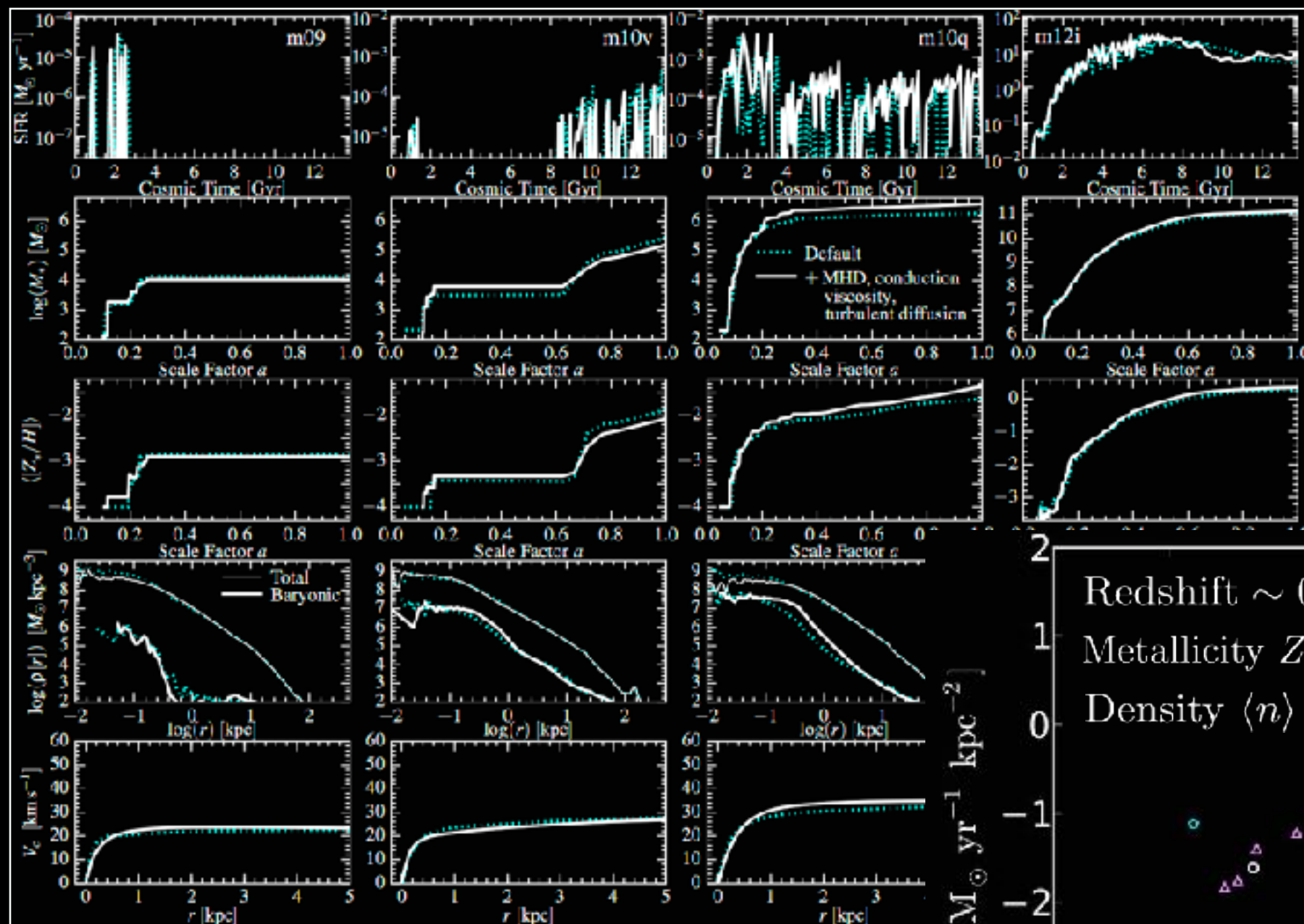






# Galaxy SFRs (sub-L\*) independent of MHD+diffusion

MAY NOT APPLY TO COOLING IN HOT HALOS!



# Numerical Methods

(aka: why did we switch from SPH?)

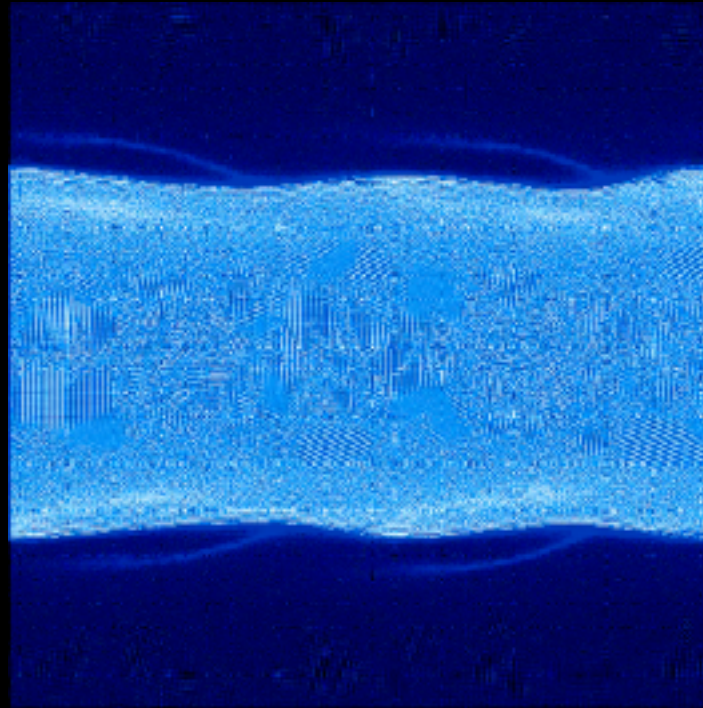


# Smoothed-Particle Hydrodynamics (SPH)

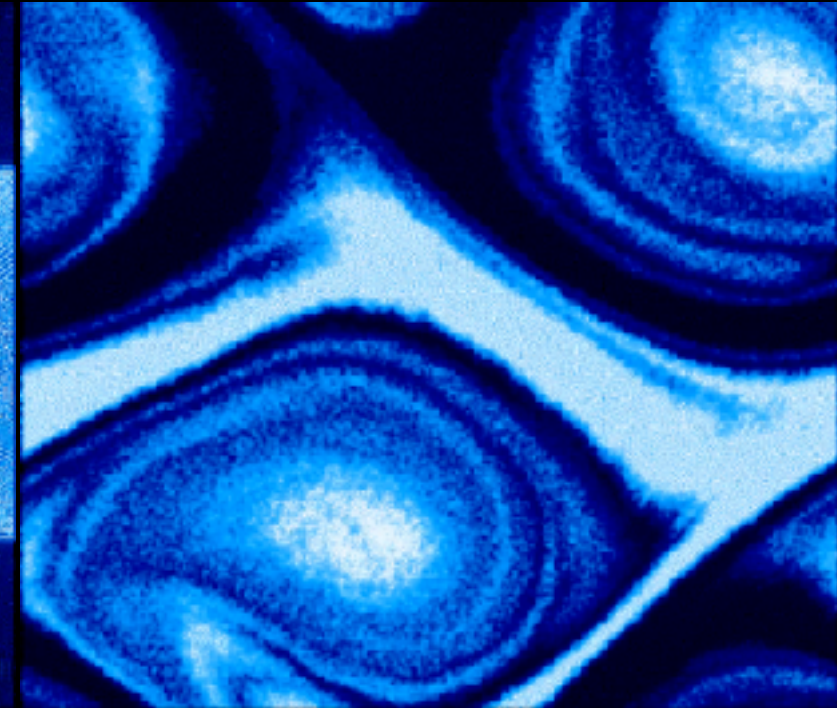
CHALLENGE: POPULAR METHODS HAVE PROBLEMS

- Lagrangian, adaptive, simple, conservative
- Artificial diffusion terms:
  - excess diffusion, viscosity

Kelvin-Helmholtz Instabilities

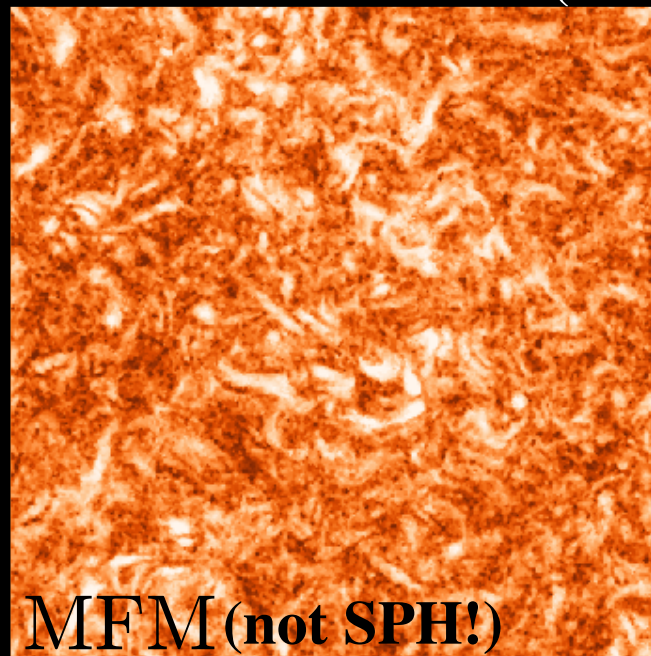


“old” SPH  
(Springel 02)

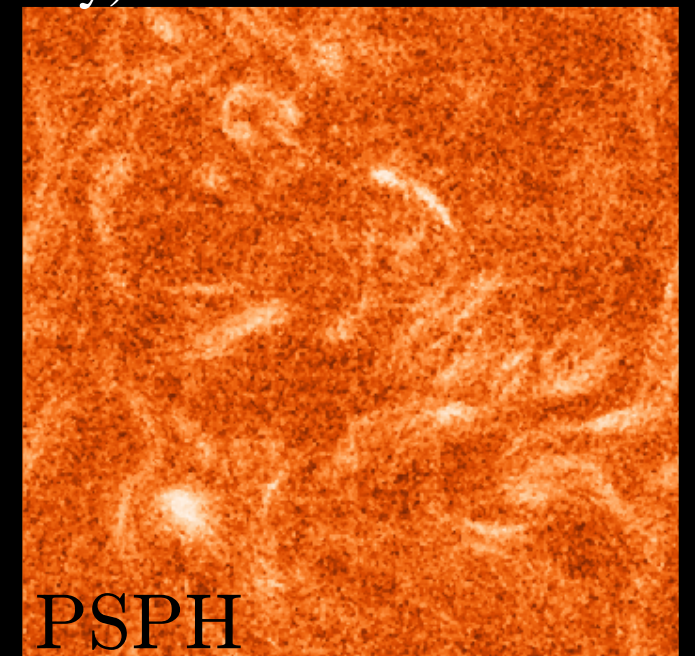


“new” SPH (PSPH)  
(Hopkins '13):  $\gg 100$  neighbors

Sub-sonic turbulence (vorticity)



MFM (not SPH!)



PSPH

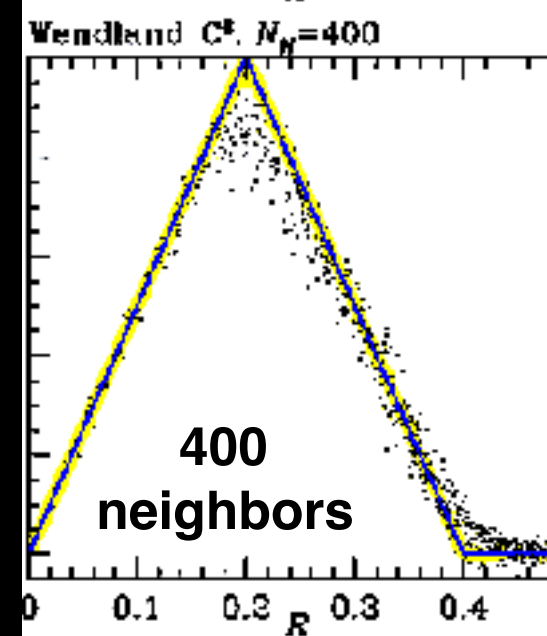
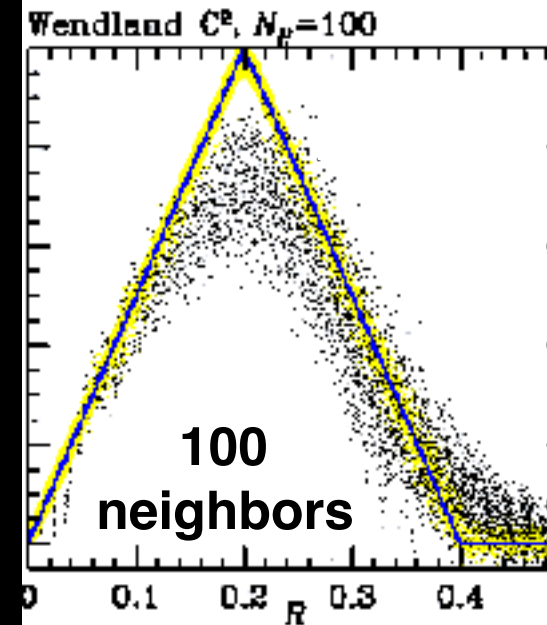
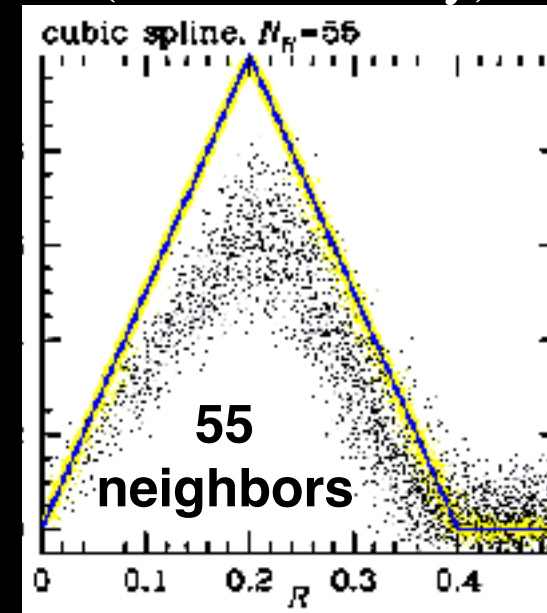


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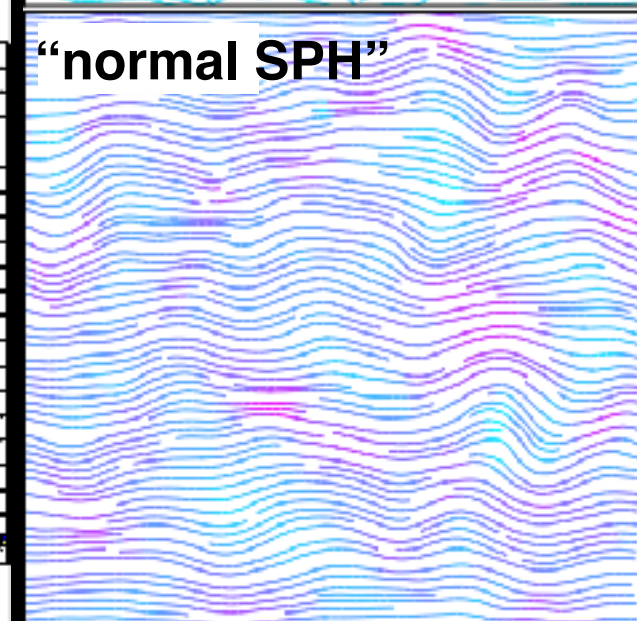
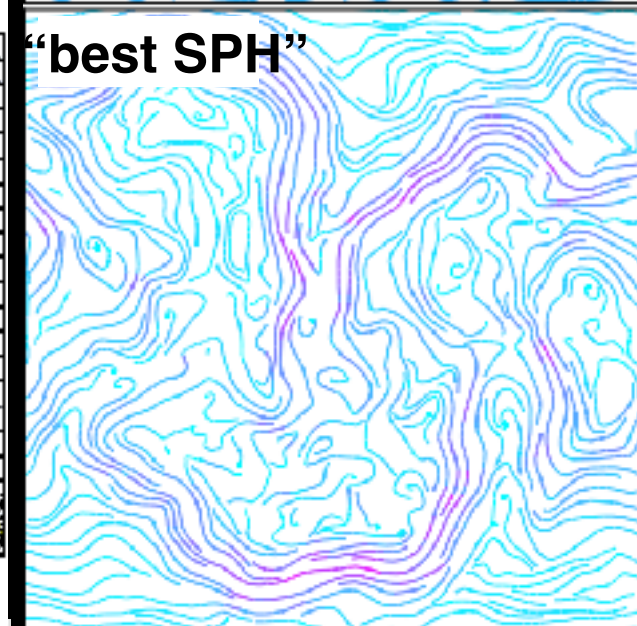
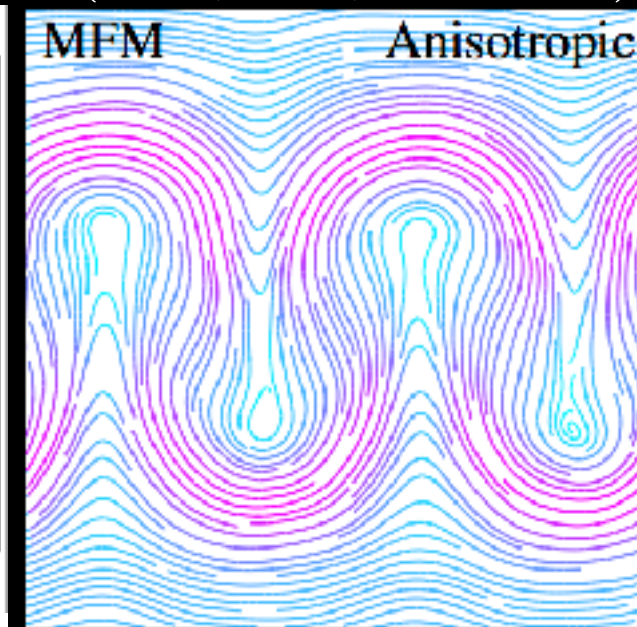
CHALLENGE: POPULAR METHODS HAVE PROBLEMS

- Fundamental low-order errors:
  - converges slowly:
    - “beat down” by increasing kernel size, but this is *not efficient*!

Gresho vortex  
(Dehnen & Aly)

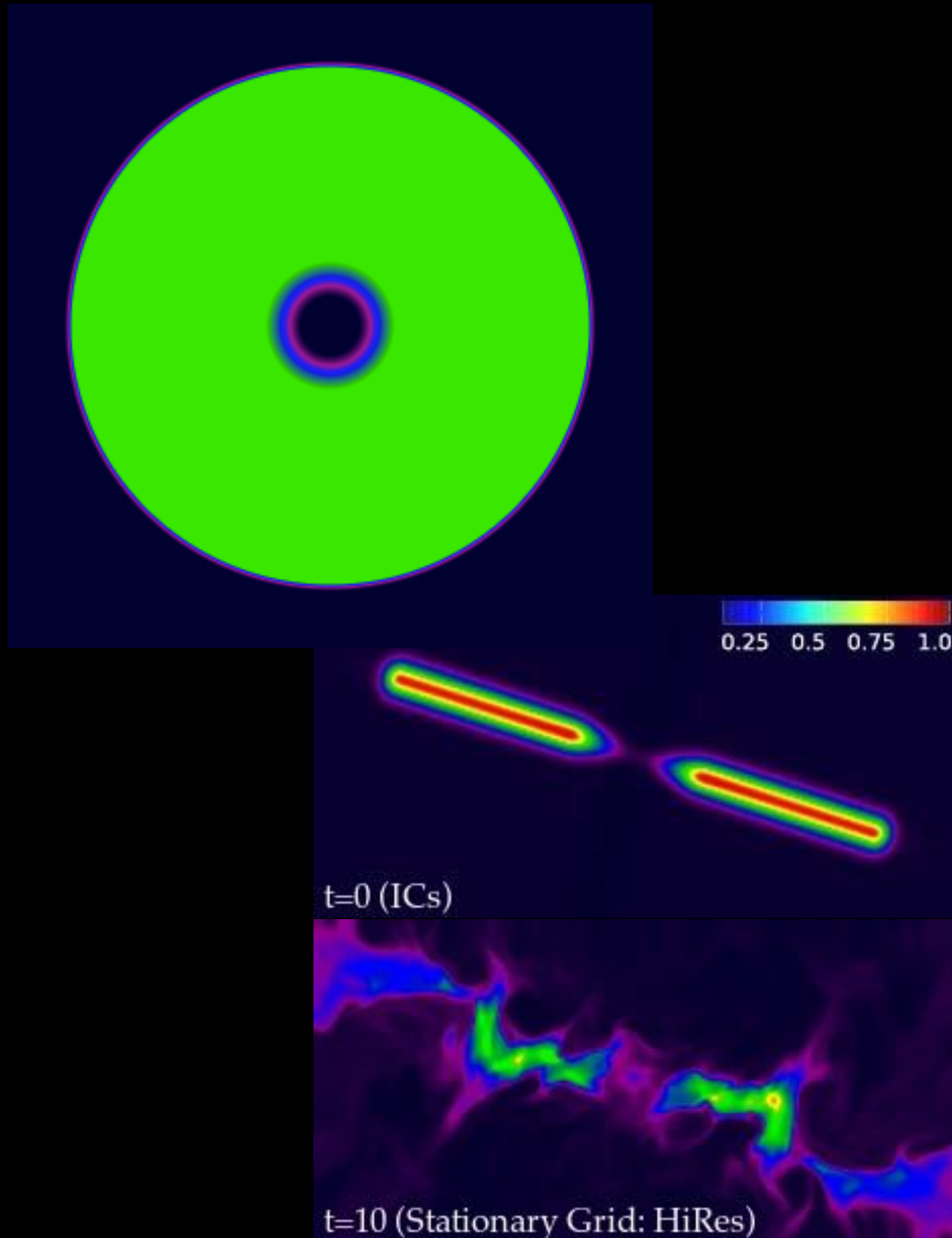


Anisotropic Conduction  
(MTI, HBI, Hall MRI)



# Adaptive Mesh Refinement (AMR)

CHALLENGE: POPULAR METHODS HAVE PROBLEMS



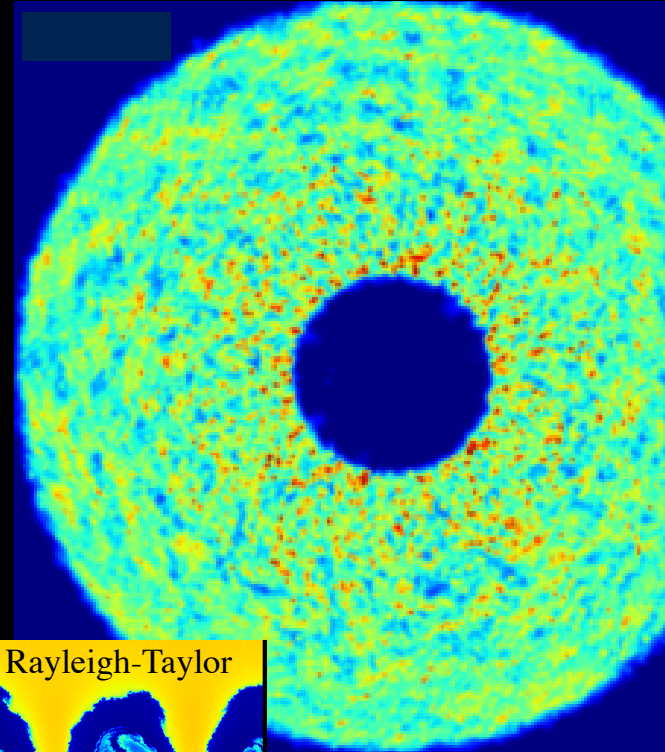
- Eulerian, well-studied, high-order
- Excessive mixing/diffusion when fluid moves over cells
- Geometric effects:
  - carbuncle instability (shocks)
  - loss of angular momentum
  - grid-alignment (disks)
- Also “beaten down” with resolution, but *expensive*
  - Hahn '10:  $\gg 512^2$  resolution to avoid grid-alignment



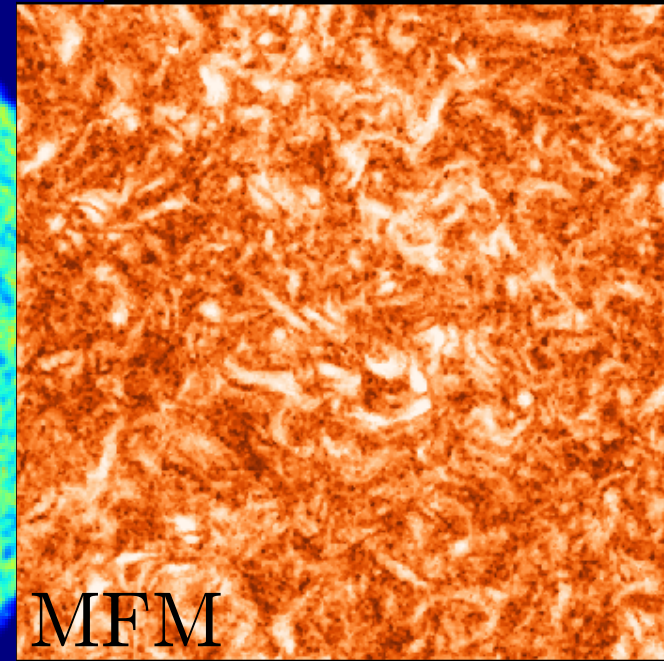
# New Methods Combine (some) Advantages of Both: (BUT REMAIN LESS WELL-TESTED)

- Moving-meshes (AREPO), meshless finite-volume (GIZMO), high-order ALE methods
- Move with flow, no preferred geometry, but also accurate, high-order, and shock-capturing
- Grid noise is more severe

GIZMO: disk after 100 orbits

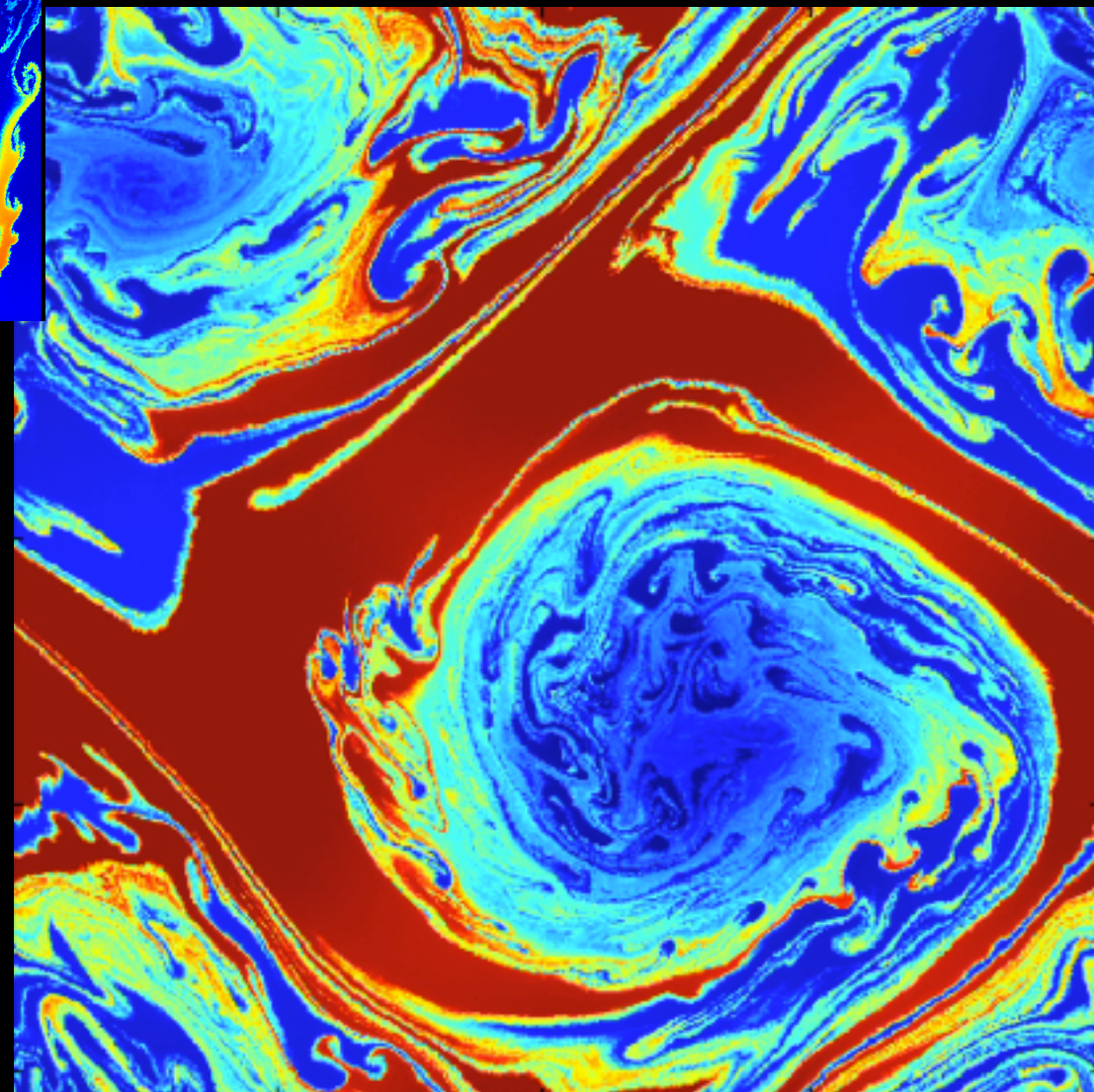
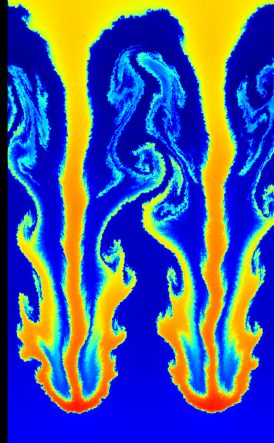


sub-sonic turbulence



MFM

Rayleigh-Taylor



AREPO: Springel 2010  
TESS/DISCO: Duffel 2011  
FVMHD3D: Gaburov 2012  
GIZMO: Hopkins 2015



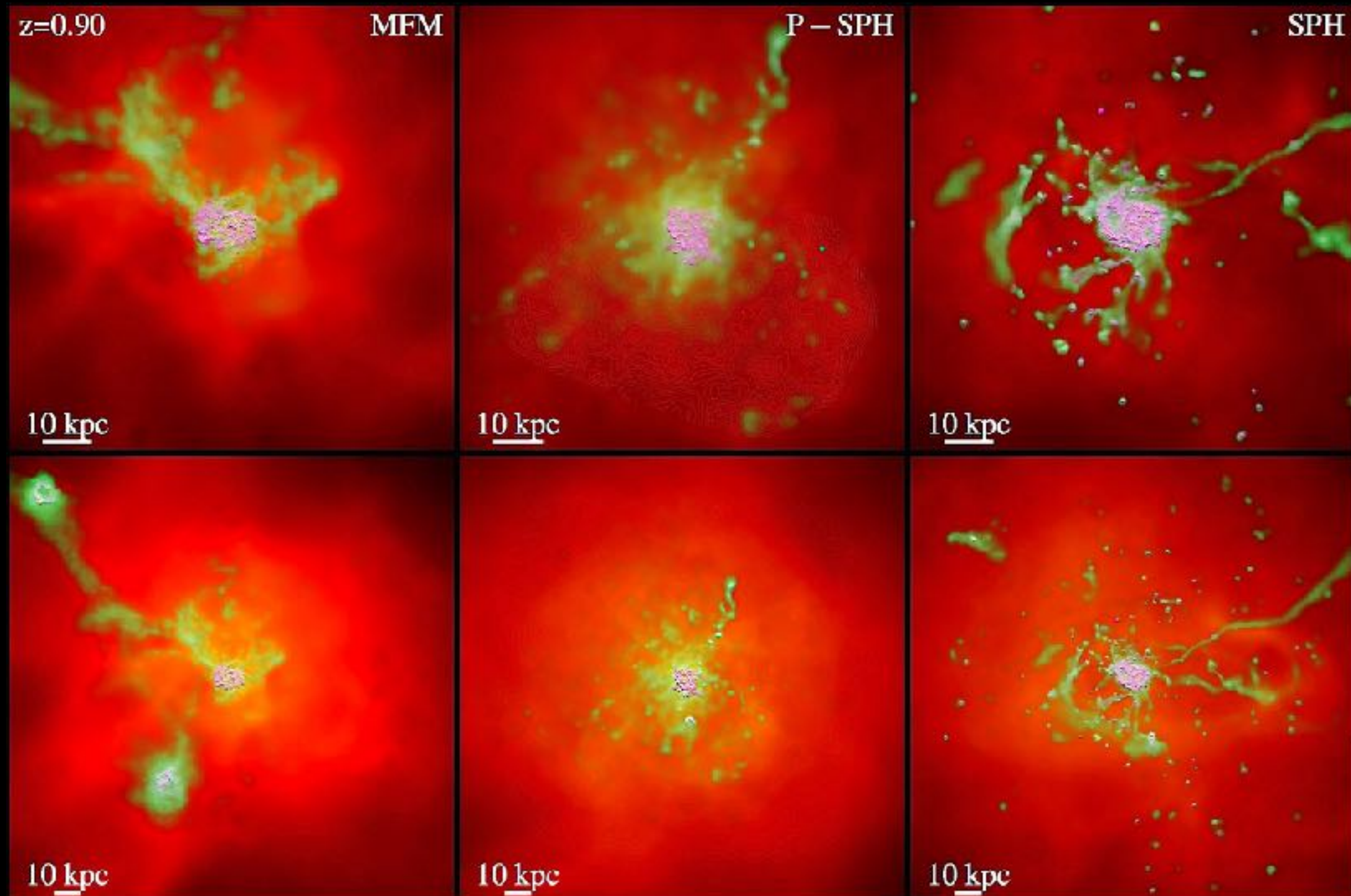
# Getting the Hydro Right Can Matter

BUT IT DEPENDS ON WHAT YOU CARE ABOUT

New Methods (GIZMO)

“New” SPH

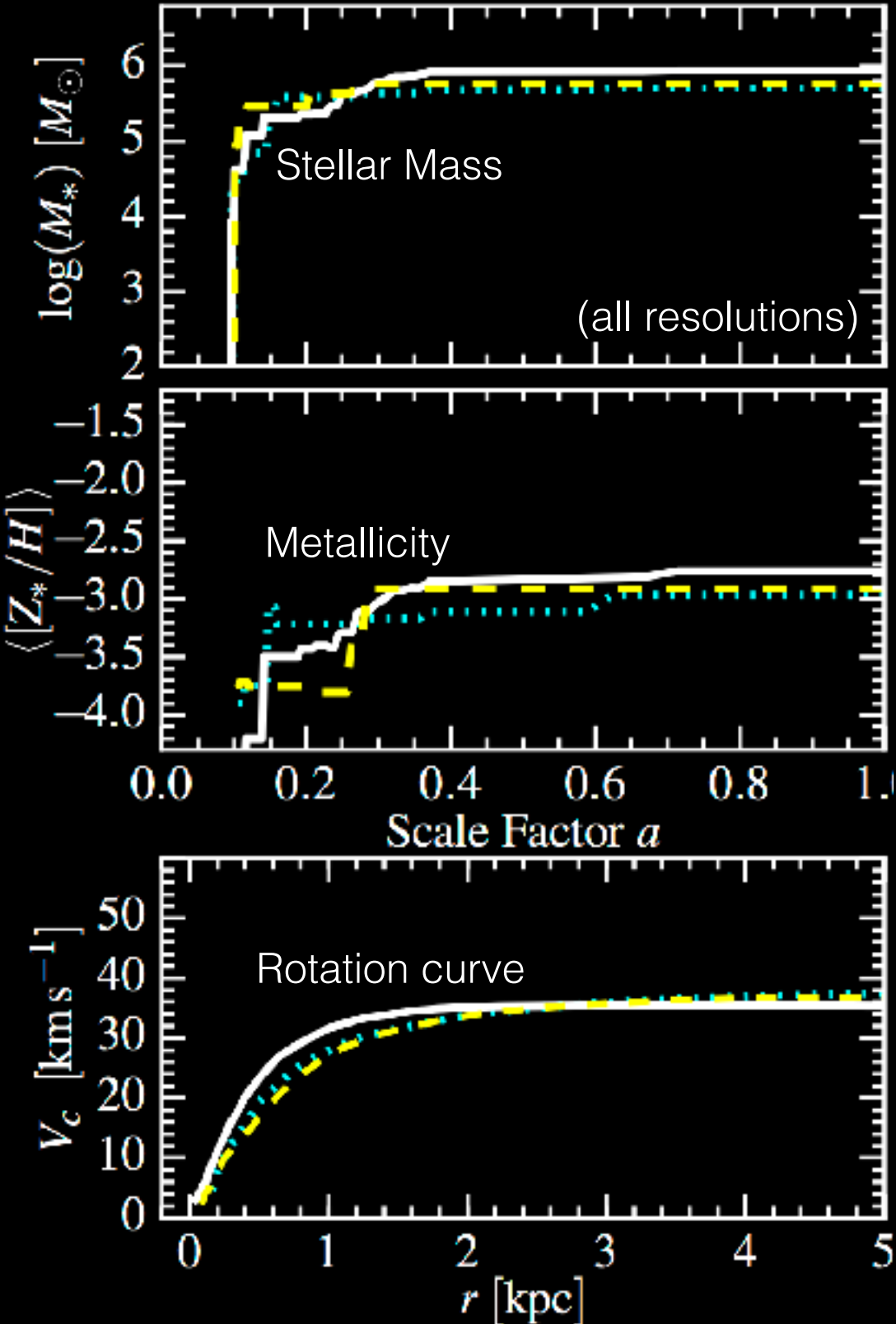
“Old” SPH



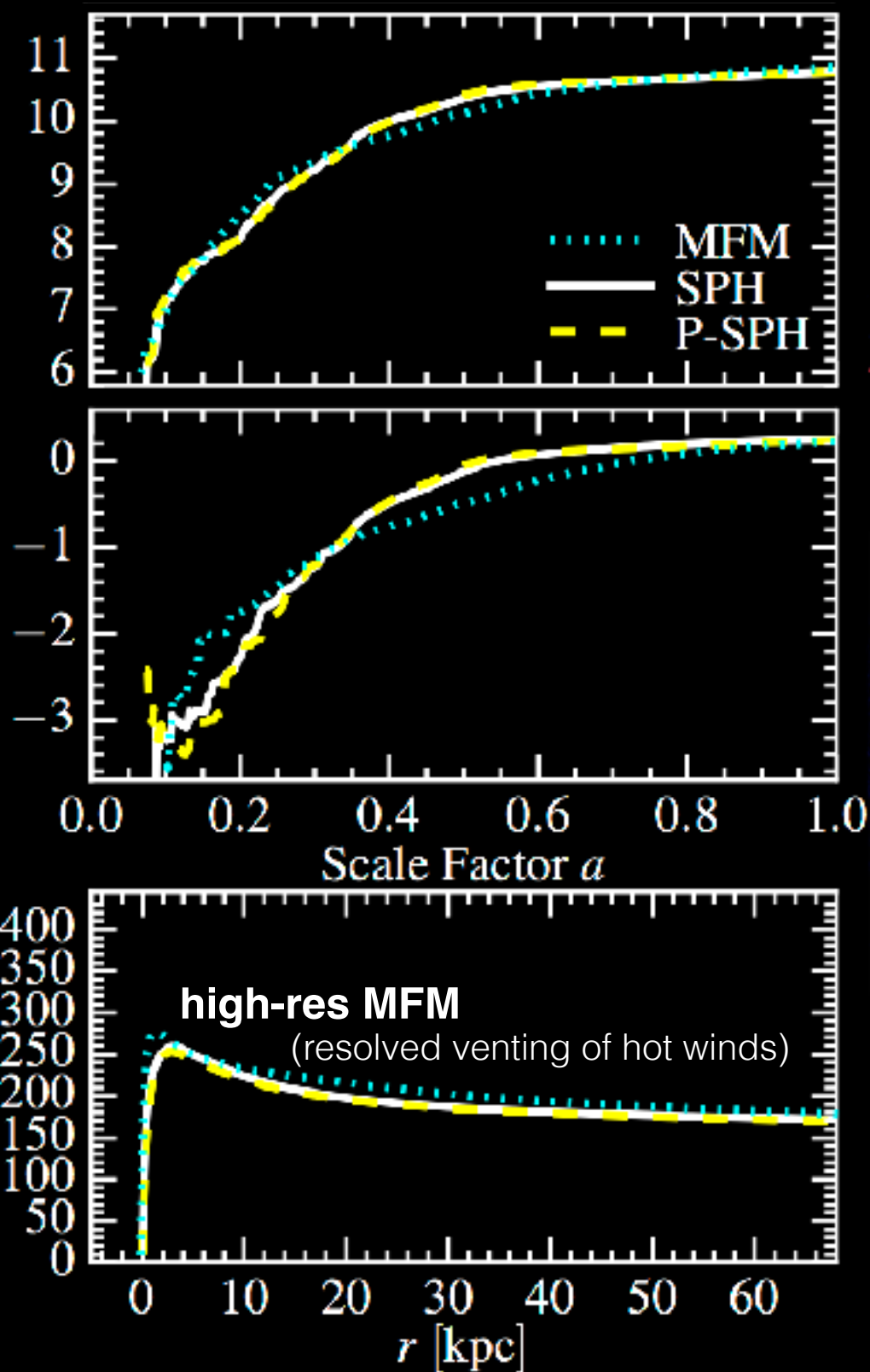
# Getting the Hydro Right Can Matter

DEPENDS ON WHAT YOU CARE ABOUT

Dwarfs (“cold mode”):  
no effects

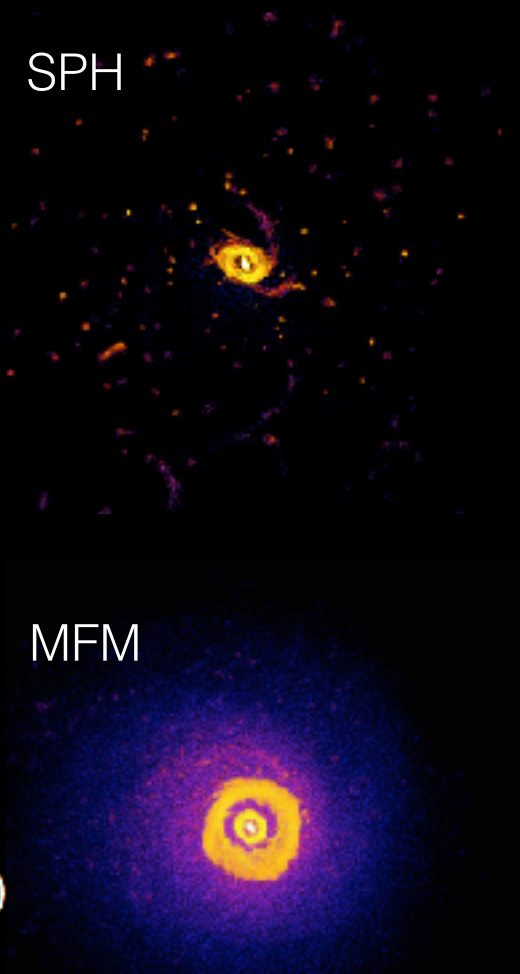


Massive Galaxies (“hot mode”):  
cooling & wind “venting”



SPH

MFM

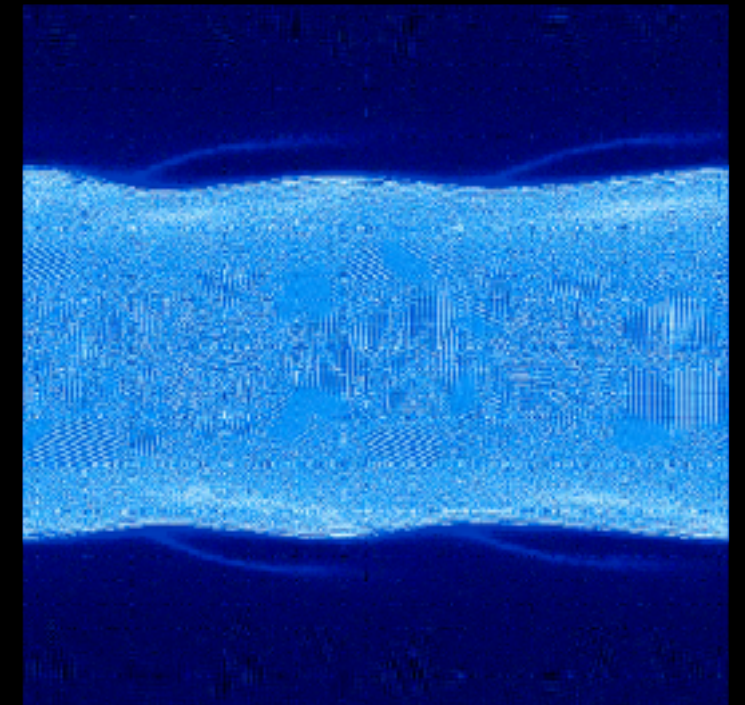
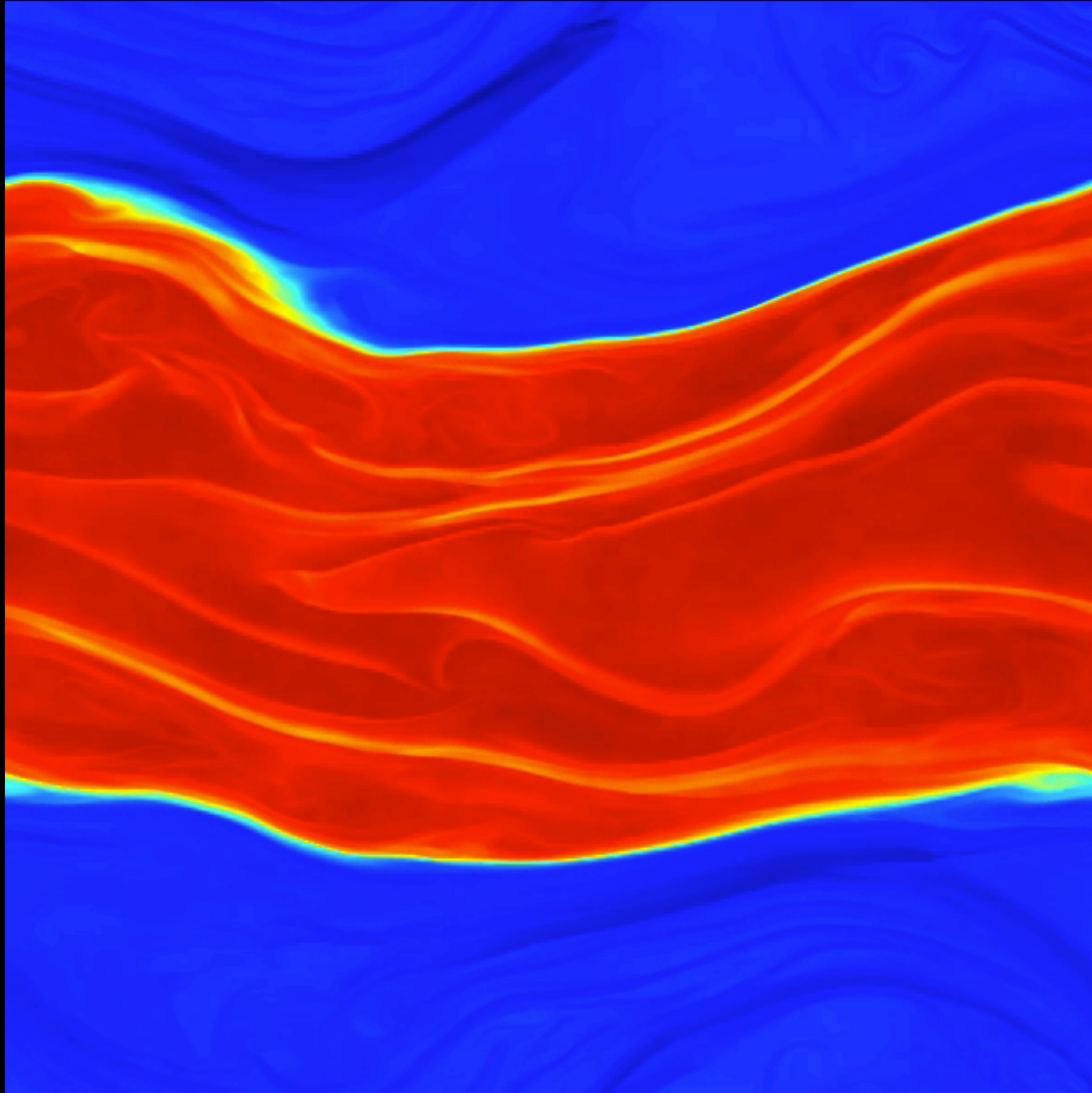


**high-res MFM**  
(resolved venting of hot winds)



# A Caution: You can get the “right” answer for the wrong reasons

DON'T MISTAKE NUMERICAL PRECISION FOR PHYSICAL ACCURACY

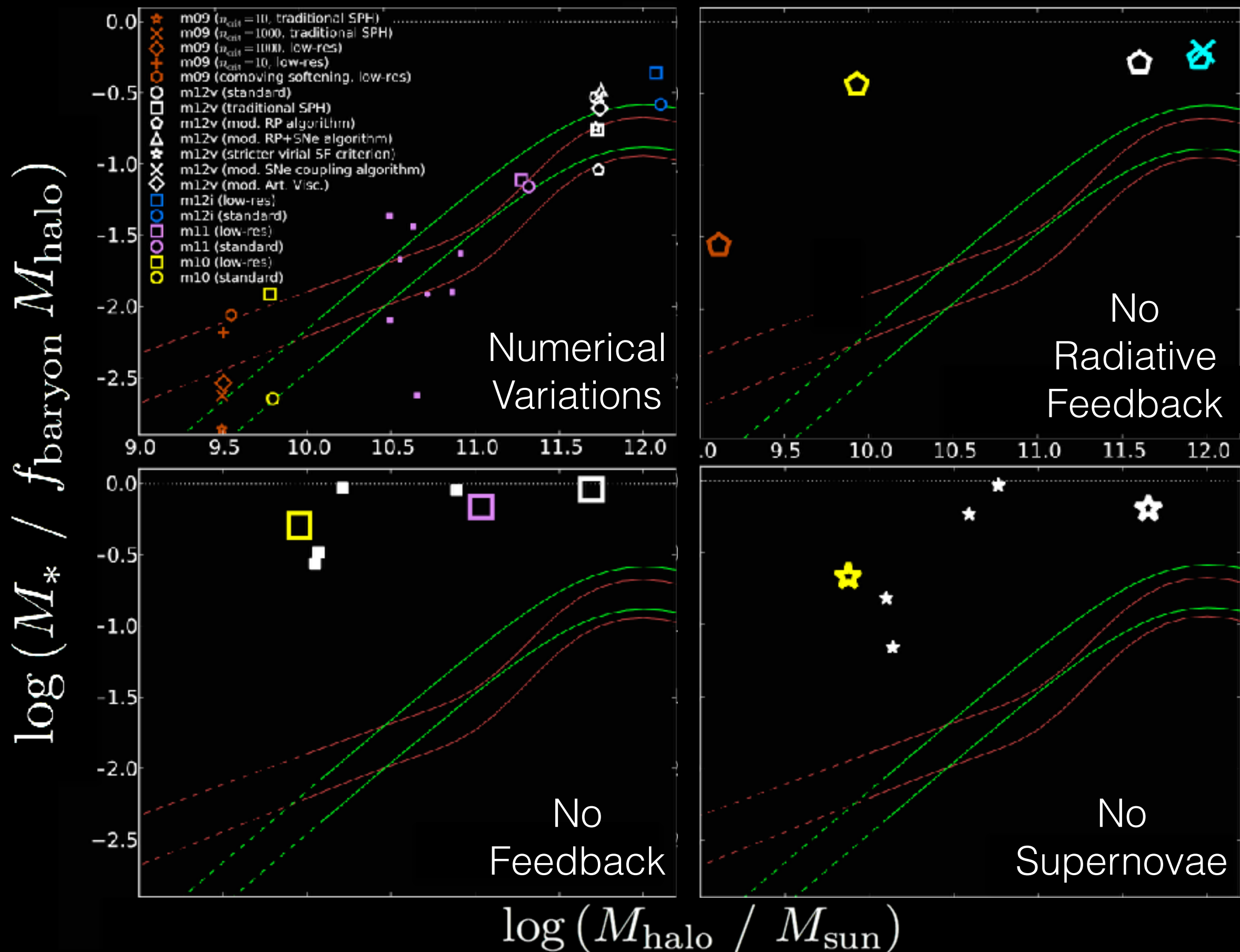


“old” SPH  
(Springel 02)

Magnetic KH  
(Equipartition field)  
with a “good” code

# Getting the Hydro Right Can Matter

DEPENDS ON WHAT YOU CARE ABOUT



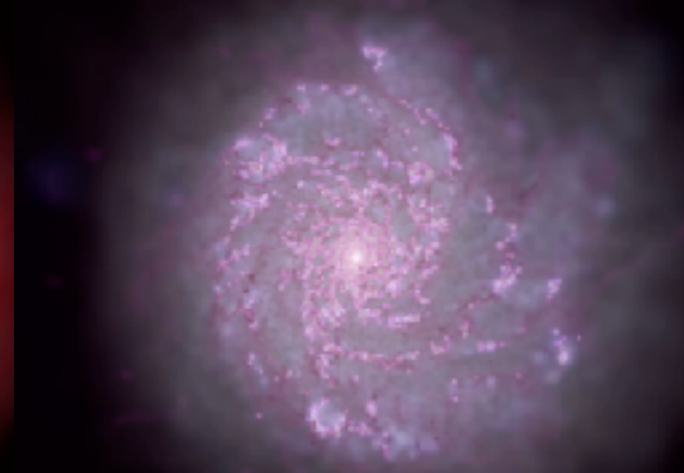
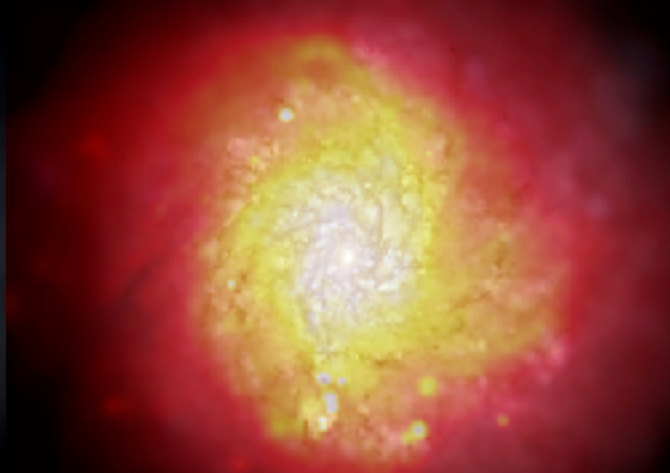
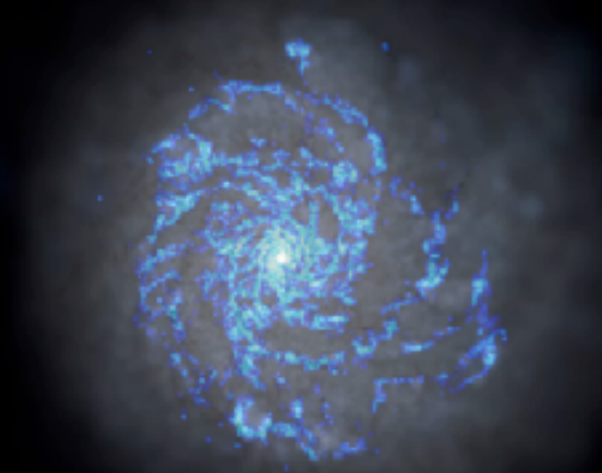


Observed Starlight

Molecular

X-Rays

Star Formation



## ➤ Numerics can be important

- SPH: is high  $N_{\text{NGB}}$  worth it? MHD, conduction, RT, issues: significant differences in “hot halos”
- Quasi-Lagrangian schemes: “grid noise” at very low Mach numbers ( $<0.01$ )
- *Physics* usually dominates

## ➤ Everything is sub-grid: but there are “good” and “bad” models, and different philosophies

- FIRE: trying to “build up” from small scales: works surprisingly well!
- Need resolution to match your physics, but also need *physics* to match your resolution (no meaning in resolving scales you don’t have the physics for)

## ➤ What is needed? Depends 100% on what you want to predict

- Resolve dense gas: resolve fragmentation (Toomre), *physics* for GMC destruction (radiative FB)
- Resolve SNe overlaps/bubbles: need to treat them explicitly, account for unresolved cooling
- SFR surprisingly insensitive to small-scale SF physics, MHD, diffusion: *feedback* dominates