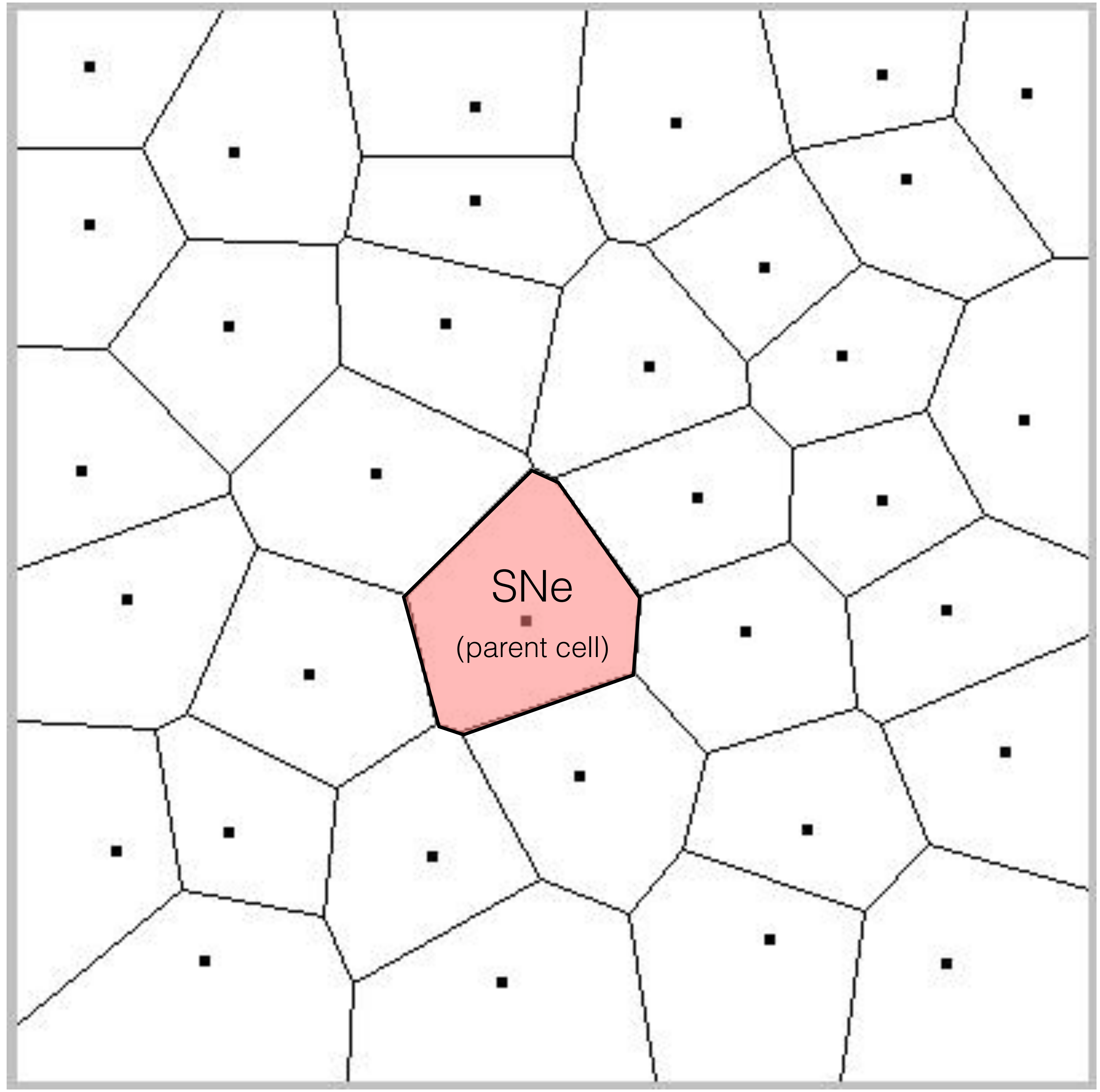
Time 0:

SNe explodes

- ejecta evolve perfectly and isotropically (expanding spherical shell) inside some volume within the "parent cell"

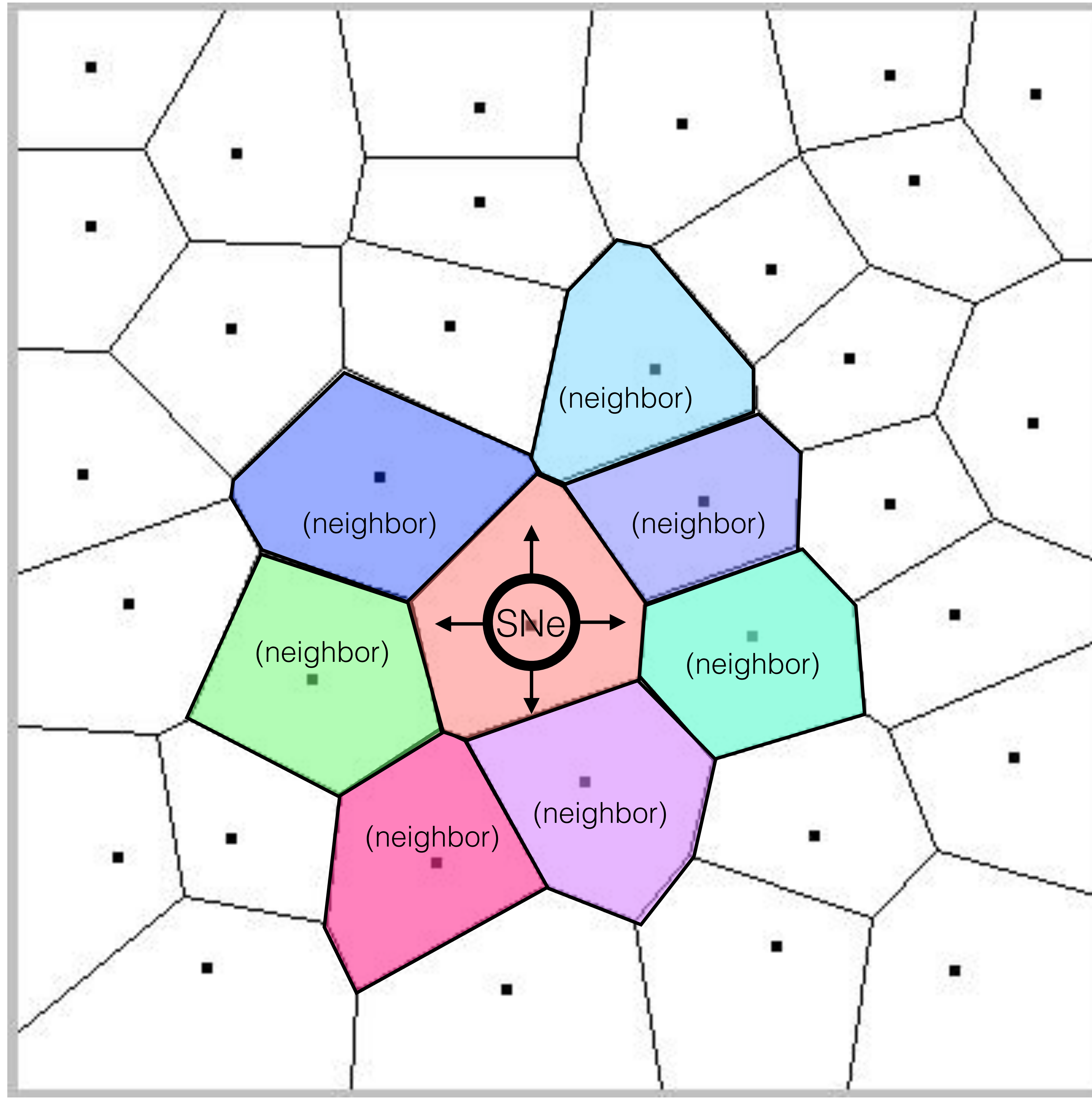
- this is unresolved: need to calc what goes into "neighbor cells"



Time 1:

Neighbors share a face

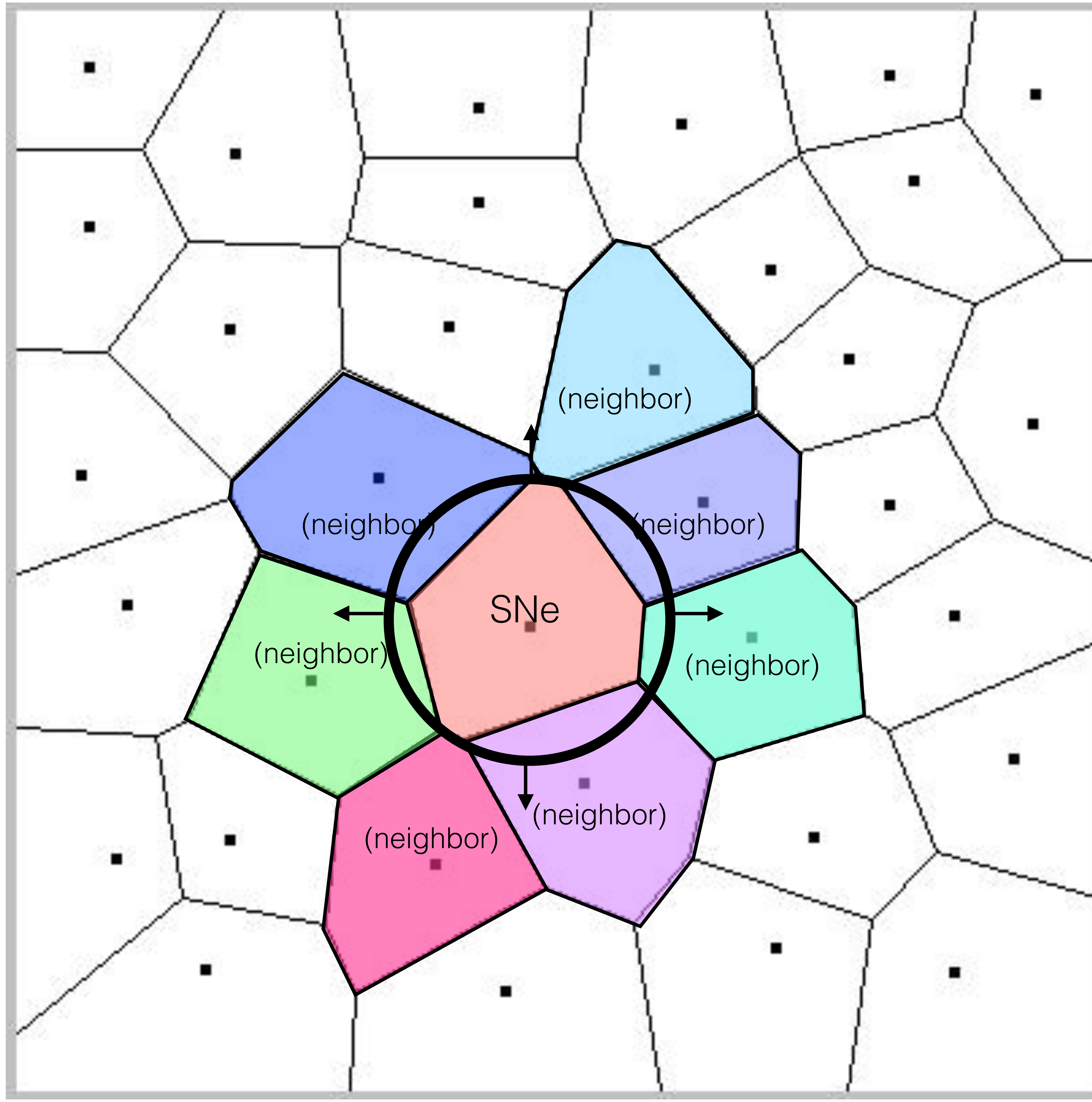
- ejecta expanding inside parent
    - eventually go into neighbors



Time 2:

Neighbors share a face

- ejecta expanding inside parent
- eventually go into neighbors
- ring centered on explosion site

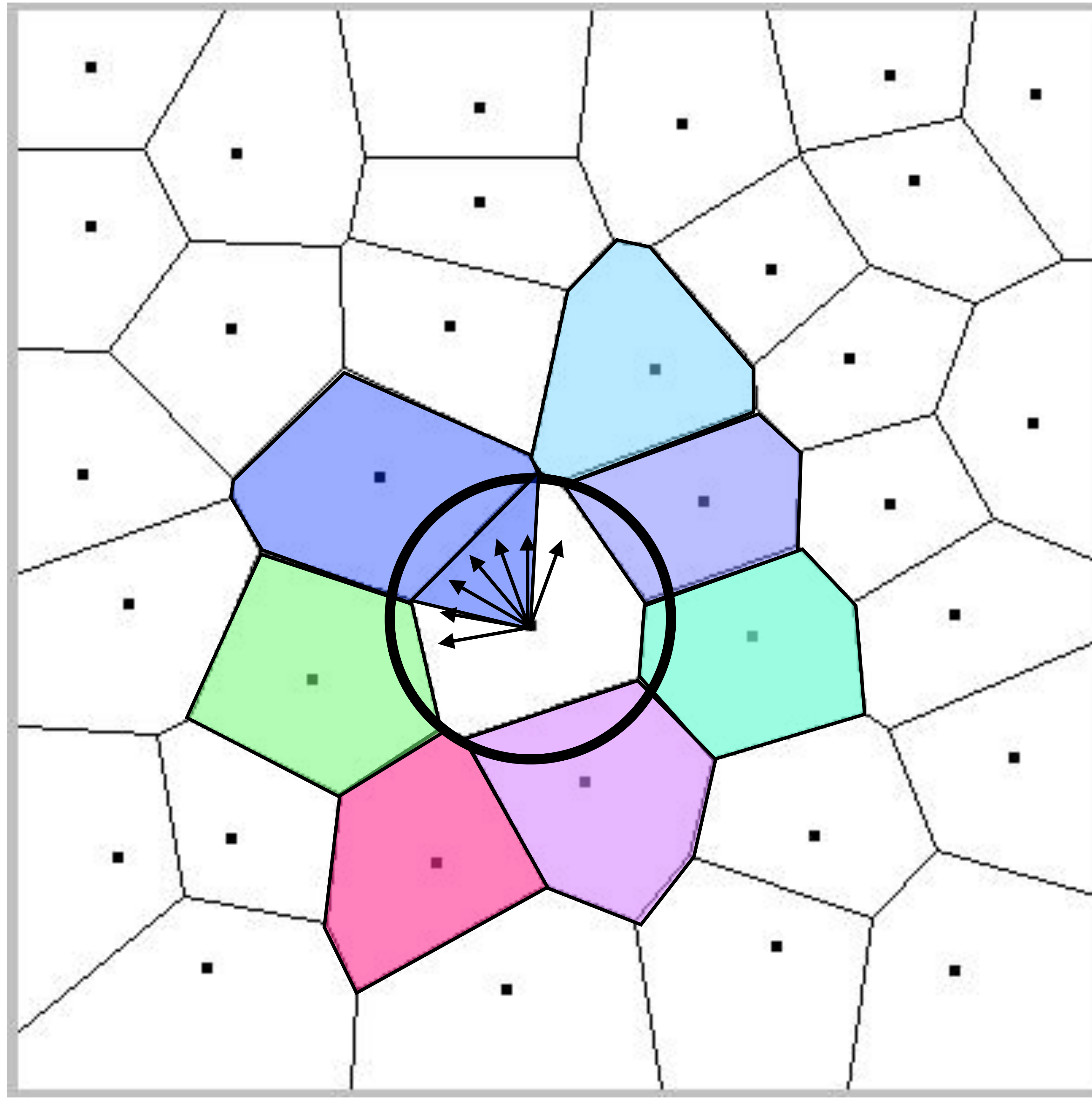


Time 2:

Ejecta isotropic:

- amount “into” each neighbor given by integrating over solid angle subtended

- vector momentum is radial, directed outward from center. NET momentum to each neighbor is also integrated over the solid angle of the face

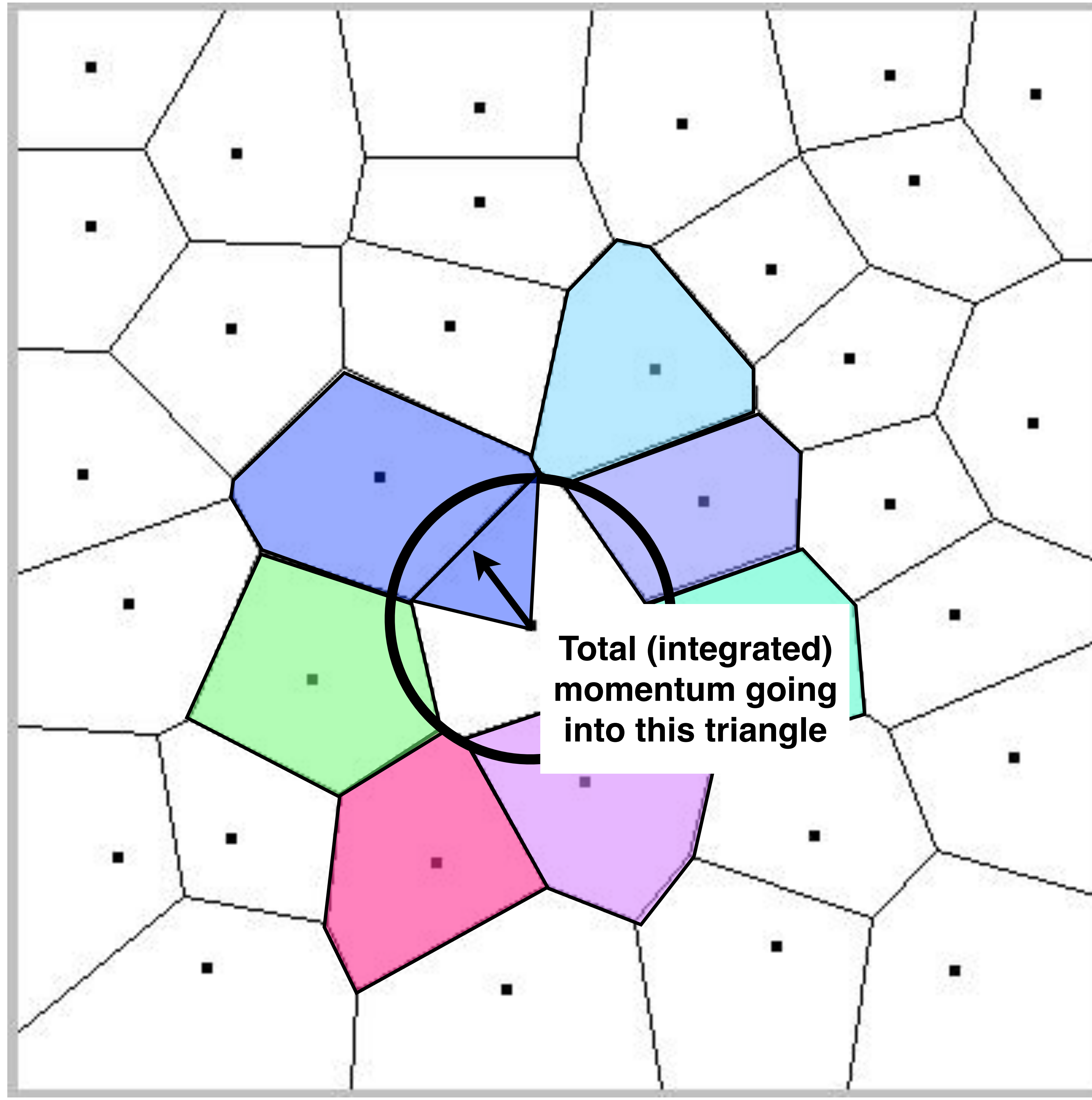


Time 2: Integrated

Ejecta isotropic:

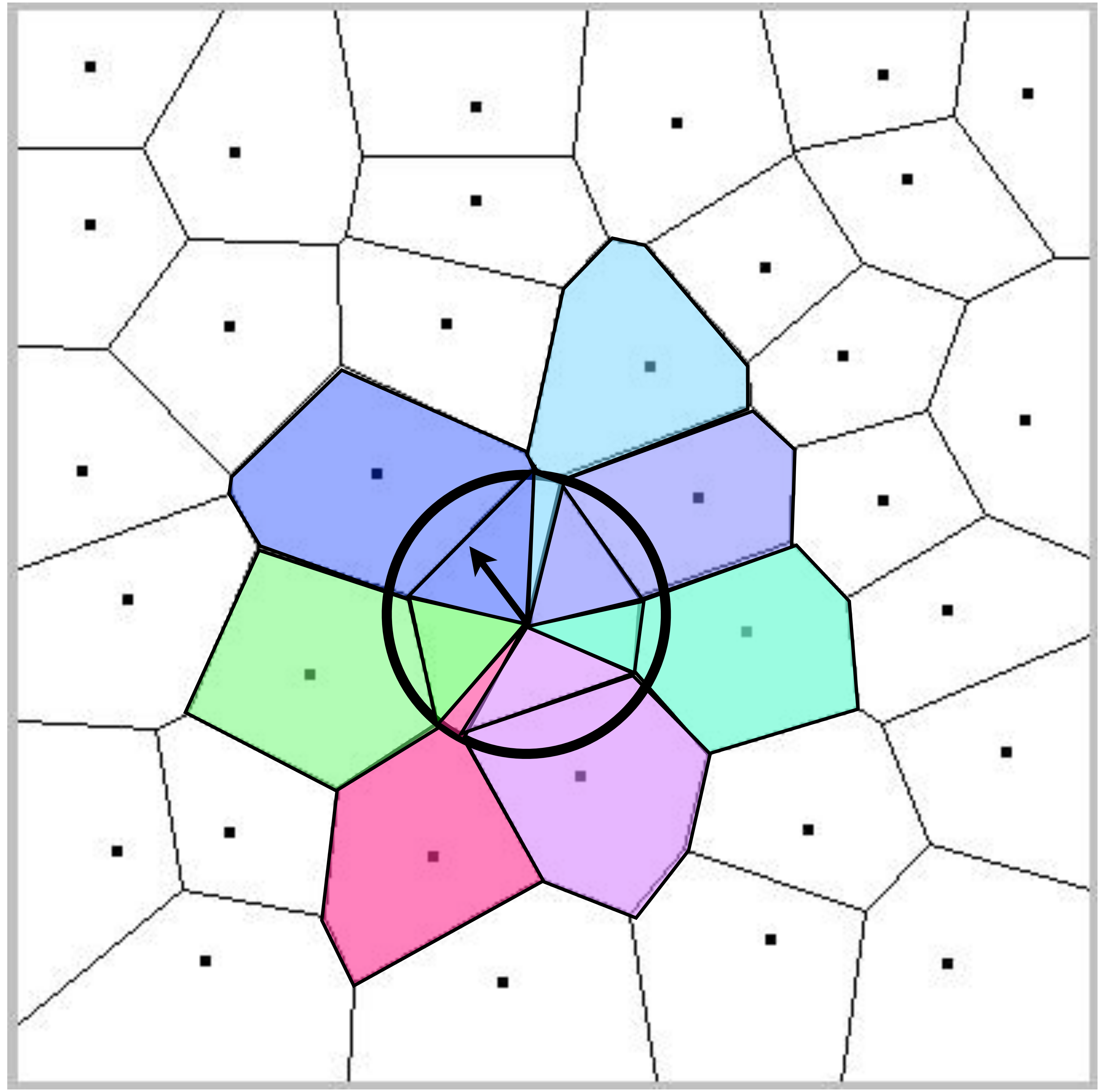
- amount “into” each neighbor given by integrating over solid angle subtended

- vector momentum is radial, directed outward from center. NET momentum to each neighbor is also integrated over the solid angle of the face



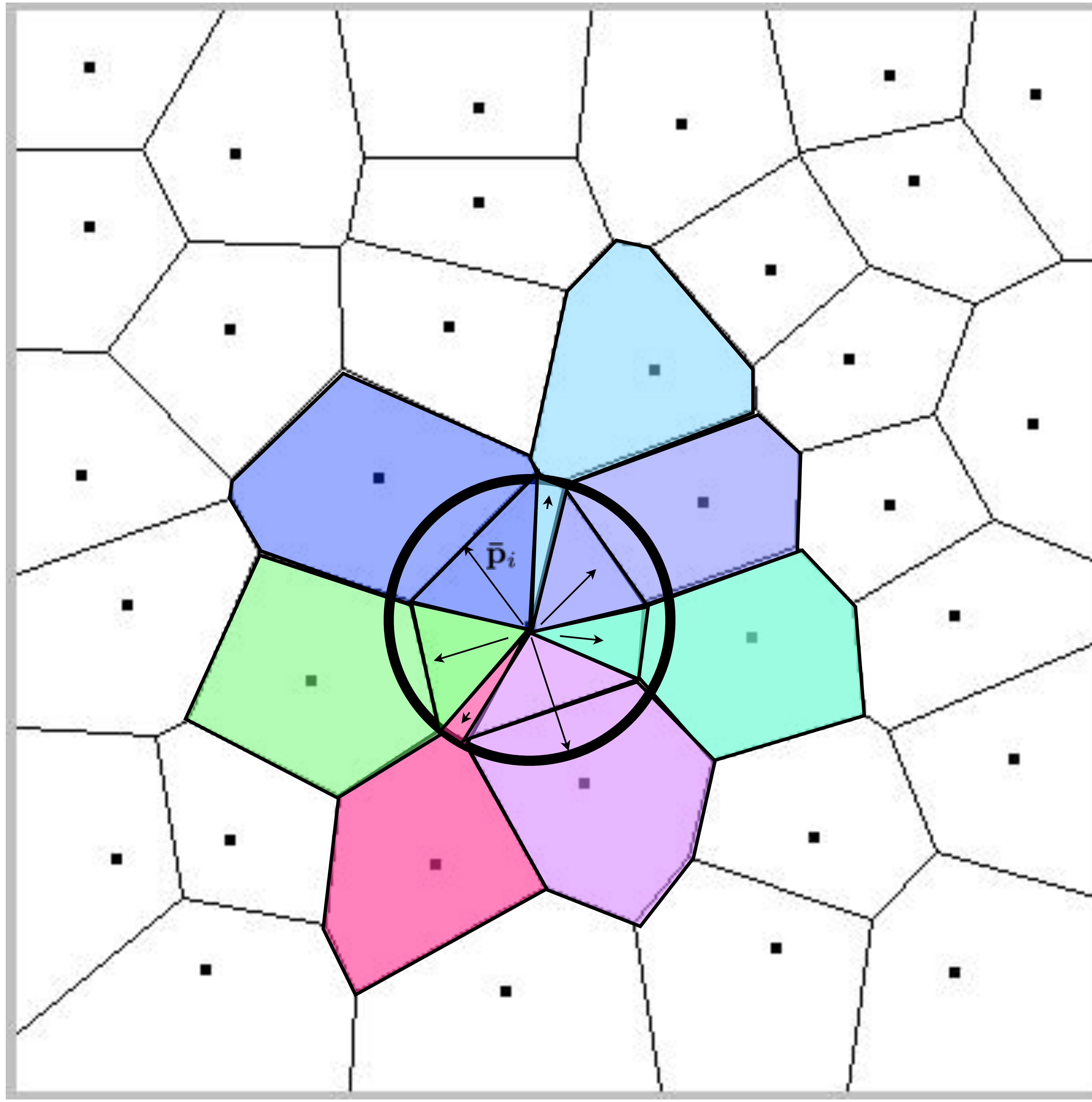
Time 2: Integrated

Do the same for each face



Time 2: Integrated

Do the same for each face:
- each vector here is properly integrated. points in direction of the total momentum integrated over the solid angle (2D here so integrating over the circle) of the face.
  size represents the total solid angle subtended (total mass/momentum into face)



Time 2: Integrated

Easily verified:

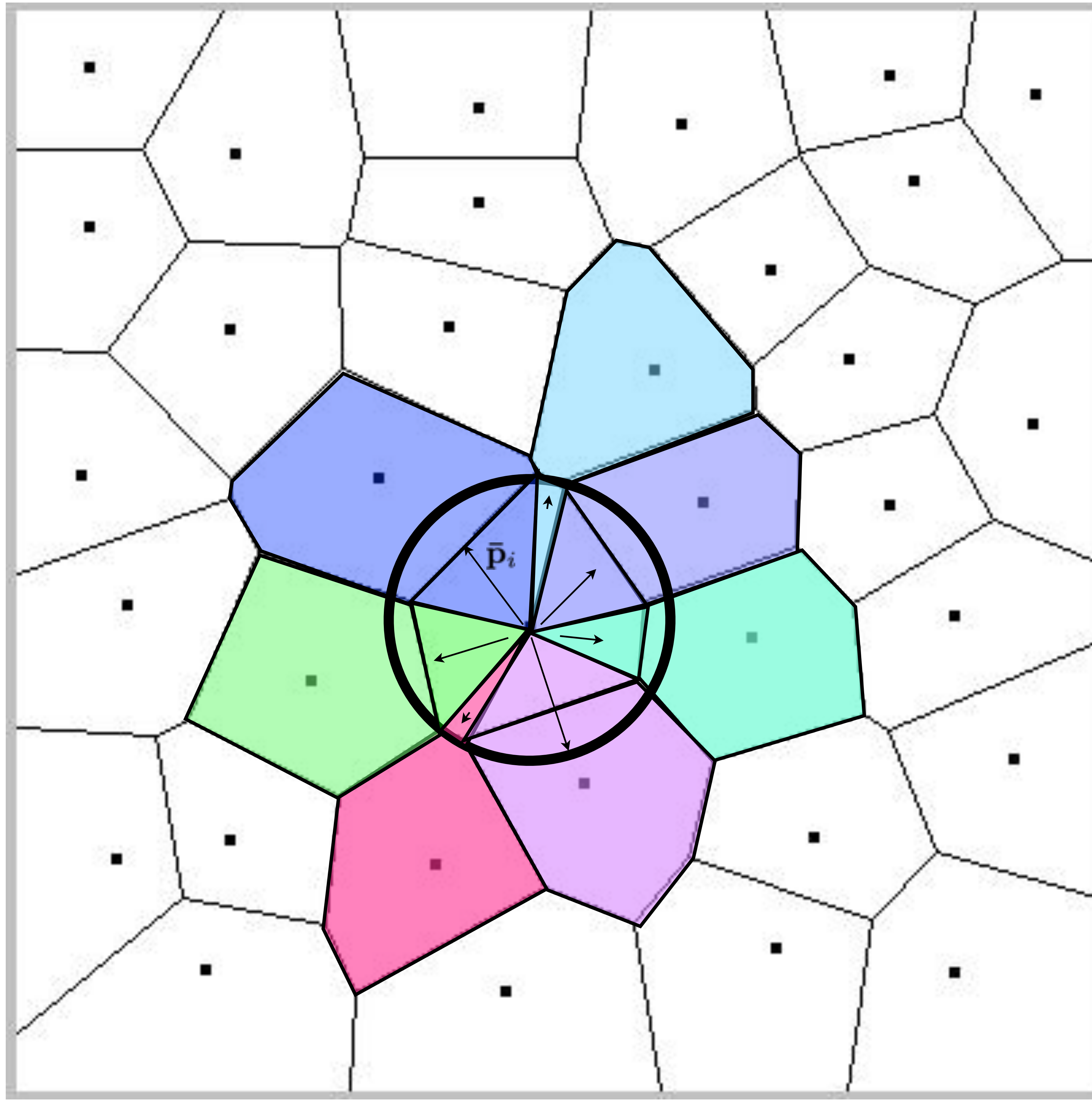
$$\sum_i \bar{\mathbf{p}}_i = \mathbf{0}$$

Net momentum (vector sum) is exactly zero

BUT, in general:

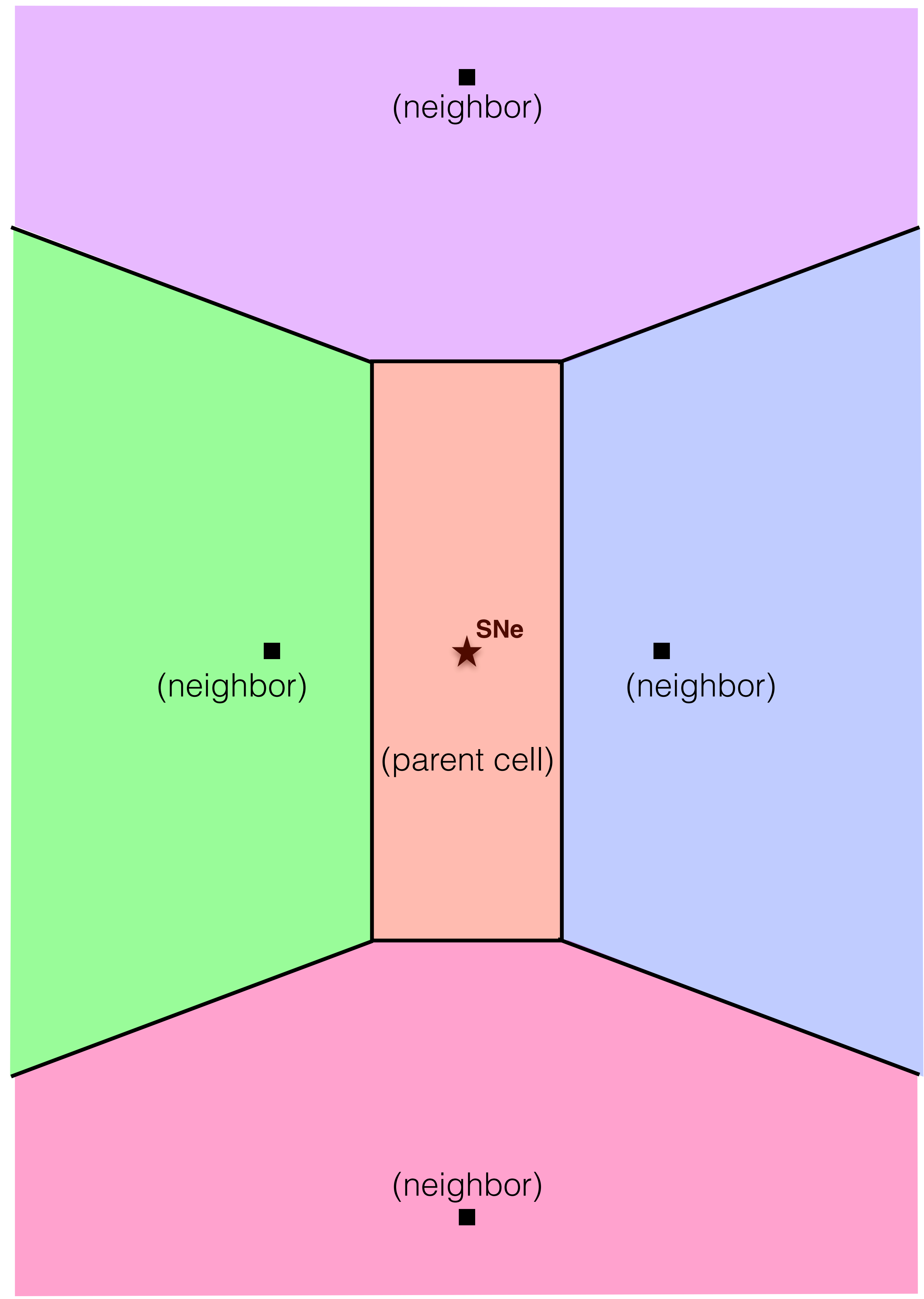
$$\sum_i |\bar{\mathbf{p}}_i \cdot \hat{x}| \neq \sum_i |\bar{\mathbf{p}}_i \cdot \hat{y}|$$

(vectors are no longer isotropic -unless- faces are isotropic)



A more extreme example:

You can check, for this simple mesh-generating point (particle) configuration, this is the correct Voronoi tesselation



In general:

$$\sum_i |\bar{\mathbf{p}}_i \cdot \hat{x}| \neq \sum_i |\bar{\mathbf{p}}_i \cdot \hat{y}|$$

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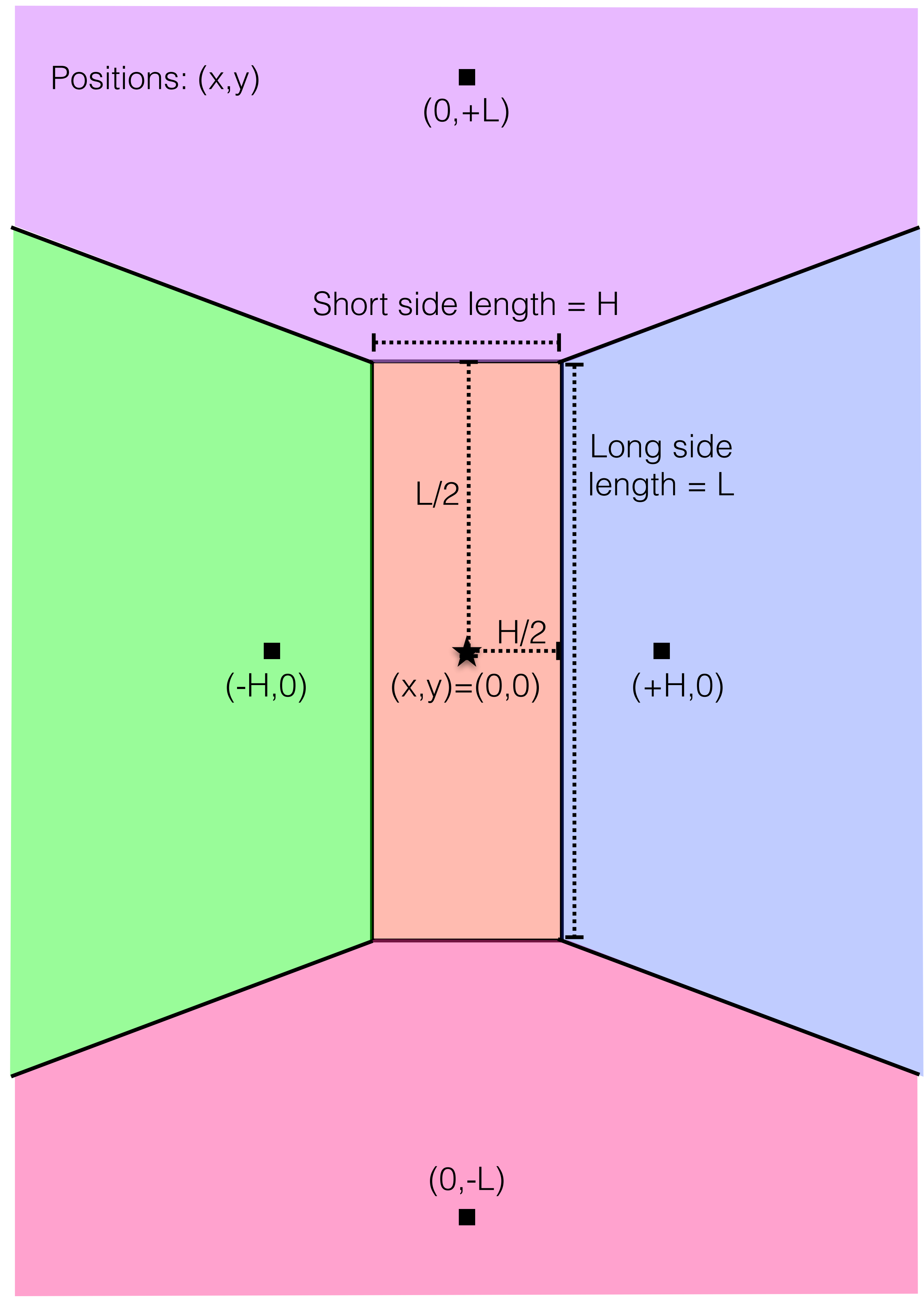
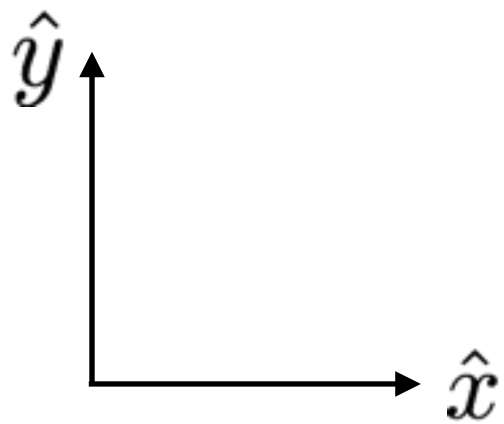
A more extreme example:



Notation for the geometry:
(this is all relevant in a moment, just being specific)



A more extreme example:

If initial “parcel” of ejecta is launched into solid angle $d\phi$ with momentum

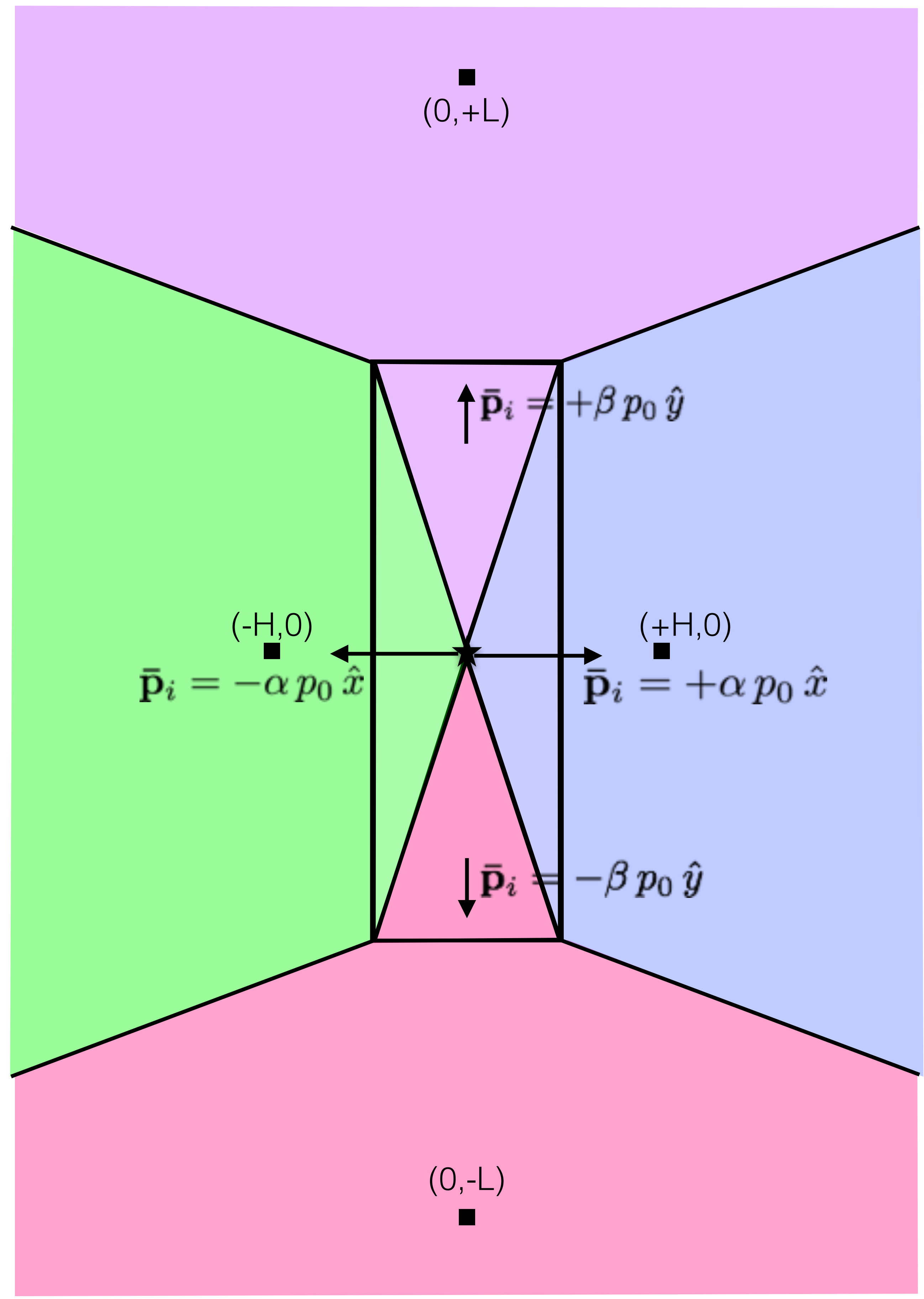
$$d\bar{\mathbf{p}} = (p_0\, d\phi)\, \hat{r}$$

(i.e. the ejecta is in an isotropic, radially expanding shell)

Then we can solve exactly for the total momentum flux into each face, given by labeled vectors here, with:

$$\alpha = \frac{2\,L}{\sqrt{L^2 + H^2}}$$

$$\beta = \frac{2\,H}{\sqrt{L^2 + H^2}}$$



A more extreme example:

$$\alpha = \frac{2\,L}{\sqrt{L^2 + H^2}}$$

$$\beta = \frac{2\,H}{\sqrt{L^2 + H^2}}$$

means:

$$\frac{\sum_i |\bar{\mathbf{p}}_\mathbf{i} \cdot \hat{\mathbf{x}}|}{\sum_i |\bar{\mathbf{p}}_\mathbf{i} \cdot \hat{\mathbf{y}}|} = \frac{L}{H} \gg 1$$

if L >> H