Theory of the IMF at the Galaxy-Scale



with Eliot Quataert, Norm Murray, Lars Hernquist, Dusan Keres, Todd Thompson, Desika Narayanan, Dan Kasen, T. J. Cox, Chris Hayward, Kevin Bundy, & more

The Turbulent ISM IMPORTANT ON (ALMOST) ALL SCALES





- **Gravity**
- Turbulence
- Magnetic, Thermal, Cosmic Ray, Radiation Pressure
- Cooling (atomic, molecular, metal-line, free-free)
- Star & BH Formation/Growth
- "Feedback": Massive stars, SNe, BHs, external galaxies, etc.

Tuesday, December 25, 12

The ISM YET THERE IS SURPRISING REGULARITY



The ISM YET THERE IS SURPRISING REGULARITY



The ISM YET THERE IS SURPRISING REGULARITY



Extended Press-Schechter / Excursion-Set Formalism

- Press & Schechter '74:
 - r Fluctuations a Gaussian random field
 - Know linear power spectrum P(k~1/r): variance ~ k³ P(k)





Extended Press-Schechter / Excursion-Set Formalism

- Press & Schechter '74:
 - r Fluctuations a Gaussian random field
 - Know linear power spectrum P(k~1/r): variance ~ k³ P(k)

- "Count" mass above critical fluctuation: "Halos"
 - > Turnaround & gravitational collapse $ar{
 ho}(< R \sim 1/k) >
 ho_{
 m crit}$





Extended Press-Schechter / Excursion-Set Formalism

- Press & Schechter '74:
 - r Fluctuations a Gaussian random field
 - Know linear power spectrum P(k~1/r): variance ~ k³ P(k)

- "Count" mass above critical fluctuation: "Halos"
 - > Turnaround & gravitational collapse $ar{
 ho}(< R \sim 1/k) >
 ho_{
 m crit}$

 Generalize to conditional probabilities,
 N-point statistics, resolve "cloud in cloud" problem (e.g. Bond et al. 1991)





Turbulence BASIC EXPECTATIONS

 $(k E(k) \sim u_t(k)^2)$ Velocity: $E(k) \propto k^{-p}$

Turbulence BASIC EXPECTATIONS



Turbulence BASIC EXPECTATIONS



Super-Sonic Turbulence BASIC EXPECTATIONS

$$dp(\ln \rho | R) = \frac{1}{\sqrt{2\pi S(R)}} \exp\left[\frac{-(\ln \rho - \langle \ln \rho \rangle)^2}{2 S(R)}\right]$$

Super-Sonic Turbulence BASIC EXPECTATIONS



Super-Sonic Turbulence BASIC EXPECTATIONS

$$dp(\ln \rho | R) = \frac{1}{\sqrt{2\pi S(R)}} \exp\left[\frac{-(\ln \rho - \langle \ln \rho \rangle)^2}{2 S(R)}\right]$$

$$S_k = \ln\left[1 + \alpha \mathcal{M}(k)^2\right]$$

$$Lemaster & Stone 2009$$

$$1 \qquad 2 \qquad 3$$

$$\ln(1 + 0.5 \text{ Mach}^2)$$

$$S(R) = \int d\ln k S_k |W(k, R)|^2$$

$$\omega^2 = \kappa^2 + c_s^2 k^2 + u_t(k)^2 k^2 - \frac{4\pi G \rho |k|h}{1 + |k|h}$$

Chandrasekhar '51, Vandervoort '70, Toomre '77

$$\omega^2 = \kappa^2 + c_s^2 \, k^2 + u_t(k)^2 \, k^2 - \frac{4\pi \, G \, \rho \, |k| h}{1 + |k| h}$$
 Angular Momentum

 $\kappa \sim \frac{V_{\rm disk}}{R_{\rm disk}}$

Chandrasekhar '51, Vandervoort '70, Toomre '77



Chandrasekhar '51, Vandervoort '70, Toomre '77



Chandrasekhar '51, Vandervoort '70, Toomre '77



Chandrasekhar '51, Vandervoort '70, Toomre '77



Mode Grows (Collapses) when w<0:

$$\rho > \rho_c(k) = \rho_0 \left(1 + |kh| \right) \left[\left(\mathcal{M}_h^{-2} + |kh|^{1-p} \right) kh + \frac{2}{|kh|} \right]$$

Chandrasekhar '51, Vandervoort '70, Toomre '77

"Counting" Collapsing Objects **EVALUATE DENSITY FIELD vs. "BARRIER"** Averaging Scale R [pc] 1000 100 0.1 10 15 10 Log[Density / Mean 5 0 -5 0.01 10 100 1000 0.1 lkhl

PFH 2011





PFH 2011



PFH 2011













Evolve the Fluctuations in Time CONSTRUCT "MERGER/FRAGMENTATION" TREES

$$p(\delta \mid \tau) = \frac{1}{\sqrt{2\pi S \left(1 - \exp\left[-2\tau\right]\right)}} \exp\left[-\frac{(\delta - \delta(t = 0) \exp\left[-\tau\right])^2}{2 S \left(1 - \exp\left[-2\tau\right]\right)}\right]$$

Evolve the Fluctuations in Time CONSTRUCT "MERGER/FRAGMENTATION" TREES



The "First Crossing" Mass Function VS GIANT MOLECULAR CLOUDS



The "First Crossing" Mass Function **VS GIANT MOLECULAR CLOUDS**

 $r_{
m sonic} \ll r \ll h$ $S(r) \sim S_0$





 $r_{
m sonic} \ll r \ll h$ $S(r) \sim S_0$

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto M^{-\alpha} \, e^{-(M/M_J)^{\beta}}$$






Tuesday, December 25, 12



Tuesday, December 25, 12





For $r \ll \ell_{\rm GMC} \ll h$ this becomes Hennebelle-Chabrier theory:

PFH 2012

$$\mathcal{N}(\tilde{M}) = 2\mathcal{N}_{0} \frac{1}{\tilde{R}^{3}} \frac{1}{1 + (2\eta + 1)\mathcal{M}_{*}^{2}\tilde{R}^{2\eta}} \frac{1 + (1 - \eta)\mathcal{M}_{*}^{2}\tilde{R}^{2\eta}}{\left(1 + \mathcal{M}_{*}^{2}\tilde{R}^{2\eta}\right)^{3/2}} \\ \times \exp\left\{-\frac{\left[\ln(\tilde{M}/\tilde{R}^{3})\right]^{2}}{2\sigma^{2}}\right\} \frac{\exp(-\sigma^{2}/8)}{\sqrt{2\pi}\sigma},$$

For $r \ll \ell_{GMC} \ll h$ this becomes Hennebelle-Chabrier theory:

$$\mathcal{N}(\tilde{M}) = 2\mathcal{N}_{0} \frac{1}{\tilde{R}^{3}} \frac{1}{1 + (2\eta + 1)\mathcal{M}_{*}^{2}\tilde{R}^{2\eta}} \frac{1 + (1 - \eta)\mathcal{M}_{*}^{2}\tilde{R}^{2\eta}}{\left(1 + \mathcal{M}_{*}^{2}\tilde{R}^{2\eta}\right)^{3/2}} \\ \times \exp\left\{-\frac{\left[\ln(\tilde{M}/\tilde{R}^{3})\right]^{2}}{2\sigma^{2}}\right\} \frac{\exp(-\sigma^{2}/8)}{\sqrt{2\pi}\sigma},$$

...*BUT*,

For $r \ll \ell_{GMC} \ll h$ this becomes Hennebelle-Chabrier theory: ind have not $\mathcal{N}(\tilde{M}) = 2\mathcal{N}_0 \frac{1}{\tilde{R}^3} \frac{1}{1 + (2\eta + 1)\mathcal{M}_*^2 \tilde{R}^{2\eta}} \frac{1 + (1 - \eta)\mathcal{M}_*^2 \tilde{R}^{2\eta}}{\left(1 + \mathcal{M}_*^2 \tilde{R}^{2\eta}\right)^{3/2}}$ 1.5 M_m/M_m+115 M___/M__=-295 w__/W_=51% N., 424 1.0 0.5 0.0 $\times \exp\left\{-\frac{\left[\ln\left(\tilde{M}/\tilde{R}^{3}\right)\right]^{2}}{2\sigma^{2}}\right\}\frac{\exp(-\sigma^{2}/8)}{\sqrt{2\pi}\sigma},$ -0.5 n. - 4.3+10% 4-1.3x10'm -4.3+10'04 1.5 A. Marth M_-305 1.0 0.5 ... **BUT**, 0.0 -0.5n,=4.3x10/cm n=4.3x10⁴cm a. a.d. 3a10³cm 1.5 N___/N___-875 M___/M__-29% w___/W___=+19% N...+30 - 410 z 1.0 0.5 Padoan & Nordlund L_c=1 pc 0.0 _ L_=10 pc -0.5 - Lo=100 pc n.=4.3.10 km n.+4.3+10 lon/ 4,=4.3x10⁴0% 0 Log[M / M_⊗] 0 1 1000 1.5 M___/M__=7% M___/M__=258 w___/W___+46% 1.0 ĝ 0.5 ω N (m^{-3/(4-\$)} Log[dN/dlogM] [M_{sac}/M_{sock}] 100 0.0 -0.5 n=4.3+10/cm n,=4.3x10/cm -2 -1 0 1 10910 M [Ma] 1.5 N___/N__-8% M_1/M_2=27% N-#117 N_+295 1.0 10 Jappsen -1 0.5 CMF: 0.0 Predicted -3 -2 -1 0 1 -3 -2 -1 0 2 Ophiuchus 10.00 100.00 0.01 0.10 1.00 log₁₀ M [N₀] 10910 M [Ma] Ophiuchus (Enoch) m [m_o] Perseus Taurus **Bate & Bonnell 2005 (Accretion-Ejection)** -2 0 -1 Larson 1992 (Fractal collapse) Log[M / Macoic] **Elmegreen 1997 (Fractal GMCs)** Padoan & Nordlund (Turb. Frag.) Hennebelle & Chabrier (Press-Schechter) Veltchev Veltchev+ 2011 (Clump mass-density + turb + accretion)

PFH 2012

Tuesday, December 25, 12

"Void" Abundance VS HI "HOLES" IN THE ISM



Tuesday, December 25, 12

Structural Properties of "Clouds" LARSON'S LAWS EMERGE NATURALLY



Tuesday, December 25, 12

Structural Properties of "Clouds" LARSON'S LAWS EMERGE NATURALLY



Tuesday, December 25, 12









Clustering of Stars: Predicted vs. Observations PREDICT N-POINT CORRELATION FUNCTIONS





Tuesday, December 25, 12

General, Flexible Theory: EXTREMELY ADAPTABLE TO MOST CHOICES

- Complicated, multivariable gas equations of state
- Accretion
- Magnetic Fields
- Time-Dependent Background Evolution/Collapse
- Intermittency
- Correlated, multi-scale driving



What Can We Say About Galactic-Scale IMF Variation?



Most theories predict IMF *locally*:





Most theories predict IMF *locally*:



Larson 2005: It's all Jeans mass

Hethew Bate Unversity of Exeter

Most theories predict IMF *locally*:



Larson 2005: It's all Jeans mass

Dave 2008: Extrapolate to galaxies....



Most theories predict IMF *locally*:



Larson 2005: It's all Jeans mass

Dave 2008: Extrapolate to galaxies....

 $M_{IMF} \sim T_{min}^{1.7-2.3}$



Most theories predict IMF *locally*:



Larson 2005: It's all Jeans mass

Dave 2008: Extrapolate to galaxies....

- $M_{\rm IMF} \sim T_{\rm min}^{1.7-2.3}$
- > T_{CMB} is not interesting at z<6

Hatthew Bate University of Exeter

10

Most theories predict IMF *locally*:



Larson 2005: It's all Jeans mass

Dave 2008: Extrapolate to galaxies....

- $M_{IMF} \sim T_{min}^{1.7-2.3}$
- > T_{CMB} is not interesting at z<6







Narayanan 2012: estimate "mean" thermal state of clouds

High-z: Higher SFR, more heating (CRs & photons)



- High-z: Higher SFR, more heating (CRs & photons)
 - Also lower-metallicity: less cooling (Marks et al., others)



- High-z: Higher SFR, more heating (CRs & photons)
 - Also lower-metallicity: less cooling (Marks et al., others)
- Mergers (bulge-makers) tend to higher T_{min}



- High-z: Higher SFR, more heating (CRs & photons)
 - Also lower-metallicity: less cooling (Marks et al., others)
- Mergers (bulge-makers) tend to higher T_{min}
 - Observed (Downes & Solomon, Bryant & Scoville)





Kroupa, Weidner & Kroupa, et al. :



Kroupa, Weidner & Kroupa, et al.:

Maximum M_{star} in cluster scales with M_{cluster}



Kroupa, Weidner & Kroupa, et al.:

Maximum M_{star} in cluster scales with M_{cluster}



IF physical (not sampling), then 'IGIMF' depends on cluster MF:

Kroupa, Weidner & Kroupa, et al.:

Maximum M_{star} in cluster scales with M_{cluster}



IF physical (not sampling), then 'IGIMF' depends on cluster MF:
Maximum M_{cluster} hence "top-heaviness" increases with:

Kroupa, Weidner & Kroupa, et al.:

Maximum M_{star} in cluster scales with M_{cluster}



- **IF** physical (not sampling), then 'IGIMF' depends on cluster MF:
 - Maximum M_{cluster} hence "top-heaviness" increases with:
 - Increasing SFR (higher-z, merger/starbursts) -- may itself be sampling!
Variation in the Mass Function & the Jeans Mass

Kroupa, Weidner & Kroupa, et al.:

Maximum M_{star} in cluster scales with M_{cluster}



- **IF** physical (not sampling), then 'IGIMF' depends on cluster MF:
 - Maximum M_{cluster} hence "top-heaviness" increases with:
 - Increasing SFR (higher-z, merger/starbursts) -- may itself be sampling!
 - Lower metallicity? (less clear)

Variation in the Mass Function & the Jeans Mass

Kroupa, Weidner & Kroupa, et al.:

Maximum M_{star} in cluster scales with M_{cluster}



- **IF** physical (not sampling), then 'IGIMF' depends on cluster MF:
 - Maximum M_{cluster} hence "top-heaviness" increases with:
 - Increasing SFR (higher-z, merger/starbursts) -- may itself be sampling!
 - Lower metallicity? (less clear)
 - Similar to "mean galaxy Jeans mass" case





Jeans Length & Mass:
$$\ell_{\text{Jeans}} \sim \frac{c_s}{\sqrt{G \rho}}$$
 $M_{\text{Jeans}} \sim \frac{c_s^3}{G^{3/2} \rho^{1/2}}$

PFH 2012

Jeans Length & Mass:
$$\ell_{\text{Jeans}} \sim \frac{c_s}{\sqrt{G \rho}}$$
 $M_{\text{Jeans}} \sim \frac{c_s^3}{G^{3/2} \rho^{1/2}}$

Sonic Length & Mass: $R_{\text{Sonic}} \sim R(\mathcal{M} = 1) \sim h_{\text{disk}} \mathcal{M}(h_{\text{disk}})^{-2}$

PFH 2012

2

Jeans Length & Mass:
$$\ell_{\text{Jeans}} \sim \frac{c_s}{\sqrt{G \rho}}$$
 $M_{\text{Jeans}} \sim \frac{c_s^3}{G^{3/2} \rho^{1/2}}$

Sonic Length & Mass: $R_{\text{Sonic}} \sim R(\mathcal{M} = 1) \sim h_{\text{disk}} \mathcal{M}(h_{\text{disk}})^{-2}$

$$M_{\rm sonic} \sim \frac{c_s^2 R_{\rm sonic}}{G} \sim \frac{c_s^4}{G^2 Q_{\rm disk} \Sigma_{\rm disk}}$$

2











BUT, What About Starbursts?

MW: $T_{\text{cold}} \sim 10 \, K$ $\sigma_{\text{gas}} \sim 10 \, \text{km s}^{-1}$ $(Q \sim 1 \text{ for } \Sigma_{\text{gas}} \sim 10 \, M_{\odot} \, \text{pc}^{-2})$



BUT, What About Starbursts?

MW: $T_{\text{cold}} \sim 10 \, K$ $\sigma_{\text{gas}} \sim 10 \, \text{km s}^{-1}$ $(Q \sim 1 \text{ for } \Sigma_{\text{gas}} \sim 10 \, M_{\odot} \, \text{pc}^{-2})$

ULIRG: $T_{\text{cold}} \sim 70 \, K$ $\sigma_{\text{gas}} \sim 80 \, \text{km s}^{-1}$ $(Q \sim 1 \text{ for } \Sigma_{\text{gas}} \sim 1000 \, M_{\odot} \, \text{pc}^{-2})$









BUT, What About Starbursts? BOTTOM-HEAVY: TURBULENCE WINS!



BUT, What About Starbursts? BOTTOM-HEAVY: TURBULENCE WINS!





We can make some guesses for other galaxies... HOWEVER, CAUTION IS NEEDED!



We can make some guesses for other galaxies...

HOWEVER, CAUTION IS NEEDED!



1. What Maintains the Turbulence?

1. What Maintains the Turbulence?



1. What Maintains the Turbulence?

Efficient Cooling: $\dot{P}_{\rm diss} \sim \frac{M_{\rm gas} v_{\rm turb}}{t_{\rm crossing}}$

2. Why Doesn't Everything Collapse?

1. What Maintains the Turbulence?

Efficient Cooling: $\dot{P}_{\rm diss} \sim \frac{M_{\rm gas} v_{\rm turb}}{t_{\rm crossing}}$

2. Why Doesn't Everything Collapse?

"Top-down" turbulence can't stop collapse once self-gravitating

Fast Cooling:
$$\dot{M}_* \sim \frac{M_{\rm gas}}{t_{\rm freefall}}$$

Summary:

* ISM *statistics* are far more fundamental than we typically assume *

Turbulence + Gravity: ISM structure follows

- Lognormal density PDF is not critical
- > ANALYTICALLY understand:
 - GMC Mass Function & Structure ("first crossing")
 - Core MF ("last crossing") & Linewidth-Size-Mass
 - Clustering of Stars (correlation functions)

Feedback Regulates & Sets Efficiencies of Star Formation

- K-S Law: 'enough' stars to offset dissipation (set by gravity)
 - Independent of small-scale star formation physics (how stars form)