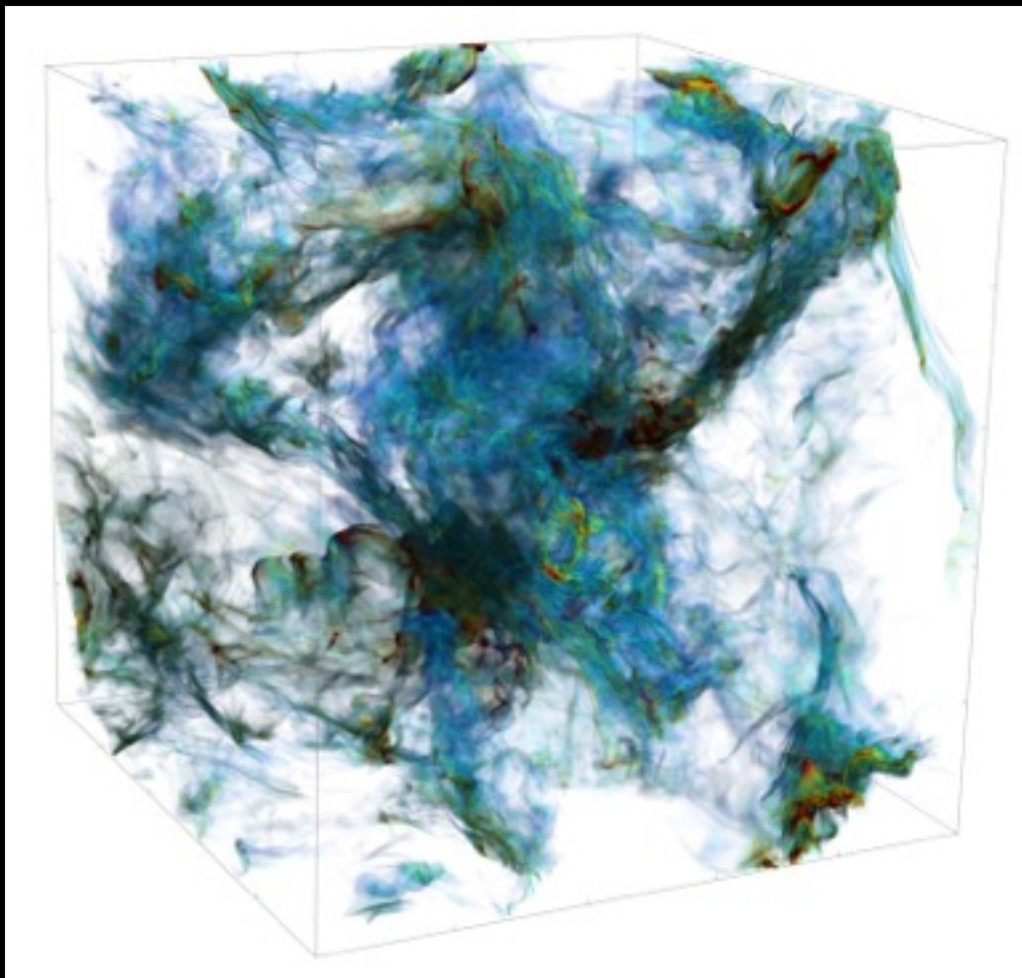
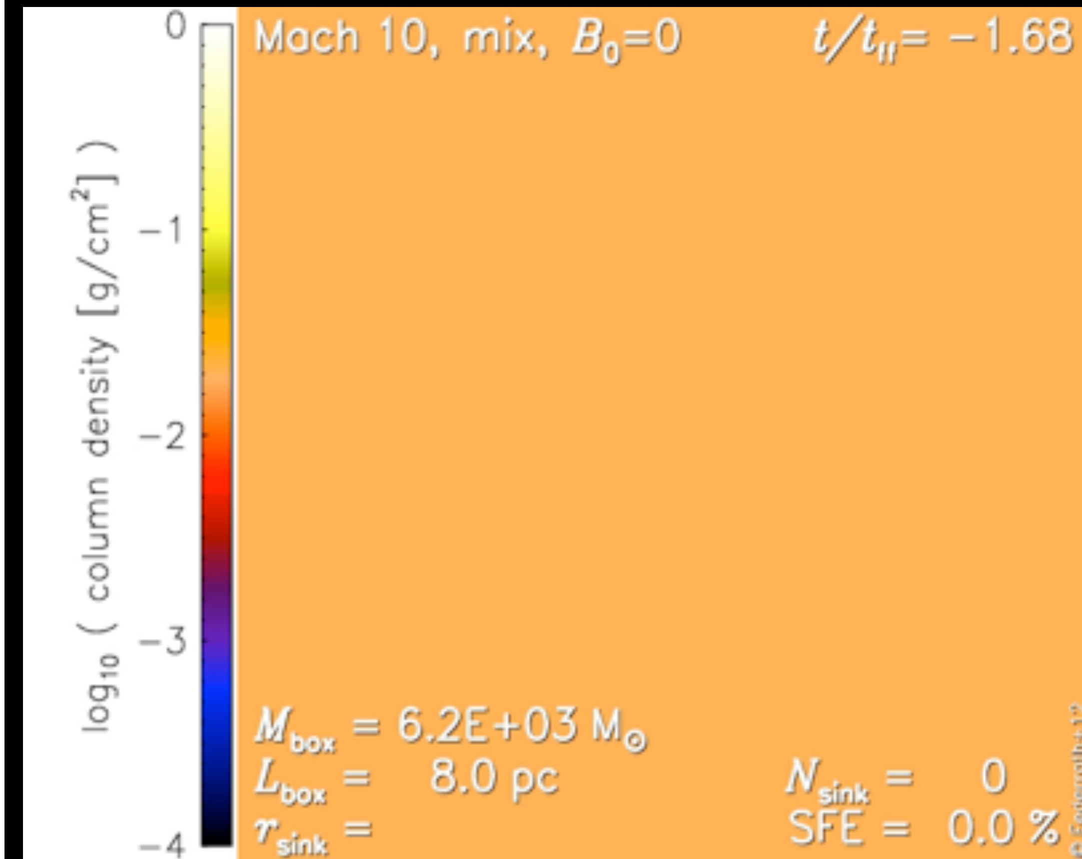
A New Approach to Turbulence:

Origins of ISM Structure, Stellar Clustering & the IMF, and (perhaps?) Planet Formation

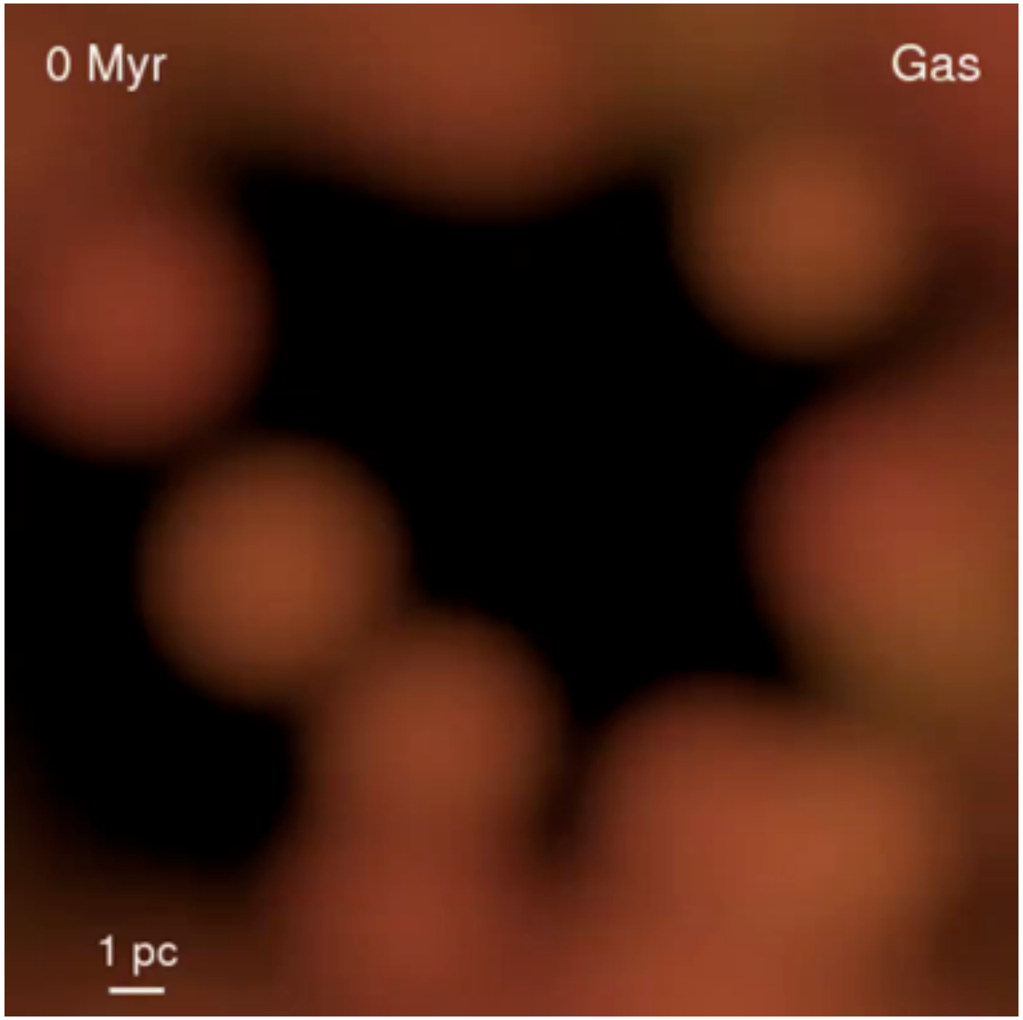


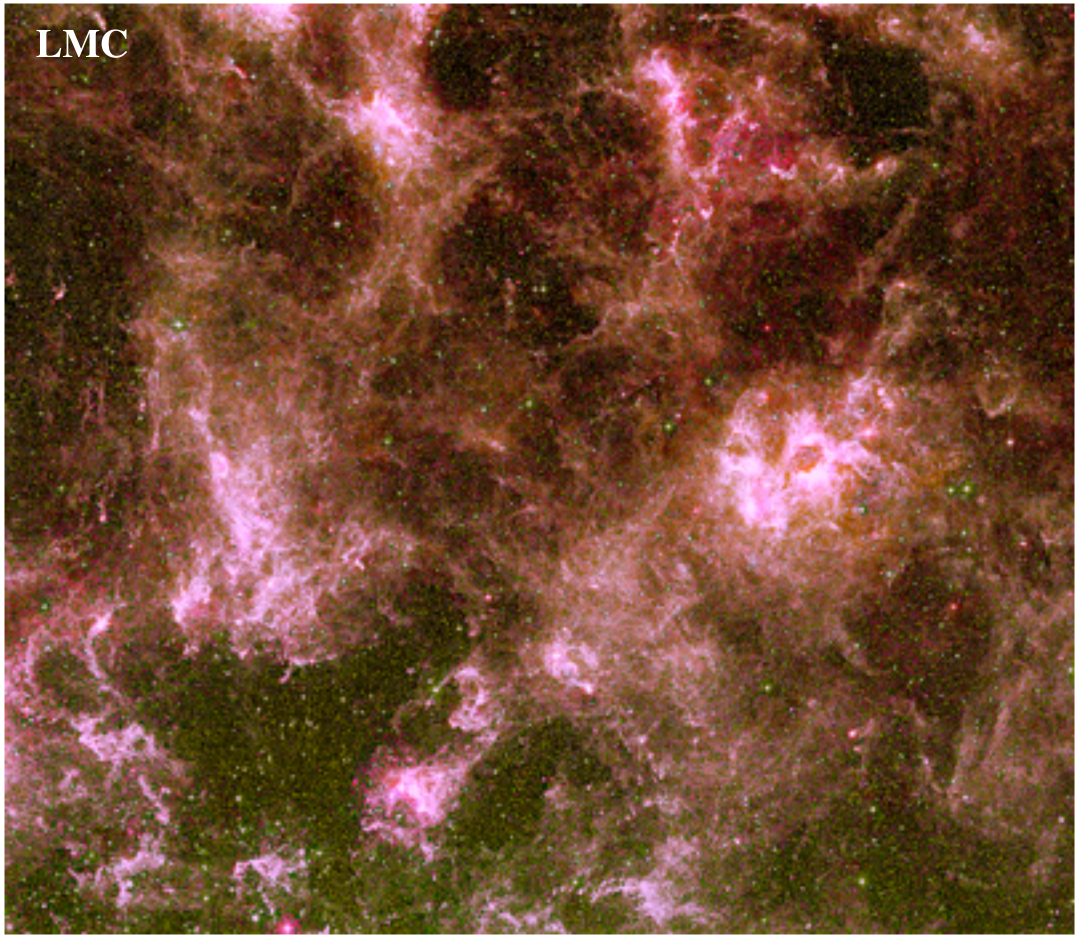


Philip Hopkins

The Turbulent ISM

IMPORTANT ON
(ALMOST) ALL SCALES



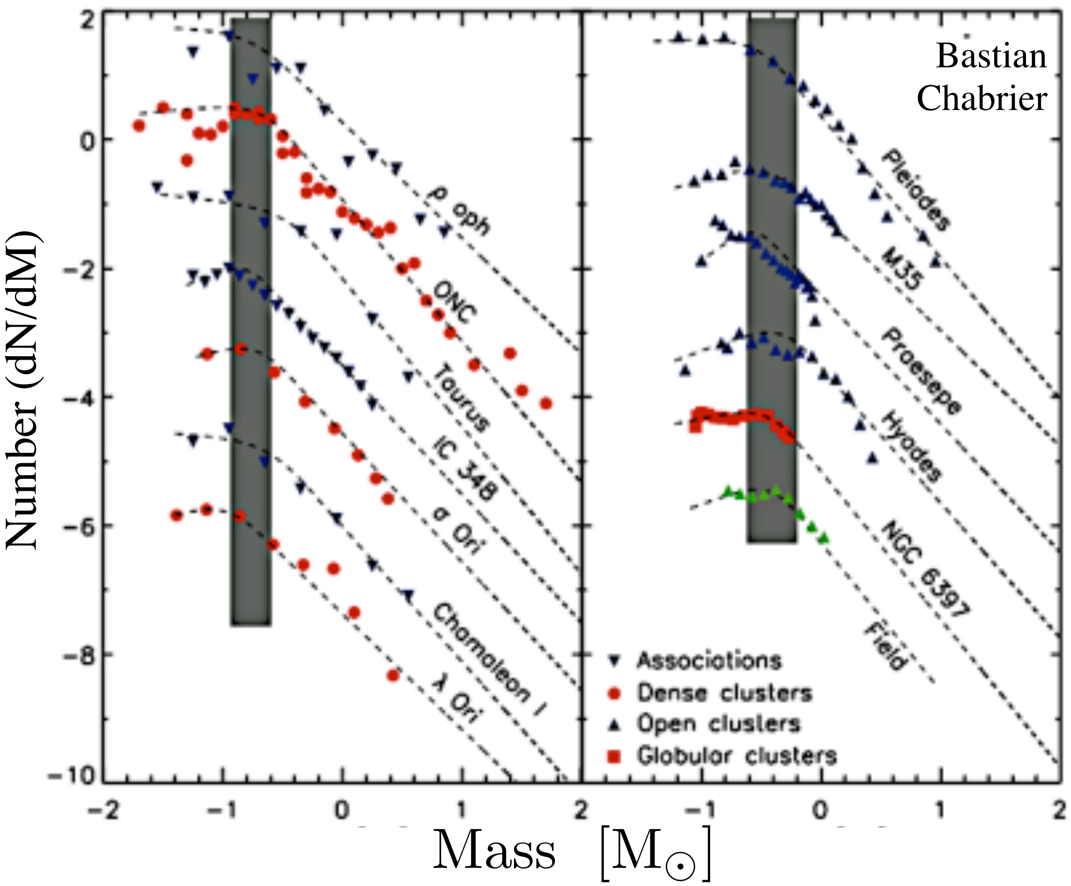


- Gravity
- Turbulence
- Magnetic, Thermal, Cosmic Ray, Radiation Pressure
- Cooling (atomic, molecular, metal-line, free-free)
- Star & BH Formation/Growth
- "Feedback": Massive stars, SNe, BHs,
  external galaxies, etc.

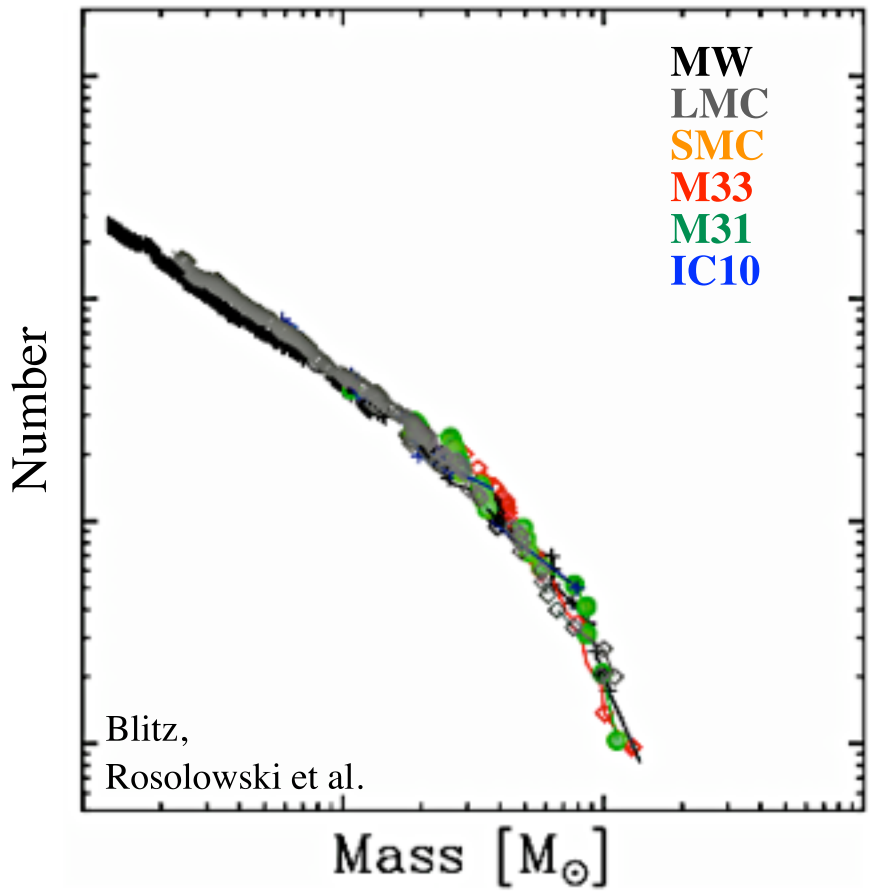
The ISM

YET THERE IS SURPRISING REGULARITY

Stars & Pre-Stellar Gas Cores:



Giant Molecular Clouds:



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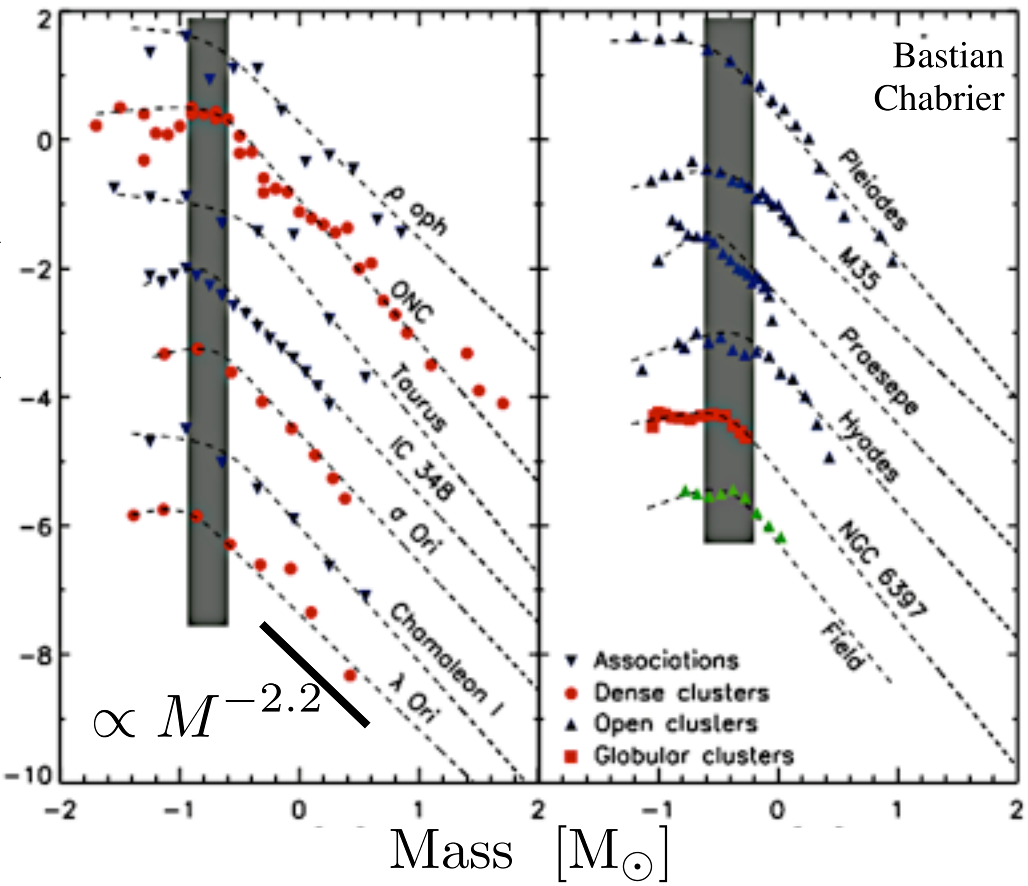

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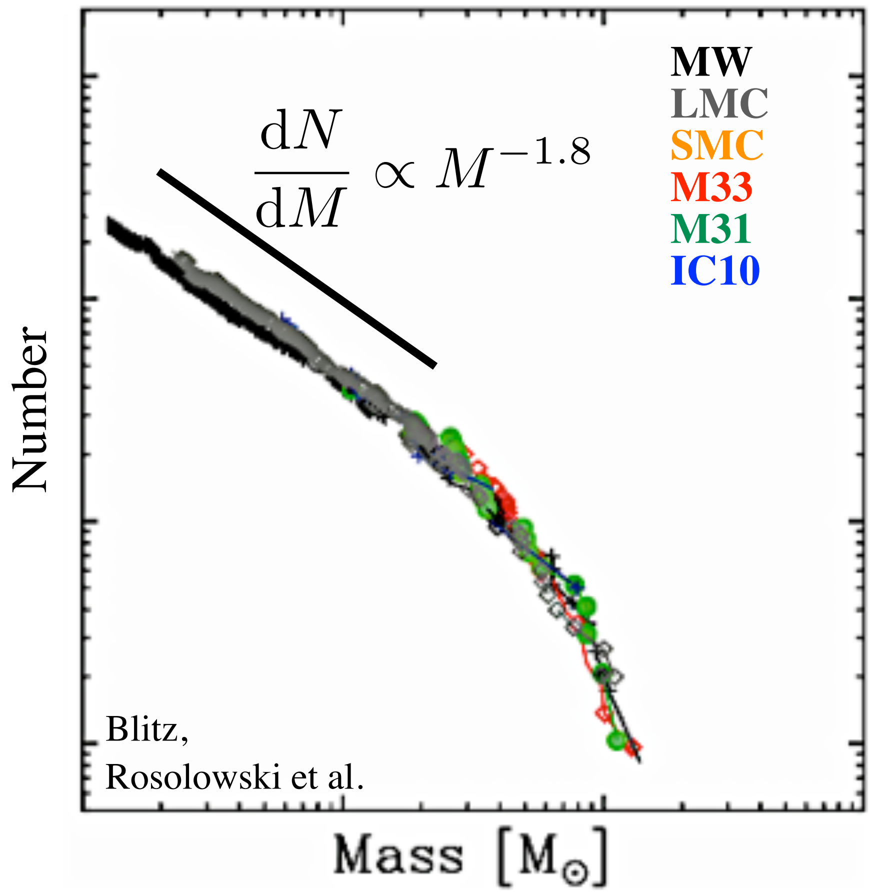

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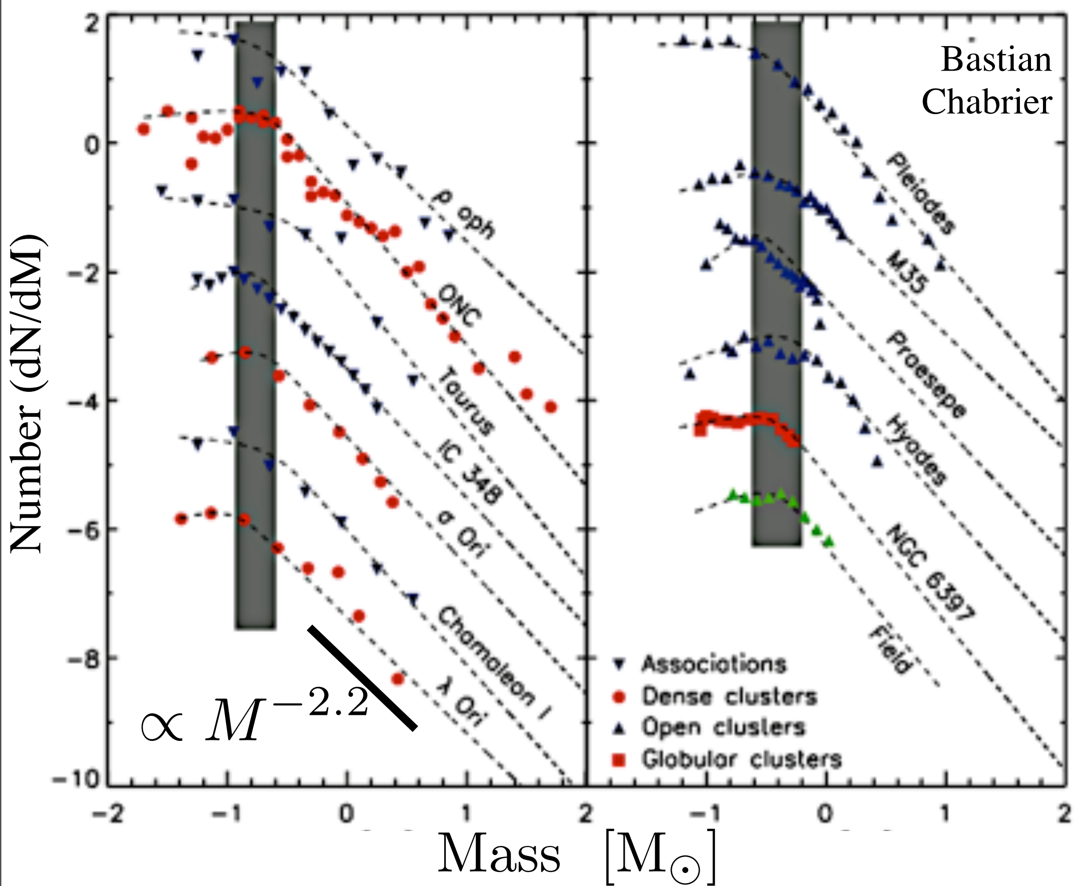

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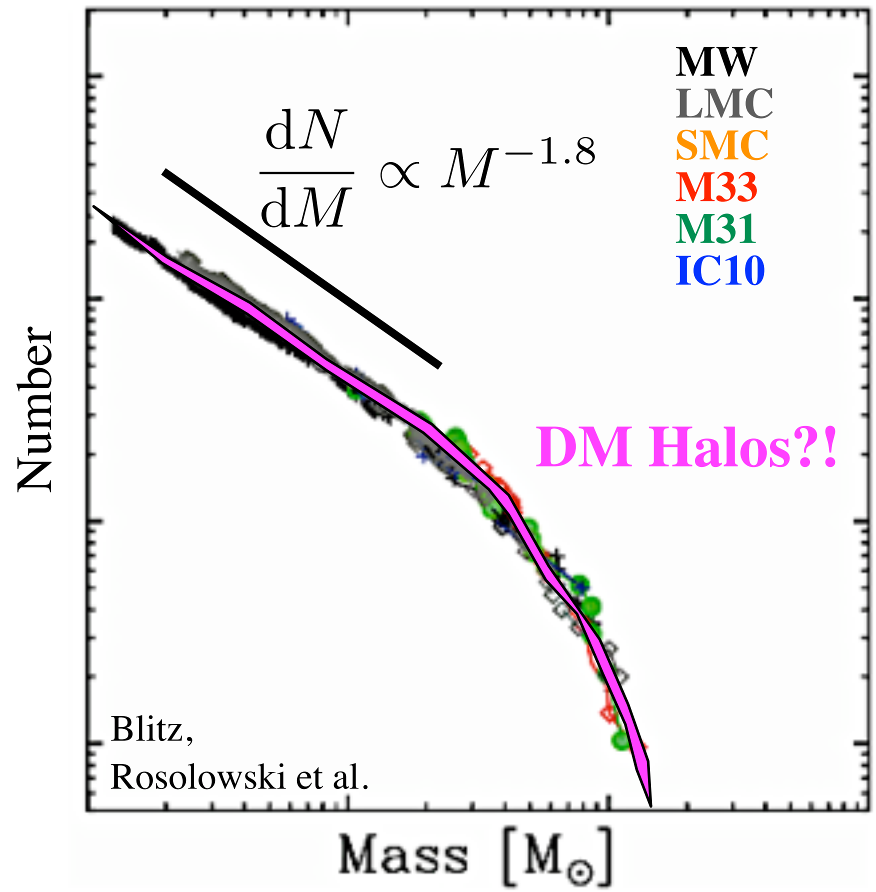
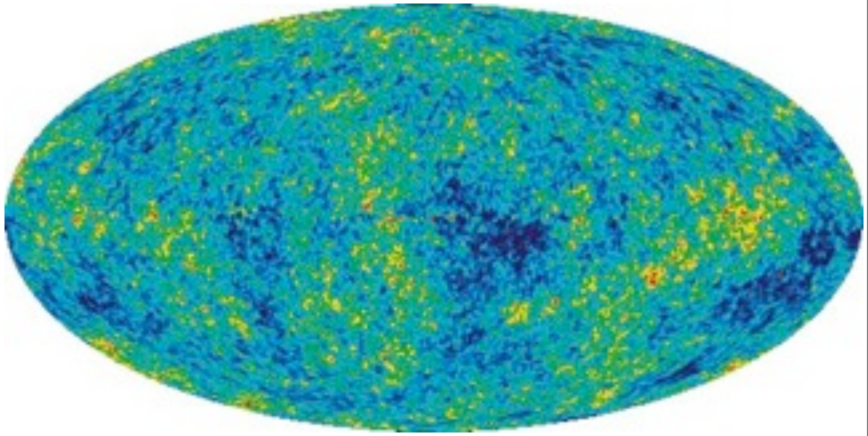
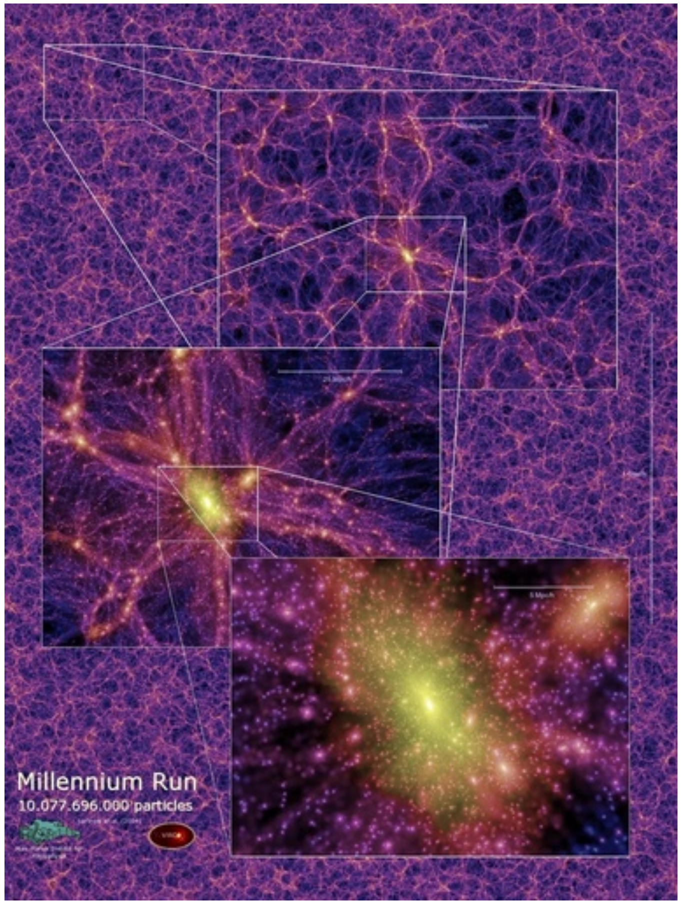


Extended Press-Schechter / Excursion-Set Formalism



- Press & Schechter ‘74:
  - ρ Fluctuations a Gaussian random field
  - Know linear power spectrum P(k~1/r):
    variance $\sim k^3\,P(k)$



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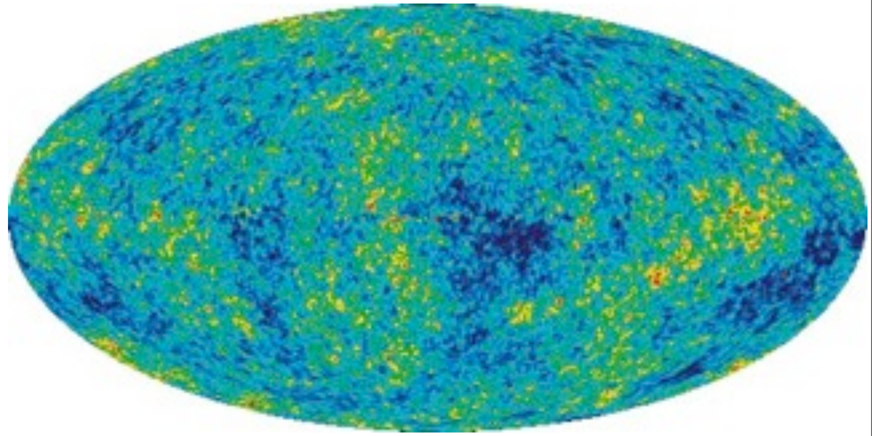

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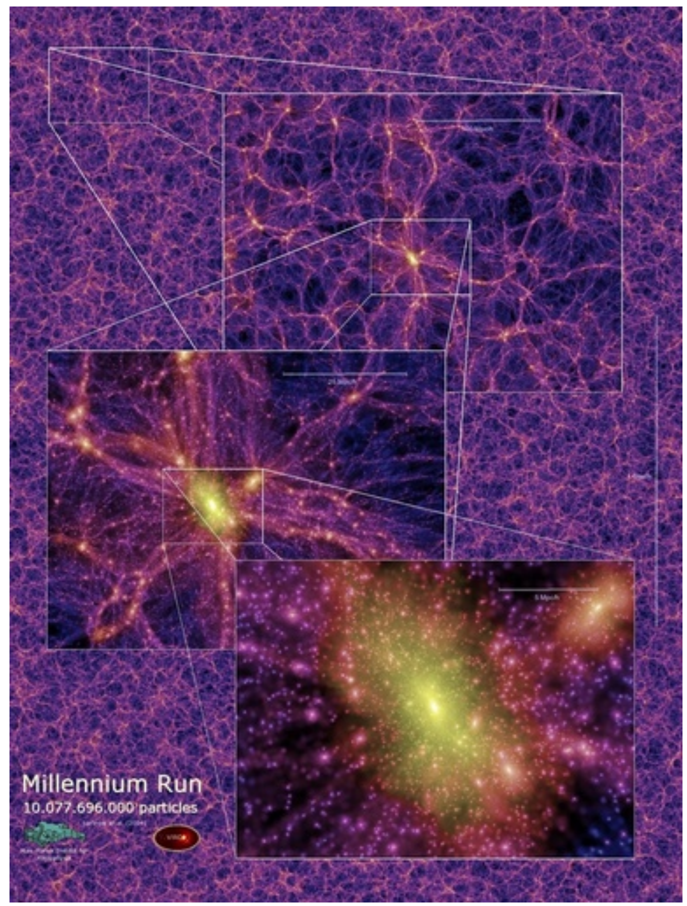

- "Count" mass above critical fluctuation: "Halos"
  - Turnaround & gravitational collapse

$$\bar{\rho}(< R \sim 1/k) > \rho_{\rm crit}$$

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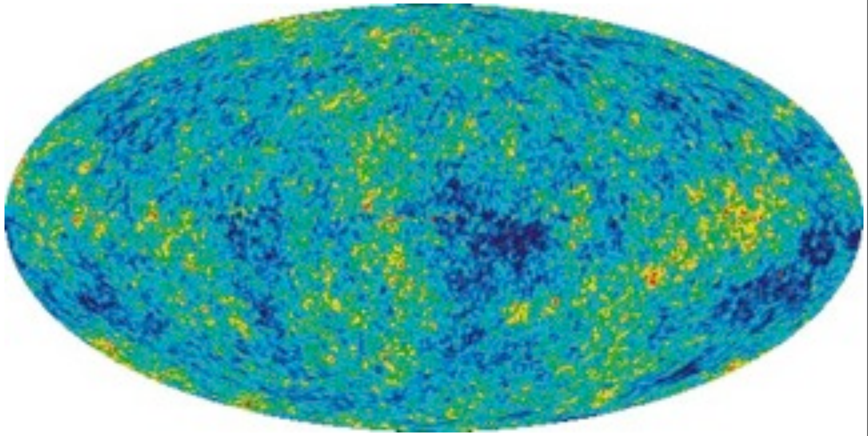

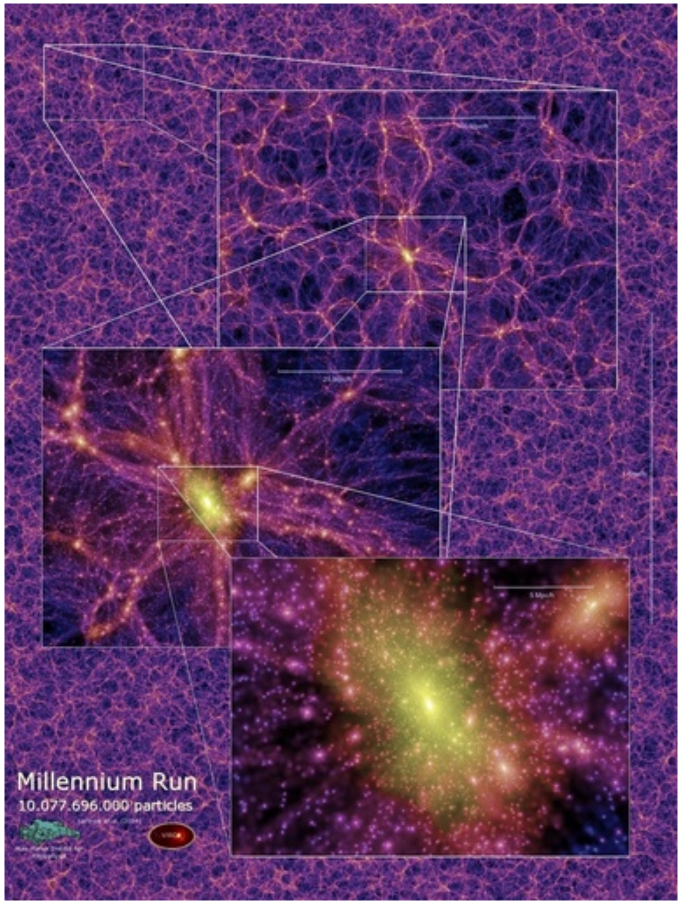

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- Generalize to conditional probabilities,
  N-point statistics, resolve "cloud in cloud" problem
  (e.g. Bond et al. 1991)

BASIC EXPECTATIONS

Velocity: $E(k) \propto k^{-p}$ $\quad (k\,E(k) \sim u_t(k)^2)$

Turbulence

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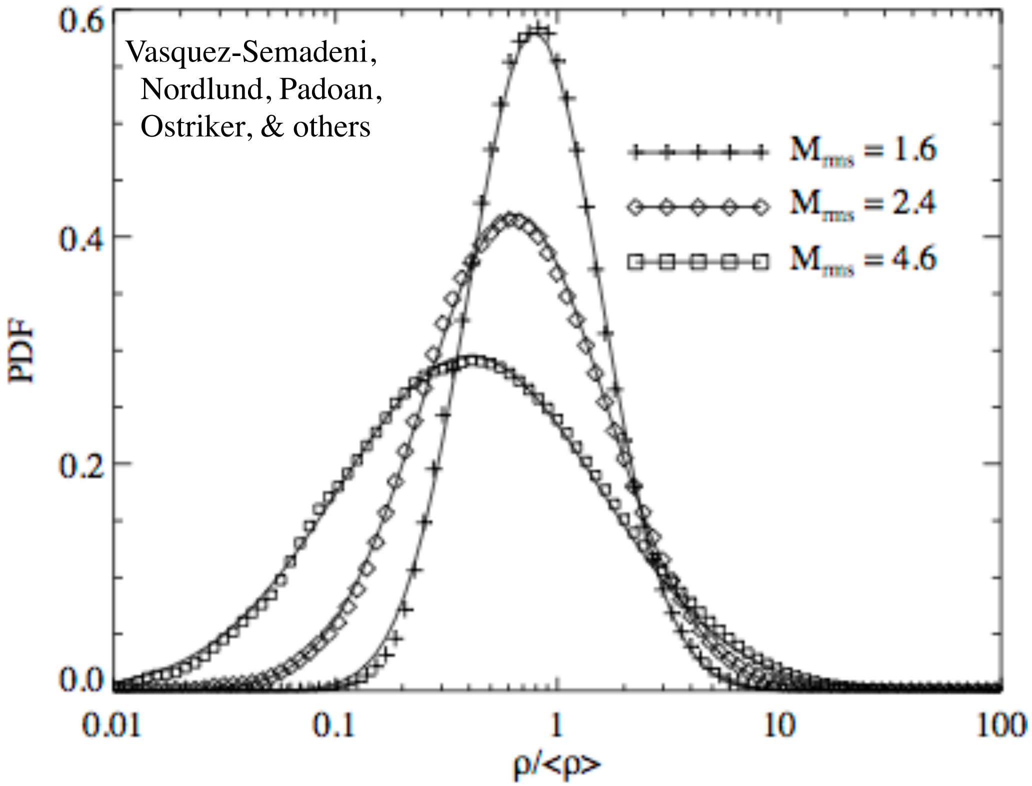

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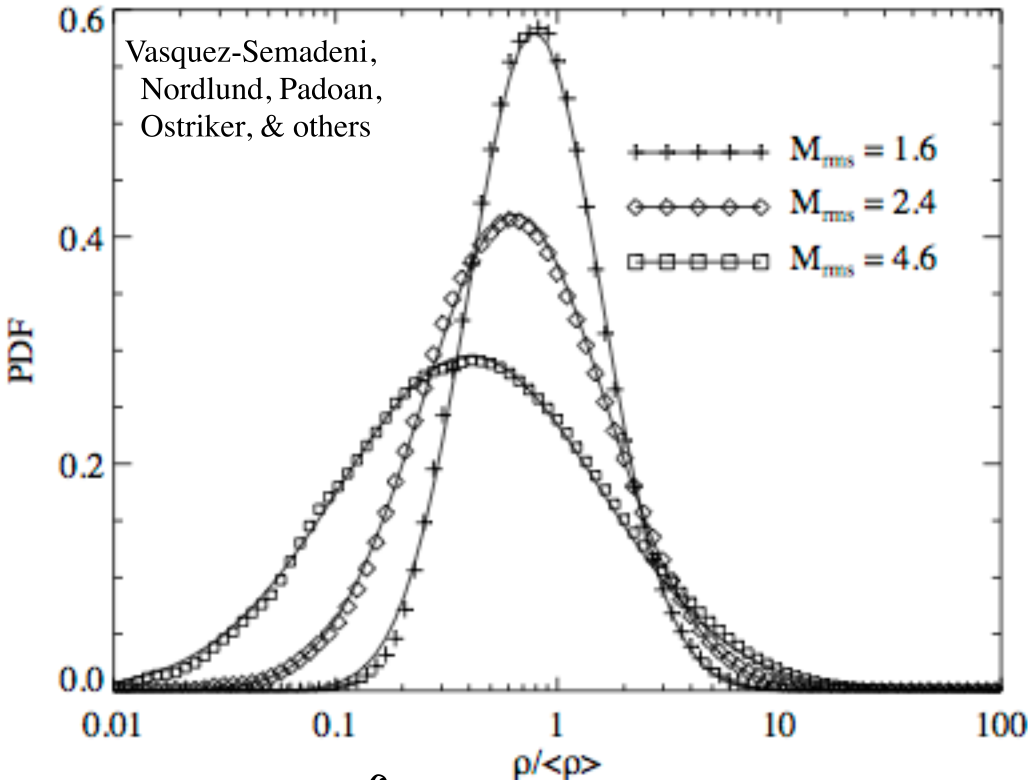

$$S(R) = \int \mathrm{d}\ln k\,S_k\,|W(k,\,R)|^2$$

Super-Sonic Turbulence

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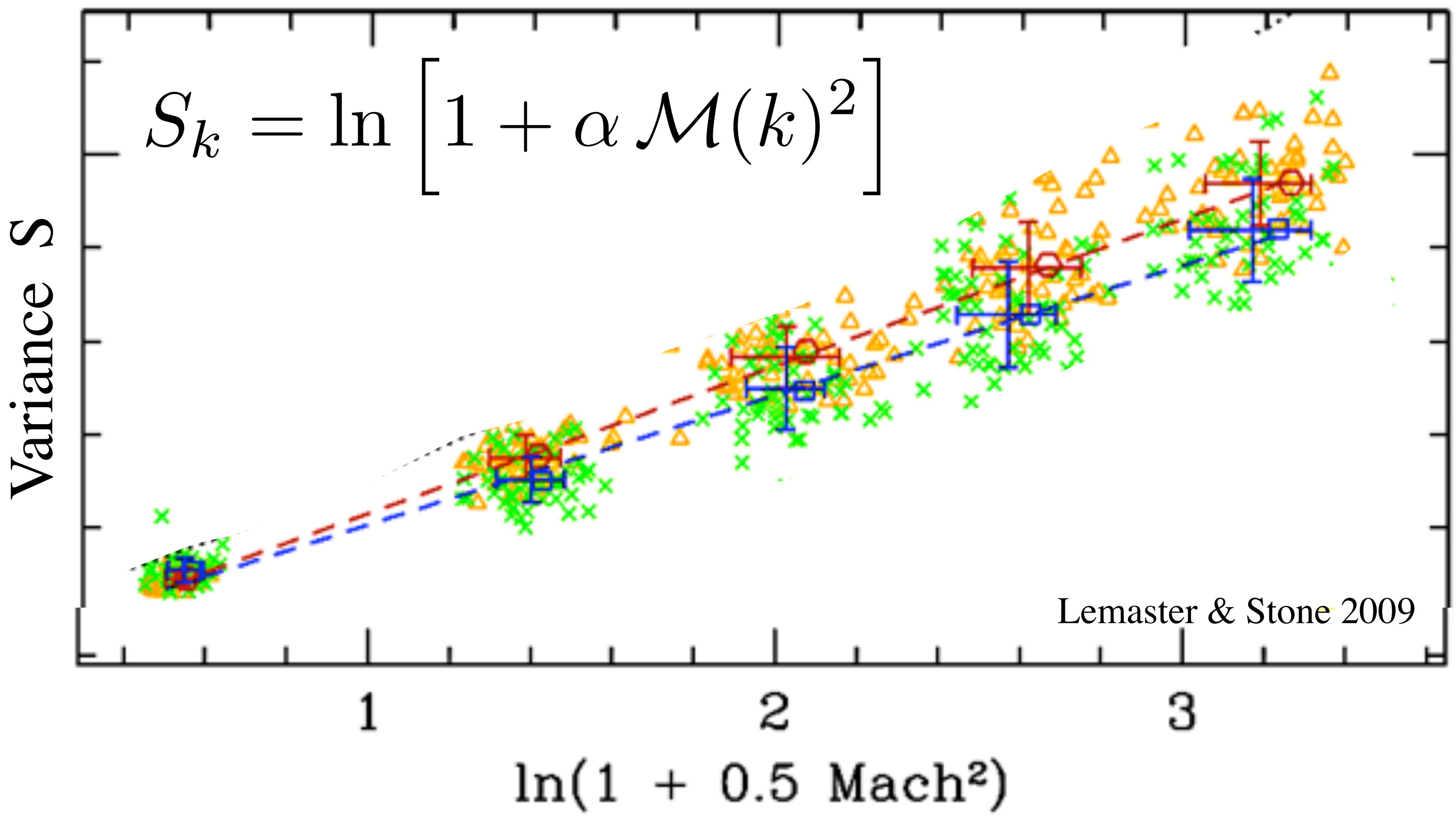

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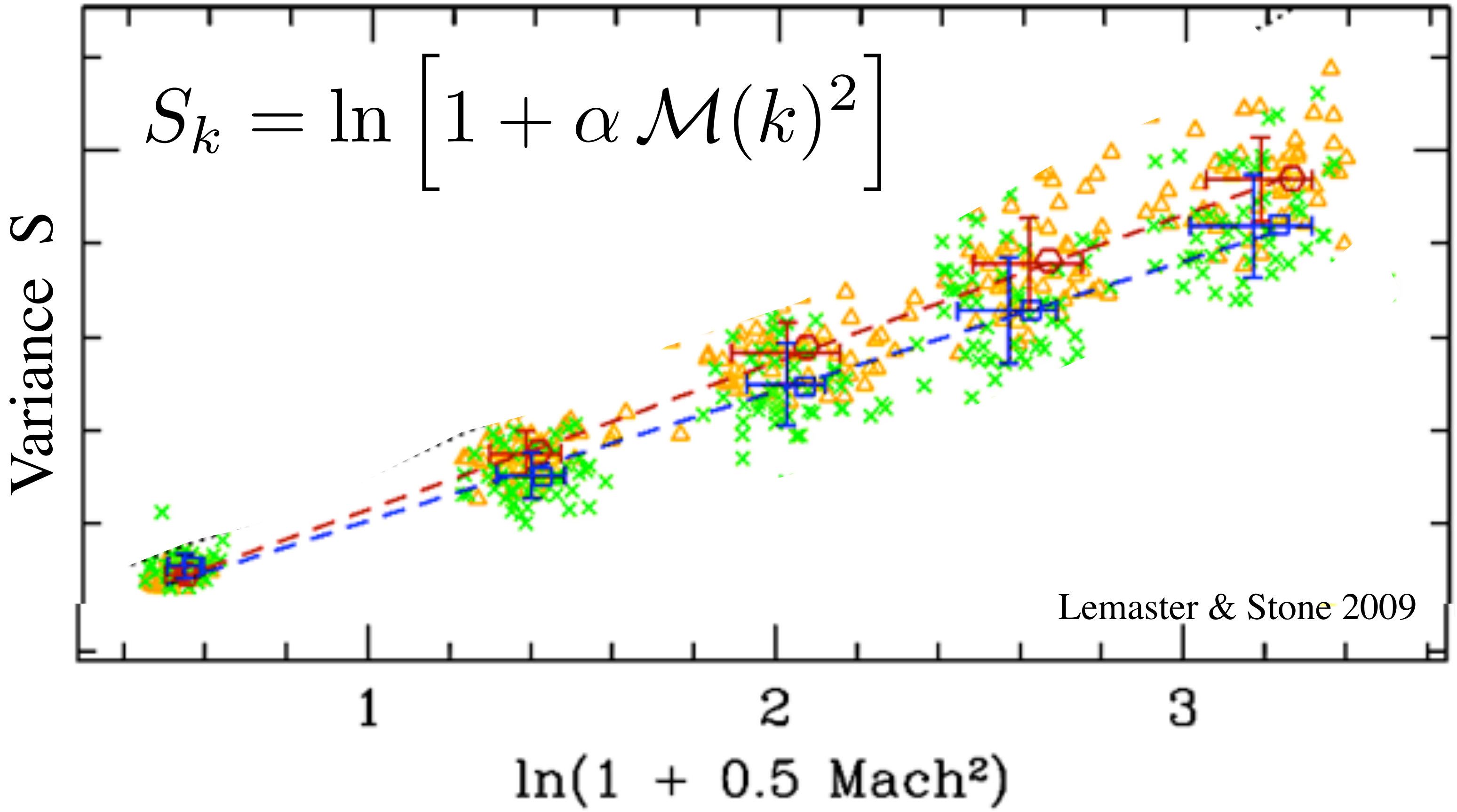

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What Defines a Fluctuation of Interest?

DISPERSION RELATION:

$$\omega^2 = \kappa^2 + c_s^2\, k^2 + u_t(k)^2\, k^2 - \frac{4\pi\, G\, \rho\, |k| h}{1 + |k| h}$$

Chandrasekhar '51, Vandervoort '70, Toomre '77

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Gravity

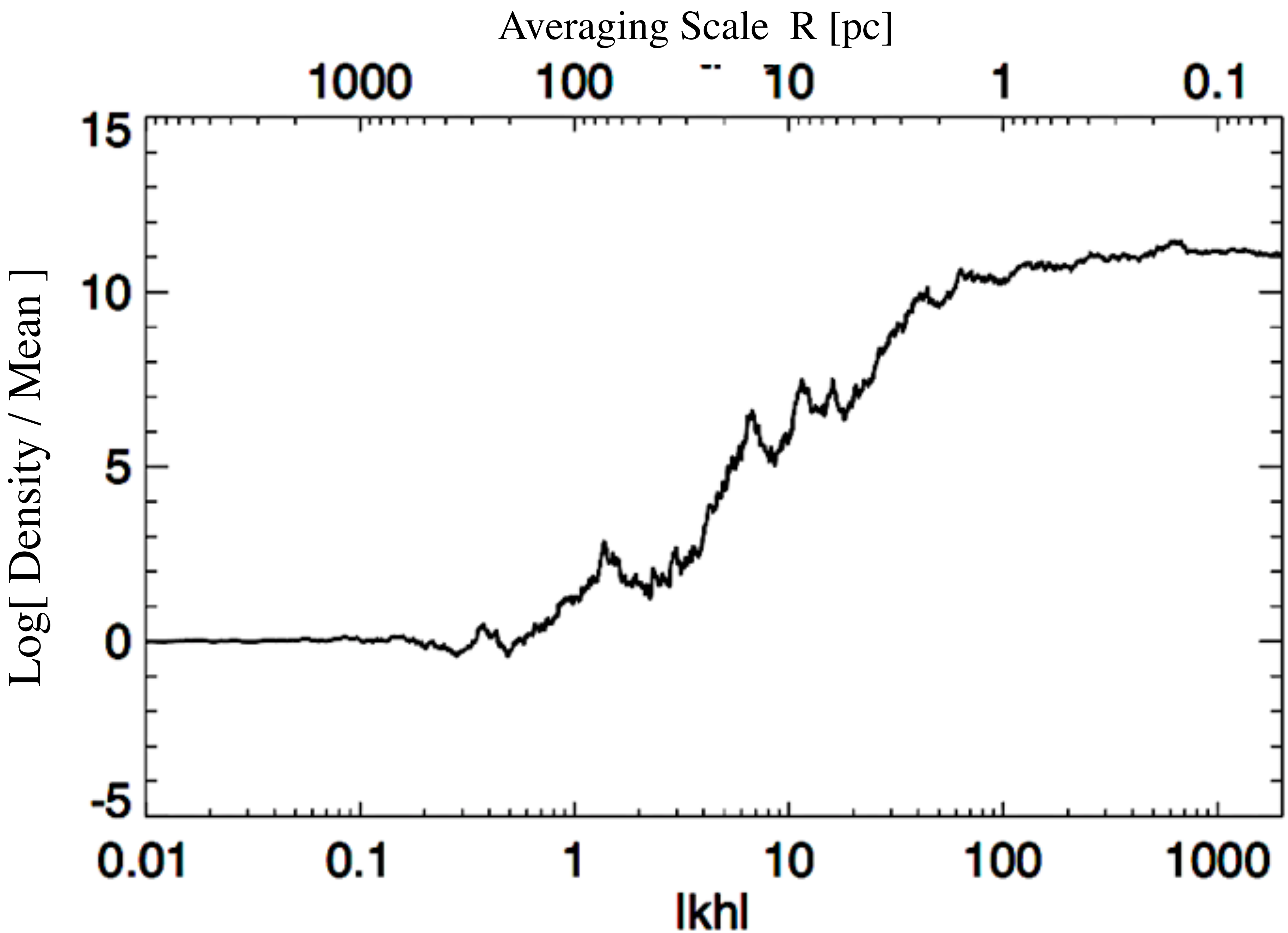
Mode Grows (Collapses) when ω<0:

$$\rho > \rho_c(k) = \rho_0\,(1 + |kh|)\left[\left(\mathcal{M}_h^{-2} + |kh|^{1-p}\right) kh + \frac{2}{|kh|}\right]$$

Chandrasekhar ‘51, Vandervoort ‘70, Toomre ‘77

"Counting" Collapsing Objects

EVALUATE DENSITY FIELD vs. "BARRIER"



PFH 2011

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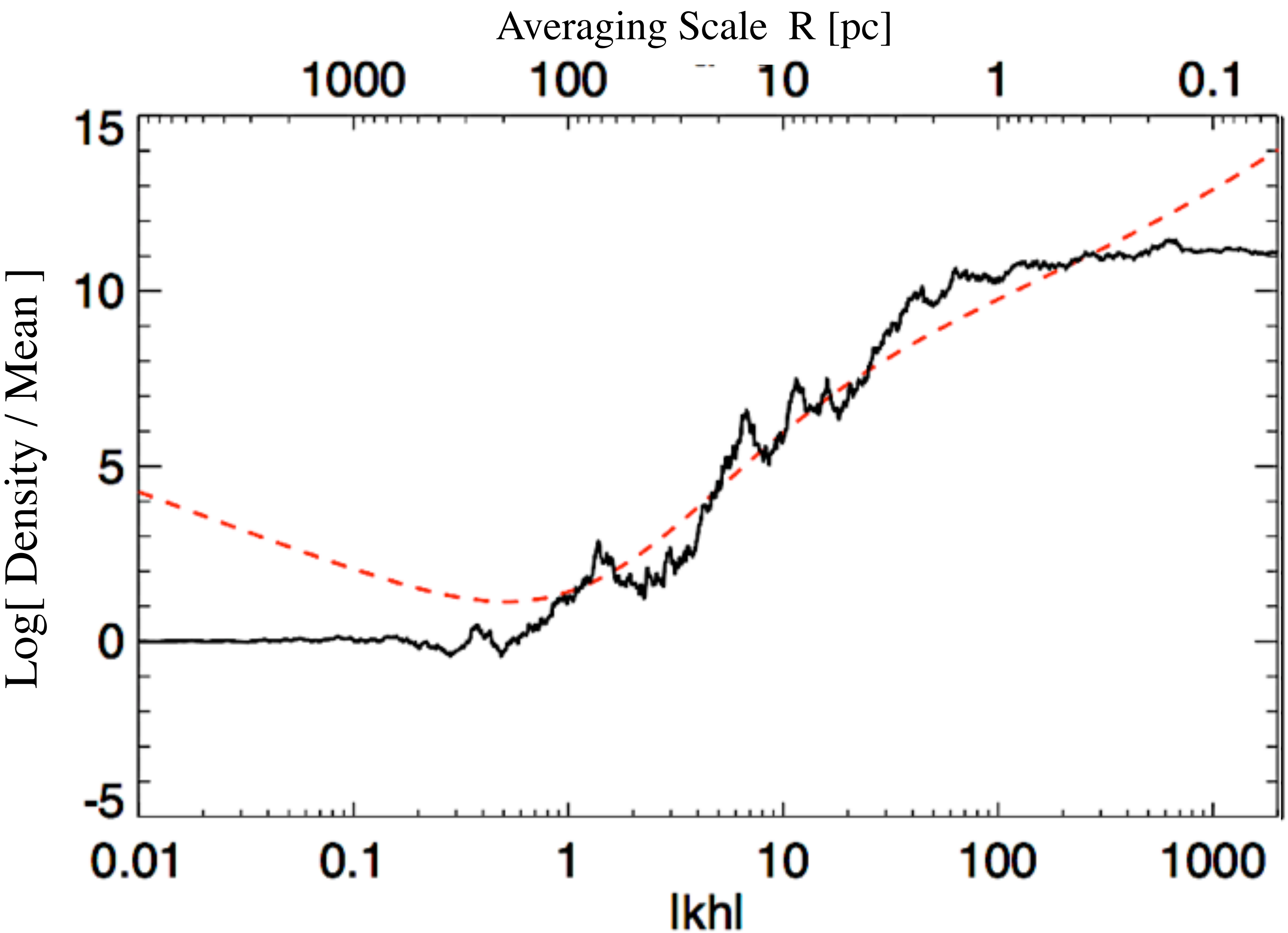

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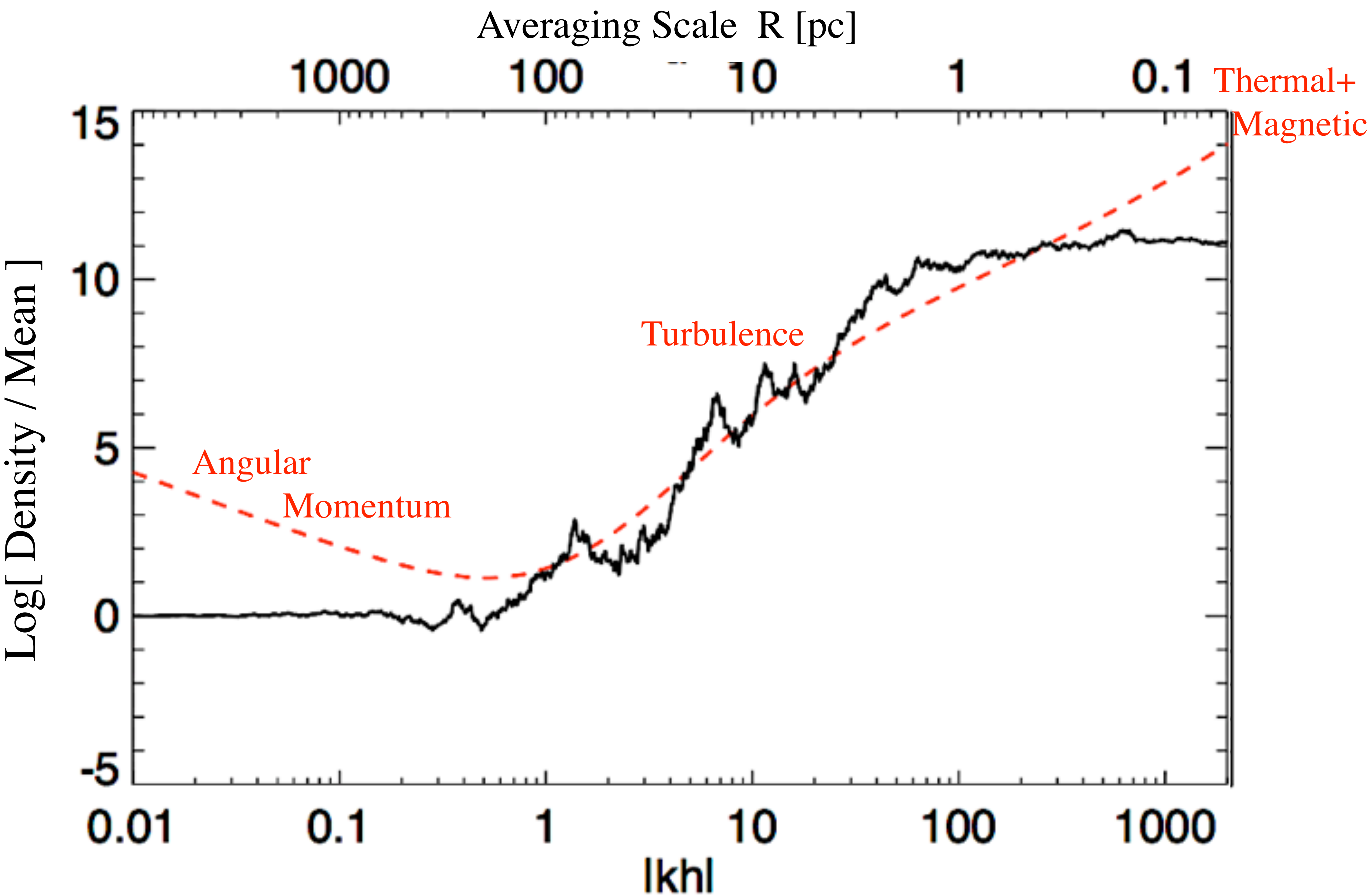

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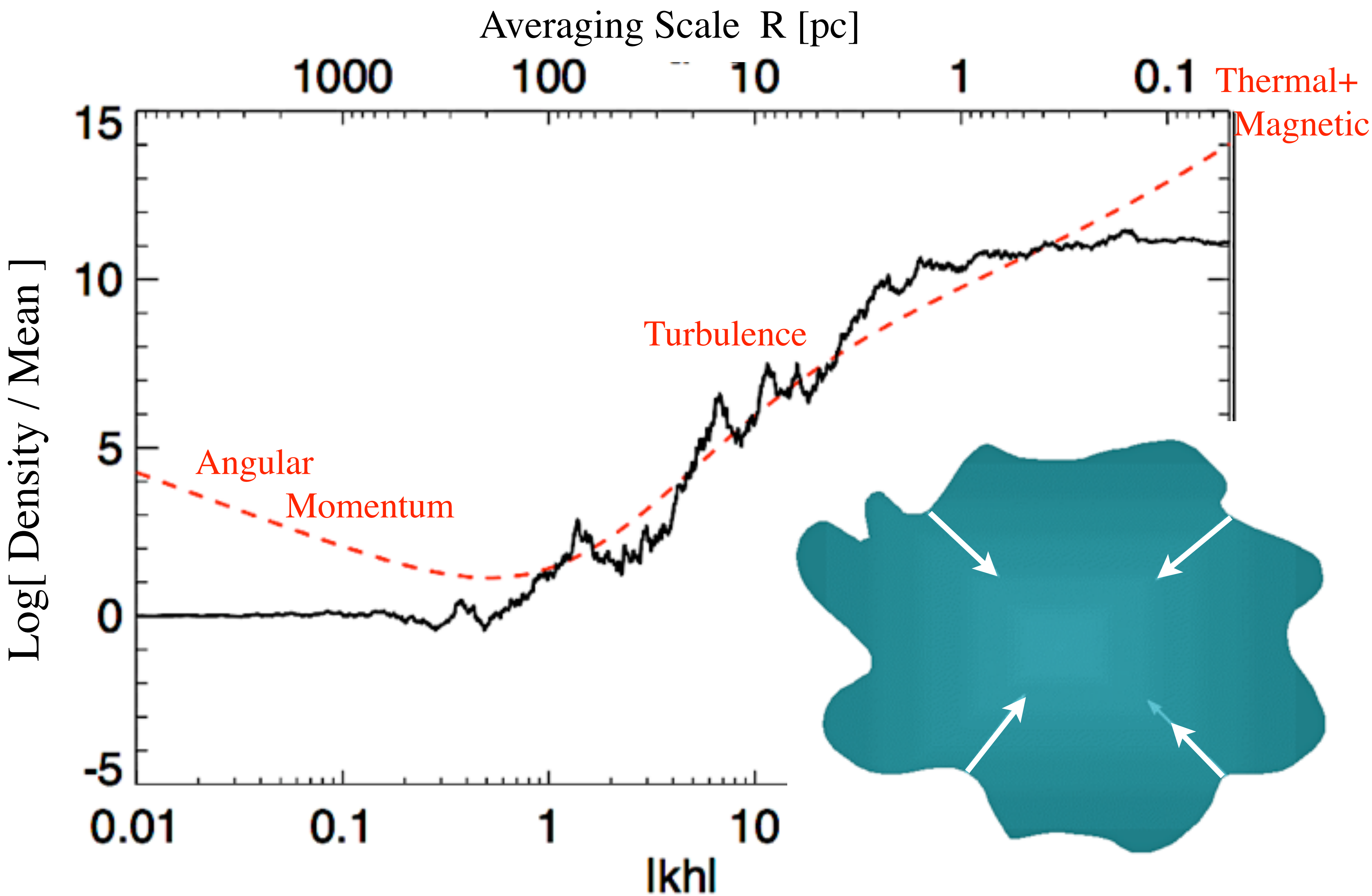

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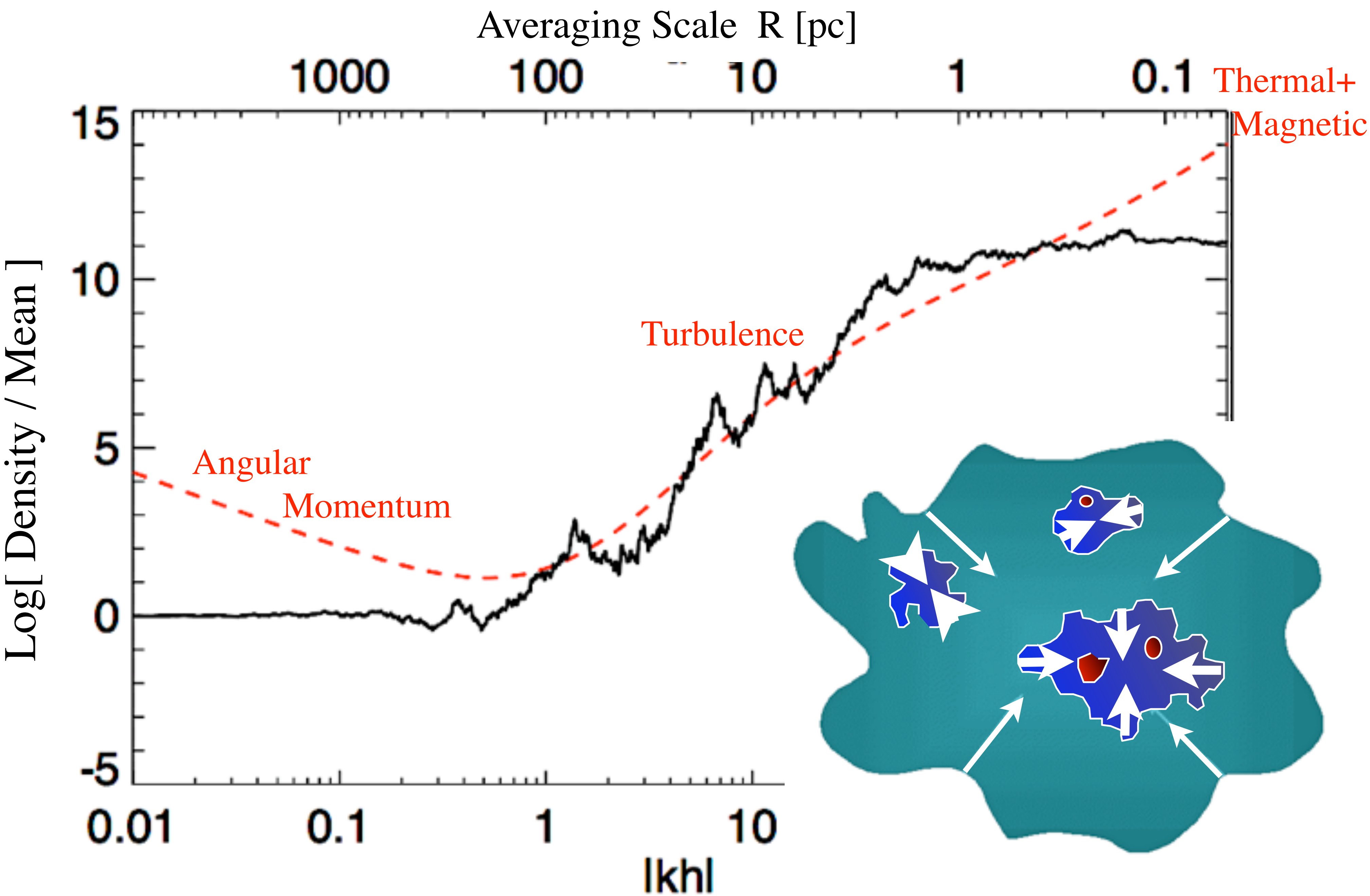

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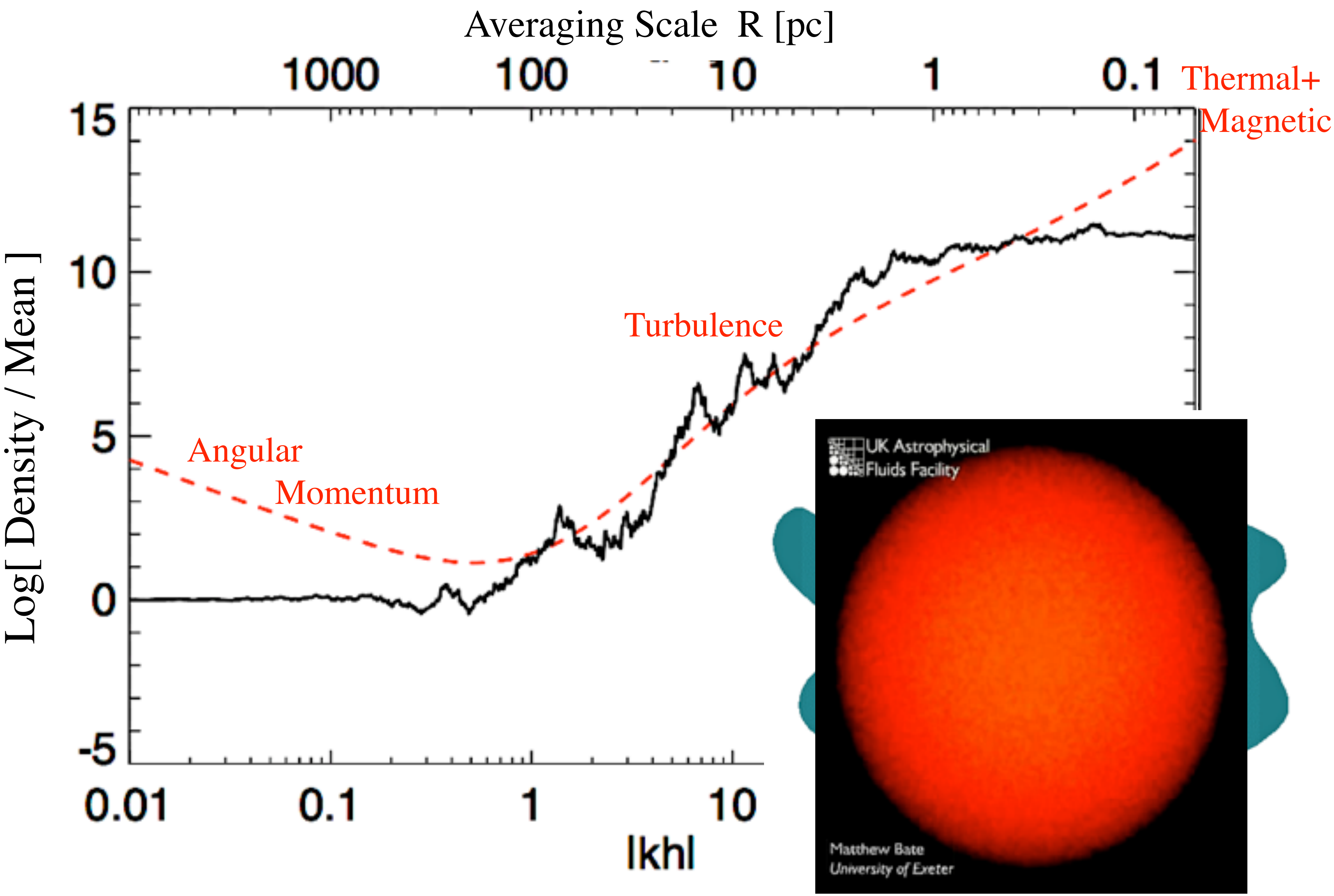

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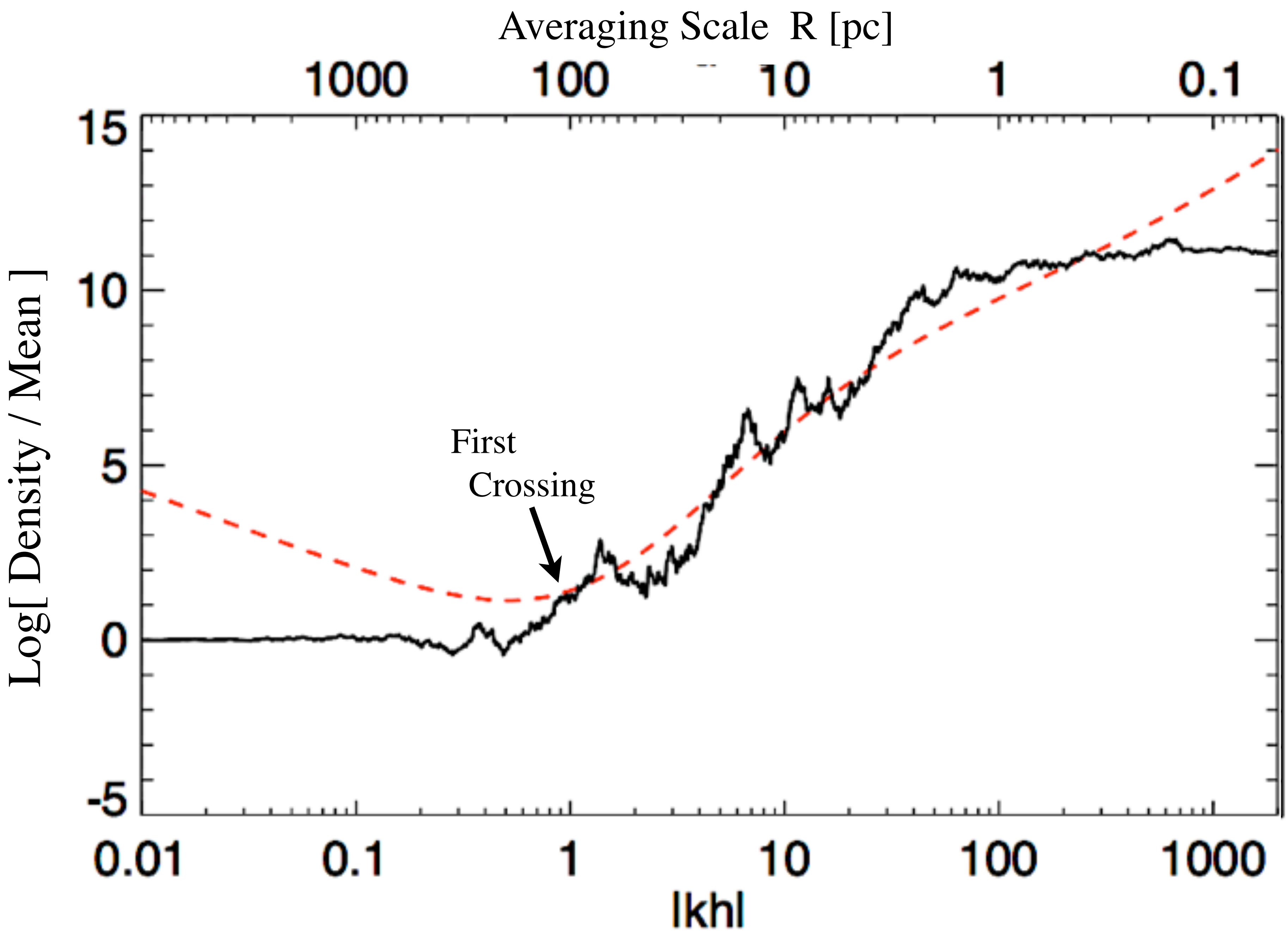

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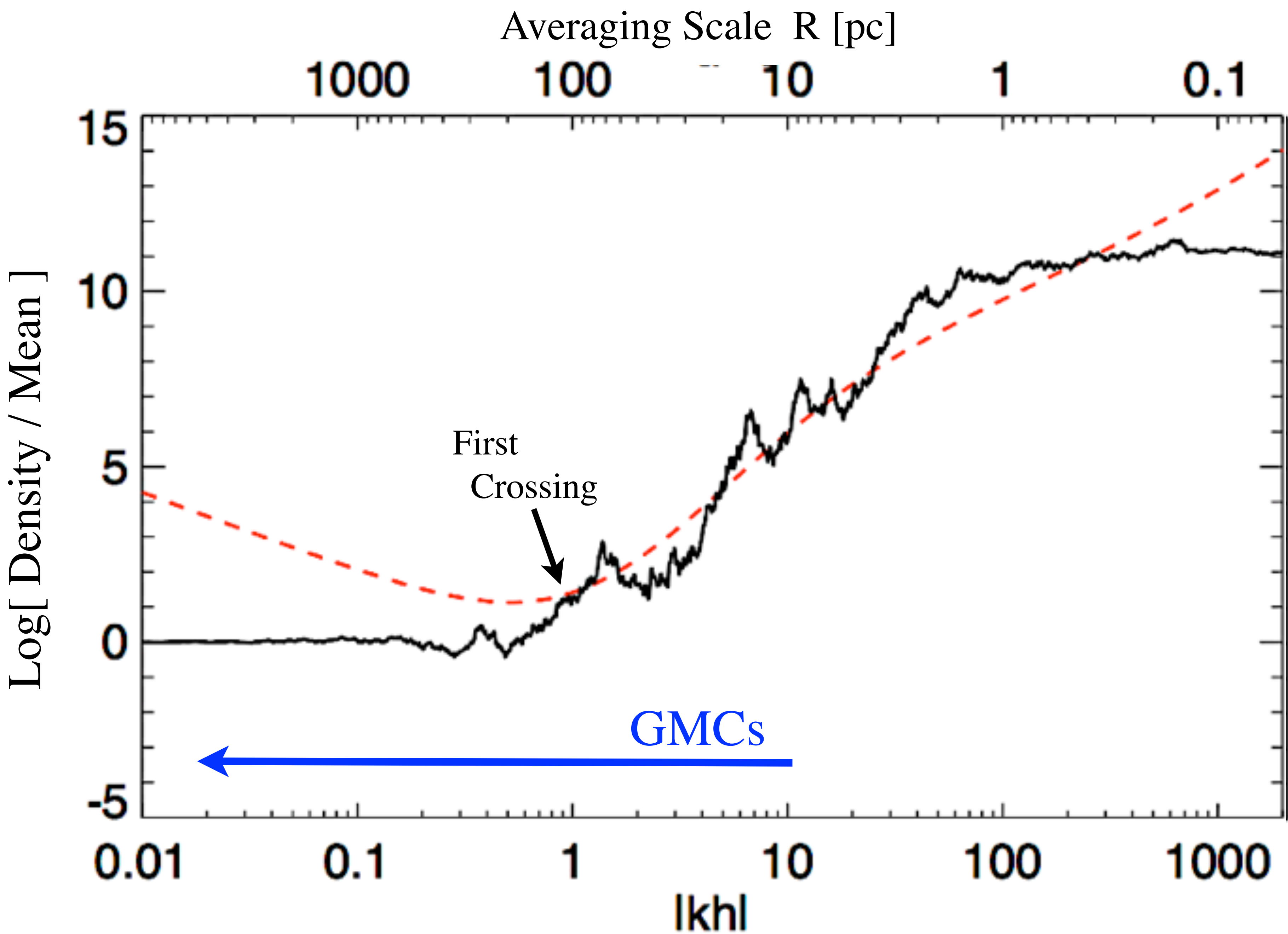

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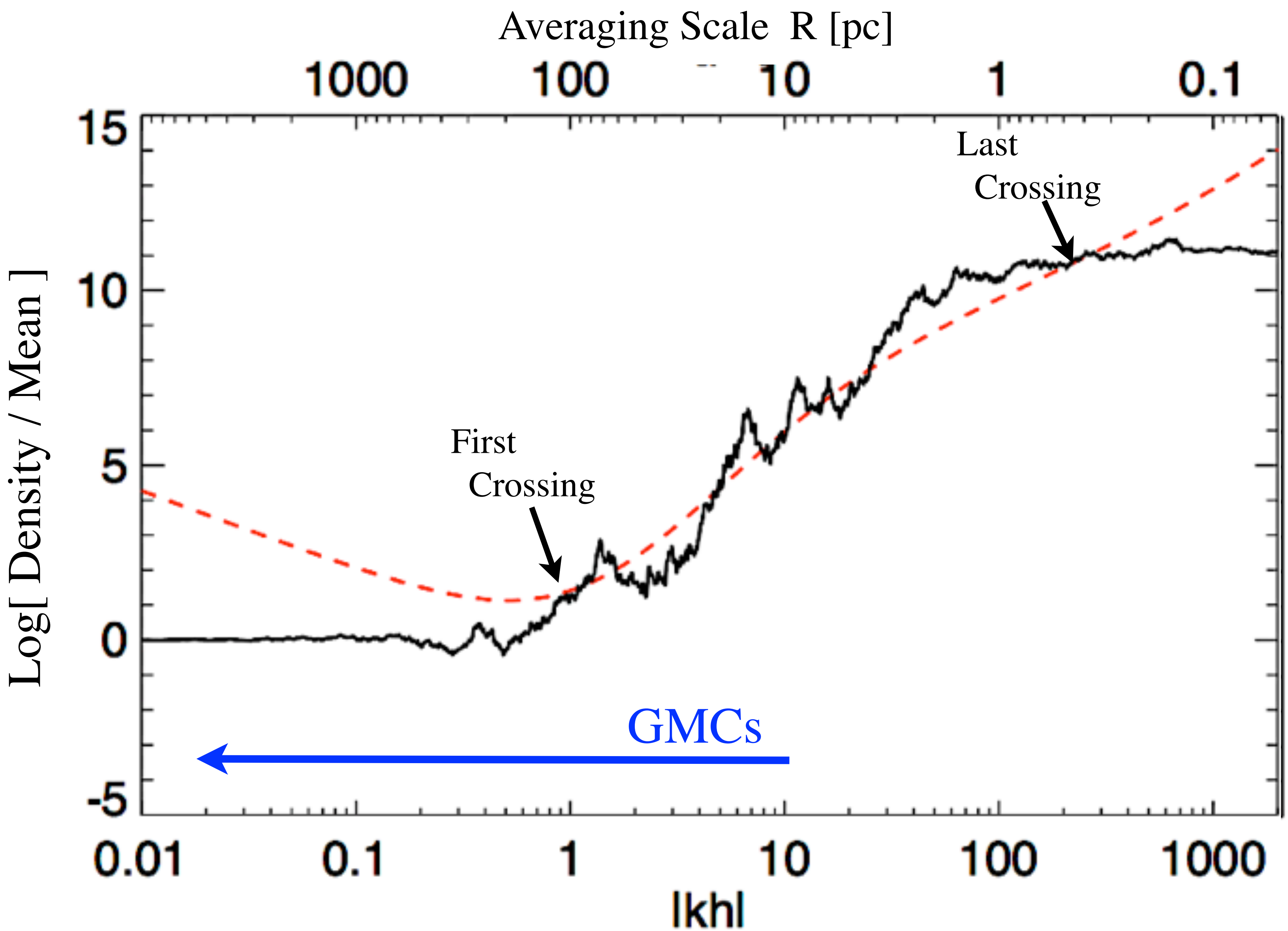

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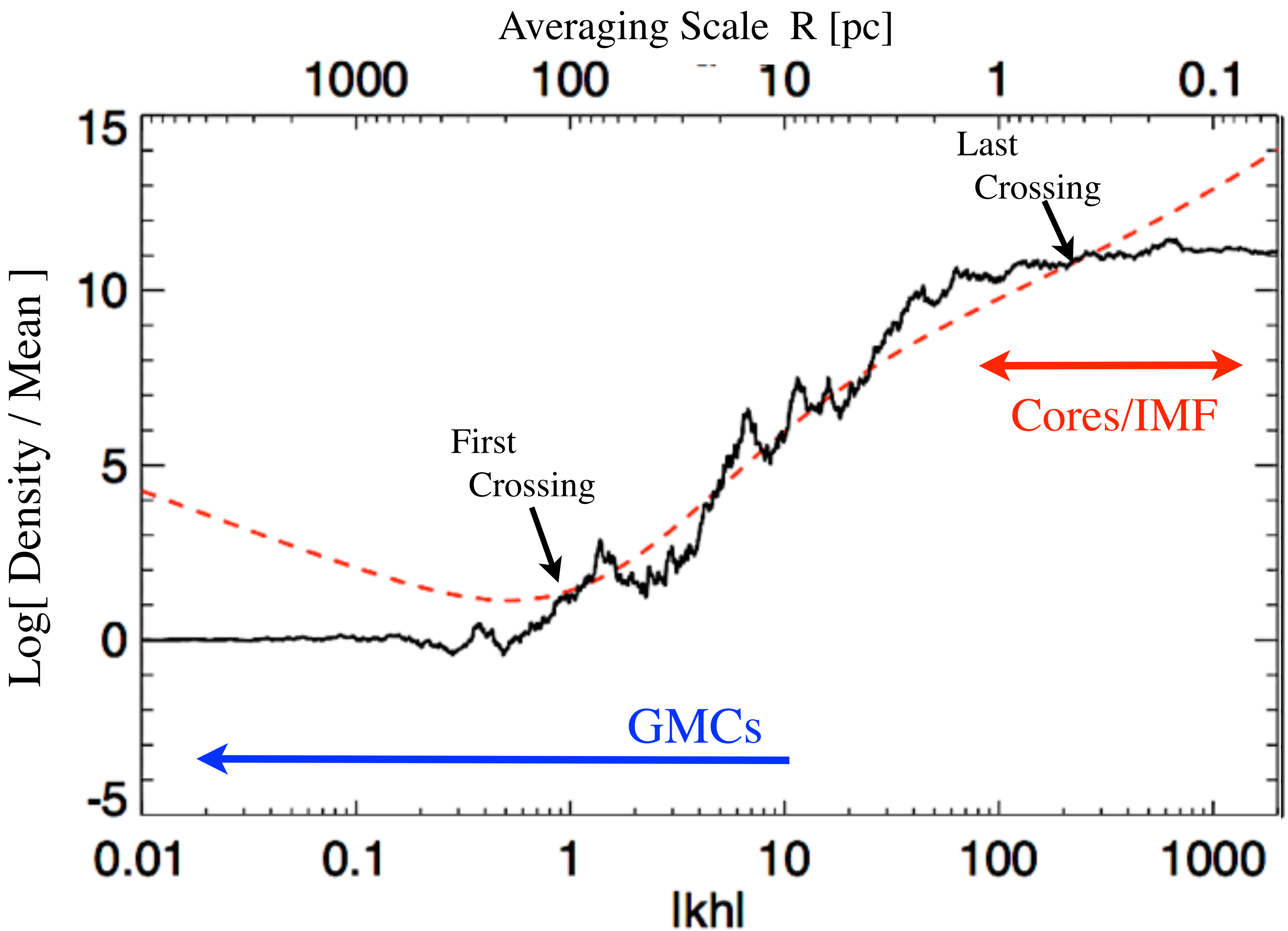
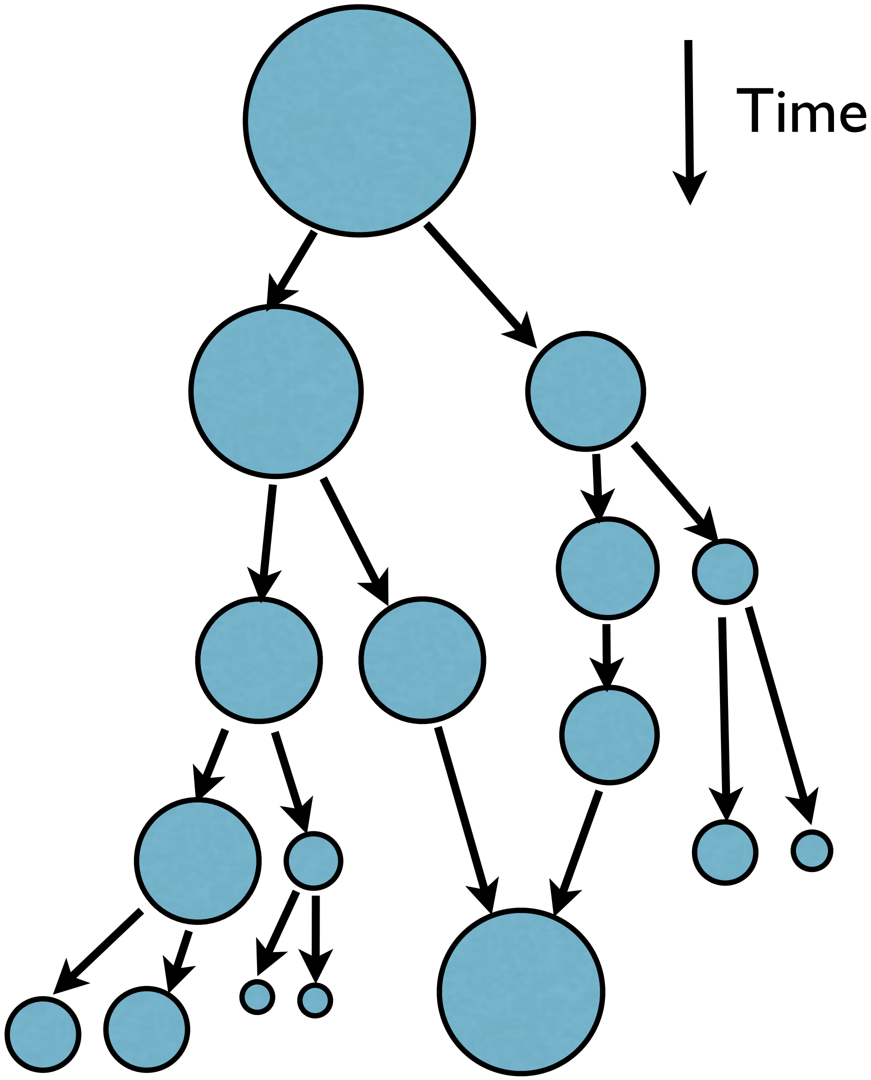


Evolve the Fluctuations in Time

CONSTRUCT "MERGER/FRAGMENTATION" TREES

$$p(\delta\,|\,\tau) = \frac{1}{\sqrt{2\pi\,S\,(1-\exp{[-2\tau]})}}\,\exp\left[-\frac{(\delta-\delta(t=0)\,\exp{[-\tau]})^2}{2\,S\,(1-\exp{[-2\tau]})}\right]$$



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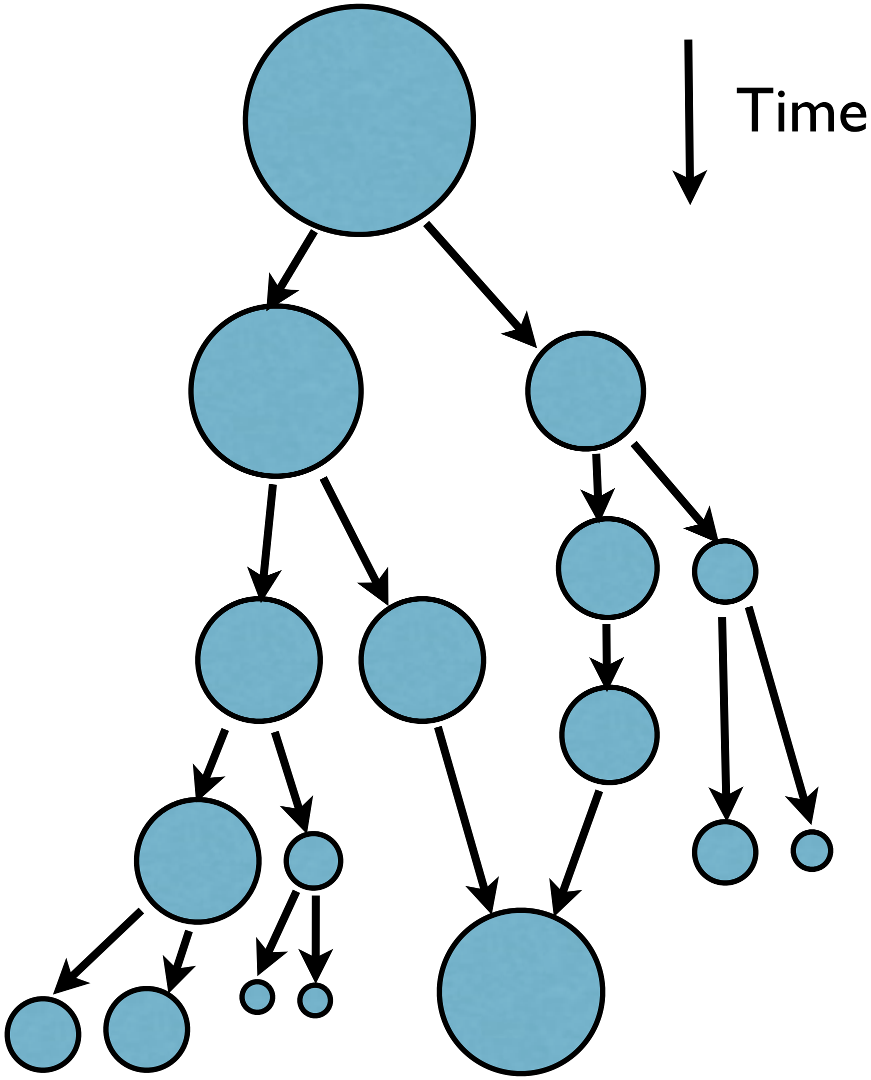
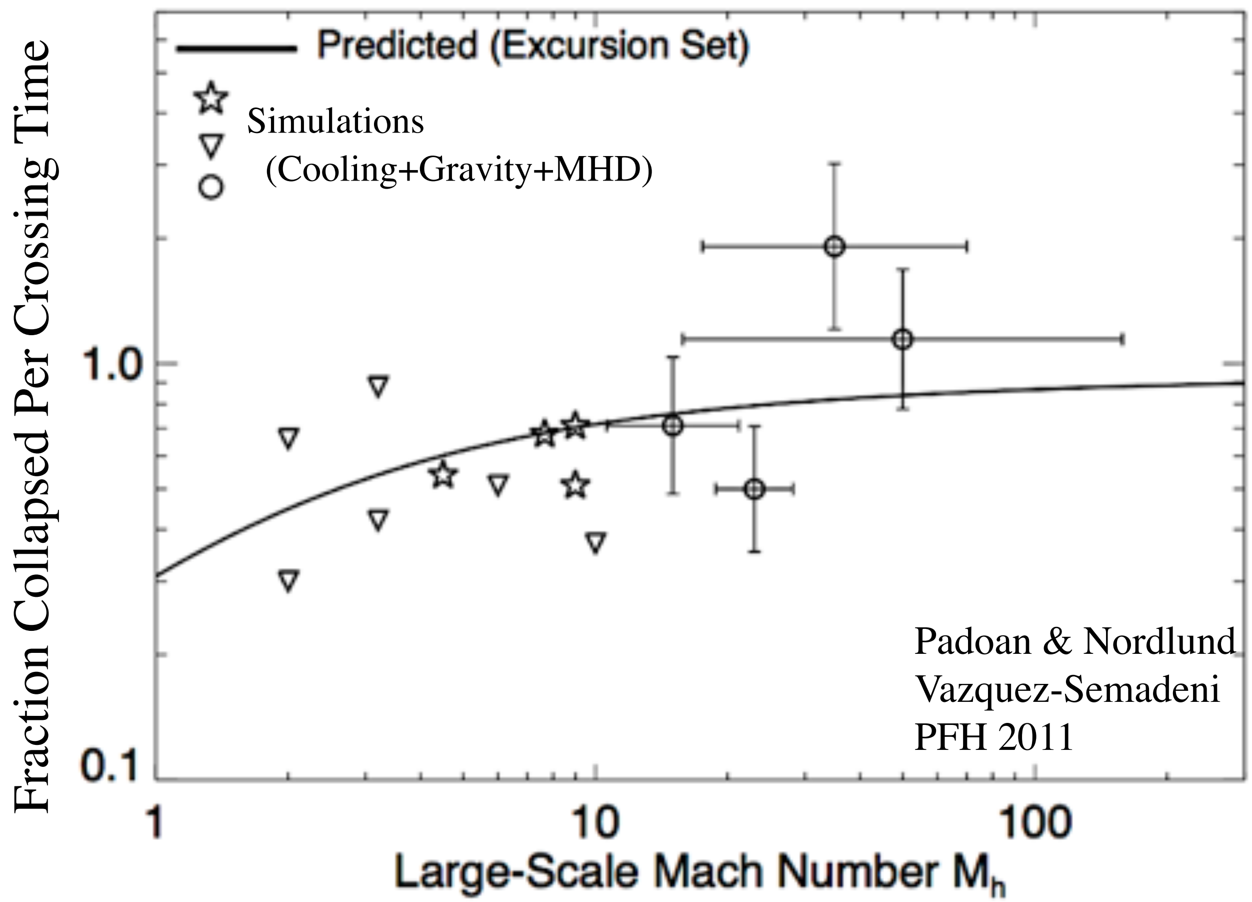
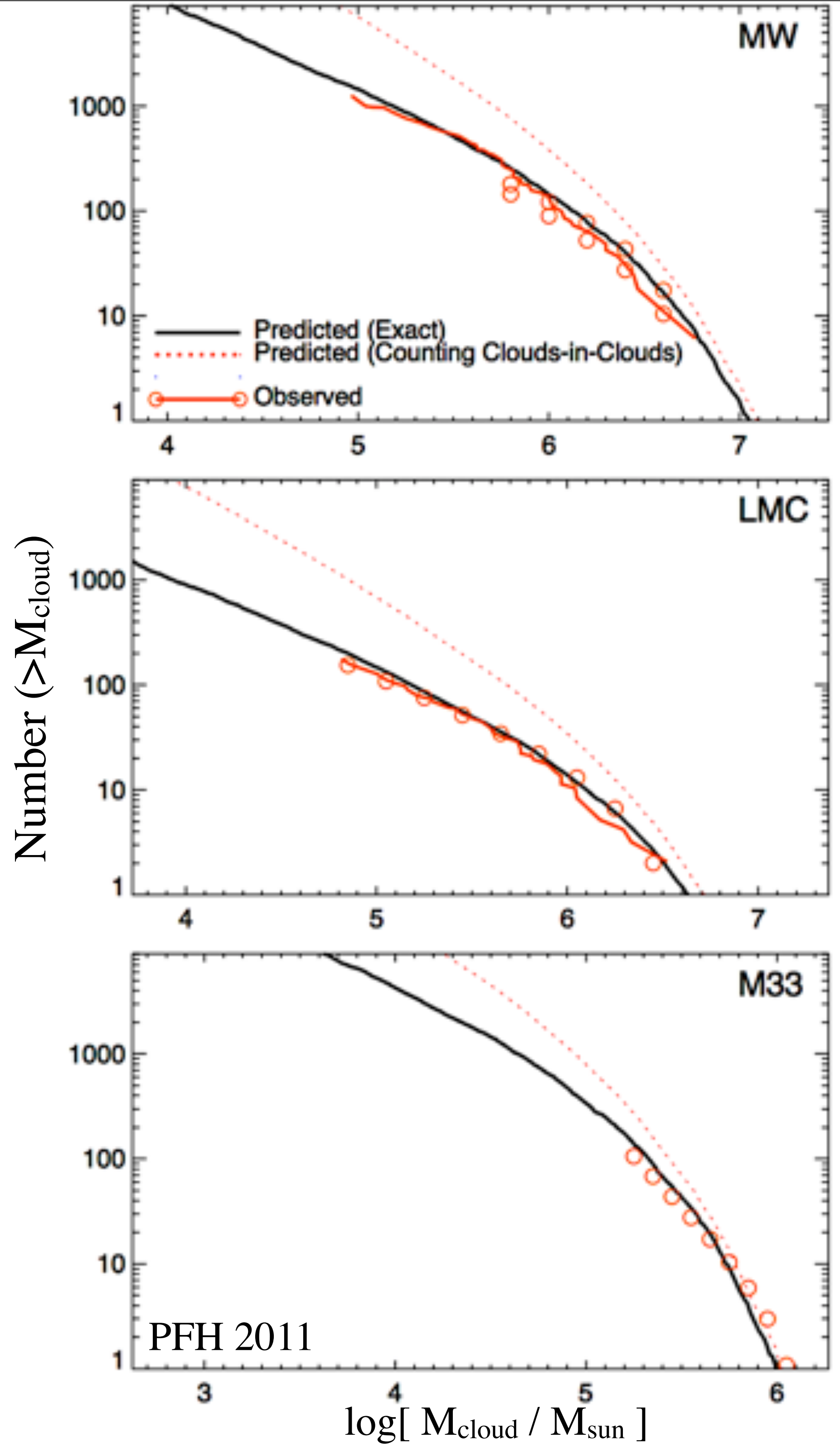




The "First Crossing" Mass Function

VS GIANT MOLECULAR CLOUDS



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$$r_{\rm sonic} \ll r \ll h$$

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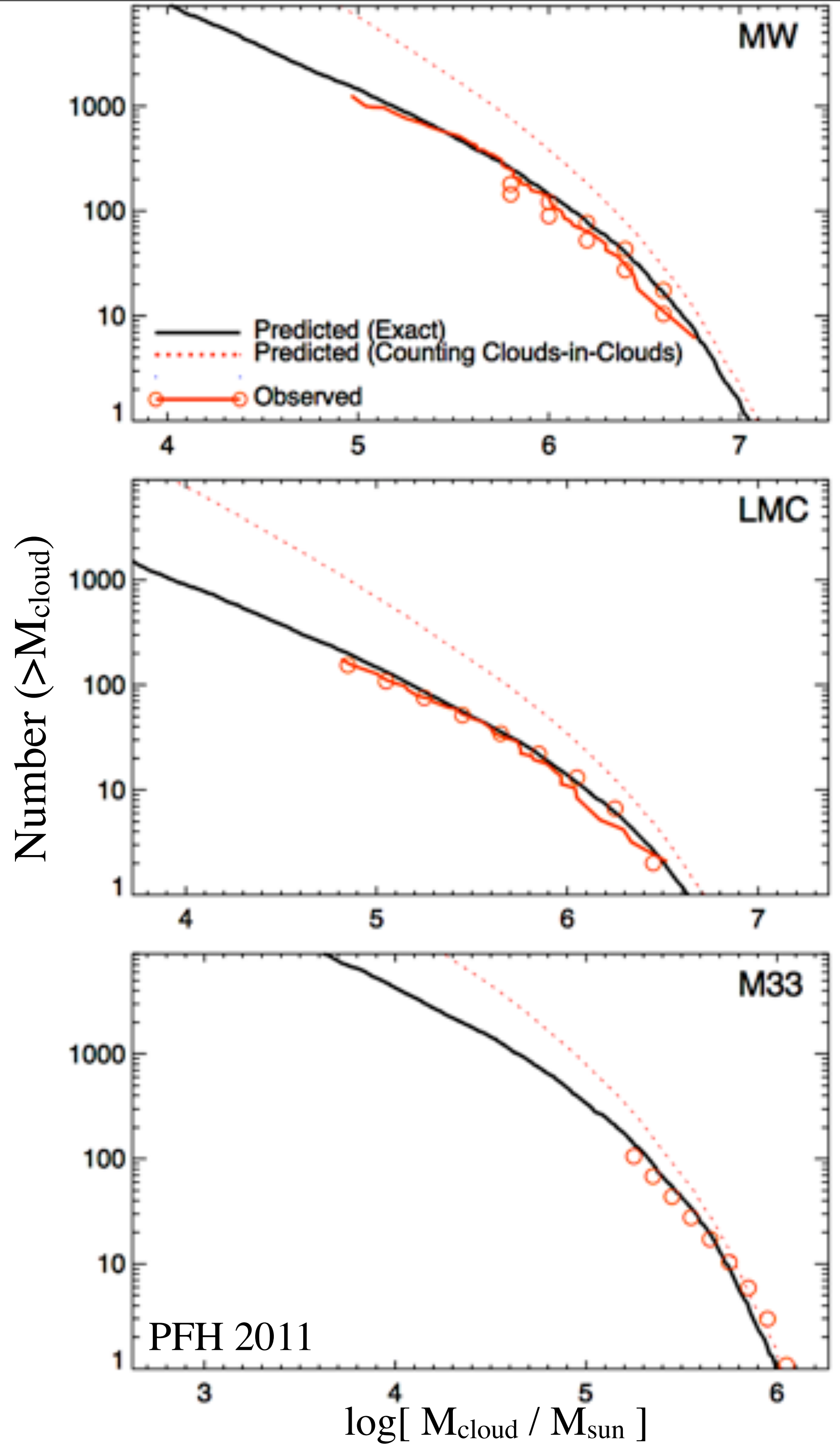

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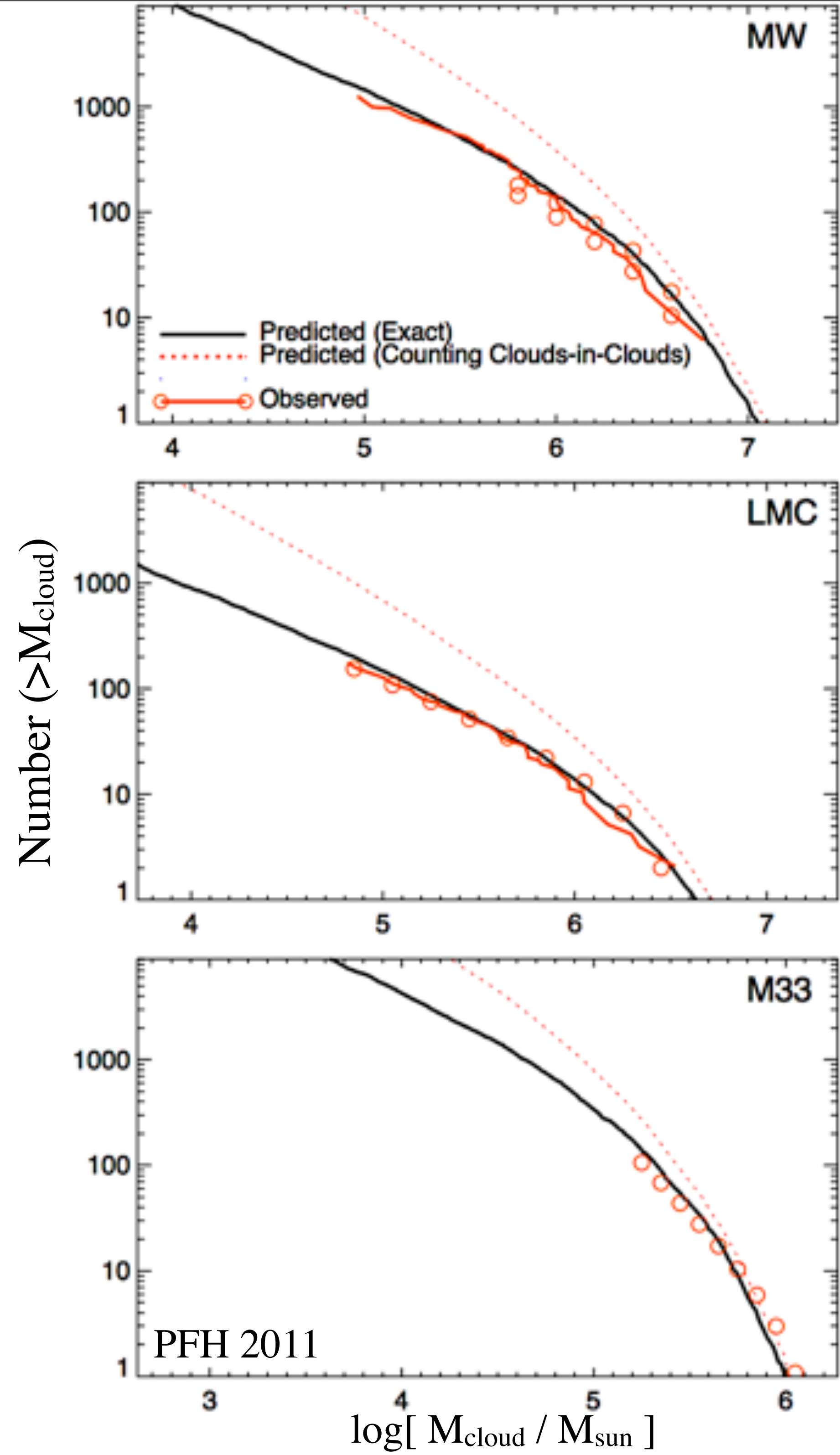

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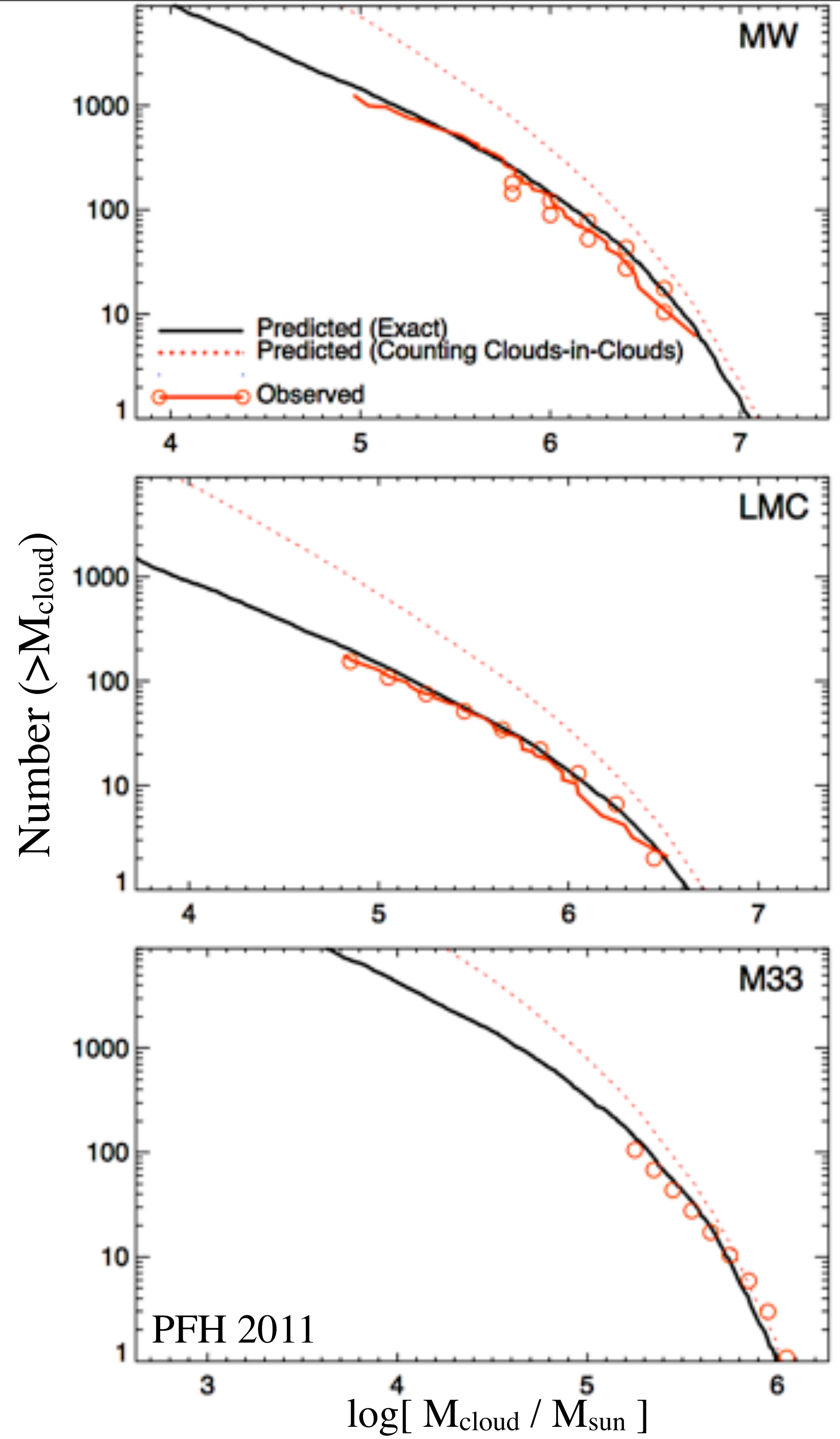
$$\alpha \approx -2 + \frac{(3-p)^2}{2\,S\,p^2}\ln\left(\frac{M_J}{M}\right)$$

$$\approx -2 + 0.1\,\log\left(\frac{M_J}{M}\right)$$



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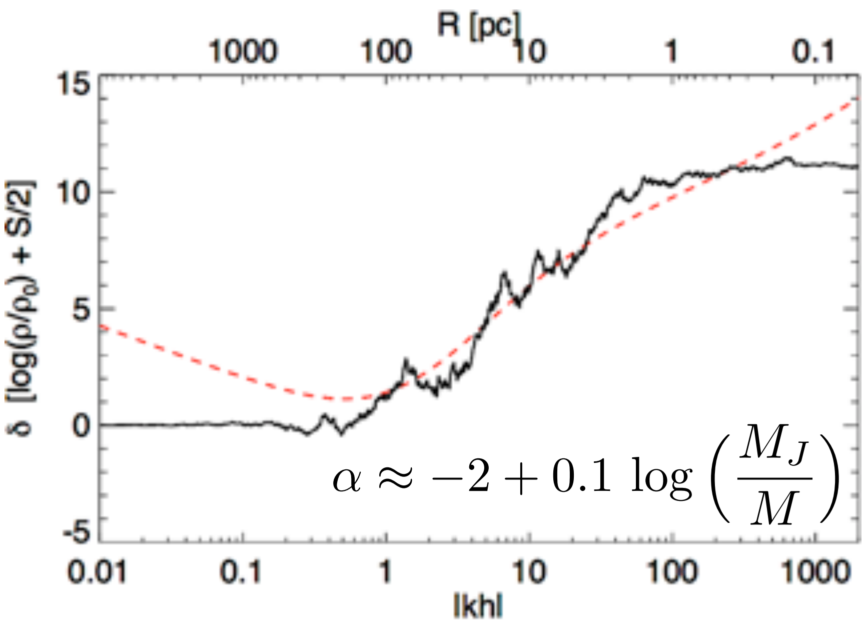

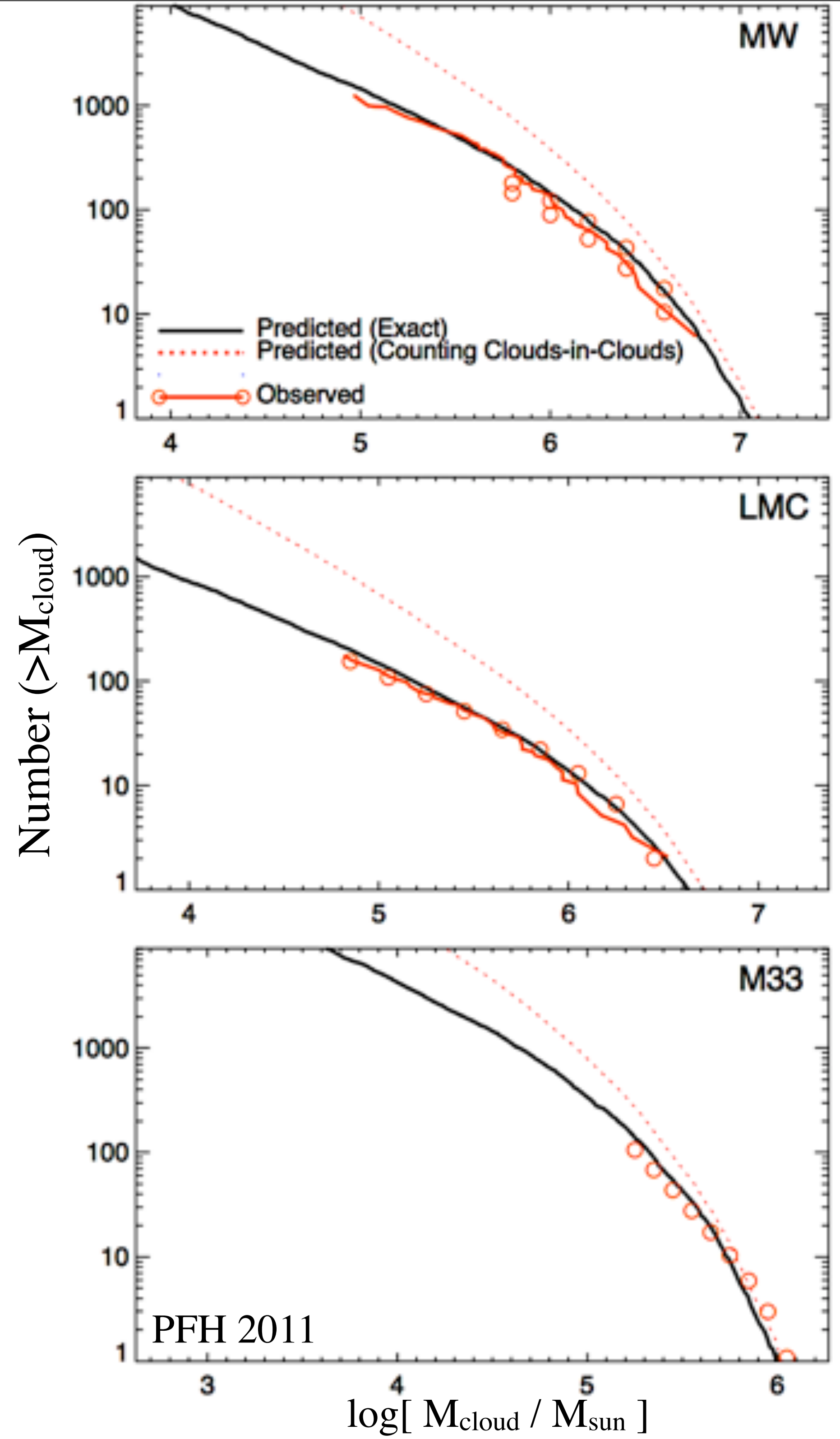

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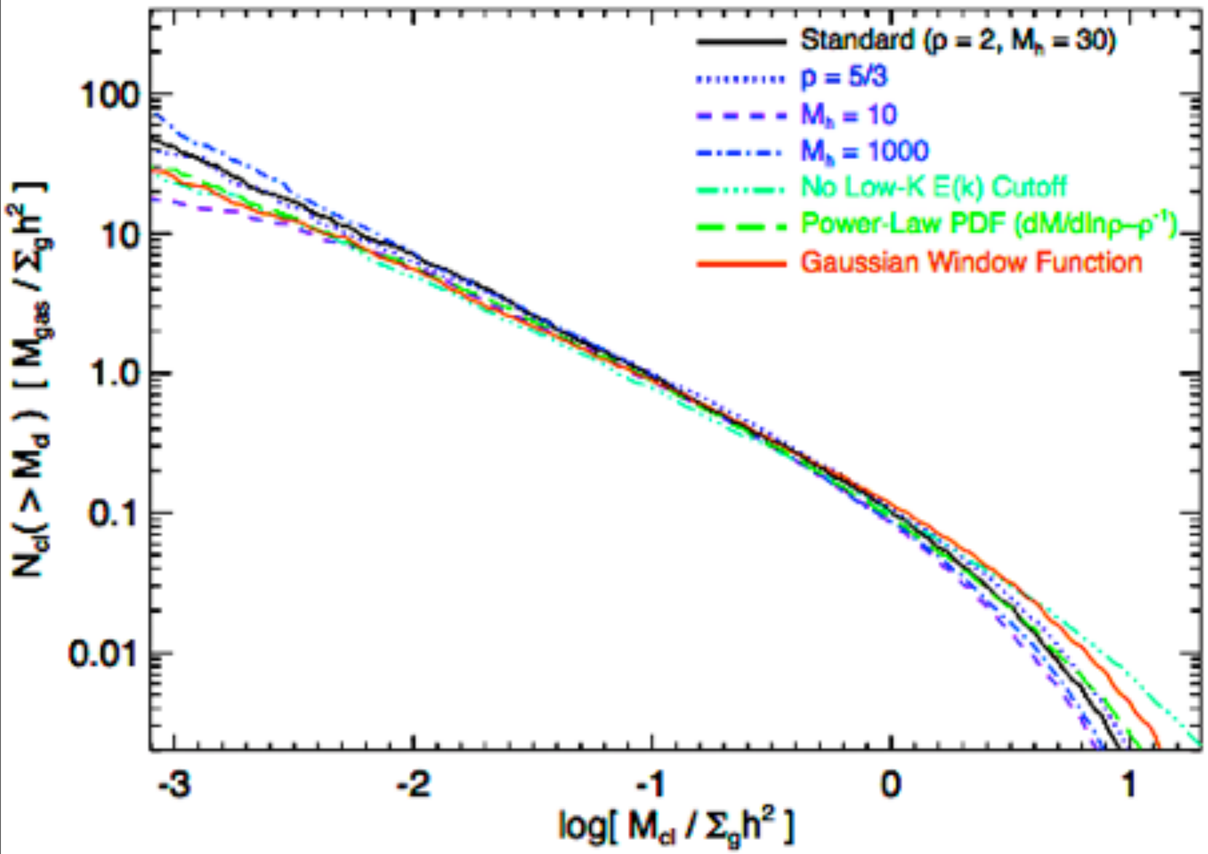
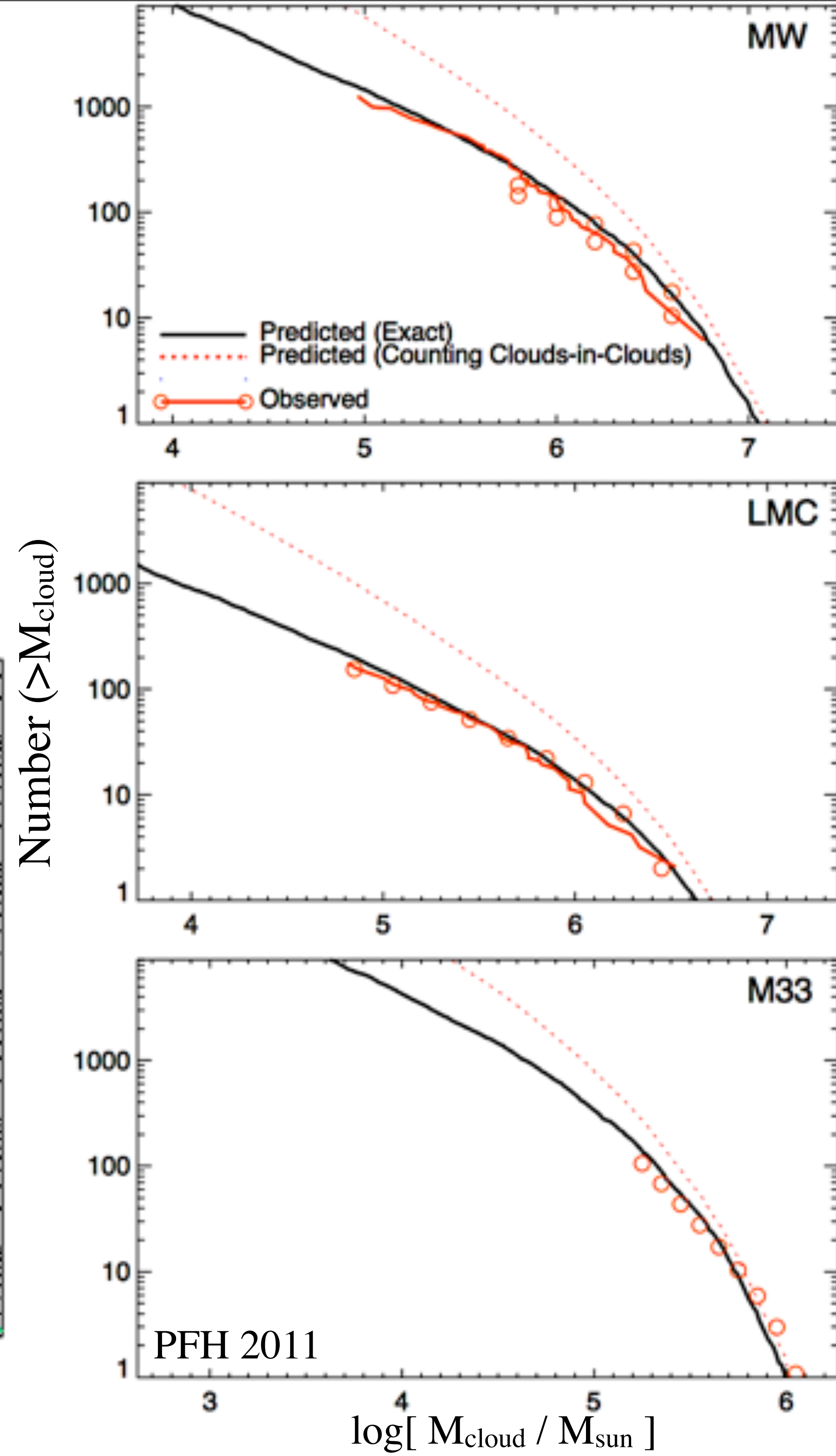
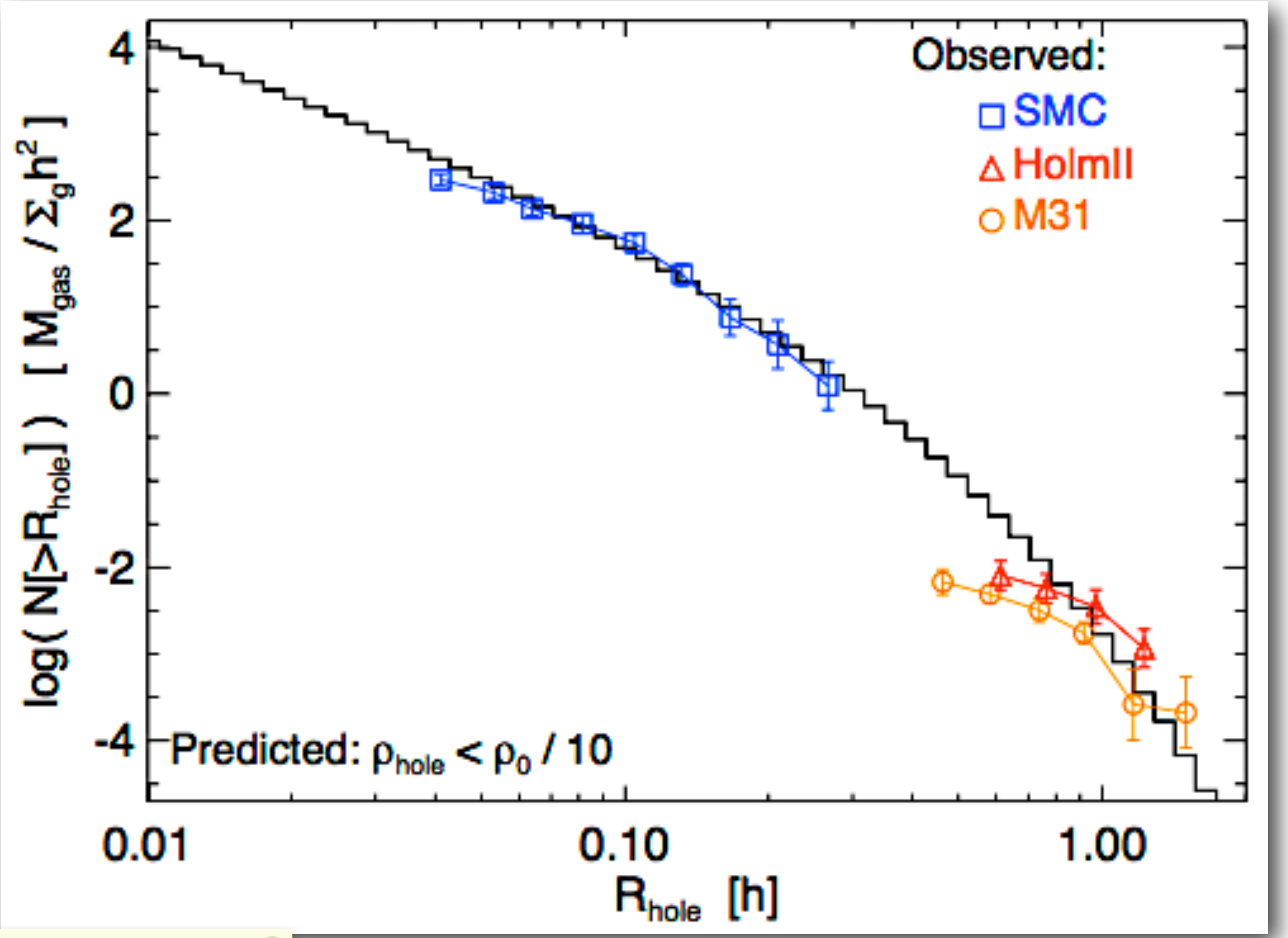




"Void" Abundance

VS HI "HOLES" IN THE ISM

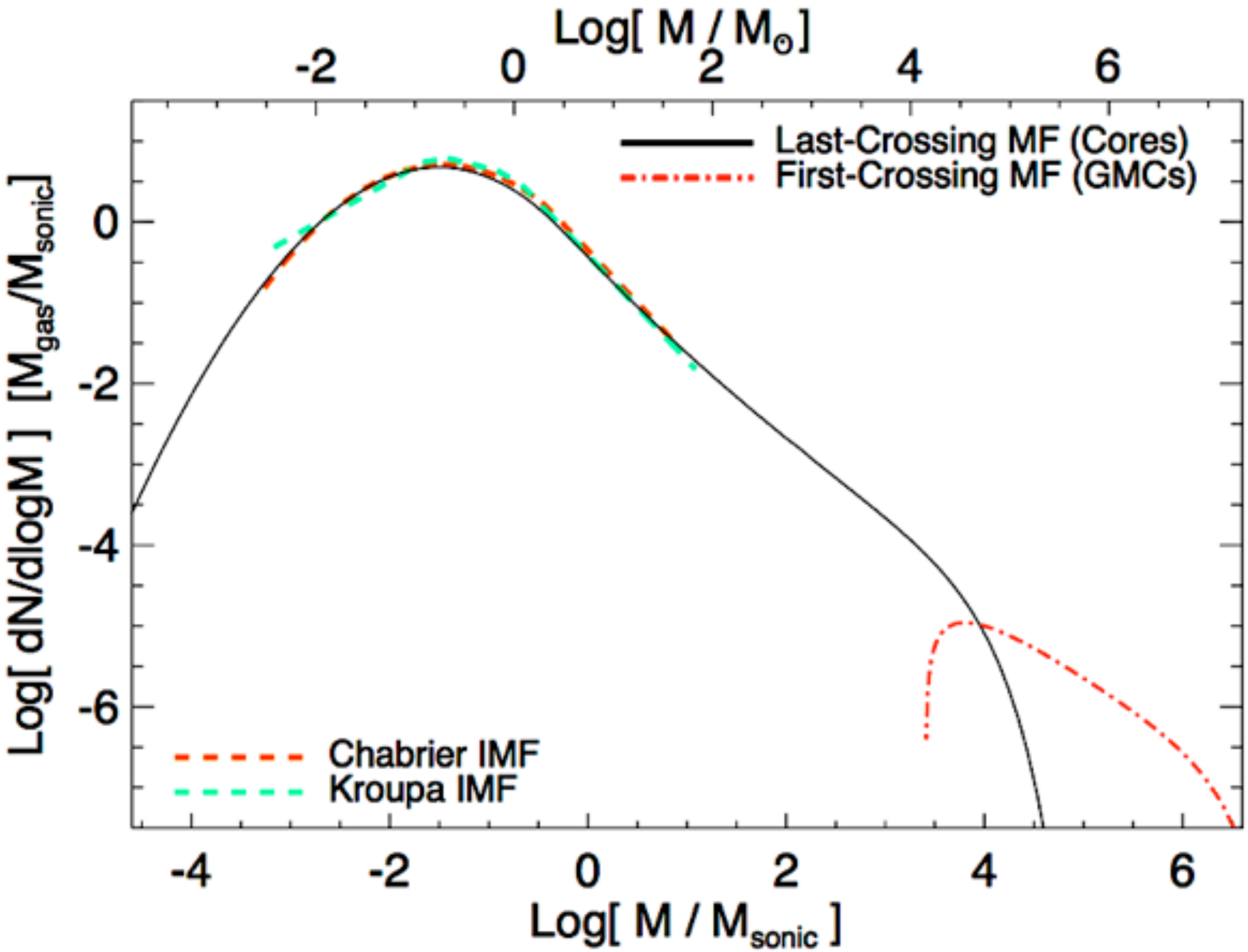


KEEP?

Don't need SNe to "carve out" voids

The "Last Crossing" Mass Function

VS PROTOSTELLAR CORES & THE STELLAR IMF



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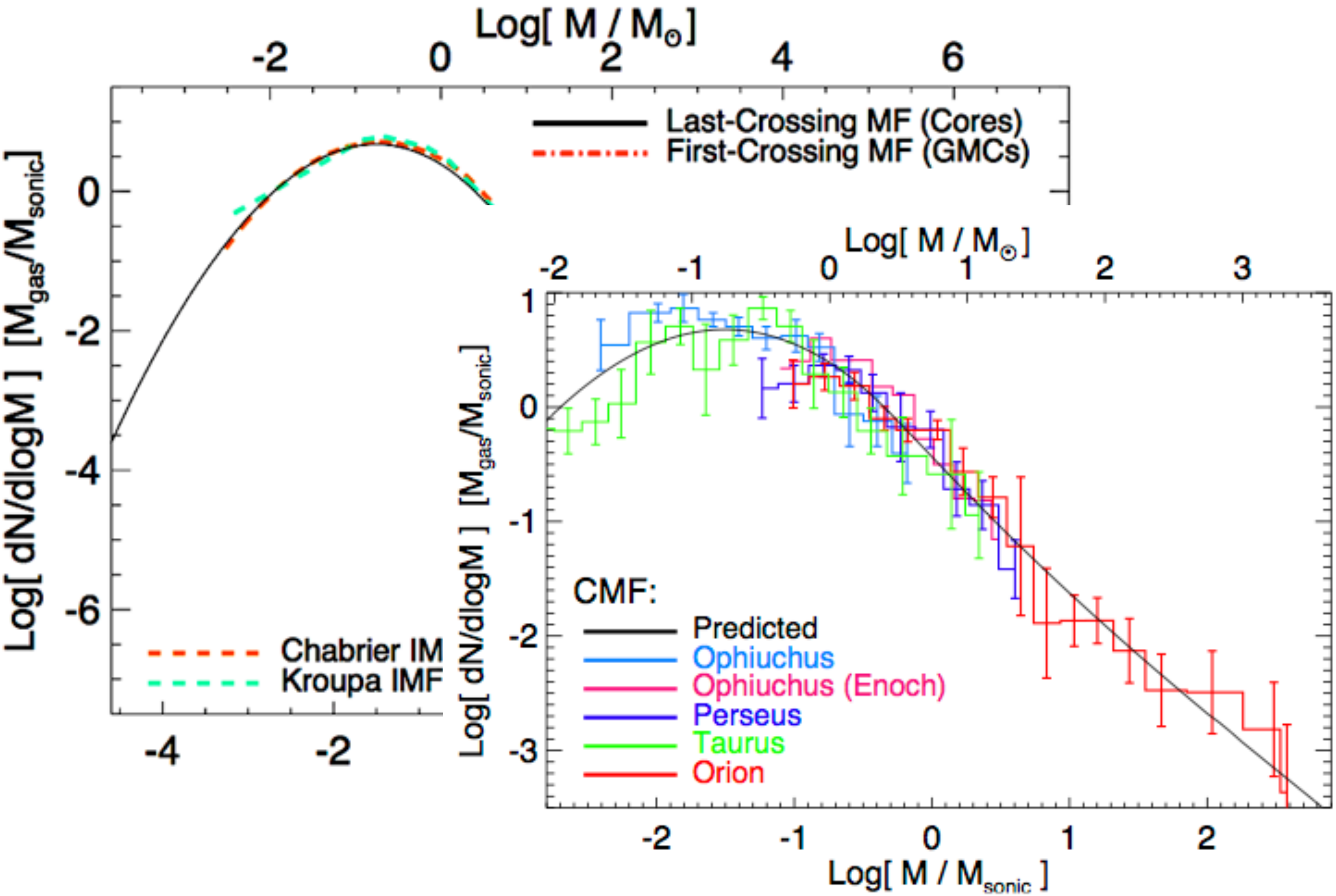
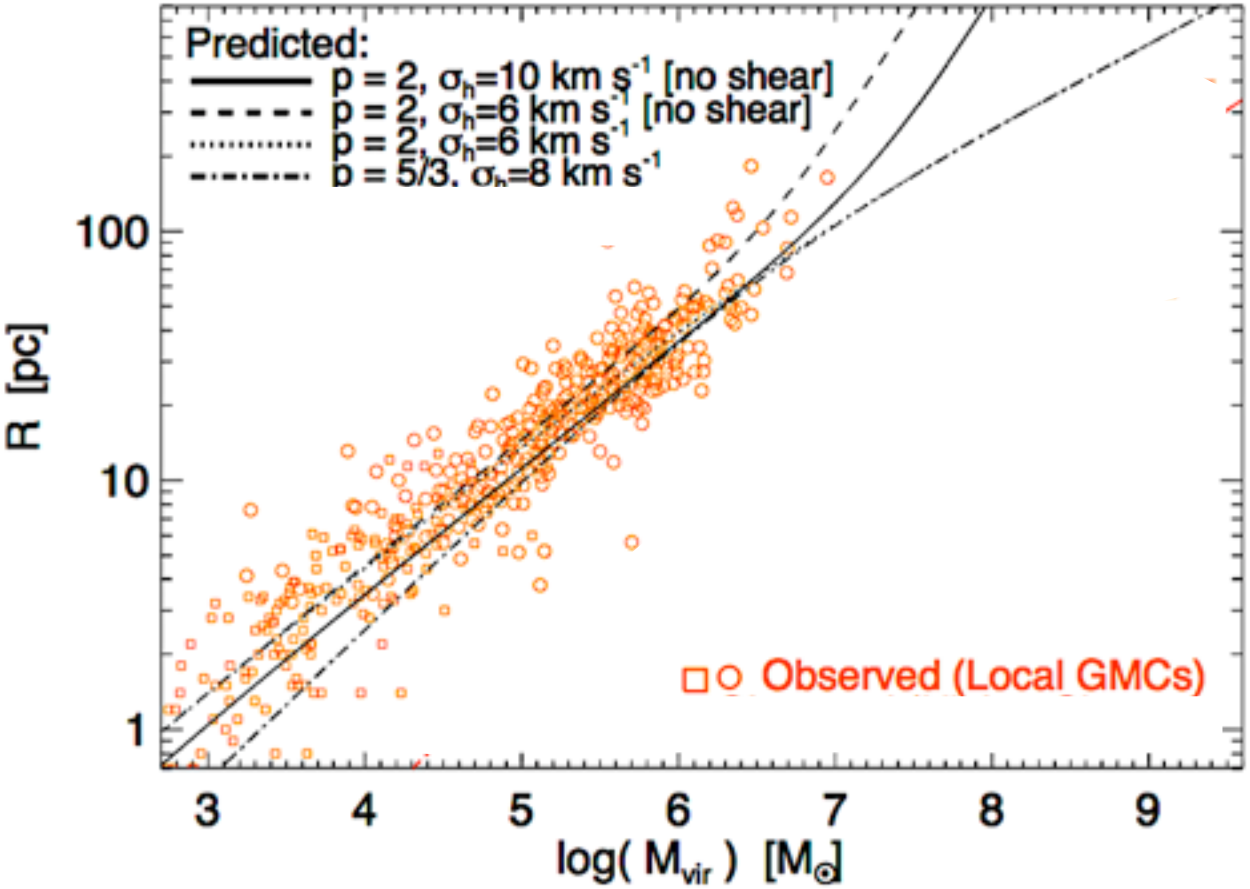


Structural Properties of “Clouds”

LARSON’S LAWS EMERGE NATURALLY



KEEP?

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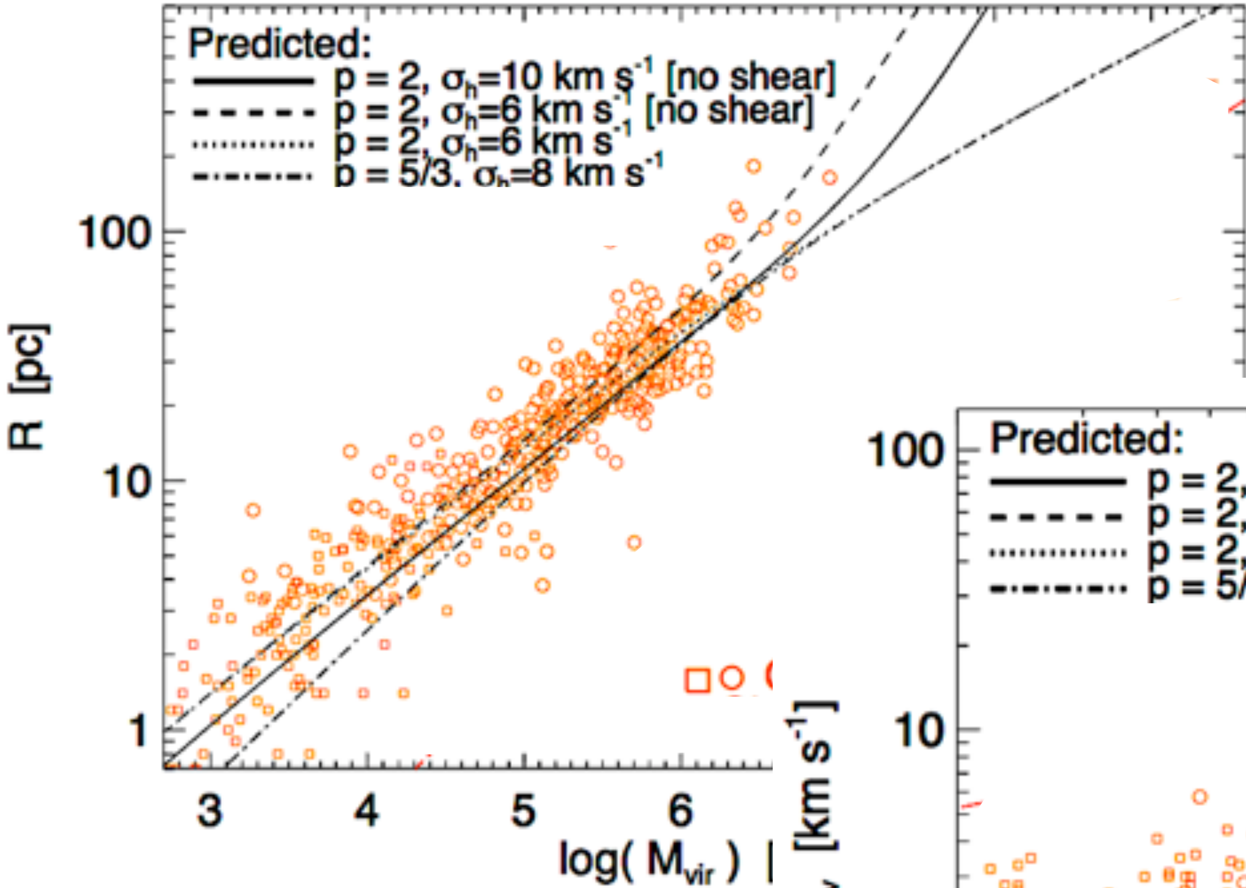
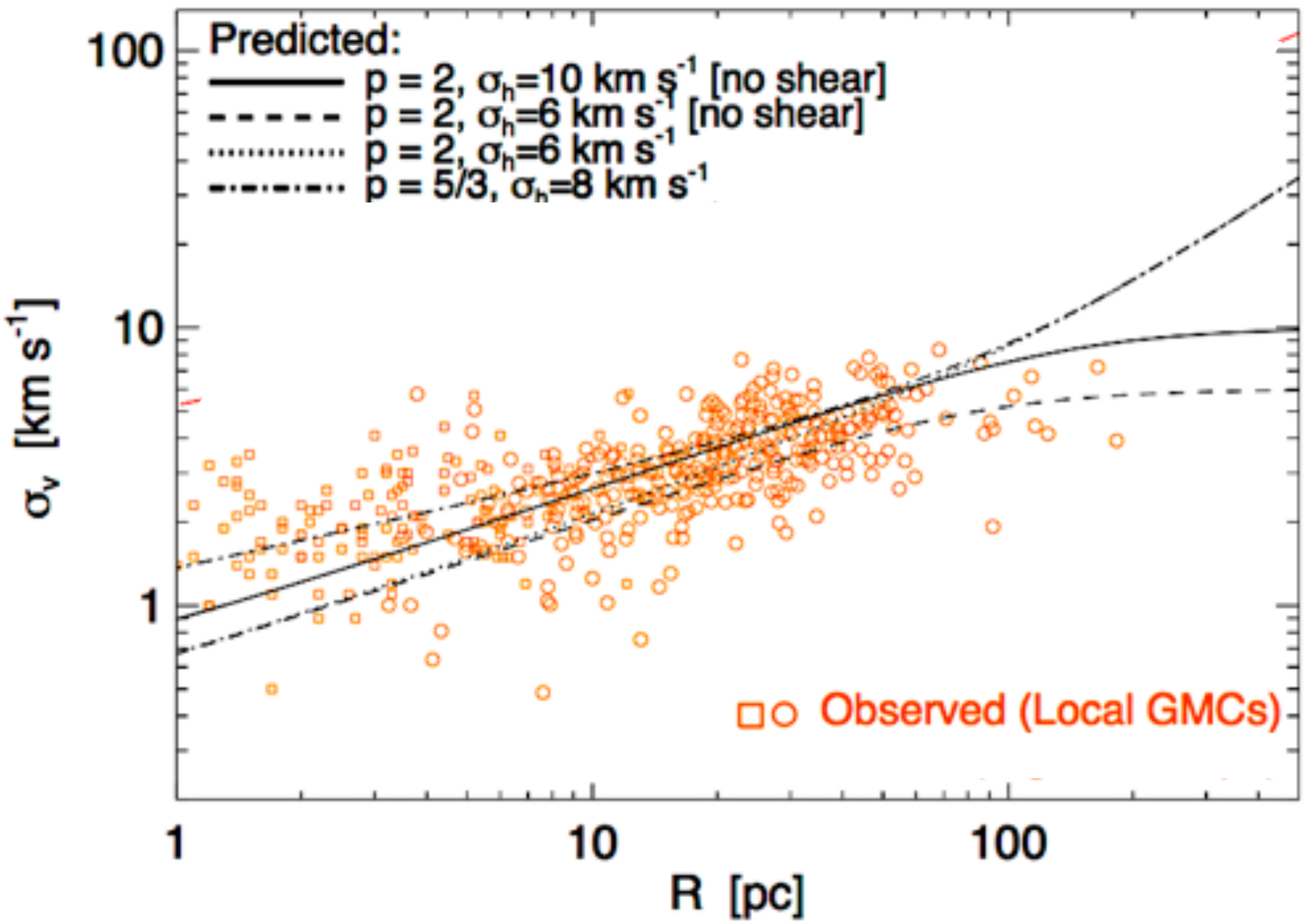




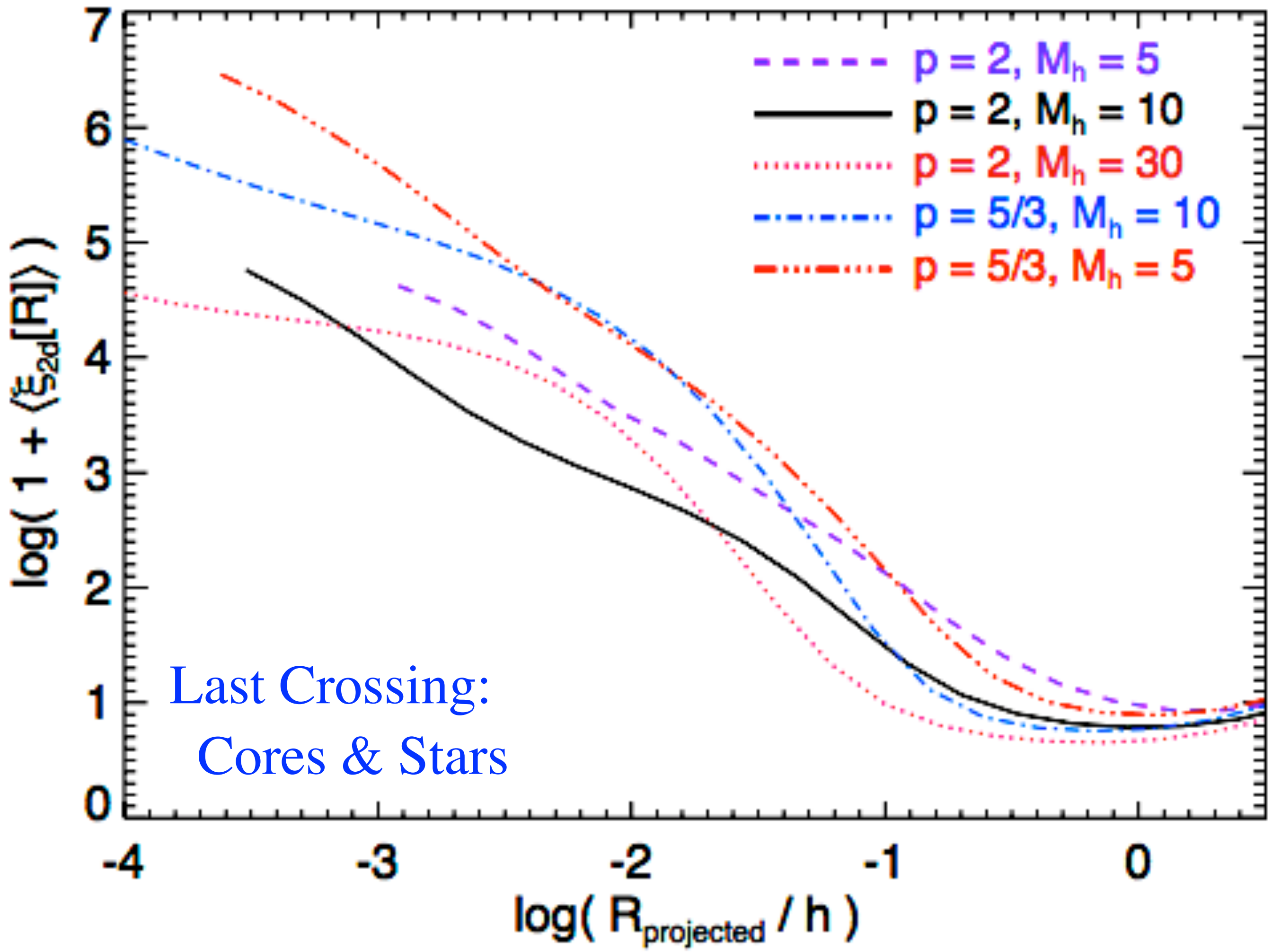
KEEP?

Clustering

PREDICT N-POINT CORRELATION FUNCTIONS

$$1 + \xi(r\,|\,M) \equiv \frac{\langle n[M\,|\,r' < r]\rangle}{\langle n[M]\rangle}$$

PFH 2012b



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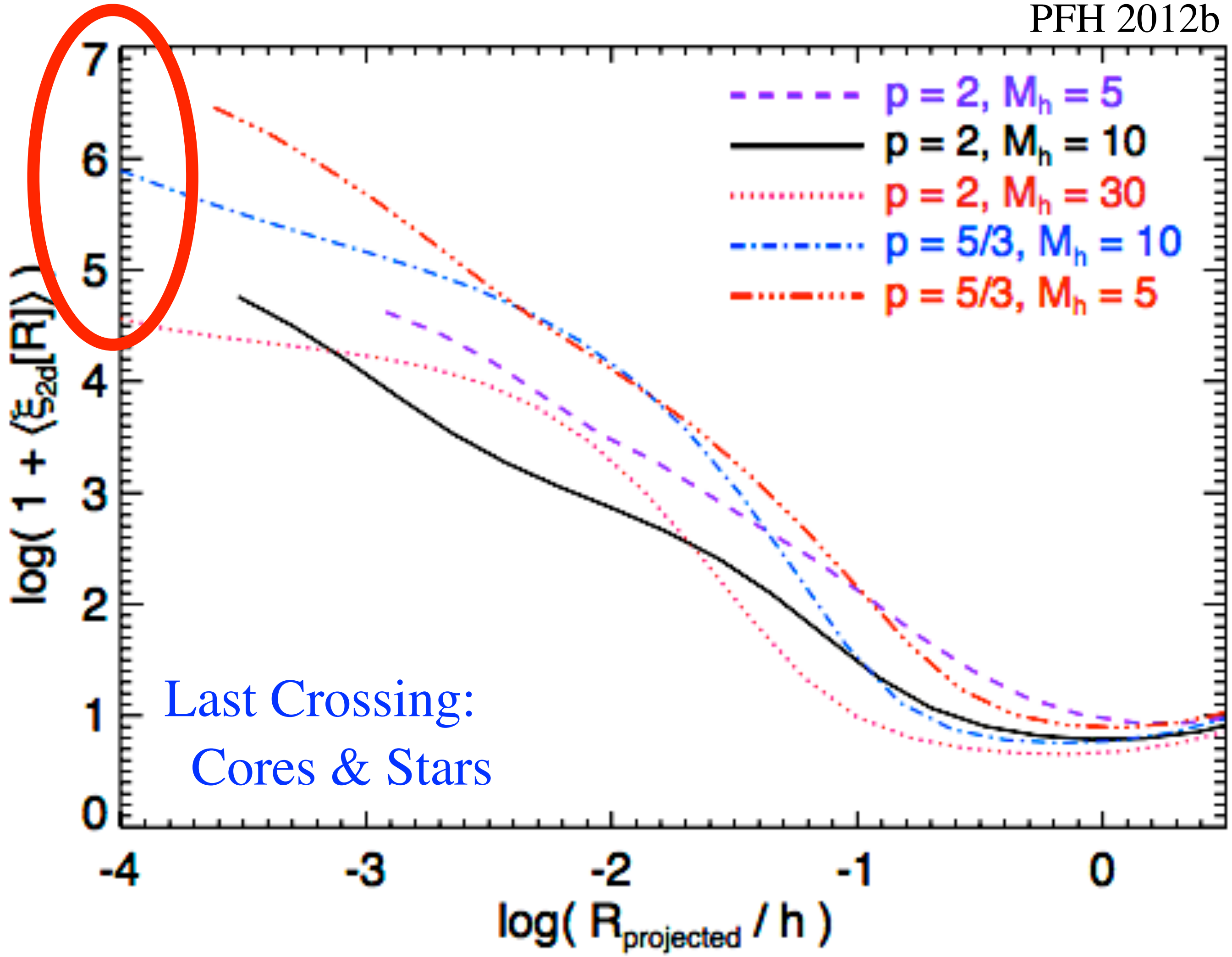
PFH 2012b



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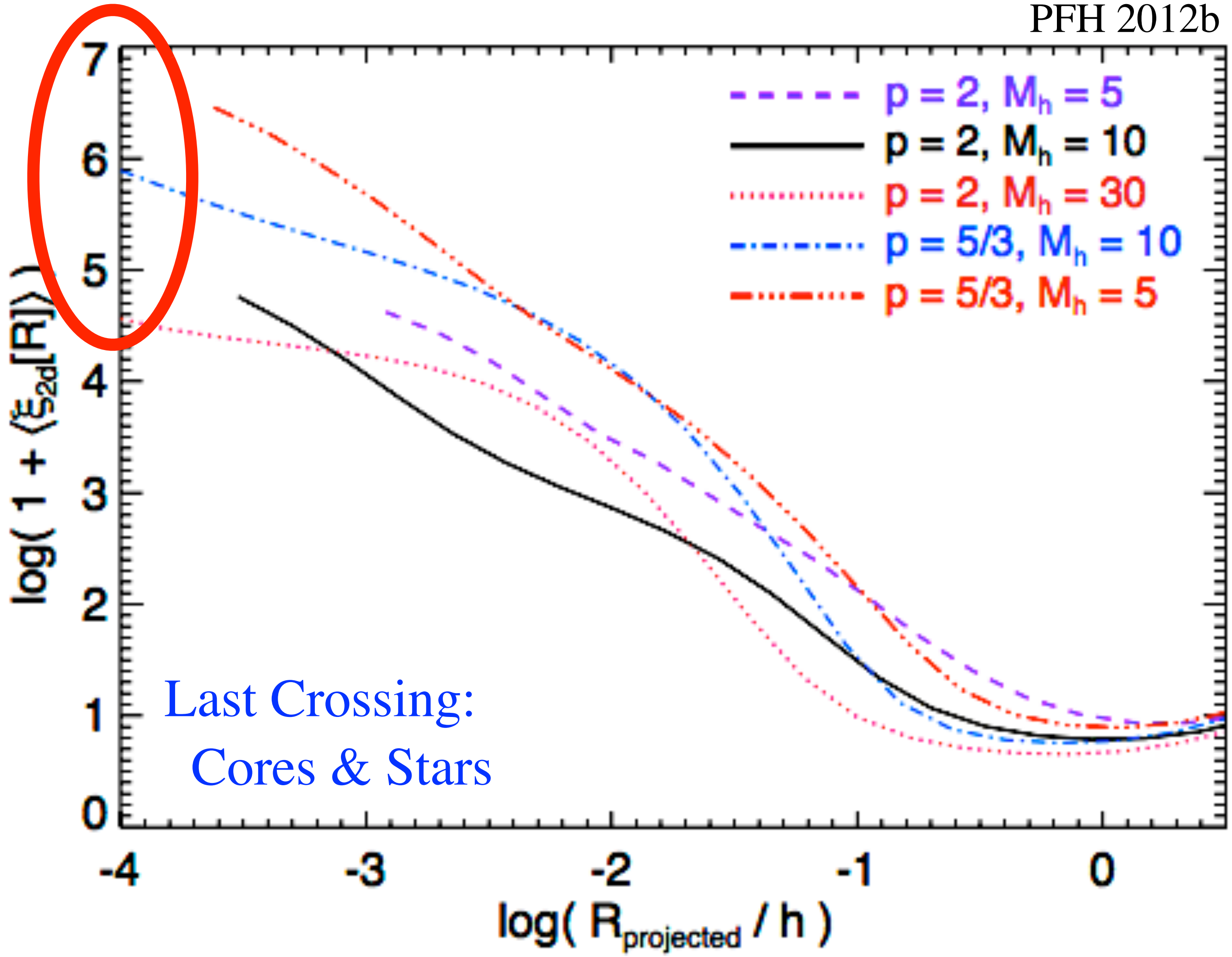
PFH 2012b

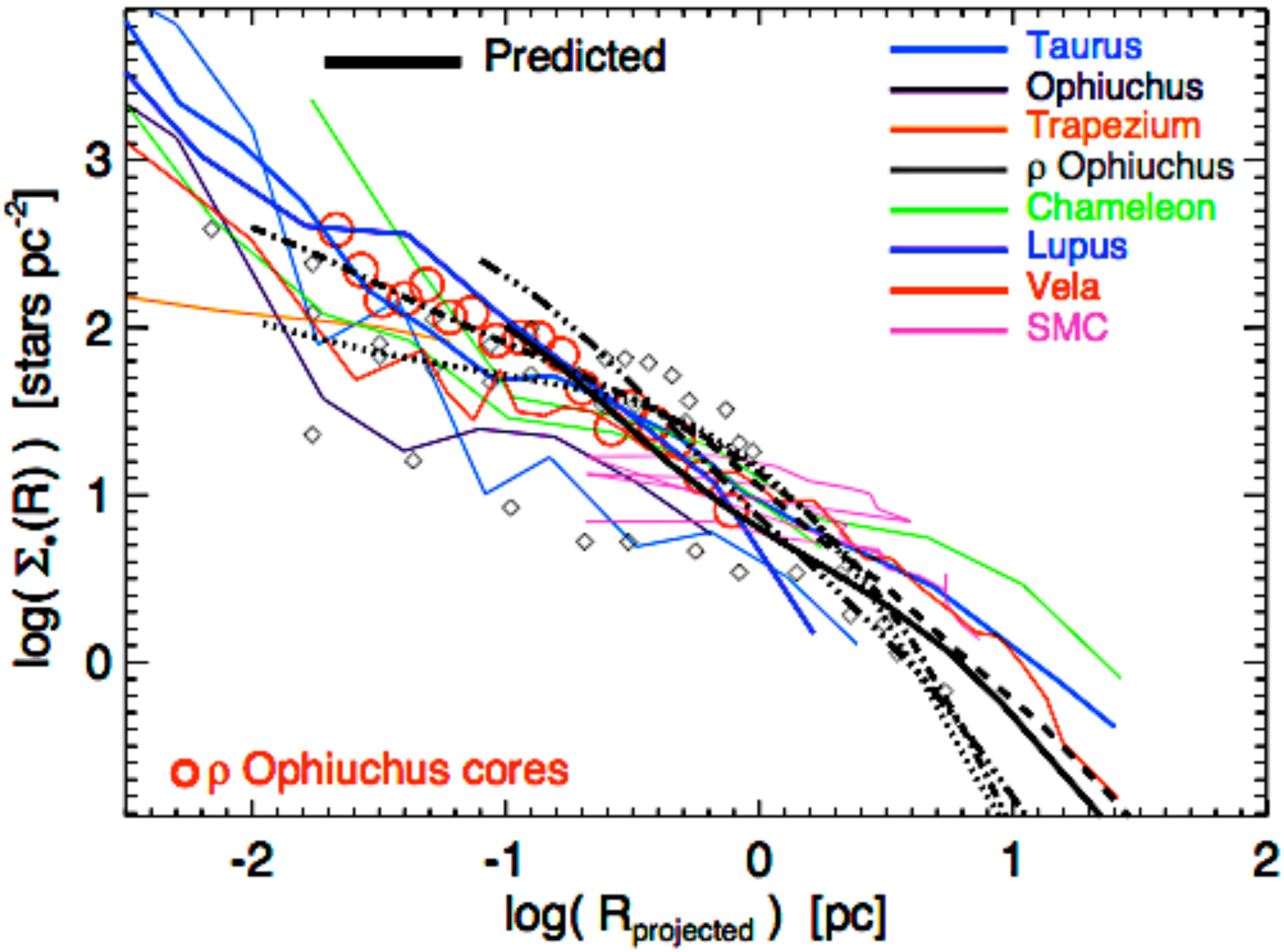
Why is Star Formation Clustered?

$$S \sim \ln \mathcal{M}(k)^2$$
$$\sim \ln r^{3-p}$$



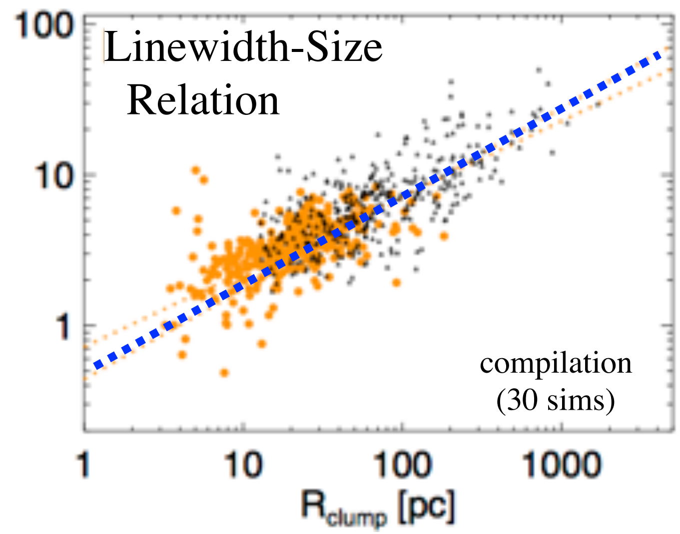
Clustering of Stars: Predicted vs. Observations

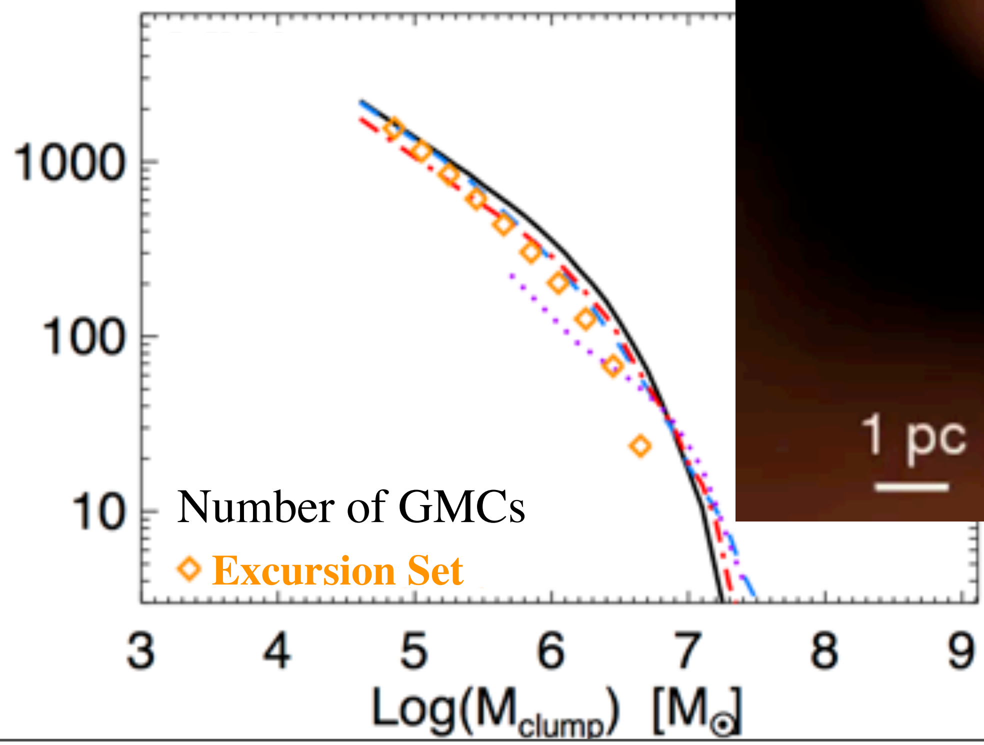
PREDICT N-POINT CORRELATION FUNCTIONS

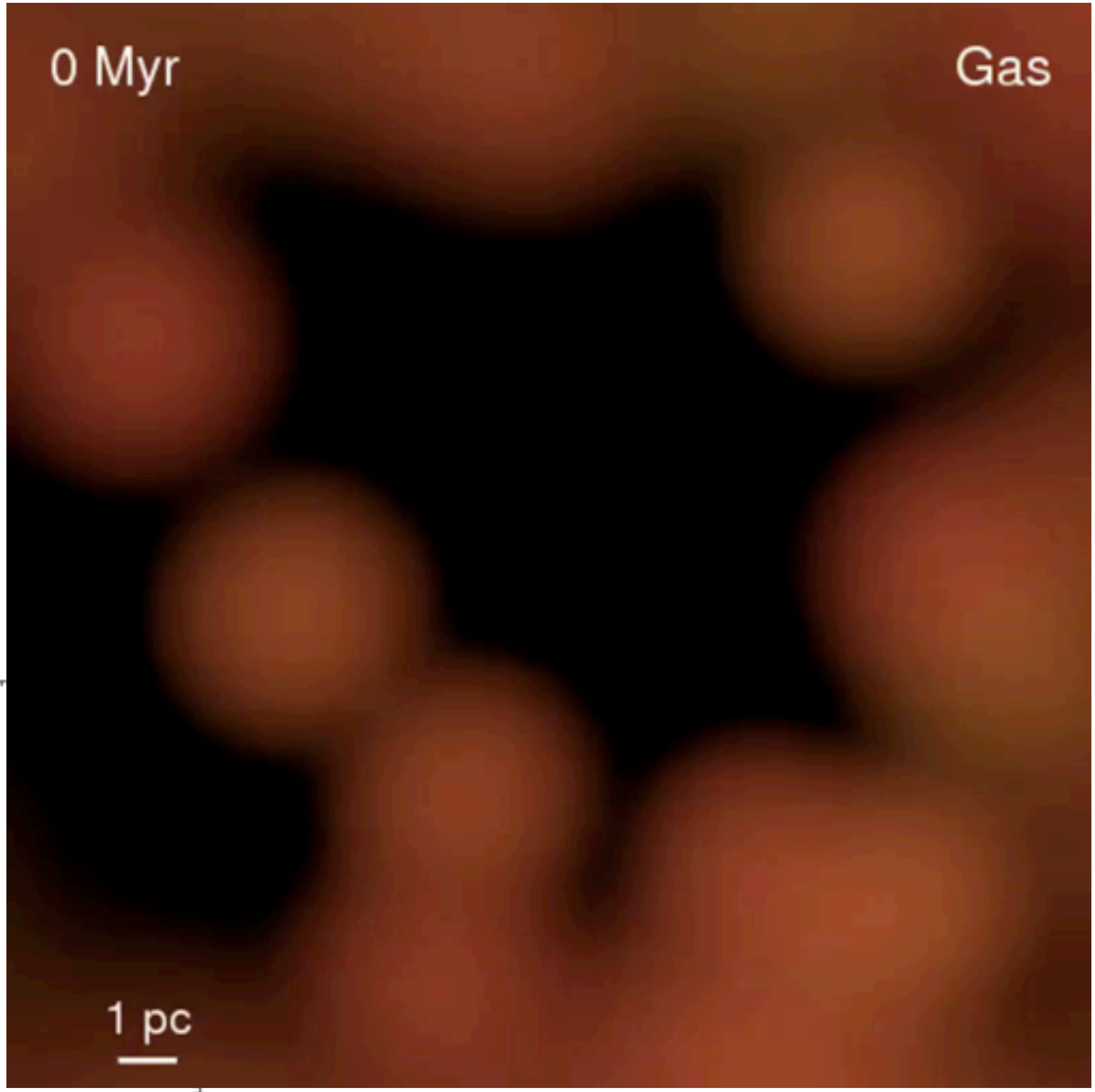


Testing the Analytics

vs. NUMERICAL SIMULATIONS





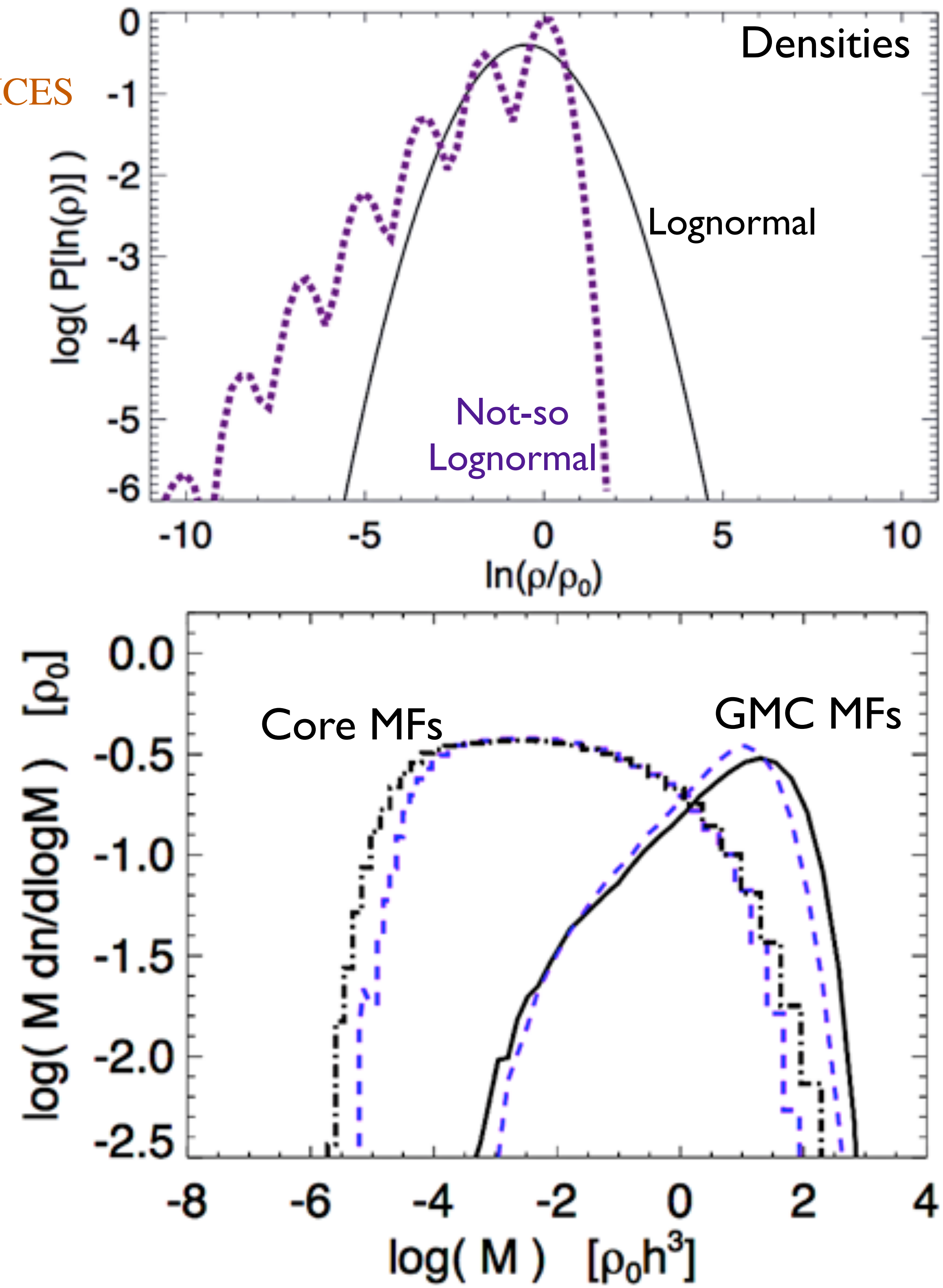


PFH

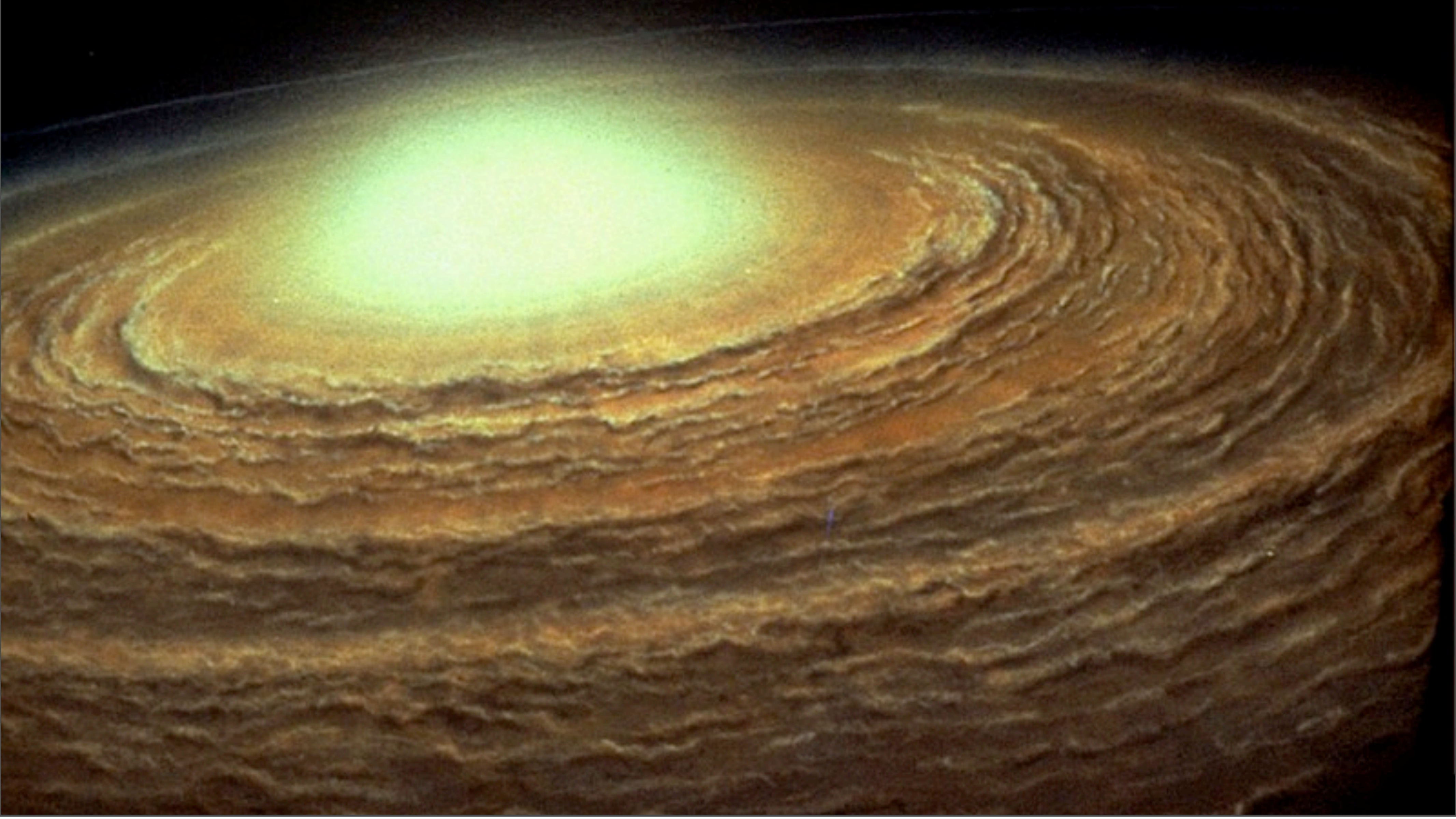
General, Flexible Theory:

EXTREMELY ADAPTABLE TO MOST CHOICES

- Complicated, multivariable gas equations of state
- Accretion
- Magnetic Fields
- Time-Dependent Background Evolution/Collapse
- Intermittency
- Correlated, multi-scale driving

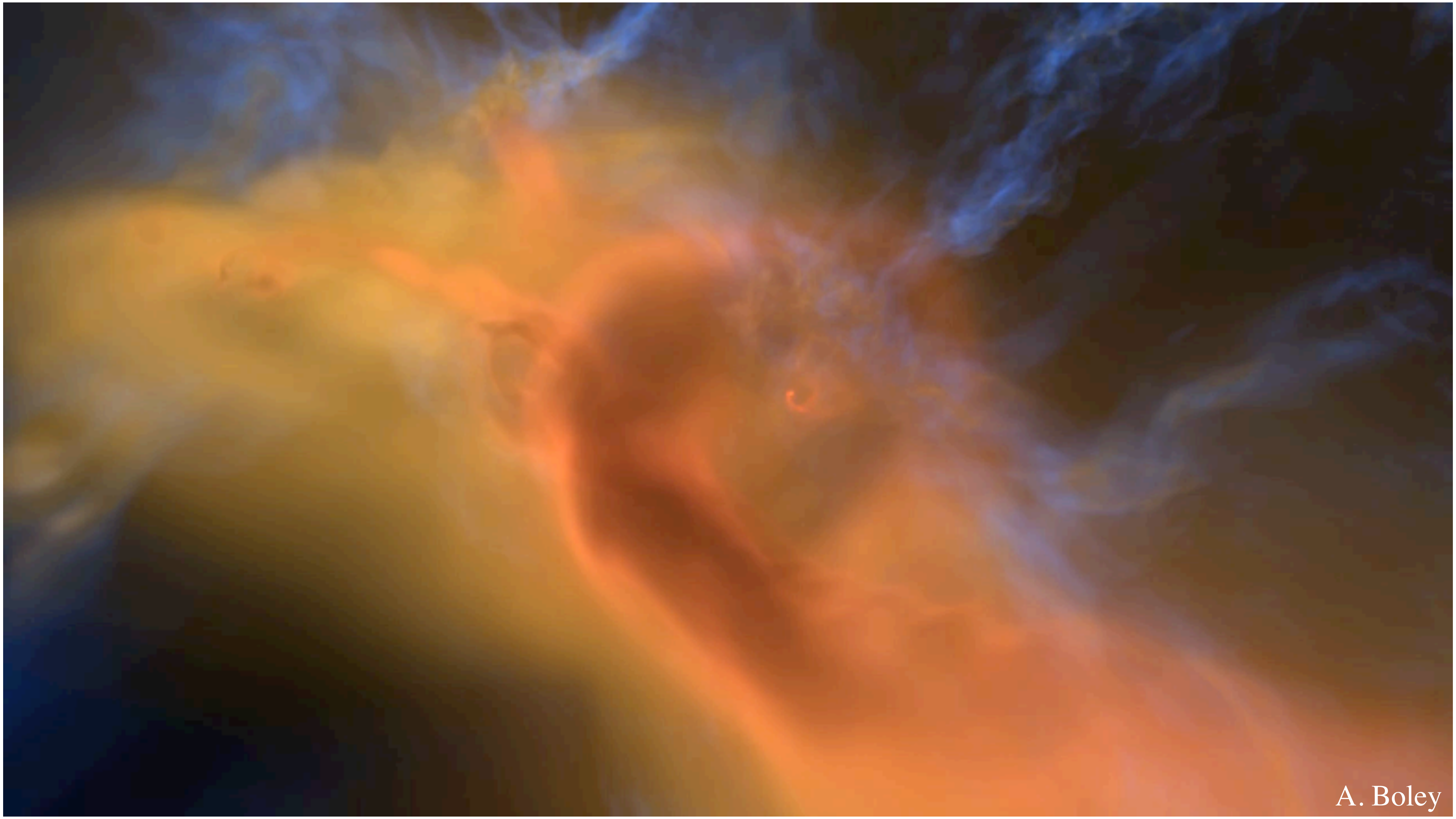


What About Planets?

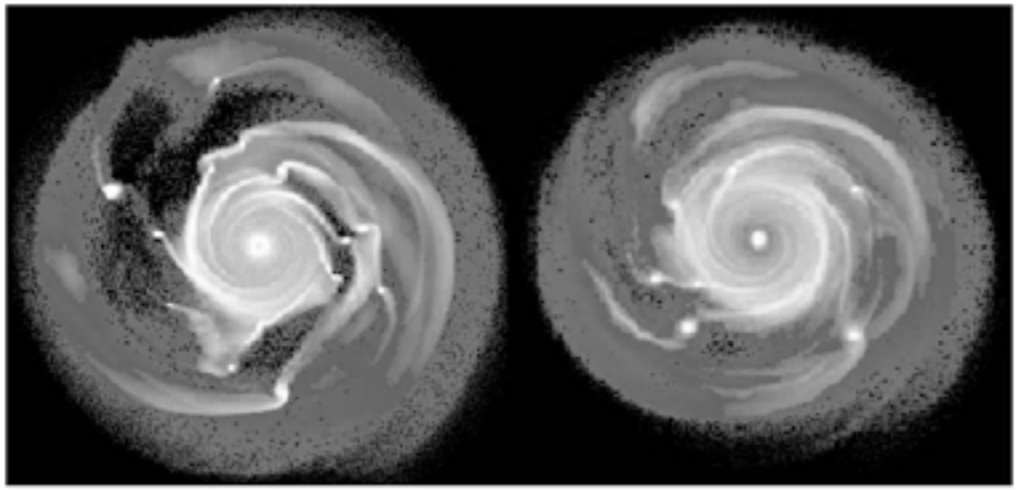


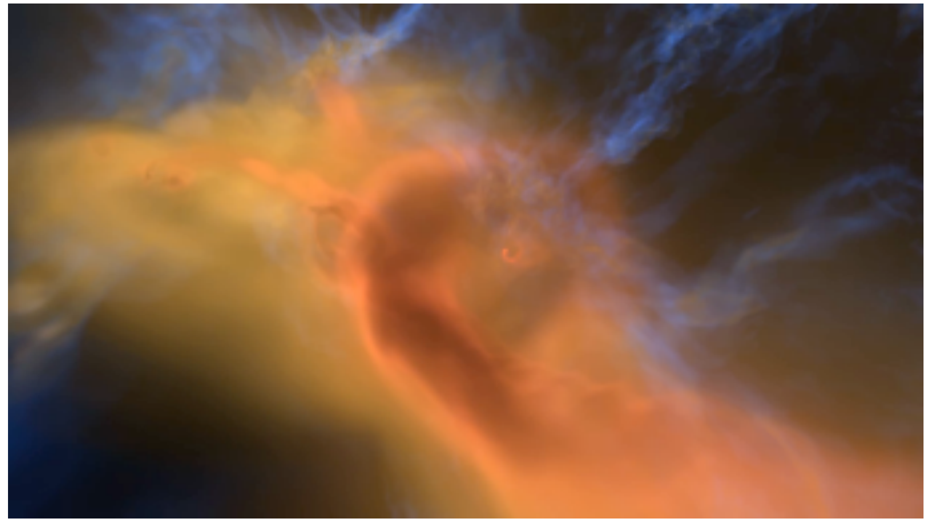
Planet Formation?

- Two channels:
  - (1) “Core accretion”
  - (2) “Direct Collapse”



Standard (Toomre) Criterion for Direct Collapse:





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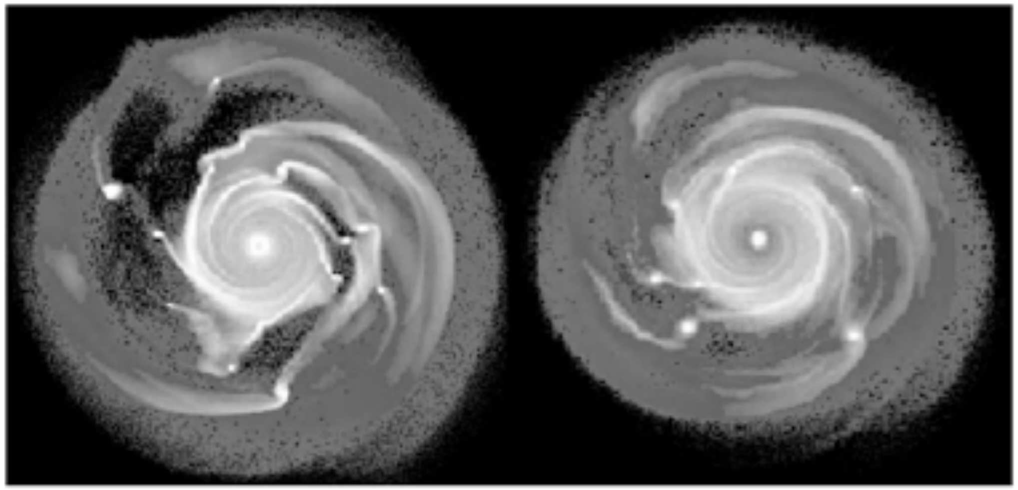

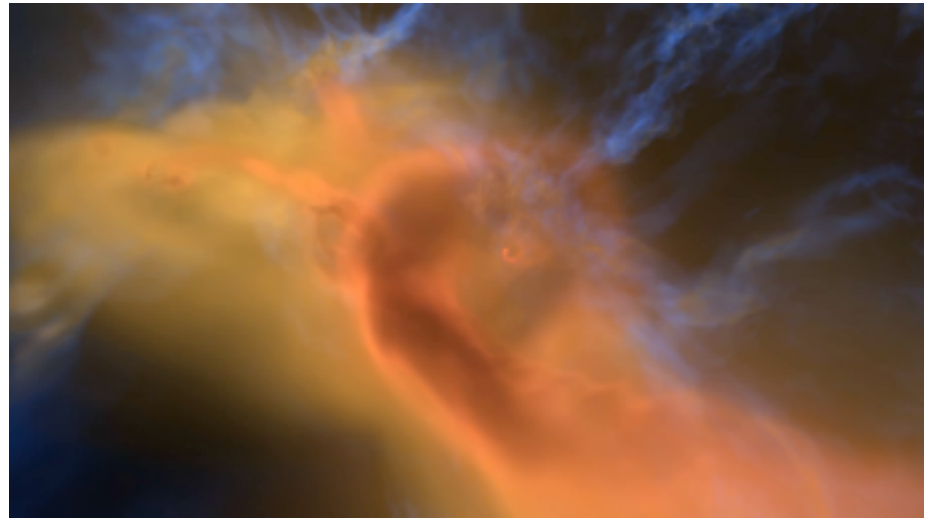

$$Q = \frac{c_s\,\Omega}{\pi\,G\,\Sigma_{\rm gas}} \sim \frac{1}{\rho}\,\frac{M_*}{r_*^3}$$

$$Q \sim 100\left(\frac{\Sigma_{\rm gas}}{\Sigma_{\rm MMSN}}\right)^{-1} r_{*,\,{\rm AU}}^{-1/4}$$

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- Q~100, *M*~0.1 :: $P_p \sim 10^{-7}$ is small!

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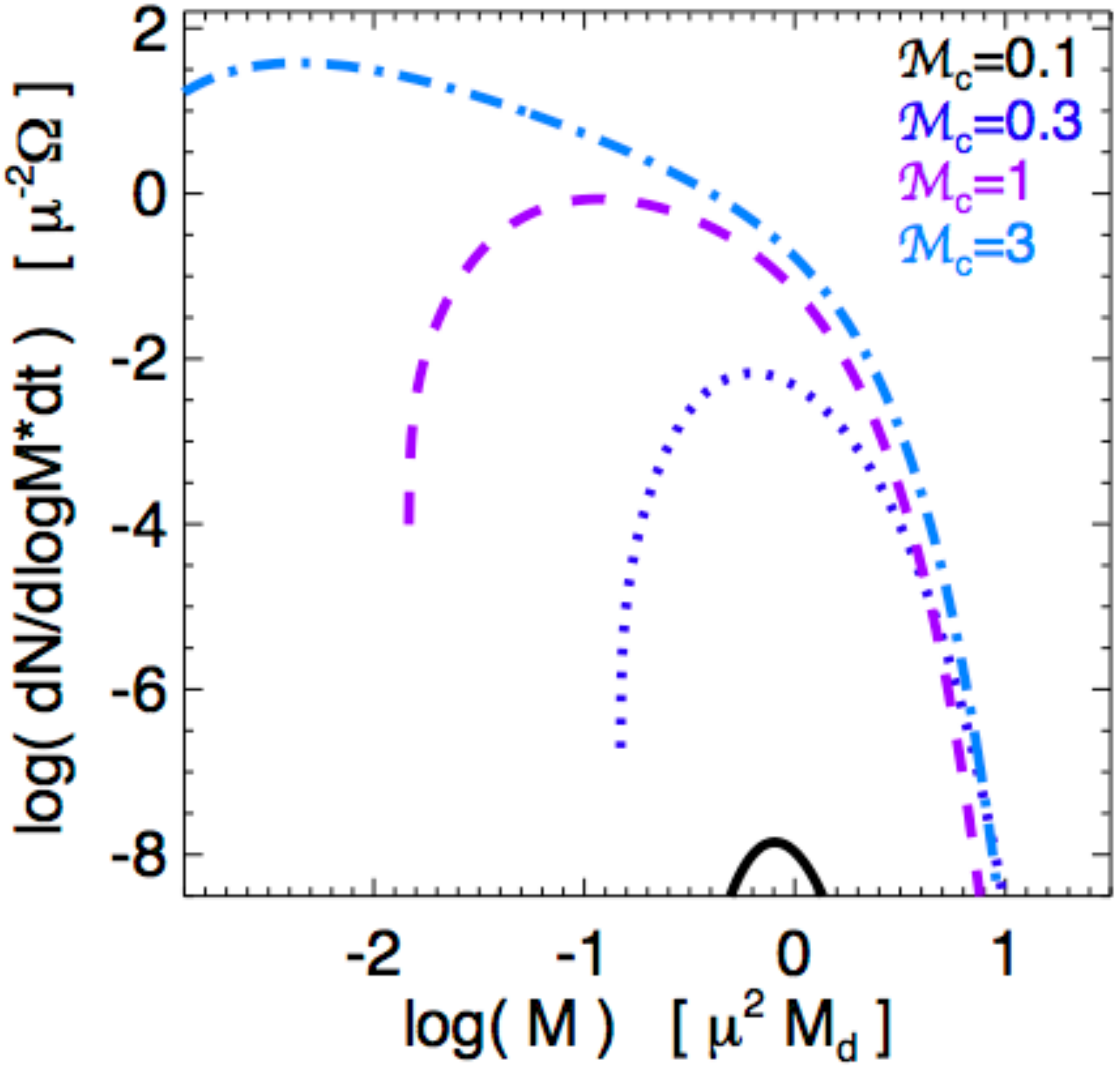
Mass Function of "Stochastic" Direct Collapse Events

RIGOROUSLY CALCULATE RATE OF EVENTS VS MASS



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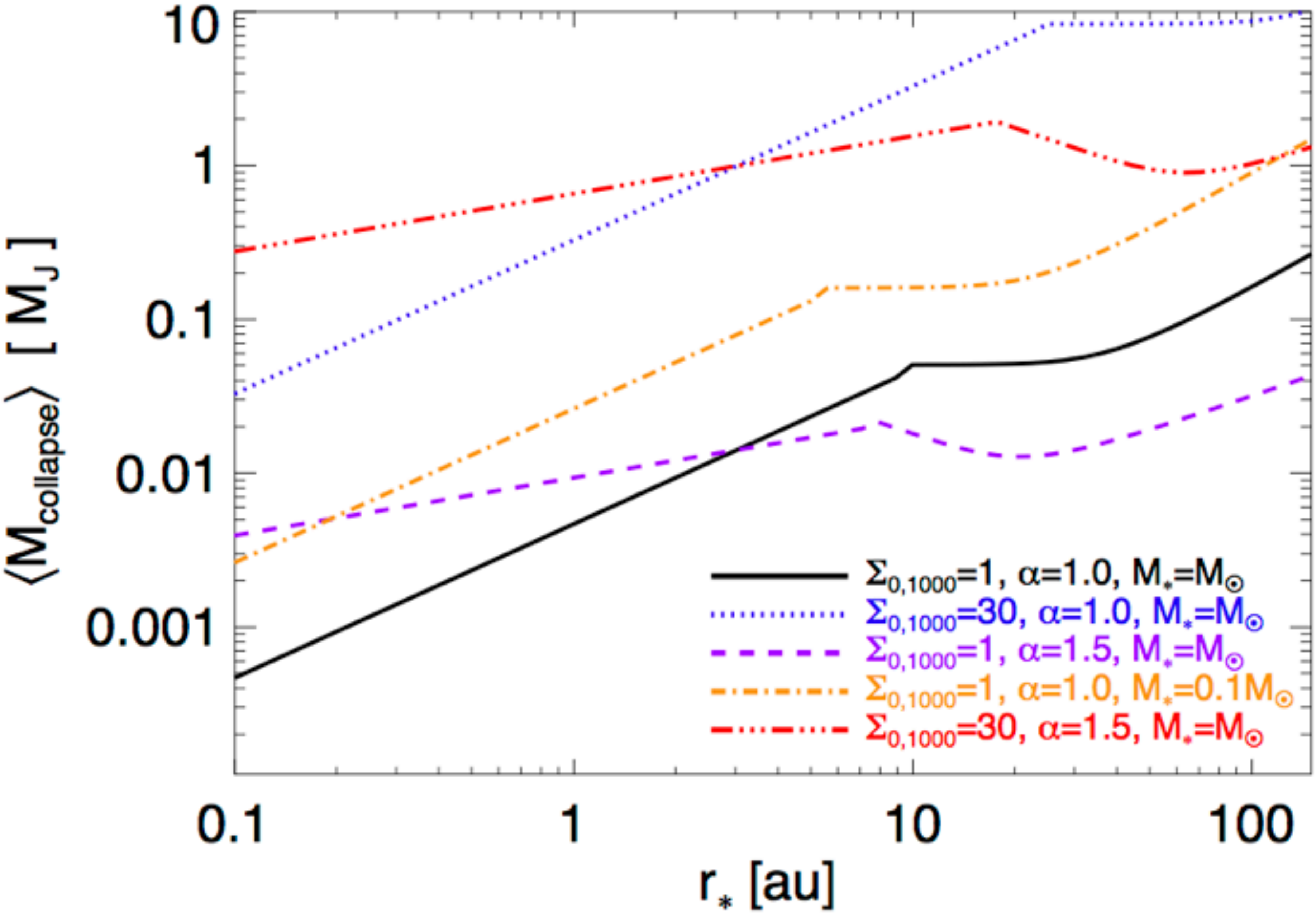
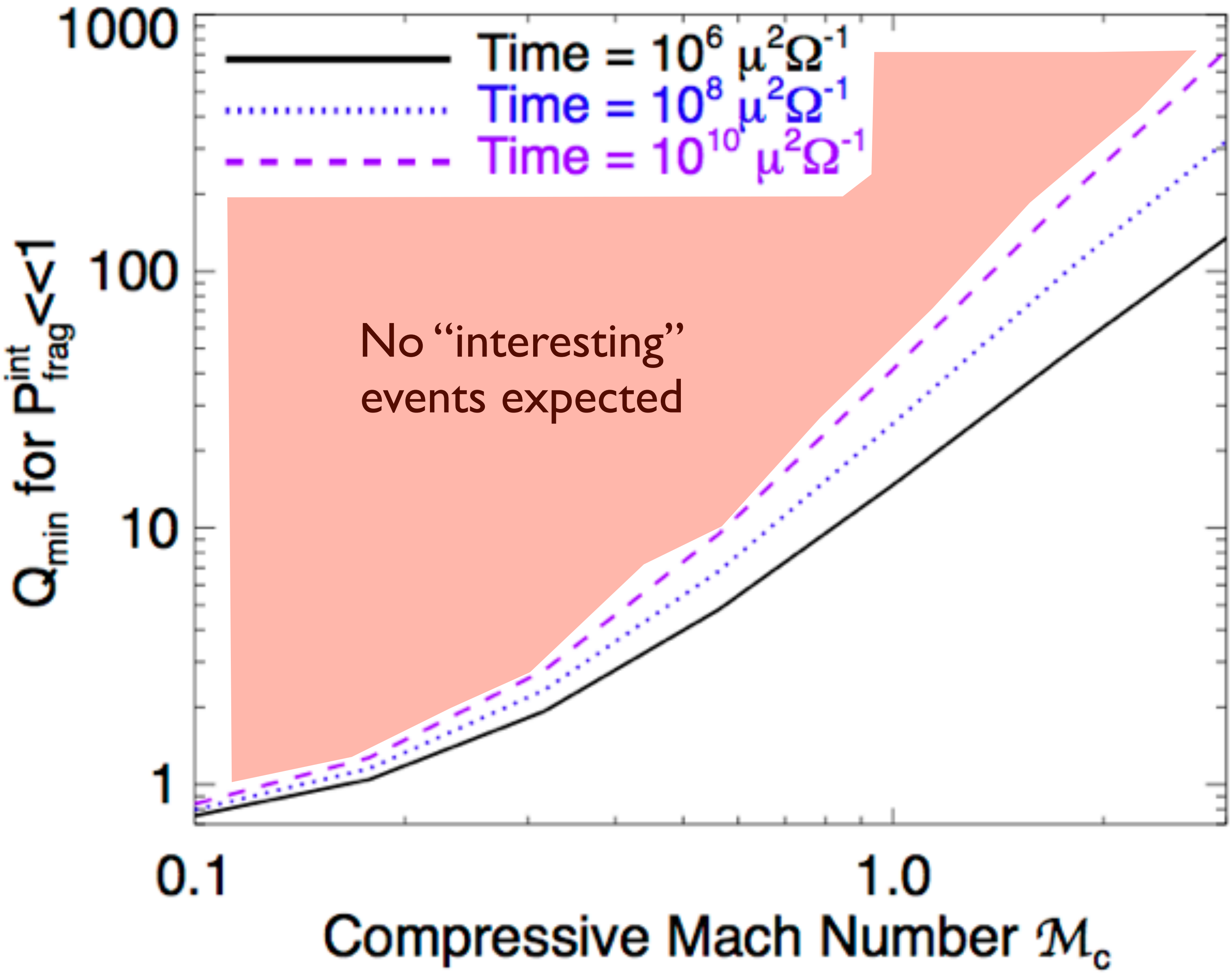


What is the real "threshold" for an event?

(FOR A GIVEN DISK LIFETIME)



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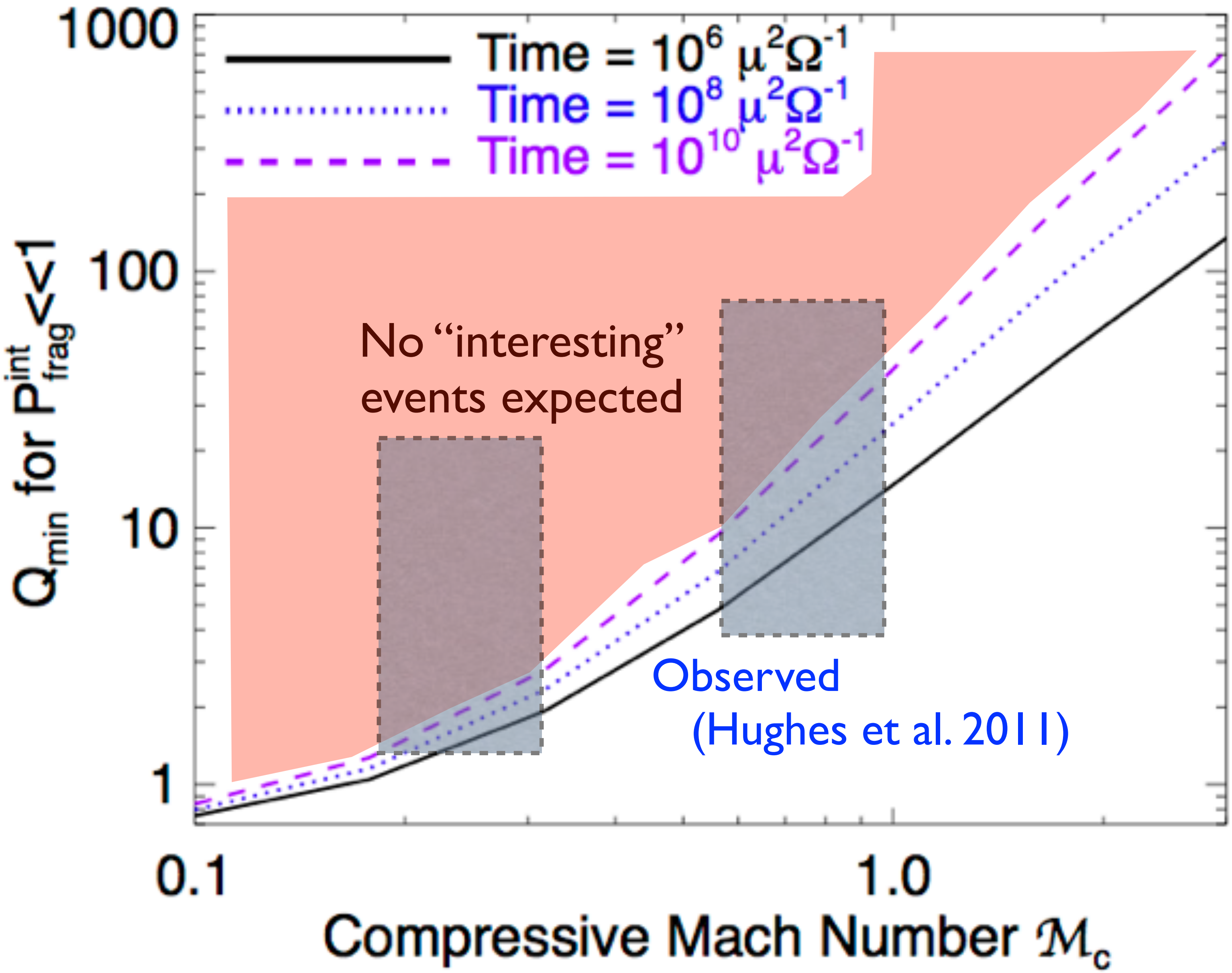
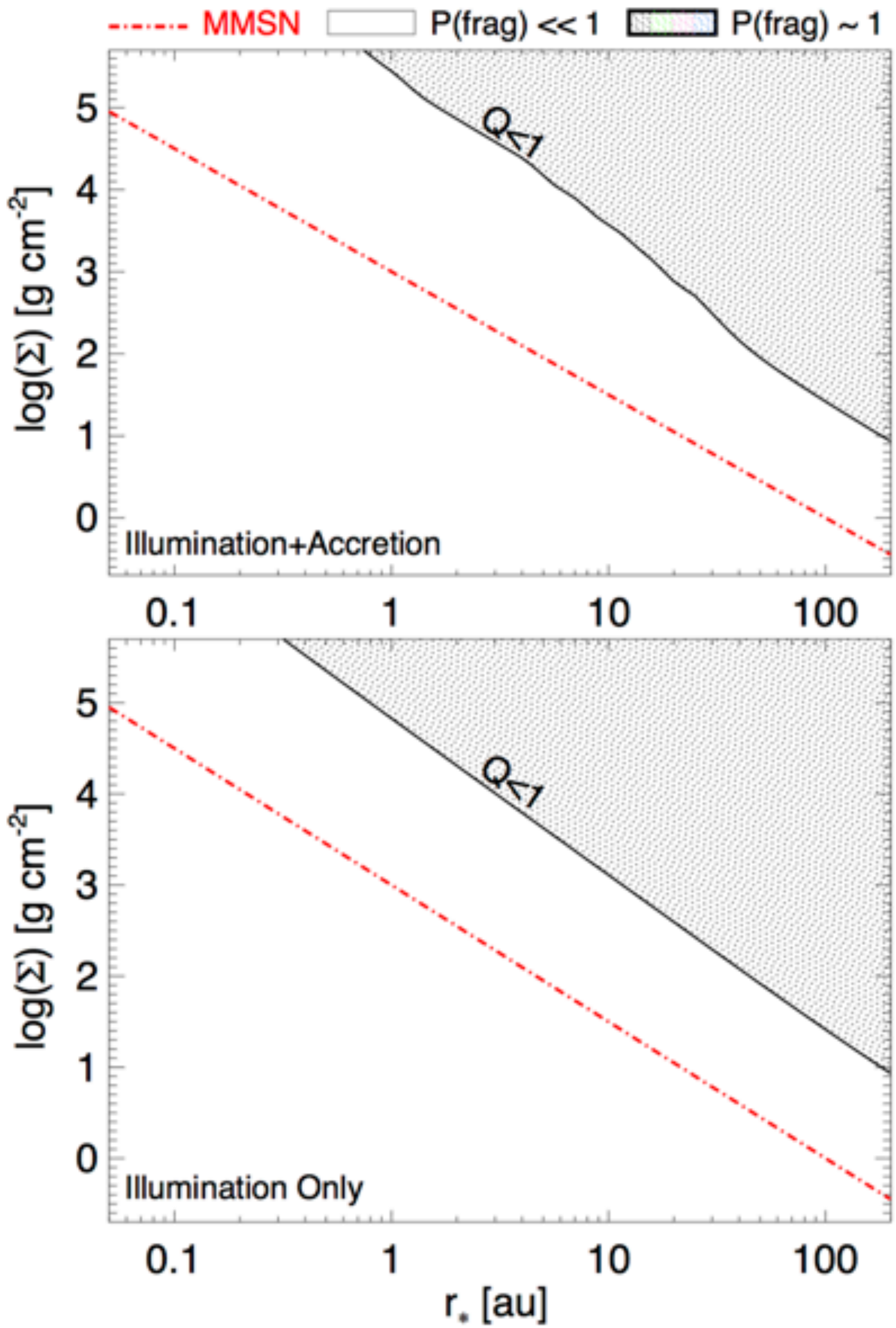


What is Theoretically Expected?

"TYPICAL" DISK AROUND
A SOLAR-TYPE STAR

- Self-consistently calculate temperatures, etc.

- Different drivers of turbulence:
  - Convection
  - Magneto-Rotational Instability
  - "Gravito-Turbulence"



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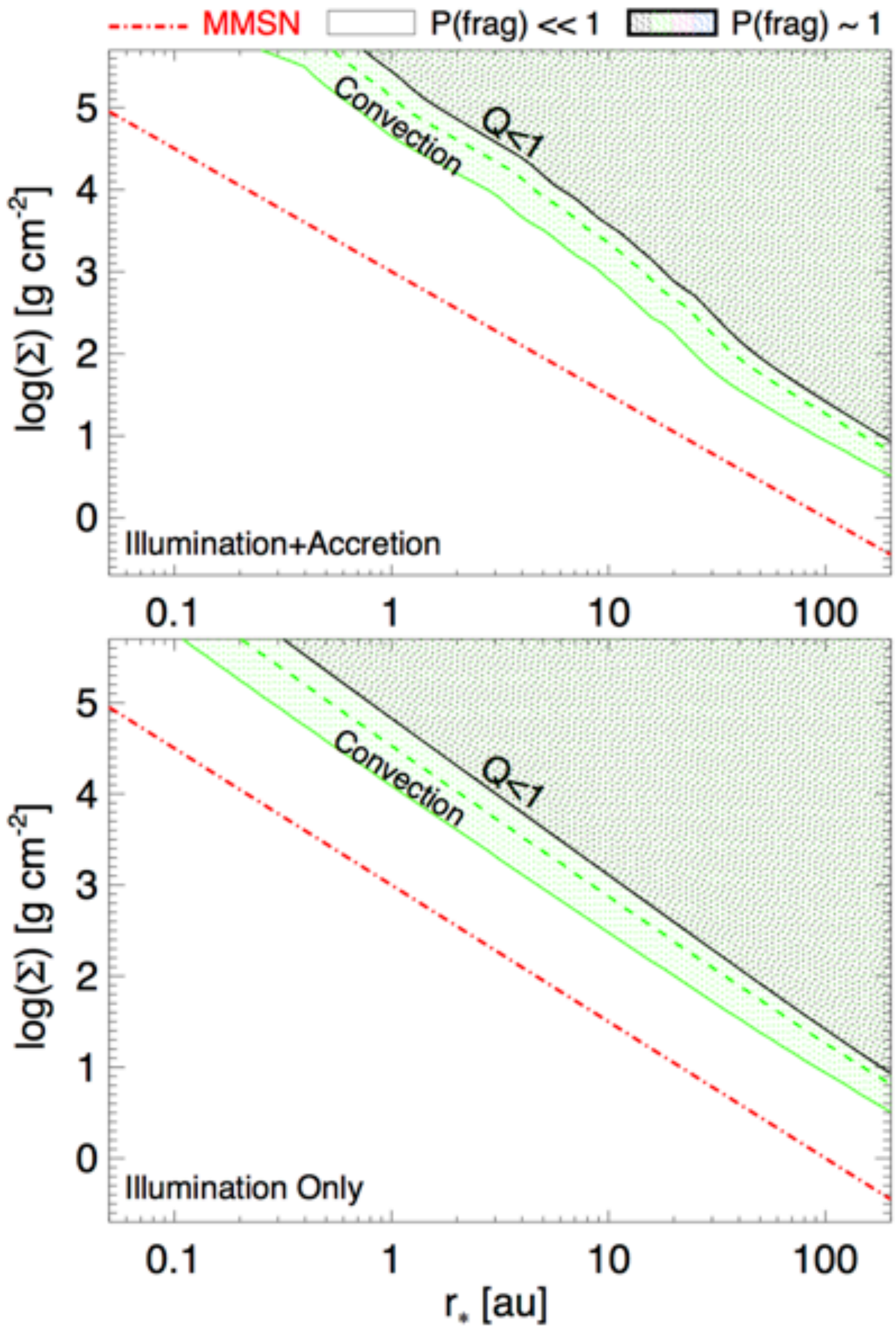

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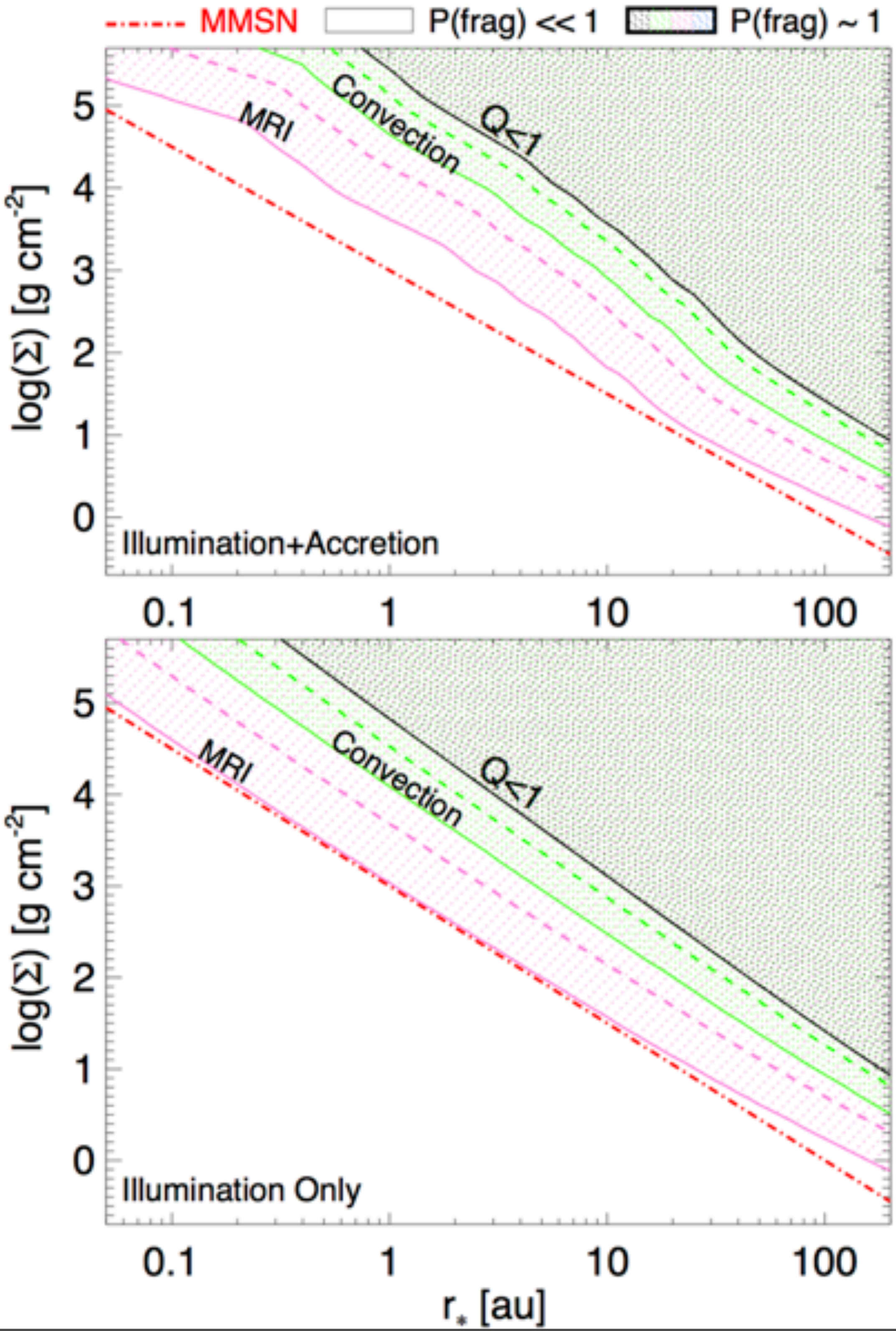

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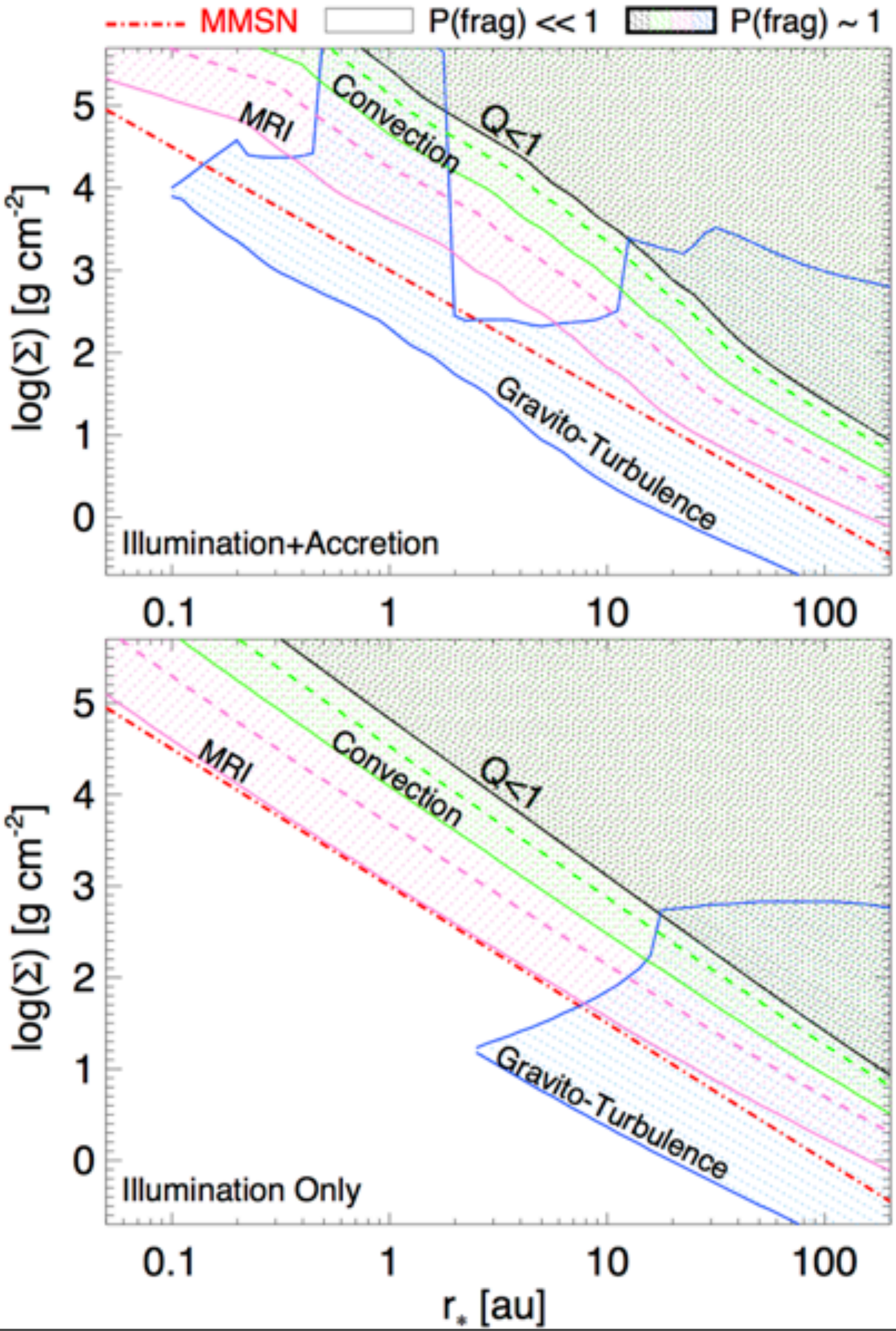

Summary:

*** ISM *statistics* are far more fundamental than we typically assume ***

- **Turbulence + Gravity:** ISM structure follows
  - Lognormal density PDF is ***not*** critical
  - *ANALYTICALLY* understand:
    - **GMC Mass Function & Structure** ("first crossing")
    - **Core MF** ("last crossing") & **Linewidth-Size-Mass**
    - **Clustering** of Stars (correlation functions)

- **Planet Formation in Direct Collapse:**
  - Modest turbulence (Mach >0.1) is sufficient for ~1 event(s)
  - Applies to grains as well?

- **Non-Gaussian Statistics**: not dominant in calculations above
  - But very interesting probes of the structure of turbulence!
  - Indicates Mach-density connection generalizes over entire cascade

What If The Statistics Aren't Gaussian?

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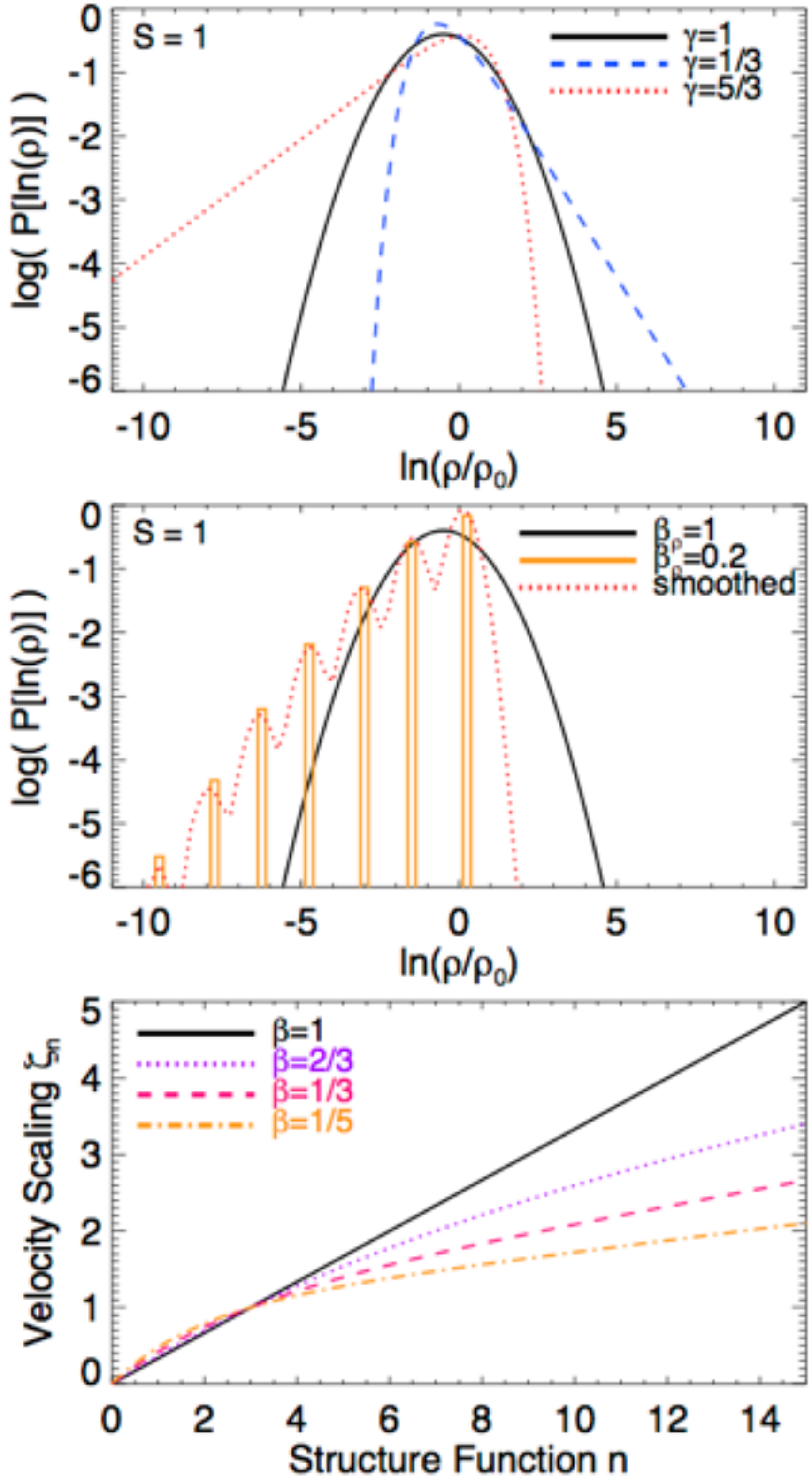
... actually, they never are, and that's great!

Many kinds of Non-Gaussianity Appear:

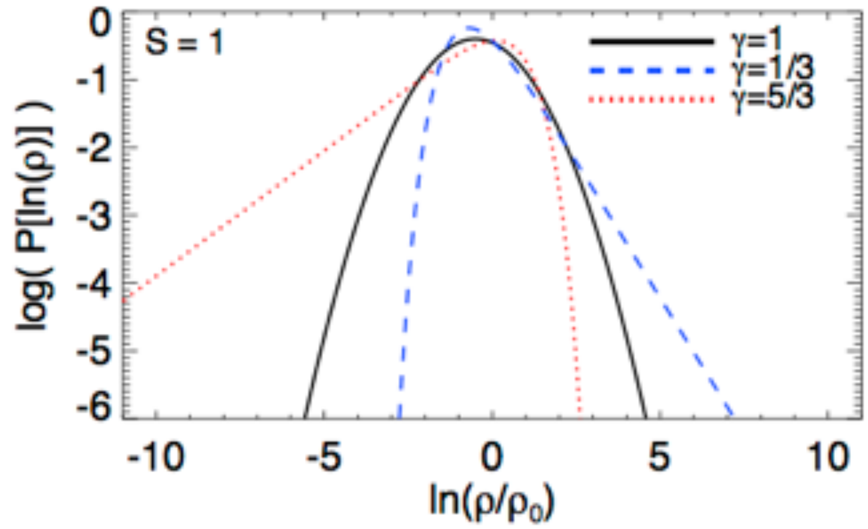
BUT THESE ARE TRACTABLE!

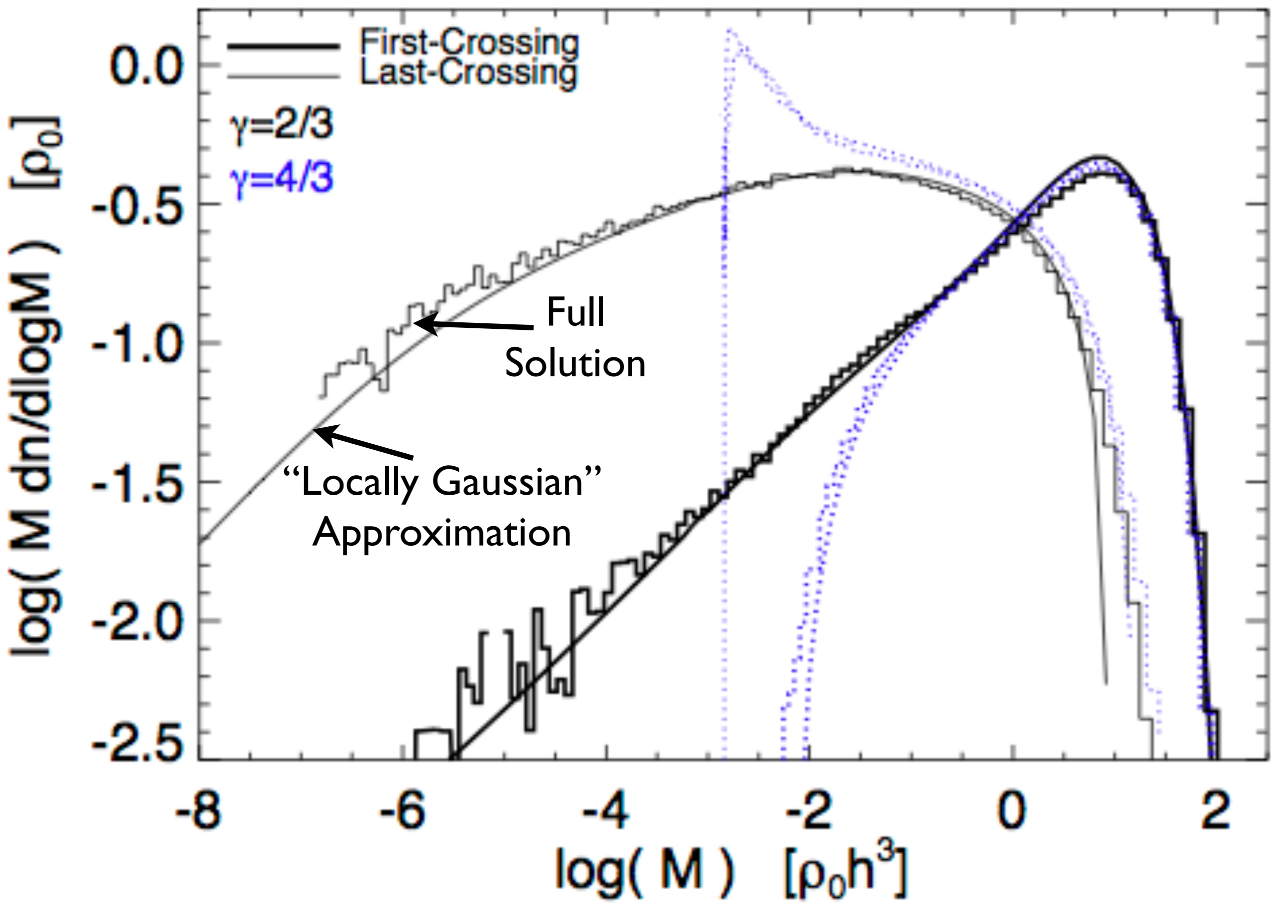
- Non-isothermal equations of state
- Long-range forces (gravity)
- Intermittency
  (non-self similarity)
  in the turbulence



Example: Non-isothermal equations of state

APPLY COSMOLOGICAL METHODS FOR NON-GAUSSIAN FIELDS

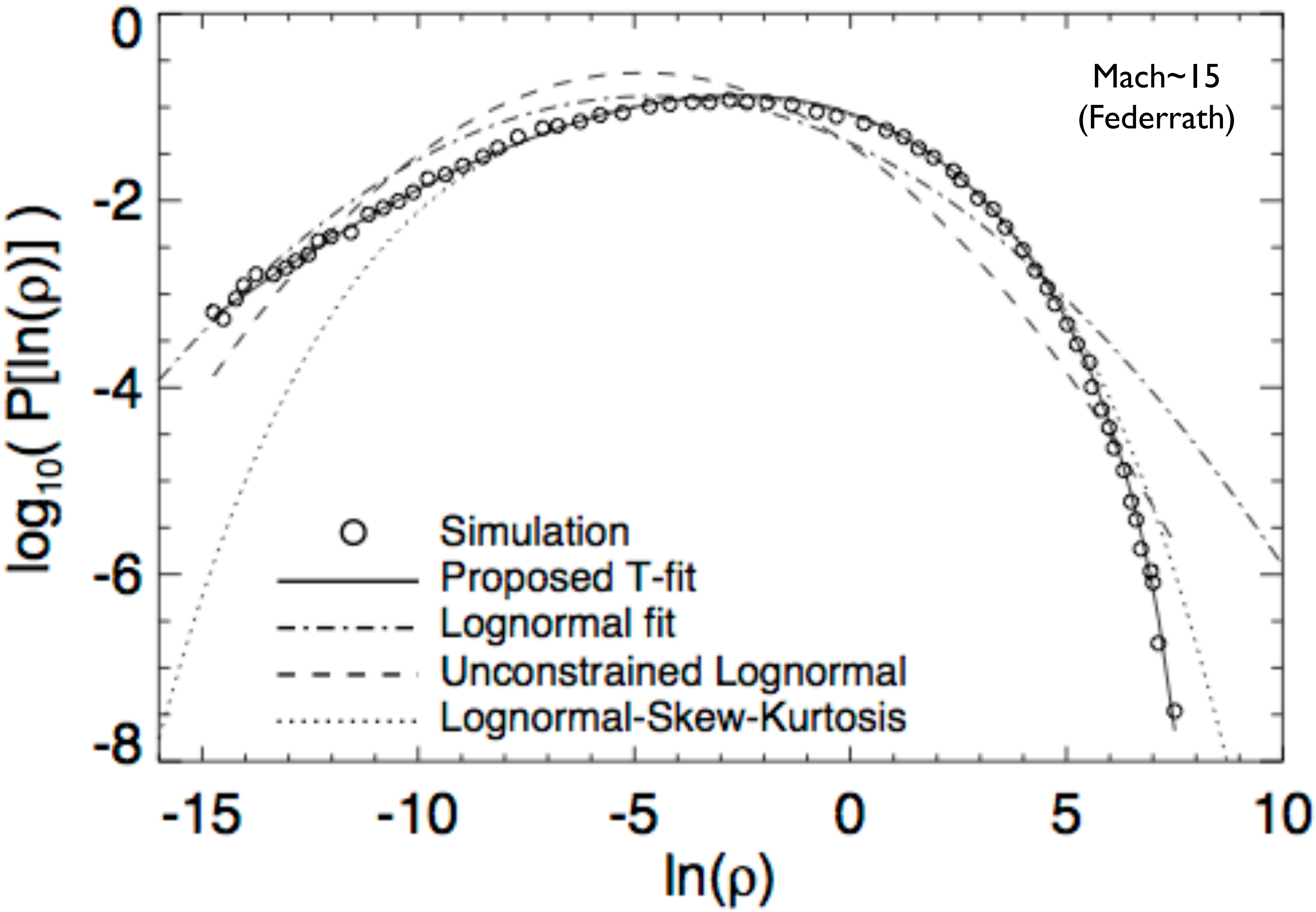




More Interesting: Even Isothermal Gas is Not Lognormal!

MASS CONSERVATION & INTERMITTENCY PREVENT IT

- **Lognormal:** $$\mathrm{d}p(\ln\rho\,|\,R) = \frac{1}{\sqrt{2\pi\,S(R)}}\,\exp\left[\frac{-(\ln\rho - \langle\ln\rho\rangle)^2}{2\,S(R)}\right]$$



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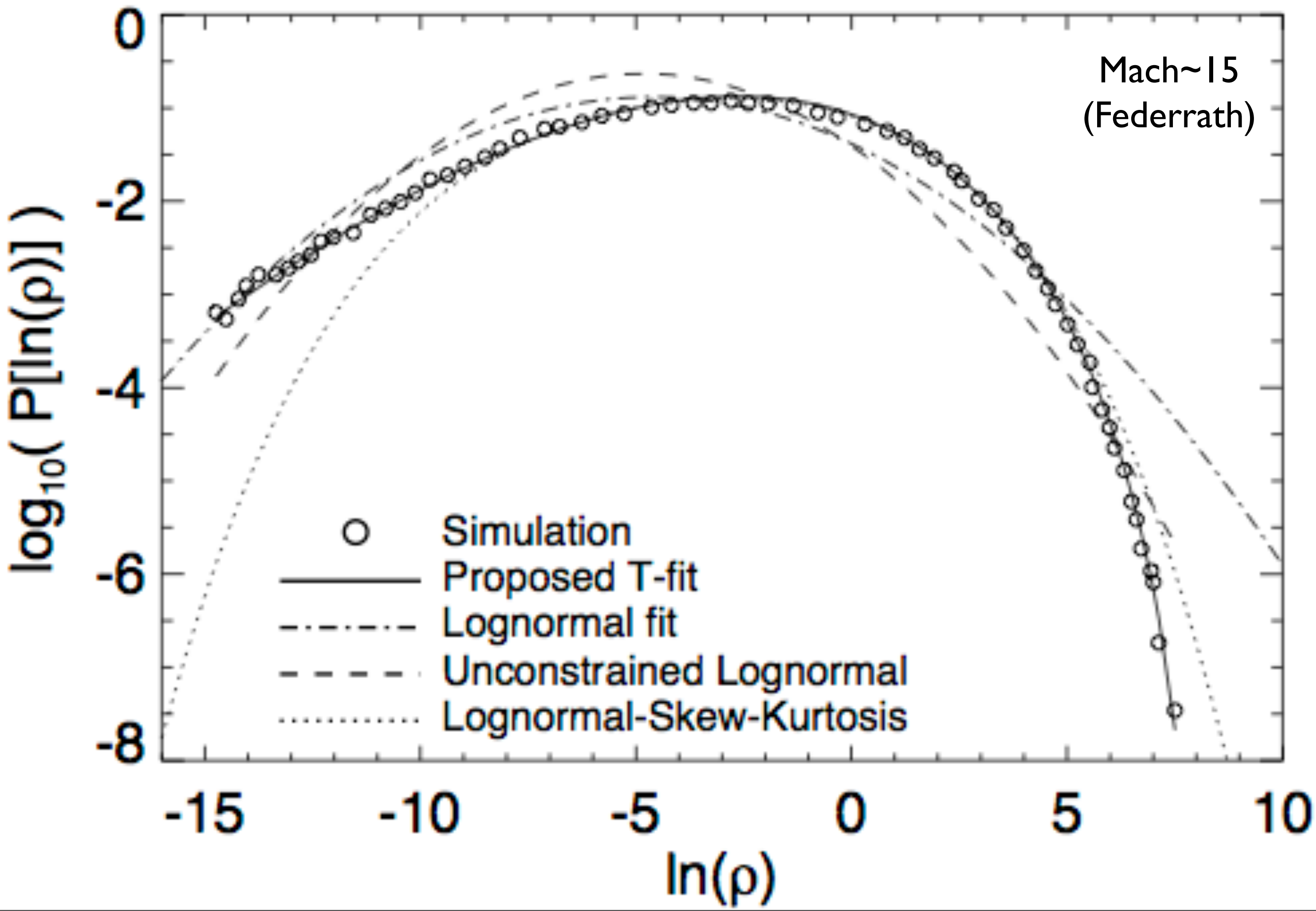
- T-Function: $P_V(\ln\rho)\,\mathrm{d}\ln\rho = I_1(2\sqrt{\lambda u})\exp[-(\lambda+u)]\sqrt{\frac{\lambda}{u}}\,\mathrm{d}u$

  (Castaing 1996)

$$= \sum_{m=0}^{\infty} \frac{\lambda^m e^{-\lambda}}{m!} \frac{u^{m-1} e^{-u}}{(m-1)!}\,\mathrm{d}u \qquad (u \propto \ln\rho)$$



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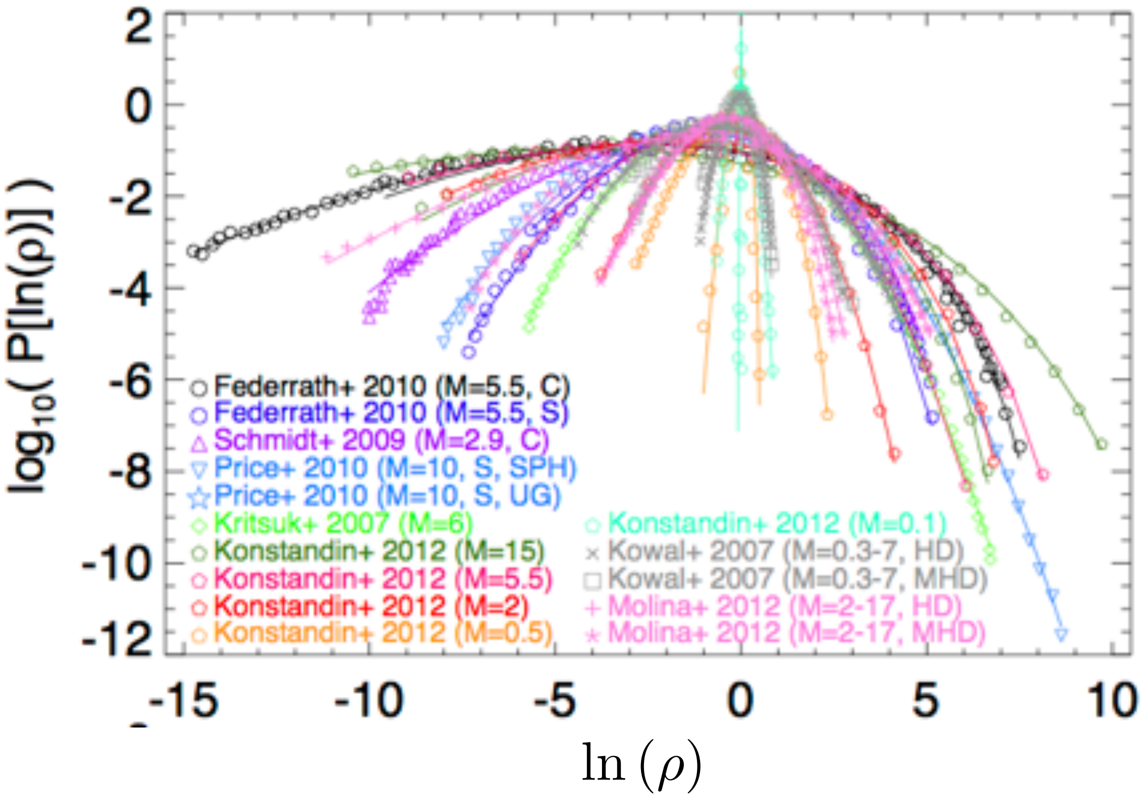
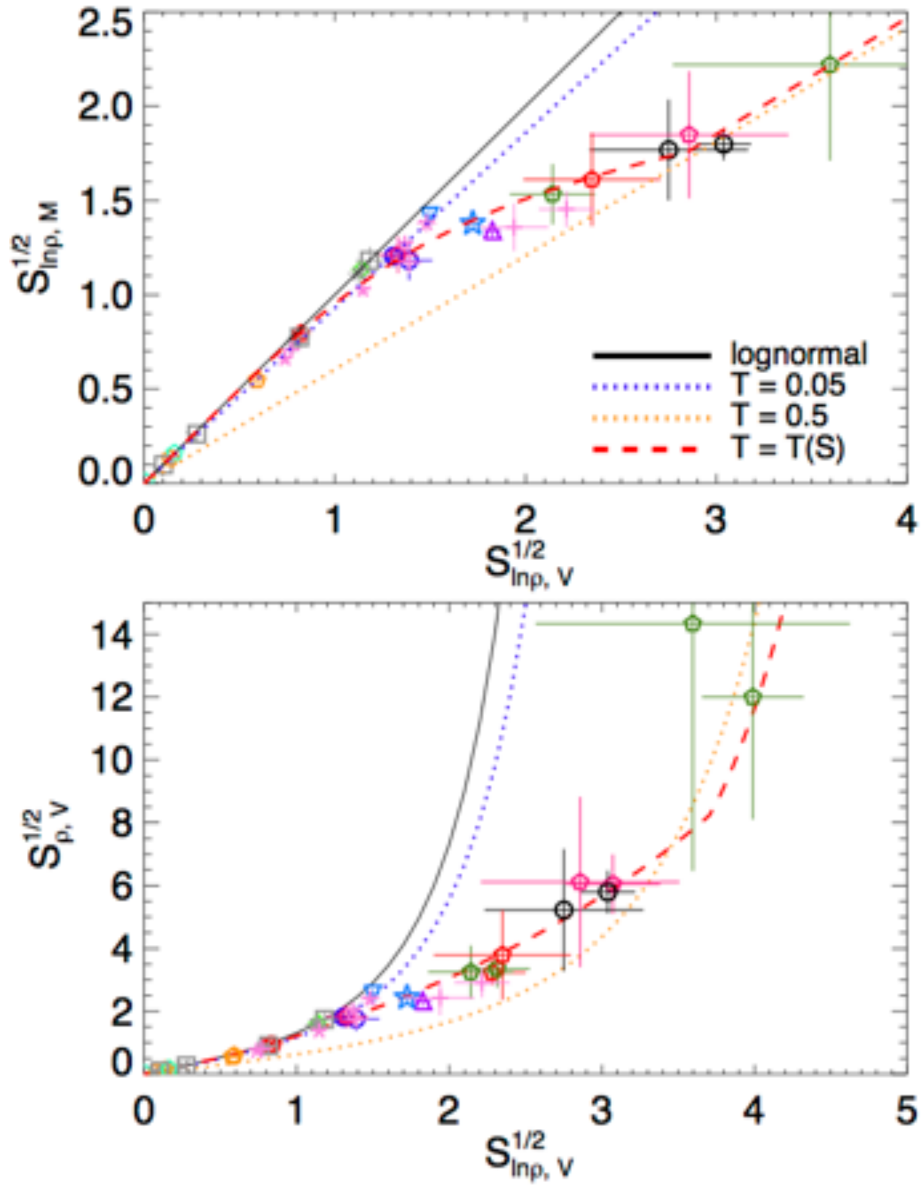
EXPLAINS MANY DISCREPANCIES IN SIMULATIONS & METHODS





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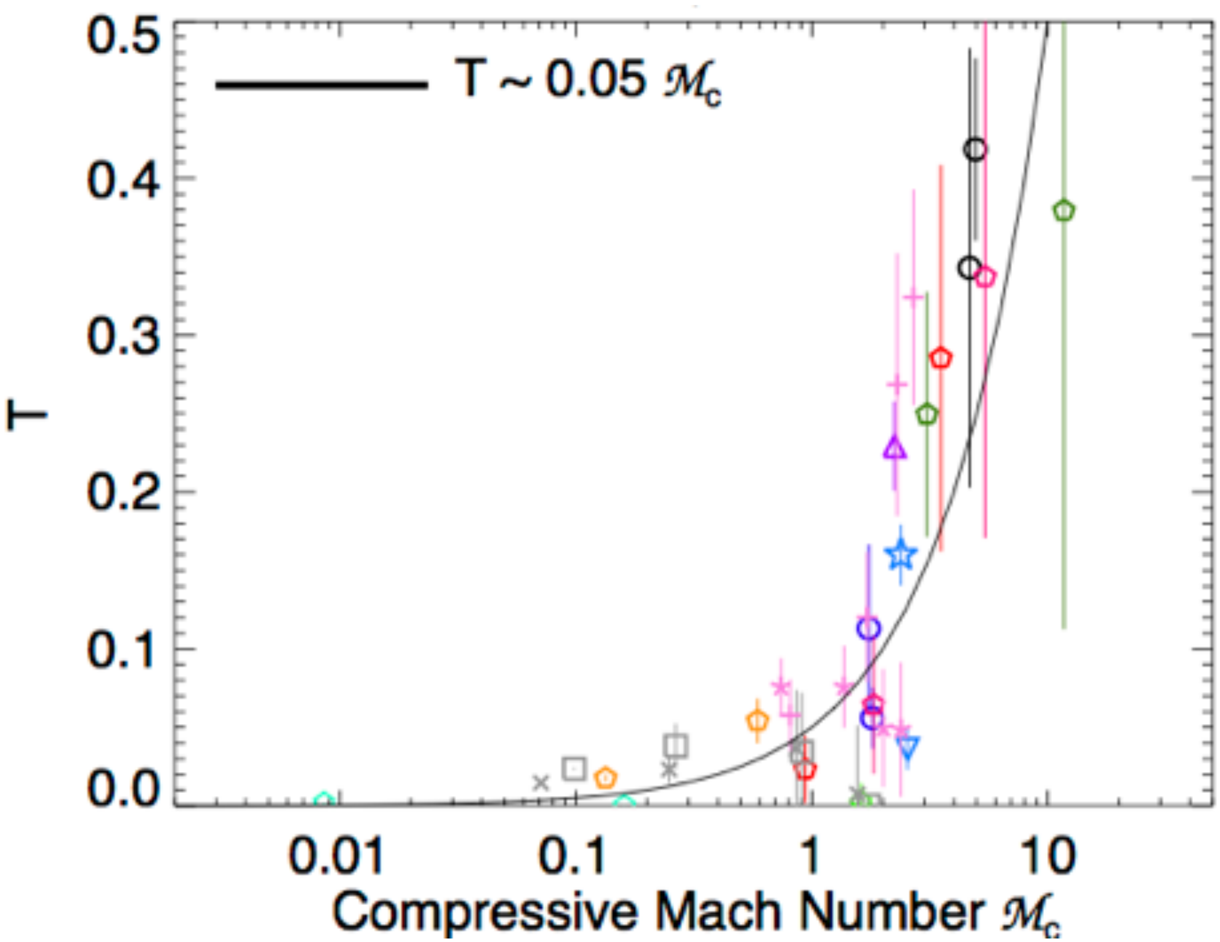
MASS CONSERVATION & INTERMITTENCY PREVENT IT

- Parameter T = 0-1 represents the "degree of intermittency"
    - *Fundamental* parameter of multi-fractal/cascade models of turbulence



- Same values for *T* derived from density PDF or velocity statistics