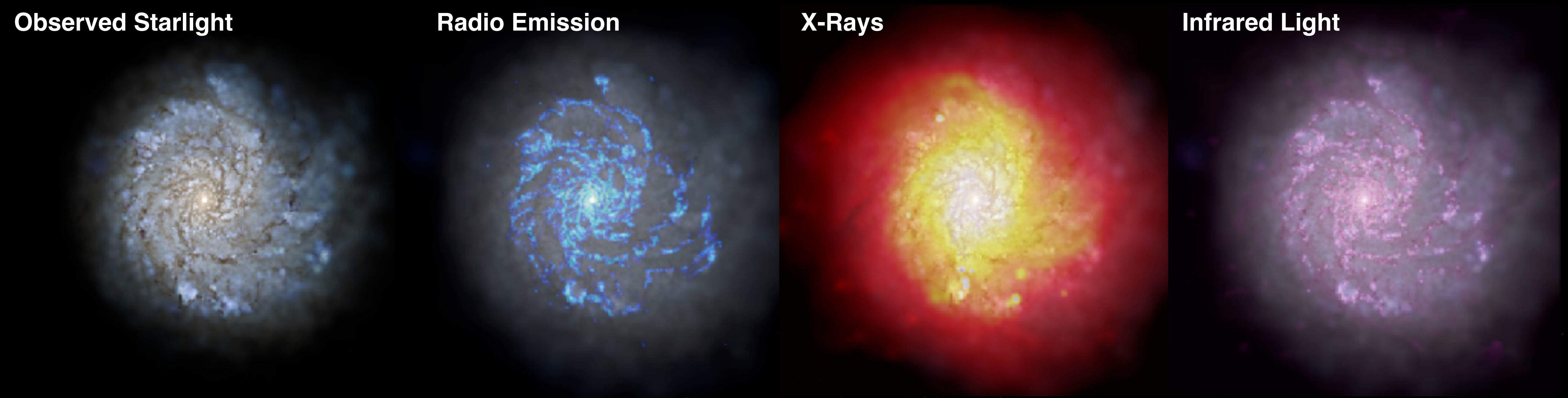
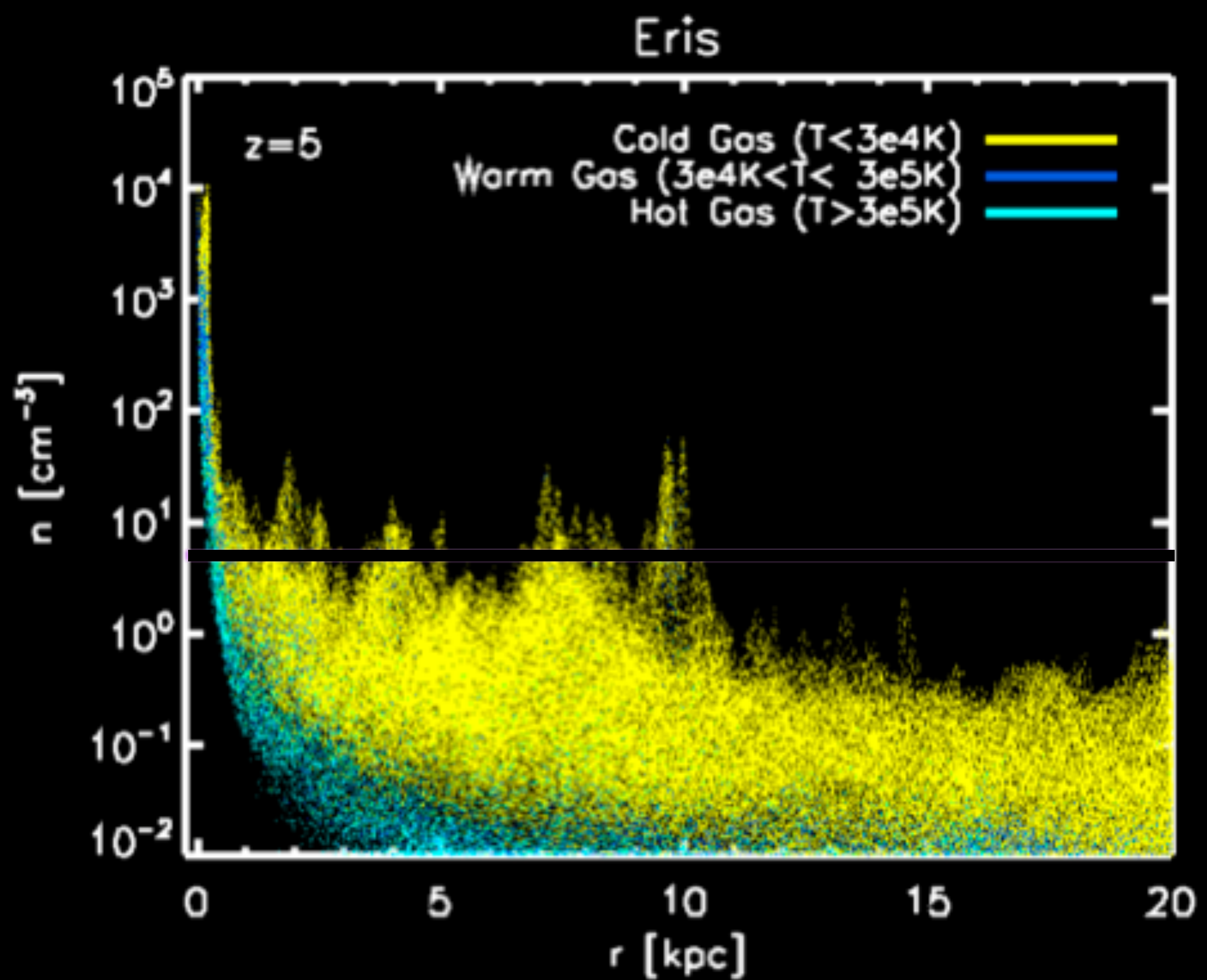


Painful Numerical Lessons

Phil Hopkins (Caltech)
www.tapir.caltech.edu/~phopkins

Gravity:

CAREFUL WITH SF THRESHOLDS AND "REAL" RESOLUTION!



Some choices set a maximum density where self-gravity can be resolved:

- Fixed minimum softening / AMR refinement / smoothing $\epsilon_{\rm min}$

$$\rho_{\rm max} \sim \frac{\Delta m_i}{\epsilon^3_{\rm min}} \sim 0.1\,{\rm cm}^{-3} \left[\frac{\Delta m_i}{10^4\,{\rm M}_\odot}\right] \left[\frac{100\,{\rm pc}}{\epsilon^{\rm Plummer}_{\rm min}}\right]^3$$

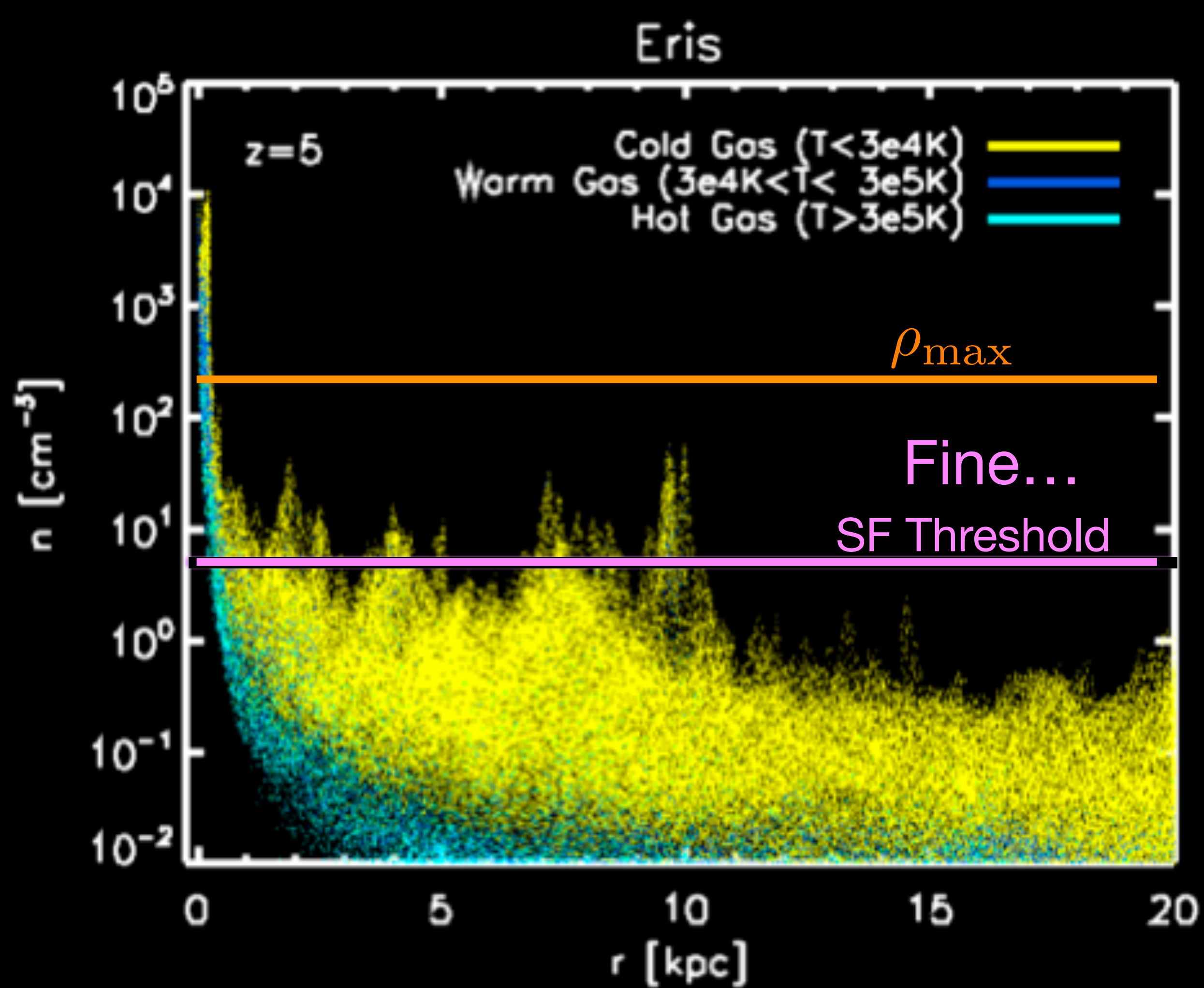
- "Pressure Floor" or "Effective EOS"

$$\lambda^{\rm min}_{\rm Jeans} \sim 3\,{\rm kpc} \left[\frac{T_{\rm min}}{10^4\,{\rm K}}\right]^{1/2} \left[\frac{10^4\,{\rm M}_\odot}{\Delta m_i}\right]^{1/2} \left[\frac{\epsilon^{\rm Plummer}_{\rm min}}{100\,{\rm pc}}\right]^{3/2}$$

- Always use adaptive softening for gas (gravity = hydro)
- "Correct" softening for collisionless undefined (but much less important)
  - Best guess: ~inter-particle separation in region of interest → $\left[\epsilon_i^{\rm Plummer} \sim 30\,{\rm pc}\left(\frac{\Delta m_i}{10^4 {\rm M}_\odot}\right)^{1/3}\right]$
- Don't use pressure floors with "sink particle" (self-gravity)-based SF

Gravity:

CAREFUL WITH SF THRESHOLDS AND "REAL" RESOLUTION!



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↓

$$\rho_{\rm max} \sim \frac{\Delta m_i}{\epsilon_{\rm min}^3} \sim 0.1\,{\rm cm}^{-3} \left[\frac{\Delta m_i}{10^4\,{\rm M}_\odot}\right] \left[\frac{100\,{\rm pc}}{\epsilon_{\rm min}^{\rm Plummer}}\right]^3$$

- "Pressure Floor" or "Effective EOS"

↓

$$\lambda_{\rm Jeans}^{\rm min} \sim 3\,{\rm kpc} \left[\frac{T_{\rm min}}{10^4\,{\rm K}}\right]^{1/2} \left[\frac{10^4\,{\rm M}_\odot}{\Delta m_i}\right]^{1/2} \left[\frac{\epsilon_{\rm min}^{\rm Plummer}}{100\,{\rm pc}}\right]^{3/2}$$

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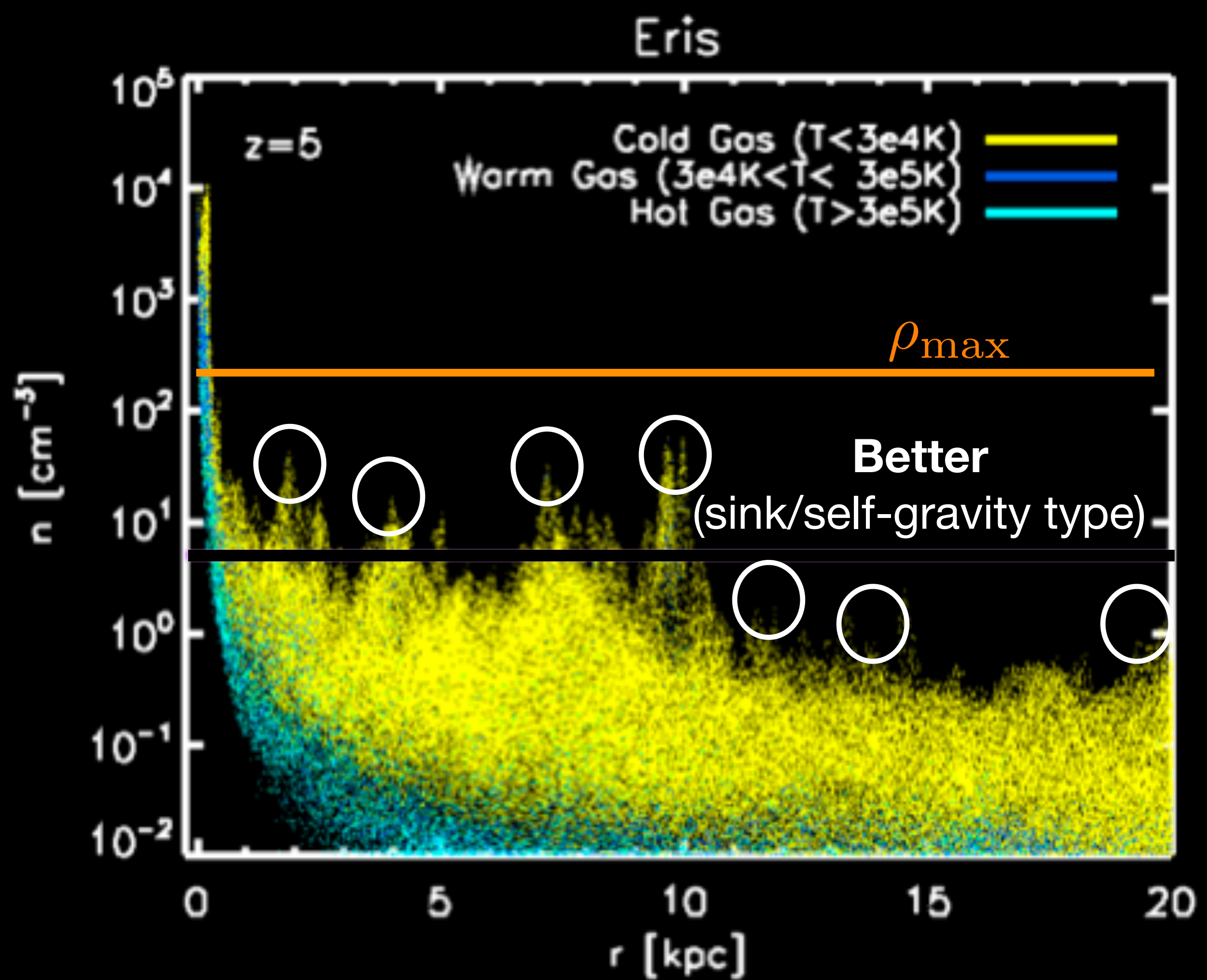

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Gravity:

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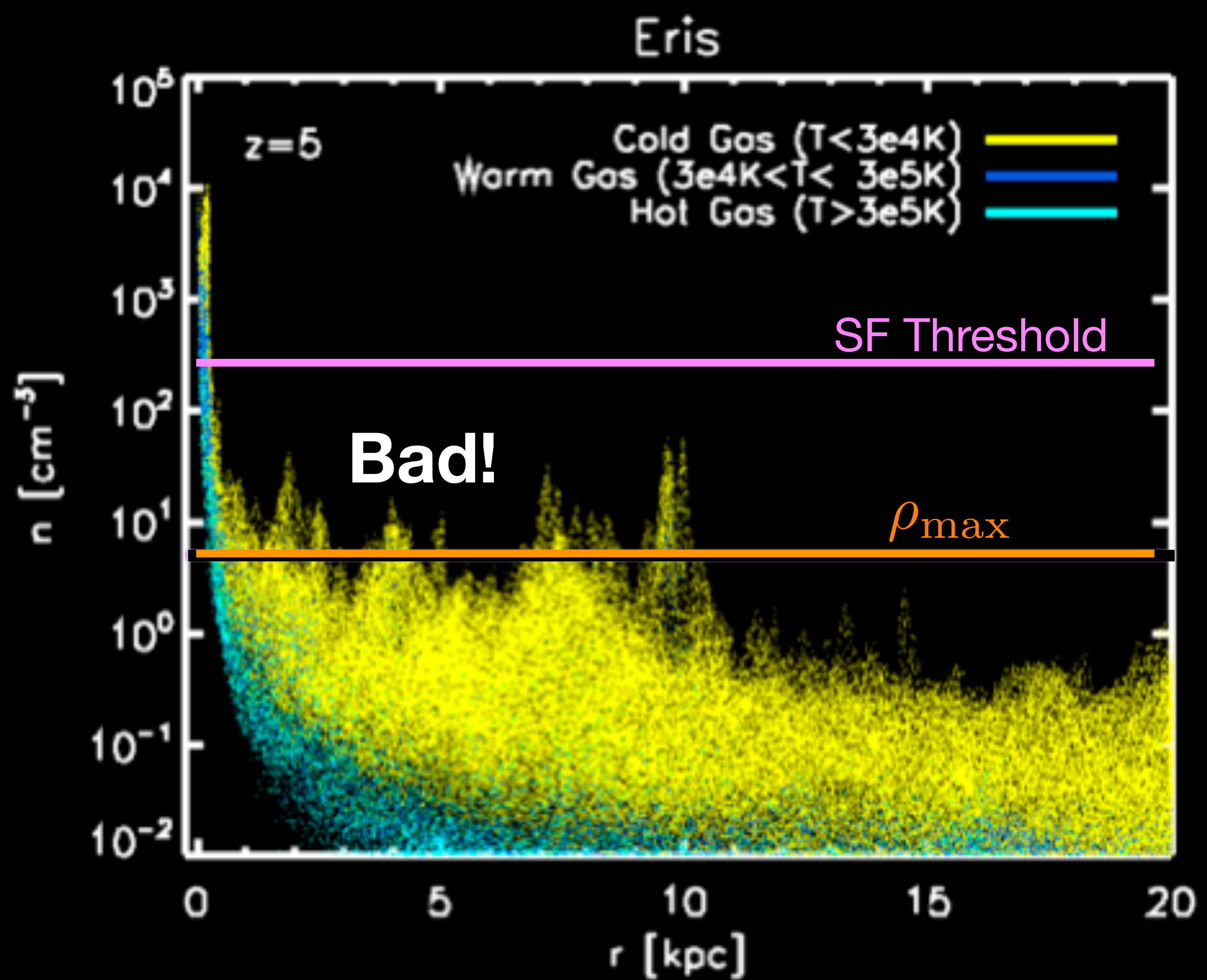

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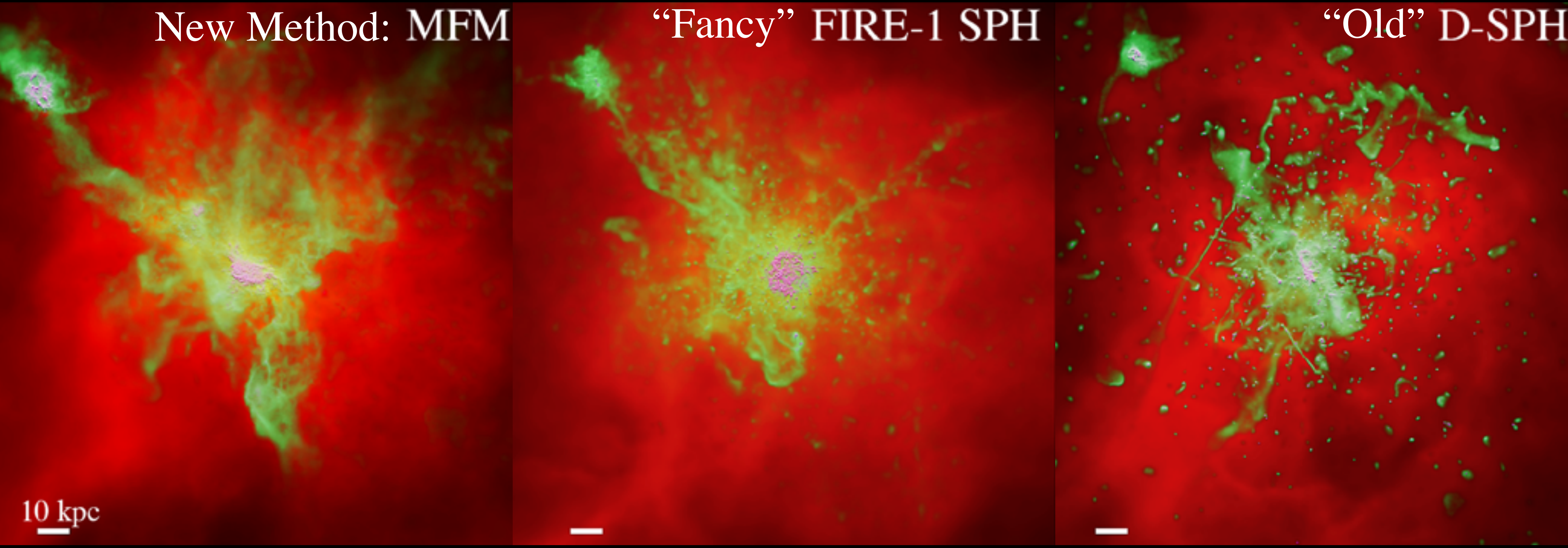
- Don’t use pressure floors with “sink particle” (self-gravity)-based SF

SPH: Non-Convergent Errors Still an Issue

PFH+ (1702.06148)

SPH errors in Outflows/CGM (not so much for integrated galaxy properties)



only ~50% different M_{stars}

+ Agertz 07 & many others

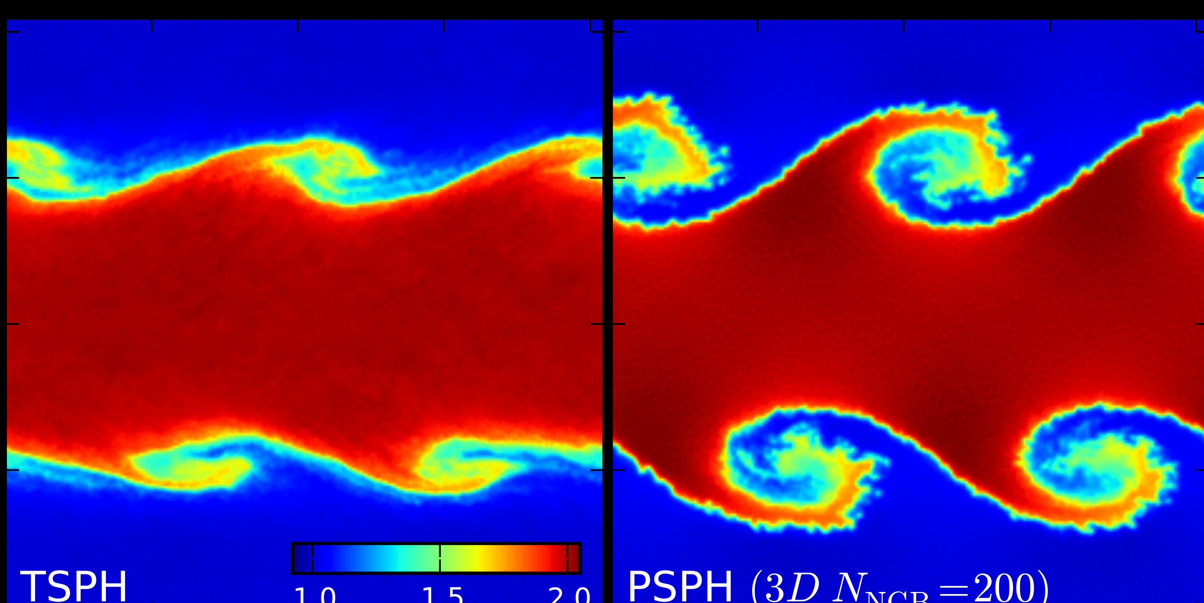
SPH: *Non-Convergent* Errors

CANNOT BE ELIMINATED IN SPH

- *Fundamental* (sub)-0th-order errors:
  - (1) Abandon SPH
  - (2) “beat down” with larger kernels: *not efficient!*

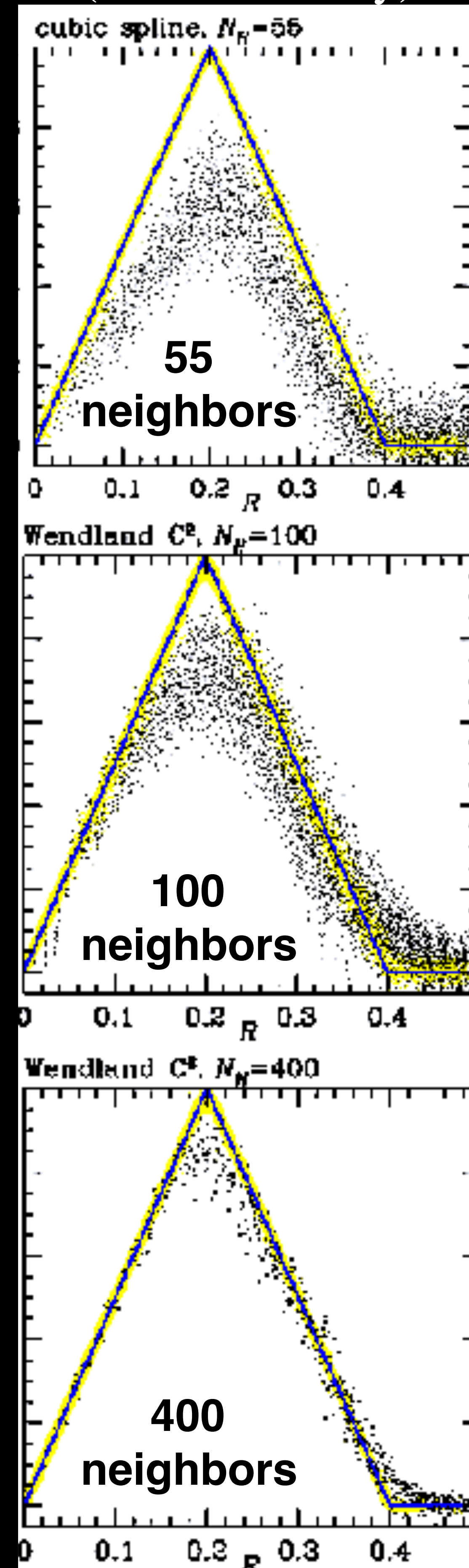
$$(N_{\rm neighbors} \propto N_{\rm particles}^{0.5-1.0})$$

- MHD & anisotropic diffusion operators ill-posed

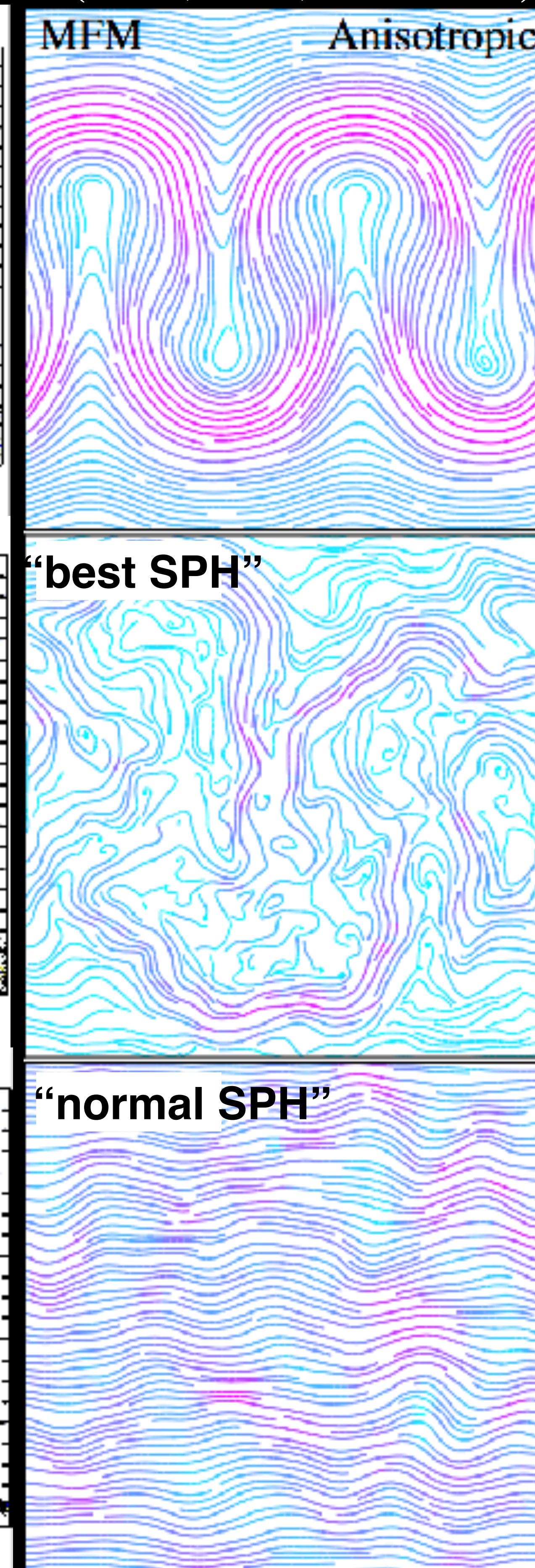


Dehnen & Aly, Rosswog, Hopkins, Tricco & Price, Read, Zhu

Gresho vortex (Dehnen & Aly)

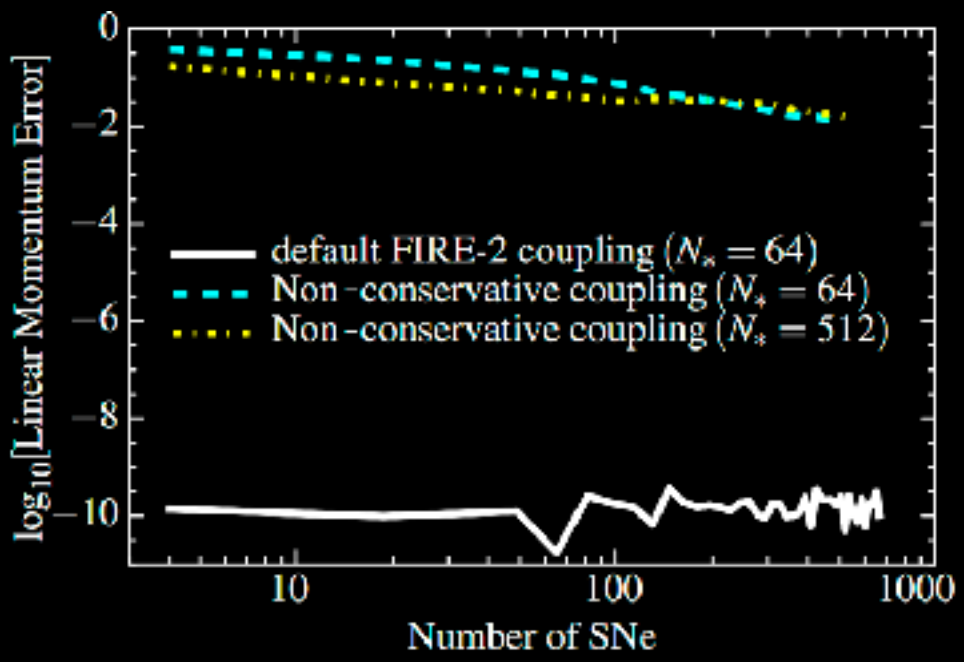


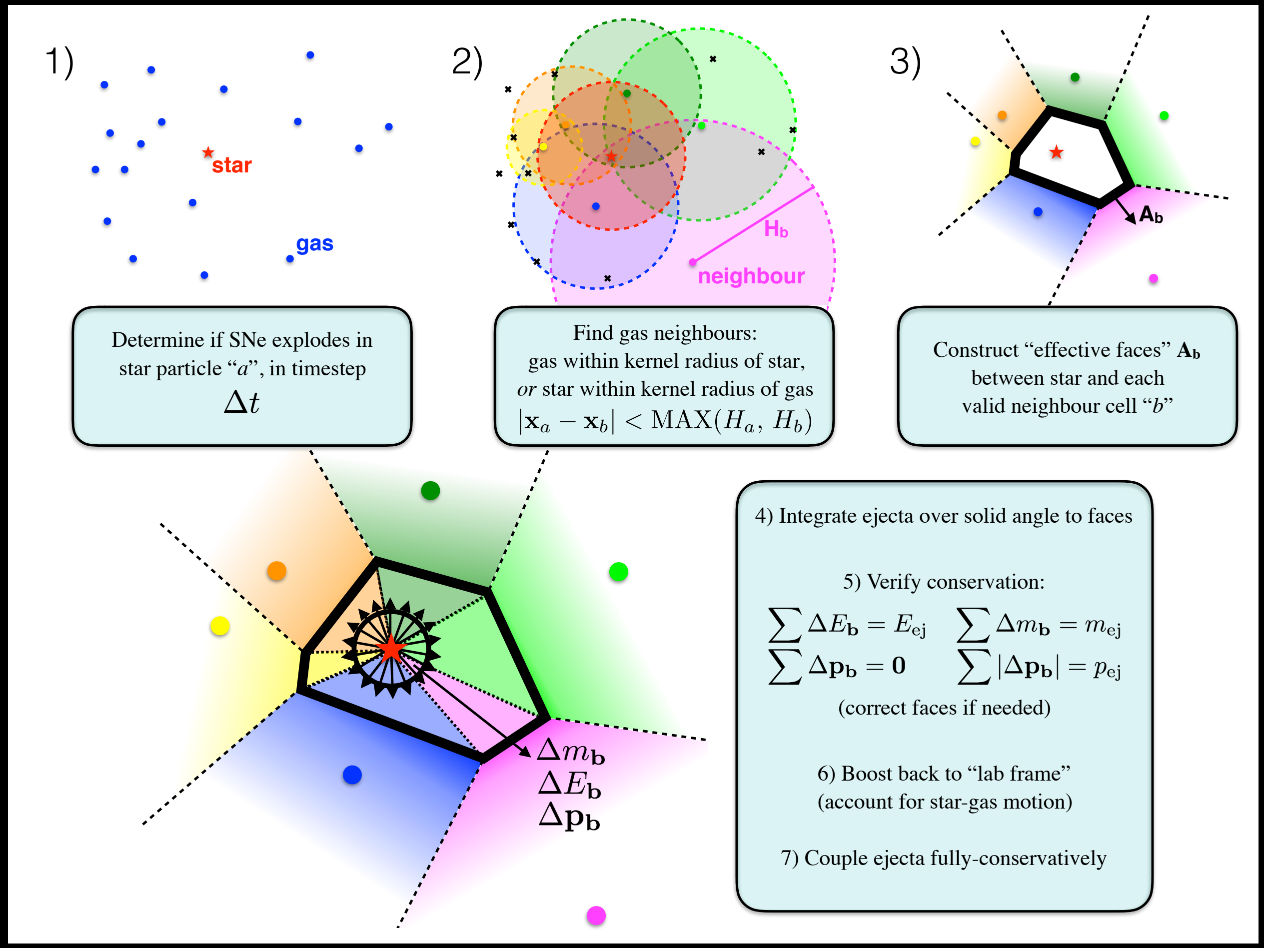
Anisotropic Conduction (MTI, HBI, Hall MRI)

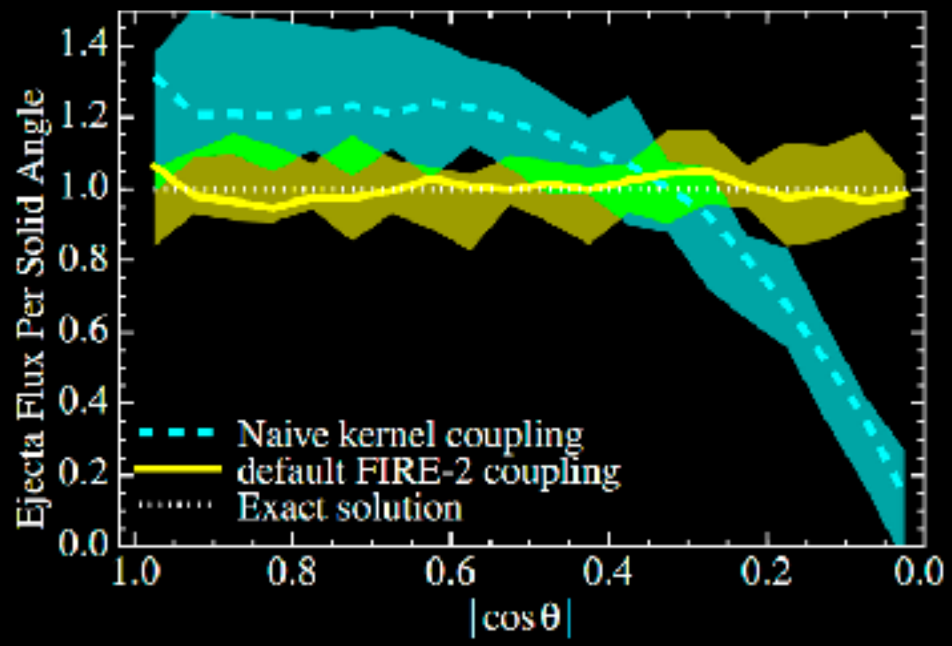


SNe/Mechanical FB:
ISOTROPY & CONSERVATION

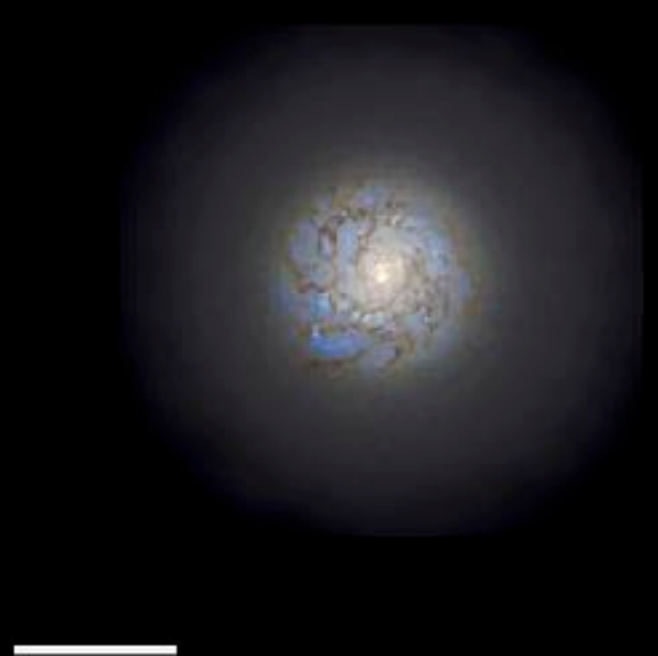
PFH+ (1707.07010)







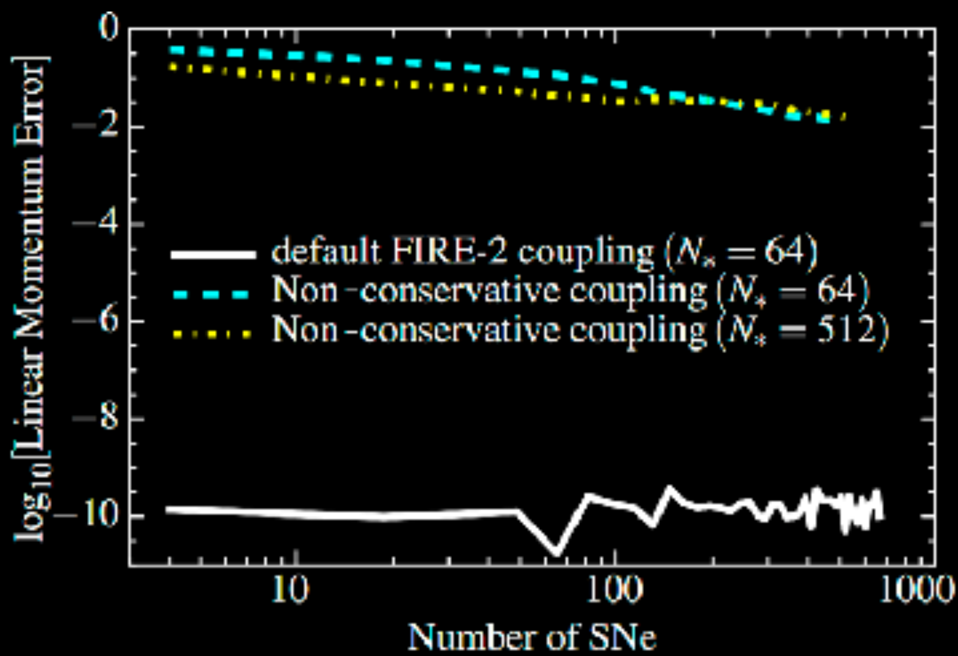
Careful (isotropic, conservative) Coupling

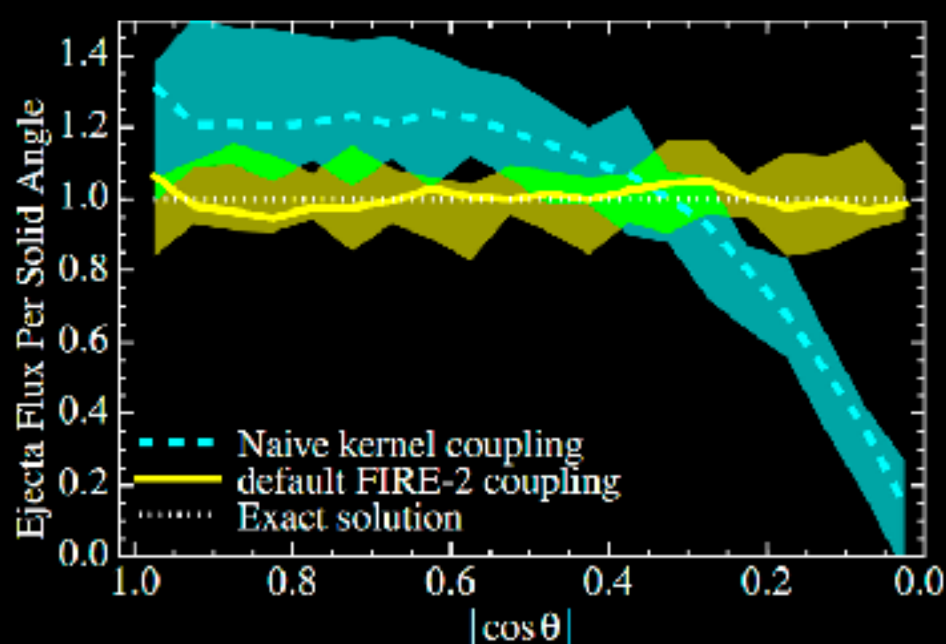


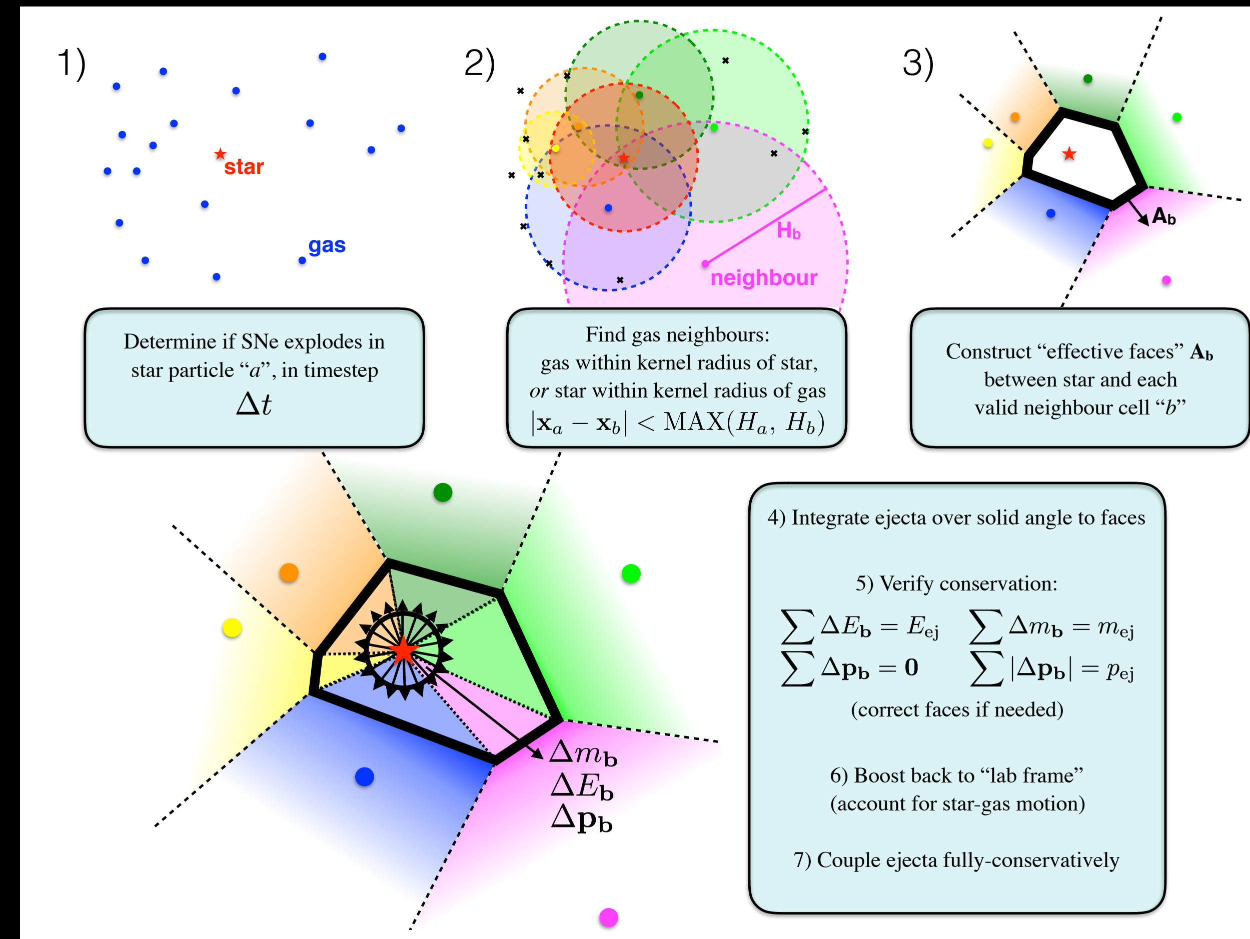
Simplest (non-conservative) Coupling

PFH+ (1707.07010)

SNe/Mechanical FB:
ISOTROPY & CONSERVATION

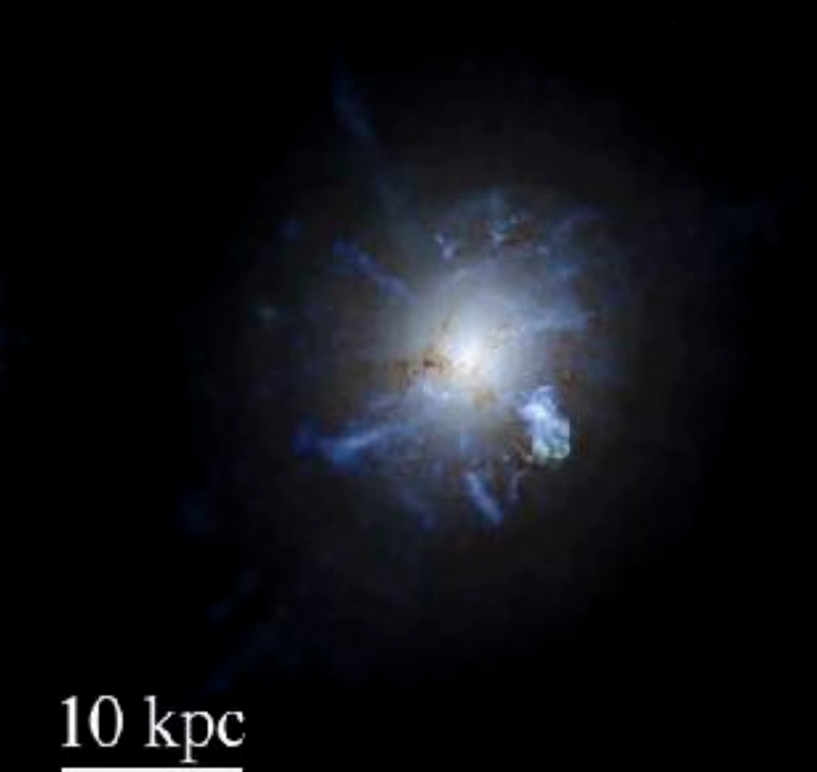






Careful (isotropic, conservative) Coupling

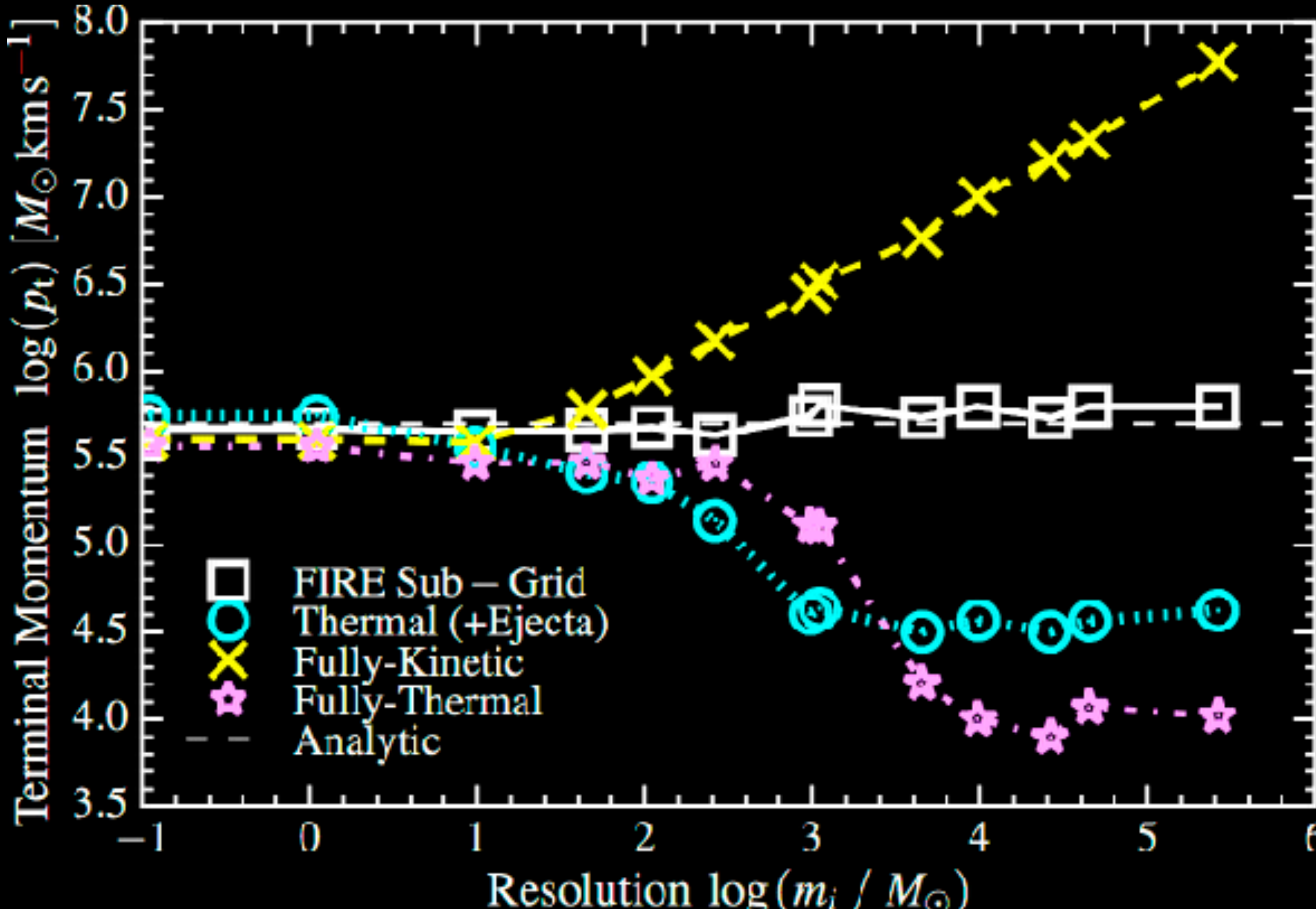
Simplest (non-conservative) Coupling



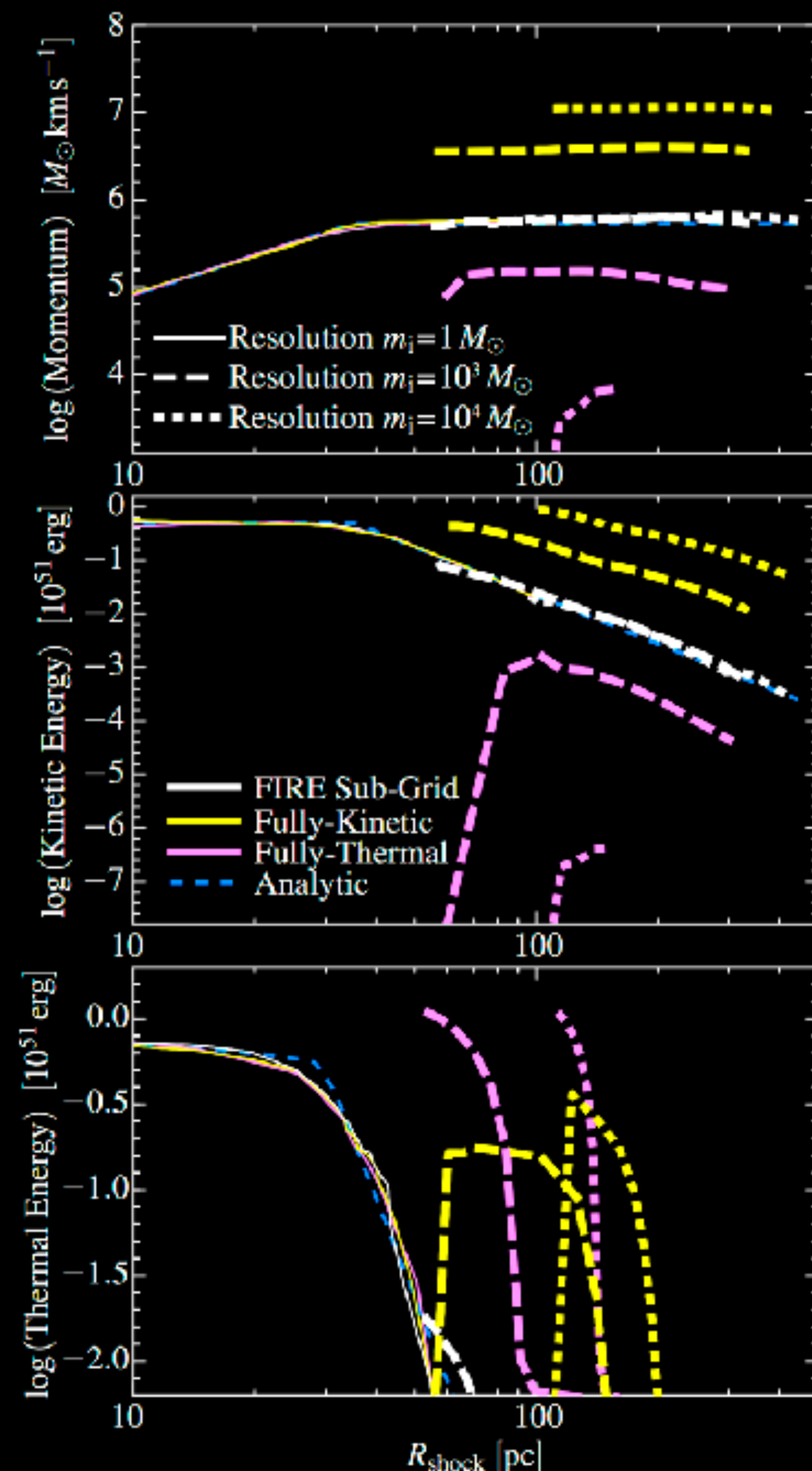
SNe/Mechanical FB:

MOMENTUM VS. ENERGY - A “RIGHT” ANSWER

Simple single-explosion test:



Want a model that matches high-res solution *at the radius/mass where you couple*

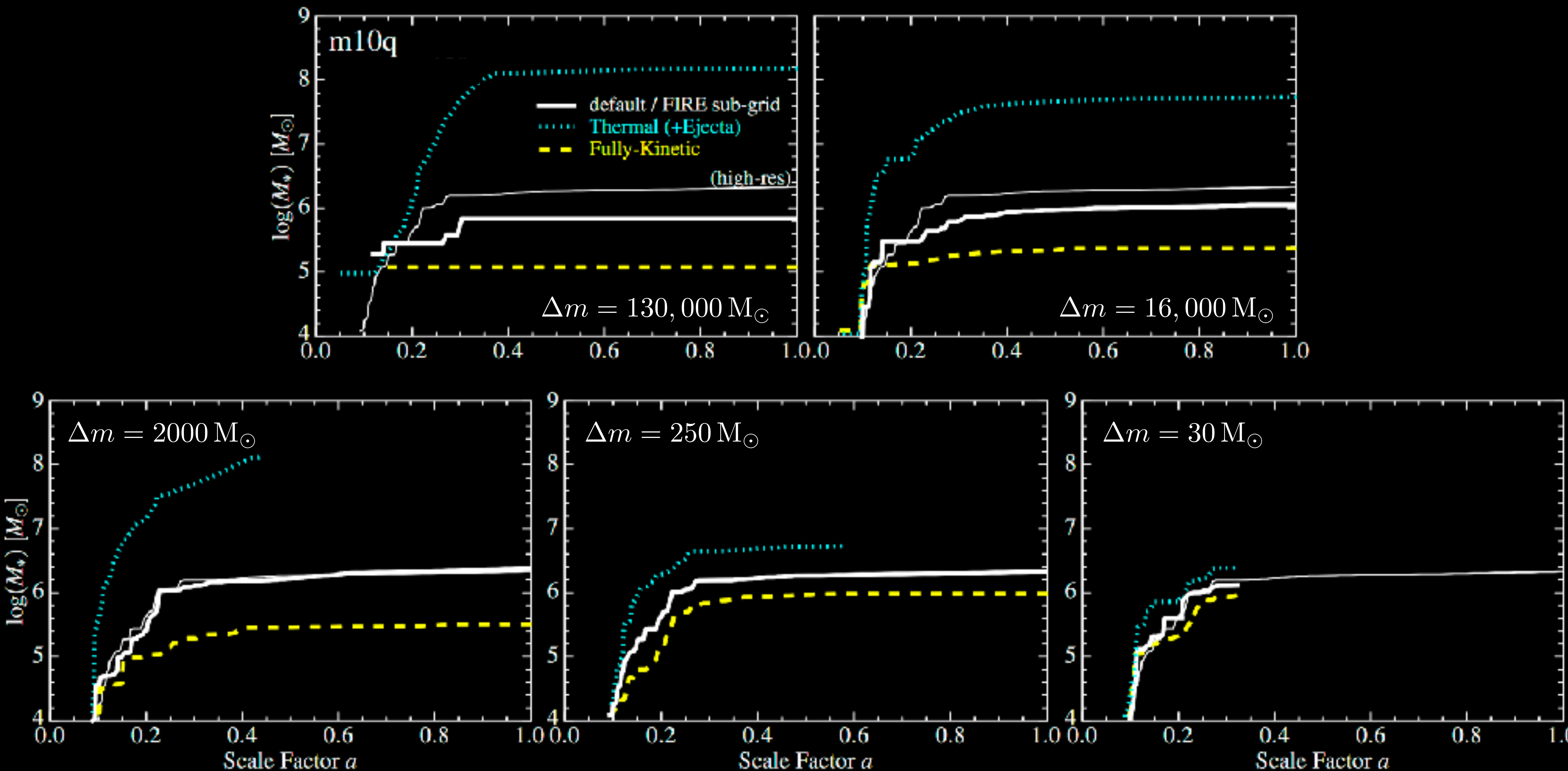


PFH+ (1707.07010)

SNe/Mechanical FB:

MOMENTUM VS. ENERGY - A "RIGHT" ANSWER

Stable behavior above/below resolution of individual blastwaves:

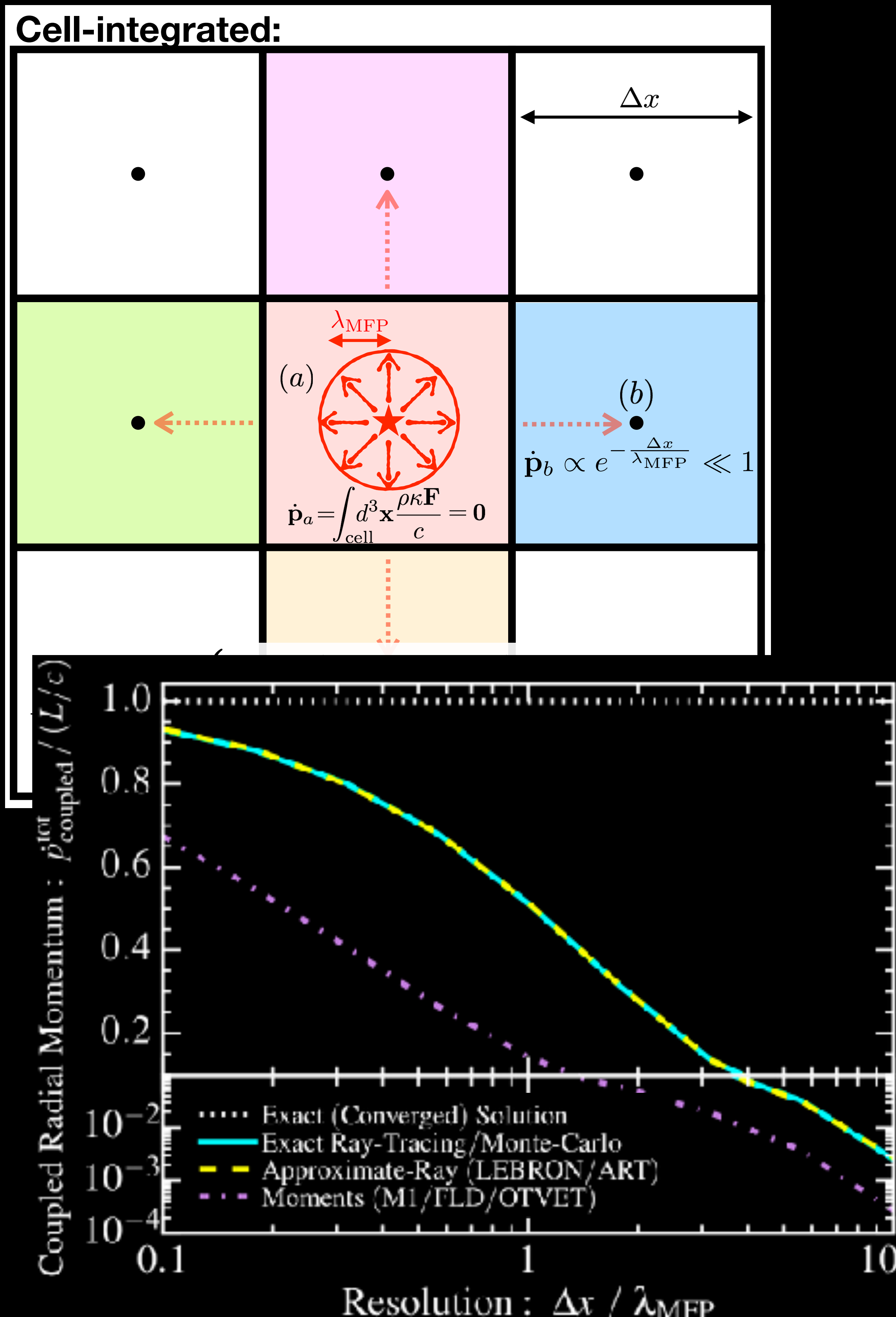


PFH+ (1707.07010)

Radiative FB:

DEALING WITH UNRESOLVED RADIATION PRESSURE (*ANY* RHD METHOD)

PFH & Grudic (tomorrow)



"Brute-force" Resolution Required:

$$\Delta x \ll \lambda^{\rm photon}_{\rm MFP}$$

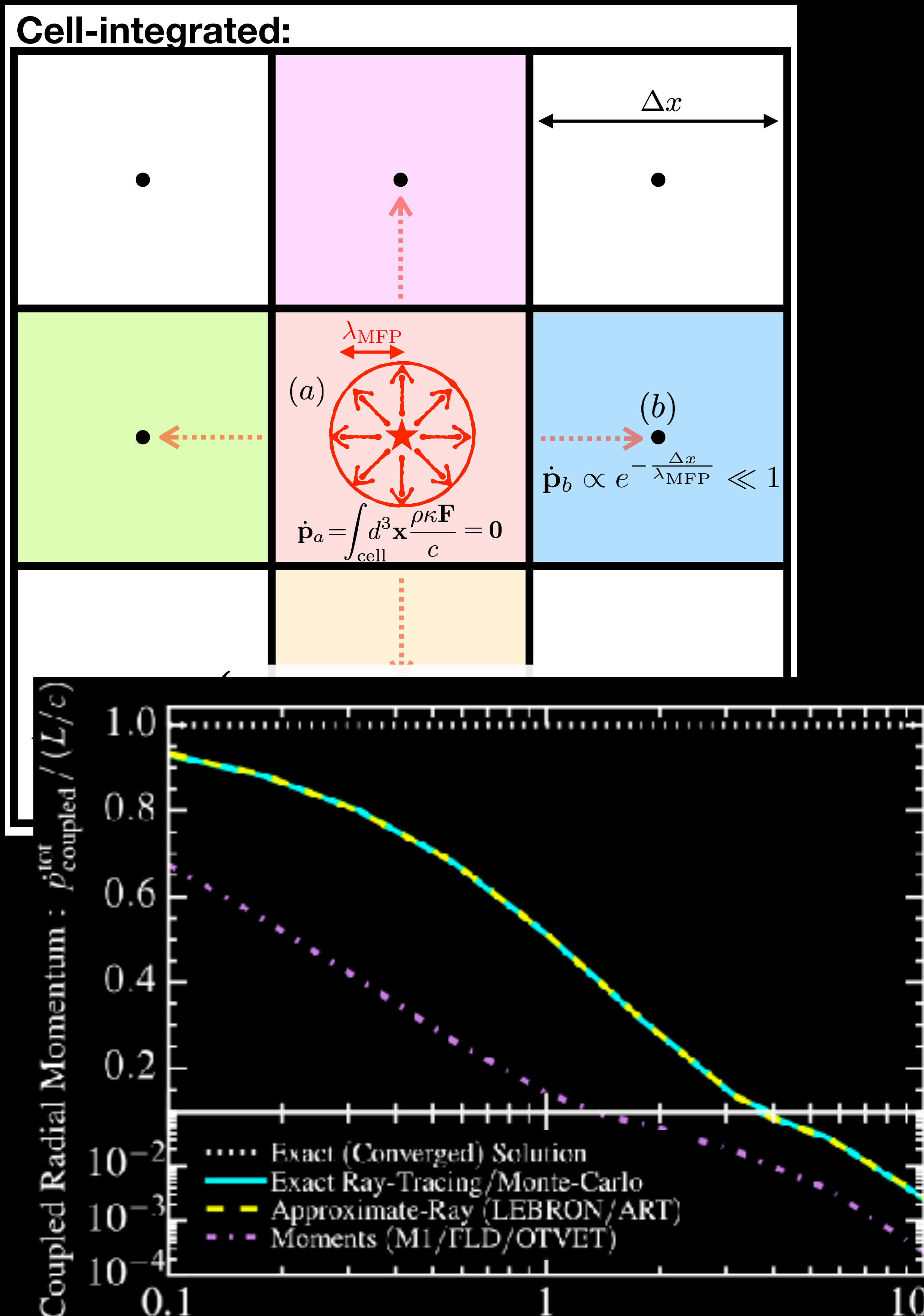
$$\Delta m_i \ll \frac{\lambda^2_{\rm MFP}}{\kappa_\nu} \sim 10^{-13} {\rm M}_\odot \left[\frac{10^4\,{\rm cm}^{-3}}{n_{\rm gas}}\right]^2$$

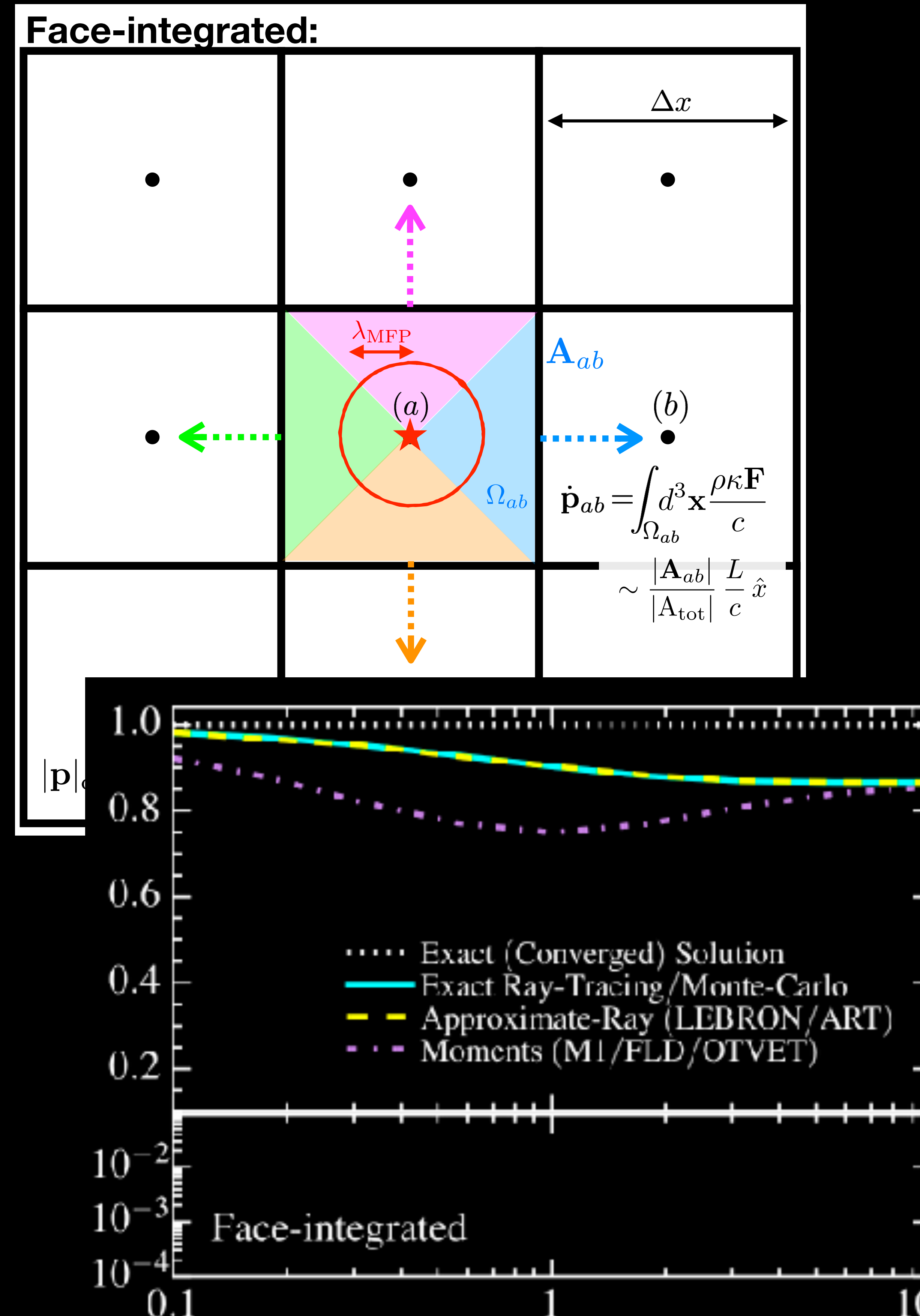
(ionizing photons)

PFH & Grudic (tomorrow)

Radiative FB:

DEALING WITH UNRESOLVED RADIATION PRESSURE (*ANY* RHD METHOD)





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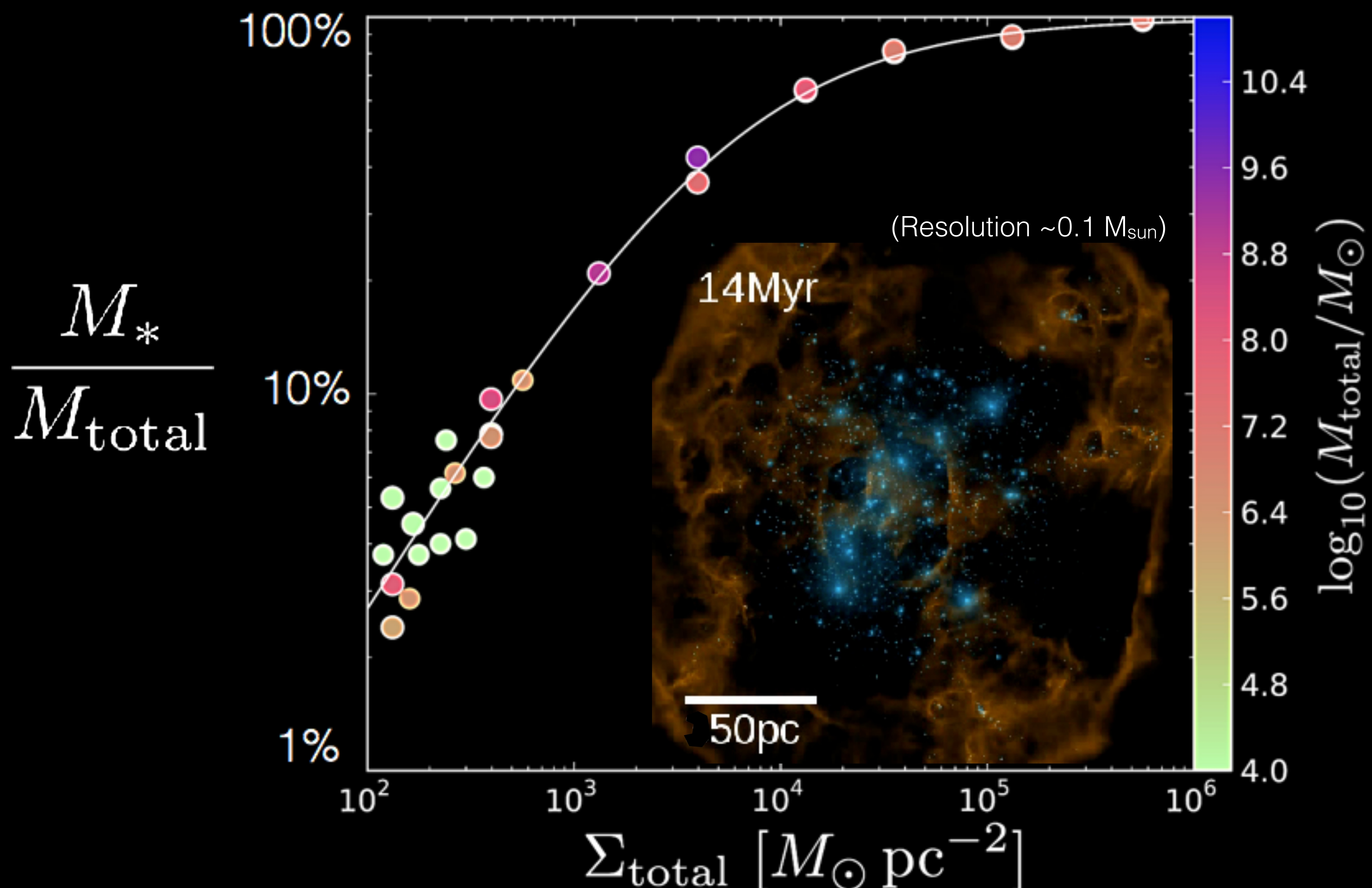
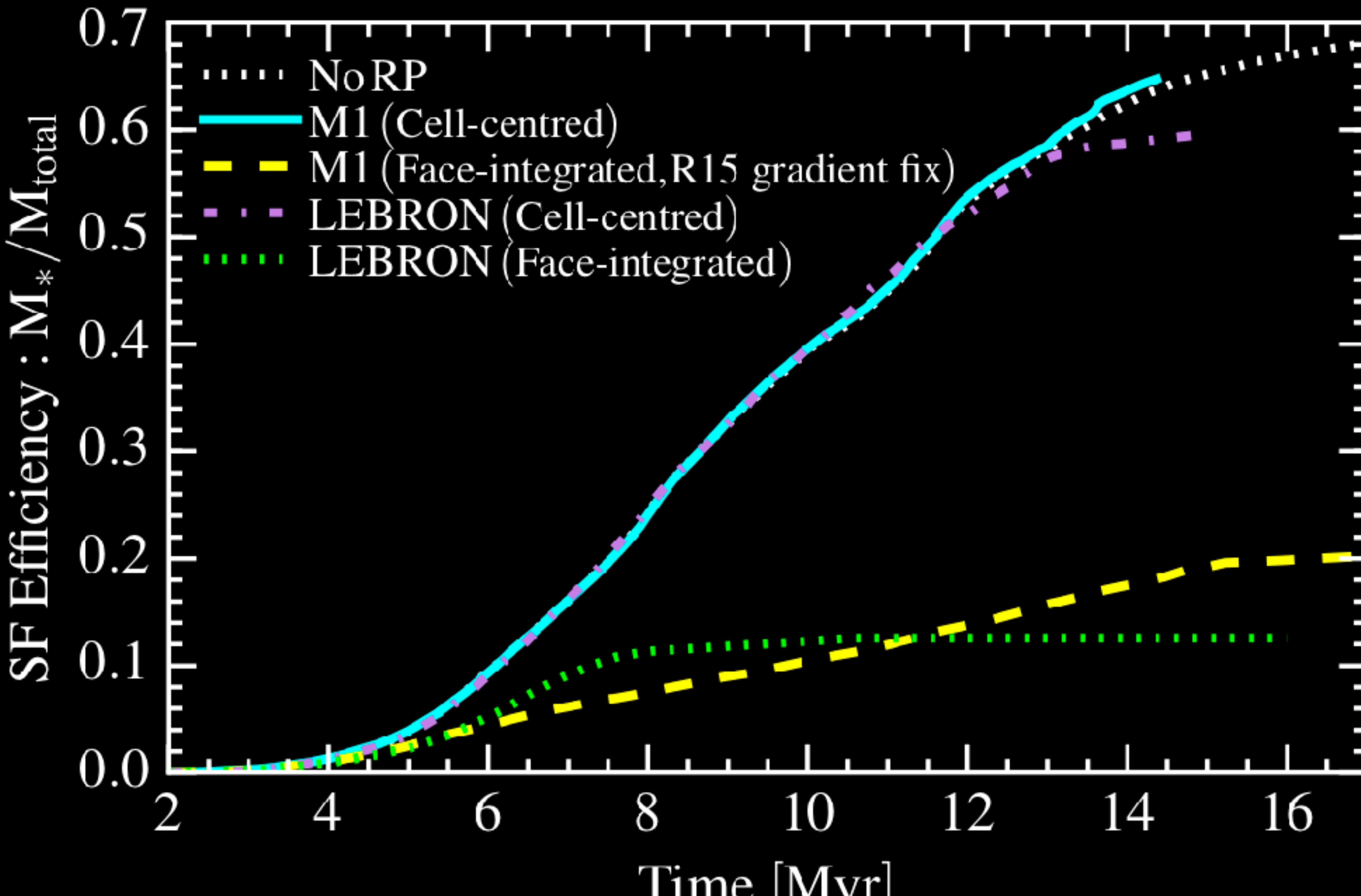


$$\text{Gravity} \sim \frac{G\,M_{\rm tot}\,M_{\rm gas}}{R^2} \propto M_{\rm tot}\,\Sigma_{\rm gas} \quad \text{vs.} \quad \text{Feedback} \sim \frac{\text{Momentum}}{\text{Time}} \propto (\ldots)\,M_*$$

PFH & Grudic (tomorrow)

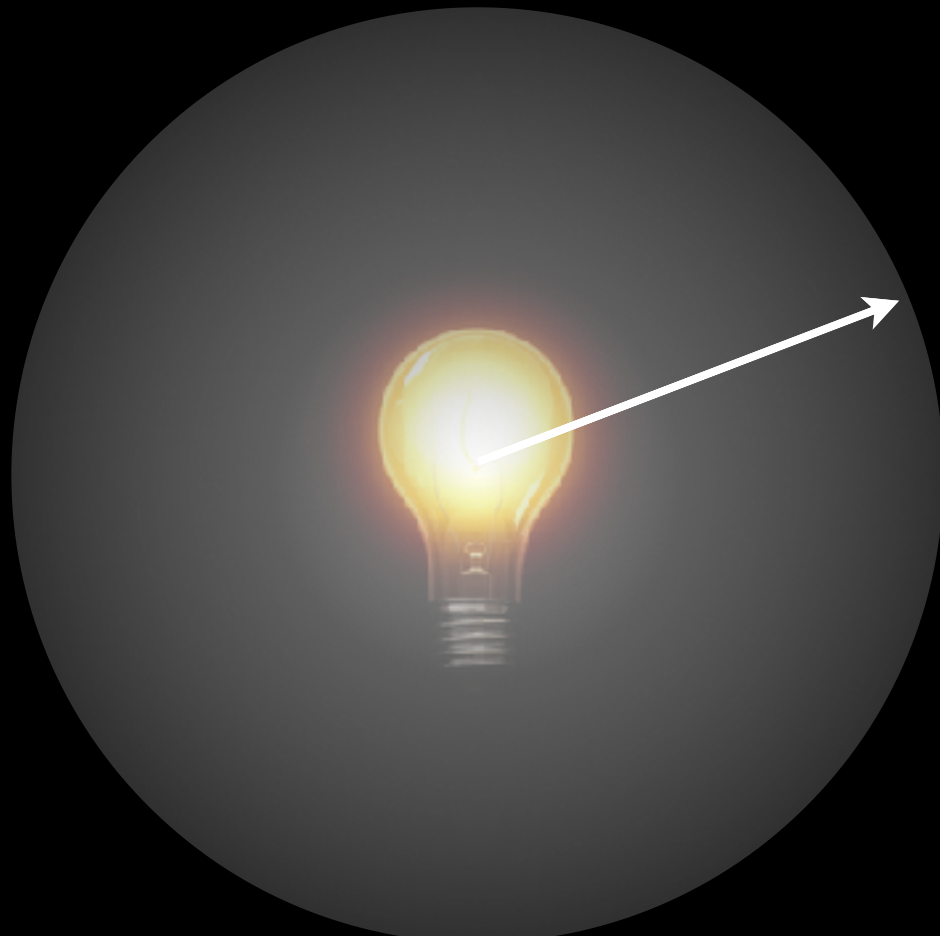
Radiative FB:

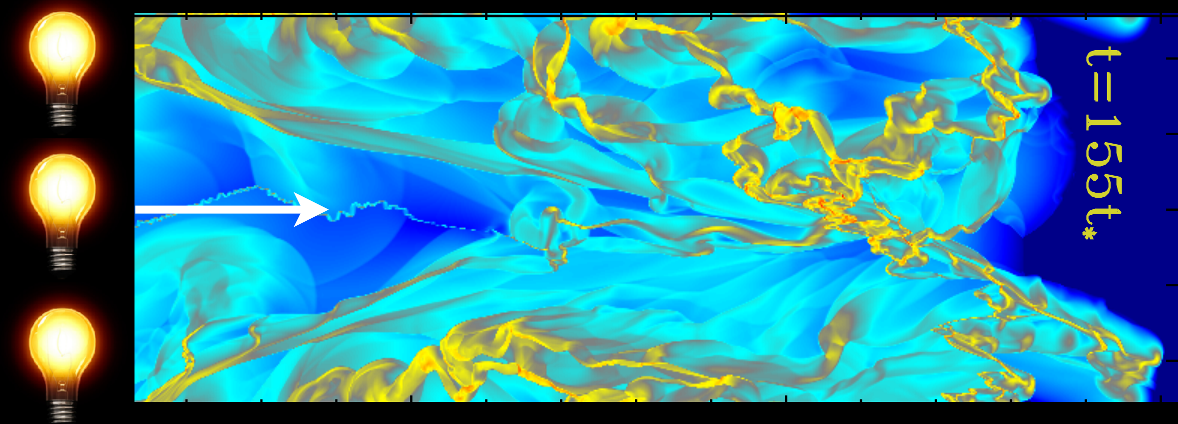
DEALING WITH UNRESOLVED RADIATION PRESSURE (*ANY* RHD METHOD)

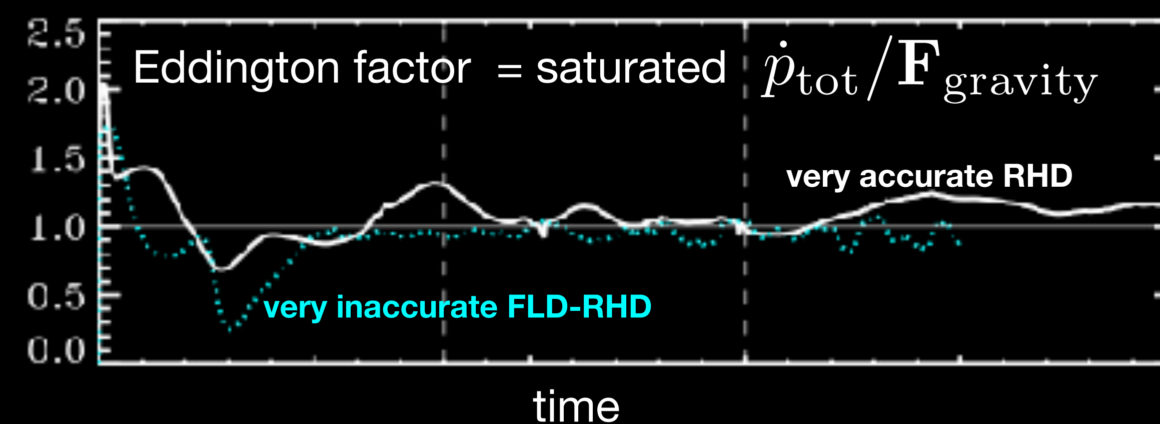


Radiative FB:

MULTIPLE-SCATTERING: Actual Numerical Differences Are Small!







Smooth density profile has *exact* solution:

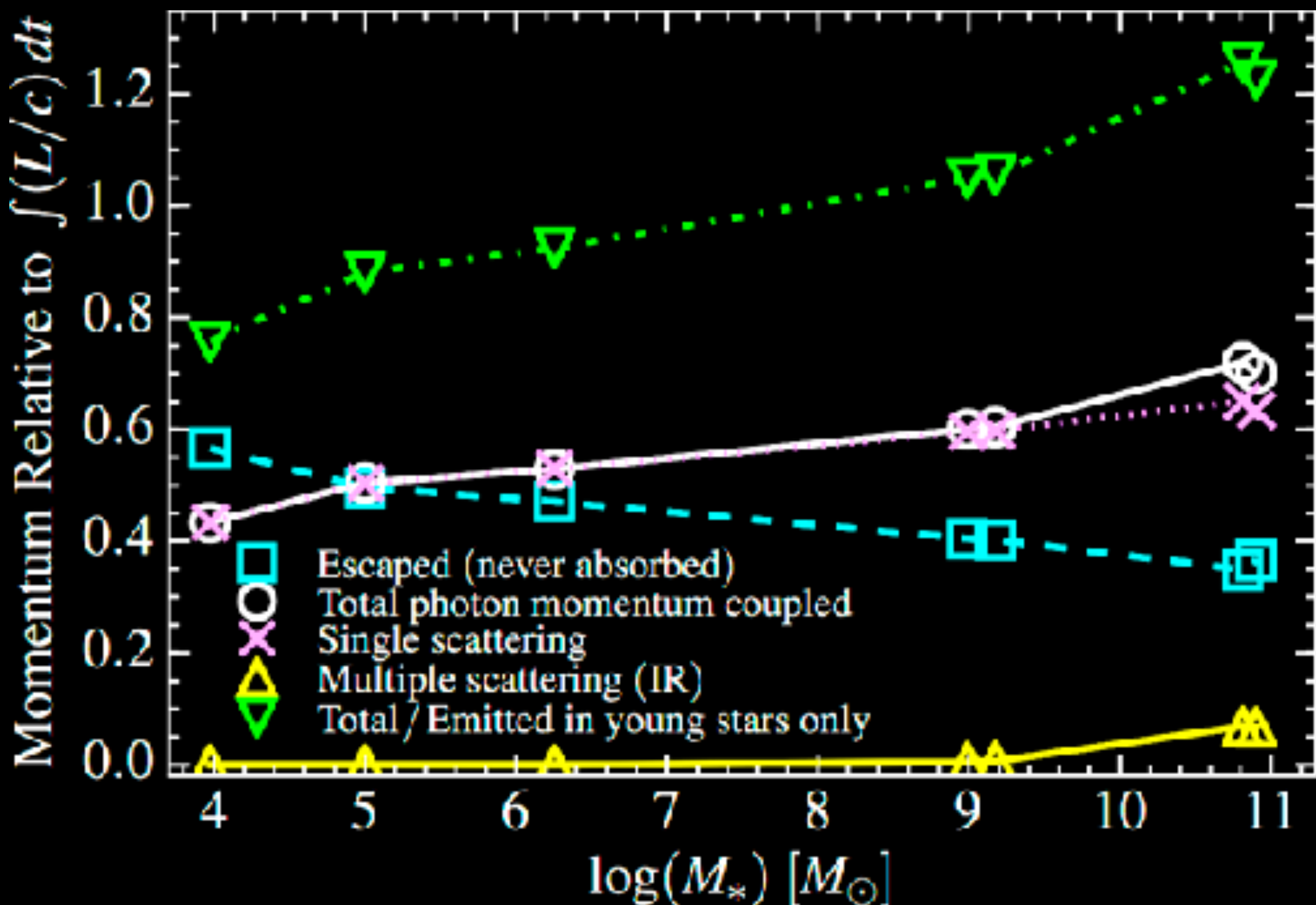
$$\dot{p}_{\rm tot} = \tau_{\rm eff}\, L/c$$

What if it's "lumpy"?

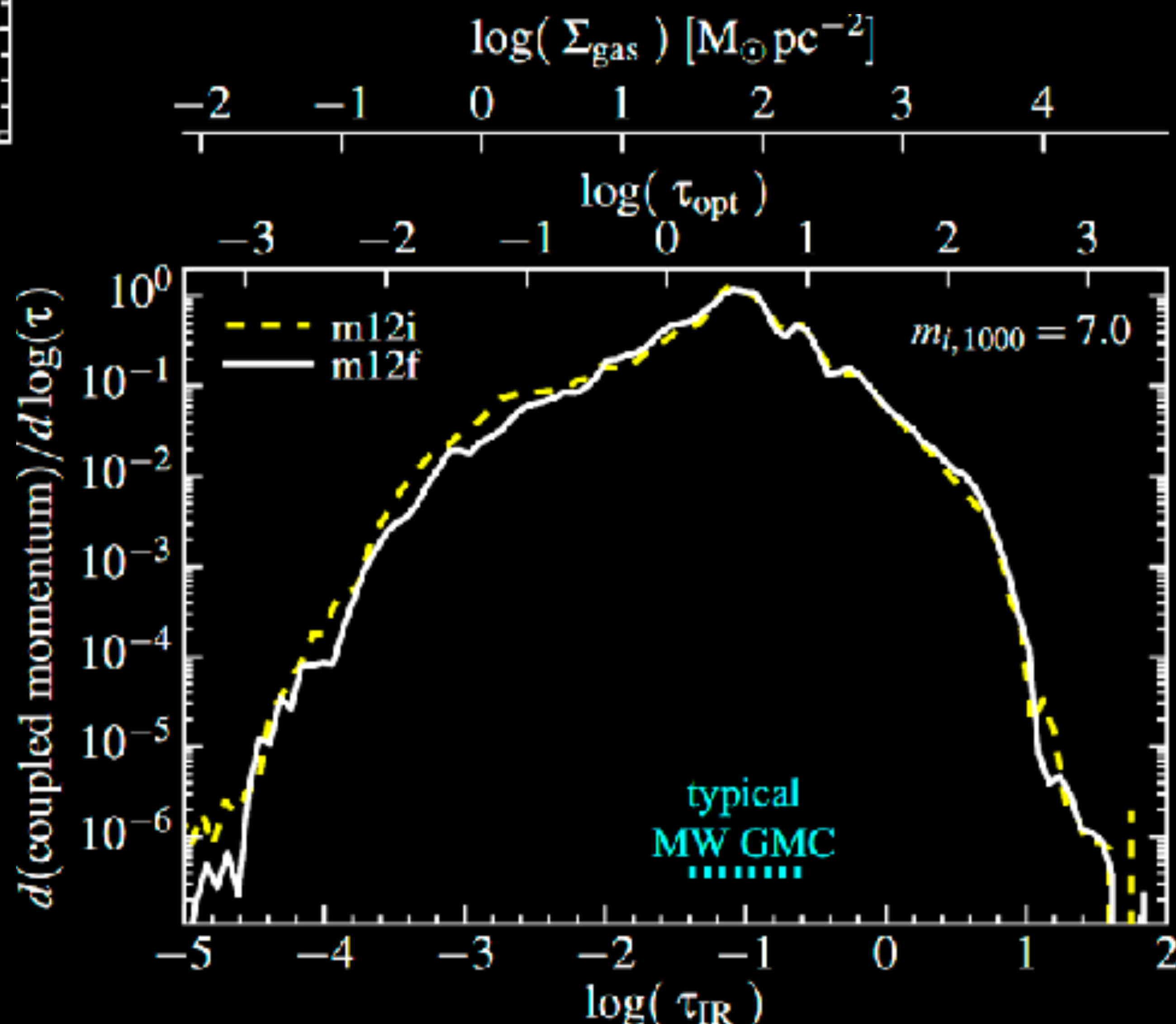
Study:	$\frac{\dot{p}_{\rm tot}}{\tau L/c}$	Saturates at…
Davis+ 14 (short-char):	~0.90	~ 20x ~ Eddington
Hopkins+ 11 (no RT!):	~1.0	~ 30x ~ Eddington
Costa+ 18 (M1):	~1.0	~ 50x ~ Eddington
Krumholz+ 12 (FLD):	~0.75	~ 15x ~ Eddington
Zhang+ 17 (rays):	~0.5-0.9	> 10x ~ Eddington
Jiang+ 15 (rays-line):	~1.0	> 50x ~ Eddington
Tsang+ 17 (monte carlo):	~0.97	> 250x ~ Eddington

* $\tau_{\rm eff} = \sqrt{\langle N^{\rm total}_{\rm scatterings}\rangle}$ (depends on opacity curve/spectrum/etc)

Radiative FB:
ACTUAL MULTIPLE-SCATTERING in FIRE



~1/2 light absorbed
[UV from young stars]



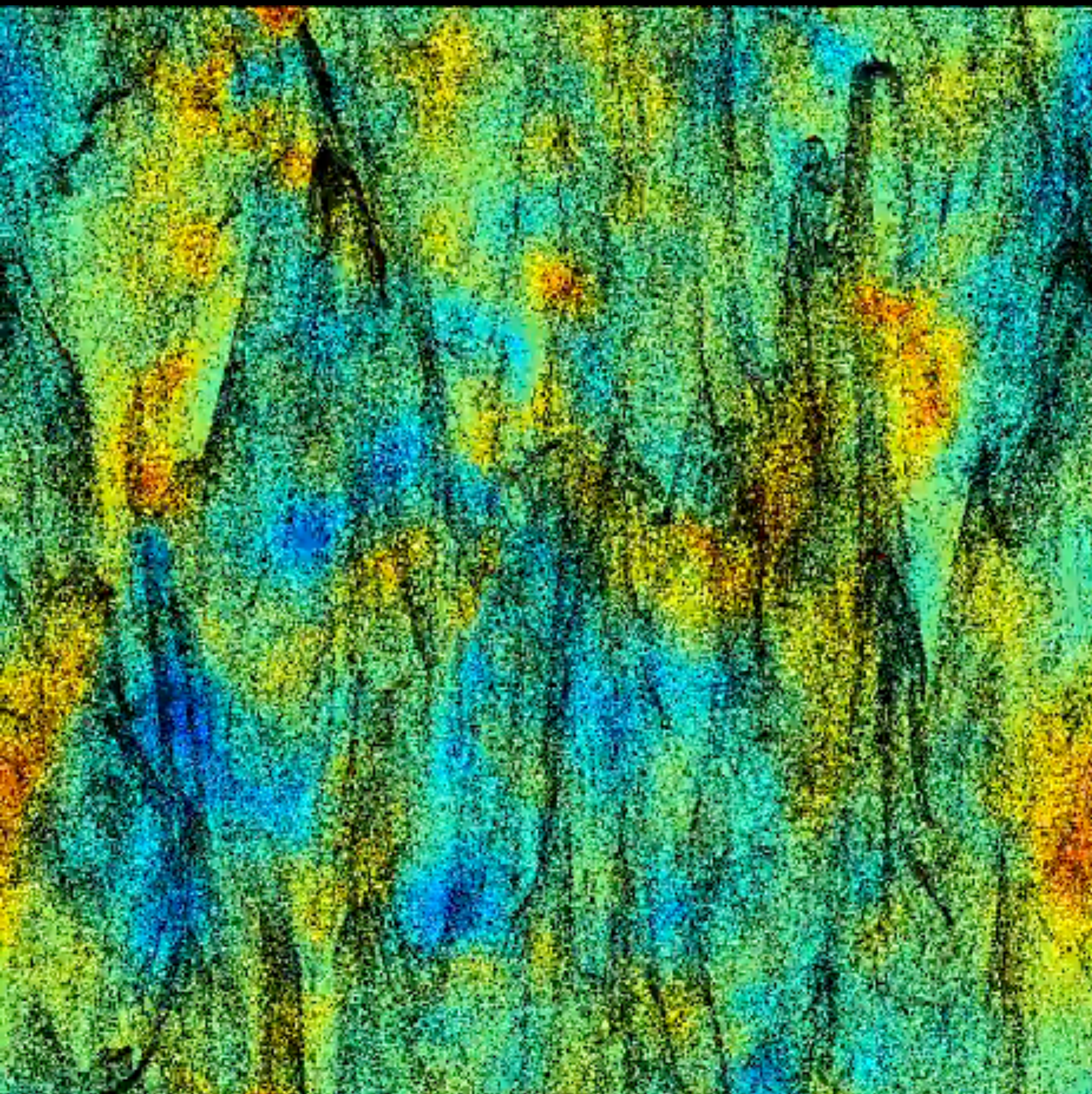
Mostly in normal GMCs with $\tau_{\rm IR}$~0.1

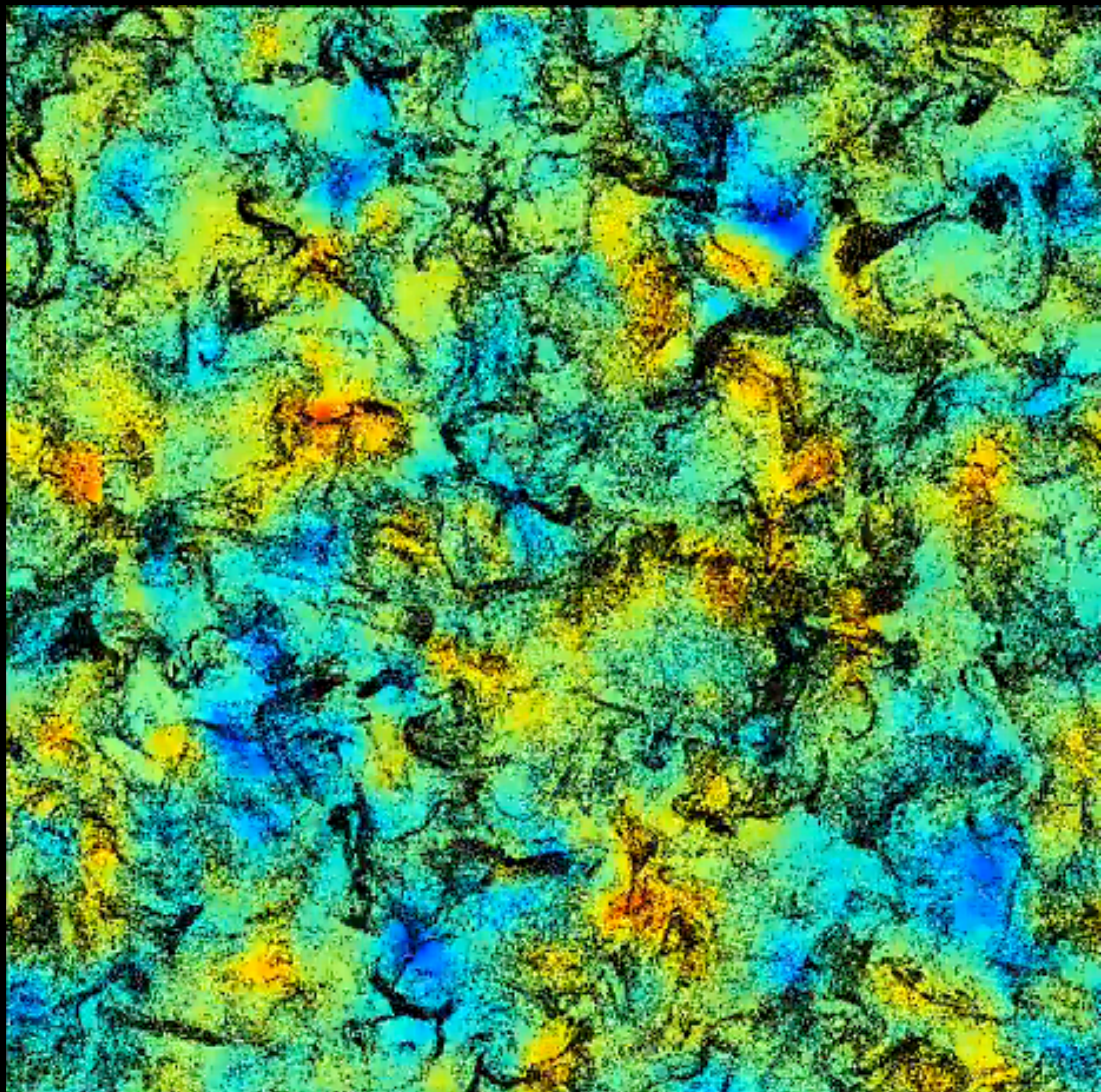
Radiative FB & the RDI

BEWARE: DUST DOES NOT MOVE WITH GAS!

Squire & Hopkins
(1706.05020, 1707.02997,
1711.03975, 1801.10166)

$|\mathbf{w}|_{\rm drift} \approx 3\,c_s$ $L_{\rm box} \sim 30\,c_s\langle t_s\rangle$ $\Delta t \sim 15\,\langle t_s\rangle$





z drift x

0.1x Gas Density 10x

y ⊗ drift x

Some examples:

Brunt-Vaisala RDI

Settling RDI, Epicyclic/Streaming RDI

Acoustic RDI, Fast/Slow Magnetosonic RDI

Alfven RDI, Gyro RDIs, CR-Type (Bell) RDIs

Radiative FB & the RDI

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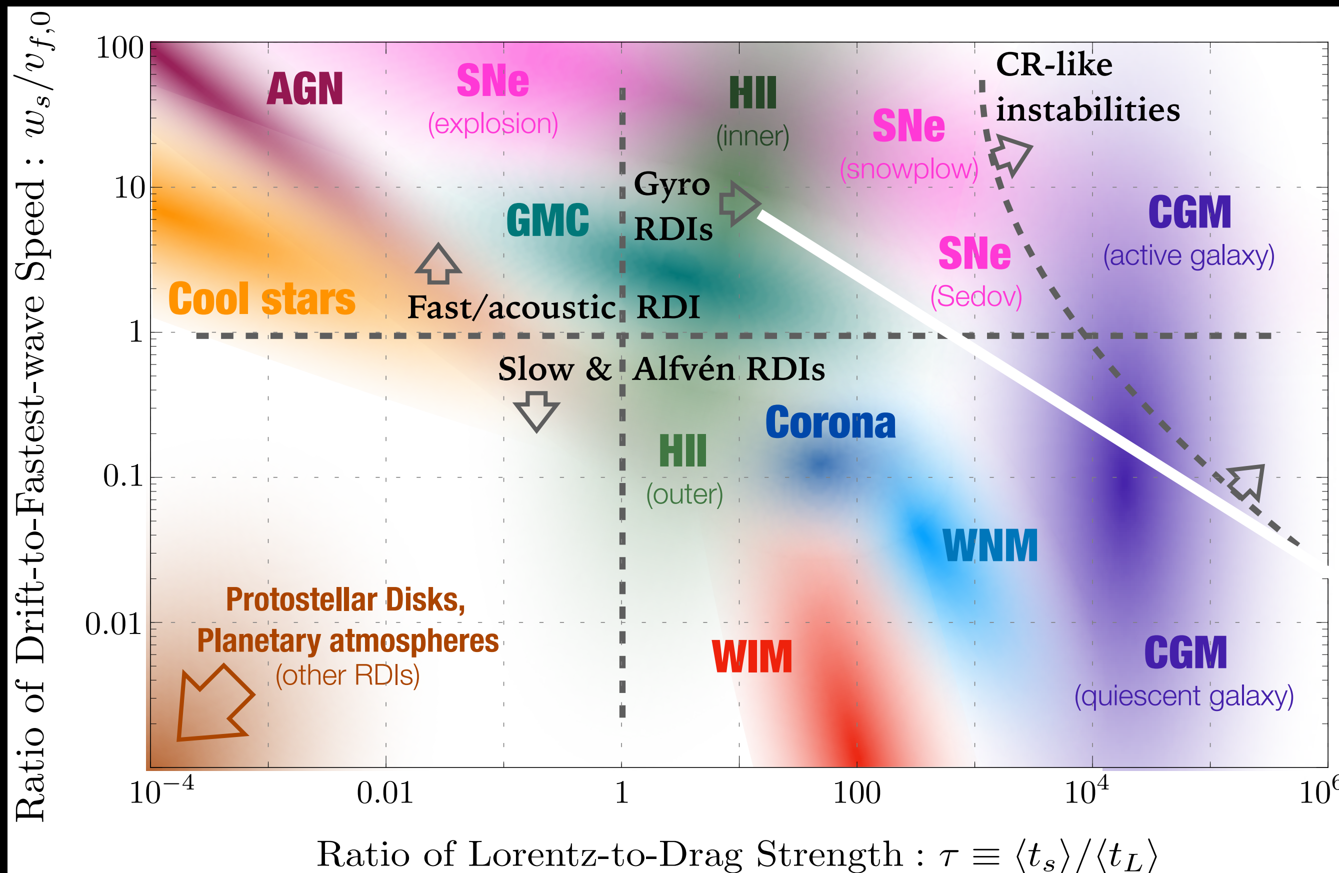
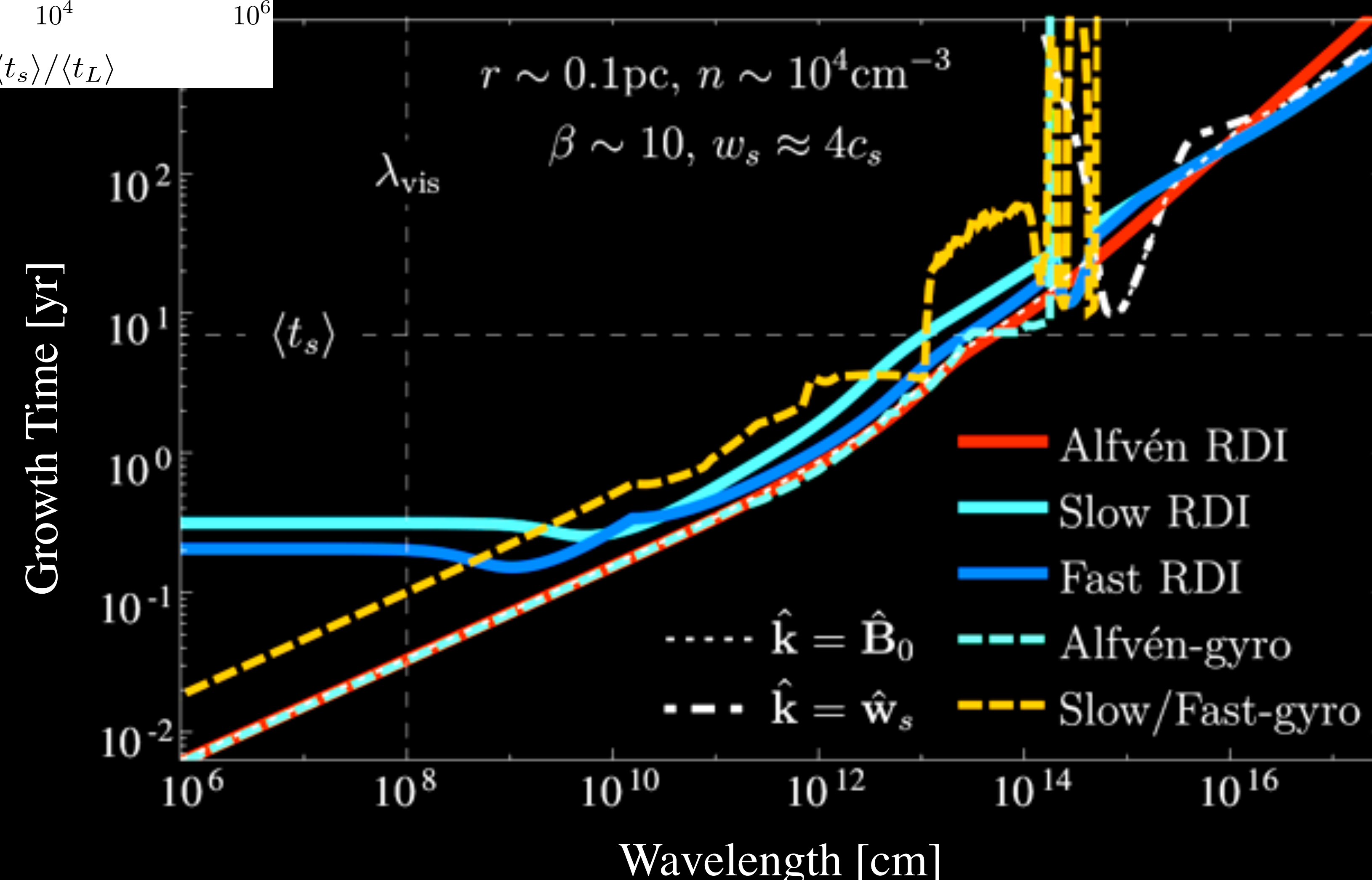
(1706.05020, 1707.02997,
1711.03975, 1801.10166)





Cosmic Rays:

MOMENT APPROXIMATIONS: TIMESTEPS & ACCURACY AT HIGH RESOLUTION

0th-moment expansion (FLD-like):

$$\frac{\partial e_{\rm cr}}{\partial t} = \nabla \cdot \left(\bar{\boldsymbol{\kappa}}_{\rm eff} \cdot \nabla e_{\rm cr} \right)$$

(+ streaming, cooling, etc.)

- Super-luminal (when $\Delta x \lesssim {\rm pc}$)
- Timestep (explicit): $\Delta t < C \,\frac{\Delta x^2}{\kappa_{\rm eff}}$
- No streaming-diffusion transition

1st-moment expansion (M1-like):

$$\frac{\partial e_{\rm cr}}{\partial t} = \nabla \cdot \mathbf{F}_{\rm cr}$$

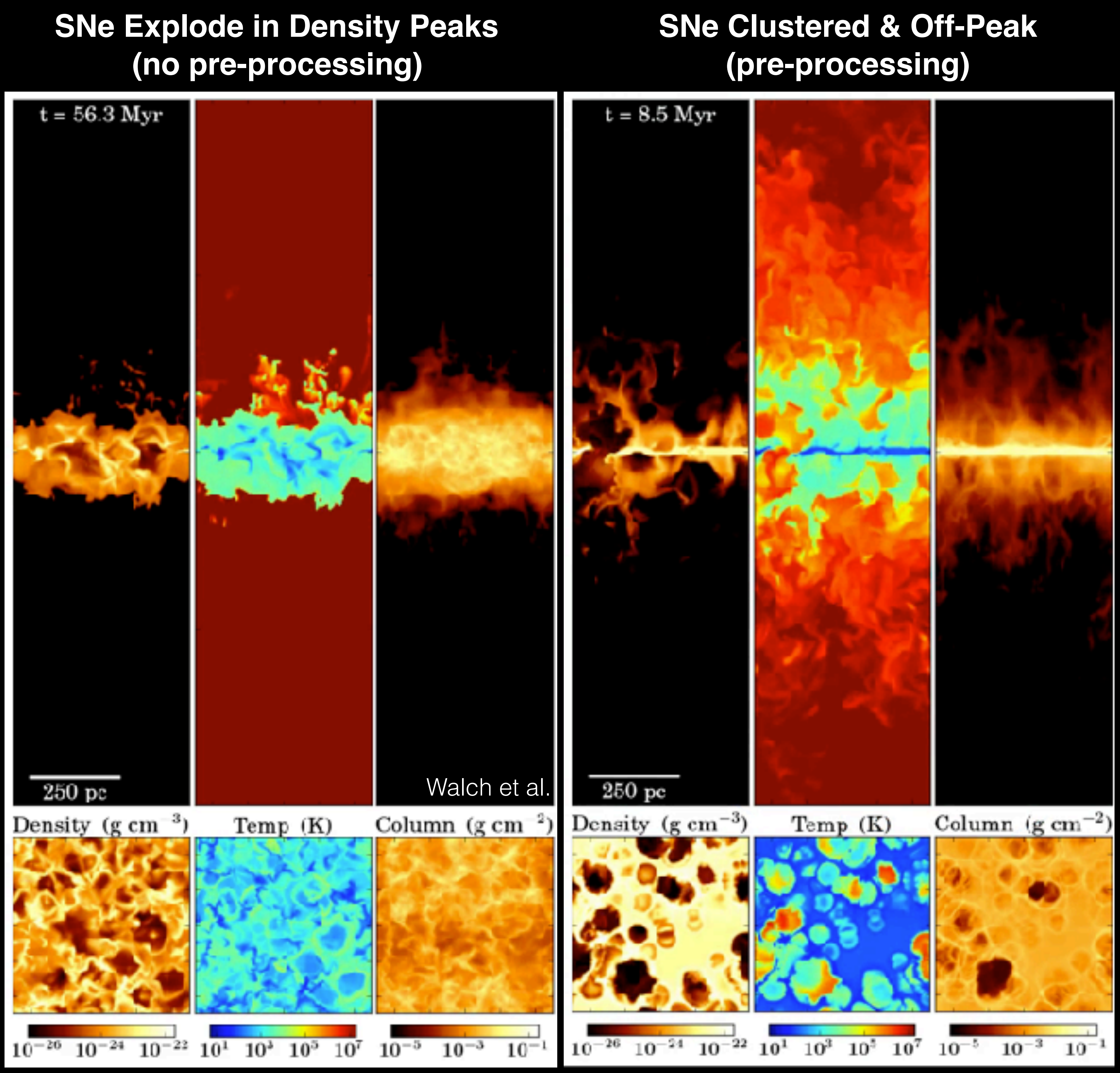
$$\frac{1}{\tilde{c}^2}\frac{\partial \mathbf{F}_{\rm cr}}{\partial t} + \nabla \cdot \mathbb{P} = \bar{\boldsymbol{\kappa}}_{\rm eff}^{-1} \cdot \mathbf{F}_{\rm cr}$$

- Maximum bulk speed = $\tilde{c}$
- Timestep (explicit): $\Delta t < C \,\frac{\Delta x}{\tilde{c}}$
- Correctly handles
  streaming-diffusion transition

e.g. Jiang & Oh, arXiv:1712.07117
(also GIZMO user notes)

Outflow/Shearing Boxes:

GEOMETRY MATTERS (cannot use shearing box and get any outflow right above ~1 scale height)

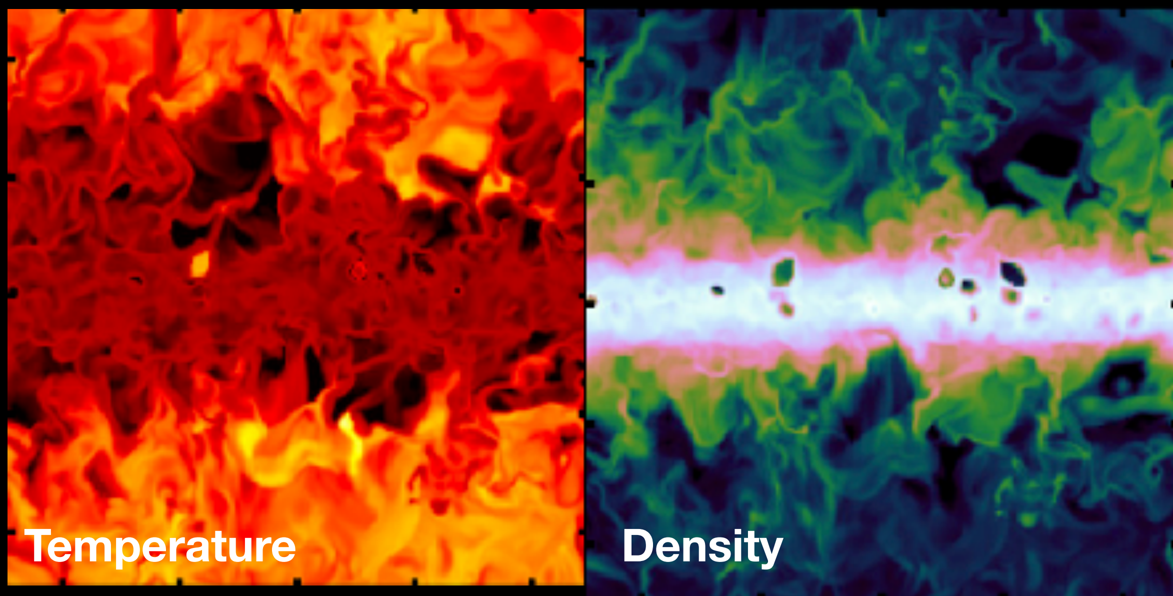


Walch et al.
(SILCC Project)

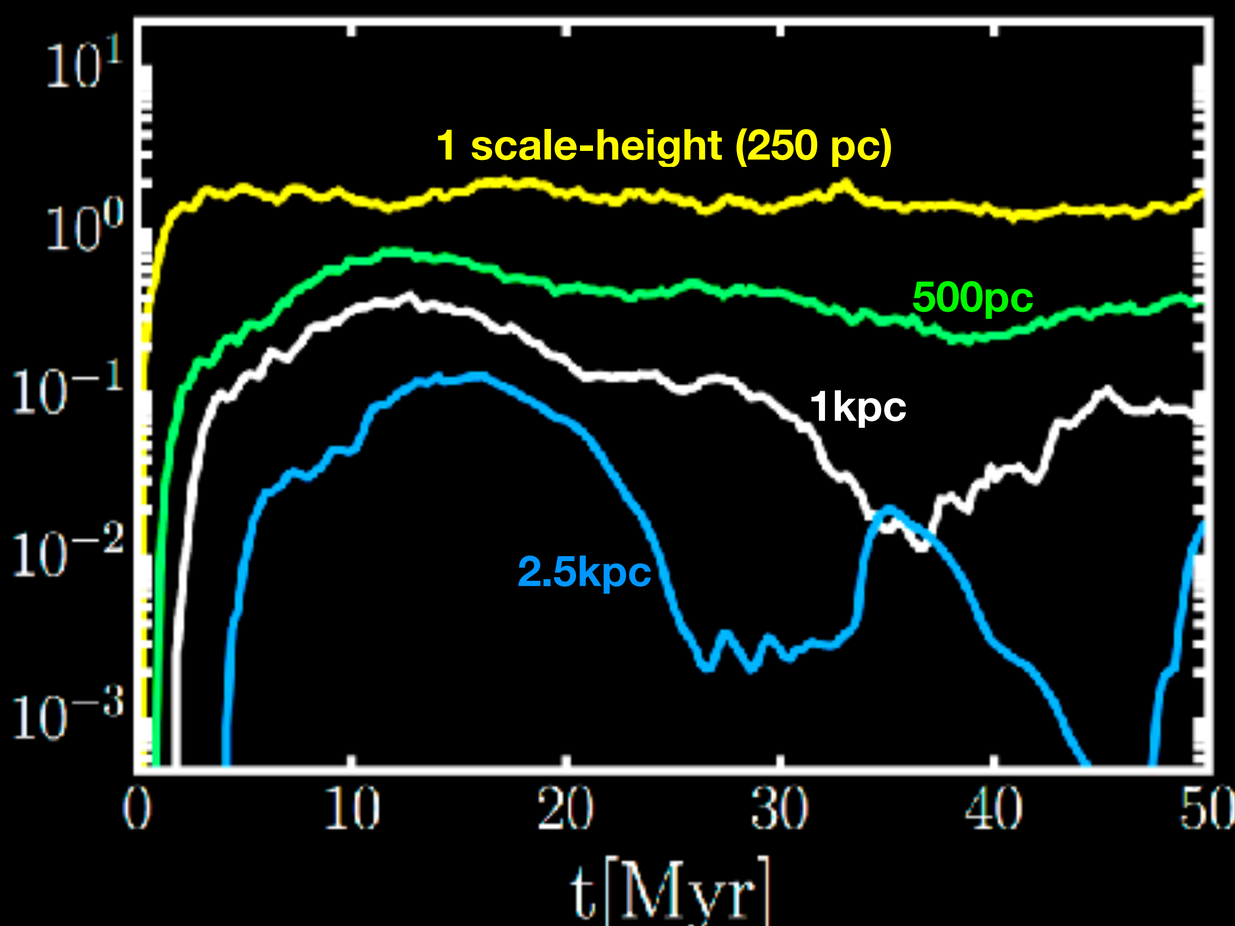
Outflow/Shearing Boxes:

GEOMETRY MATTERS (cannot use shearing box and get any outflow right above ~1 scale height)

"Stratified Box"



Wind mass-loading: drops exponentially with height



- Potential/escape velocity increases with height!
- Wind can't expand: cooling rates too high
- No subsonic-supersonic transition with adiabatic expansion
    - Chevalier (pressure-driven) wind solutions *do not exist*

Martizzi+ 16 (1601.03399), Fielding+ 17 (1704.01579)

Outflow/Shearing Boxes:

Martizzi+ 16 (1601.03399), Fielding+ 17 (1704.01579)

GEOMETRY MATTERS (cannot use shearing box and get any outflow right above ~1 scale height)

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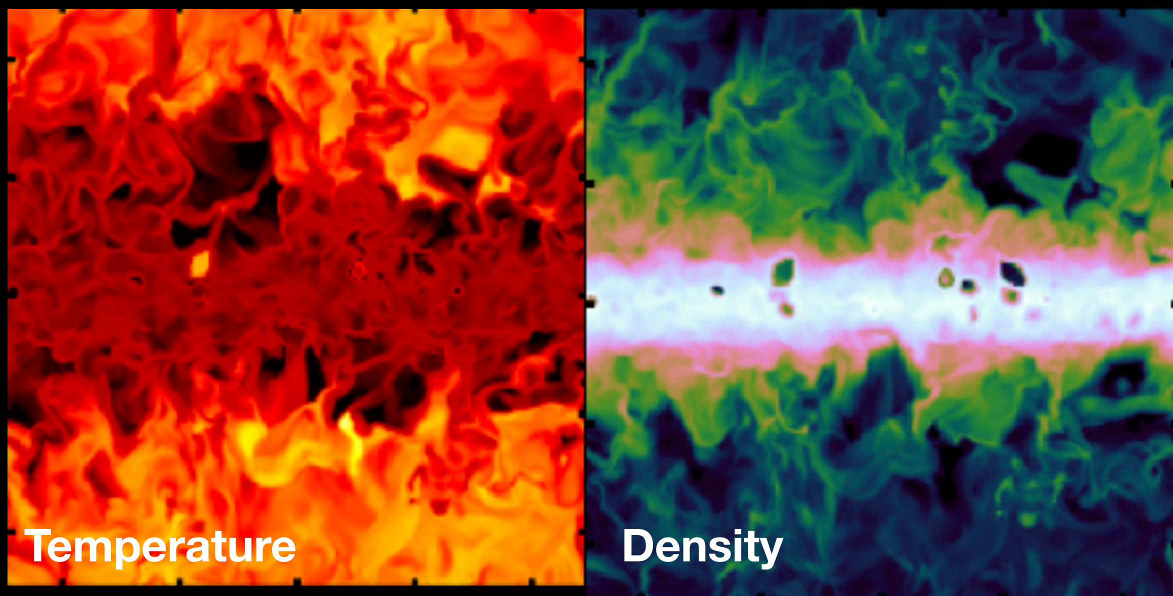
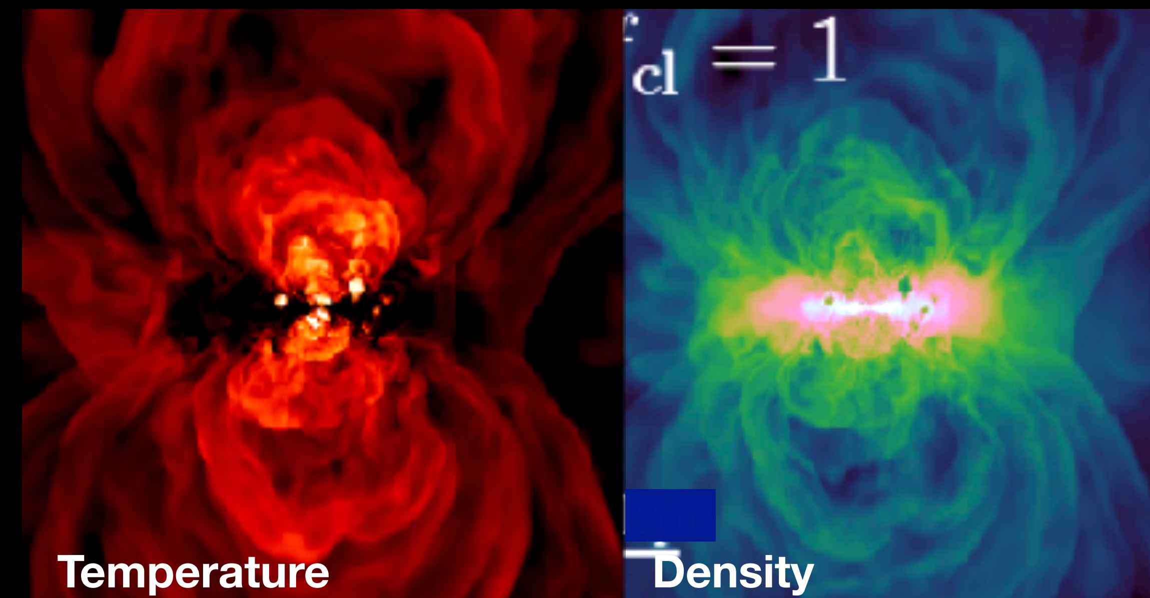


Identical physics/ code/ resolution/ ICs

Global Simulation



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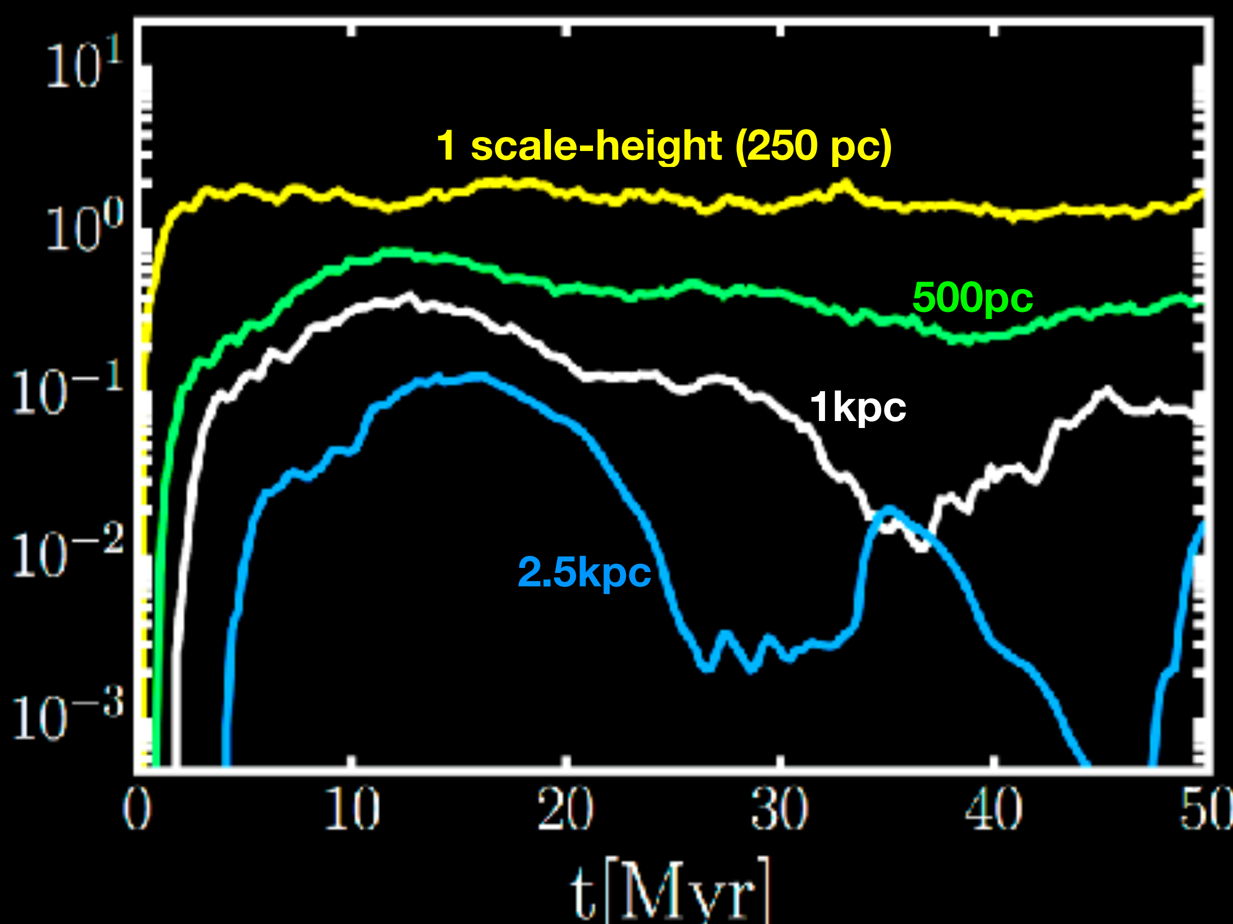
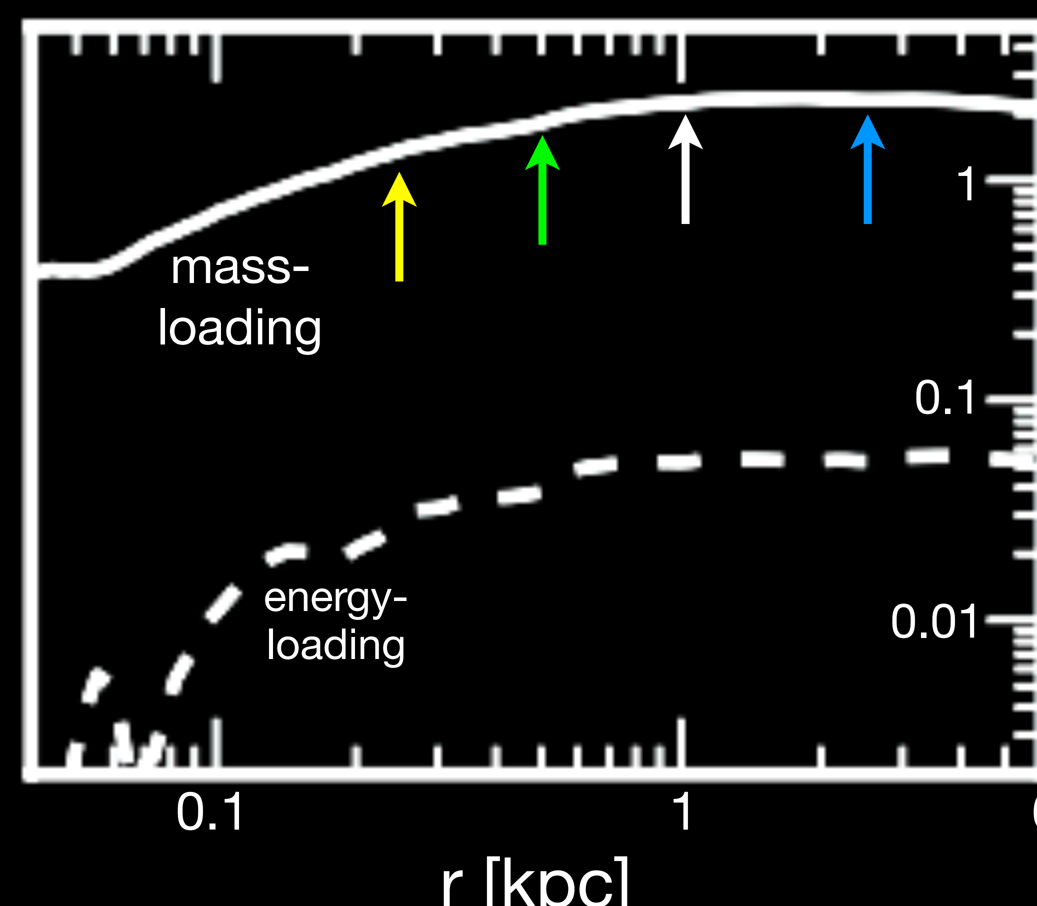
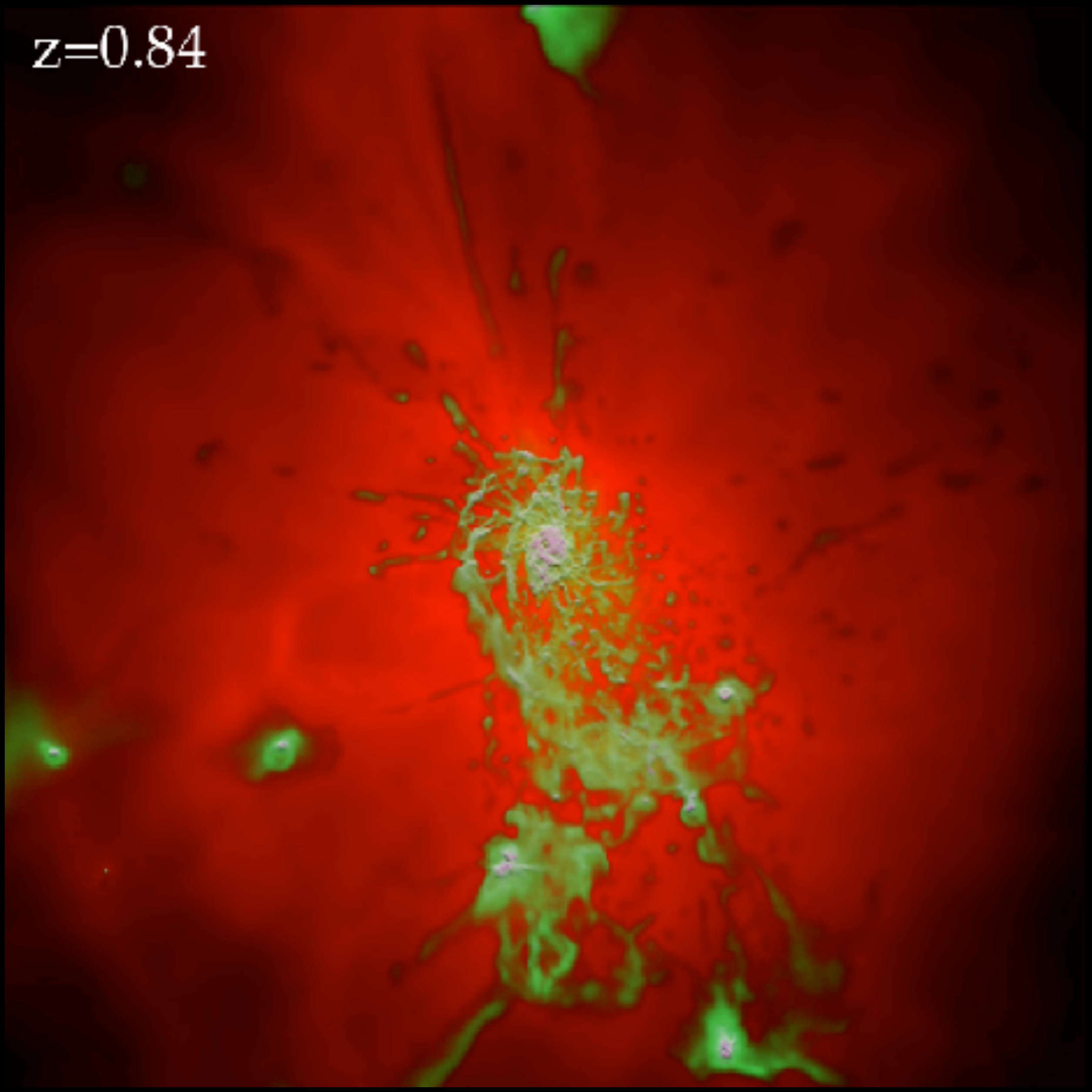


Rises then constant with height

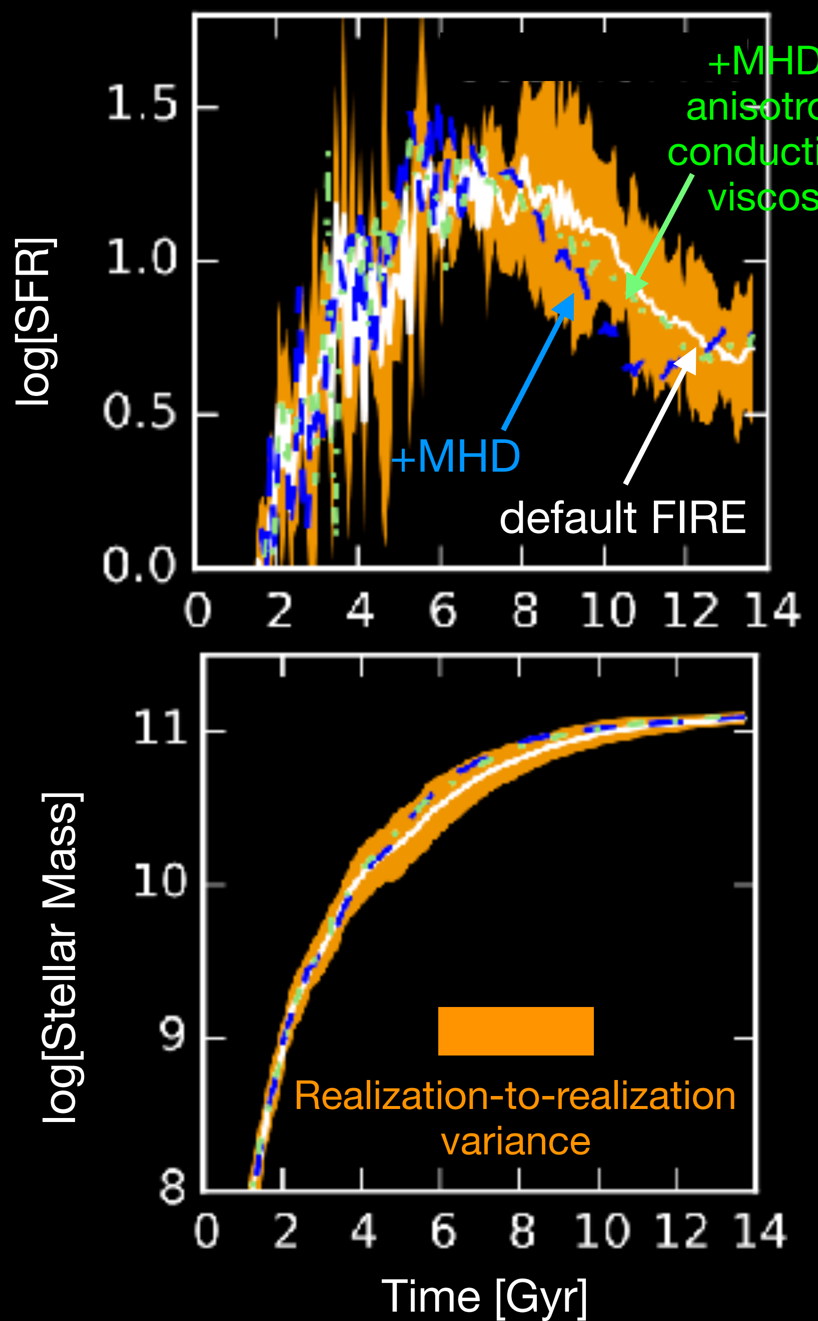


Stochasticity:

BE AWARE OF WHAT PARTS OF YOUR CODE/RESULTS DEPEND ON RANDOM NUMBERS



e.g. Su et al. (1607.05274)



- Chaotic: Lyaponov time ~ t_{dyn}
  (e.g. gravity, turbulence)

- Explicitly random events
  (e.g. SNe probabalistic)

see Keller et al. (1803.05445)

Thanks!

The State of FIRE

- Resolution (cosmological to z=0):
  - Mass: Dwarfs ~30 M_{sun}, MW-Mass & Local-Group ~800 M_{sun}
  - Spatial (in dense gas): ~ 0.1 - 1 pc
  - Densities (with resolved M_{Jeans}): ~ 1000 - 100,000 cm^{-3}
  - Time (dense/hot gas): ~50-100 yr
  - (Star clusters & GMCs with same physics: ~0.01 M_{sun}, 0.1 au)

- Stellar Feedback:
  - SNe (II & Ia)
  - Stellar Winds (O/B & AGB)
  - Photoionization (HII regions) & Photo-electric (dust)
  - Radiation Pressure (IR & UV/opt)

- Black Holes:
  - Seed models: exploring (lots of small seeds, few big seeds?)
  - Accretion models: gravitational torques & gravito-turbulent & Bondi
  - Radiative: photo-ionization & photo-electric & Compton & radiation pressure
  - "Hydrodynamic" (accretion disk winds): dM/dt~BHAR, v~30,000 km/s
  - "Non-Hydrodynamic" (jets & bubbles of cosmic rays & magnetic fields)

- Plasma Physics:
  - MHD (non-ideal in GMCs)
  - Anisotropic Viscosity & Conduction
  - Cosmic Rays (injection, streaming, anisotropic diffusion, cooling)
  - Dust dynamics (drag+Lorentz forces)
  - Dust formation/evolution
  - Explicit 5-10 band RHD

- Dark Matter Physics + Baryons:
  - Self-Interacting DM (v-dep't, anisotropic)
  - "Fuzzy" DM (quantum pressure tensor)
  - Explicit Collisionless-Boltzmann (Phase-Space) Solvers (non Monte-Carlo)