Two key equations we want to solve:

$$\left.\frac{\partial u_i}{\partial q_i}\right|_A = -\frac{P_i}{m_i}\,\frac{\partial \Delta V_i}{\partial q_i}$$

$$\phi_i(\mathbf{q}) \equiv \frac{4\pi}{3}\,h_i^3\,\frac{1}{\Delta\tilde{V}_i} - N_{\rm ngb} = 0$$

"Standard" (classical) SPH:

$$\Delta V_i = \Delta\tilde{V}_i = \frac{m_i}{\rho_i} \qquad\qquad \rho_i = \sum_j m_j\,W(r_j,\,h_i)$$

Can derive equations of motion & correction terms from Lagrangian:

$$L(\mathbf{q},\dot{\mathbf{q}}) = \frac{1}{2}\sum_{i=1}^{N} m_i\,\dot{\mathbf{r}}_i^2 - \sum_{i=1}^{N} m_i\,u_i \qquad \phi_i(\mathbf{q}) \equiv \frac{4\pi}{3}\,h_i^3\,\frac{1}{\Delta\tilde{V}_i} - N_{\mathrm{ngb}} = 0$$

- *Manifestly* energy & entropy conserving

Can derive equations of motion & correction terms from Lagrangian:

$$L(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2} \sum_{i=1}^{N} m_i \dot{\mathbf{r}}_i^2 - \sum_{i=1}^{N} m_i u_i$$

$$\phi_i(\mathbf{q}) \equiv \frac{4\pi}{3} h_i^3 \frac{1}{\Delta \tilde{V}_i} - N_{\rm ngb} = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \sum_{j=1}^{N} \lambda_j \frac{\partial \phi_j}{\partial q_i}$$

- *Manifestly* energy & entropy conserving

Can derive equations of motion & correction terms from Lagrangian:

$$L(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2} \sum_{i=1}^{N} m_i \, \dot{\mathbf{r}}_i^2 - \sum_{i=1}^{N} m_i \, u_i \qquad \phi_i(\mathbf{q}) \equiv \frac{4\pi}{3} h_i^3 \frac{1}{\Delta \tilde{V}_i} - N_{\rm ngb} = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \sum_{j=1}^{N} \lambda_j \frac{\partial \phi_j}{\partial q_i}$$

$$m_i \frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = -\sum_{j=1}^{N} x_i \, x_j \left[\frac{P_i}{y_i^2} f_{ij} \, \nabla_i W_{ij}(h_i) + \frac{P_j}{y_j^2} f_{ji} \, \nabla_i W_{ij}(h_j) \right]$$

$$f_{ij} \equiv 1 - \frac{\tilde{x}_j}{x_j} \left(\frac{h_i}{3 \, \tilde{y}_i} \frac{\partial y_i}{\partial h_i} \right) \left[1 + \frac{h_i}{3 \, \tilde{y}_i} \frac{\partial \tilde{y}_i}{\partial h_i} \right]^{-1}$$

- *Manifestly* energy & entropy conserving

Want to eliminate multi-valued pressures at contact discontinuities:

$$\bar{P}_i = y_i^\gamma = \left[\sum_{j=1}^{N} m_j A_j^{1/\gamma} W_{ij}(h_i) \right]^\gamma$$

(Ritchie & Thomas, Read et al., Price et al., Saitoh & Makino)

$$\Delta V_i = m_i \left(\frac{A_i}{\bar{P}_i} \right)^{1/\gamma} = \frac{(\gamma - 1)\, m_i\, u_i}{\bar{P}_i}$$

Want to eliminate multi-valued pressures at contact discontinuities:

$$\bar{P}_i = y_i^{\gamma} = \Big[\sum_{j=1}^{N} m_j A_j^{1/\gamma} W_{ij}(h_i)\Big]^{\gamma}$$

(Ritchie & Thomas, Read et al., Price et al., Saitoh & Makino)

$$\Delta V_i = m_i \left(\frac{A_i}{\bar{P}_i}\right)^{1/\gamma} = \frac{(\gamma - 1)\, m_i\, u_i}{\bar{P}_i}$$

But *avoid* too much dissipation & huge neighbor numbers/kernel problems:

$$\Delta \tilde{V}_i = \frac{1}{n_i} \qquad n_i = \sum_j W(r_j,\, h_i)$$

Want to eliminate multi-valued pressures at contact discontinuities:

$$\bar{P}_i = y_i^\gamma = \left[\sum_{j=1}^{N} m_j A_j^{1/\gamma} W_{ij}(h_i) \right]^\gamma$$

Want to eliminate multi-valued pressures at contact discontinuities:

$$\bar{P}_i = y_i^\gamma = \left[\sum_{j=1}^{N} m_j A_j^{1/\gamma} W_{ij}(h_i) \right]^\gamma$$

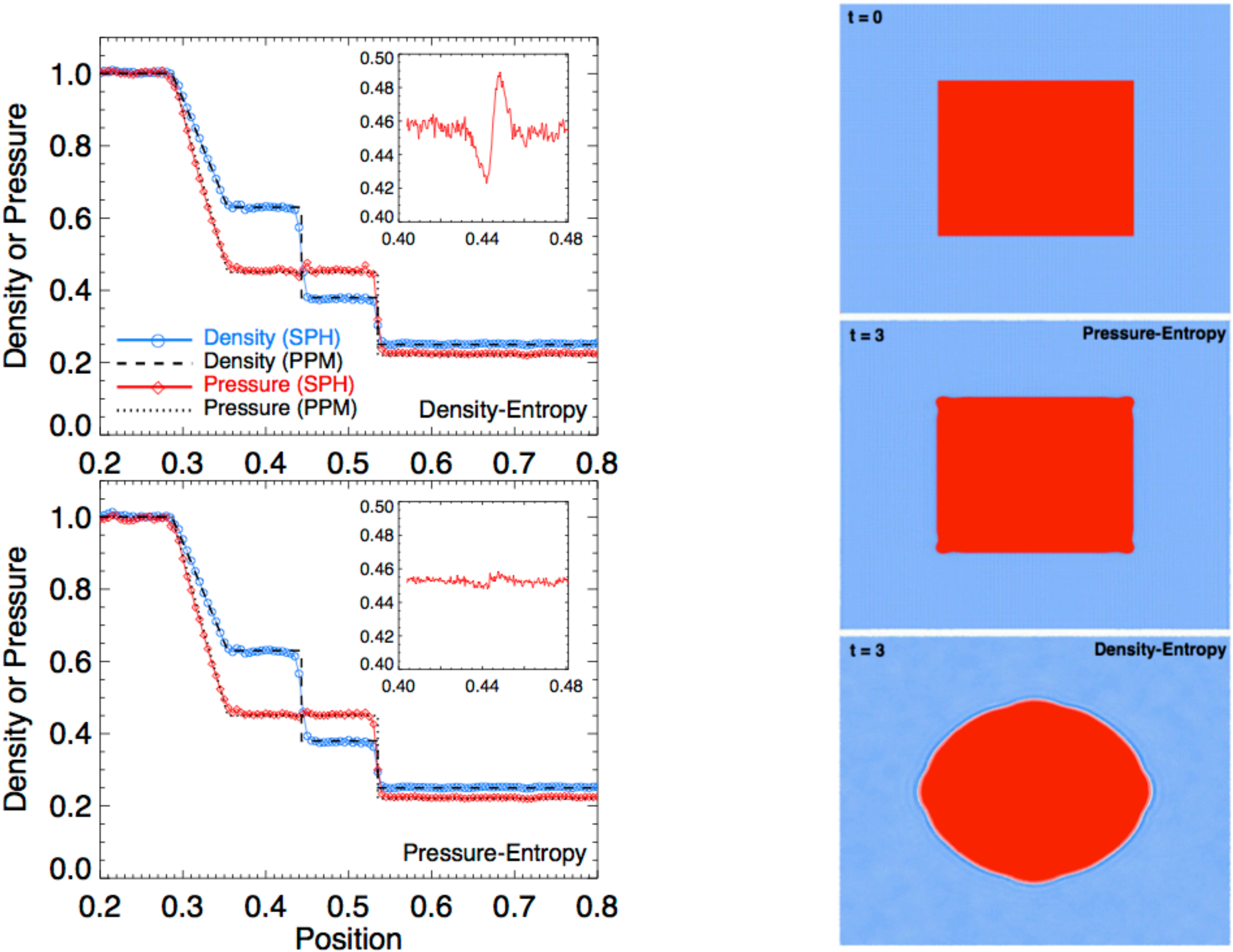
$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = -\sum_{j=1}^{N} m_j \left(A_i A_j\right)^{\frac{1}{\gamma}} \left[\frac{f_{ij}\,\bar{P}_i}{\bar{P}_i^{2/\gamma}} \nabla_i W_{ij}(h_i) + \frac{f_{ji}\,\bar{P}_j}{\bar{P}_j^{2/\gamma}} \nabla_i W_{ij}(h_j) \right]$$

Want to eliminate multi-valued pressures at contact discontinuities:

$$\bar{P}_i = y_i^{\gamma} = \left[\sum_{j=1}^{N} m_j A_j^{1/\gamma} W_{ij}(h_i) \right]^{\gamma}$$

$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = -\sum_{j=1}^{N} m_j \left(A_i A_j\right)^{\frac{1}{\gamma}} \left[\frac{f_{ij}\, \bar{P}_i}{\bar{P}_i^{2/\gamma}} \nabla_i W_{ij}(h_i) + \frac{f_{ji}\, \bar{P}_j}{\bar{P}_j^{2/\gamma}} \nabla_i W_{ij}(h_j) \right]$$

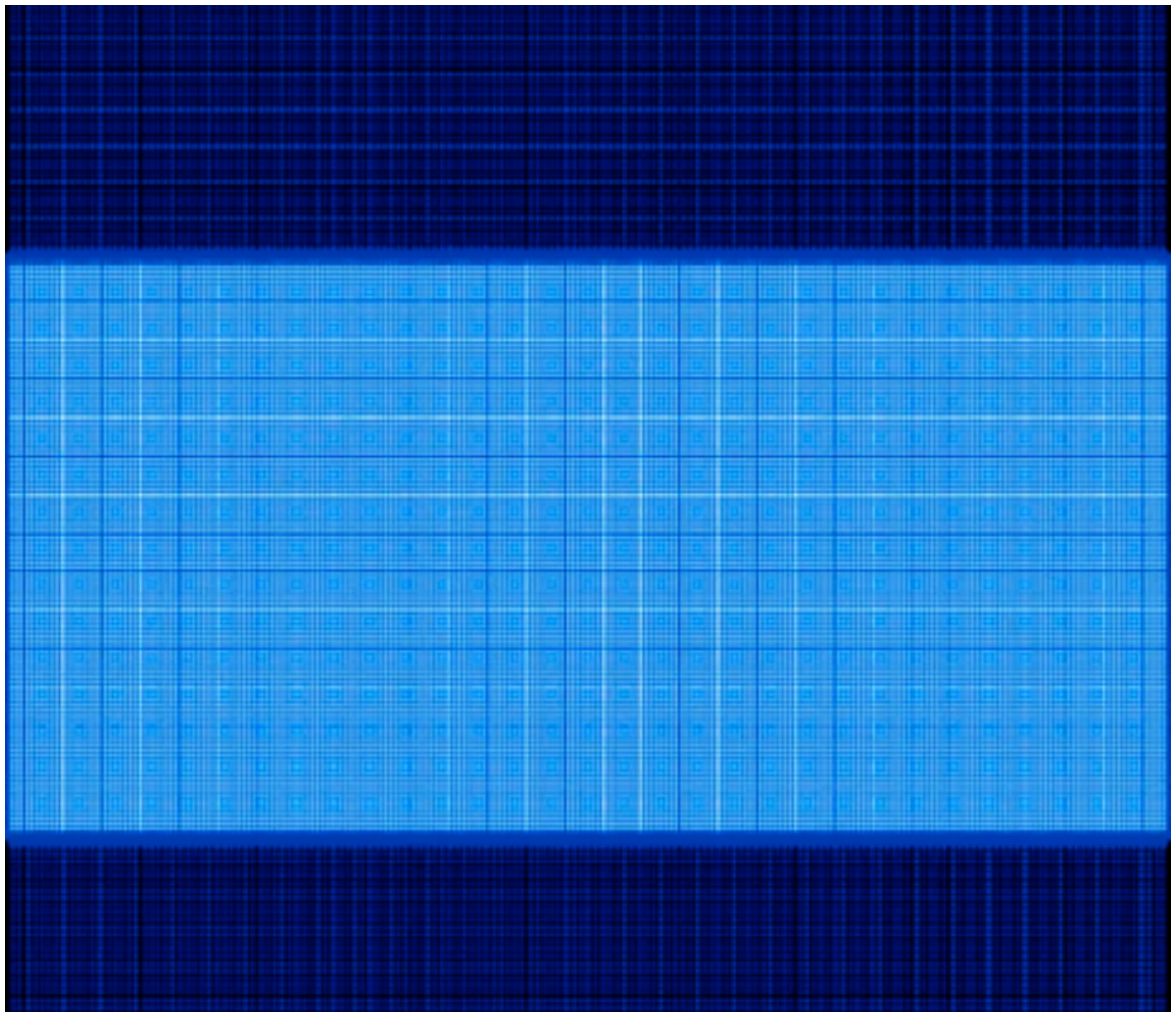
$$f_{ij} = 1 - \left(\frac{h_i}{3 A_j^{1/\gamma} m_j \bar{n}_i} \frac{\partial \bar{P}_i^{1/\gamma}}{\partial h_i} \right) \left[1 + \frac{h_i}{3\, \bar{n}_i} \frac{\partial \bar{n}_i}{\partial h_i} \right]^{-1}$$



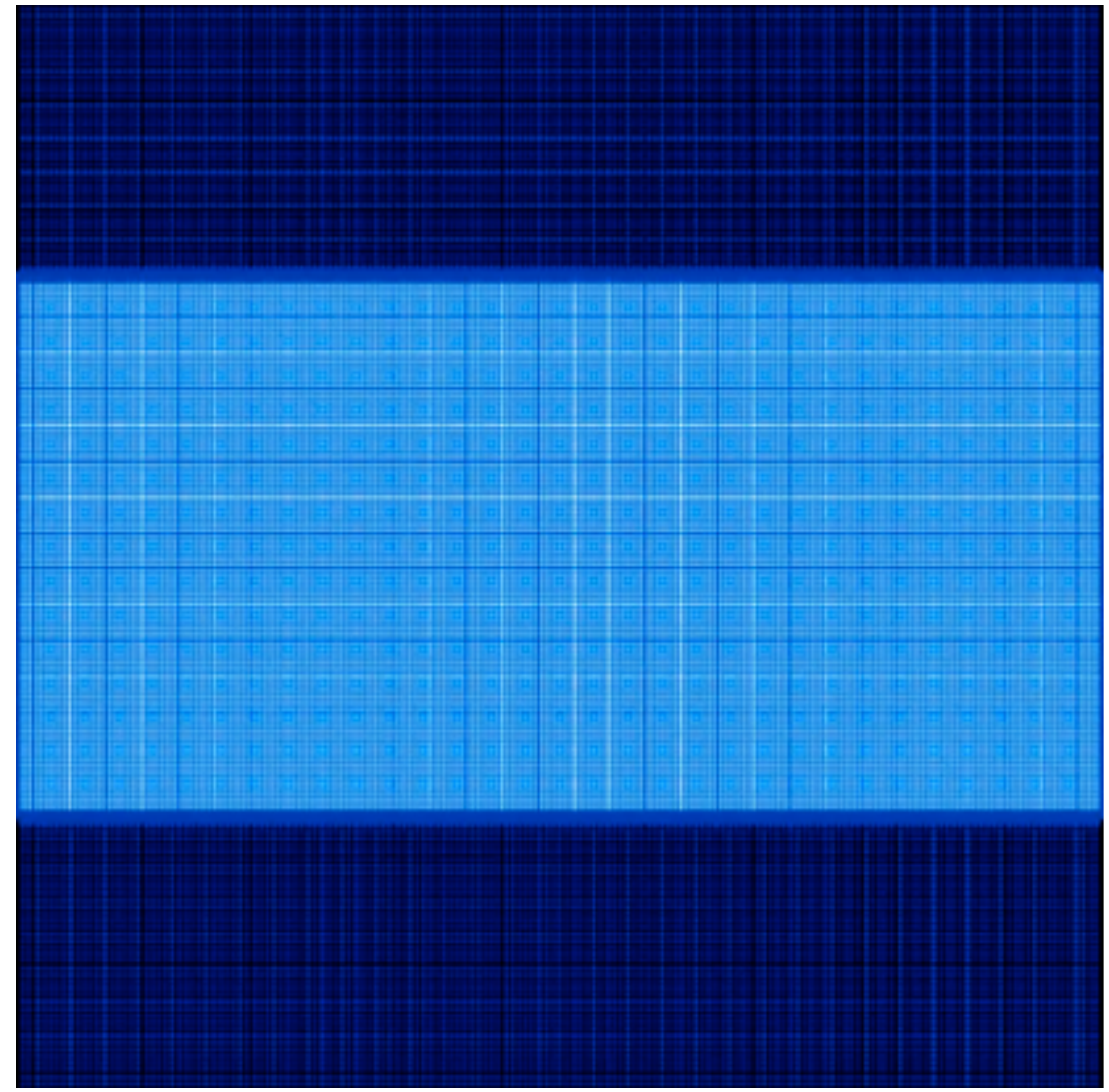
Density or Pressure
Density (SPH)
Density (PPM)
Pressure (SPH)
Pressure (PPM)
Density-Entropy
Pressure-Entropy
Position
t = 0
t = 3
Pressure-Entropy
t = 3
Density-Entropy

SPH in Pressure-Entropy Formulation

SURFACE TENSION IS FIX-ABLE



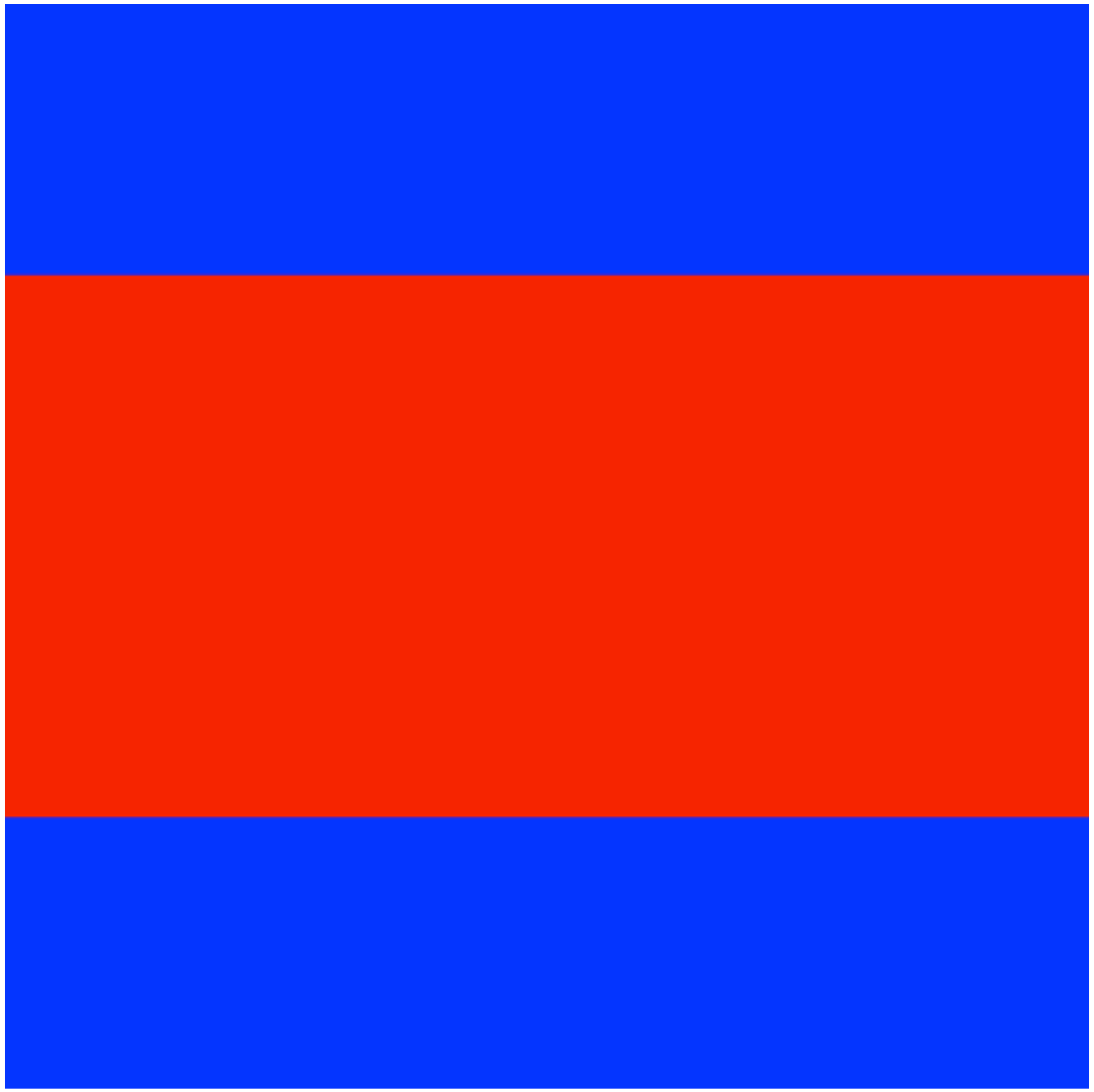
Pressure-Entropy Formulation

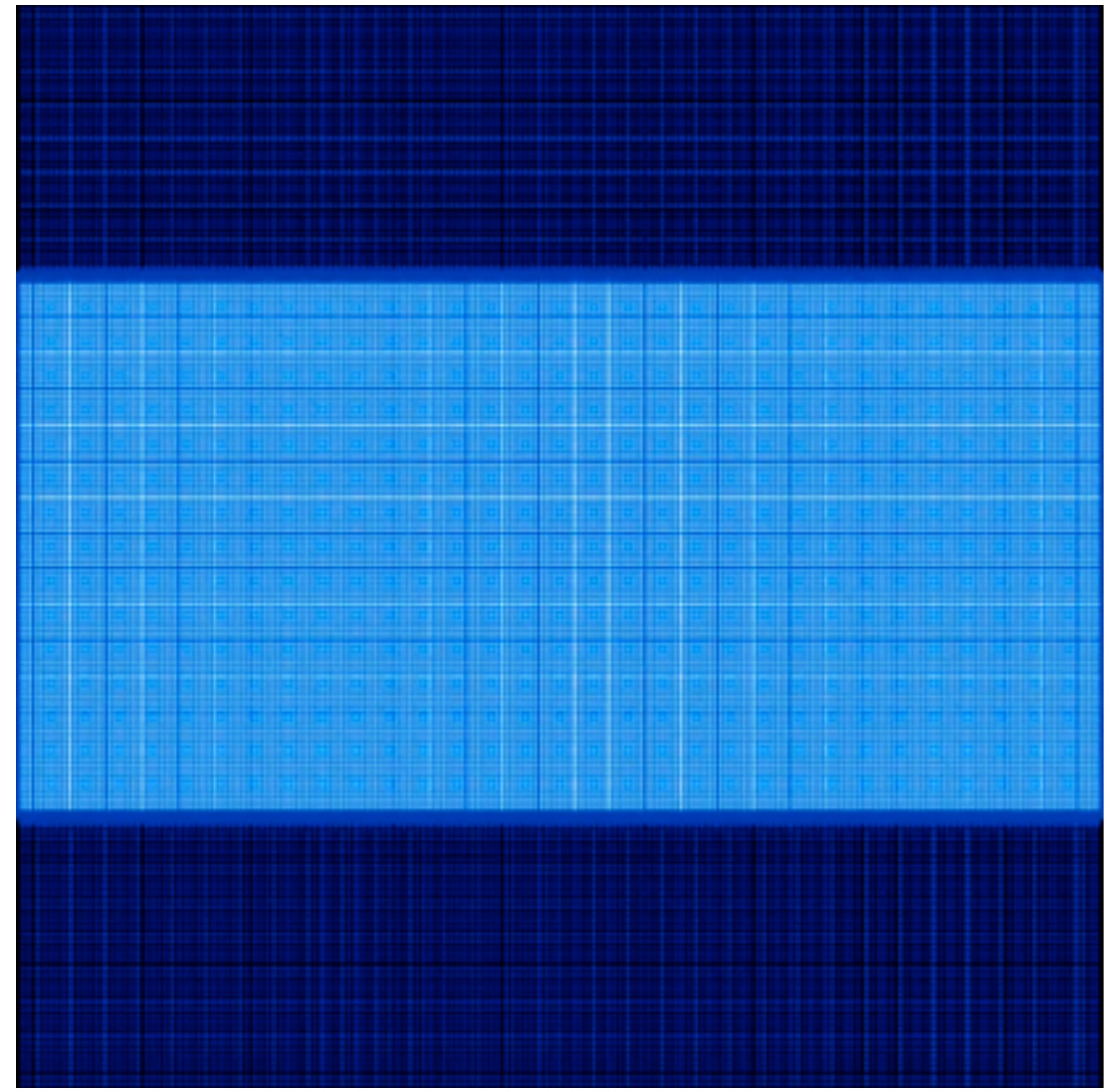


Density Formulation (Standard)

SPH in Pressure-Entropy Formulation

SURFACE TENSION IS FIX-ABLE

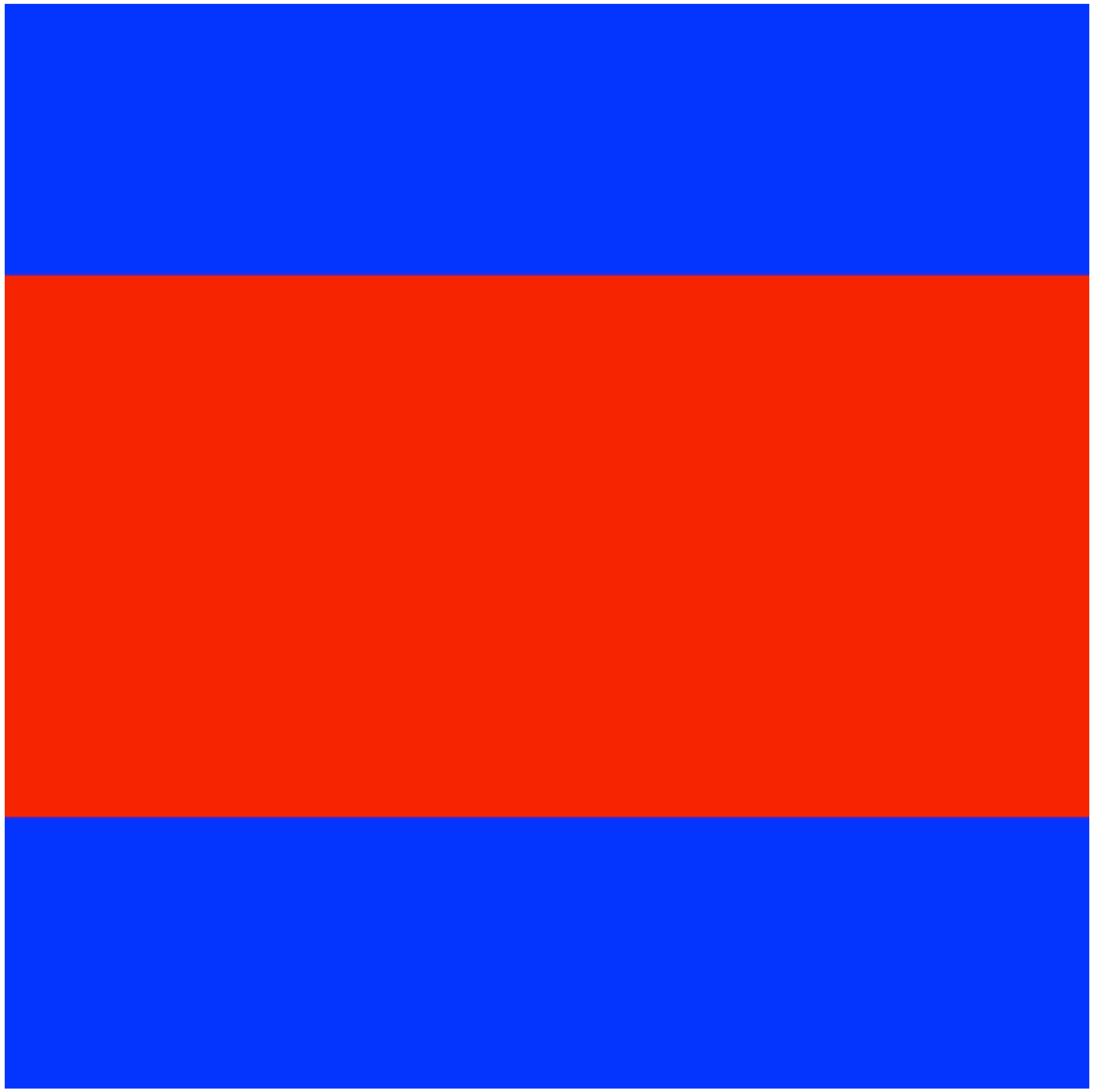




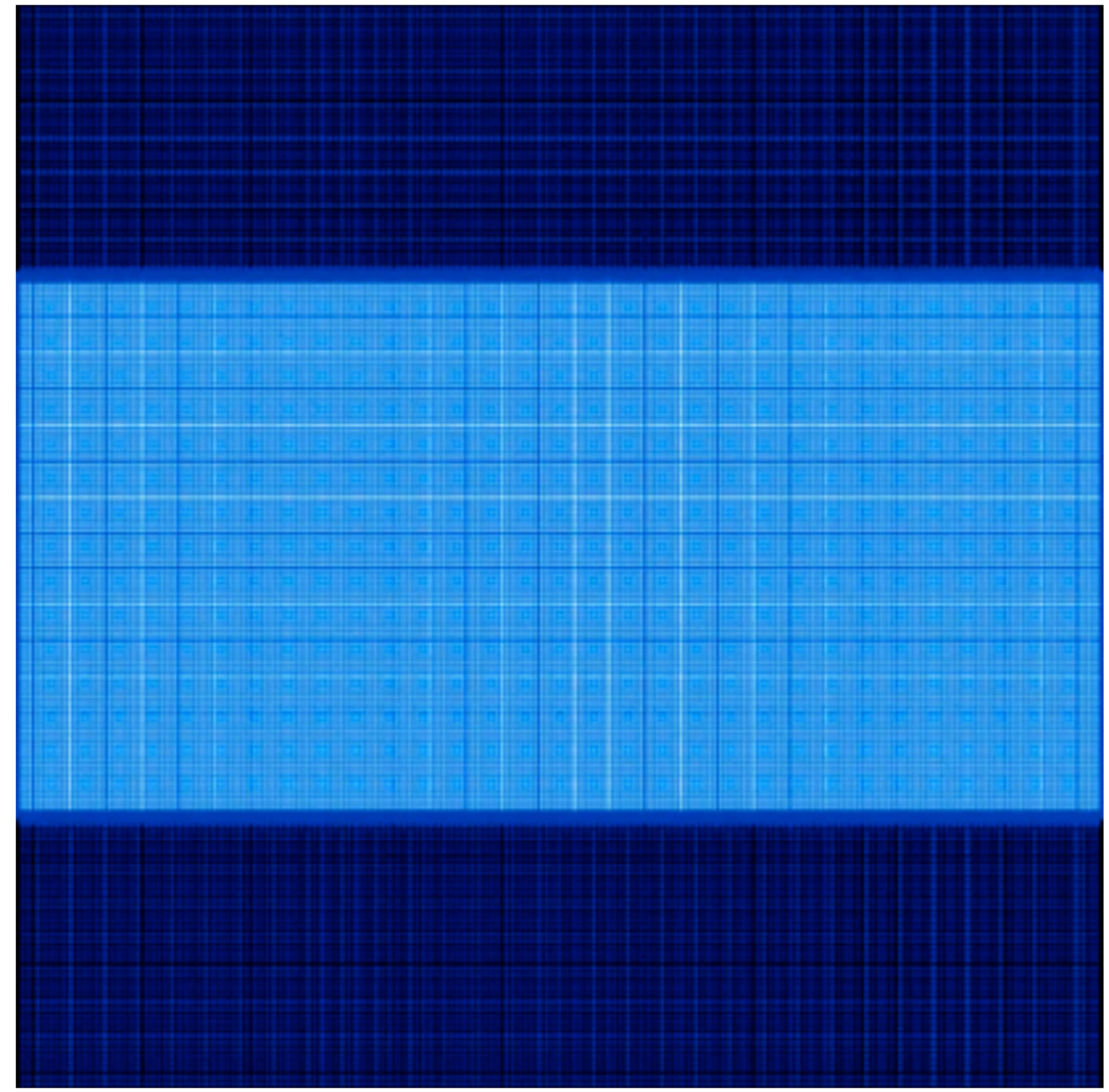
Density Formulation (Standard)

SPH in Pressure-Entropy Formulation

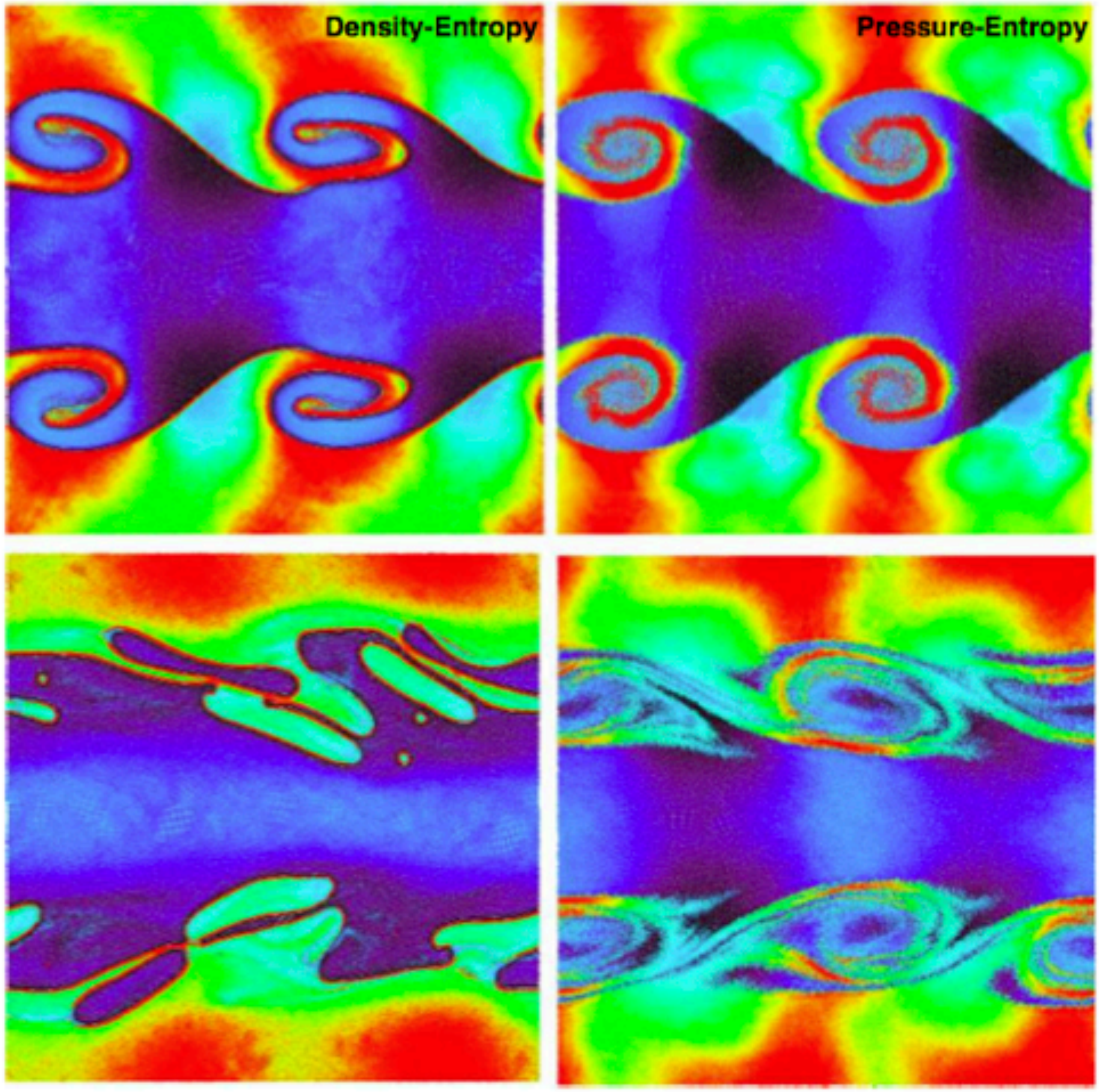
SURFACE TENSION IS FIX-ABLE

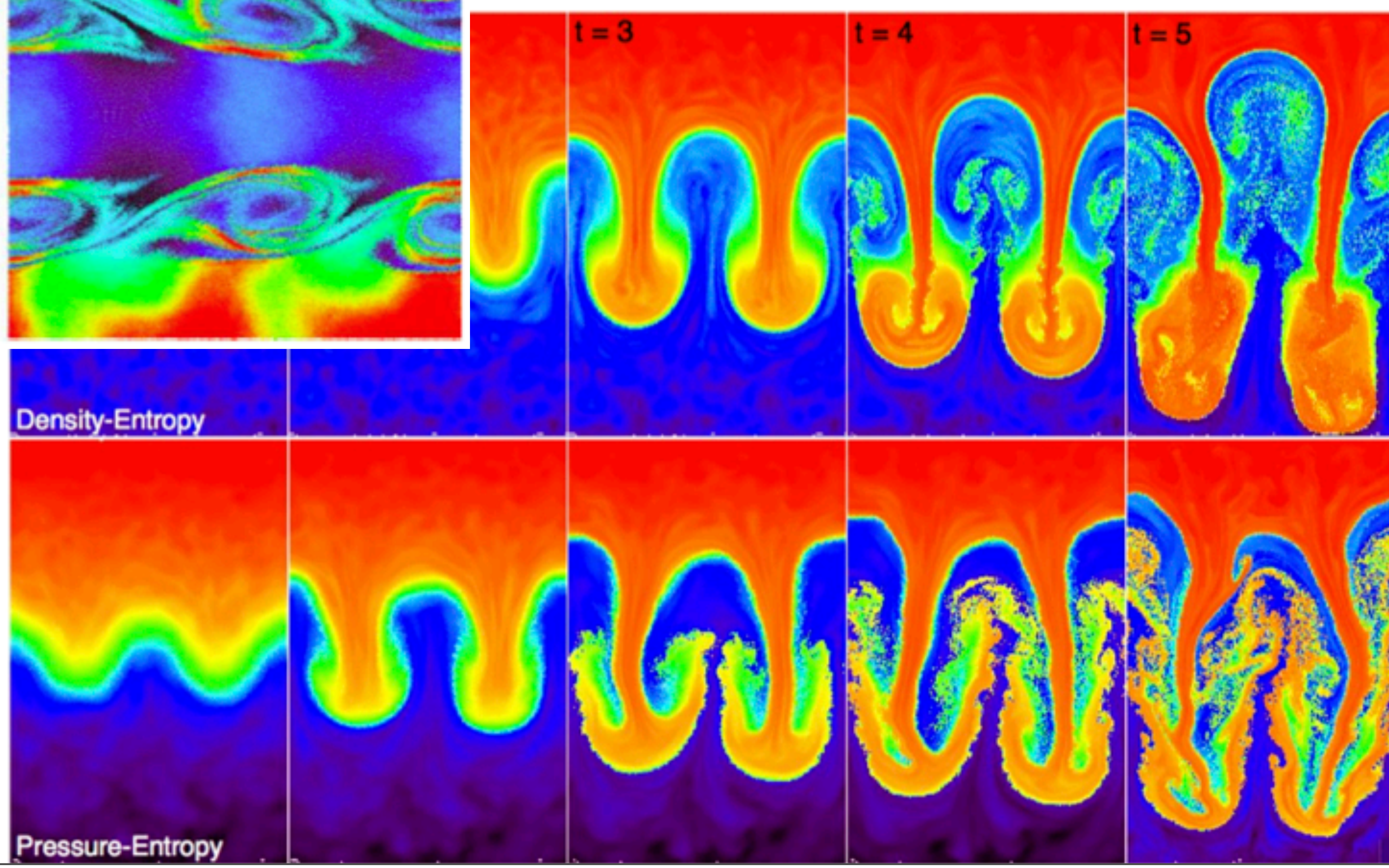


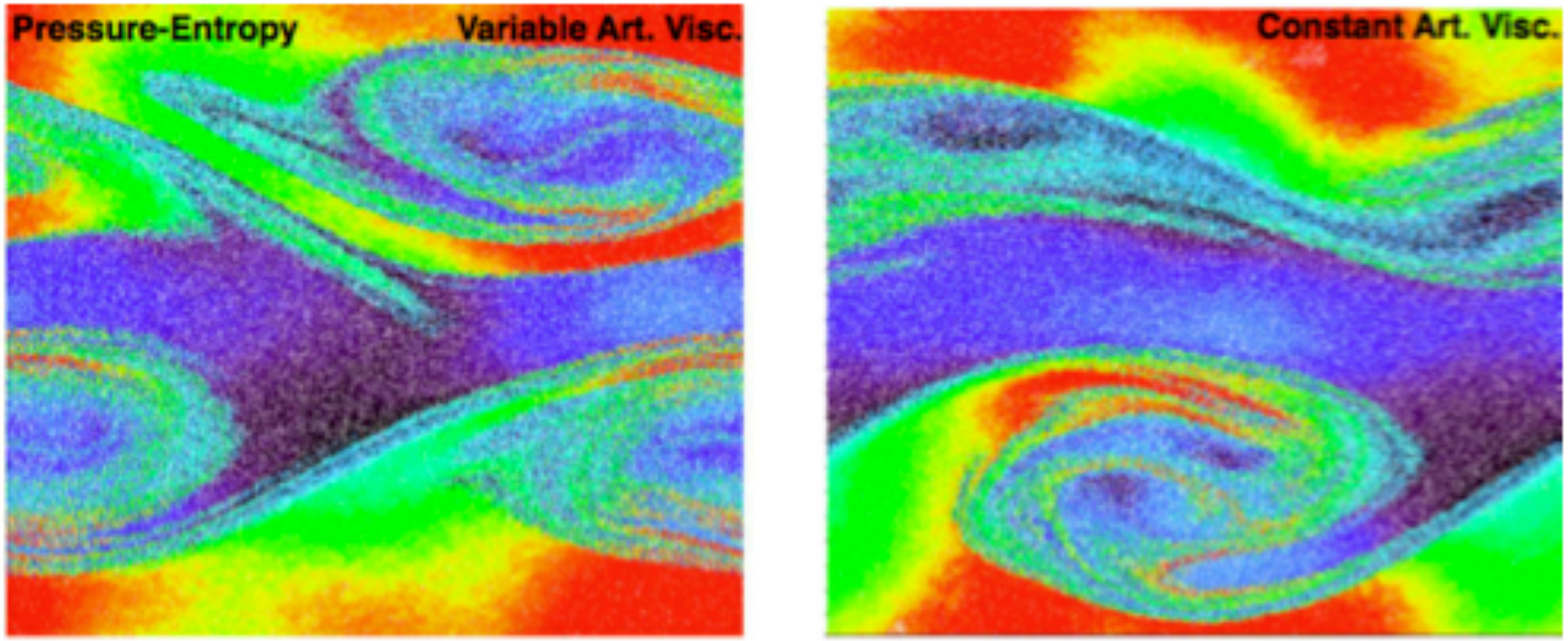
Athena (with magnetic $v_A = c_S$)

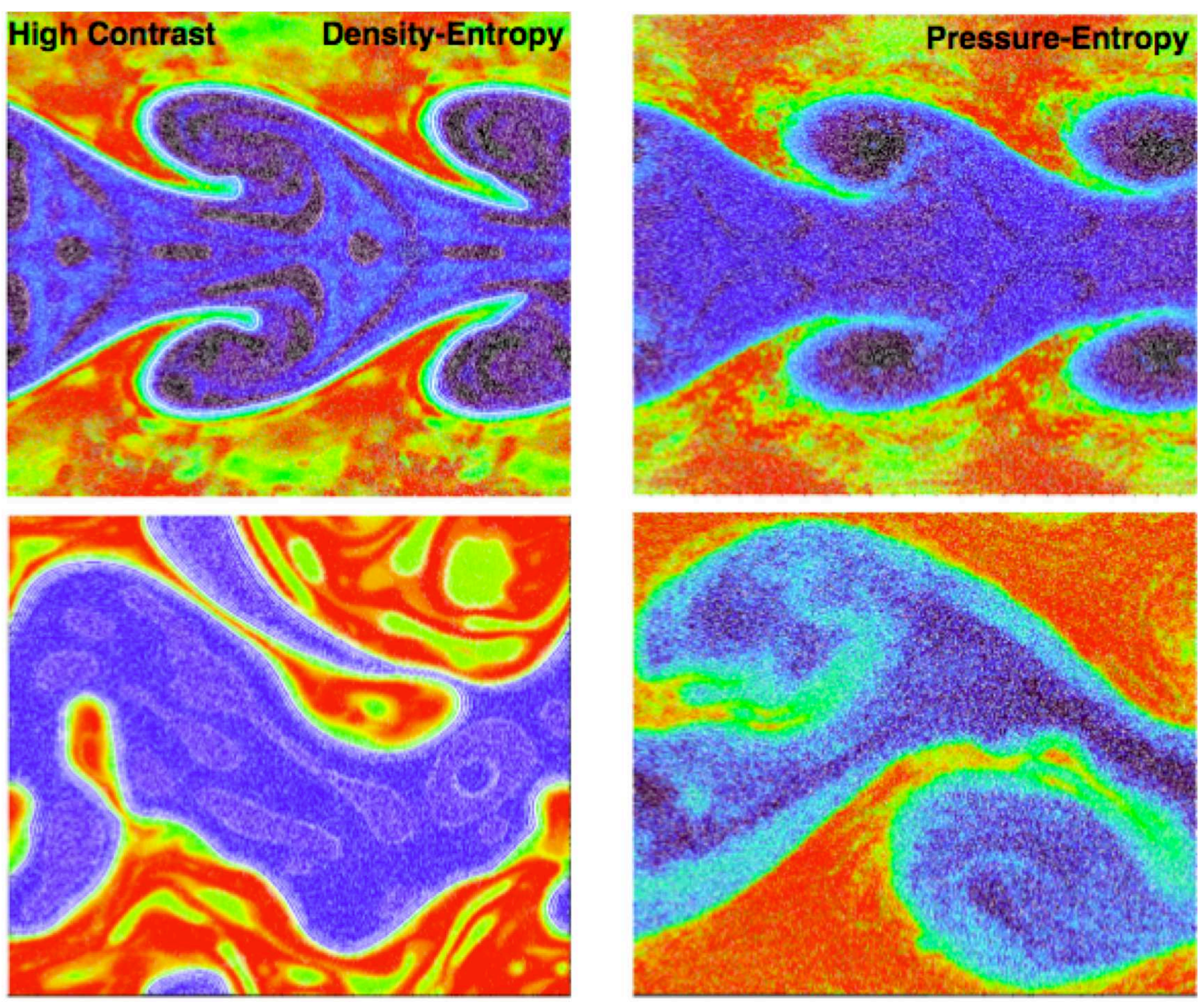


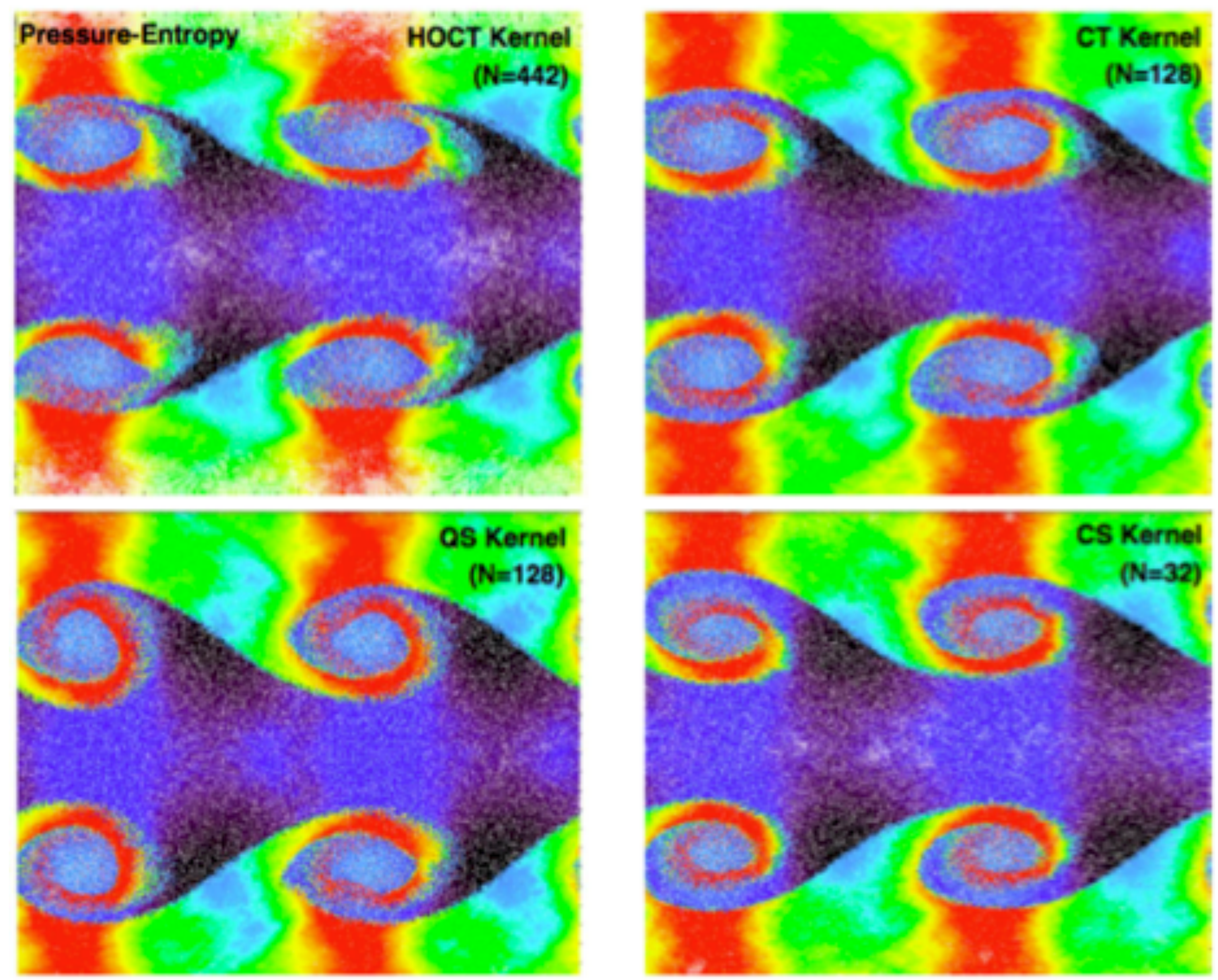
Density Formulation (Standard)

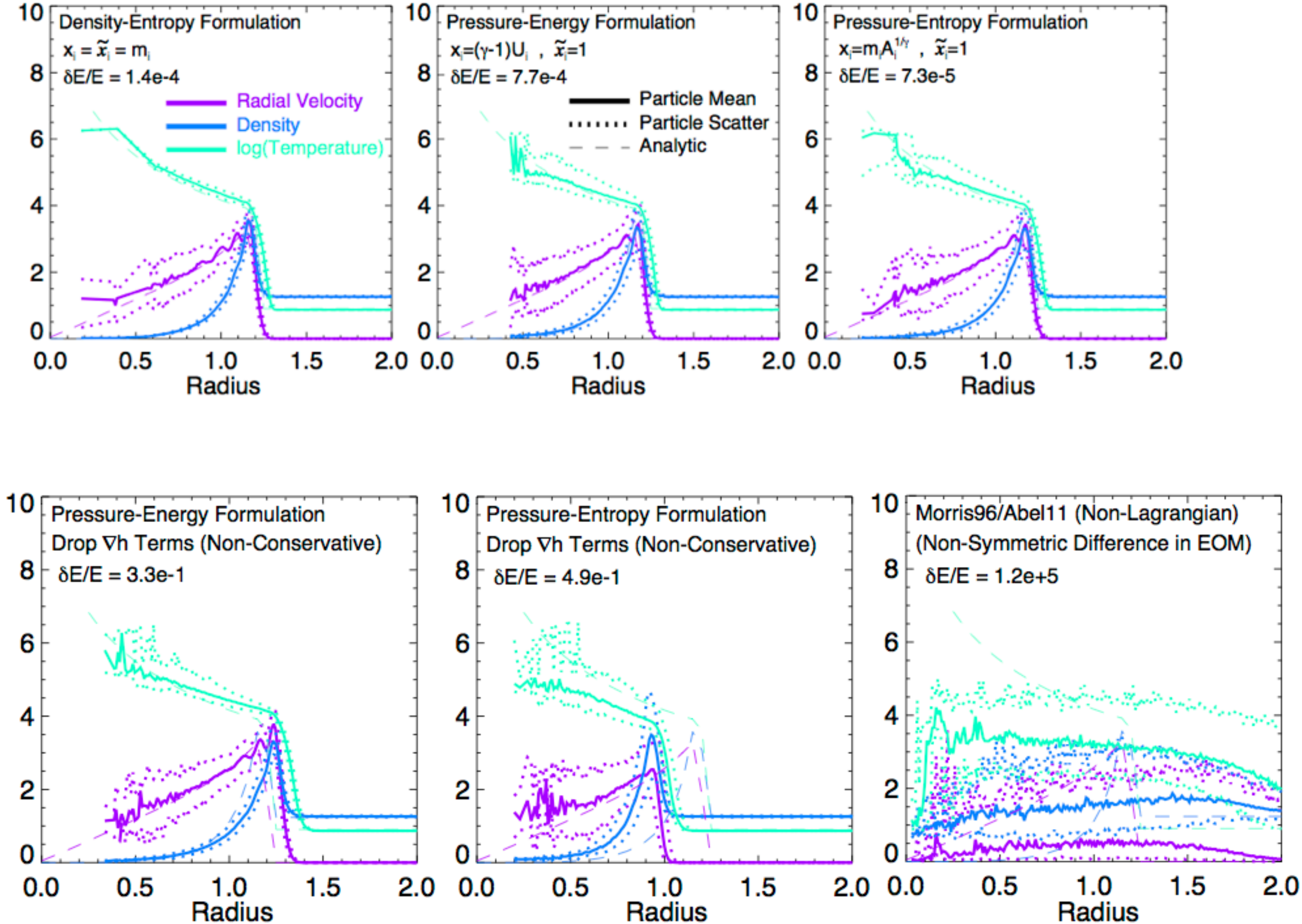

Density-Entropy
Pressure-Entropy


t = 3
t = 4
t = 5
Density-Entropy
Pressure-Entropy

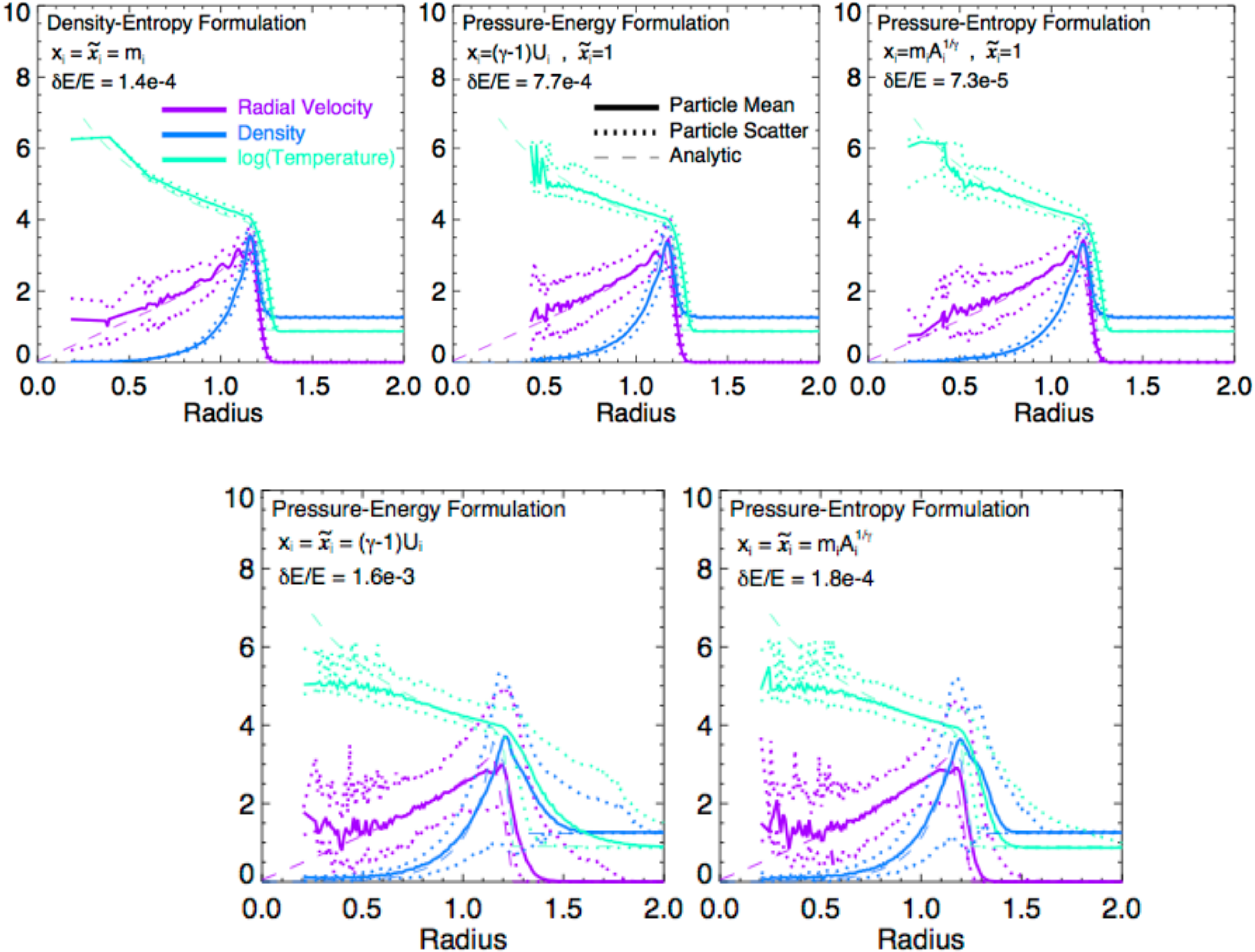

Pressure-Entropy
Variable Art. Visc.
Constant Art. Visc.


High Contrast
Density-Entropy
Pressure-Entropy

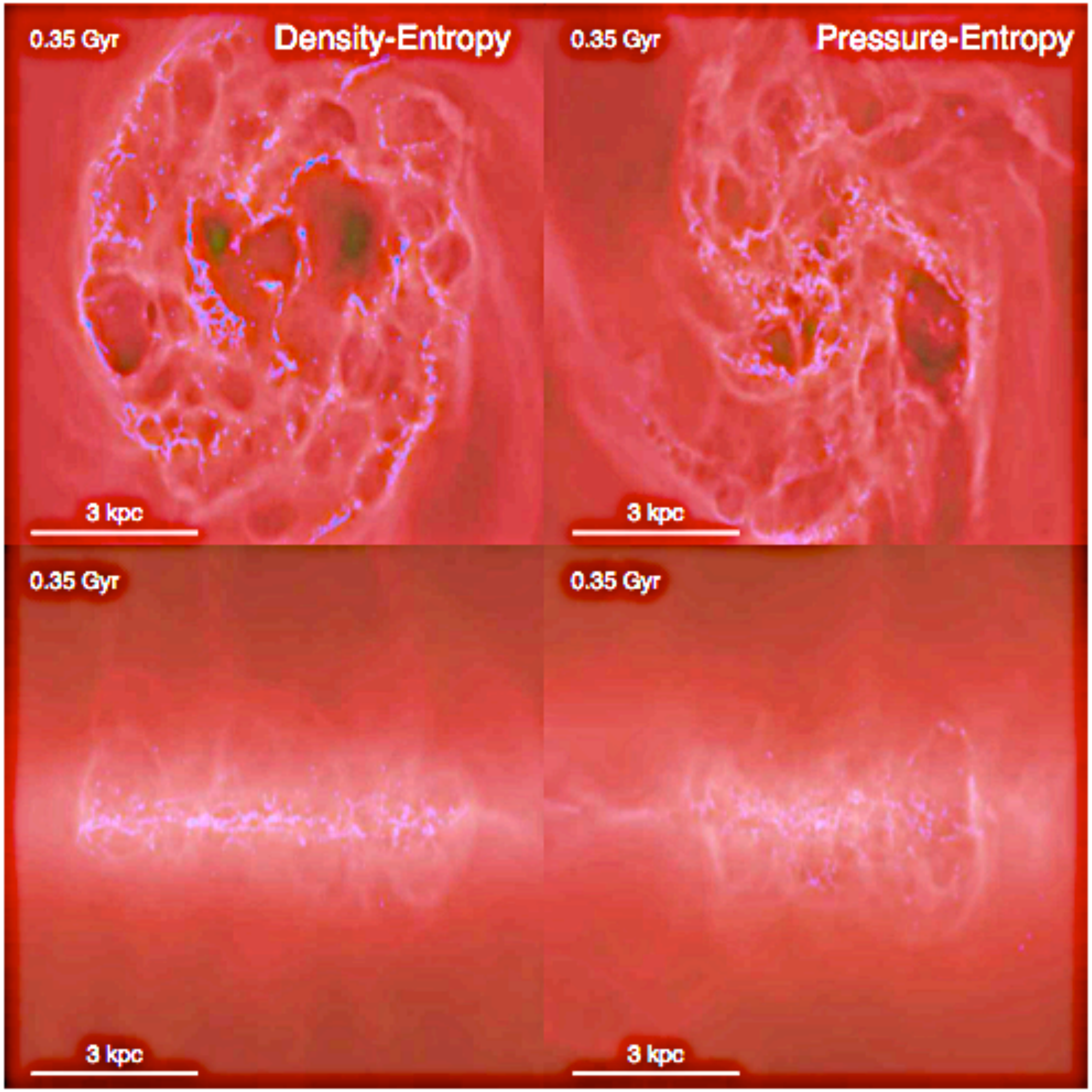

Pressure-Entropy
HOCT Kernel
(N=442)
CT Kernel
(N=128)
QS Kernel
(N=128)
CS Kernel
(N=32)



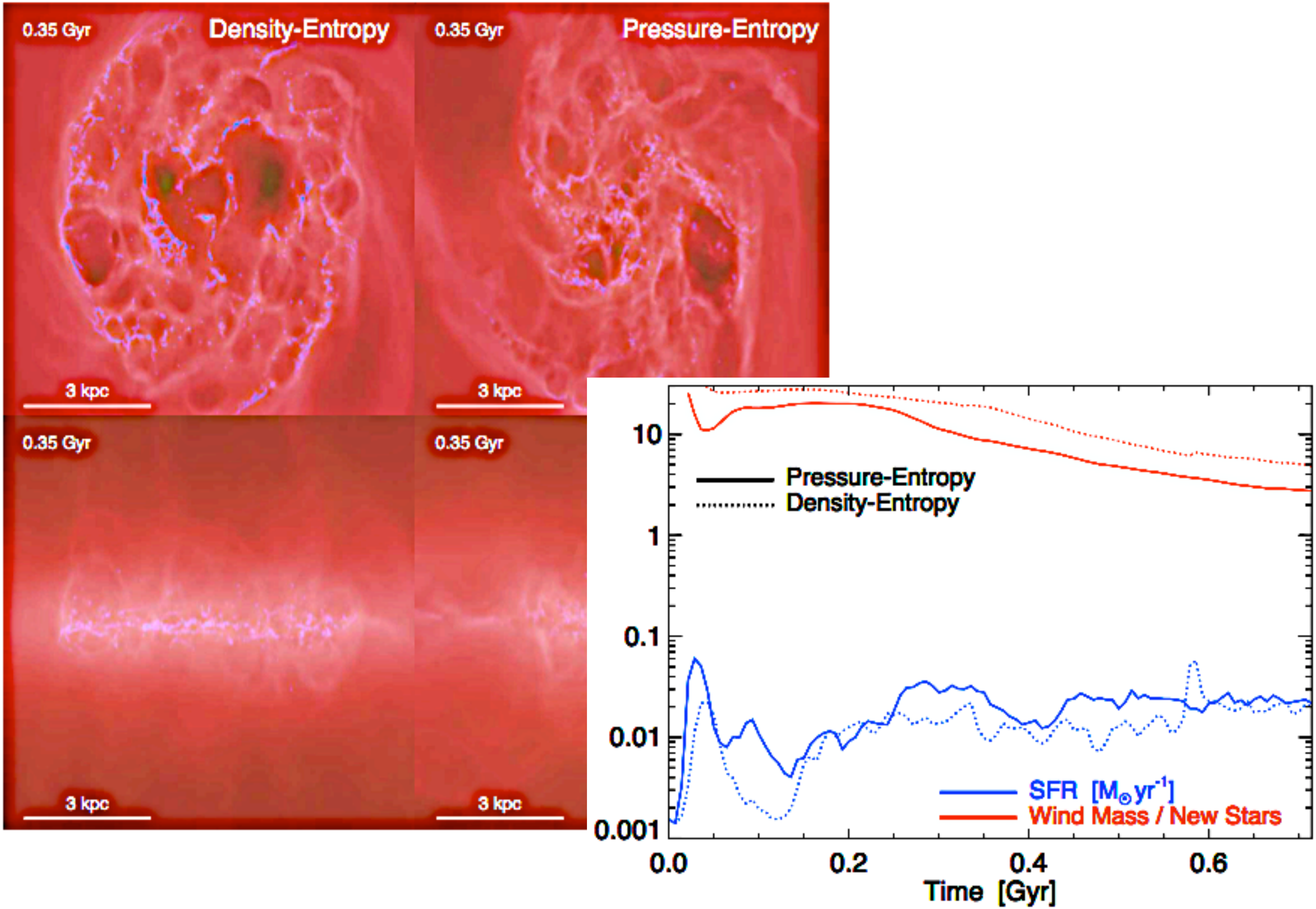
Density-Entropy Formulation
$x_i = \tilde{x}_i = m_i$
δE/E = 1.4e-4
Radial Velocity
Density
log(Temperature)
Pressure-Energy Formulation
$x_i=(\gamma-1)U_i$, $\tilde{x}_i=1$
δE/E = 7.7e-4
Particle Mean
Particle Scatter
Analytic
Pressure-Entropy Formulation
$x_i=m_iA_i^{1/\gamma}$, $\tilde{x}_i=1$
δE/E = 7.3e-5
Radius
Pressure-Energy Formulation
Drop ∇h Terms (Non-Conservative)
δE/E = 3.3e-1
Pressure-Entropy Formulation
Drop ∇h Terms (Non-Conservative)
δE/E = 4.9e-1
Morris96/Abel11 (Non-Lagrangian)
(Non-Symmetric Difference in EOM)
δE/E = 1.2e+5
Radius

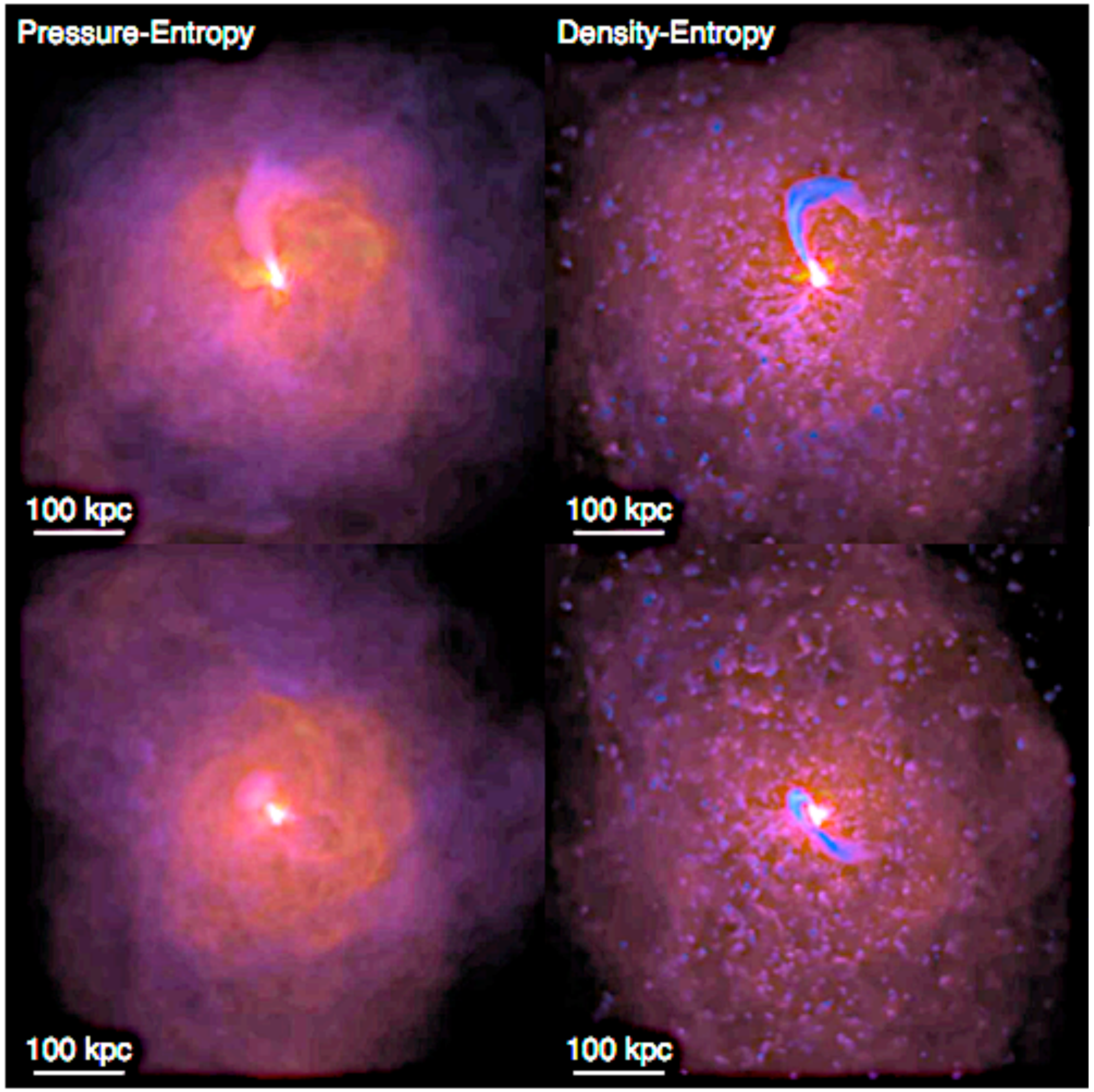

Density-Entropy Formulation
$x_i = \tilde{x}_i = m_i$
δE/E = 1.4e-4
Radial Velocity
Density
log(Temperature)
Pressure-Energy Formulation
$x_i=(\gamma-1)U_i$, $\tilde{x}_i=1$
δE/E = 7.7e-4
Particle Mean
Particle Scatter
Analytic
Pressure-Entropy Formulation
$x_i=m_iA_i^{1/\gamma}$, $\tilde{x}_i=1$
δE/E = 7.3e-5
Pressure-Energy Formulation
$x_i = \tilde{x}_i = (\gamma-1)U_i$
δE/E = 1.6e-3
Pressure-Entropy Formulation
$x_i = \tilde{x}_i = m_iA_i^{1/\gamma}$
δE/E = 1.8e-4
Radius
0.0
0.5
1.0
1.5
2.0
0
2
4
6
8
10

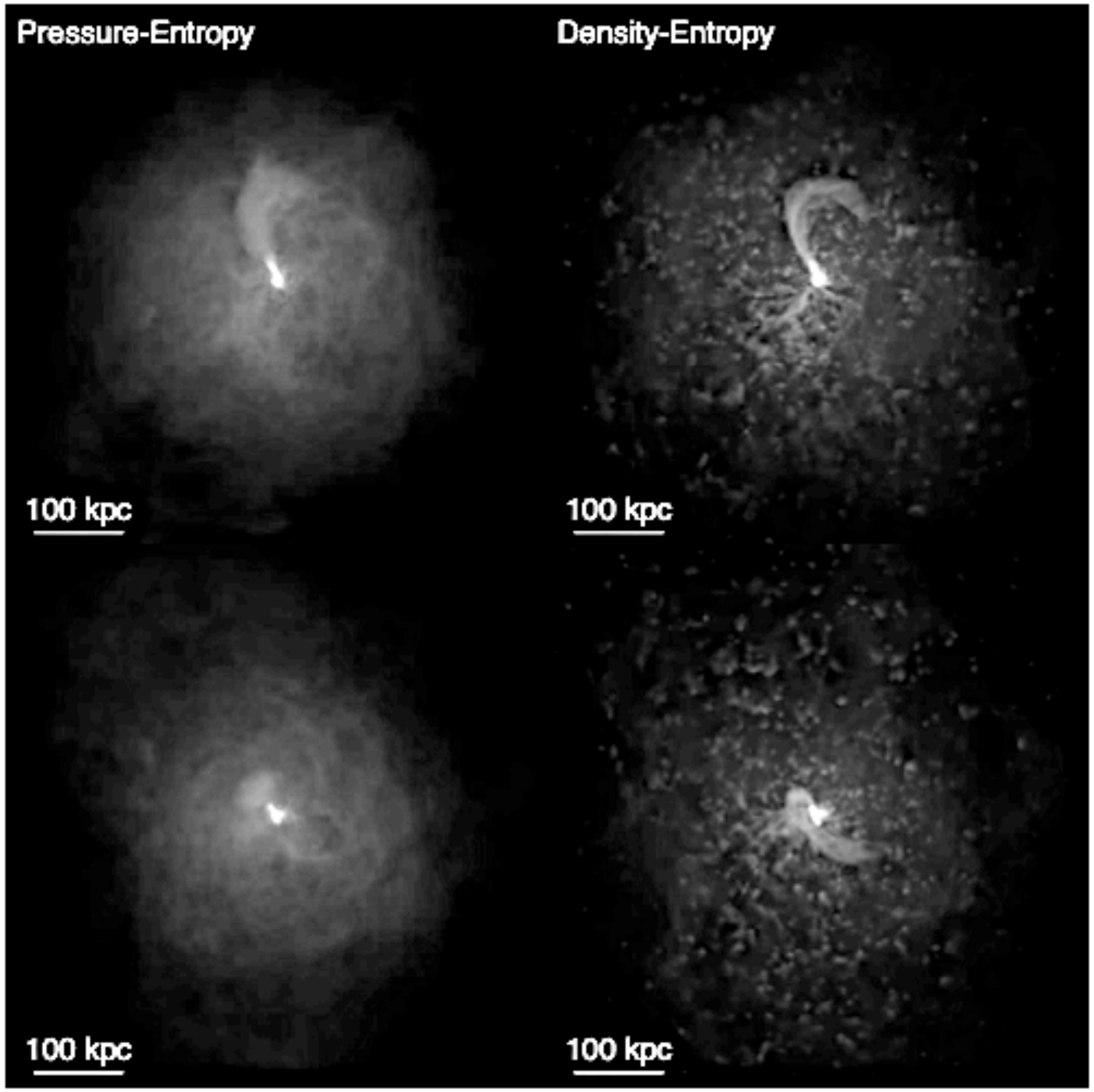
SPH in Pressure-Entropy Formulation



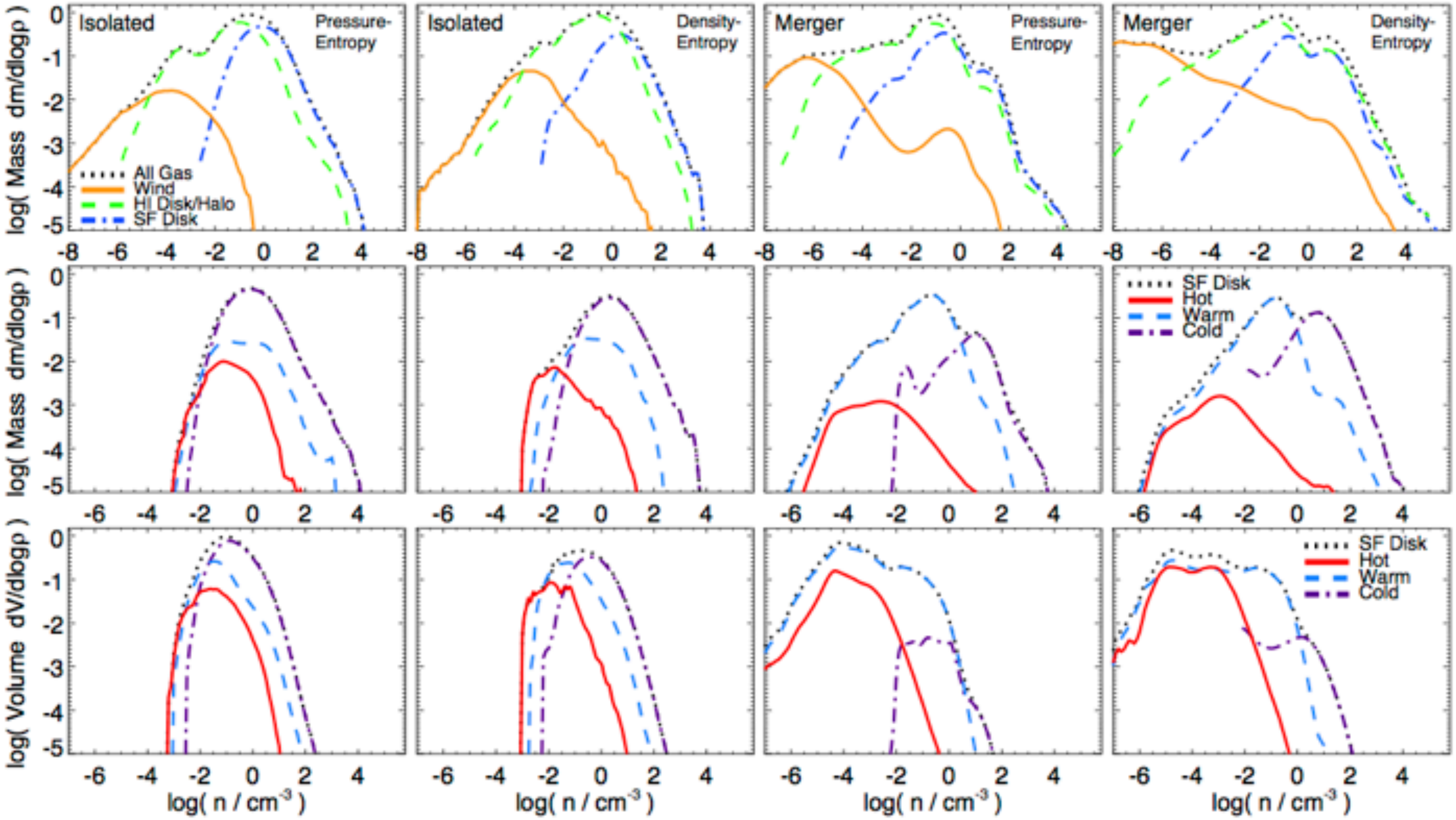
SPH in Pressure-Entropy Formulation




Pressure-Entropy
Density-Entropy
100 kpc
100 kpc
100 kpc
100 kpc

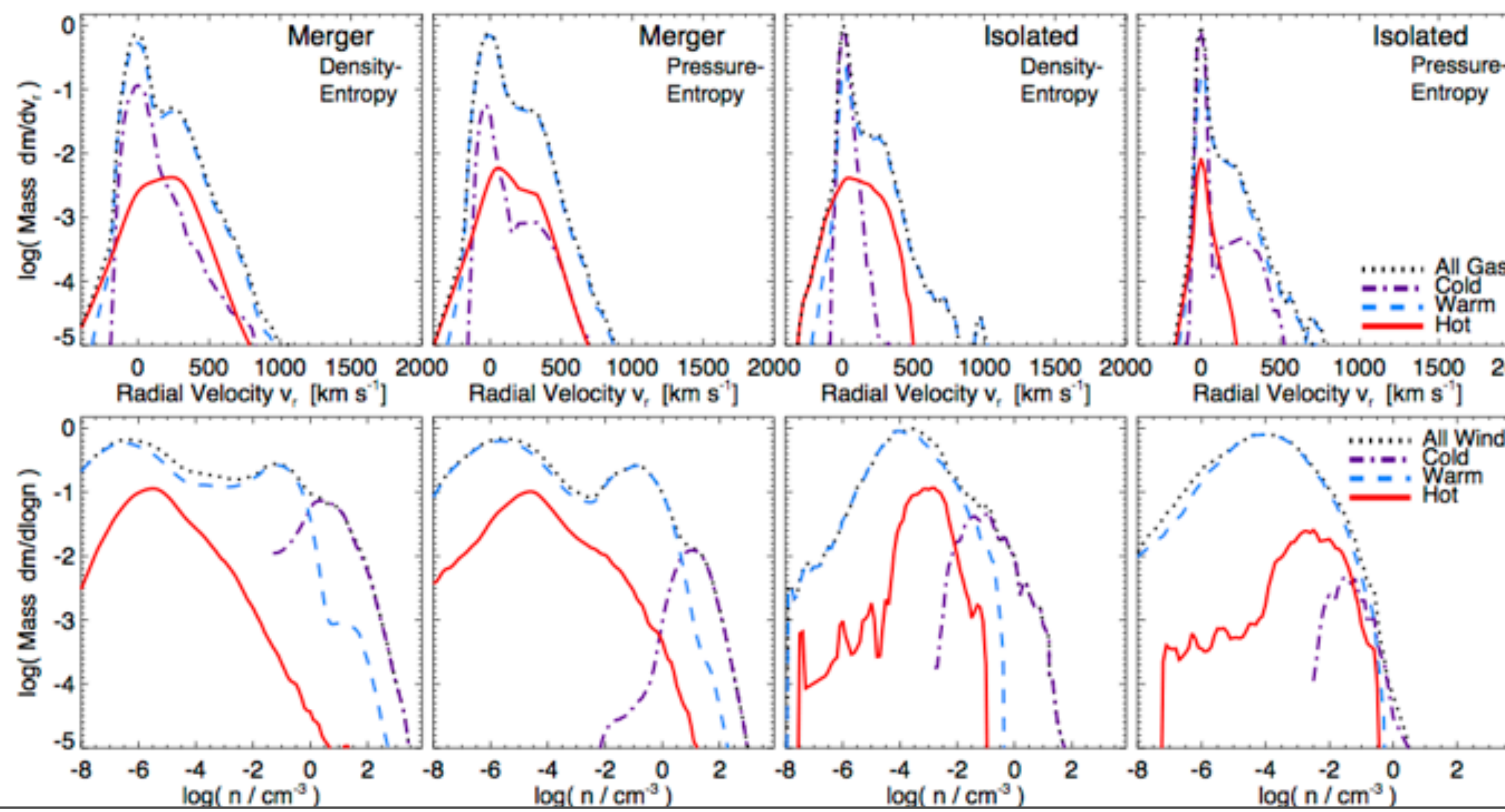

Pressure-Entropy
Density-Entropy
100 kpc
100 kpc
100 kpc
100 kpc

Disk



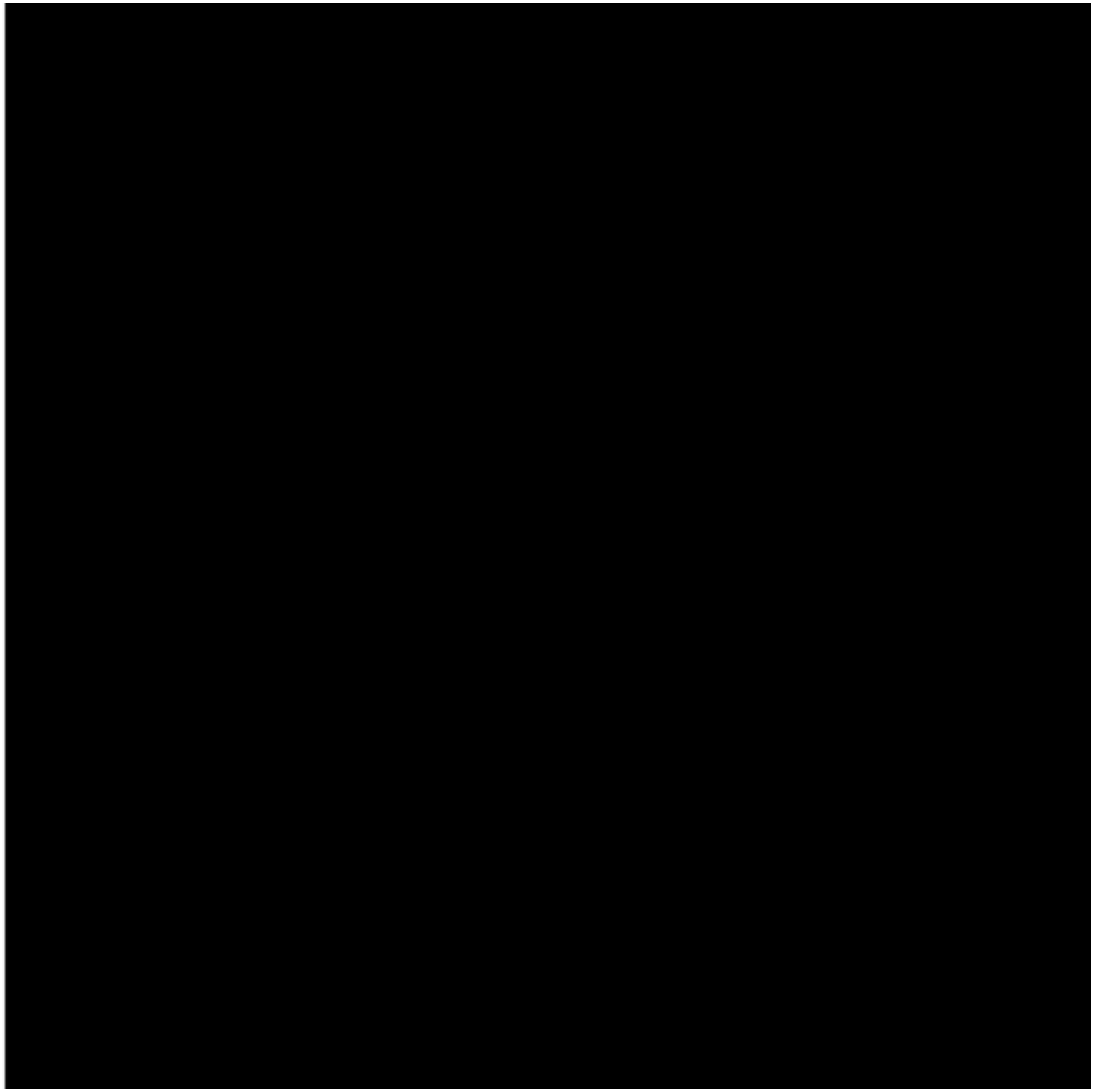
Isolated
Pressure-Entropy
Isolated
Density-Entropy
Merger
Pressure-Entropy
Merger
Density-Entropy
All Gas
Wind
HI Disk/Halo
SF Disk
SF Disk
Hot
Warm
Cold
log(Mass dm/dlogρ)
log(Volume dV/dlogρ)
log(n / cm⁻³)

Wind:

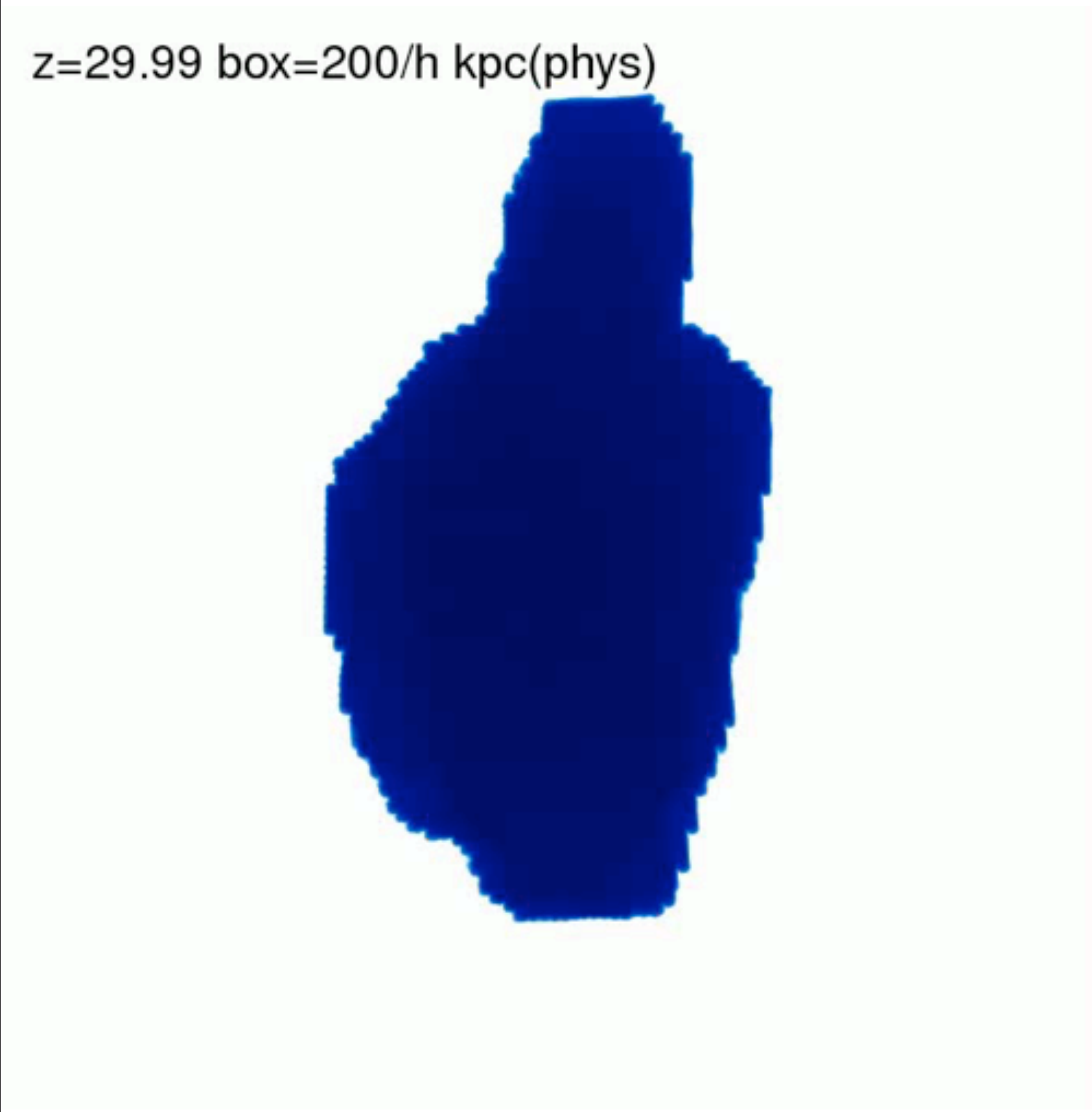


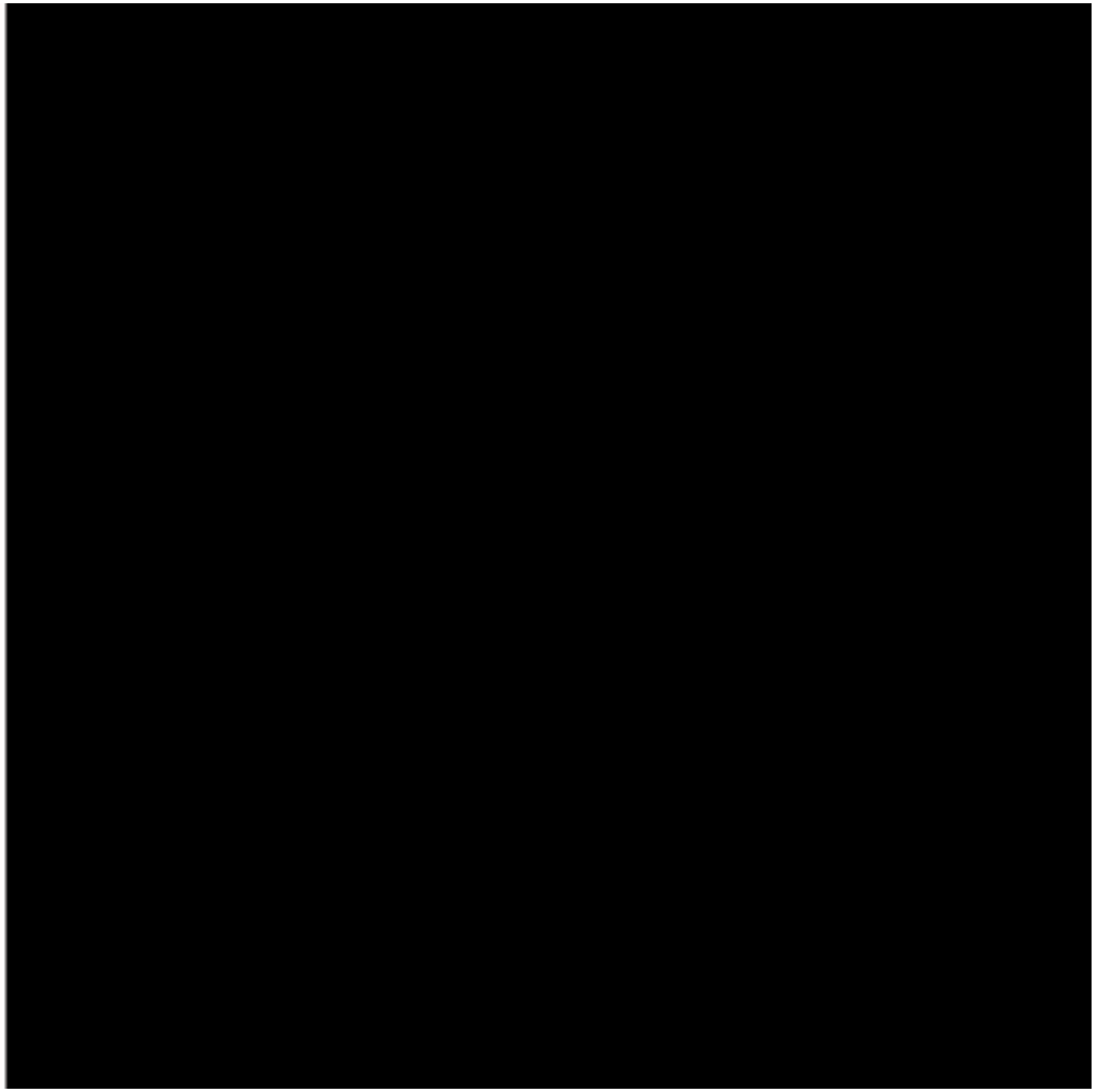
Merger
Density-Entropy
Merger
Pressure-Entropy
Isolated
Density-Entropy
Isolated
Pressure-Entropy
All Gas
Cold
Warm
Hot
All Wind
Cold
Warm
Hot
log(Mass dm/dv_r)
log(Mass dm/dlogn)
Radial Velocity v_r [km s⁻¹]
log(n / cm⁻³)

Caution: Artificial Viscosity & Particle Order



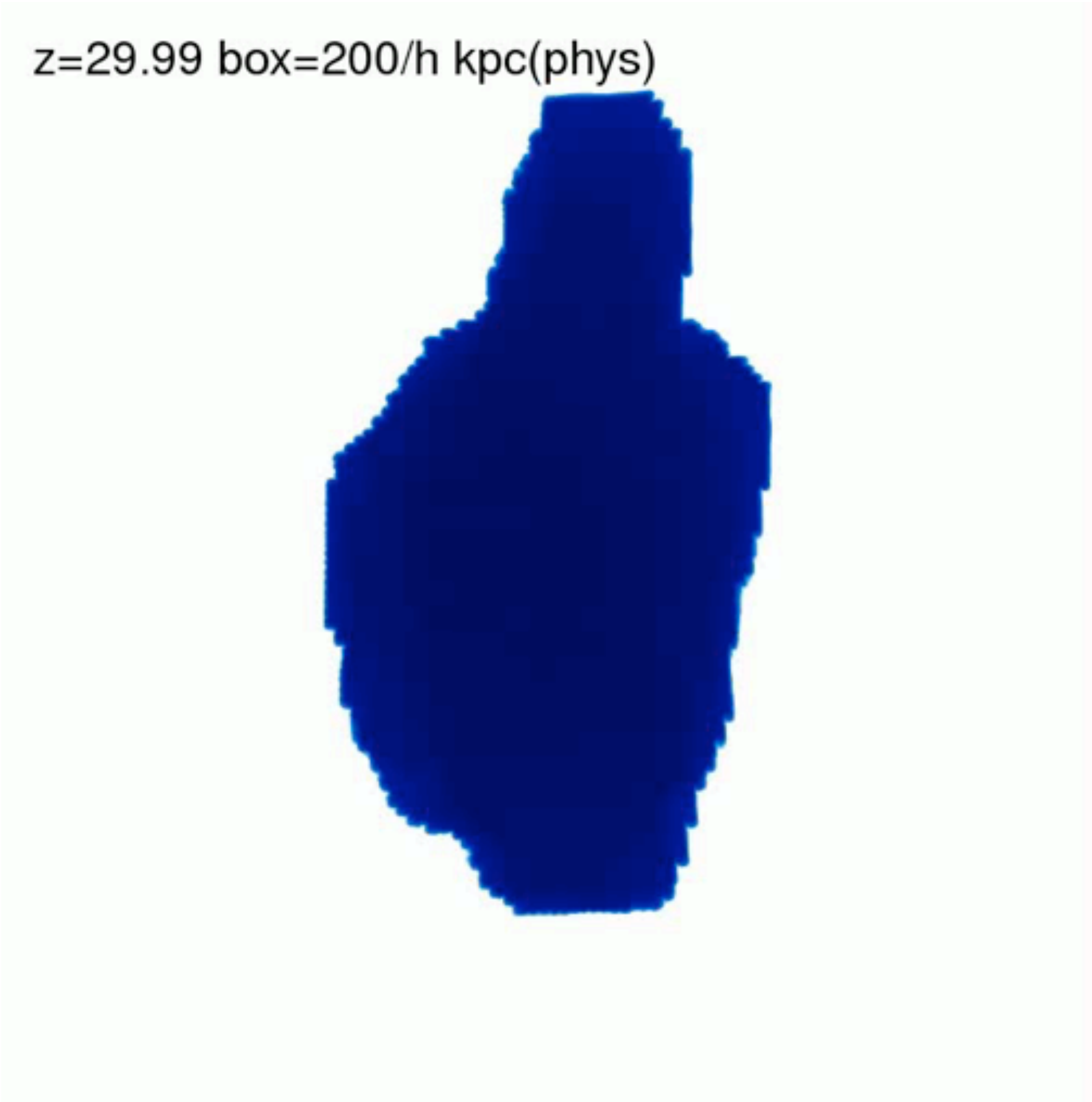
Caution: Artificial Viscosity & Particle Order

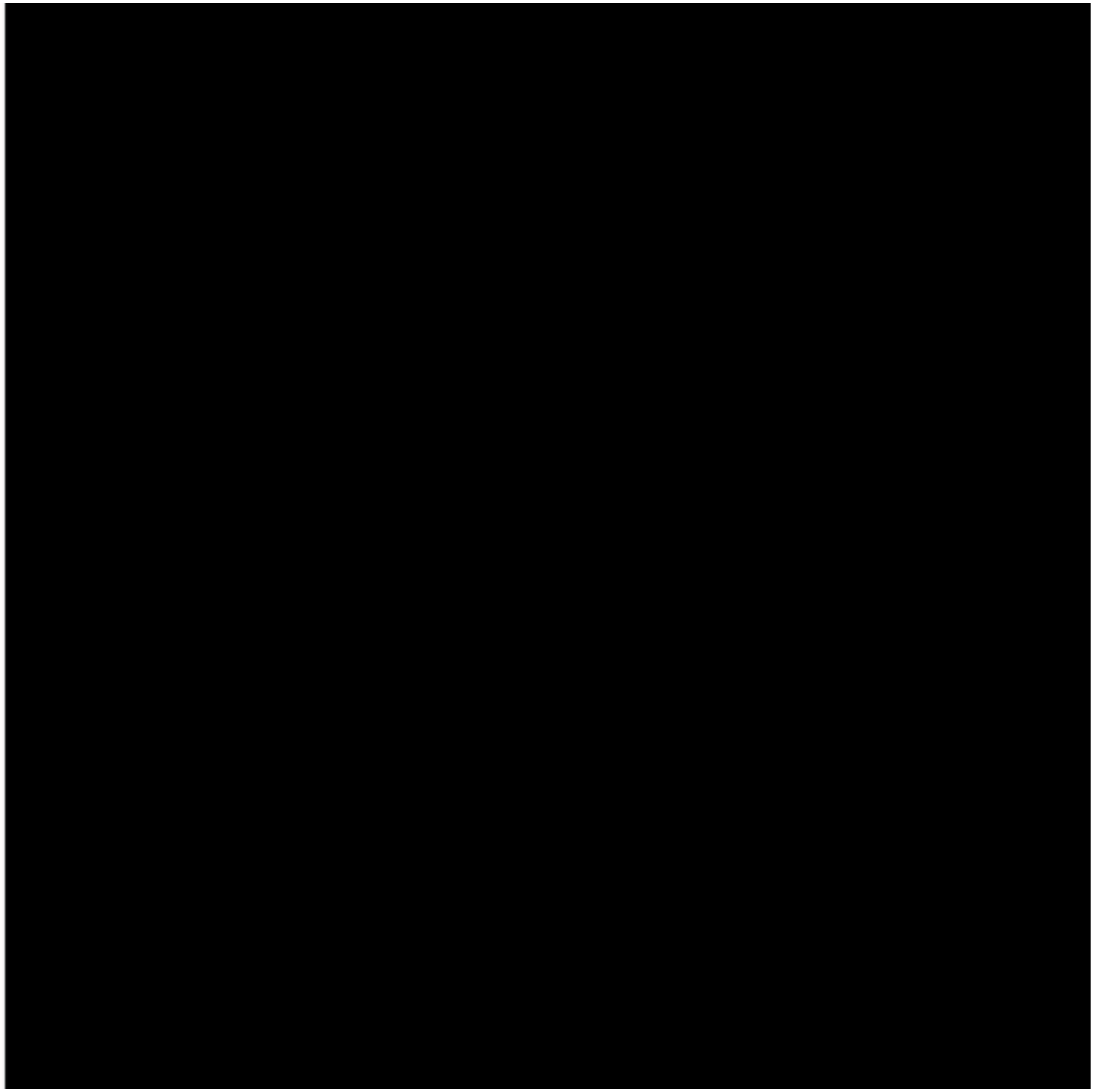




- worse if we neglect timestepping issues

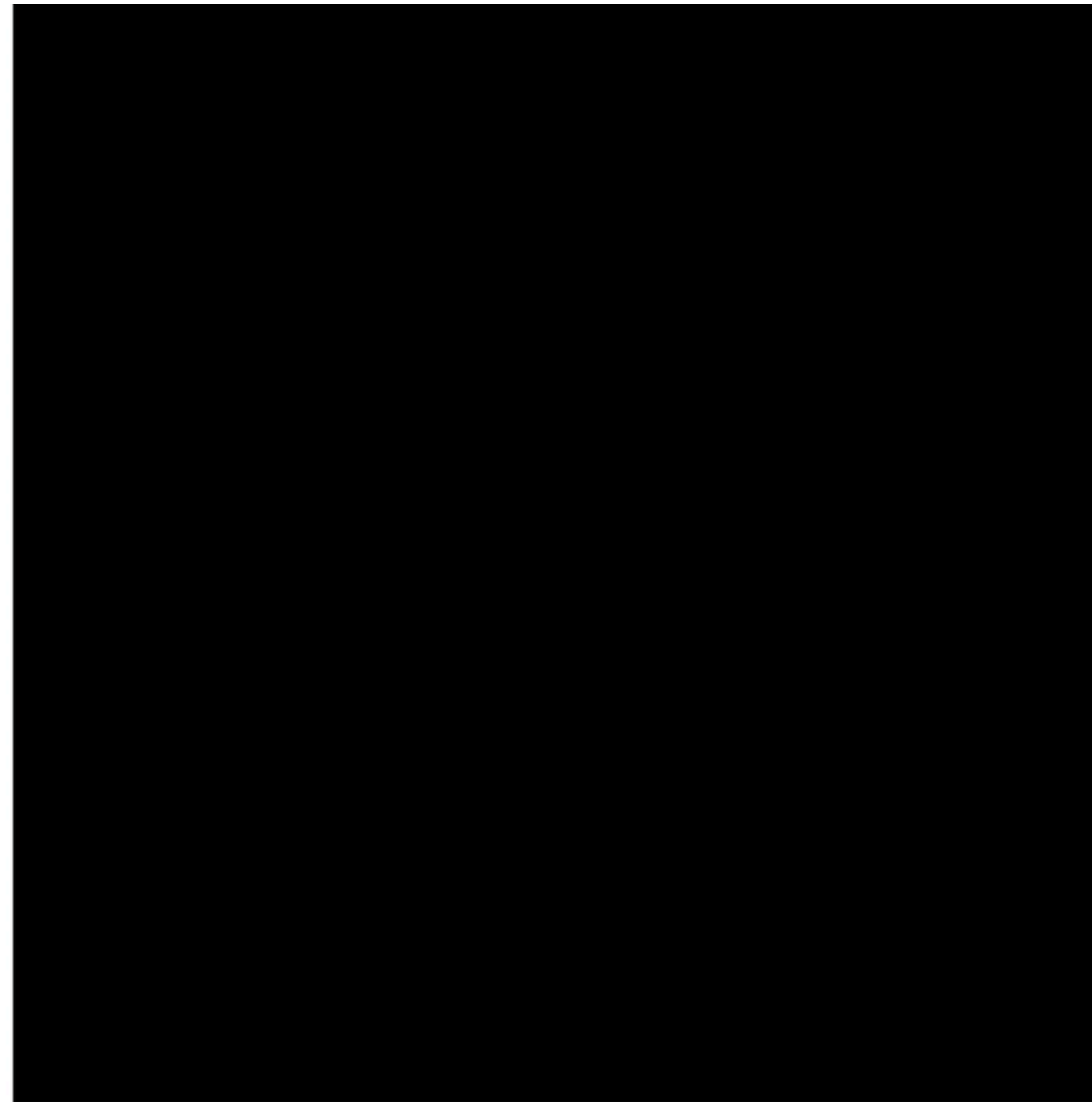
Caution: Artificial Viscosity & Particle Order





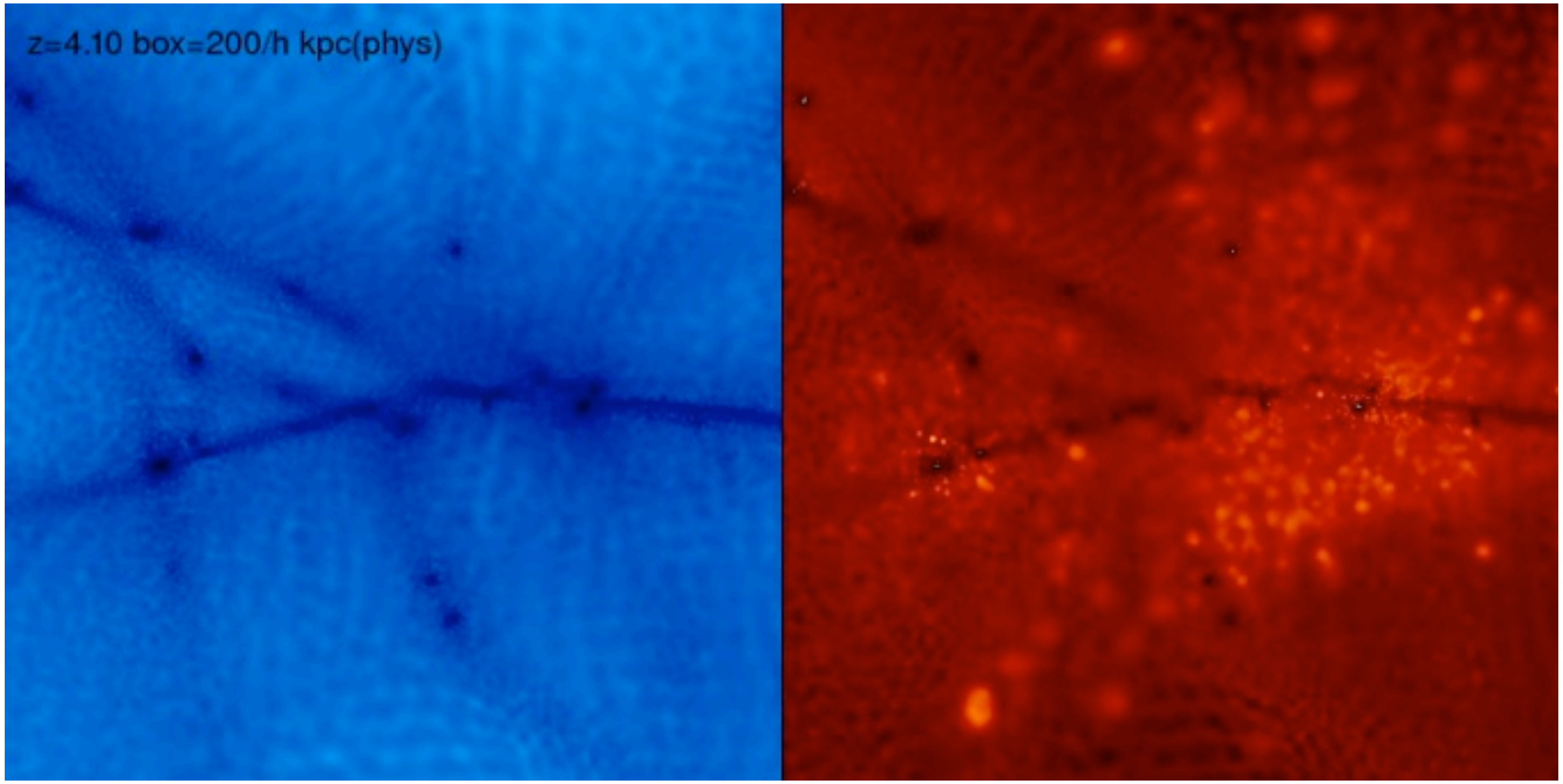
- “old” SPH instead now

Caution: Artificial Viscosity & Particle Order

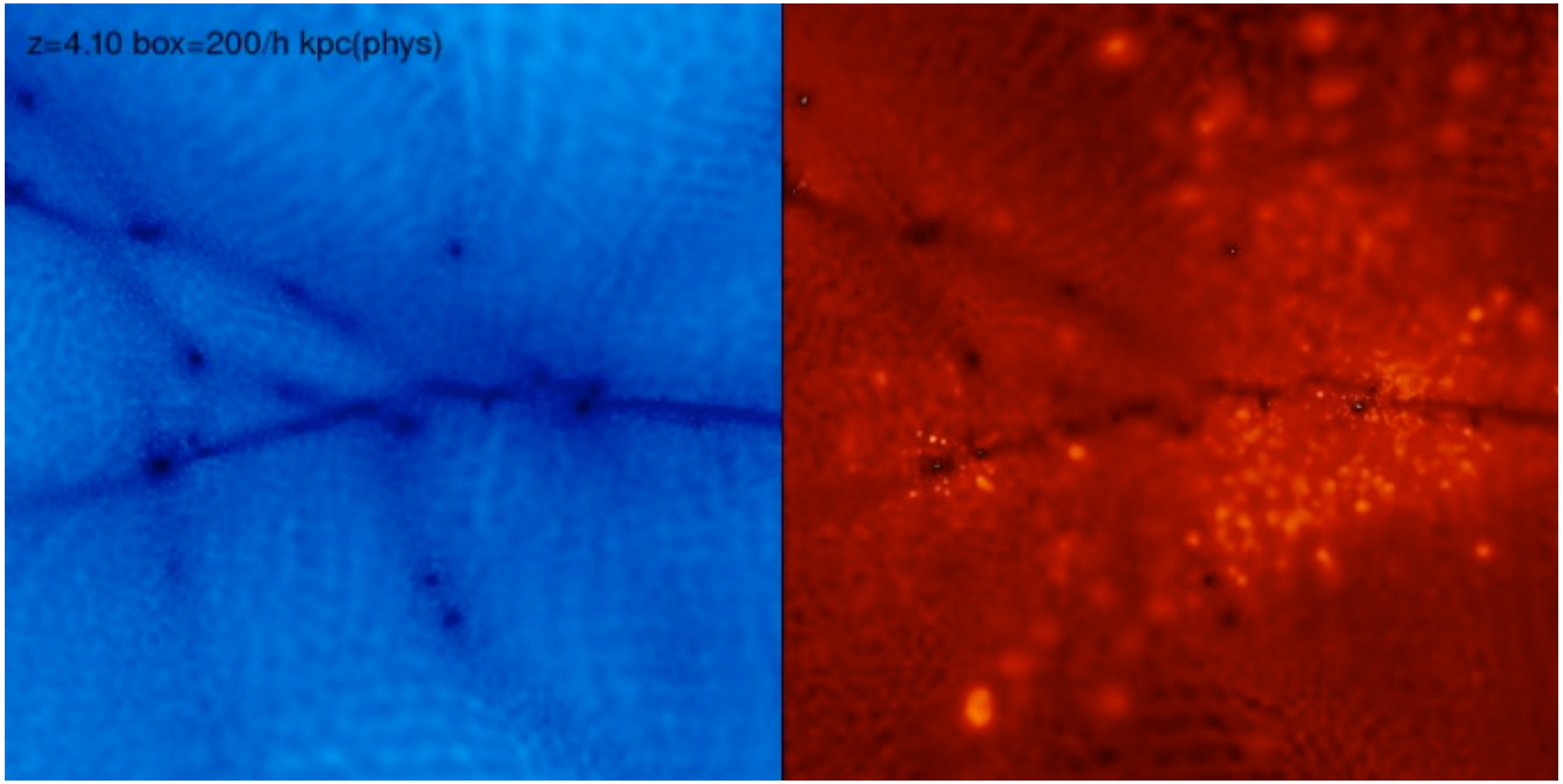


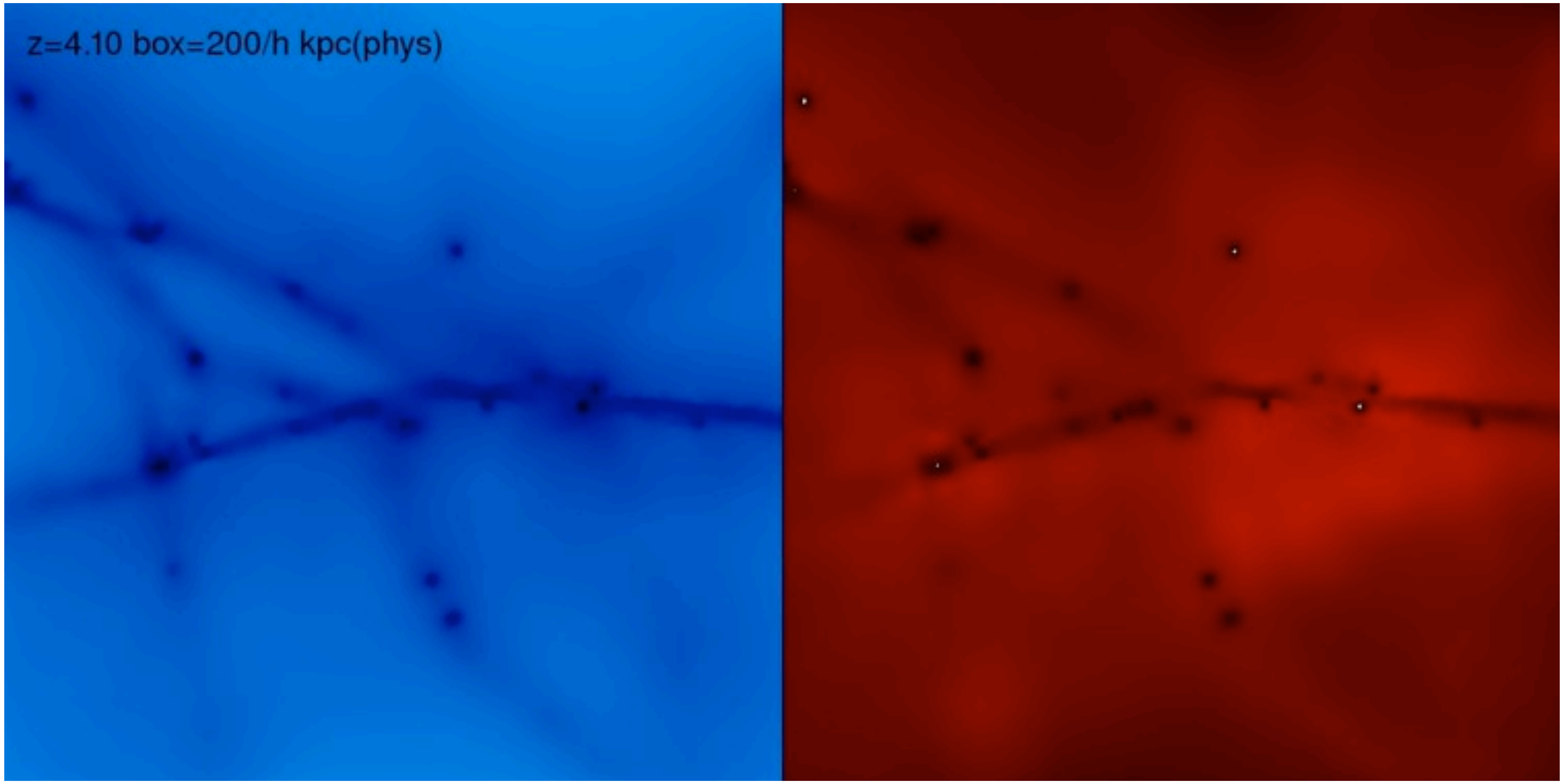
- new SPH with timestep limiter
& better artificial viscosity

Caution: Artificial Viscosity & Particle Order:

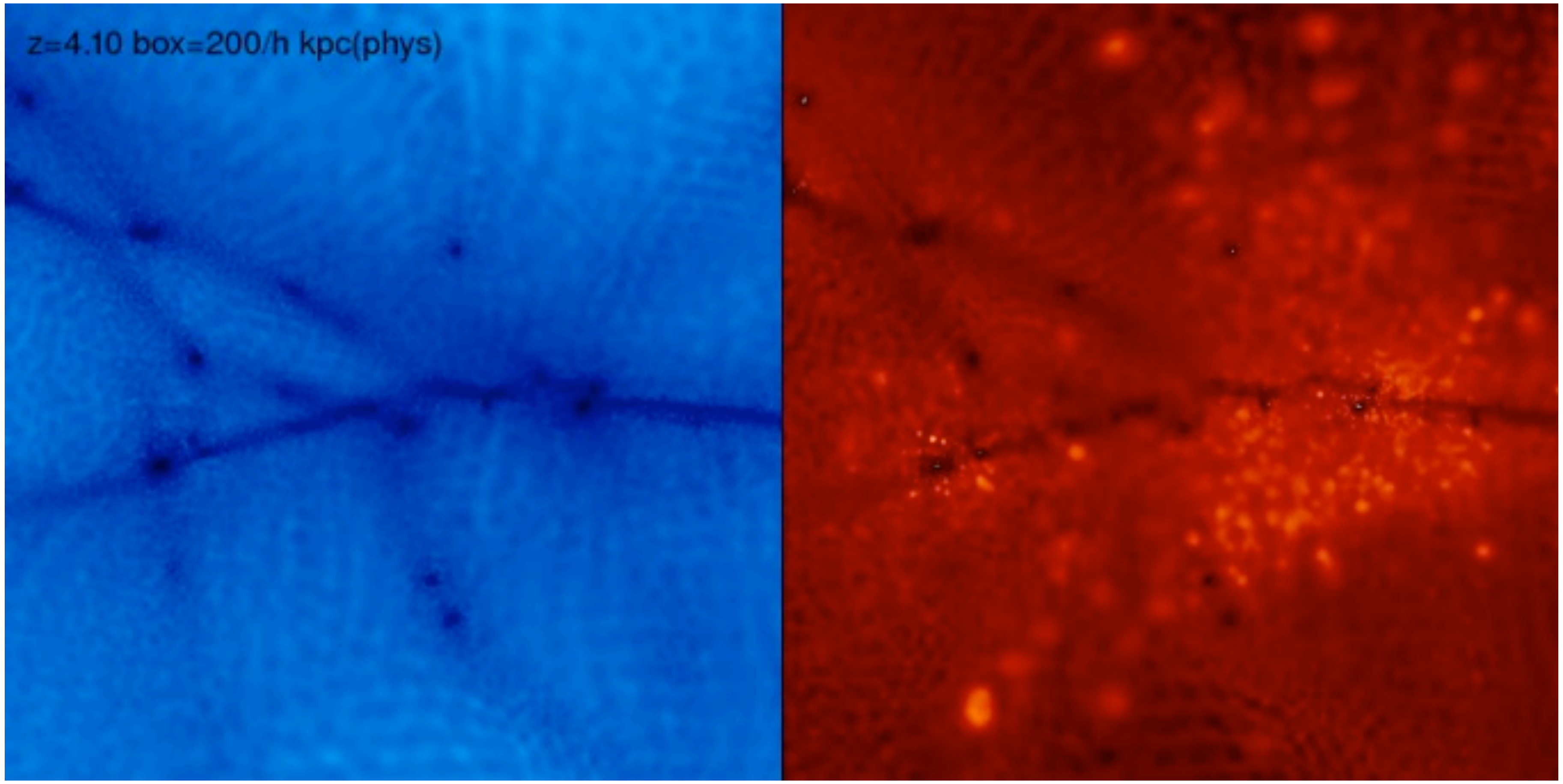


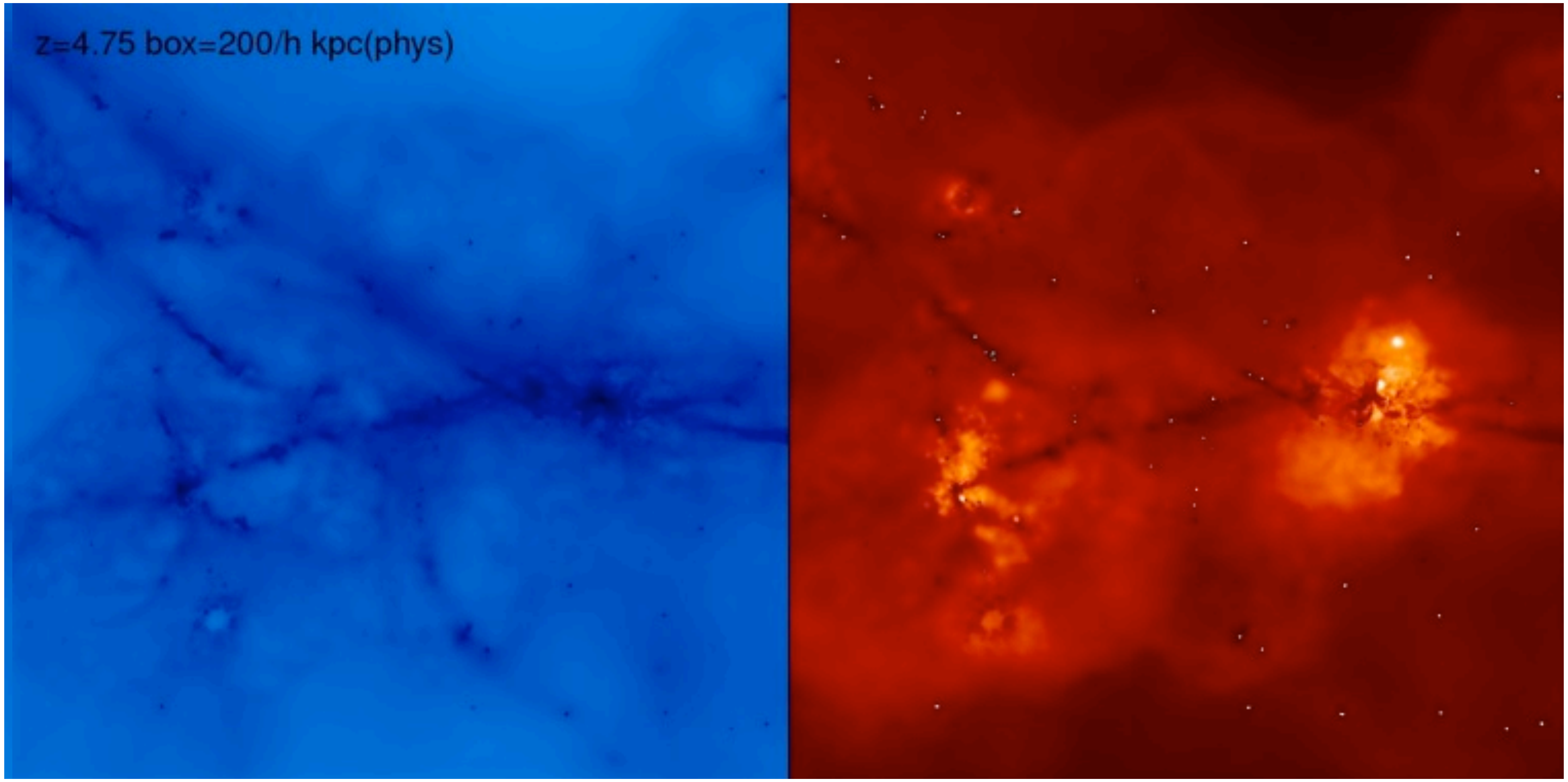
Caution: Artificial Viscosity & Particle Order:





Caution: Artificial Viscosity & Particle Order:





Mixing:

“Blobs”

K-H

Cooling:

Too Little | Too Much

C-SPH | P-SPH | Moving Mesh | AMR

Mixing:
"Blobs"
K-H
Cooling:

Too Little — Too Much

C-SPH — P-SPH — Moving Mesh — AMR

Angular Momentum:

$\mathcal{O}(1)$ errors — $\mathcal{O}(2)$ errors? — (machine accuracy)

AMR
Fixed Grid — Moving Mesh — P-SPH
C-SPH

Mixing:
"Blobs"
K-H
Cooling:

Too Little ← | → Too Much

C-SPH — P-SPH — Moving Mesh — AMR

Angular Momentum:

$\mathcal{O}(1)$ errors — $\mathcal{O}(2)$ errors? — (machine accuracy)

AMR, Fixed Grid — Moving Mesh — P-SPH, C-SPH

Galilean Invariance:

(error grows with velocity) — (re-mesh errors) — (machine accuracy)

AMR, Fixed Grid — Moving Mesh — P-SPH, C-SPH

Strong Shocks:

[Poorer] (~ smoothing) (~ cell)

Resolution

P-SPH (Adiabatic) | C-SPH | P-SPH (Radiative) | Grid Codes

Conservation Errors

$\mathcal{O}(0)$ | $\mathcal{O}(2)$ | (~machine)

C-SPH (non-Lagrangian) | Grid Codes (Reimann) | P-SPH C-SPH (Lagrangian)

Strong Shocks:

[Poorer] Resolution

(~ smoothing) (~ cell)

P-SPH (Adiabatic) C-SPH P-SPH (Radiative) Grid Codes

Conservation Errors

$\mathcal{O}(0)$ $\mathcal{O}(2)$ (~machine)

C-SPH (non-Lagrangian)

Grid Codes (Reimann)

P-SPH C-SPH (Lagrangian)

Subsonic Turbulence:

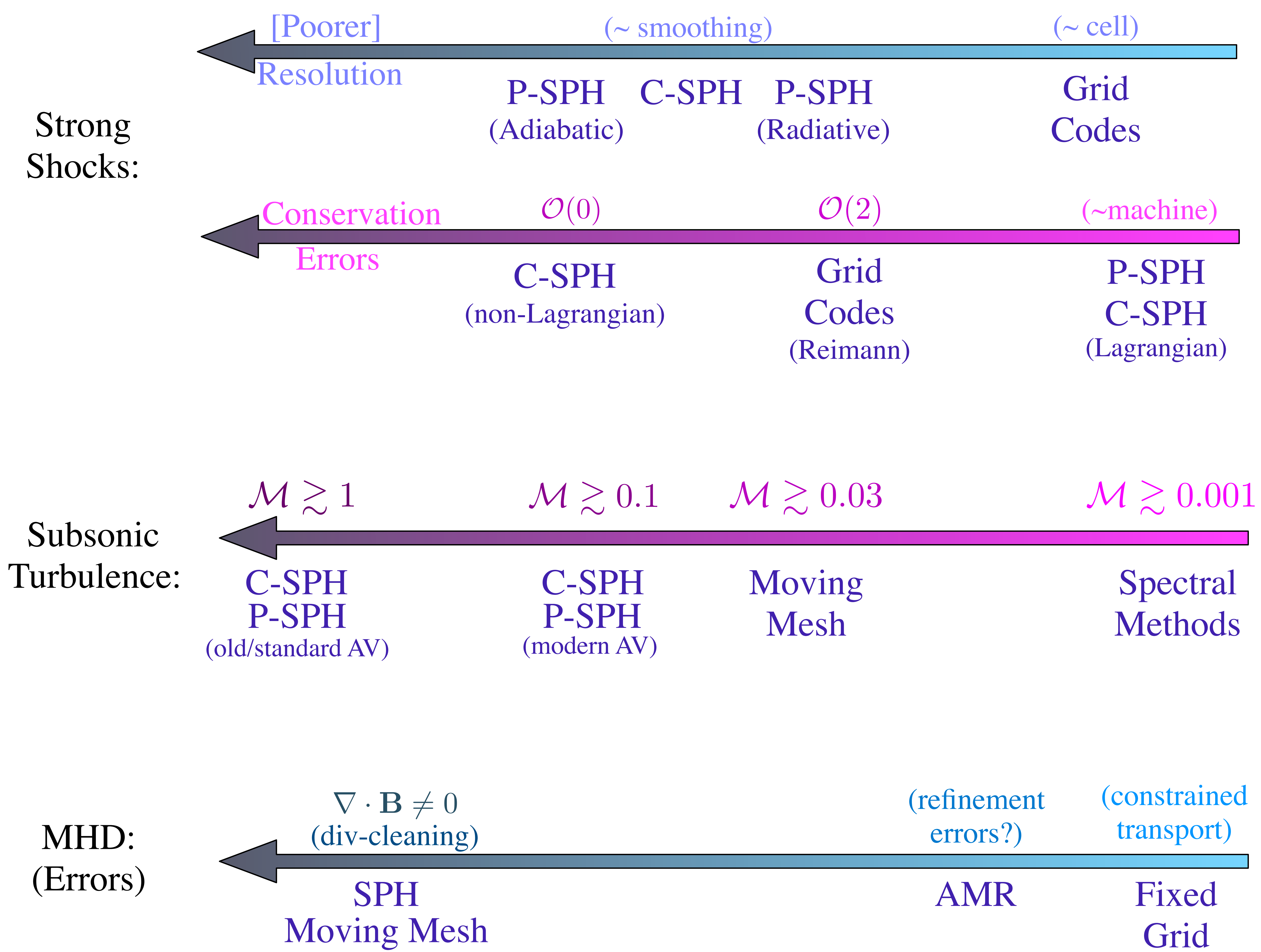
$\mathcal{M} \gtrsim 1$ $\mathcal{M} \gtrsim 0.1$ $\mathcal{M} \gtrsim 0.03$ $\mathcal{M} \gtrsim 0.001$

C-SPH P-SPH (old/standard AV)

C-SPH P-SPH (modern AV)

Moving Mesh

Spectral Methods


[Poorer]
Resolution
(~ smoothing)
(~ cell)
Strong Shocks:
P-SPH (Adiabatic)
C-SPH
P-SPH (Radiative)
Grid Codes
Conservation Errors
$\mathcal{O}(0)$
$\mathcal{O}(2)$
(~machine)
C-SPH (non-Lagrangian)
Grid Codes (Reimann)
P-SPH C-SPH (Lagrangian)
$\mathcal{M} \gtrsim 1$
$\mathcal{M} \gtrsim 0.1$
$\mathcal{M} \gtrsim 0.03$
$\mathcal{M} \gtrsim 0.001$
Subsonic Turbulence:
C-SPH P-SPH (old/standard AV)
C-SPH P-SPH (modern AV)
Moving Mesh
Spectral Methods
$\nabla \cdot \mathbf{B} \neq 0$
(div-cleaning)
(refinement errors?)
(constrained transport)
MHD: (Errors)
SPH Moving Mesh
AMR
Fixed Grid

Public version (in GADGET-2):

arXiv: 1206.5006

http://astrosim.net/code/doku.php?id=home:code:nbody:multipurpose

Not-so public version (GADGET-3): contact me