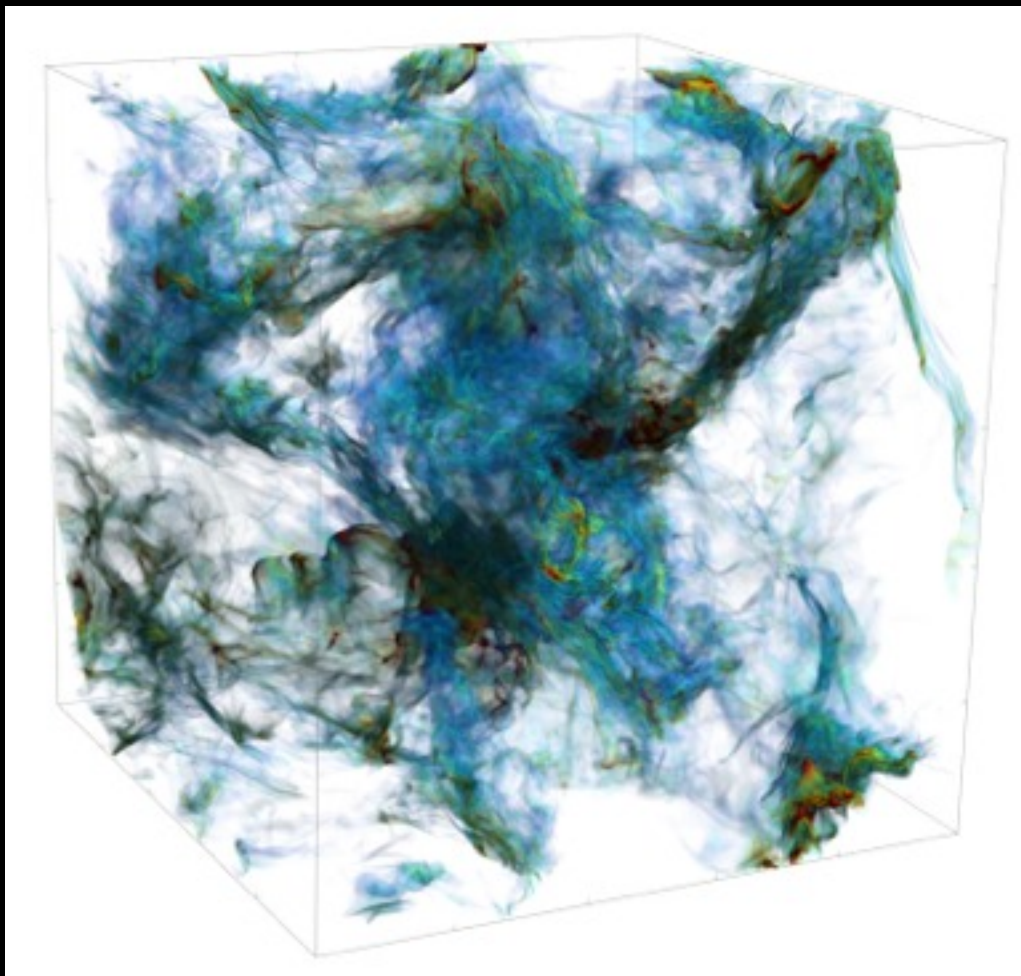
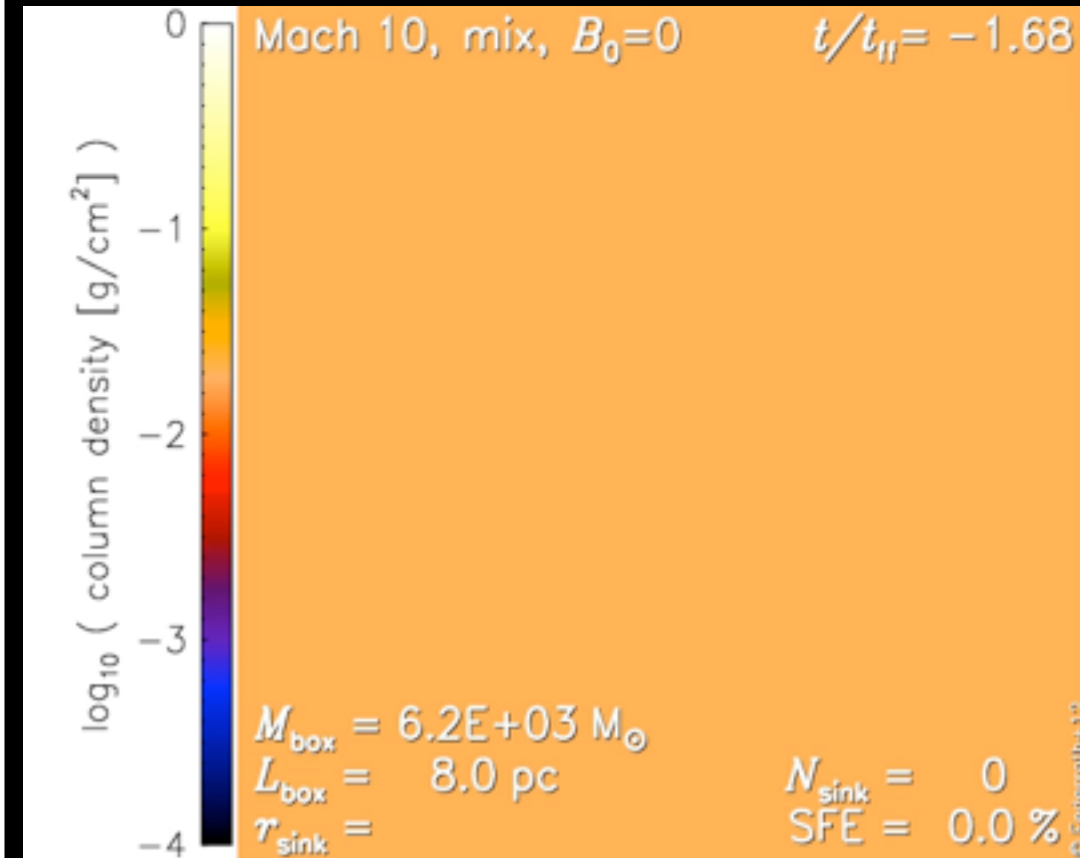
A New Approach to Turbulence:

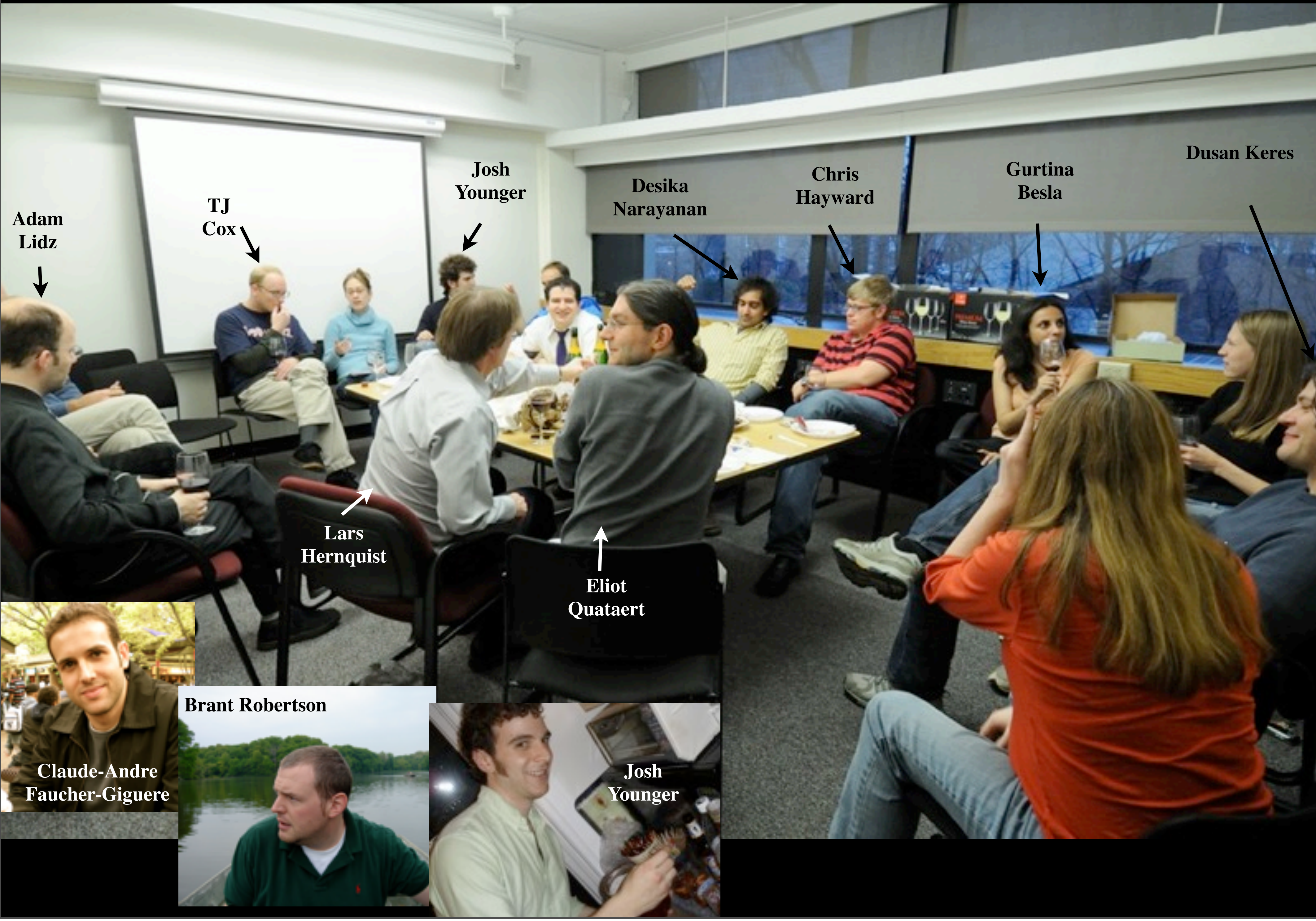
Origins of ISM Structure, Stellar Clustering & the IMF, and (perhaps?) Planet Formation





Philip Hopkins (with Jessie Christiansen)

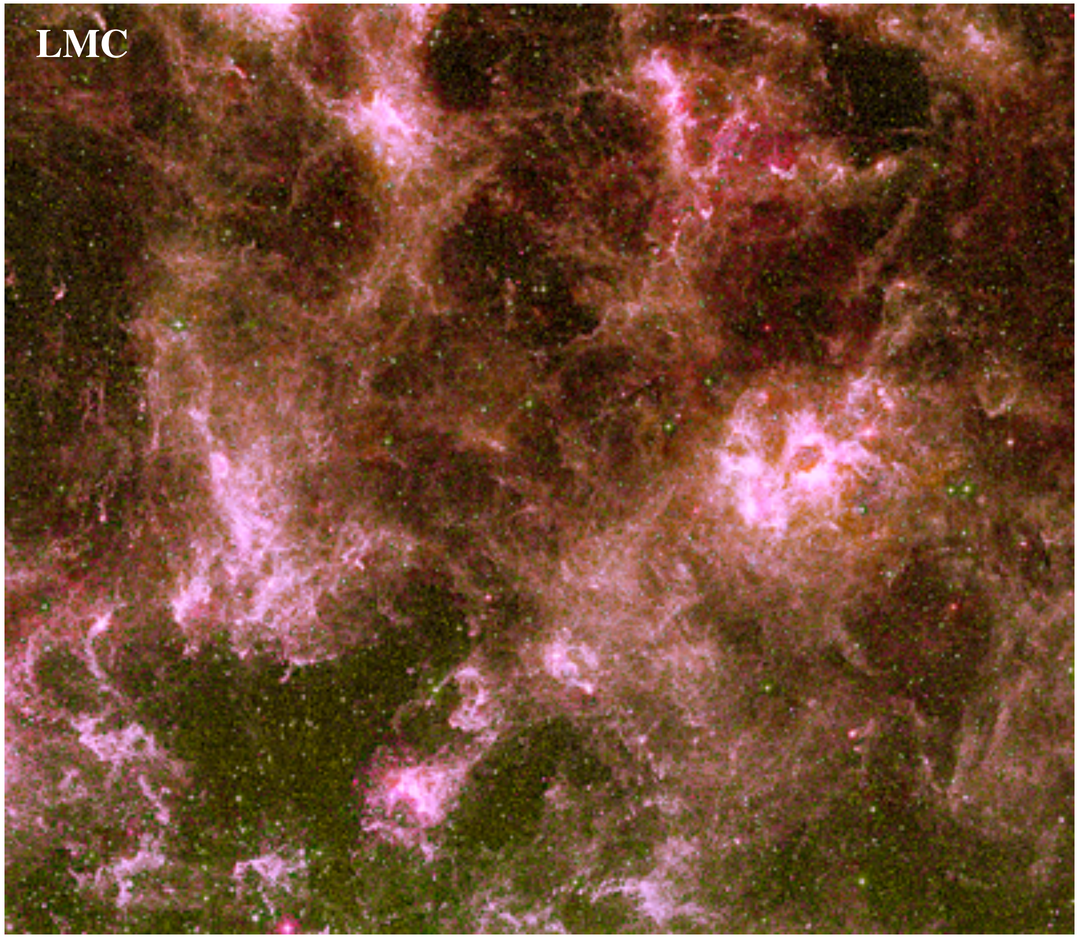
- currently UC Berkeley, soon to be CalTech -

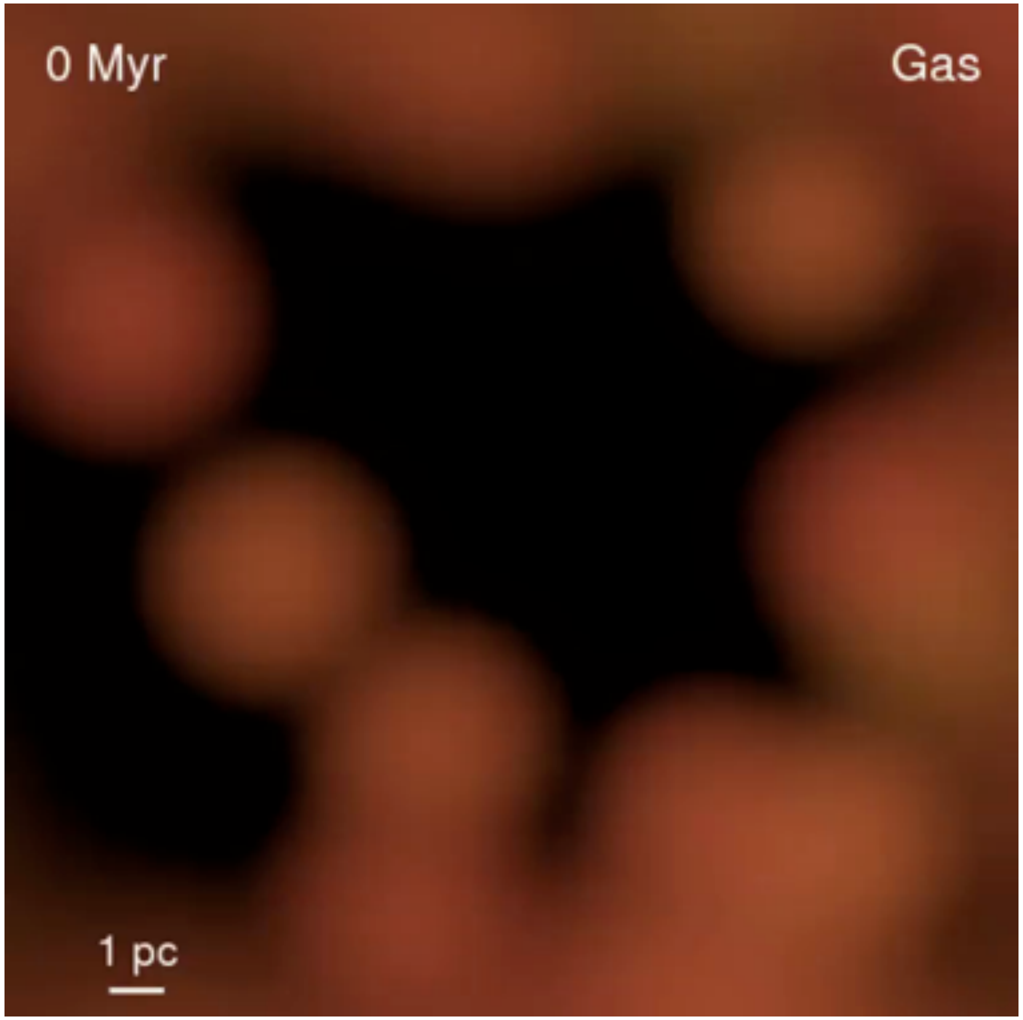

Adam Lidz
TJ Cox
Josh Younger
Desika Narayanan
Chris Hayward
Gurtina Besla
Dusan Keres
Lars Hernquist
Eliot Quataert
Claude-Andre Faucher-Giguere
Brant Robertson
Josh Younger

Jessie Christiansen

The Turbulent ISM

IMPORTANT ON
(ALMOST) ALL SCALES



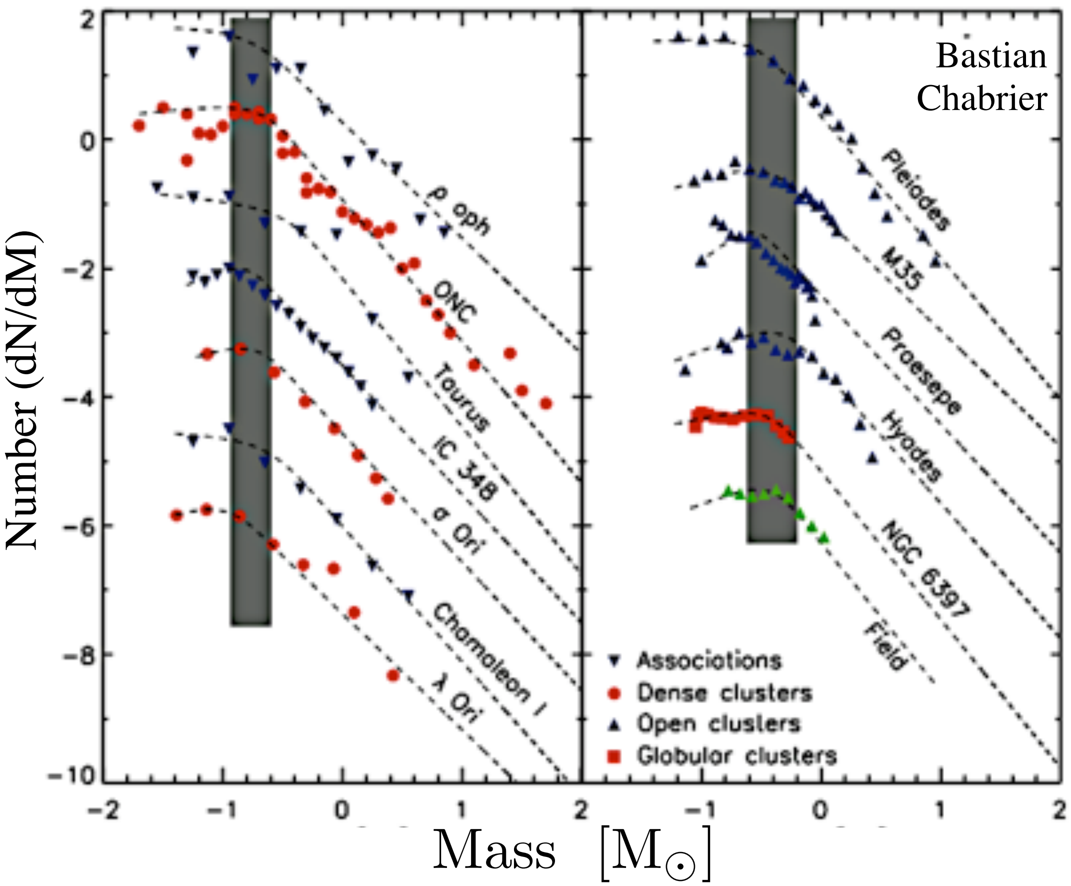


- Gravity
- Turbulence
- Magnetic, Thermal, Cosmic Ray, Radiation Pressure
- Cooling (atomic, molecular, metal-line, free-free)
- Star & BH Formation/Growth
- "Feedback": Massive stars, SNe, BHs, external galaxies, etc.

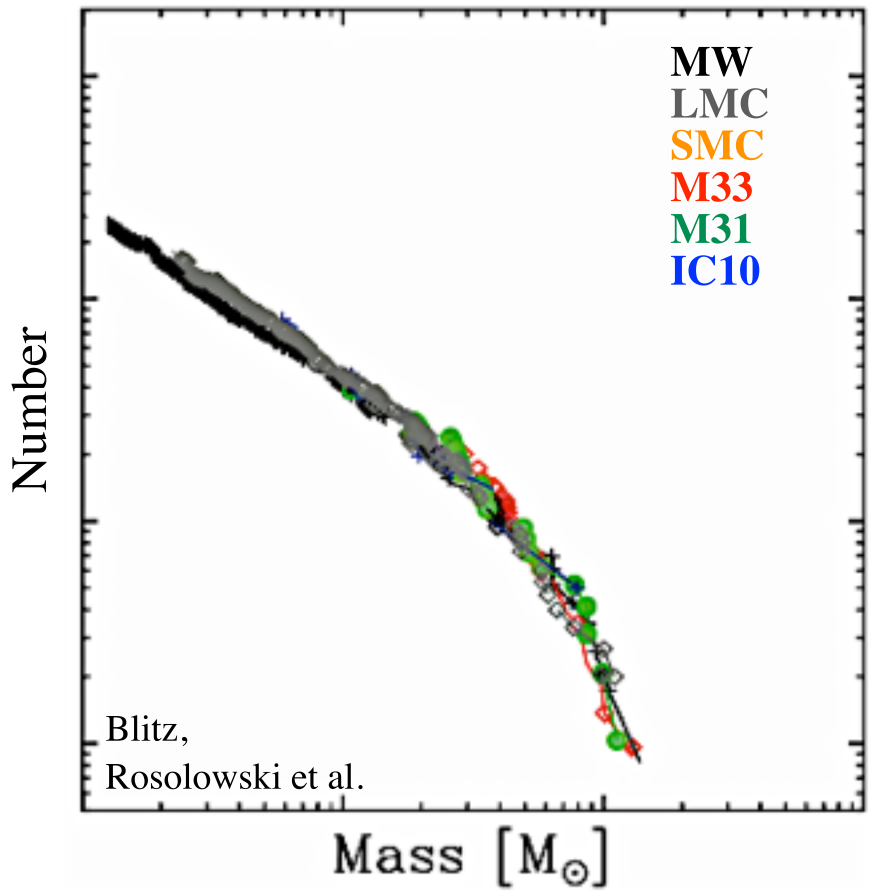
The ISM

YET THERE IS SURPRISING REGULARITY

Stars & Pre-Stellar Gas Cores:



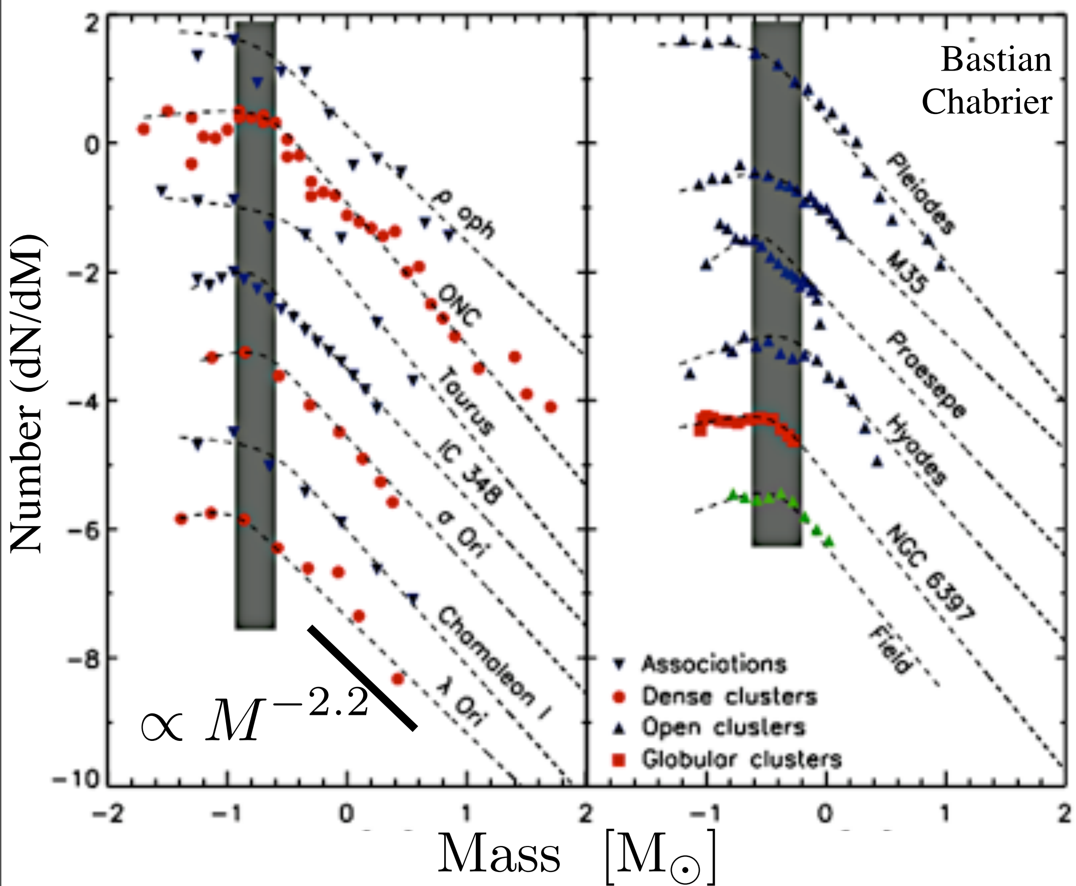
Giant Molecular Clouds:



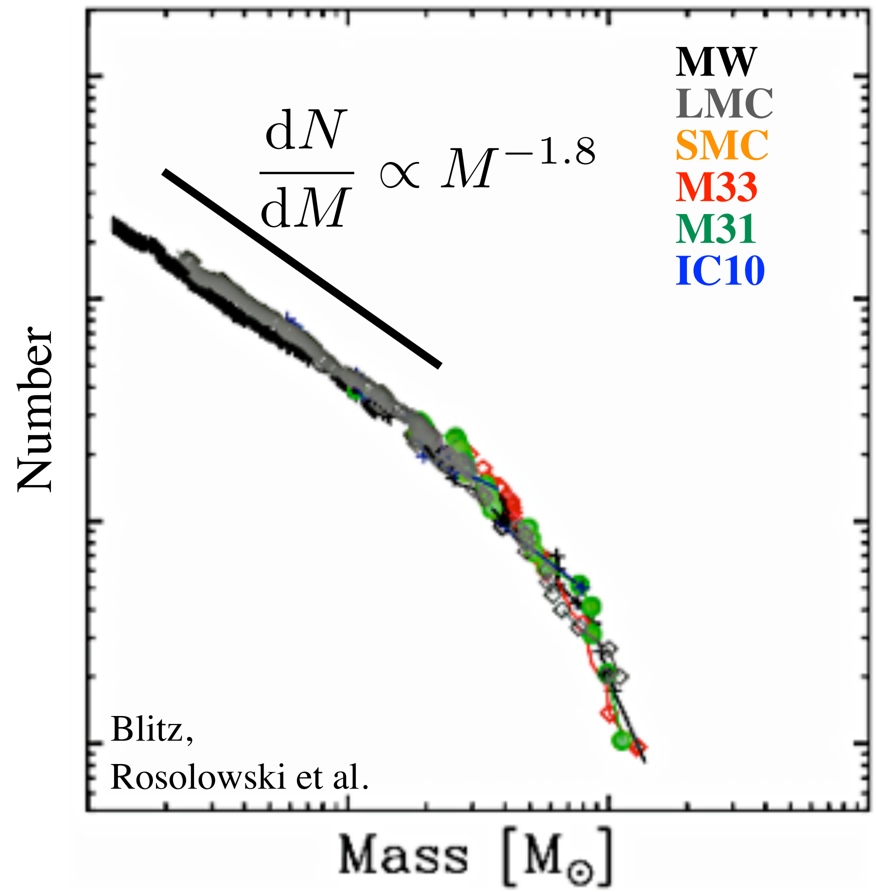
The ISM

YET THERE IS SURPRISING REGULARITY

Stars & Pre-Stellar Gas Cores:



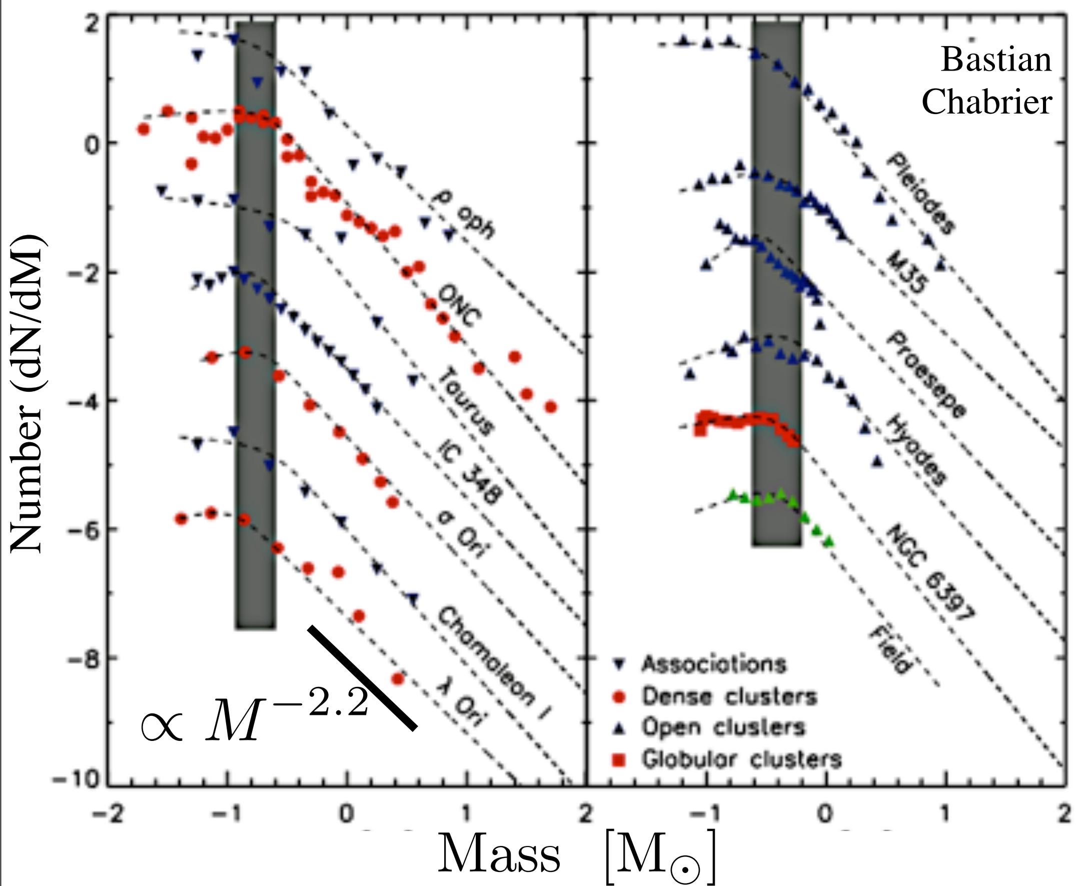
Giant Molecular Clouds:



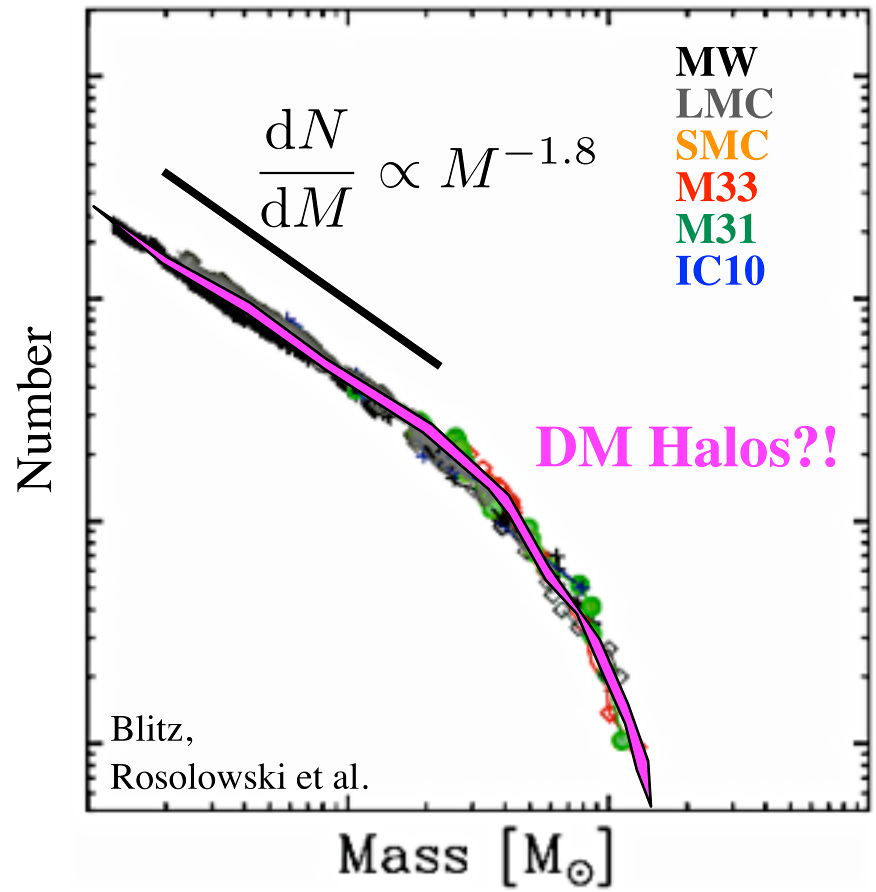
The ISM

YET THERE IS SURPRISING REGULARITY

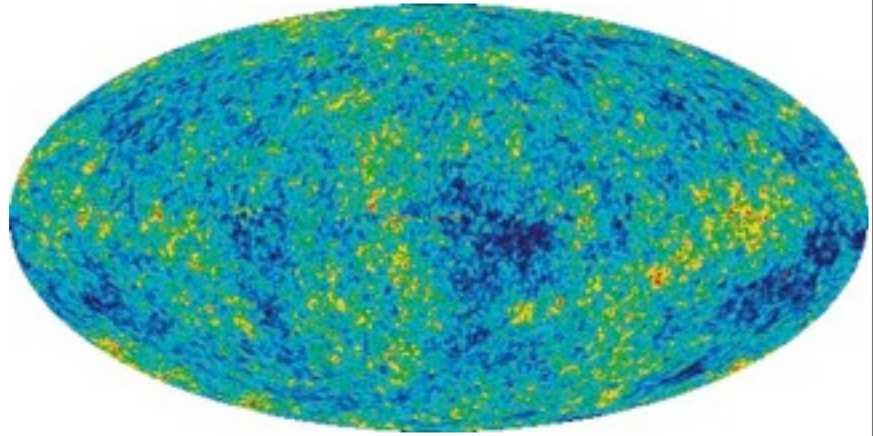
Stars & Pre-Stellar Gas Cores:



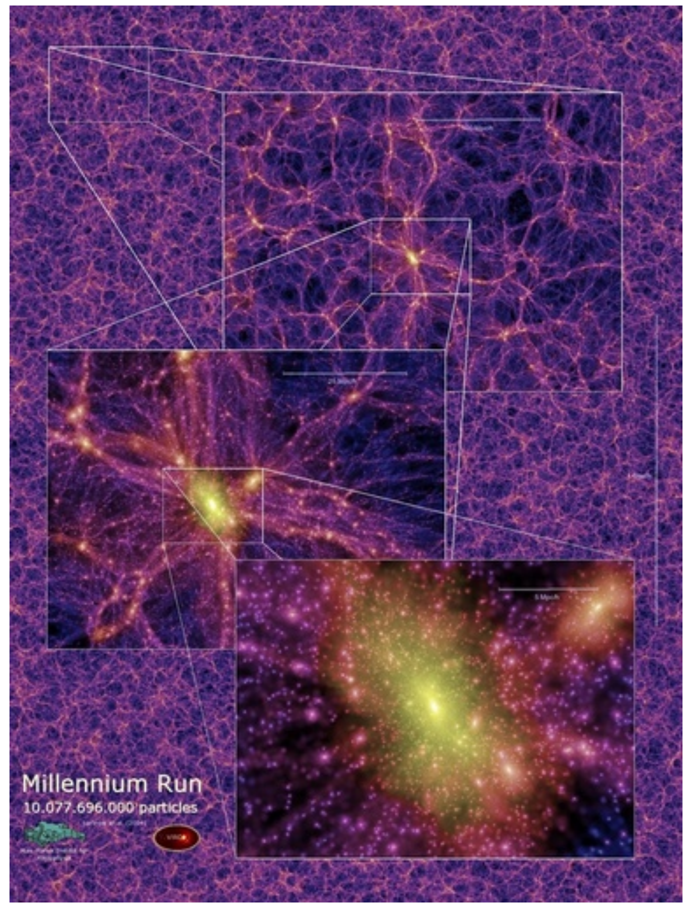
Giant Molecular Clouds:



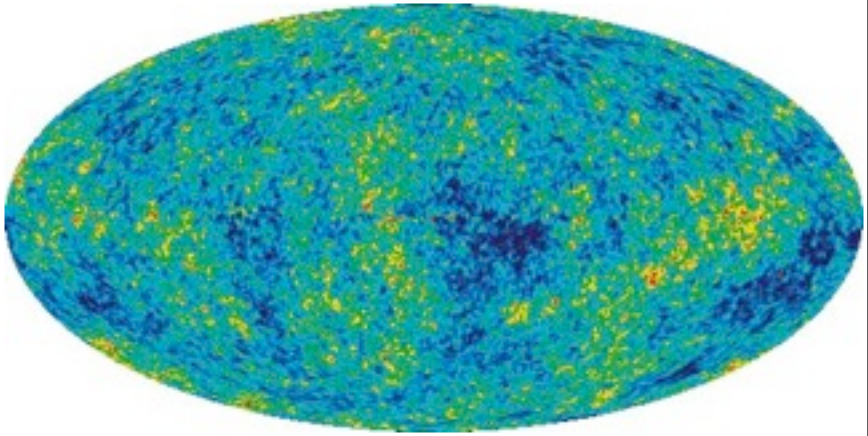
Extended Press-Schechter / Excursion-Set Formalism



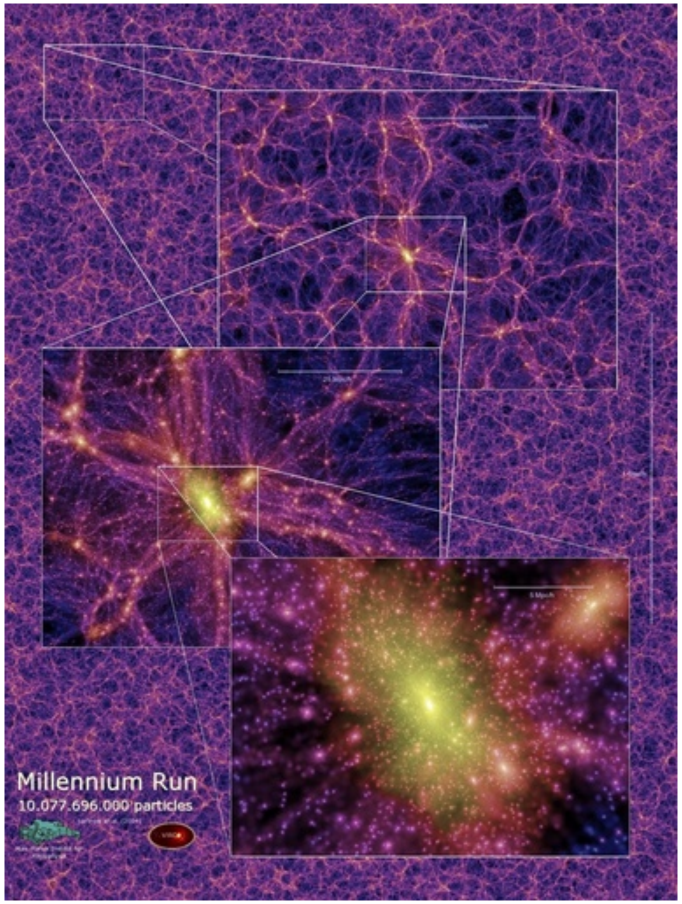
- Press & Schechter '74:
  - ρ Fluctuations a Gaussian random field
  - Know linear power spectrum P(k~1/r): variance $\sim k^3\,P(k)$



Extended Press-Schechter / Excursion-Set Formalism



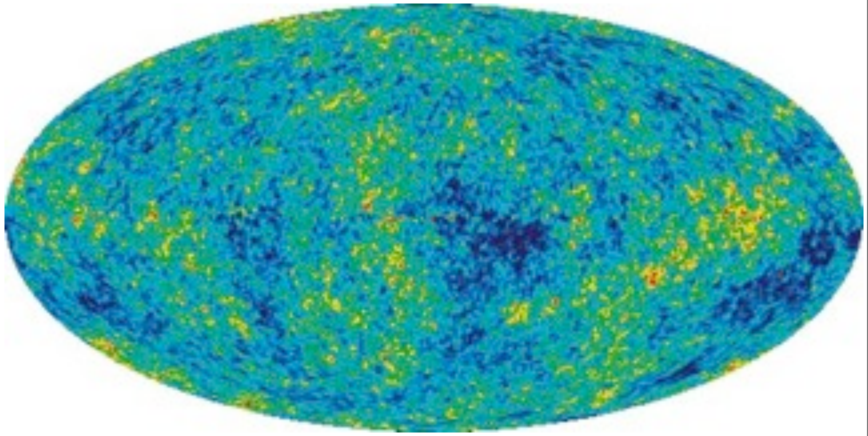
- Press & Schechter ‘74:
  - ρ Fluctuations a Gaussian random field
  - Know linear power spectrum P(k~1/r): variance ~ k^3 P(k)



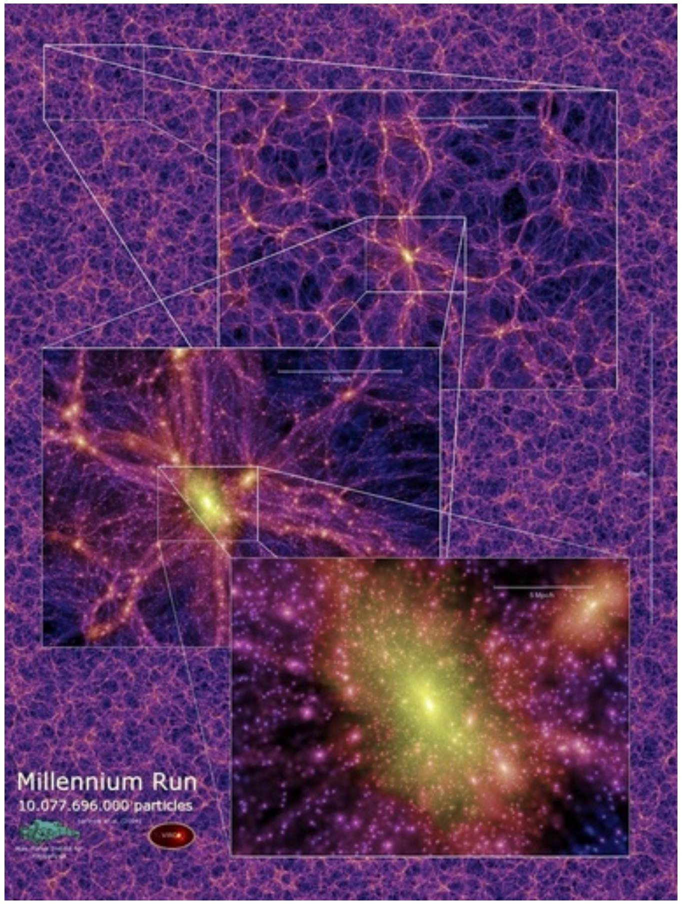
- “Count” mass above critical fluctuation: “Halos”
  - Turnaround & gravitational collapse

$$\bar{\rho}(< R \sim 1/k) > \rho_{\rm crit}$$

Extended Press-Schechter / Excursion-Set Formalism



- Press & Schechter '74:
  - ρ Fluctuations a Gaussian random field
  - Know linear power spectrum P(k~1/r):
    variance $\sim k^3$ P(k)



- "Count" mass above critical fluctuation: "Halos"
  - Turnaround & gravitational collapse

$$\bar{\rho}(< R \sim 1/k) > \rho_{\rm crit}$$

- Generalize to conditional probabilities,
  N-point statistics, resolve "cloud in cloud" problem
  (e.g. Bond et al. 1991)

Turbulence

BASIC EXPECTATIONS

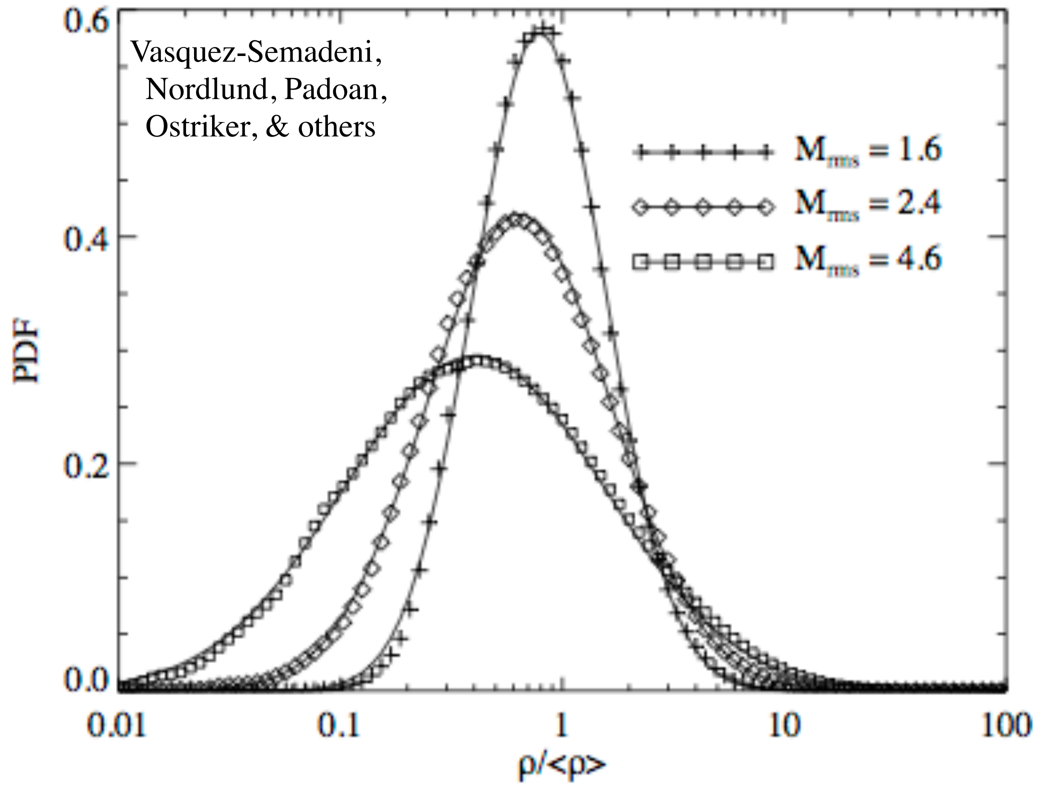
Velocity: $E(k) \propto k^{-p}$ $\quad (k\,E(k) \sim u_t(k)^2)$

Turbulence

BASIC EXPECTATIONS

Velocity: $$E(k) \propto k^{-p} \qquad (k\,E(k) \sim u_t(k)^2)$$

Density: $$\mathrm{d}p(\ln\rho\,|\,R) = \frac{1}{\sqrt{2\pi\,S(R)}}\,\exp\left[\frac{-(\ln\rho - \langle\ln\rho\rangle)^2}{2\,S(R)}\right]$$

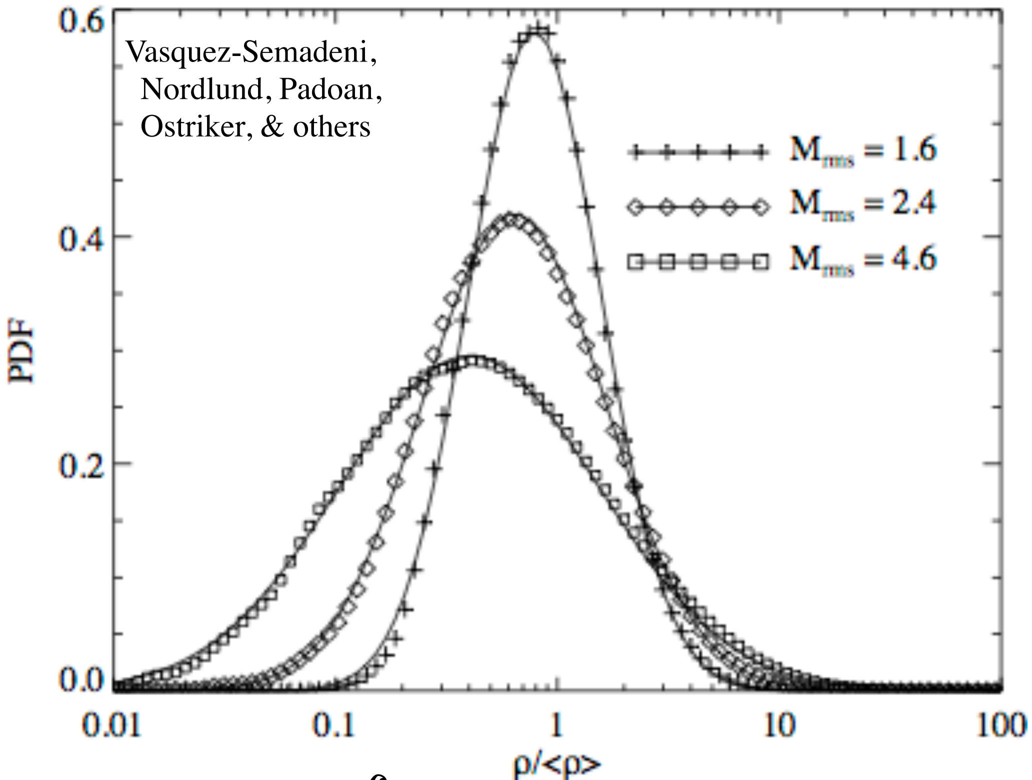


Turbulence

BASIC EXPECTATIONS

Velocity: $$E(k) \propto k^{-p} \qquad (k\,E(k) \sim u_t(k)^2)$$

Density: $$\mathrm{d}p(\ln\rho\,|\,R) = \frac{1}{\sqrt{2\pi\,S(R)}}\,\exp\left[\frac{-(\ln\rho - \langle\ln\rho\rangle)^2}{2\,S(R)}\right]$$



$$S(R) = \int \mathrm{d}\ln k\, S_k\, |W(k,\,R)|^2$$

Super-Sonic Turbulence

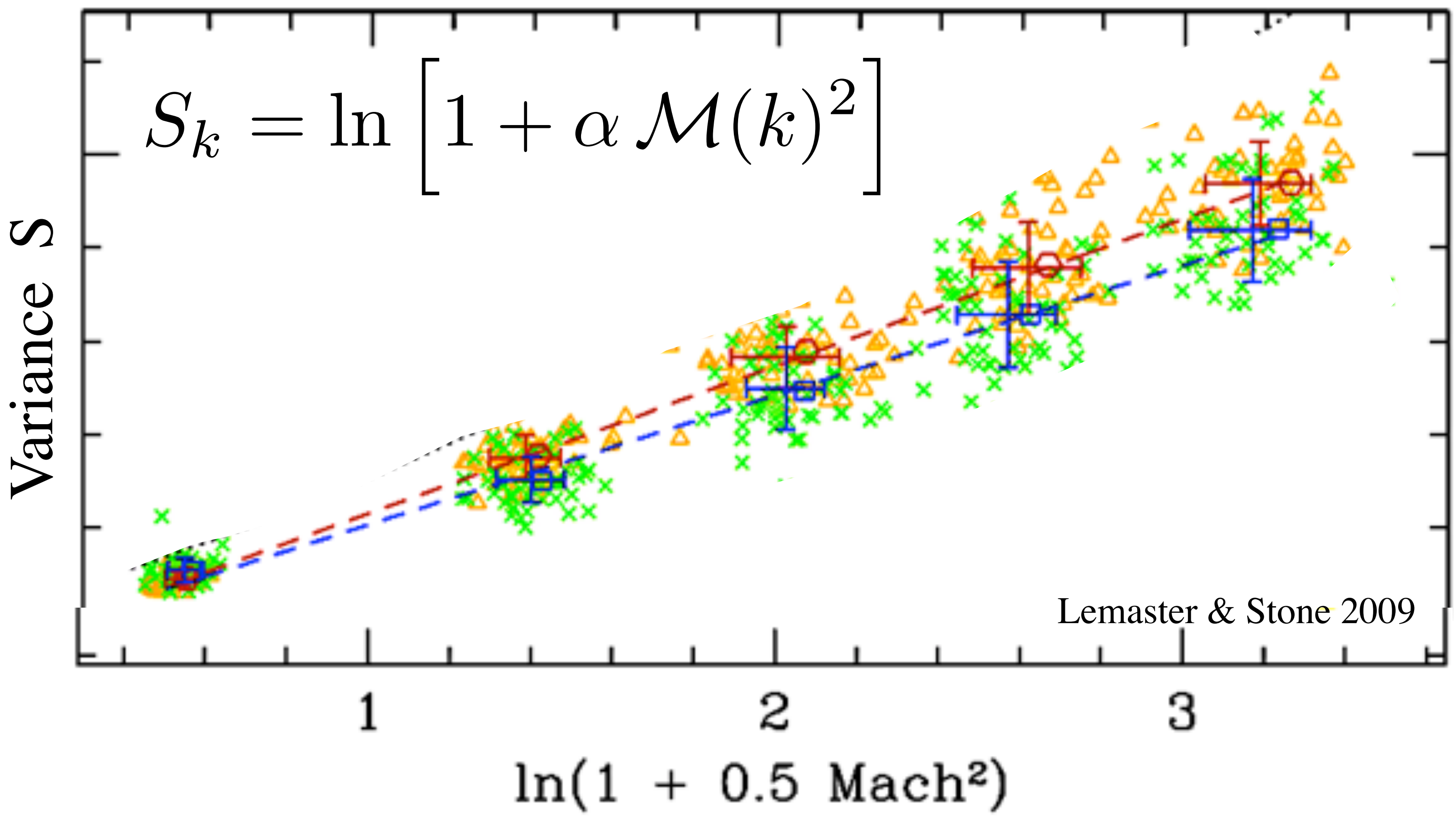
BASIC EXPECTATIONS

$$\mathrm{d}p(\ln\rho\,|\,R) = \frac{1}{\sqrt{2\pi\,S(R)}}\,\exp\left[\frac{-(\ln\rho - \langle\ln\rho\rangle)^2}{2\,S(R)}\right]$$

Super-Sonic Turbulence

BASIC EXPECTATIONS

$$\mathrm{d}p(\ln\rho\,|\,R) = \frac{1}{\sqrt{2\pi\,S(R)}}\,\exp\left[\frac{-(\ln\rho - \langle\ln\rho\rangle)^2}{2\,S(R)}\right]$$

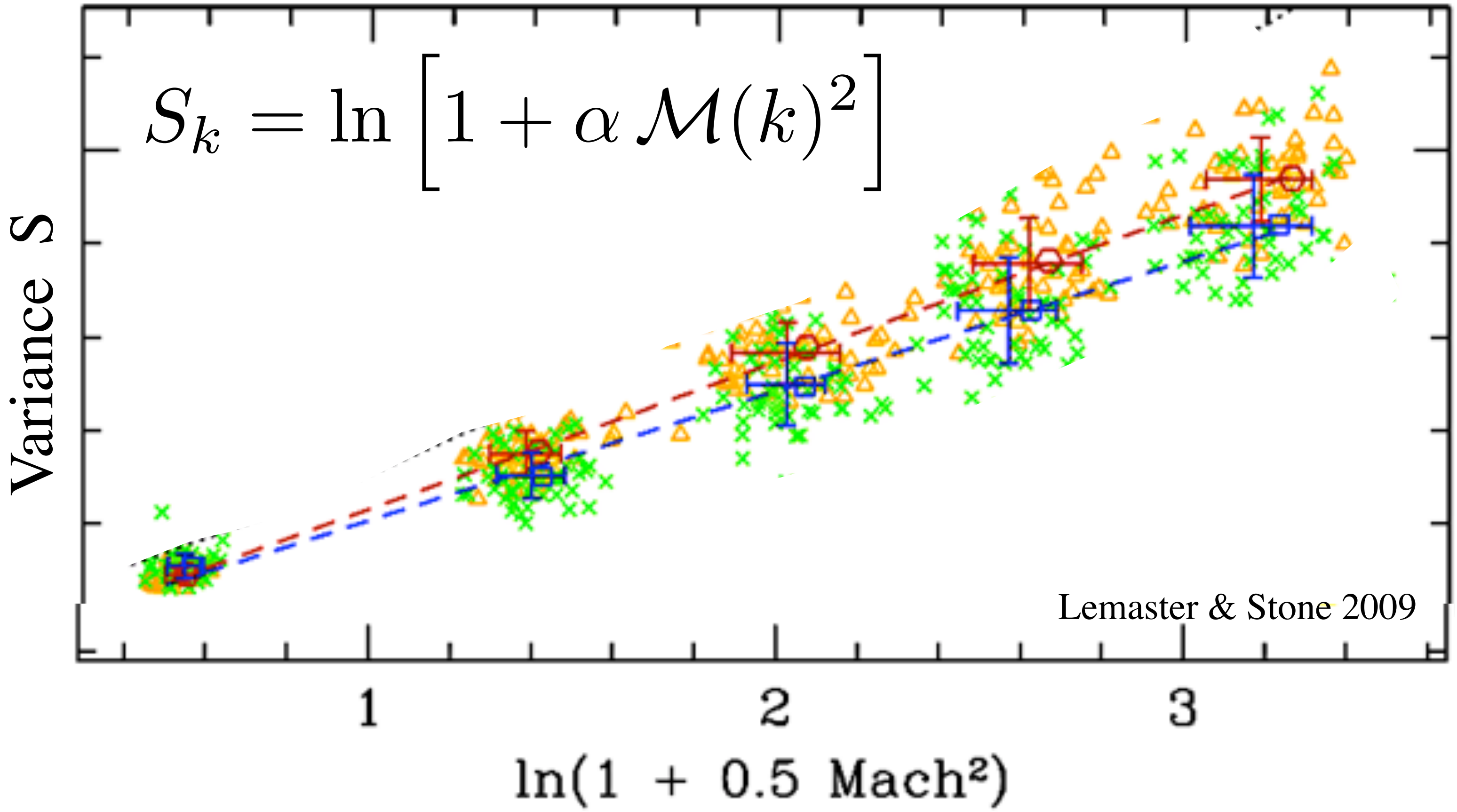


Super-Sonic Turbulence

BASIC EXPECTATIONS

$$\mathrm{d}p(\ln\rho\,|\,R) = \frac{1}{\sqrt{2\pi\,S(R)}}\,\exp\left[\frac{-(\ln\rho - \langle\ln\rho\rangle)^2}{2\,S(R)}\right]$$

$$S_k = \ln\left[1 + \alpha\,\mathcal{M}(k)^2\right]$$



$$S(R) = \int \mathrm{d}\ln k\, S_k\, |W(k,\,R)|^2$$

What Defines a Fluctuation of Interest?

DISPERSION RELATION:

$$\omega^2 = \kappa^2 + c_s^2\, k^2 + u_t(k)^2\, k^2 - \frac{4\pi\, G\, \rho\, |k| h}{1 + |k| h}$$

Chandrasekhar ‘51, Vandervoort ‘70, Toomre ‘77

What Defines a Fluctuation of Interest?

DISPERSION RELATION:

$$\omega^2 = \kappa^2 + c_s^2\, k^2 + u_t(k)^2\, k^2 - \frac{4\pi\, G\, \rho\, |k| h}{1 + |k| h}$$

Angular Momentum

$$\kappa \sim \frac{V_{\rm disk}}{R_{\rm disk}}$$

Chandrasekhar '51, Vandervoort '70, Toomre '77

What Defines a Fluctuation of Interest?

DISPERSION RELATION:

$$\omega^2 = \kappa^2 + c_s^2\, k^2 + u_t(k)^2\, k^2 - \frac{4\pi\, G\, \rho\, |k| h}{1 + |k| h}$$

Angular Momentum

$$\kappa \sim \frac{V_{\rm disk}}{R_{\rm disk}}$$

Thermal Pressure

$$\propto r^{-2}$$

Chandrasekhar ‘51, Vandervoort ‘70, Toomre ‘77

What Defines a Fluctuation of Interest?

DISPERSION RELATION:

$$\omega^2 = \kappa^2 + c_s^2\, k^2 + u_t(k)^2\, k^2 - \frac{4\pi\, G\, \rho\, |k| h}{1 + |k| h}$$

Angular Momentum

$$\kappa \sim \frac{V_{\rm disk}}{R_{\rm disk}}$$

Thermal Pressure

$$\propto r^{-2}$$

Turbulence

$$\propto r^{p-3} \sim r^{-1}$$

$$r > r_{\rm sonic} : \quad u_t^2 > c_s^2$$

Chandrasekhar '51, Vandervoort '70, Toomre '77

What Defines a Fluctuation of Interest?

DISPERSION RELATION:

$$\omega^2 = \kappa^2 + c_s^2\, k^2 + u_t(k)^2\, k^2 - \frac{4\pi\, G\, \rho\, |k| h}{1 + |k| h}$$

Angular Momentum

$$\kappa \sim \frac{V_{\rm disk}}{R_{\rm disk}}$$

Thermal Pressure

$$\propto r^{-2}$$

Turbulence

$$\propto r^{p-3} \sim r^{-1}$$

$$r > r_{\rm sonic}: \quad u_t^2 > c_s^2$$

Gravity

Chandrasekhar '51, Vandervoort '70, Toomre '77

What Defines a Fluctuation of Interest?

DISPERSION RELATION:

$$\omega^2 = \kappa^2 + c_s^2\,k^2 + u_t(k)^2\,k^2 - \frac{4\pi\,G\,\rho\,|k|h}{1+|k|h}$$

Angular Momentum

$$\kappa \sim \frac{V_{\rm disk}}{R_{\rm disk}}$$

Thermal Pressure

$$\propto r^{-2}$$

Turbulence

$$\propto r^{p-3} \sim r^{-1}$$

$$r > r_{\rm sonic}: \quad u_t^2 > c_s^2$$

Gravity

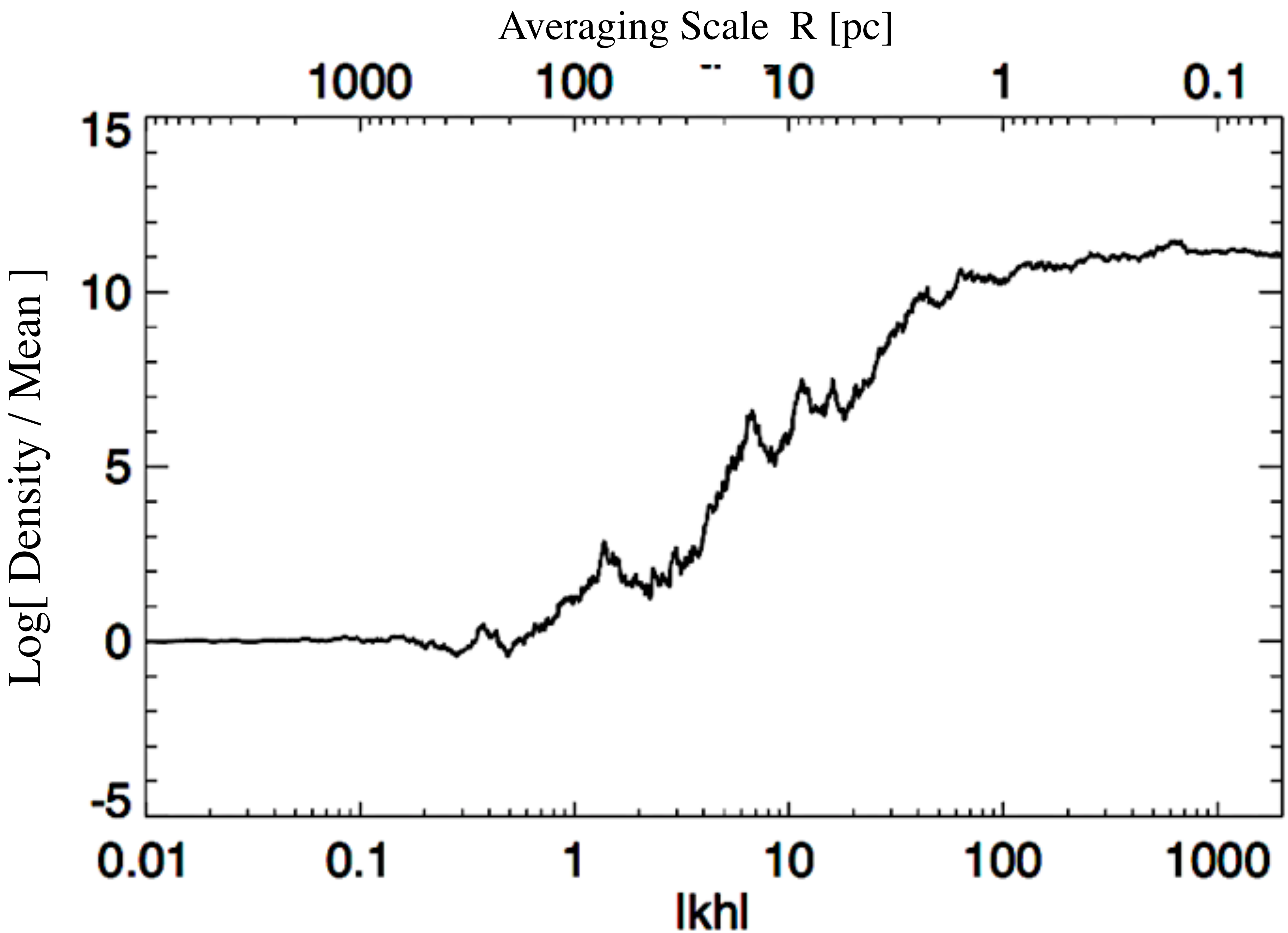
Mode Grows (Collapses) when $\omega < 0$:

$$\rho > \rho_c(k) = \rho_0\,(1+|kh|)\left[\left(\mathcal{M}_h^{-2} + |kh|^{1-p}\right)kh + \frac{2}{|kh|}\right]$$

Chandrasekhar ‘51, Vandervoort ‘70, Toomre ‘77

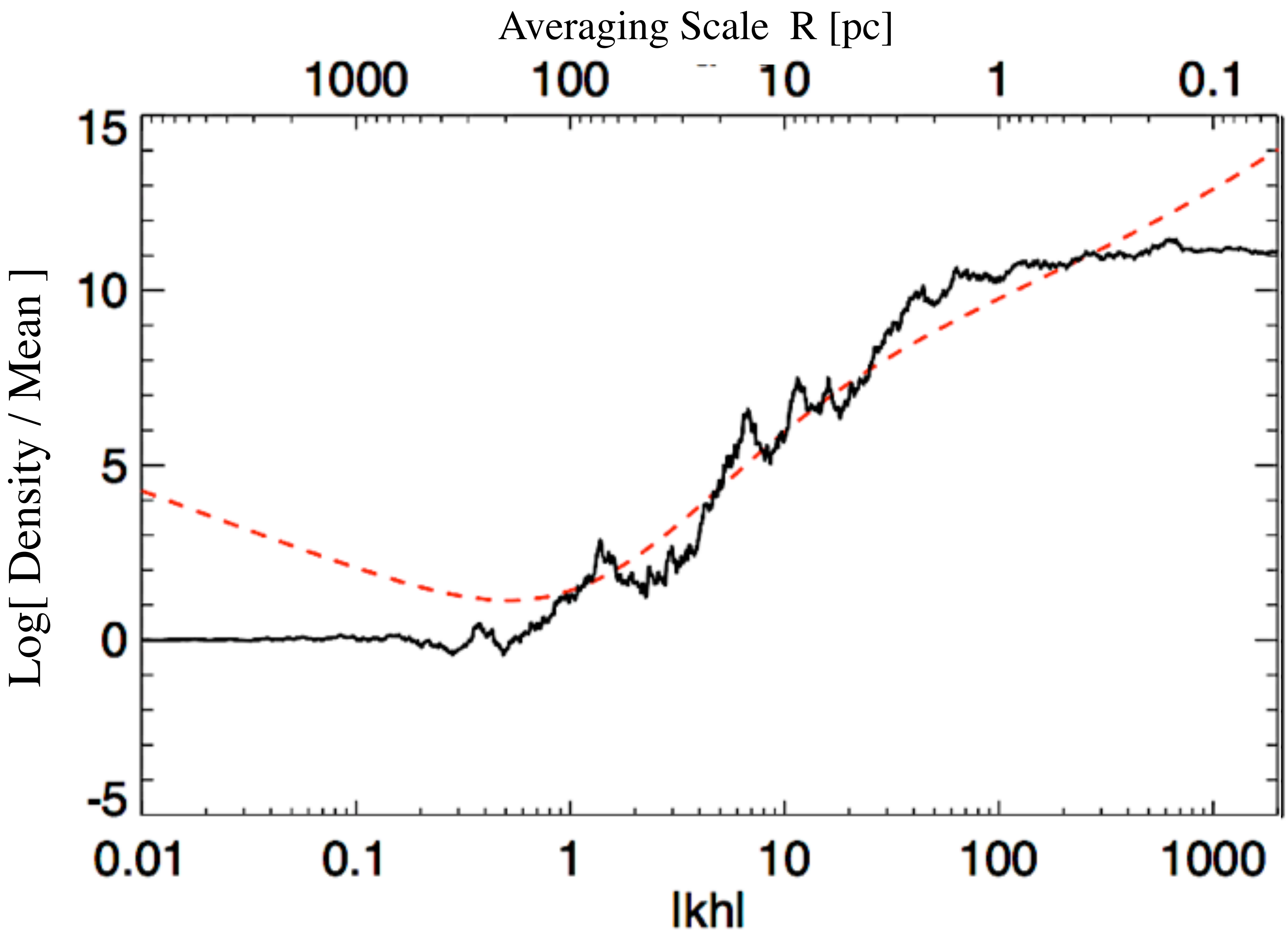
“Counting” Collapsing Objects

EVALUATE DENSITY FIELD vs. “BARRIER”



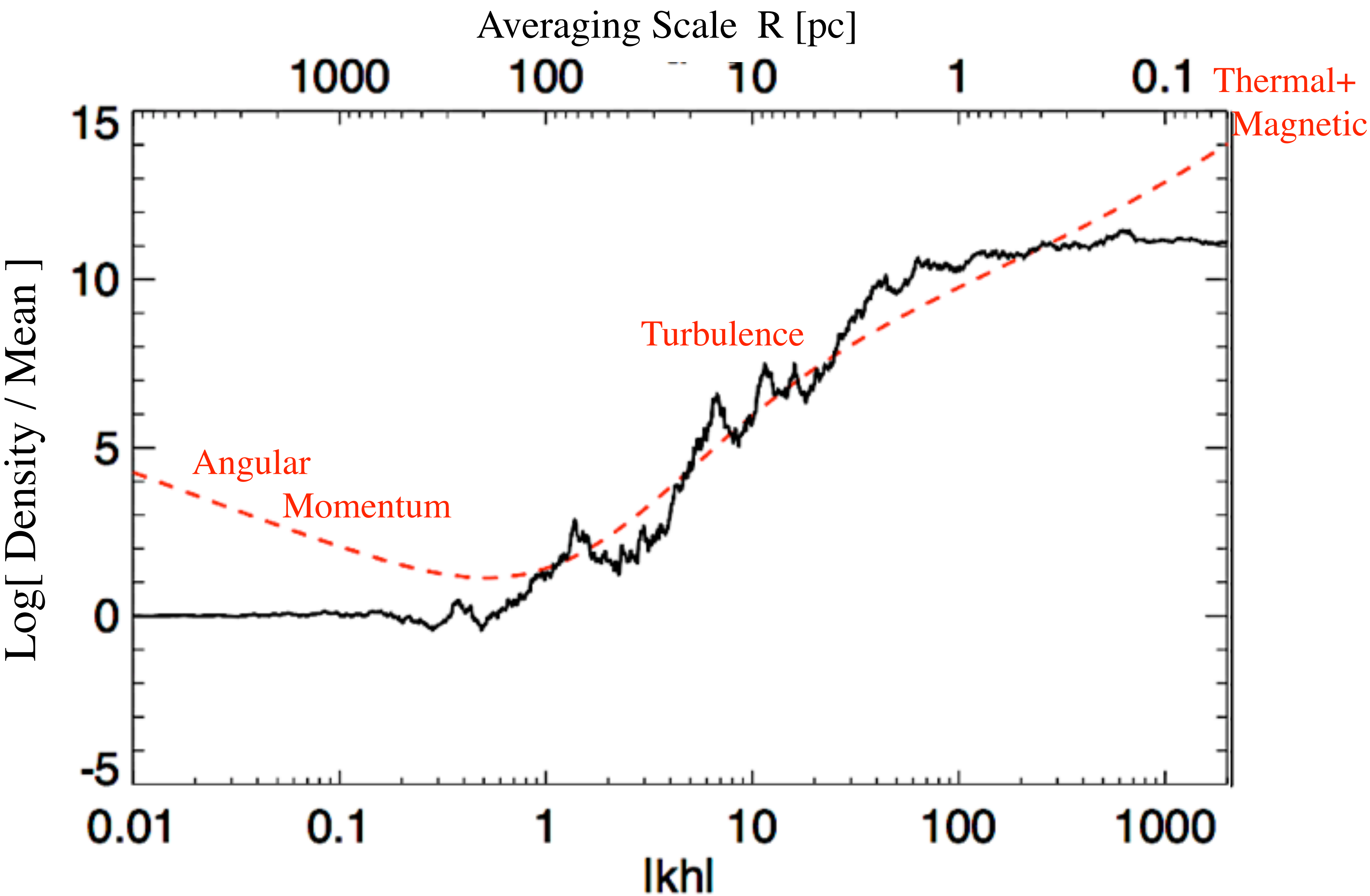
"Counting" Collapsing Objects

EVALUATE DENSITY FIELD vs. "BARRIER"



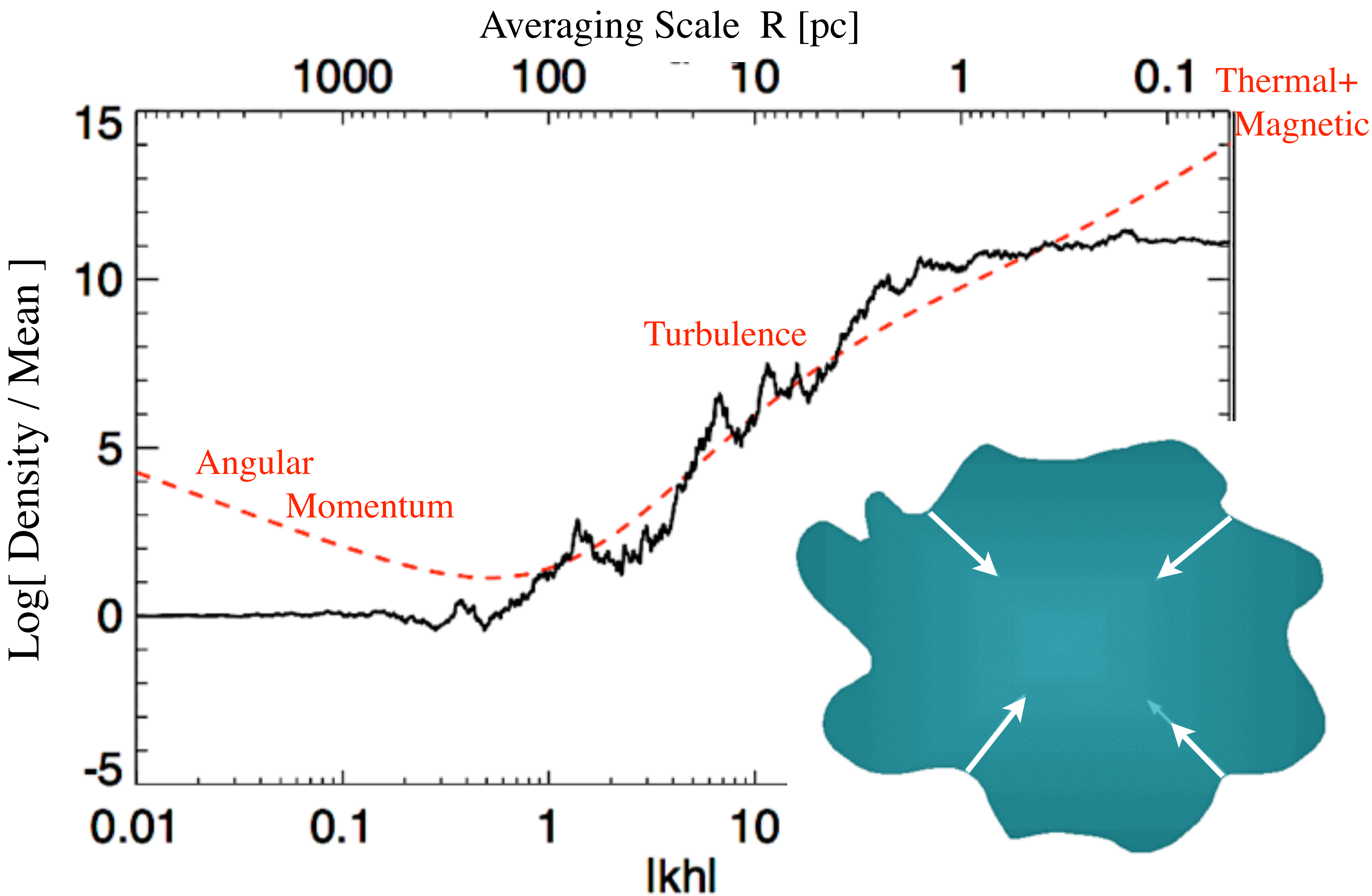
"Counting" Collapsing Objects

EVALUATE DENSITY FIELD vs. "BARRIER"



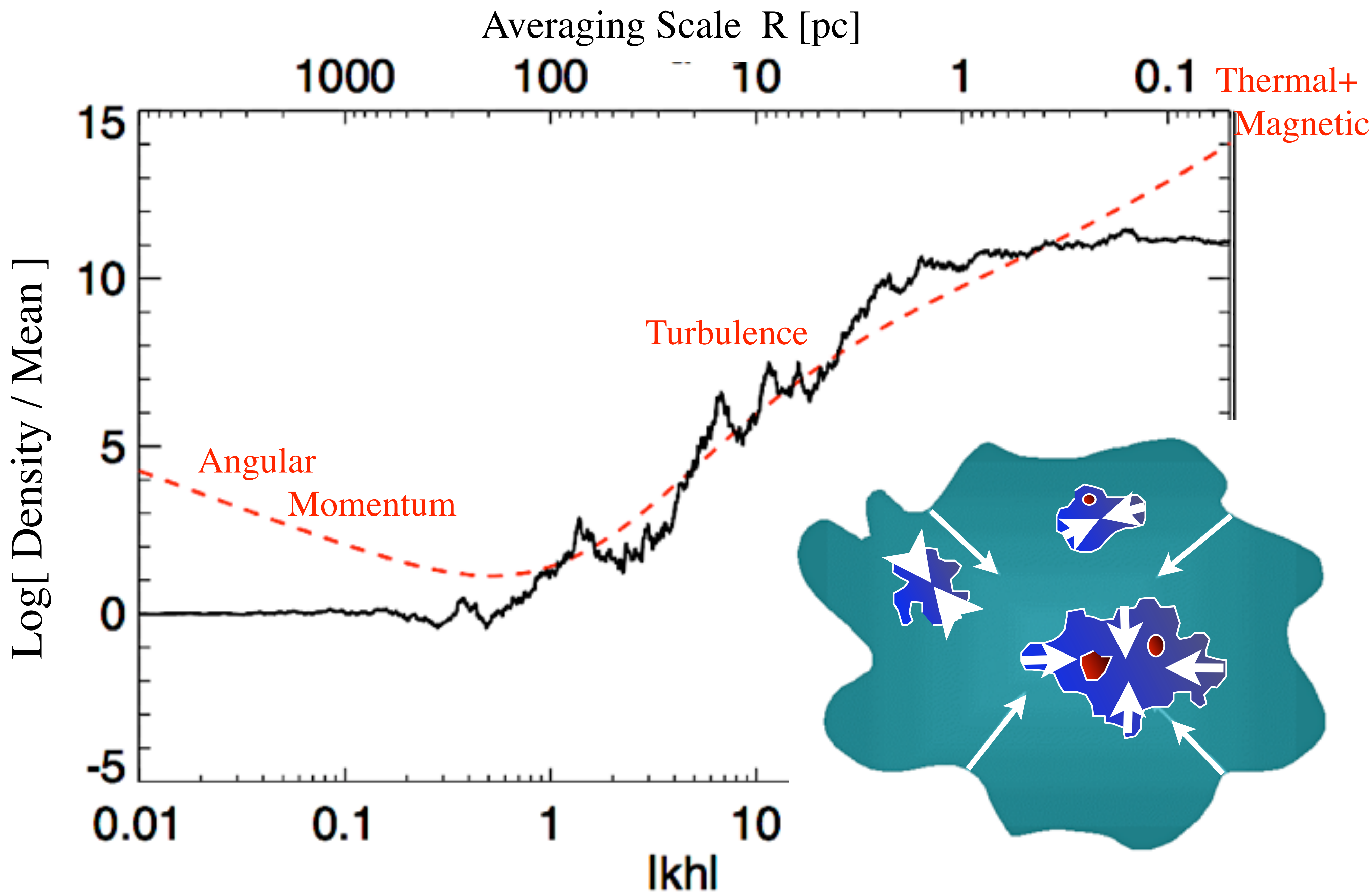
“Counting” Collapsing Objects

EVALUATE DENSITY FIELD vs. “BARRIER”



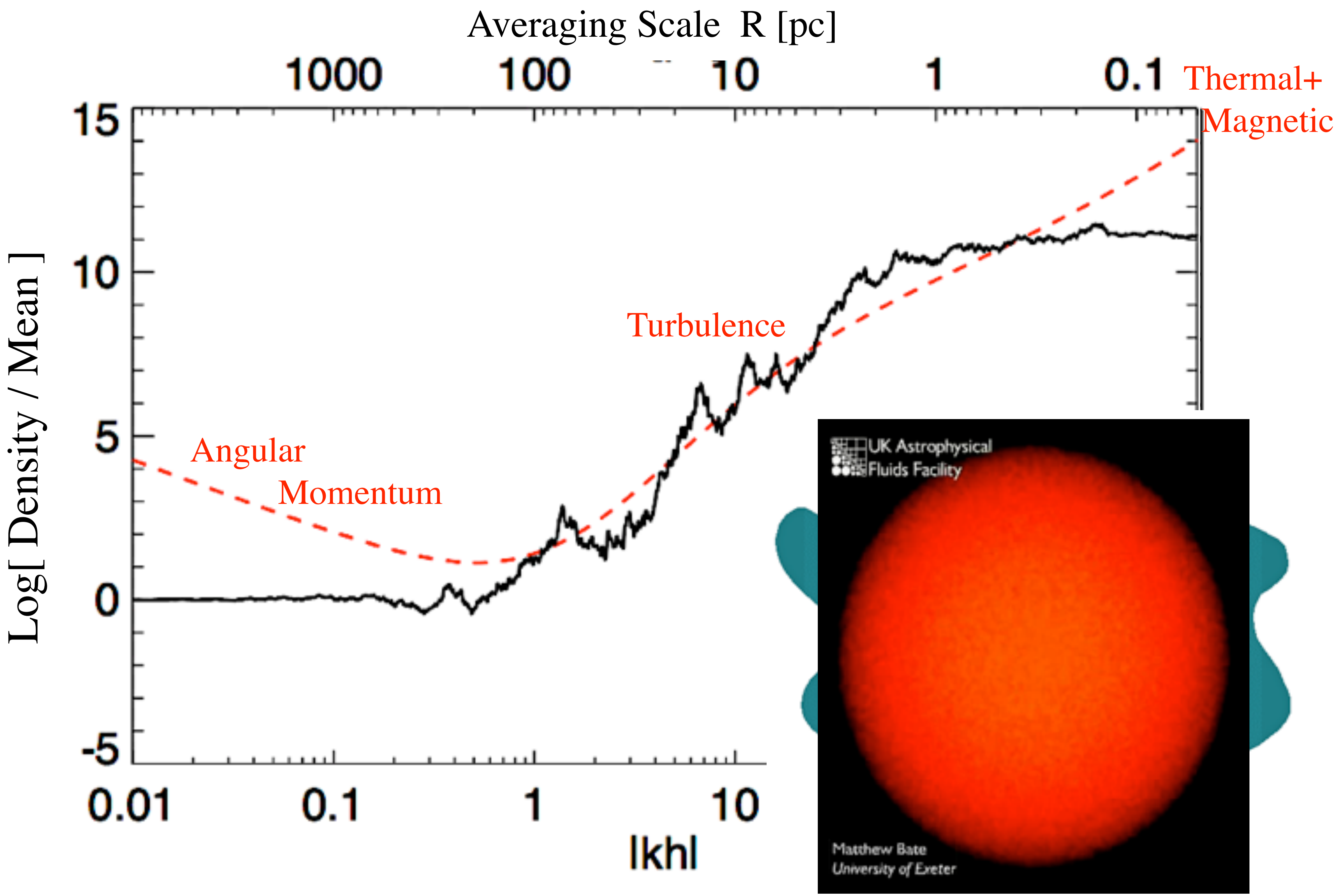
“Counting” Collapsing Objects

EVALUATE DENSITY FIELD vs. “BARRIER”



“Counting” Collapsing Objects

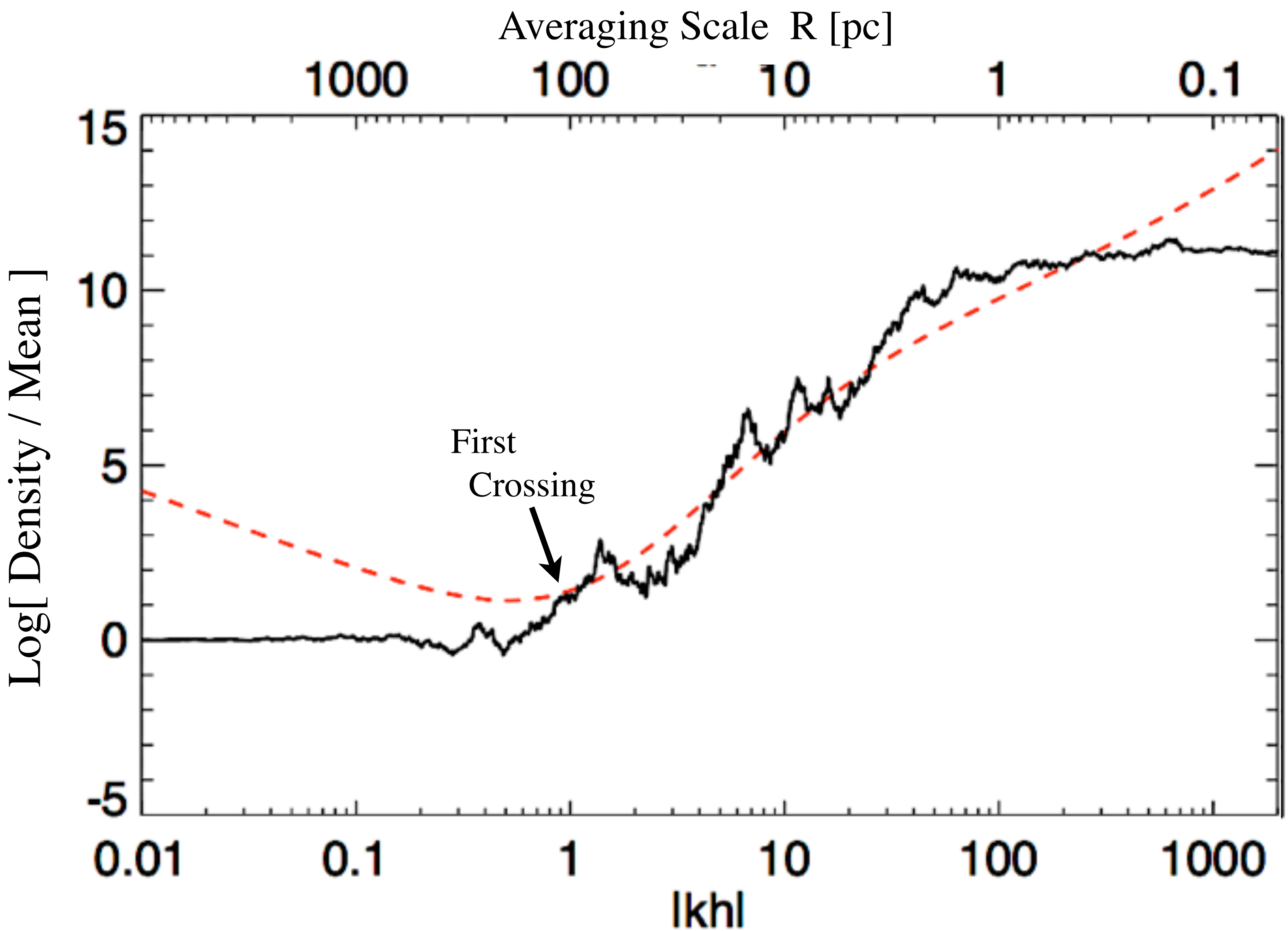
EVALUATE DENSITY FIELD vs. “BARRIER”



“Counting” Collapsing Objects

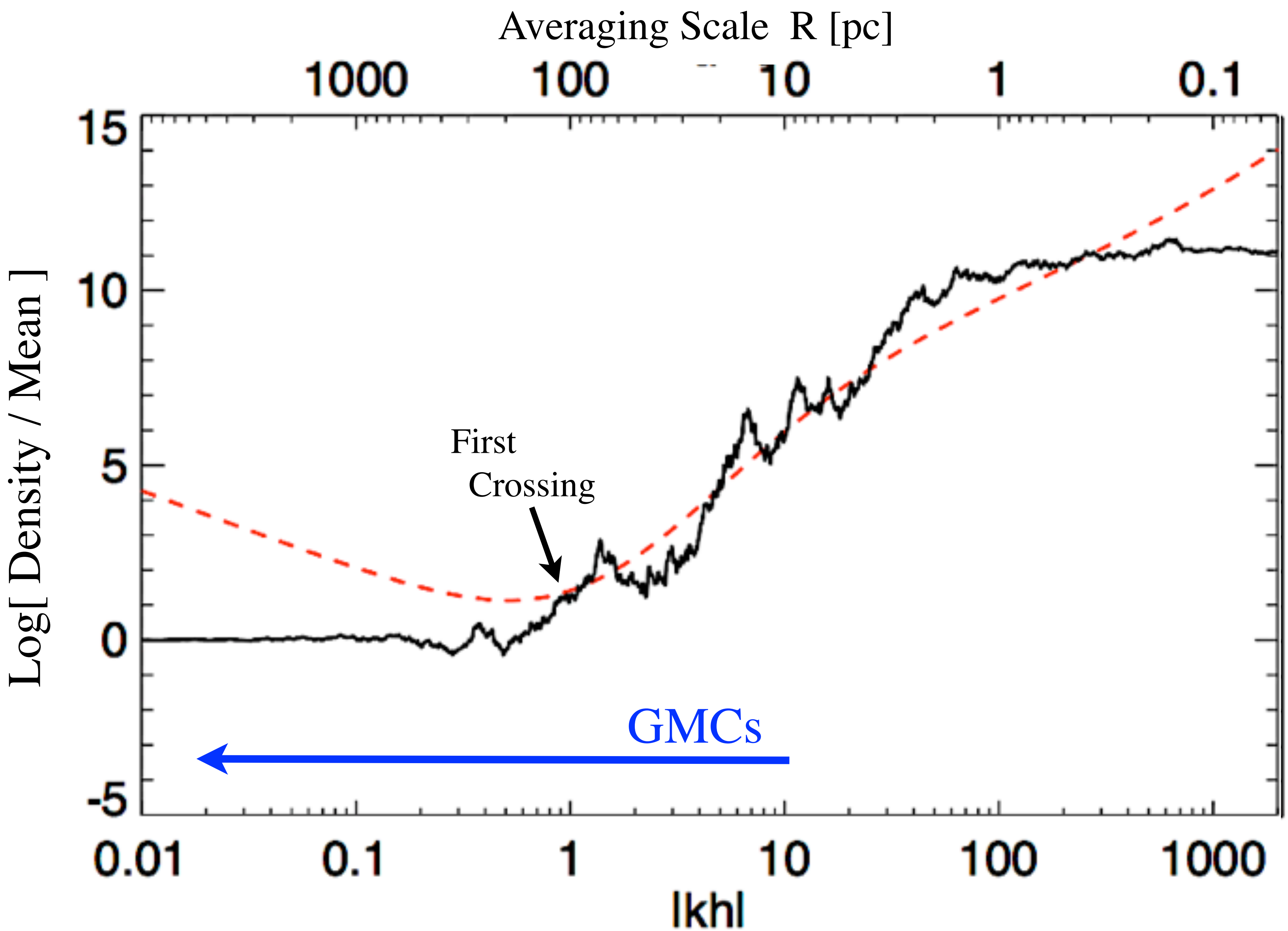
PFH 2011

EVALUATE DENSITY FIELD vs. “BARRIER”



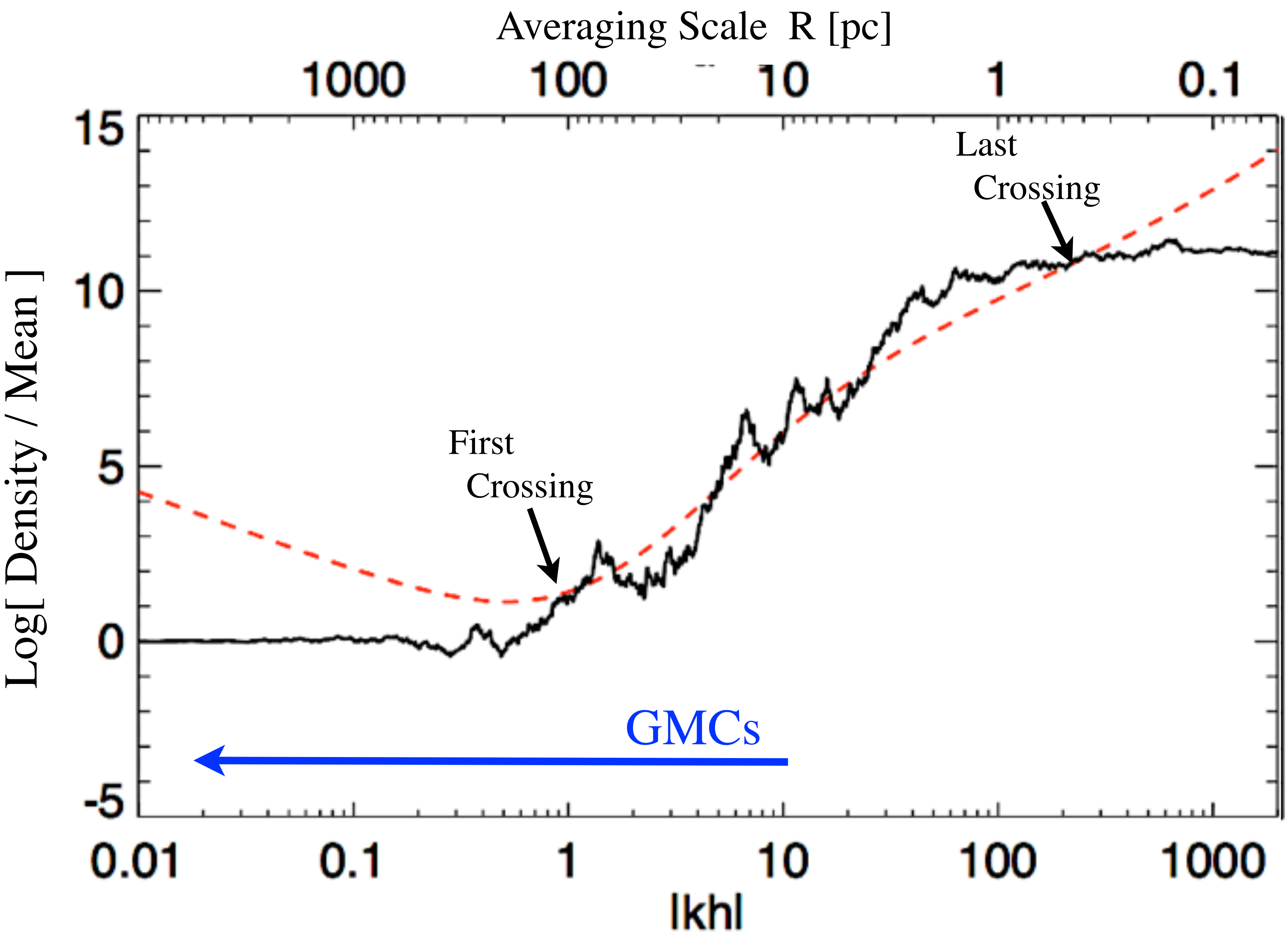
"Counting" Collapsing Objects

EVALUATE DENSITY FIELD vs. "BARRIER"



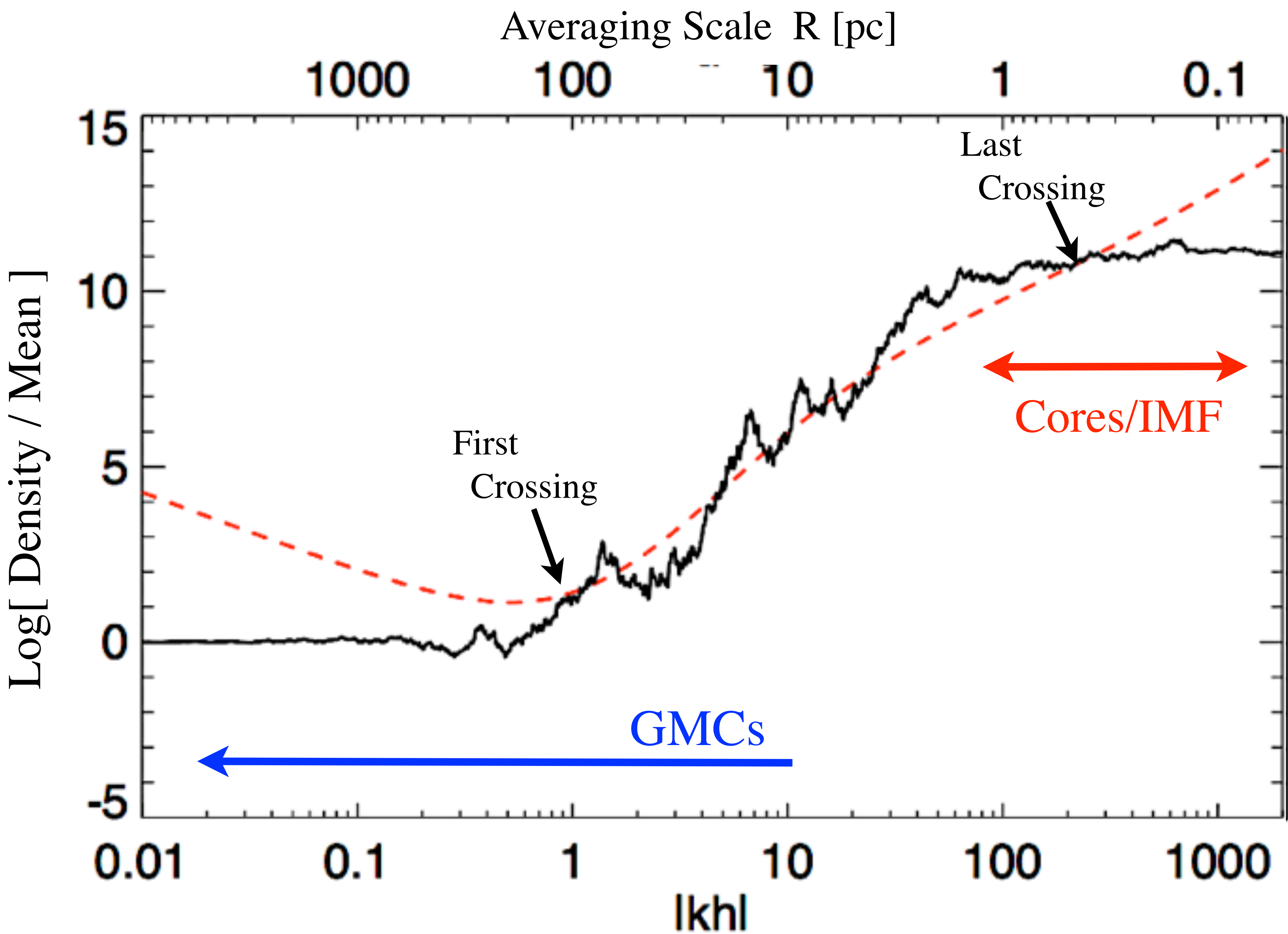
"Counting" Collapsing Objects

EVALUATE DENSITY FIELD vs. "BARRIER"



“Counting” Collapsing Objects

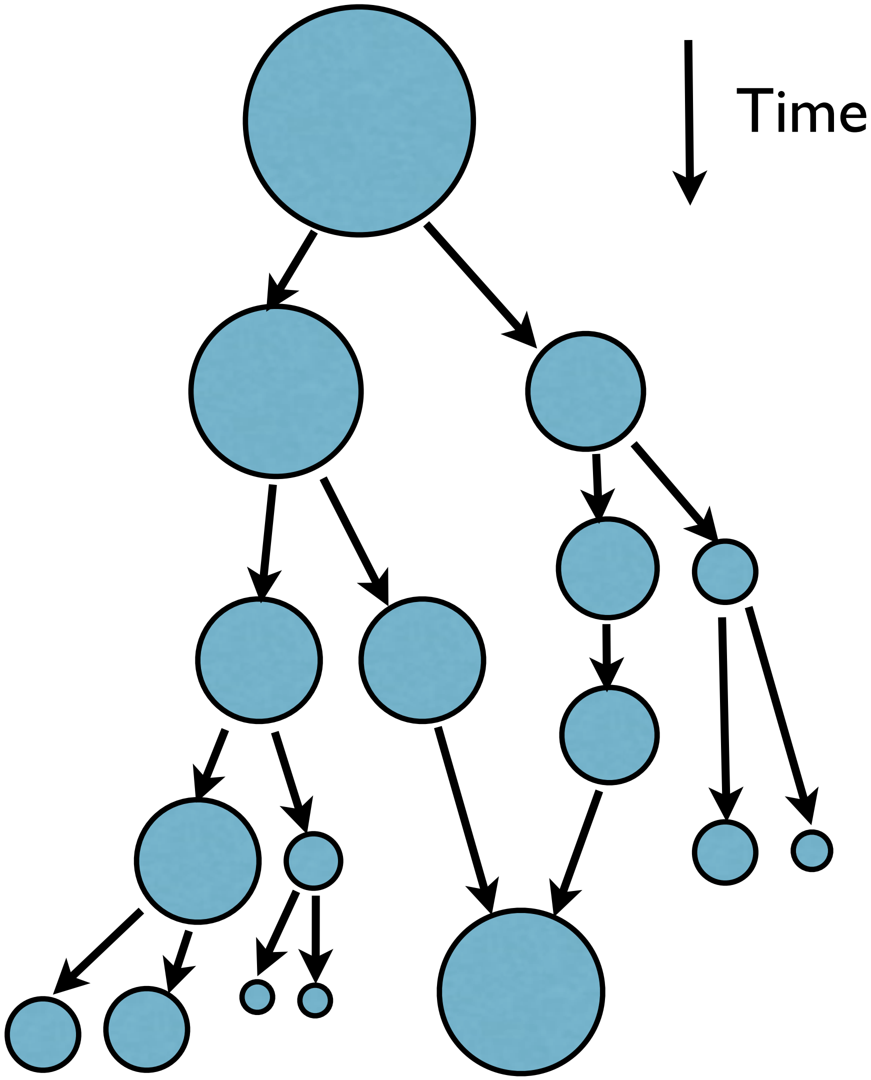
EVALUATE DENSITY FIELD vs. “BARRIER”



Evolve the Fluctuations in Time

CONSTRUCT "MERGER/FRAGMENTATION" TREES

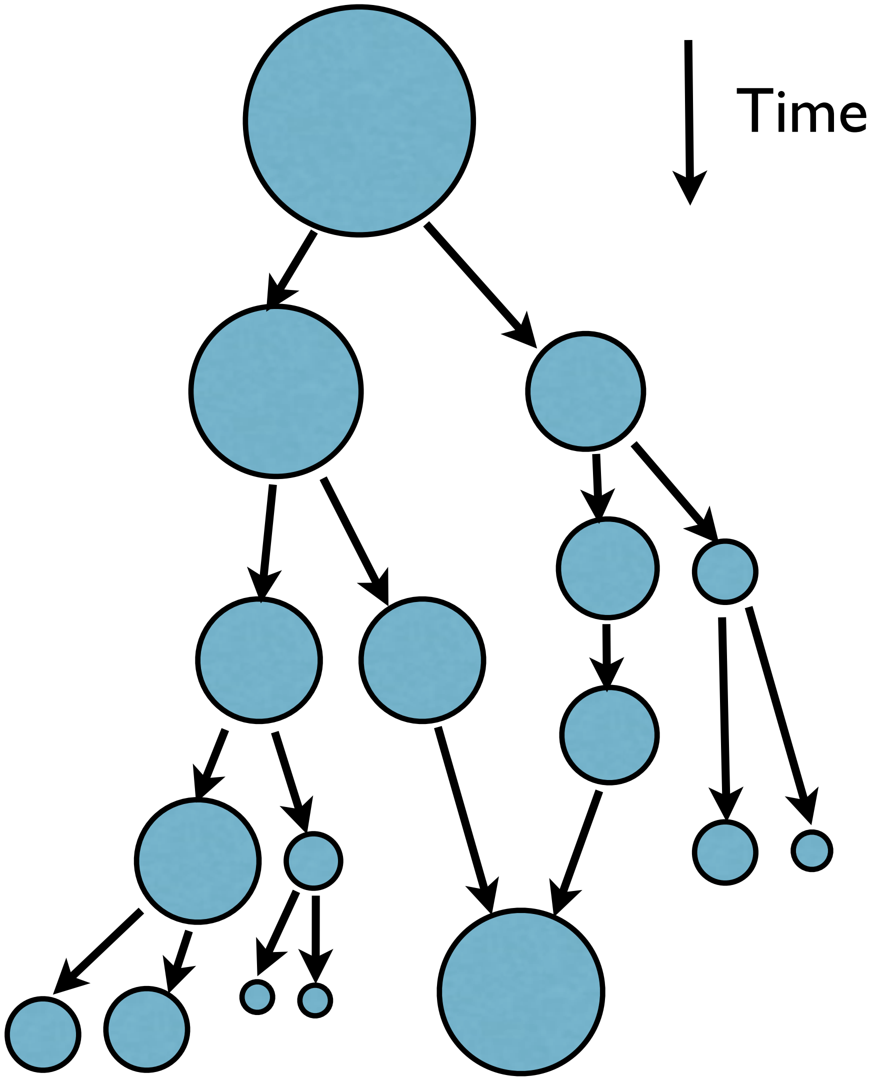
$$p(\delta\,|\,\tau) = \frac{1}{\sqrt{2\pi\, S\,(1-\exp\left[-2\tau\right])}}\,\exp\left[-\frac{(\delta-\delta(t=0)\,\exp\left[-\tau\right])^2}{2\,S\,(1-\exp\left[-2\tau\right])}\right]$$

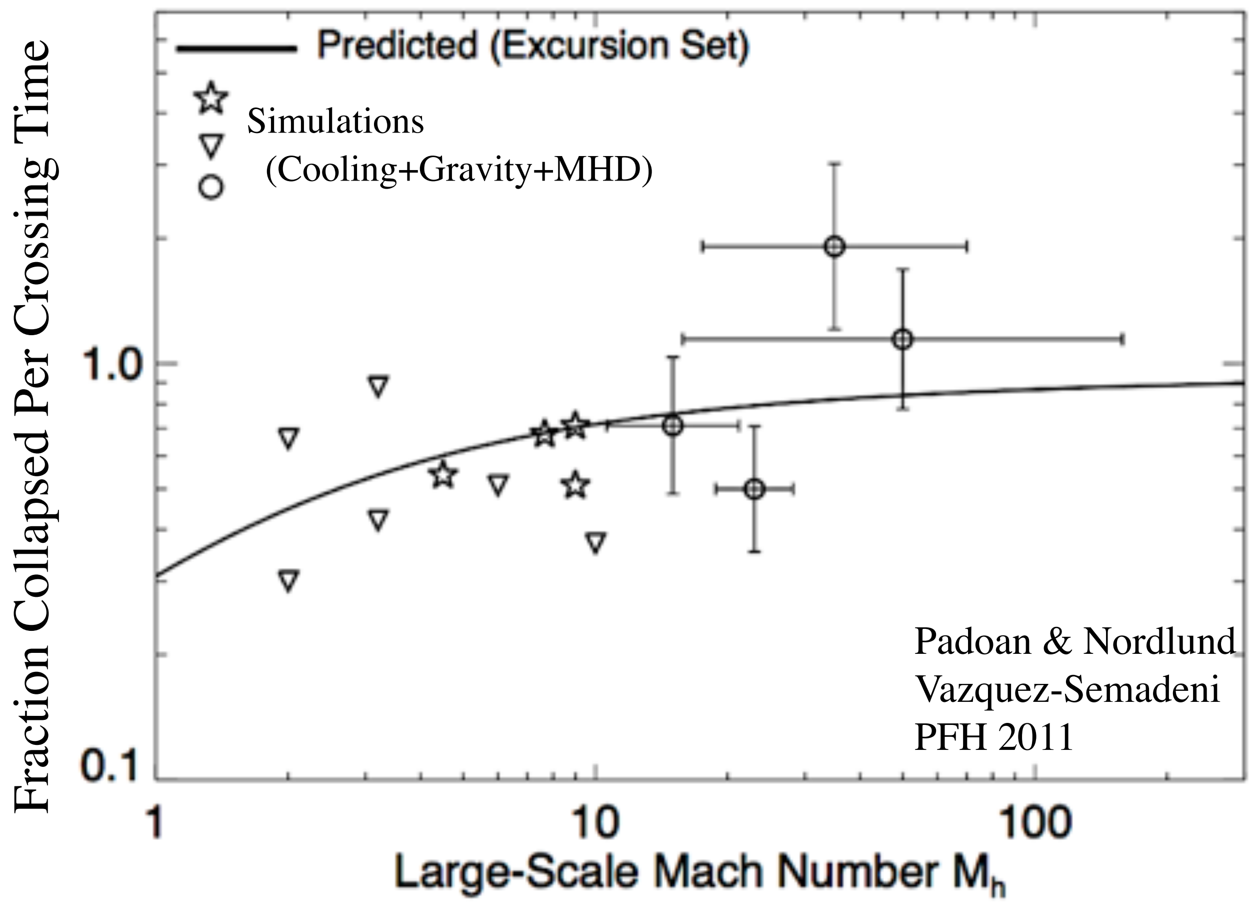


Evolve the Fluctuations in Time

CONSTRUCT "MERGER/FRAGMENTATION" TREES

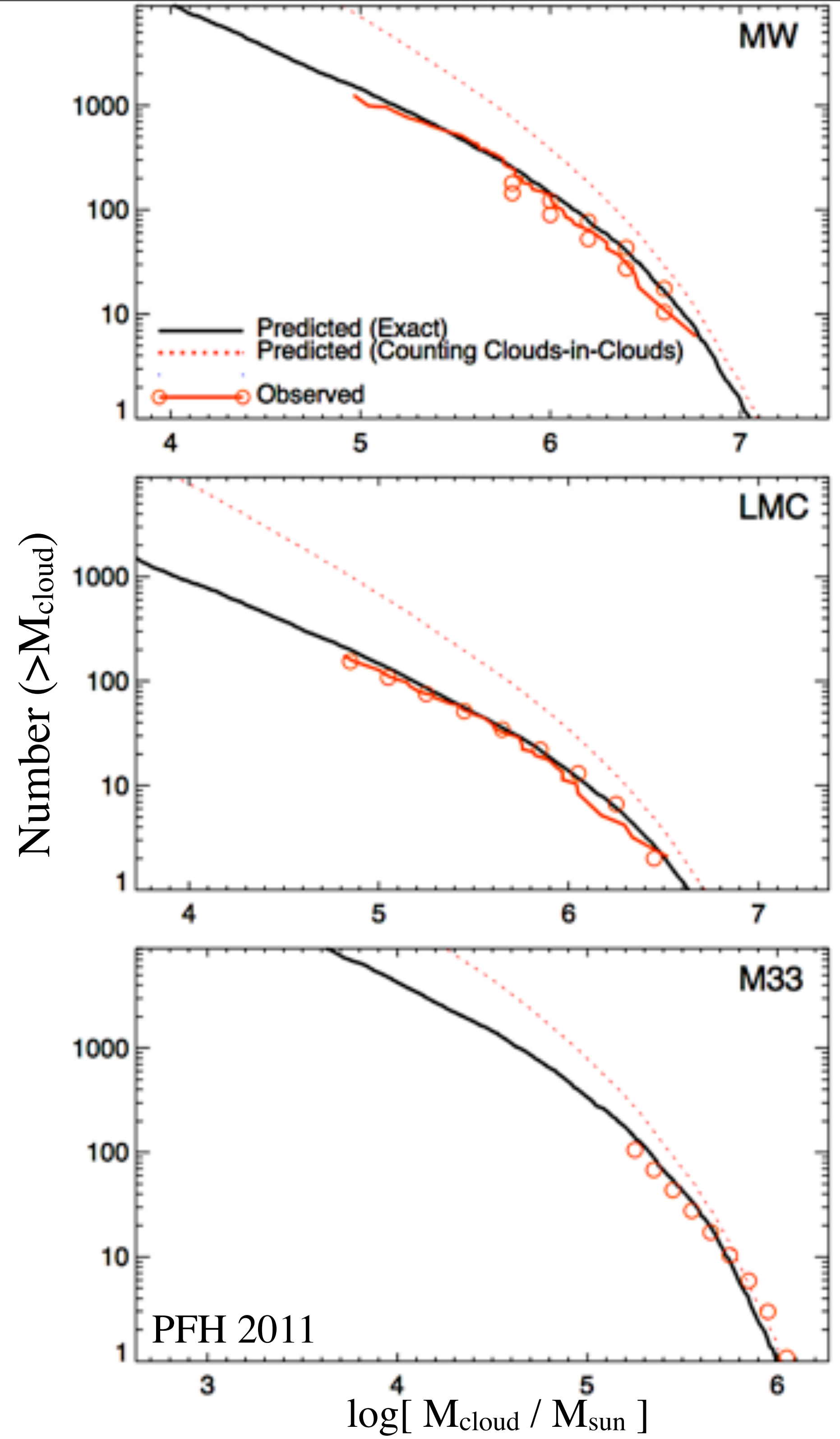
$$p(\delta\,|\,\tau) = \frac{1}{\sqrt{2\pi\,S\,(1-\exp{[-2\tau]})}}\,\exp{\left[-\frac{(\delta-\delta(t=0)\,\exp{[-\tau]})^2}{2\,S\,(1-\exp{[-2\tau]})}\right]}$$





The “First Crossing” Mass Function

VS GIANT MOLECULAR CLOUDS

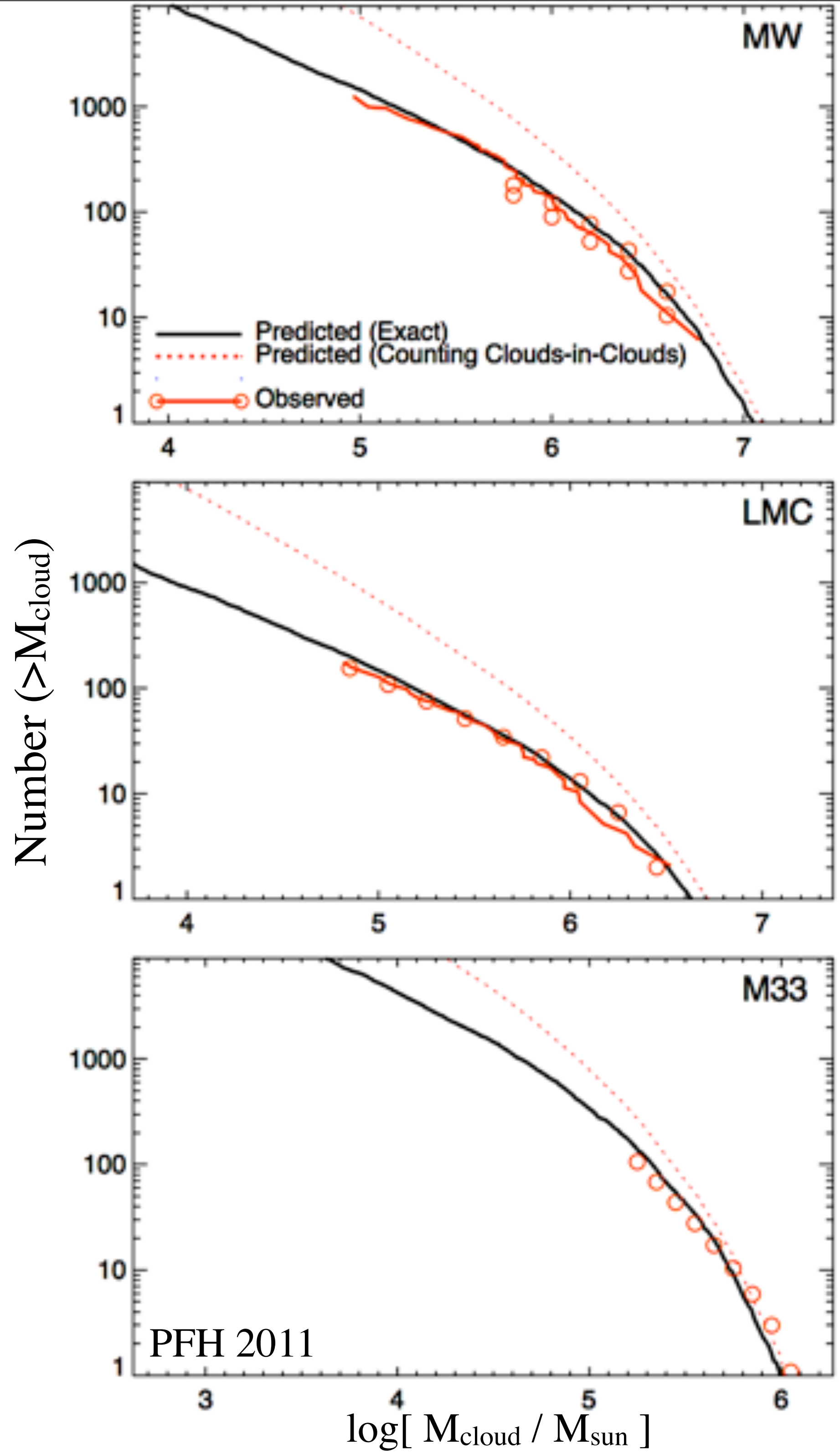


The “First Crossing” Mass Function

VS GIANT MOLECULAR CLOUDS

$$r_{\rm sonic} \ll r \ll h$$

$$S(r) \sim S_0$$



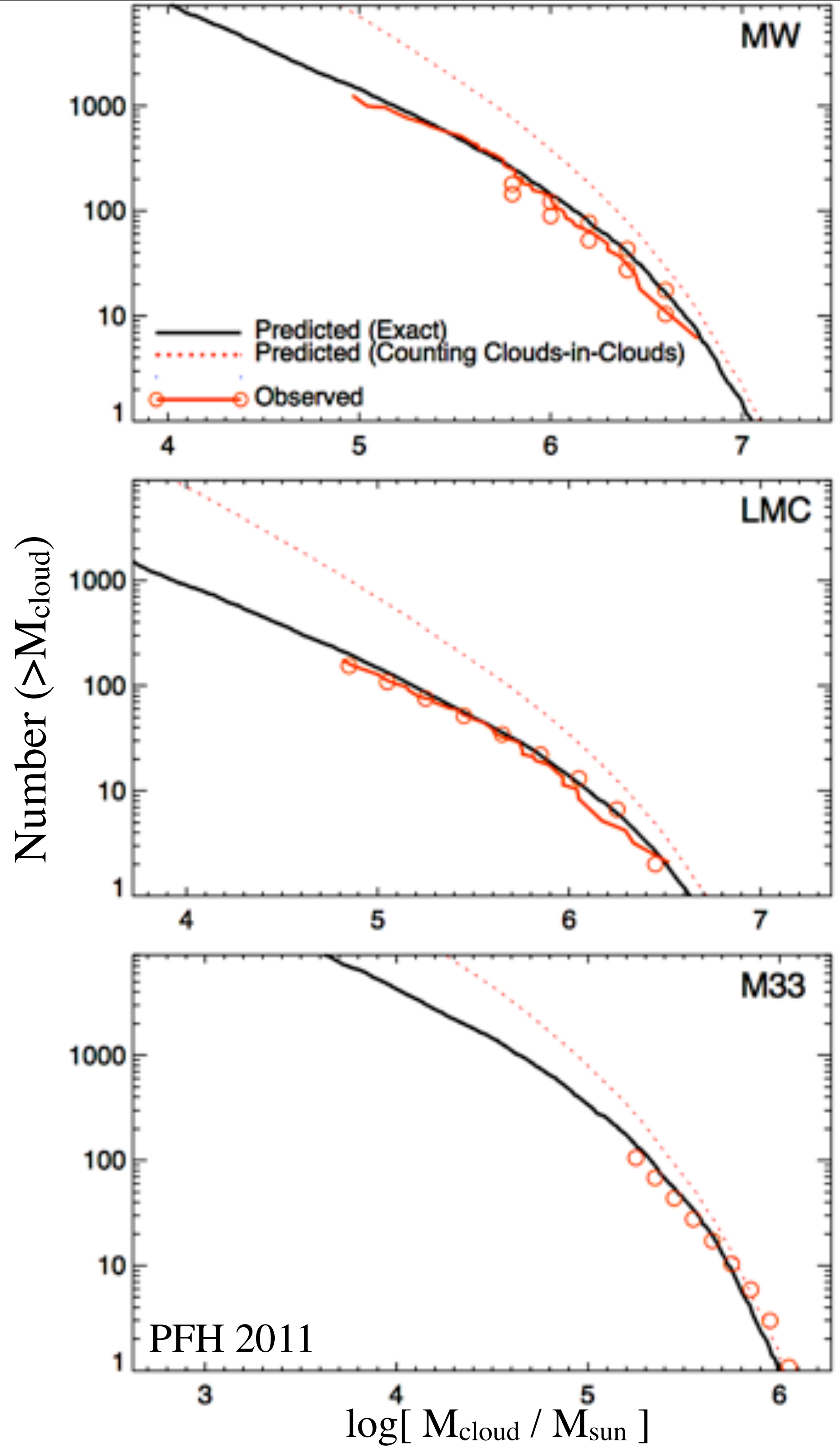
The "First Crossing" Mass Function

VS GIANT MOLECULAR CLOUDS

$$r_{\text{sonic}} \ll r \ll h$$

$$S(r) \sim S_0$$

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto M^{-\alpha}\, e^{-(M/M_J)^{\beta}}$$



The "First Crossing" Mass Function

VS GIANT MOLECULAR CLOUDS

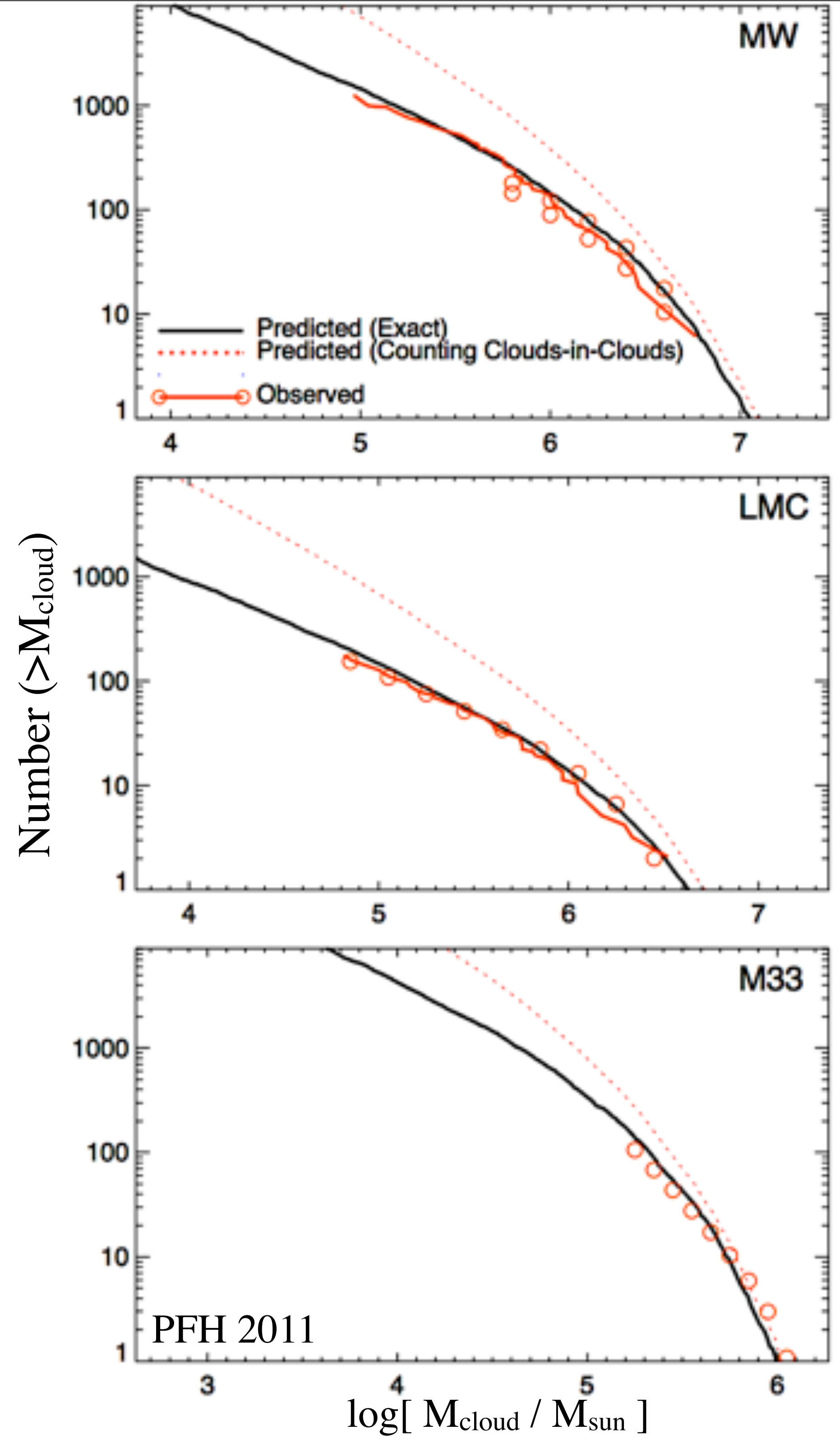
$$r_{\rm sonic} \ll r \ll h$$

$$S(r) \sim S_0$$

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto M^{-\alpha}\, e^{-(M/M_J)^{\beta}}$$

$$\alpha \approx -2 + \frac{(3-p)^2}{2\,S\,p^2}\,\ln\left(\frac{M_J}{M}\right)$$

$$\approx -2 + 0.1\,\log\left(\frac{M_J}{M}\right)$$



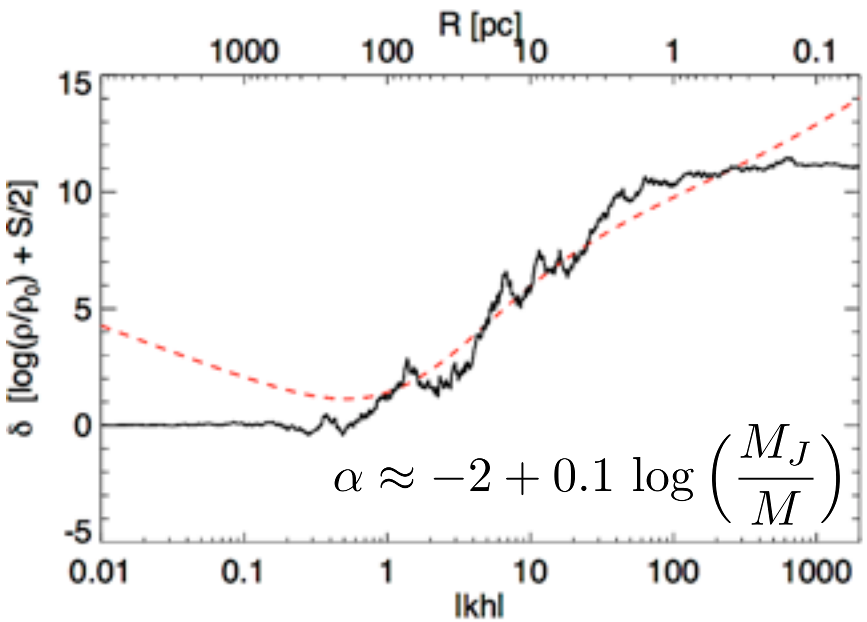
The “First Crossing” Mass Function

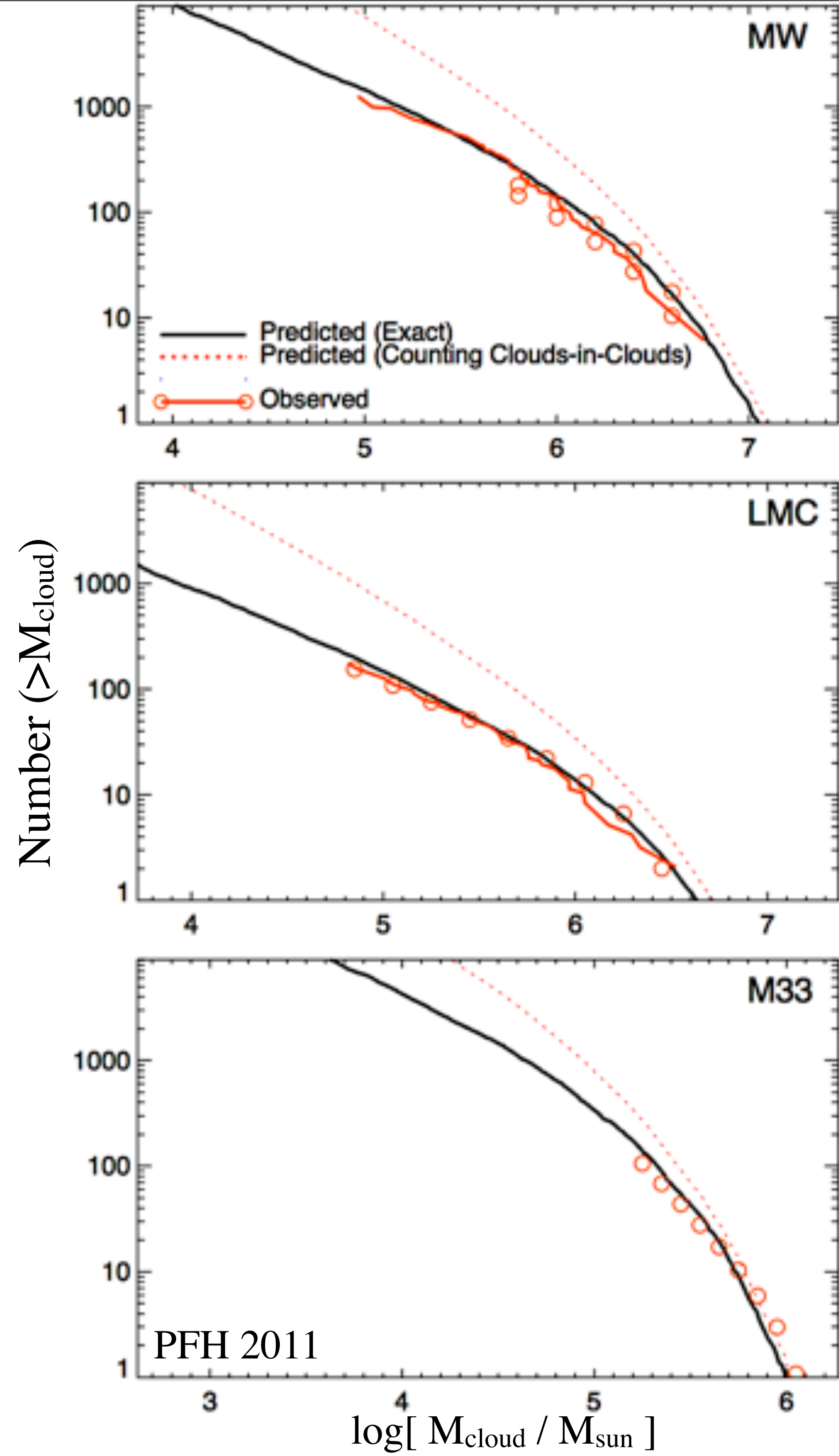
VS GIANT MOLECULAR CLOUDS

$$r_{\rm sonic} \ll r \ll h$$

$$S(r) \sim S_0$$

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto M^{-\alpha}\, e^{-(M/M_J)^{\beta}}$$



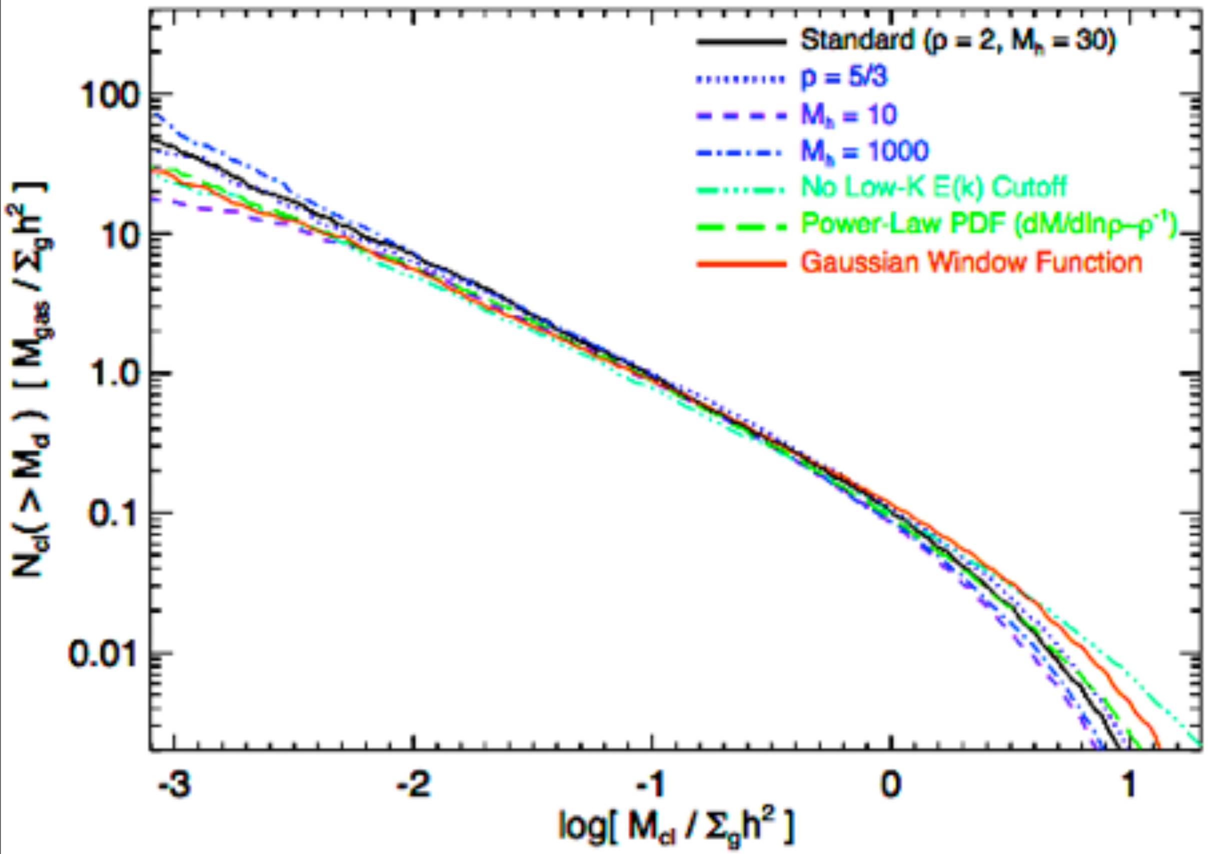


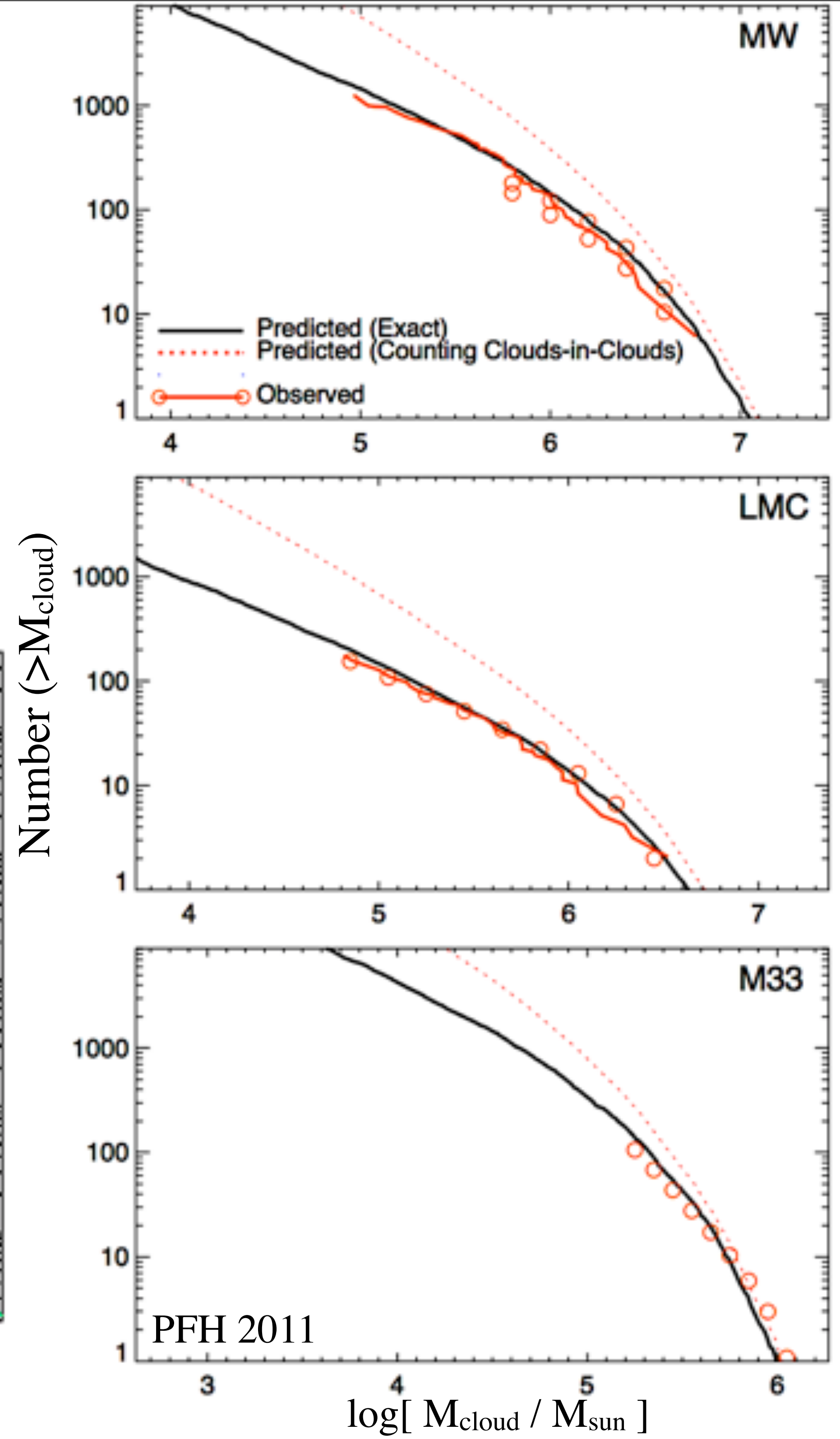
The “First Crossing” Mass Function

VS GIANT MOLECULAR CLOUDS

$$r_{\rm sonic} \ll r \ll h$$

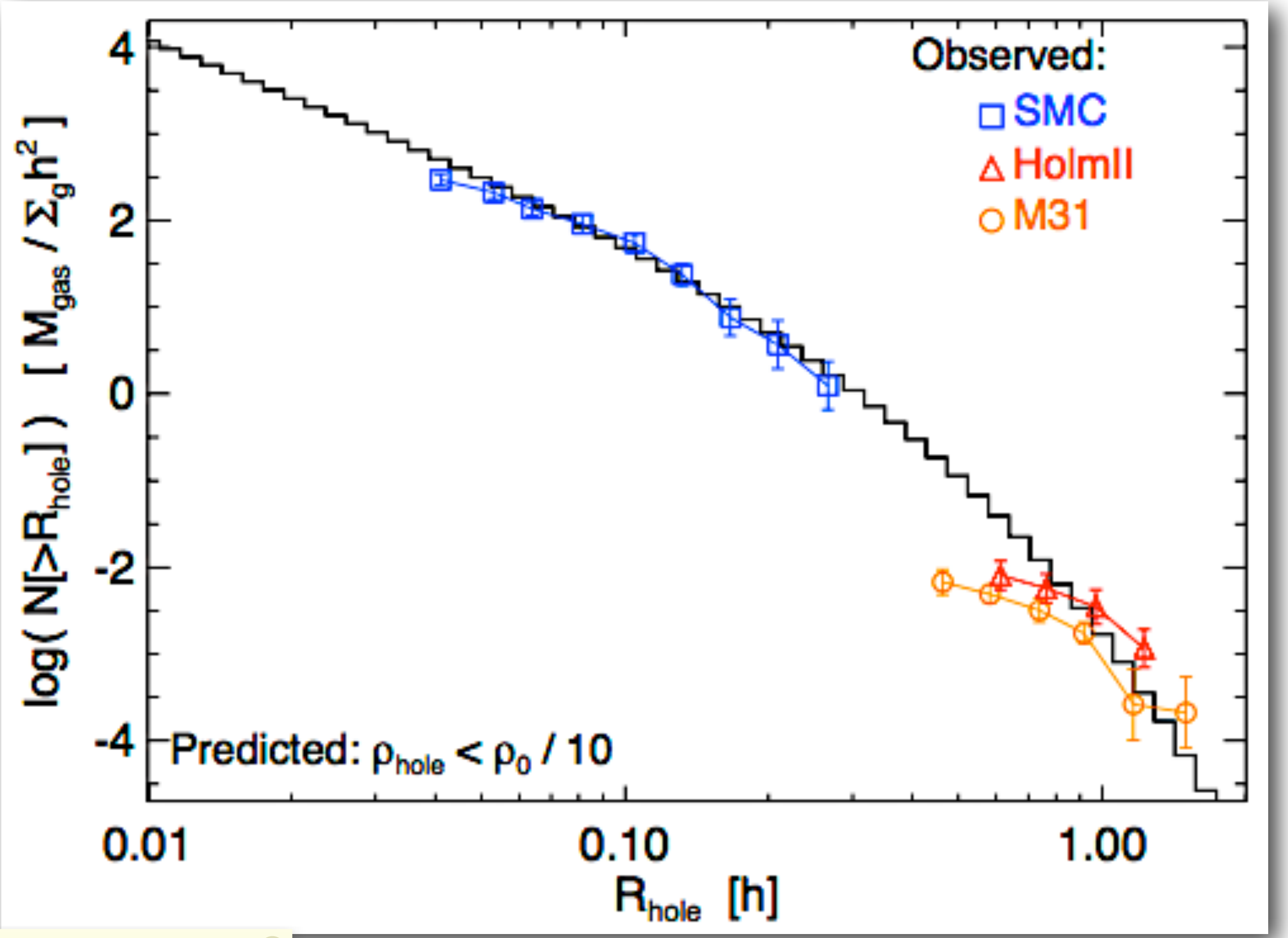
$$S(r) \sim S_0$$





"Void" Abundance

VS HI "HOLES" IN THE ISM

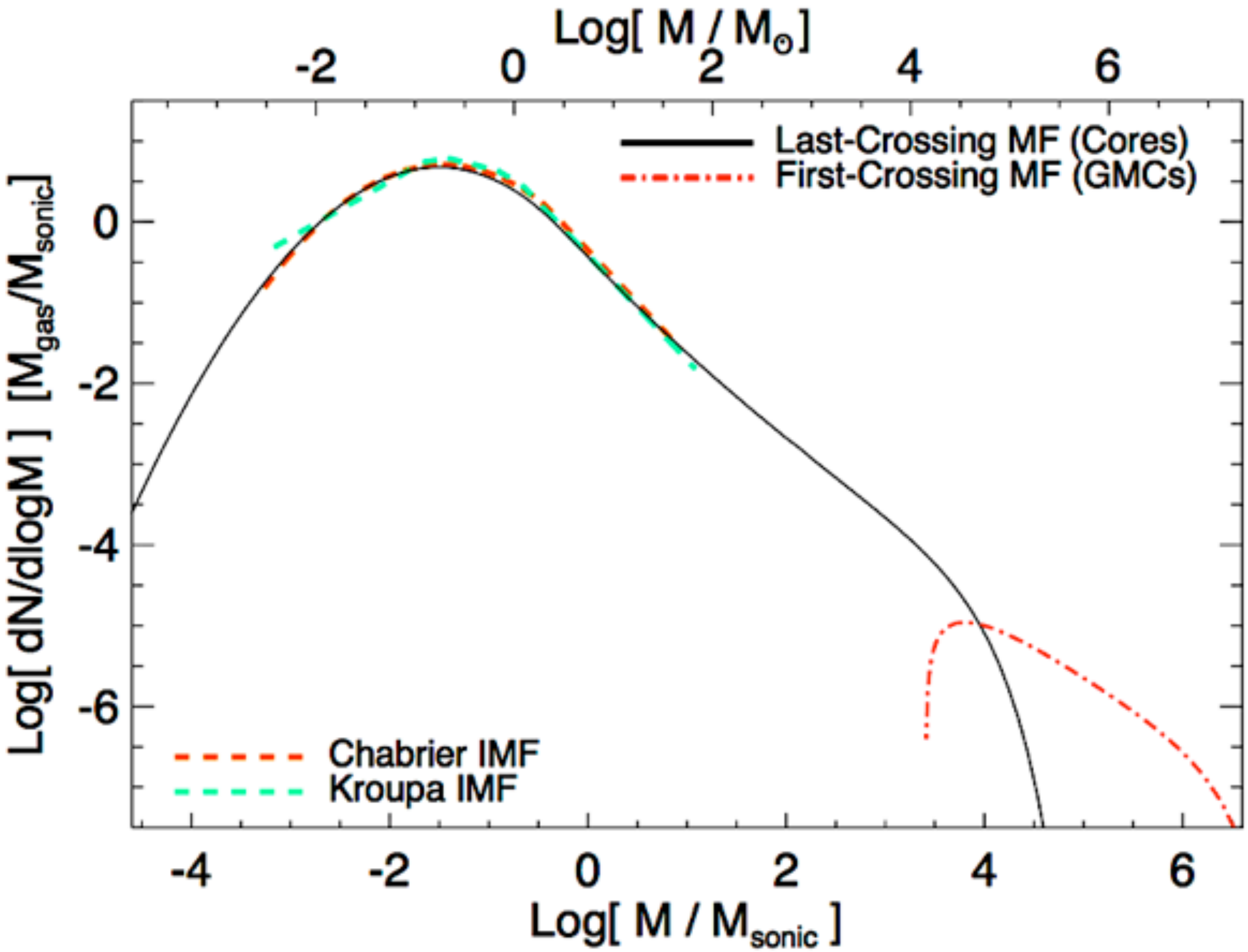


KEEP?

Don't need SNe to "carve out" voids

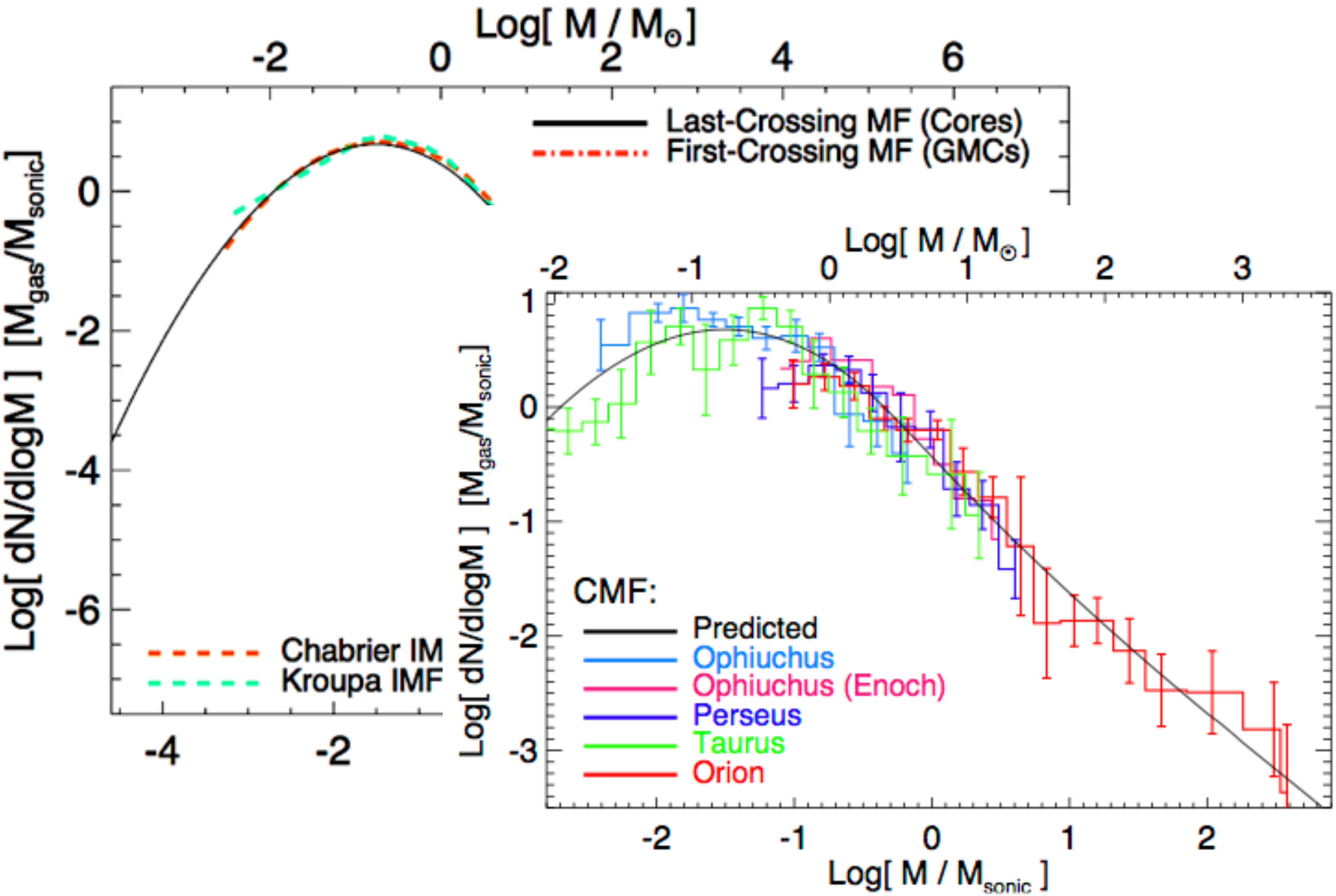
The “Last Crossing” Mass Function

VS PROTOSTELLAR CORES & THE STELLAR IMF



The “Last Crossing” Mass Function

VS PROTOSTELLAR CORES & THE STELLAR IMF



The “Last Crossing” Mass Function

VS PROTOSTELLAR CORES & THE STELLAR IMF

For $r \ll \ell_{\rm GMC} \ll h$ this becomes Hennebelle-Chabrier theory:

$$\mathcal{N}(\tilde{M}) = 2\mathcal{N}_0 \frac{1}{\tilde{R}^3} \frac{1}{1+(2\eta+1)\mathcal{M}_*^2\tilde{R}^{2\eta}} \frac{1+(1-\eta)\mathcal{M}_*^2\tilde{R}^{2\eta}}{\left(1+\mathcal{M}_*^2\tilde{R}^{2\eta}\right)^{3/2}}$$
$$\times \exp\left\{-\frac{\left[\ln\left(\tilde{M}/\tilde{R}^3\right)\right]^2}{2\sigma^2}\right\} \frac{\exp(-\sigma^2/8)}{\sqrt{2\pi}\sigma},$$

The "Last Crossing" Mass Function

VS PROTOSTELLAR CORES & THE STELLAR IMF

For $r \ll \ell_{\rm GMC} \ll h$ this becomes Hennebelle-Chabrier theory:

$$\mathcal{N}(\tilde{M}) = 2\mathcal{N}_0 \frac{1}{\tilde{R}^3} \frac{1}{1+(2\eta+1)\mathcal{M}_*^2\tilde{R}^{2\eta}} \frac{1+(1-\eta)\mathcal{M}_*^2\tilde{R}^{2\eta}}{\left(1+\mathcal{M}_*^2\tilde{R}^{2\eta}\right)^{3/2}}$$
$$\times \exp\left\{-\frac{\left[\ln\left(\tilde{M}/\tilde{R}^3\right)\right]^2}{2\sigma^2}\right\} \frac{\exp(-\sigma^2/8)}{\sqrt{2\pi}\sigma},$$

... ***BUT***,

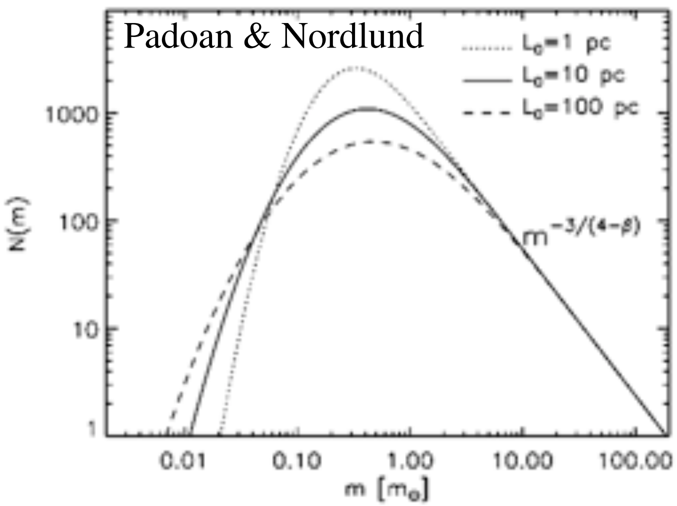
The "Last Crossing" Mass Function

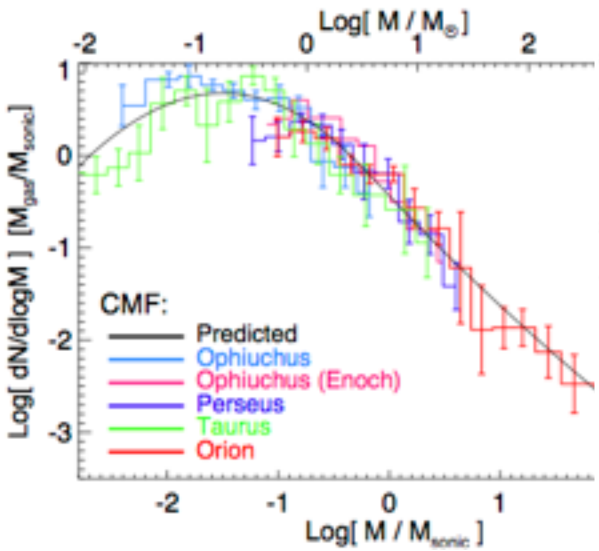
VS PROTOSTELLAR CORES & THE STELLAR IMF

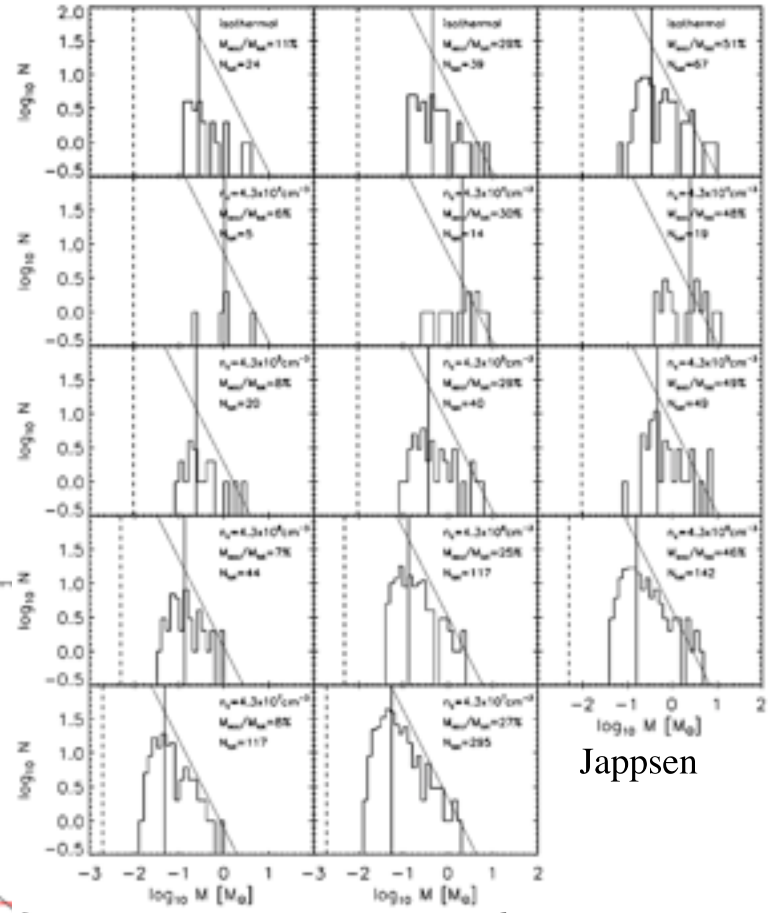
For $r \ll \ell_{\rm GMC} \ll h$ this becomes Hennebelle-Chabrier theory:

$$\mathcal{N}(\tilde{M}) = 2\mathcal{N}_0 \frac{1}{\tilde{R}^3} \frac{1}{1+(2\eta+1)\mathcal{M}_*^2 \tilde{R}^{2\eta}} \frac{1+(1-\eta)\mathcal{M}_*^2\tilde{R}^{2\eta}}{\left(1+\mathcal{M}_*^2\tilde{R}^{2\eta}\right)^{3/2}}$$
$$\times \exp\left\{-\frac{\left[\ln\left(\tilde{M}/\tilde{R}^3\right)\right]^2}{2\sigma^2}\right\} \frac{\exp(-\sigma^2/8)}{\sqrt{2\pi}\sigma},$$

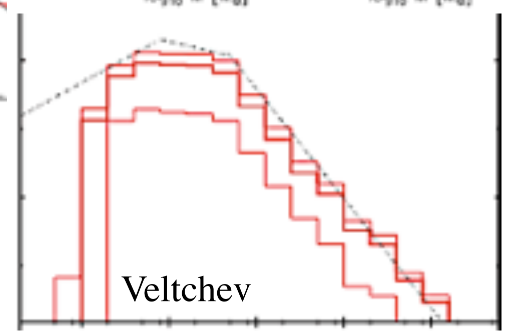
... ***BUT***,







Jappsen

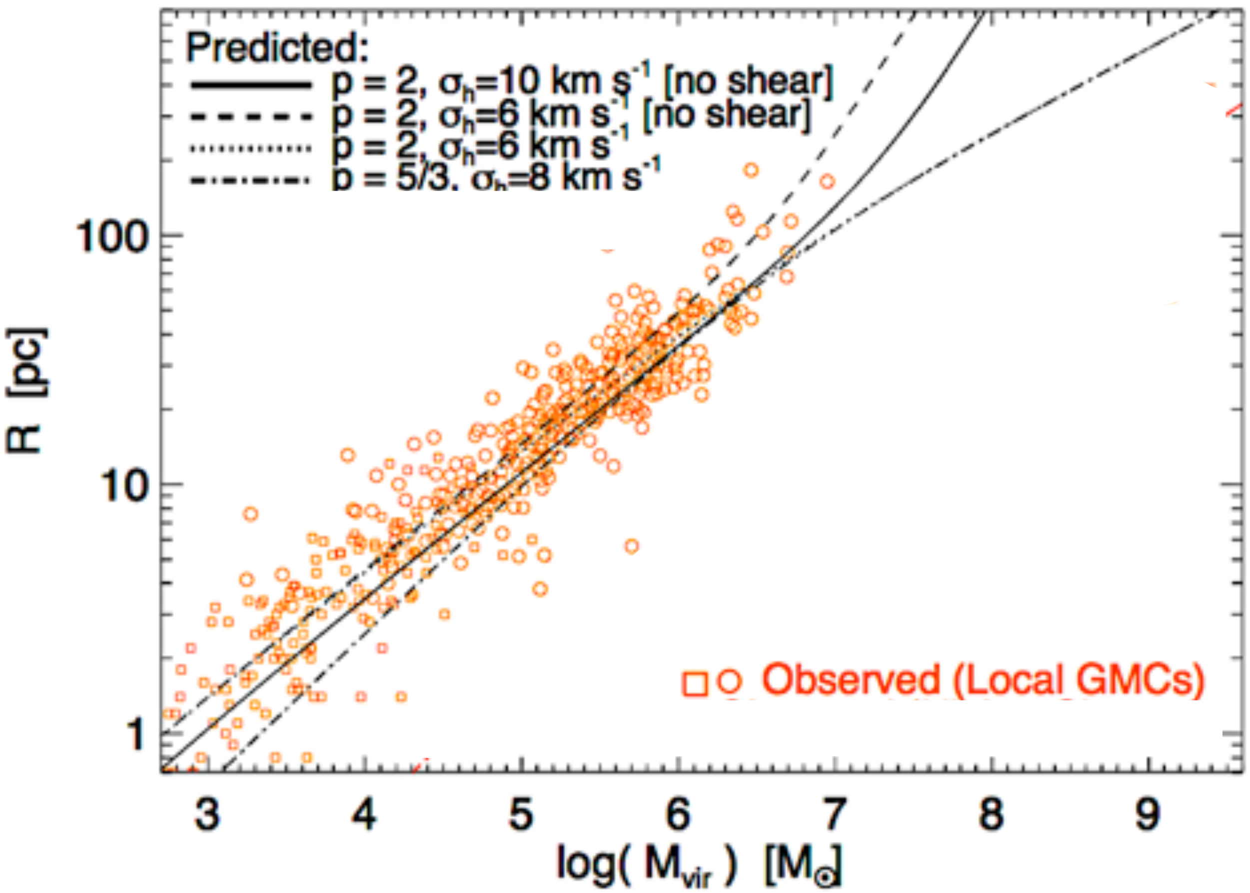


Veltchev

Bate & Bonnell 2005 (Accretion-Ejection)
Larson 1992 (Fractal collapse)
Elmegreen 1997 (Fractal GMCs)
Padoan & Nordlund (Turb. Frag.)
Hennebelle & Chabrier (Press-Schechter)
Veltchev+ 2011 (Clump mass-density + turb + accretion)

Structural Properties of “Clouds”

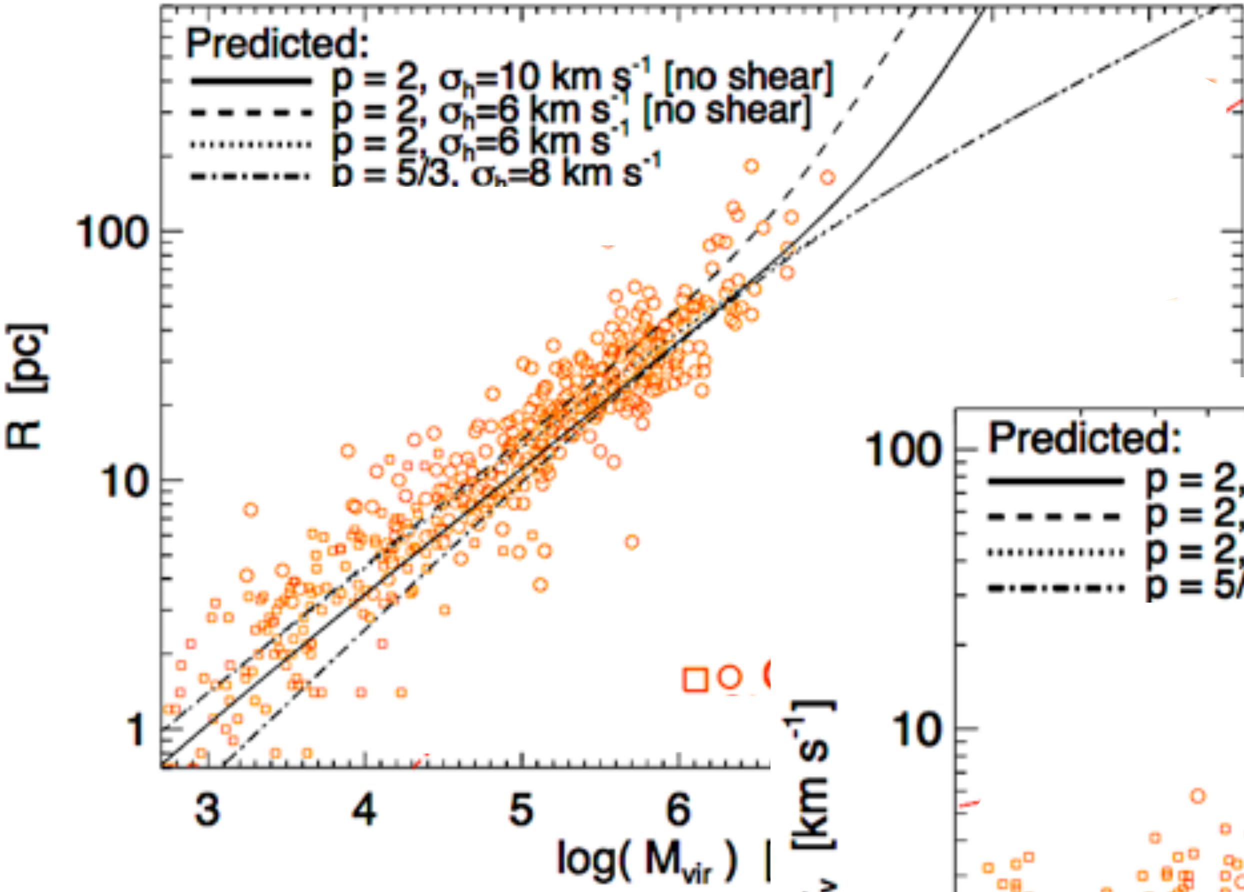
LARSON’S LAWS EMERGE NATURALLY

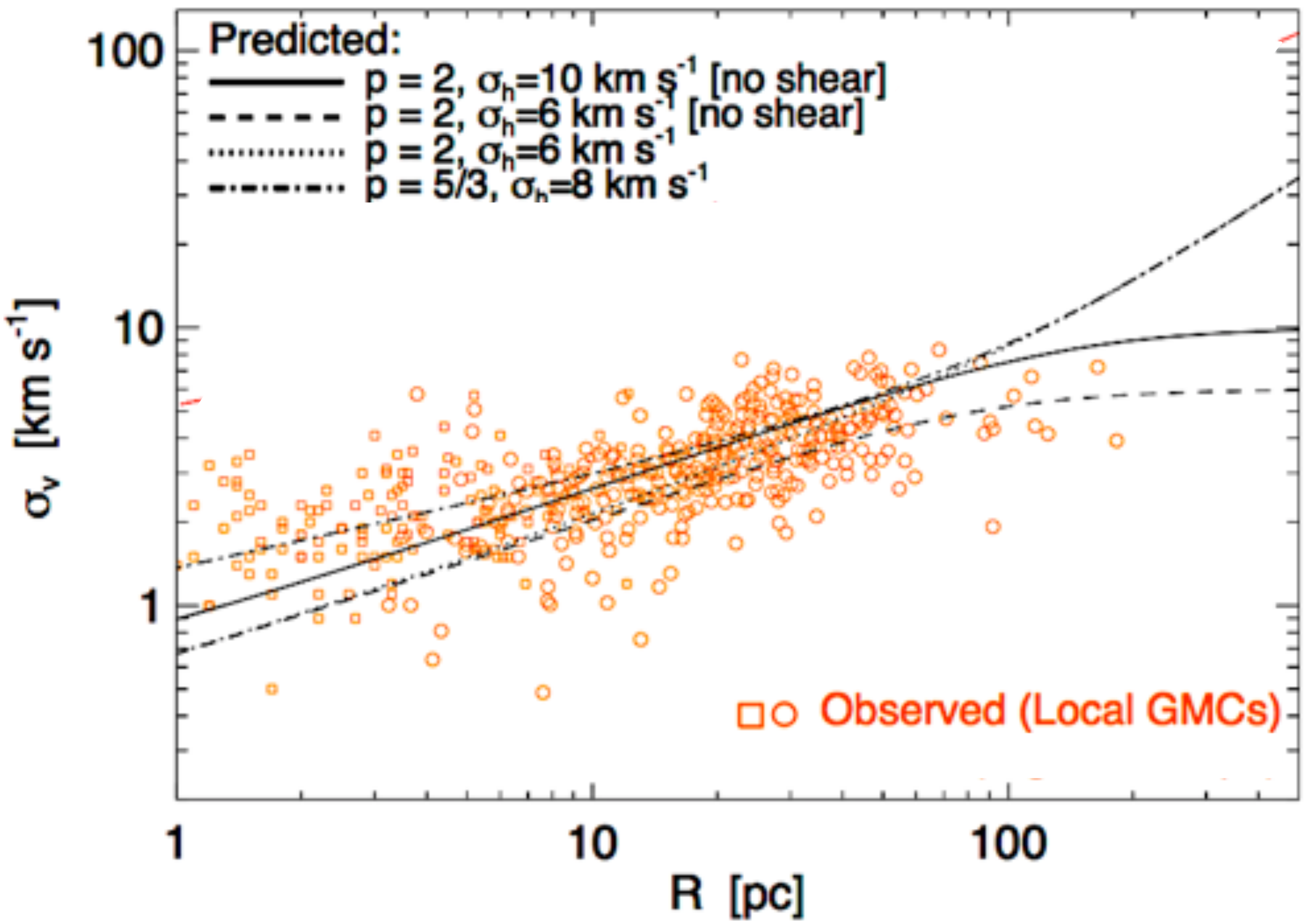


KEEP?

Structural Properties of "Clouds"

LARSON'S LAWS EMERGE NATURALLY



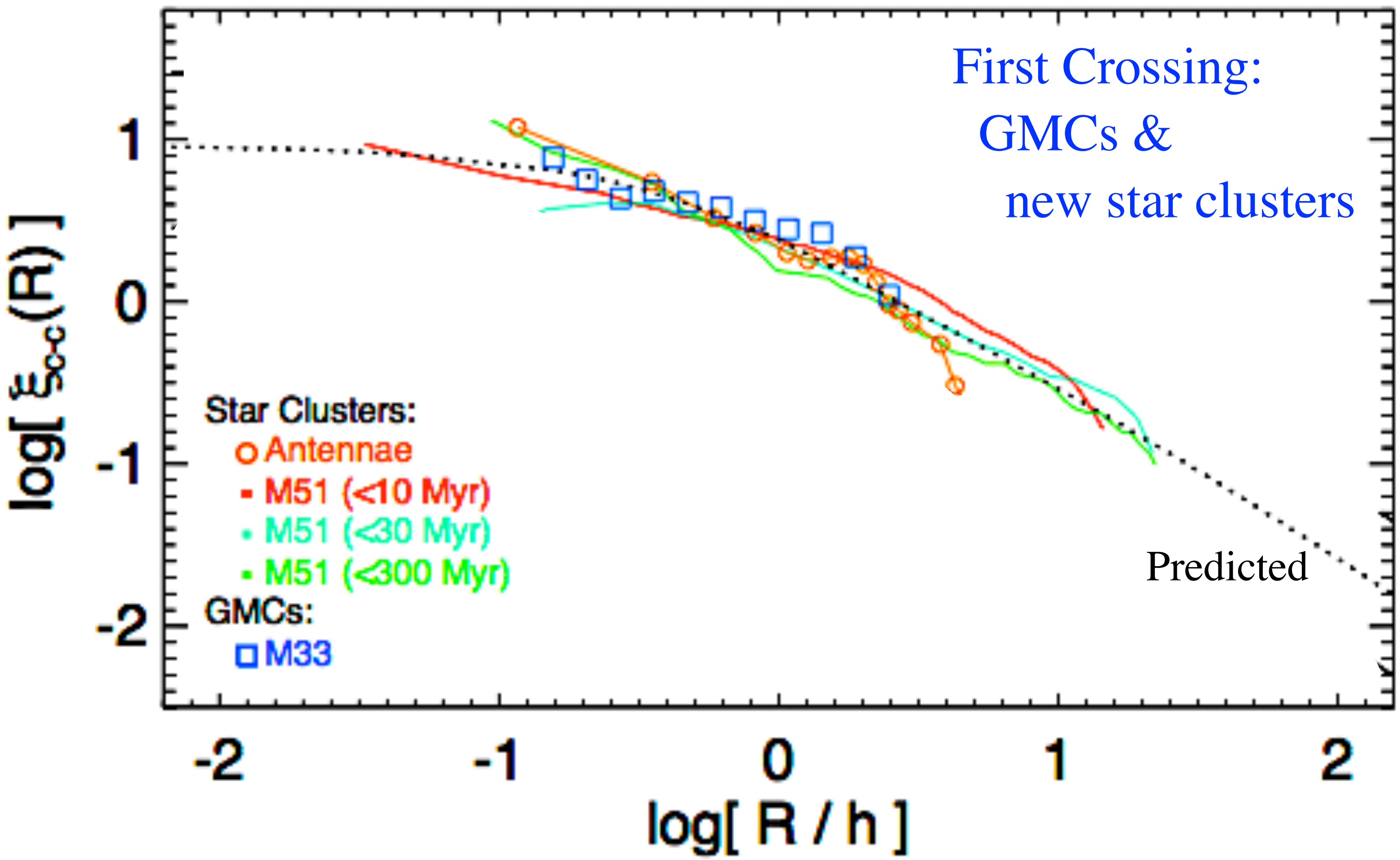


KEEP?

Clustering

PREDICT N-POINT CORRELATION FUNCTIONS

$$1 + \xi(r \,|\, M) \equiv \frac{\langle n[M \,|\, r' < r] \rangle}{\langle n[M] \rangle}$$

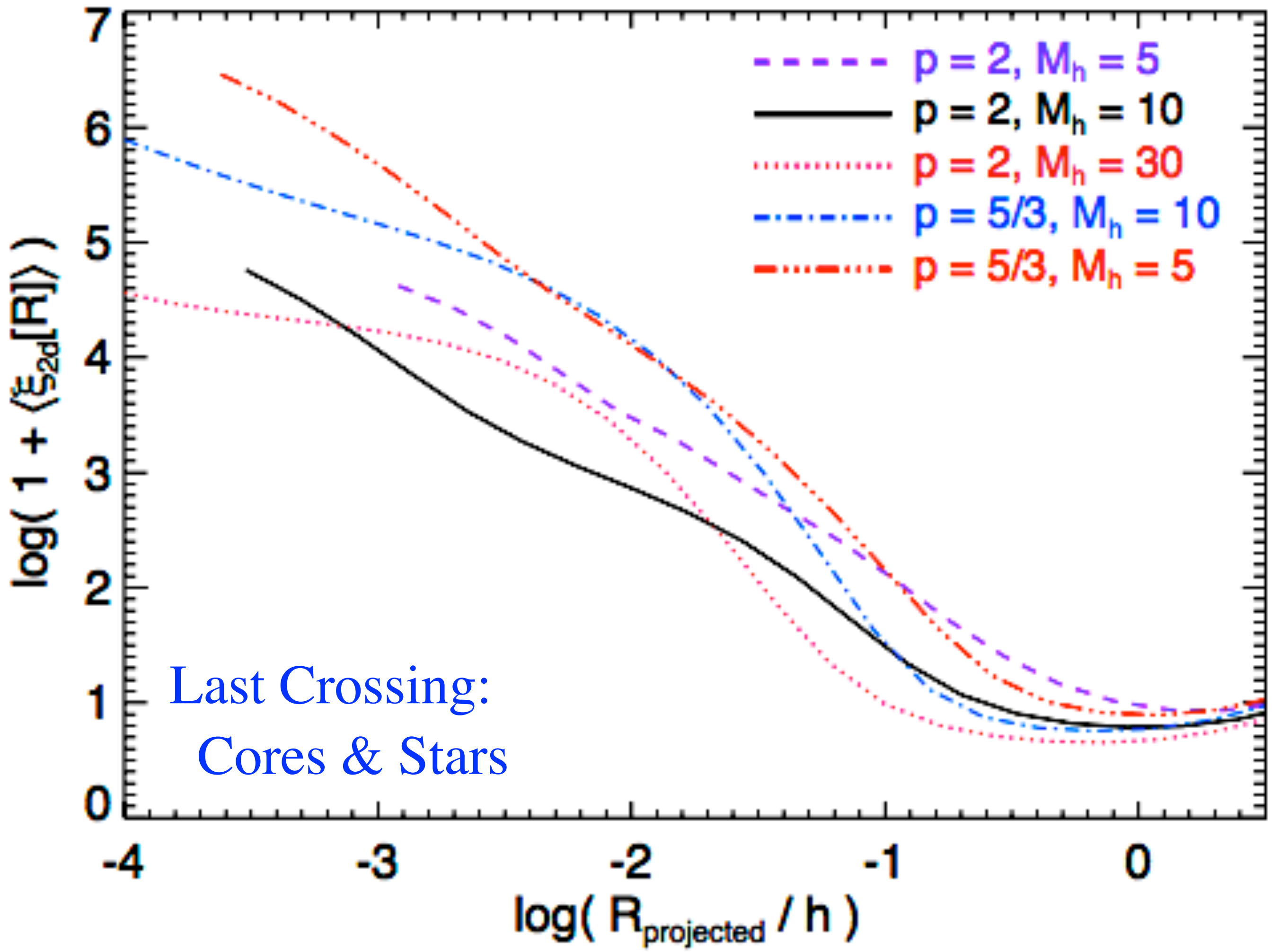


Clustering

PREDICT N-POINT CORRELATION FUNCTIONS

$$1 + \xi(r\,|\,M) \equiv \frac{\langle n[M\,|\,r' < r]\rangle}{\langle n[M]\rangle}$$

PFH 2012b

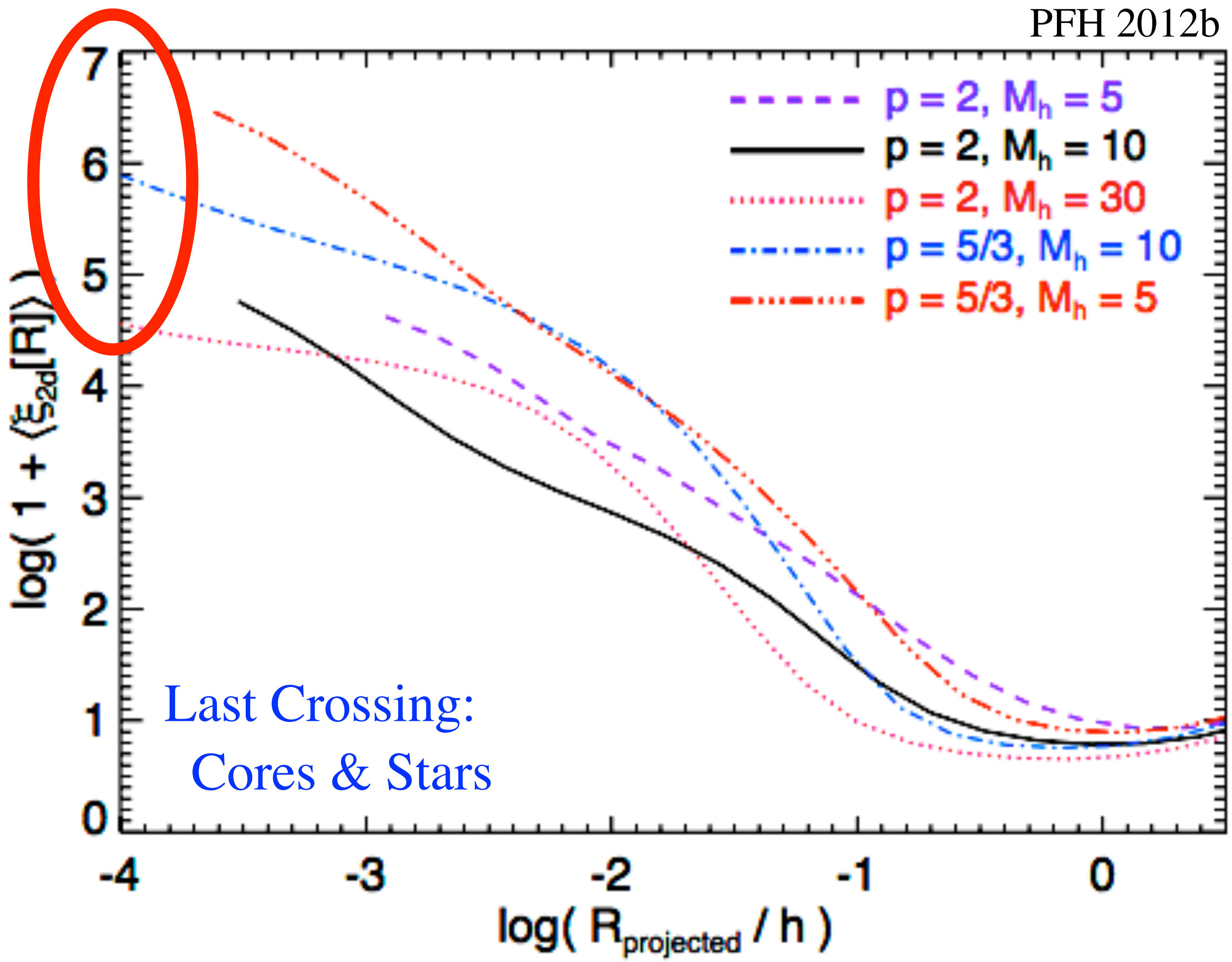


Clustering

PREDICT N-POINT CORRELATION FUNCTIONS

$$1 + \xi(r\,|\,M) \equiv \frac{\langle n[M\,|\,r' < r]\rangle}{\langle n[M]\rangle}$$

PFH 2012b



Clustering

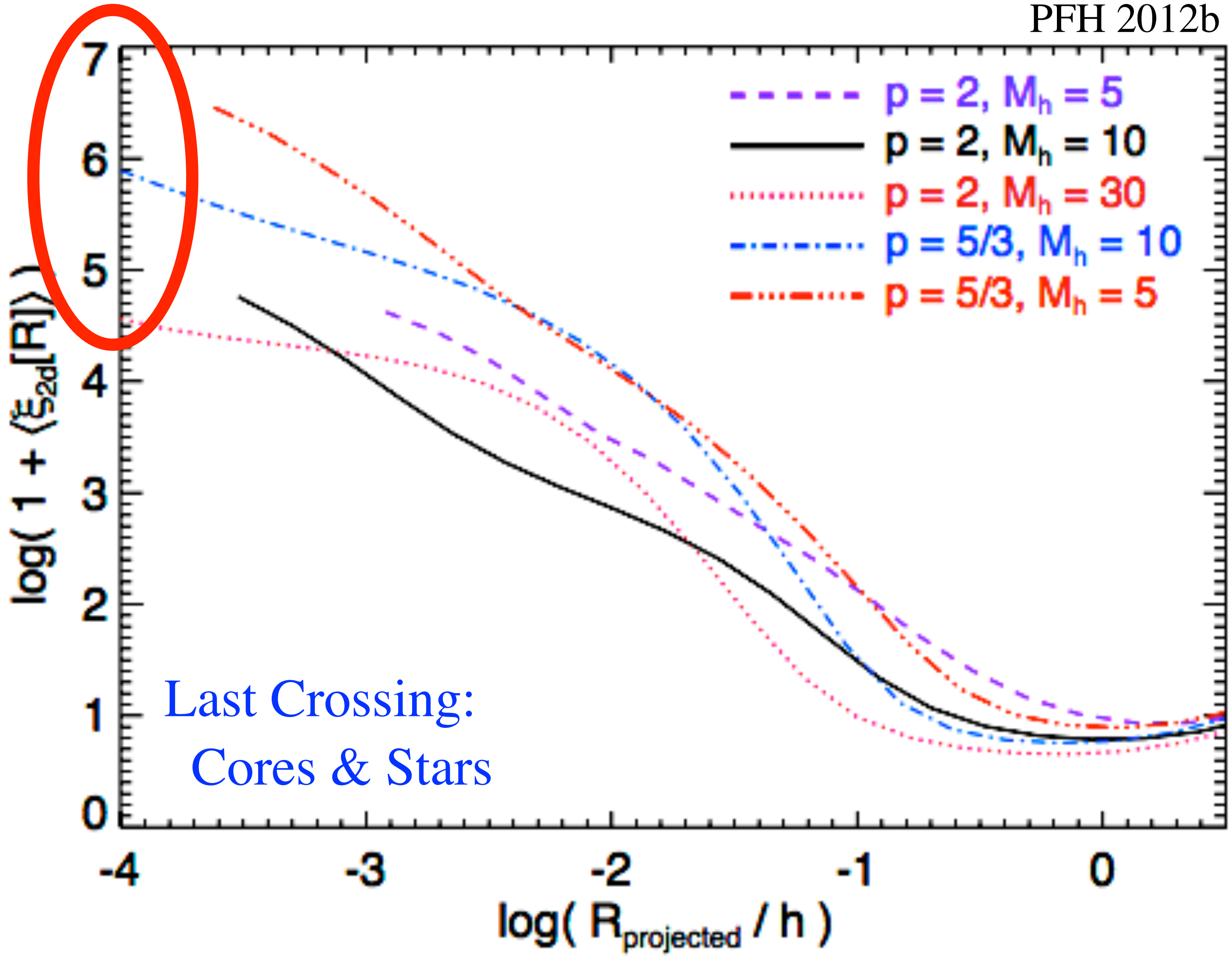
PREDICT N-POINT CORRELATION FUNCTIONS

$$1 + \xi(r \,|\, M) \equiv \frac{\langle n[M \,|\, r' < r] \rangle}{\langle n[M] \rangle}$$

Why is Star Formation Clustered?

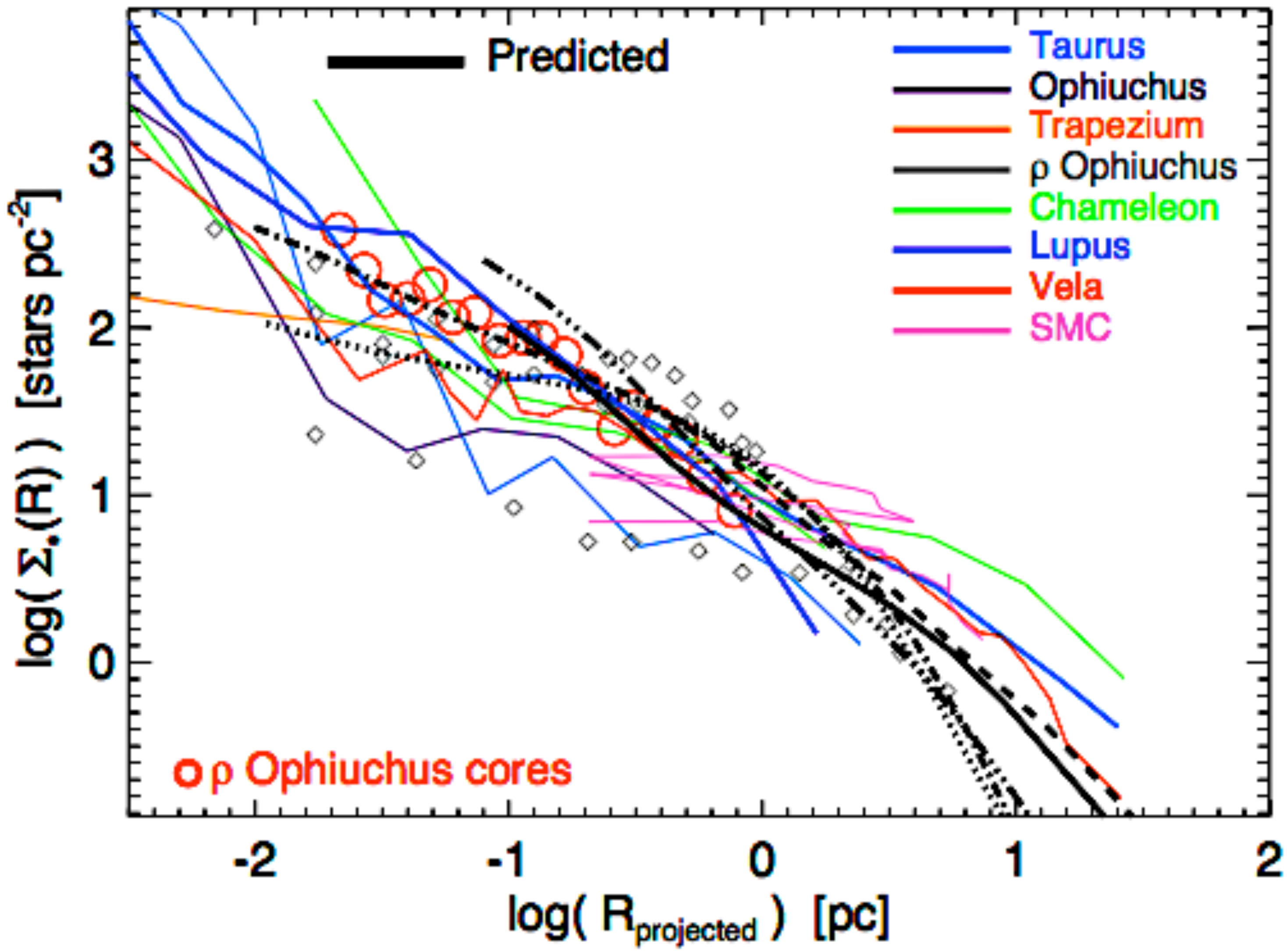
$$S \sim \ln \mathcal{M}(k)^2$$
$$\sim \ln r^{3-p}$$

PFH 2012b

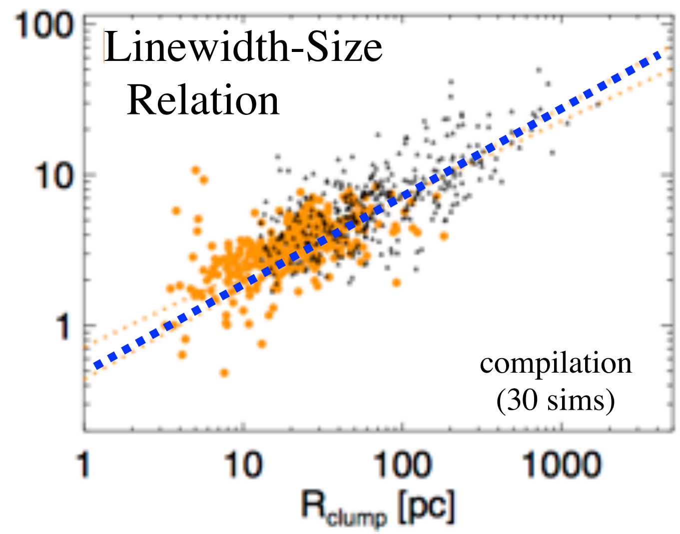


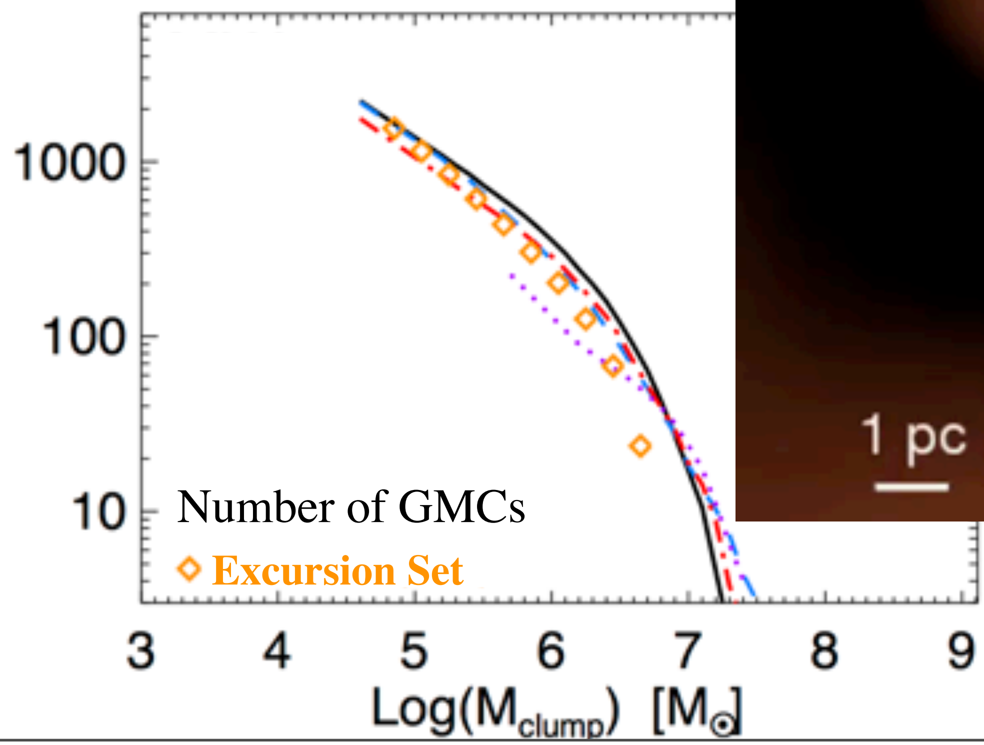
Clustering of Stars: Predicted vs. Observations

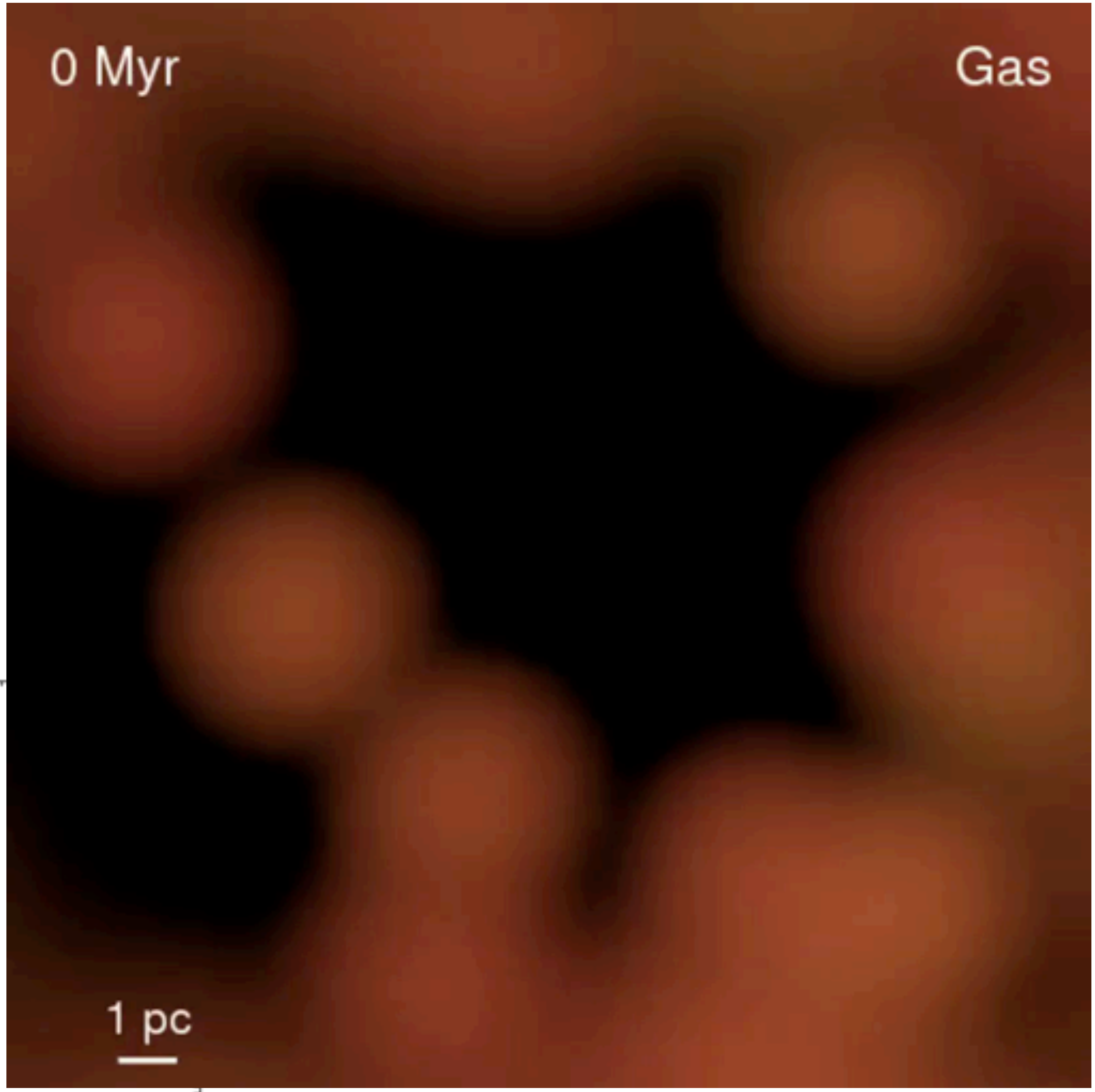
PREDICT N-POINT CORRELATION FUNCTIONS



Testing the Analytics
vs. NUMERICAL SIMULATIONS

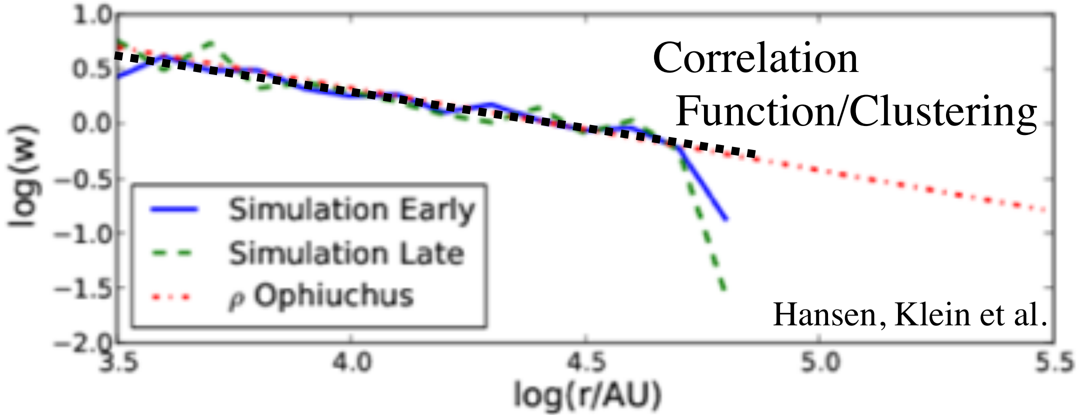


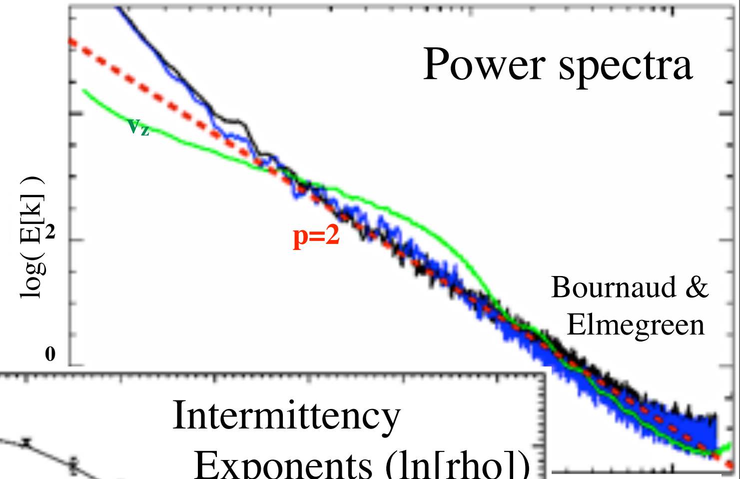


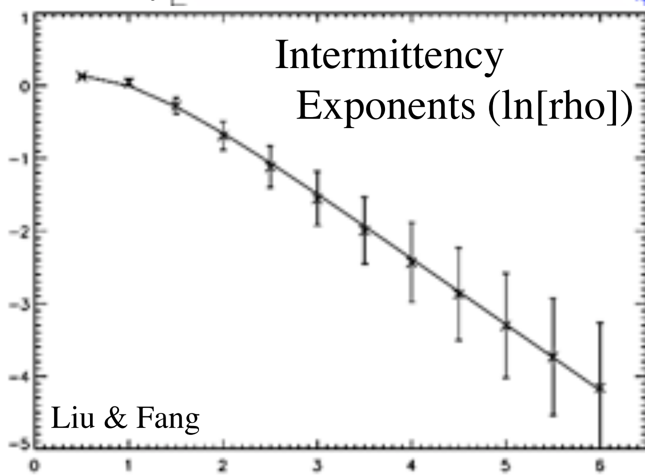


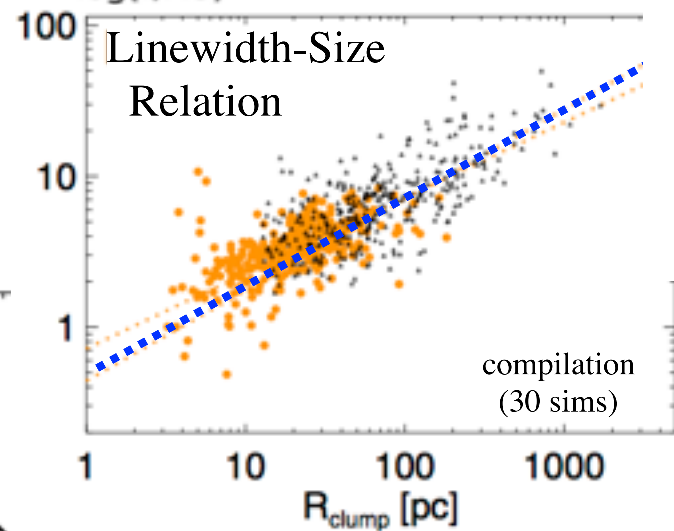
PFH

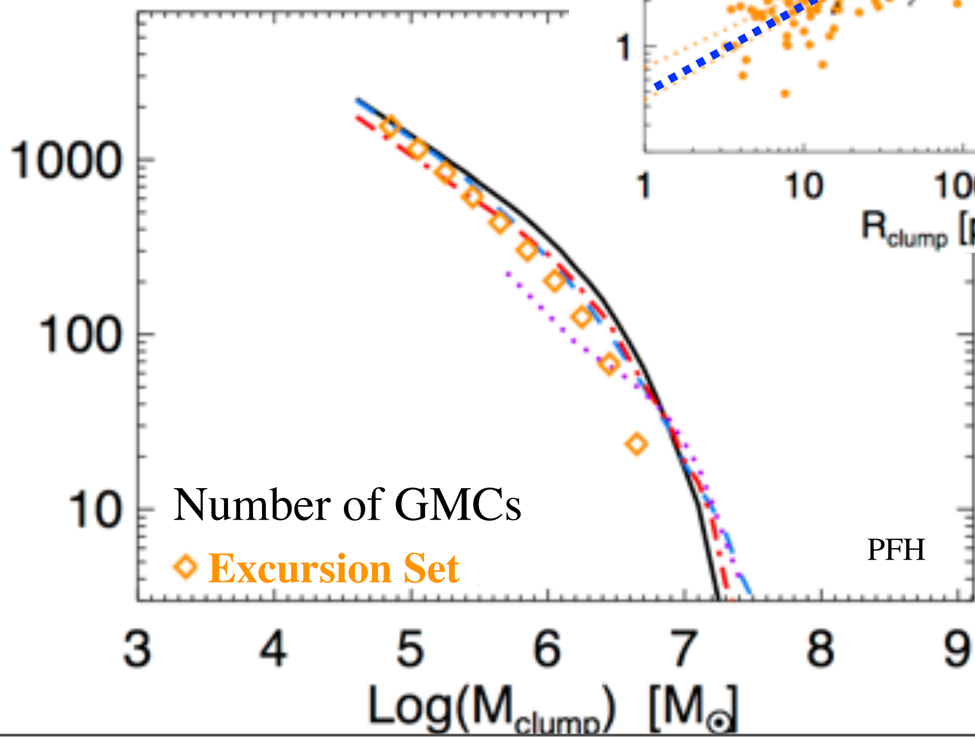
Testing the Analytics
vs. NUMERICAL SIMULATIONS

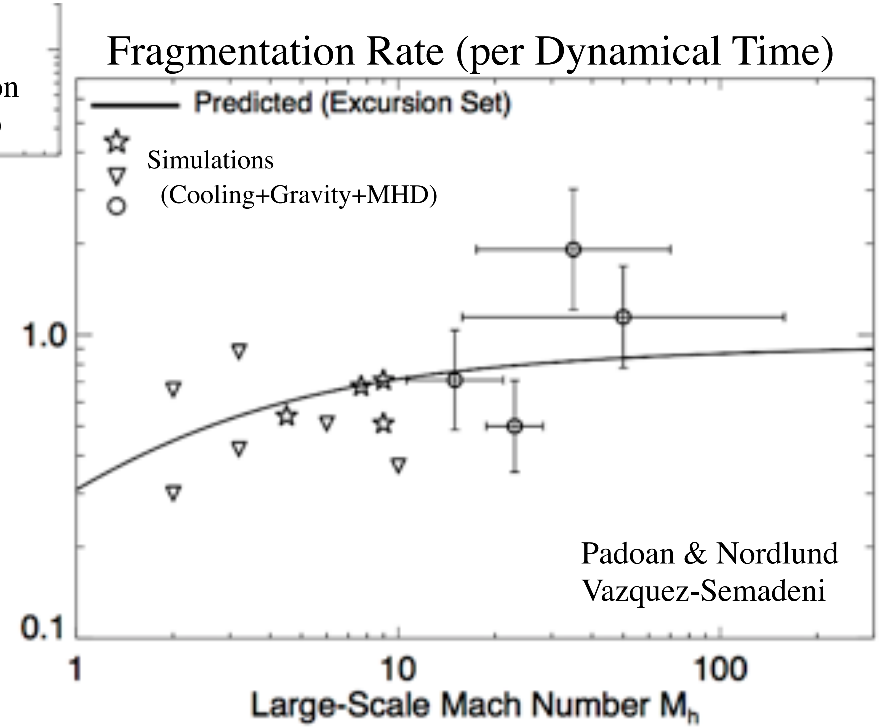








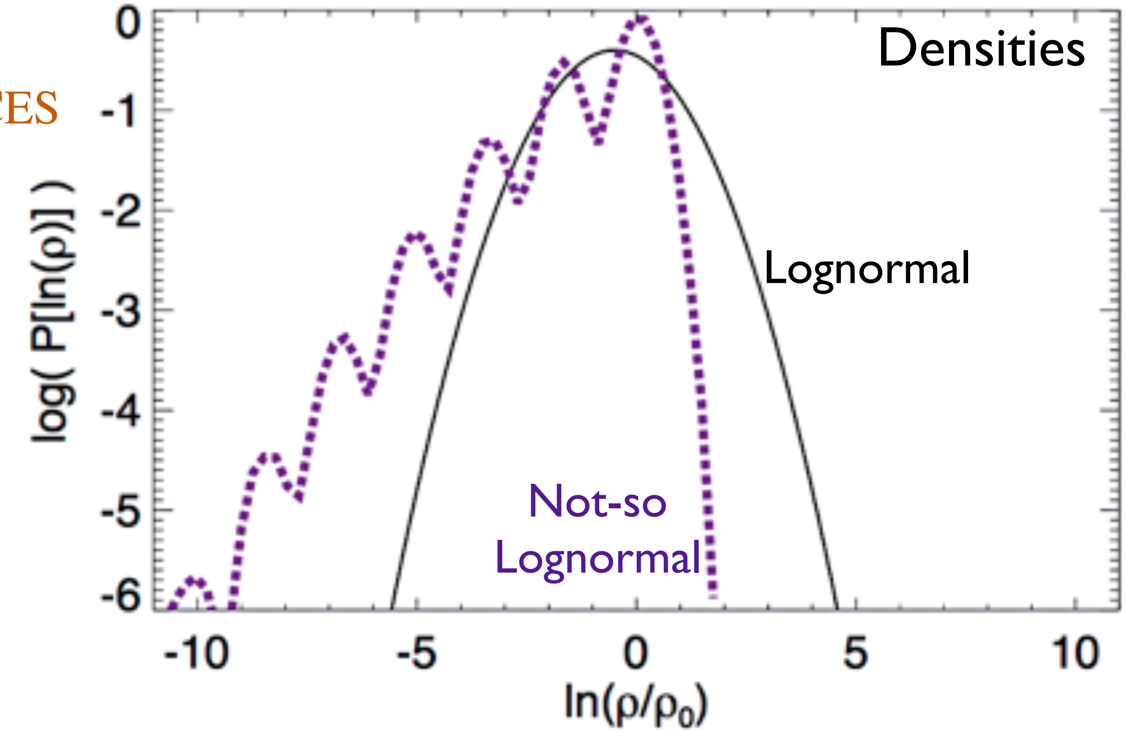


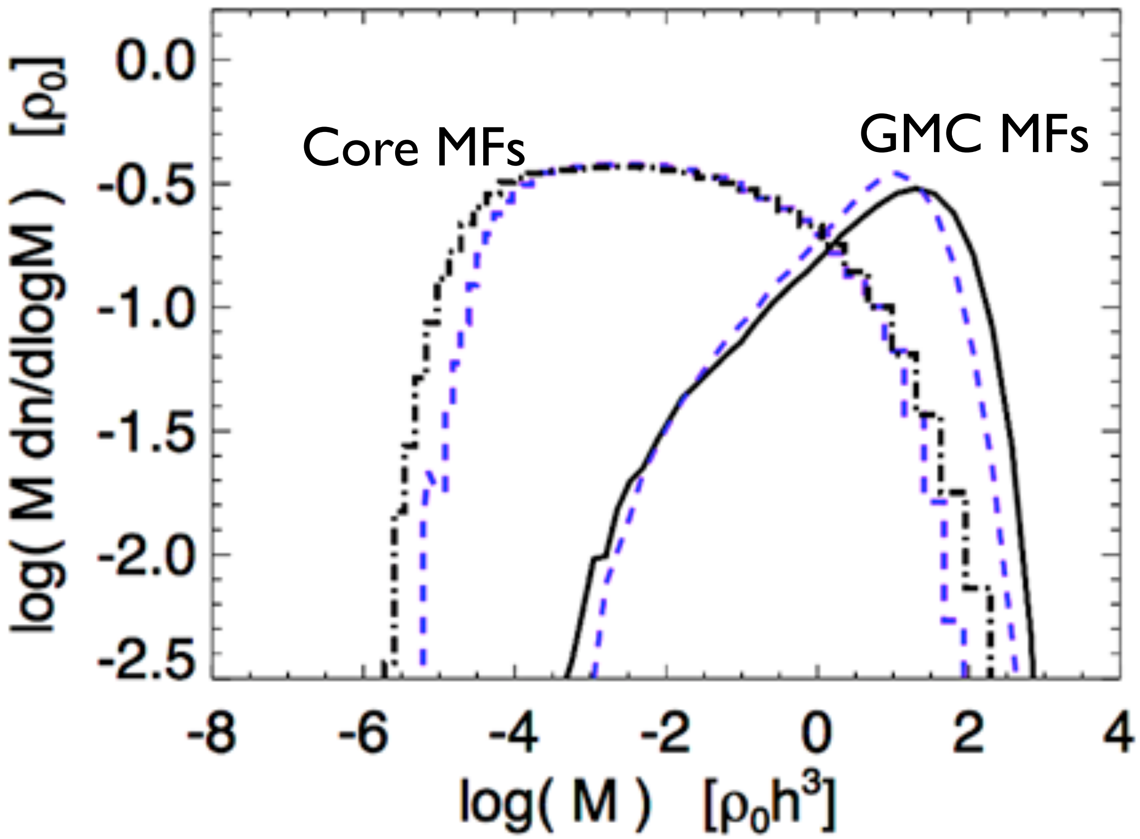


General, Flexible Theory:

EXTREMELY ADAPTABLE TO MOST CHOICES

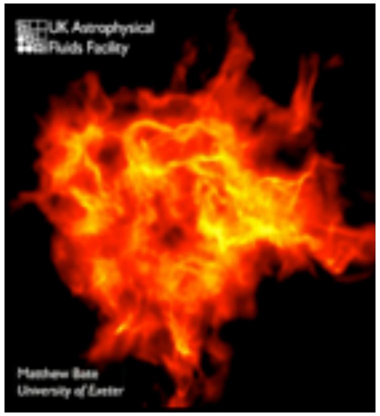
- Complicated, multivariable gas equations of state
- Accretion
- Magnetic Fields
- Time-Dependent Background Evolution/Collapse
- Intermittency
- Correlated, multi-scale driving





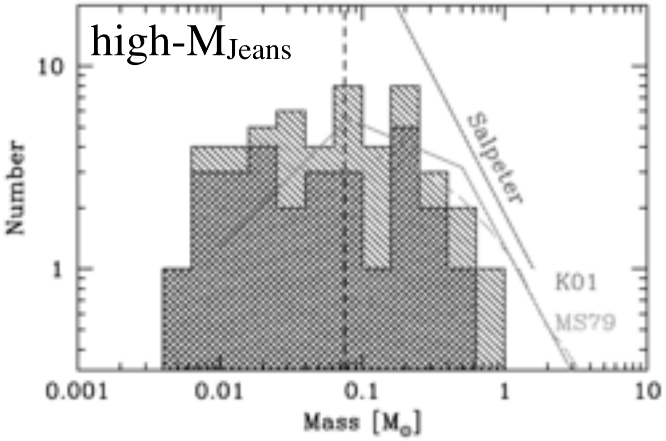
What Can We Say About Galactic-Scale IMF Variation?

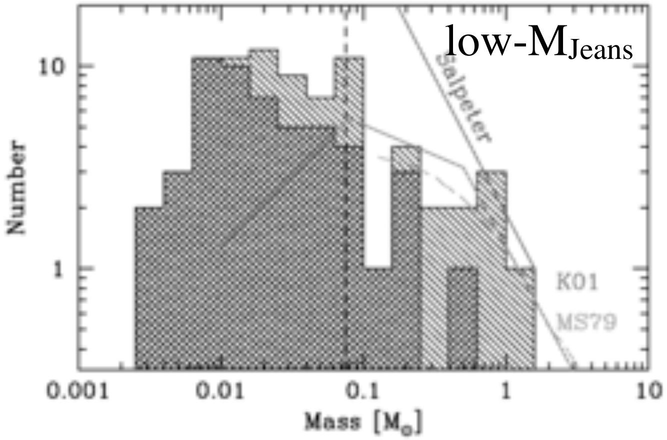
Variation in the Mass Function & the Jeans Mass



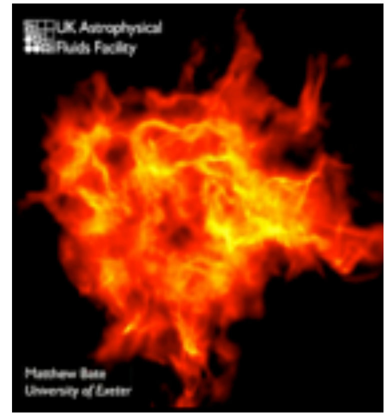
Most theories predict IMF *locally:*

Bate & Bonnell 2005
Jappsen 2005



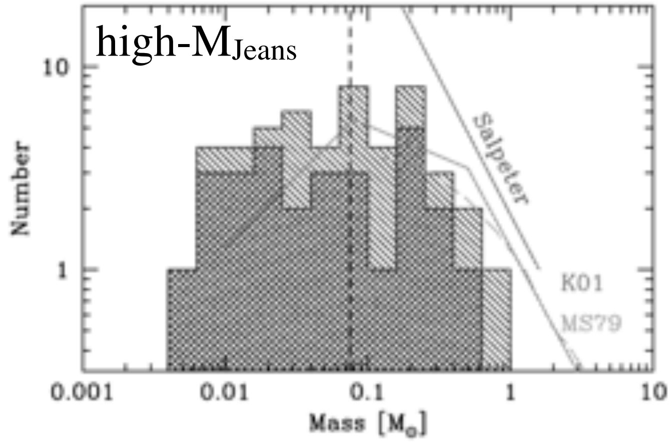


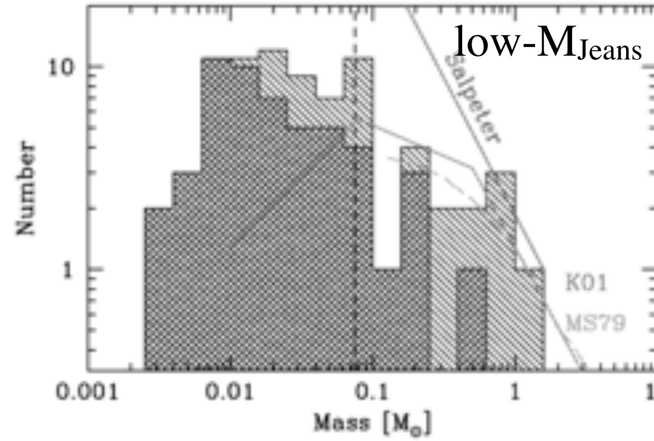
Variation in the Mass Function & the Jeans Mass



Most theories predict IMF *locally:*

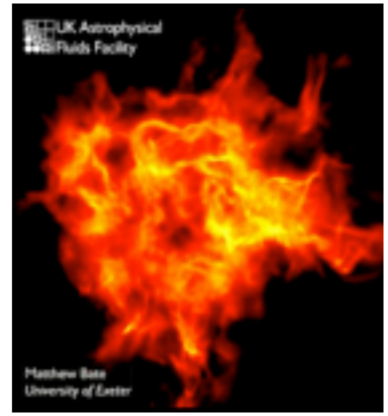
Bate & Bonnell 2005
Jappsen 2005





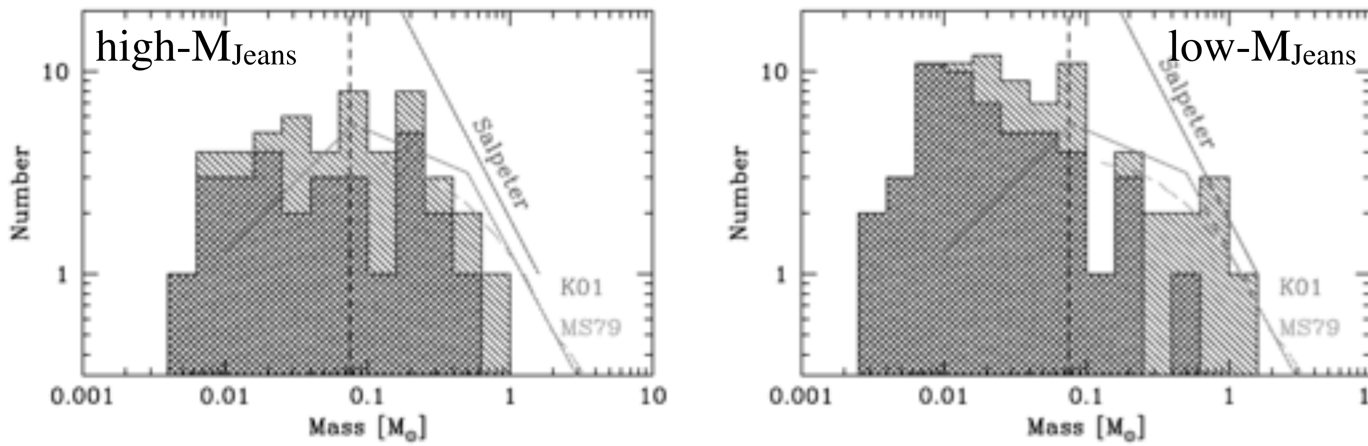
Larson 2005: It's all Jeans mass

Variation in the Mass Function & the Jeans Mass



Most theories predict IMF *locally:*

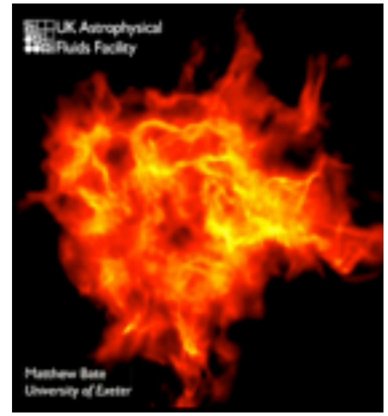
Bate & Bonnell 2005
Jappsen 2005



Larson 2005: It's all Jeans mass

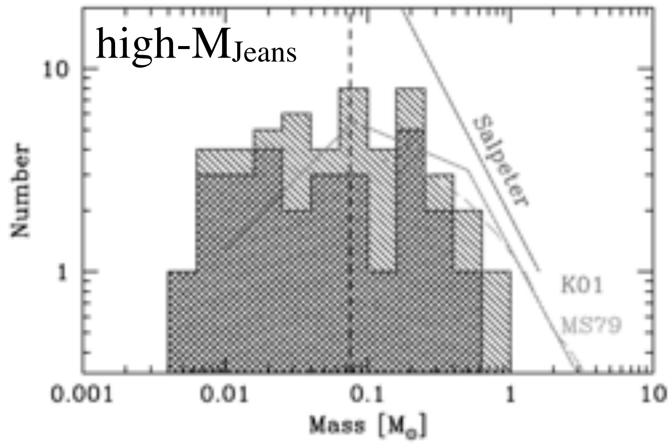
Dave 2008: Extrapolate to galaxies....

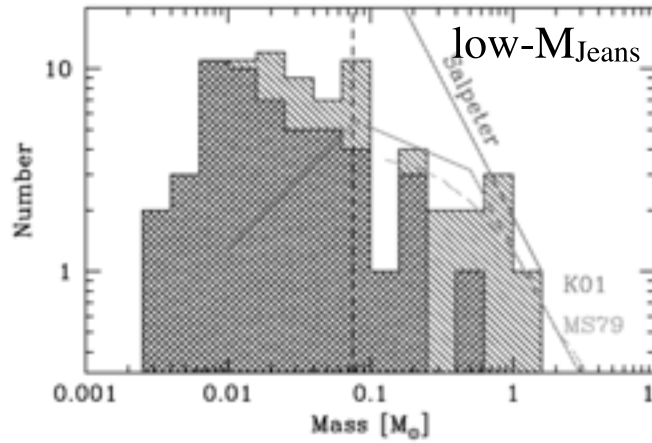
Variation in the Mass Function & the Jeans Mass



Most theories predict IMF *locally:*

Bate & Bonnell 2005
Jappsen 2005



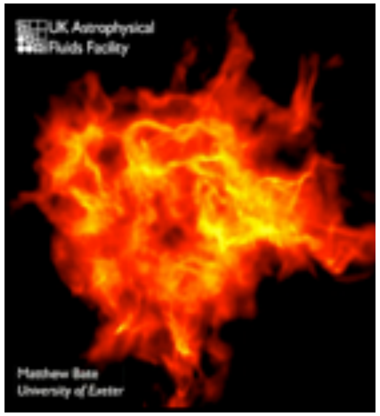


Larson 2005: It's all Jeans mass

Dave 2008: Extrapolate to galaxies....

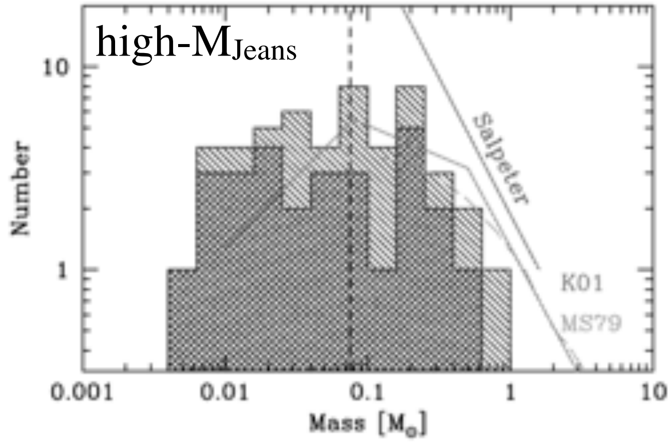
- $M_{IMF} \sim T_{min}^{1.7\text{-}2.3}$

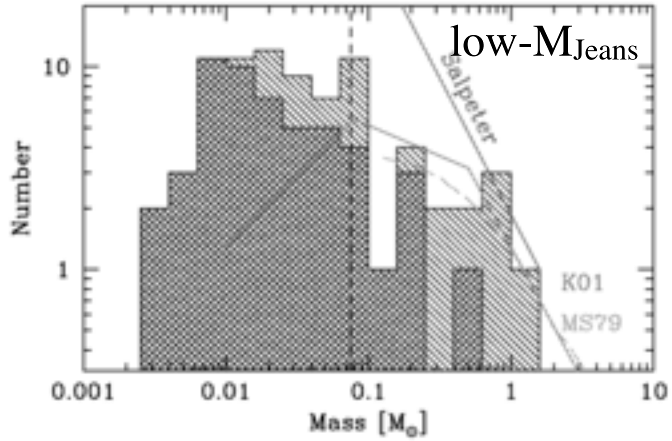
Variation in the Mass Function & the Jeans Mass



Most theories predict IMF *locally:*

Bate & Bonnell 2005
Jappsen 2005



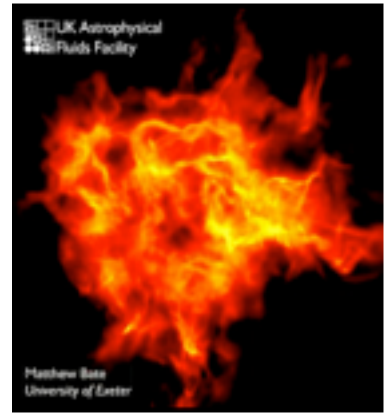


Larson 2005: It's all Jeans mass

Dave 2008: Extrapolate to galaxies....

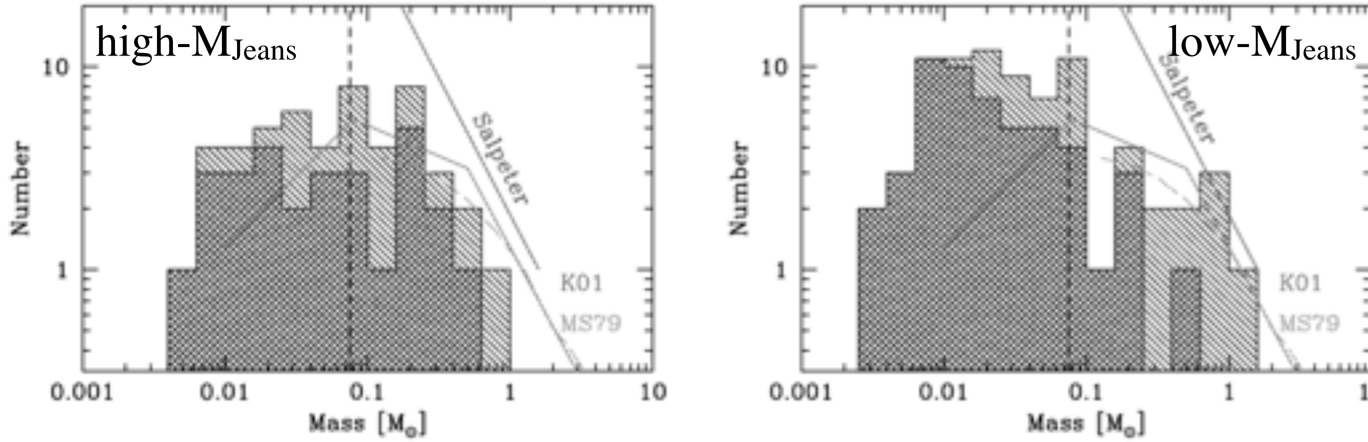
- $M_{IMF} \sim T_{min}^{1.7\text{-}2.3}$
- T_{CMB} is not interesting at z<6

Variation in the Mass Function & the Jeans Mass



Most theories predict IMF *locally:*

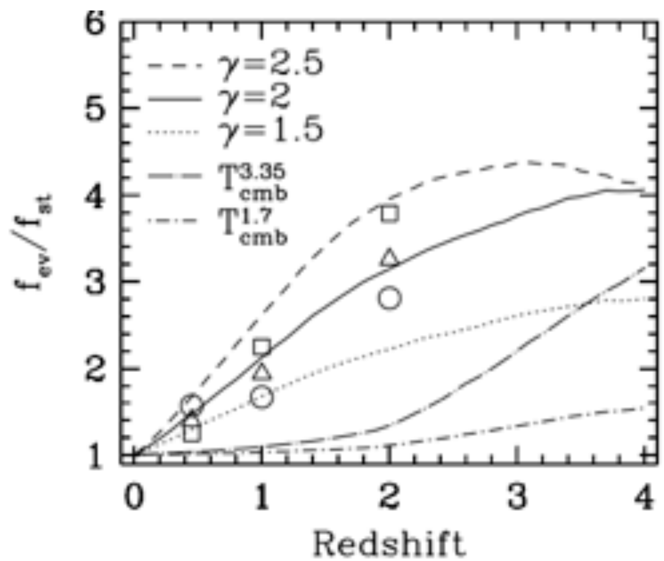
Bate & Bonnell 2005
Jappsen 2005



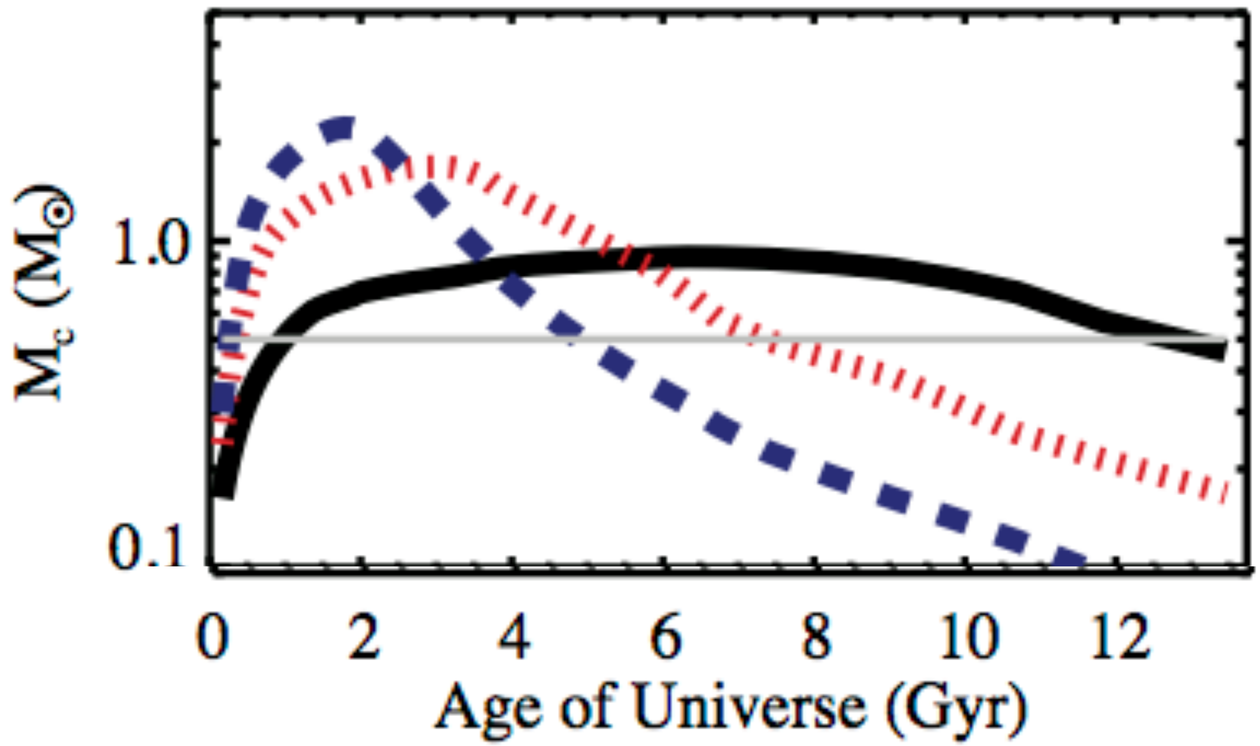
Larson 2005: It's all Jeans mass

Dave 2008: Extrapolate to galaxies....

- $M_{IMF} \sim T_{min}^{1.7\text{-}2.3}$
- T_{CMB} is not interesting at z<6

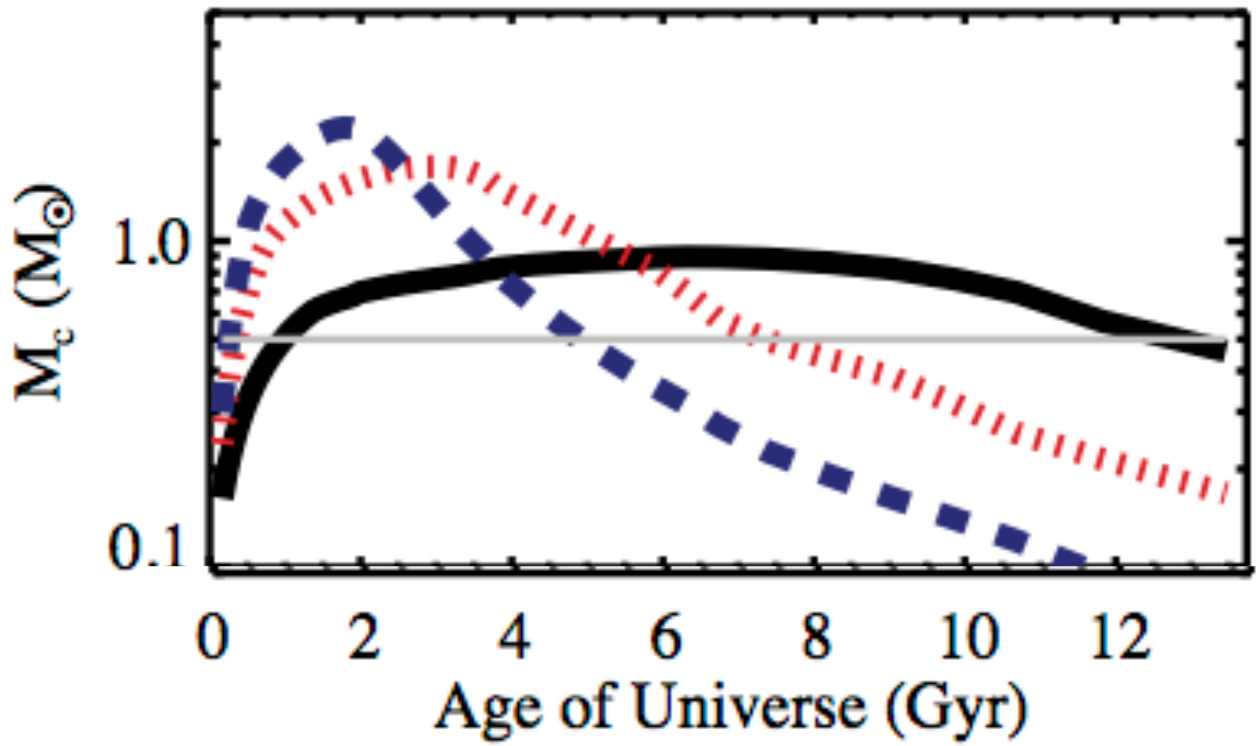


Variation in the Mass Function & the Jeans Mass



Variation in the Mass Function & the Jeans Mass

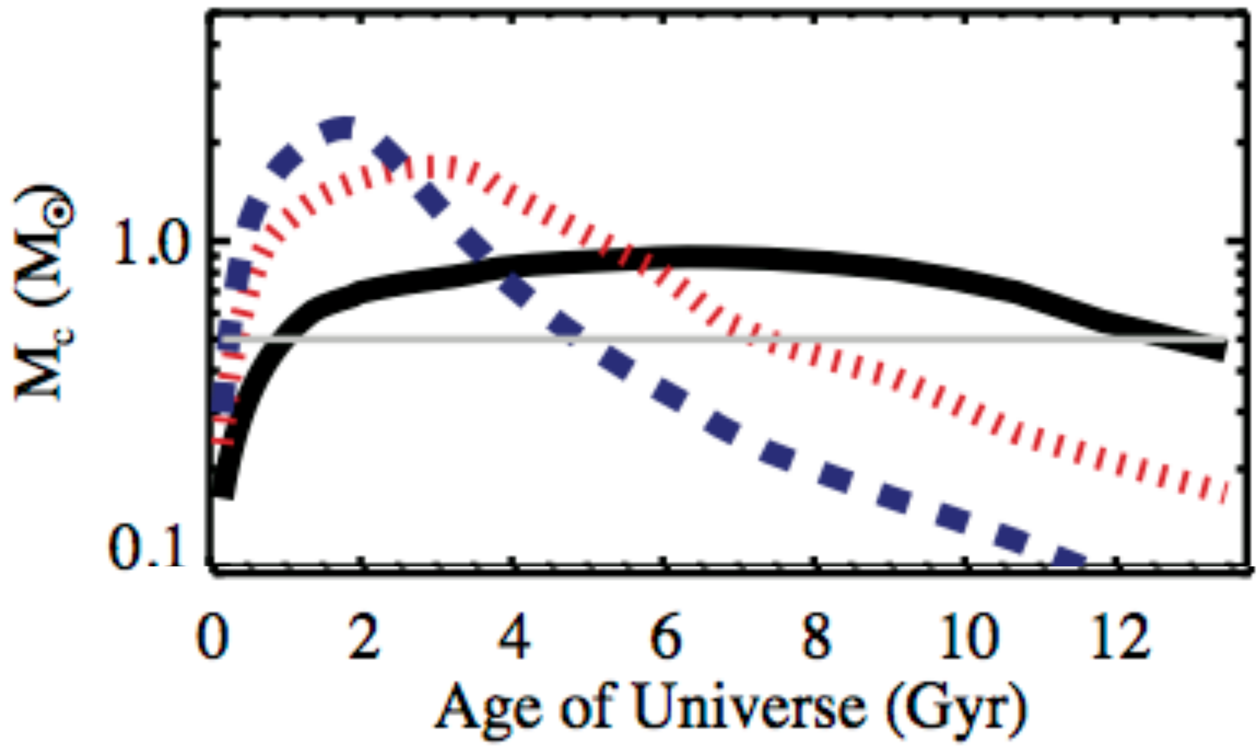
Narayanan 2012: estimate "mean" thermal state of clouds



Variation in the Mass Function & the Jeans Mass

Narayanan 2012: estimate "mean" thermal state of clouds

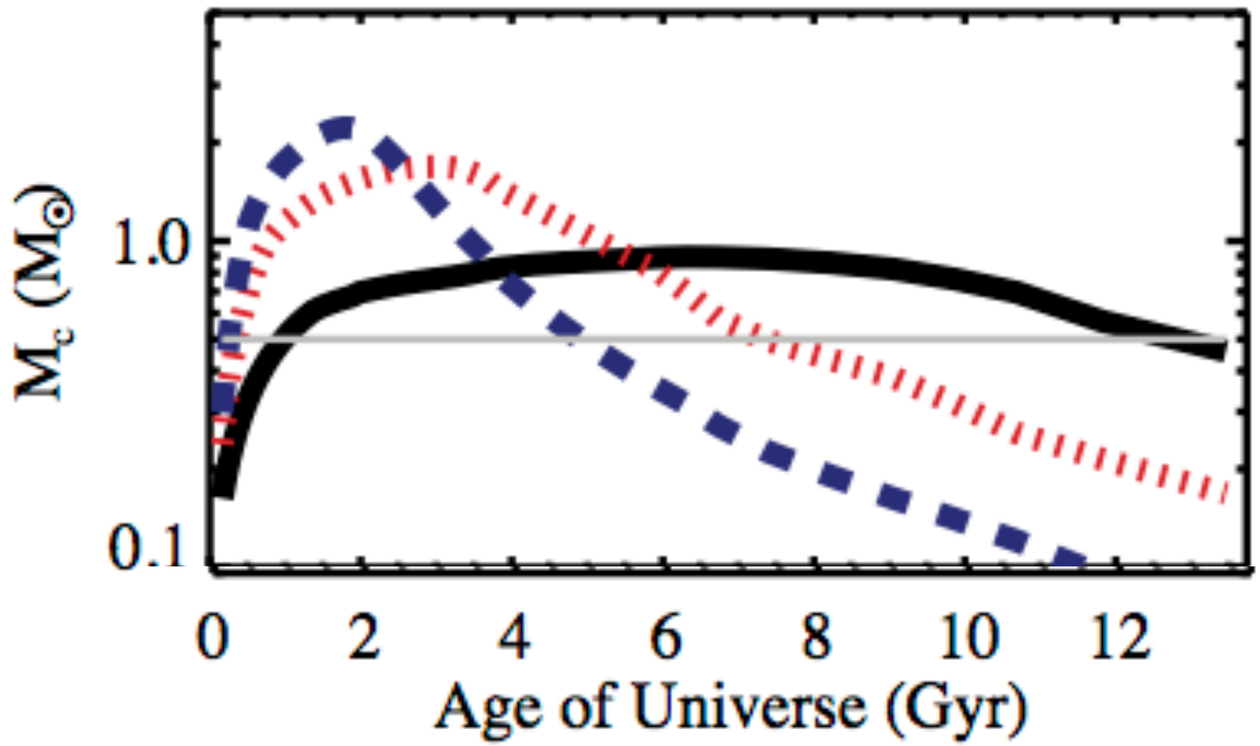
- High-z: Higher SFR, more heating (CRs & photons)



Variation in the Mass Function & the Jeans Mass

Narayanan 2012: estimate “mean” thermal state of clouds

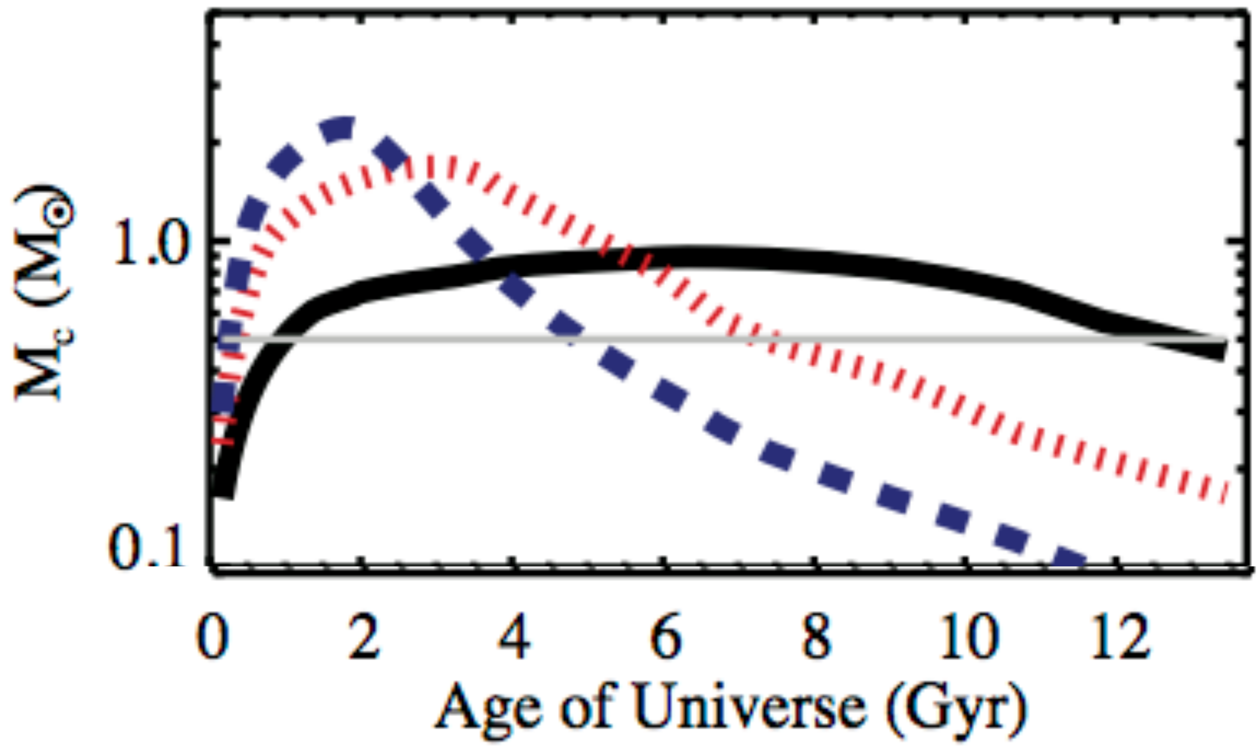
- High-z: Higher SFR, more heating (CRs & photons)
  - Also lower-metallicity: less cooling (Marks et al., others)



Variation in the Mass Function & the Jeans Mass

Narayanan 2012: estimate "mean" thermal state of clouds

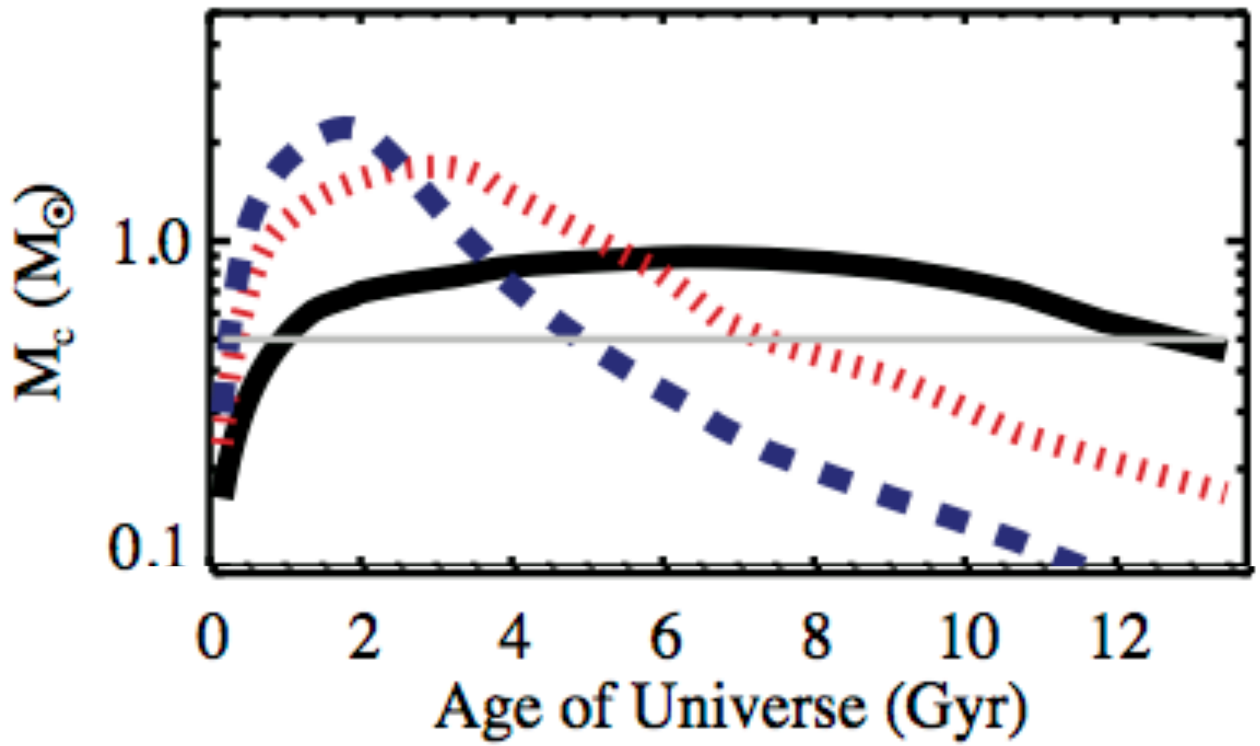
- High-z: Higher SFR, more heating (CRs & photons)
  - Also lower-metallicity: less cooling (Marks et al., others)
- Mergers (bulge-makers) tend to higher T_{min}



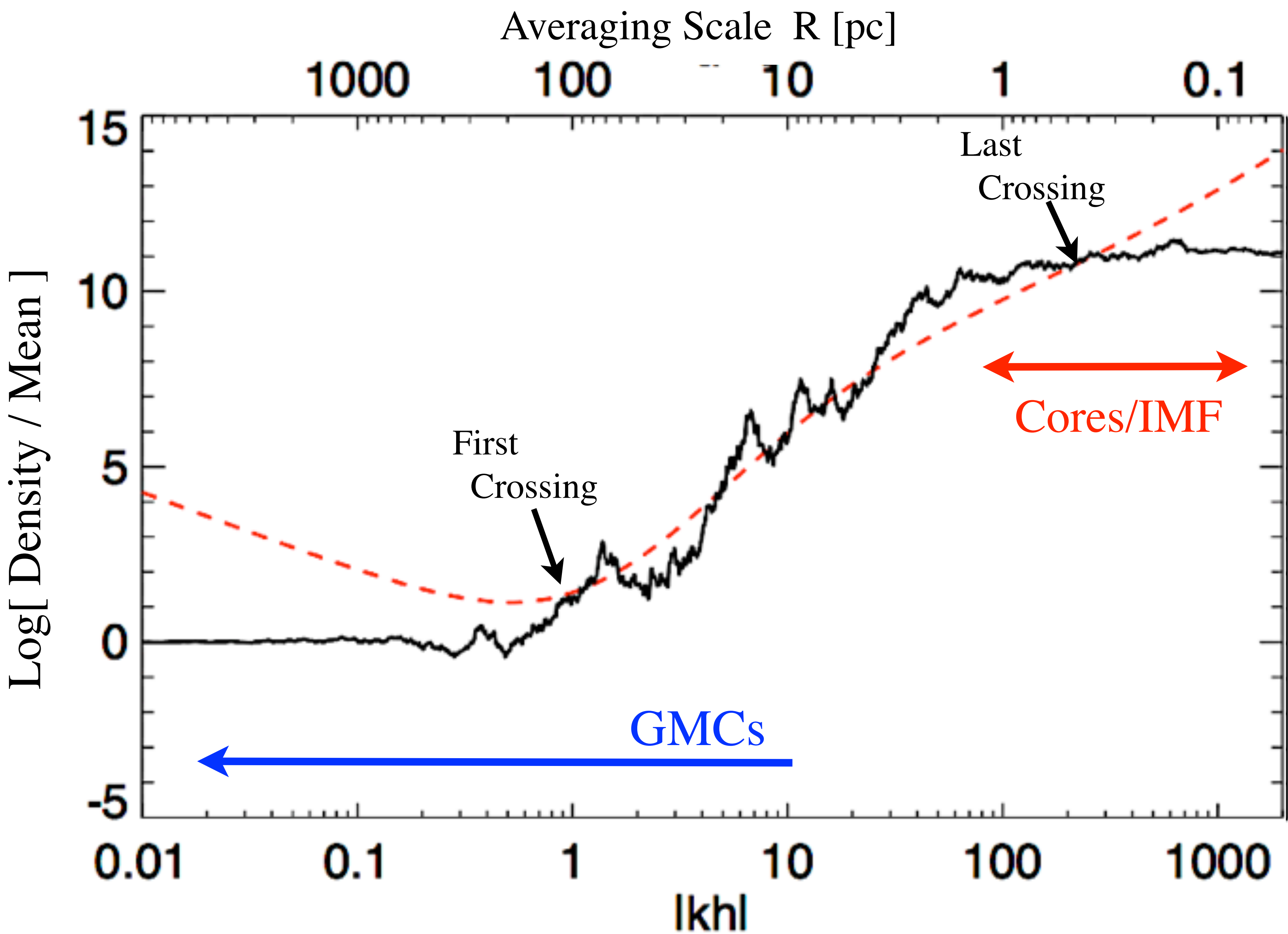
Variation in the Mass Function & the Jeans Mass

Narayanan 2012: estimate “mean” thermal state of clouds

- High-z: Higher SFR, more heating (CRs & photons)
  - Also lower-metallicity: less cooling (Marks et al., others)
- Mergers (bulge-makers) tend to higher T_{min}
  - Observed (Downes & Solomon, Bryant & Scoville)

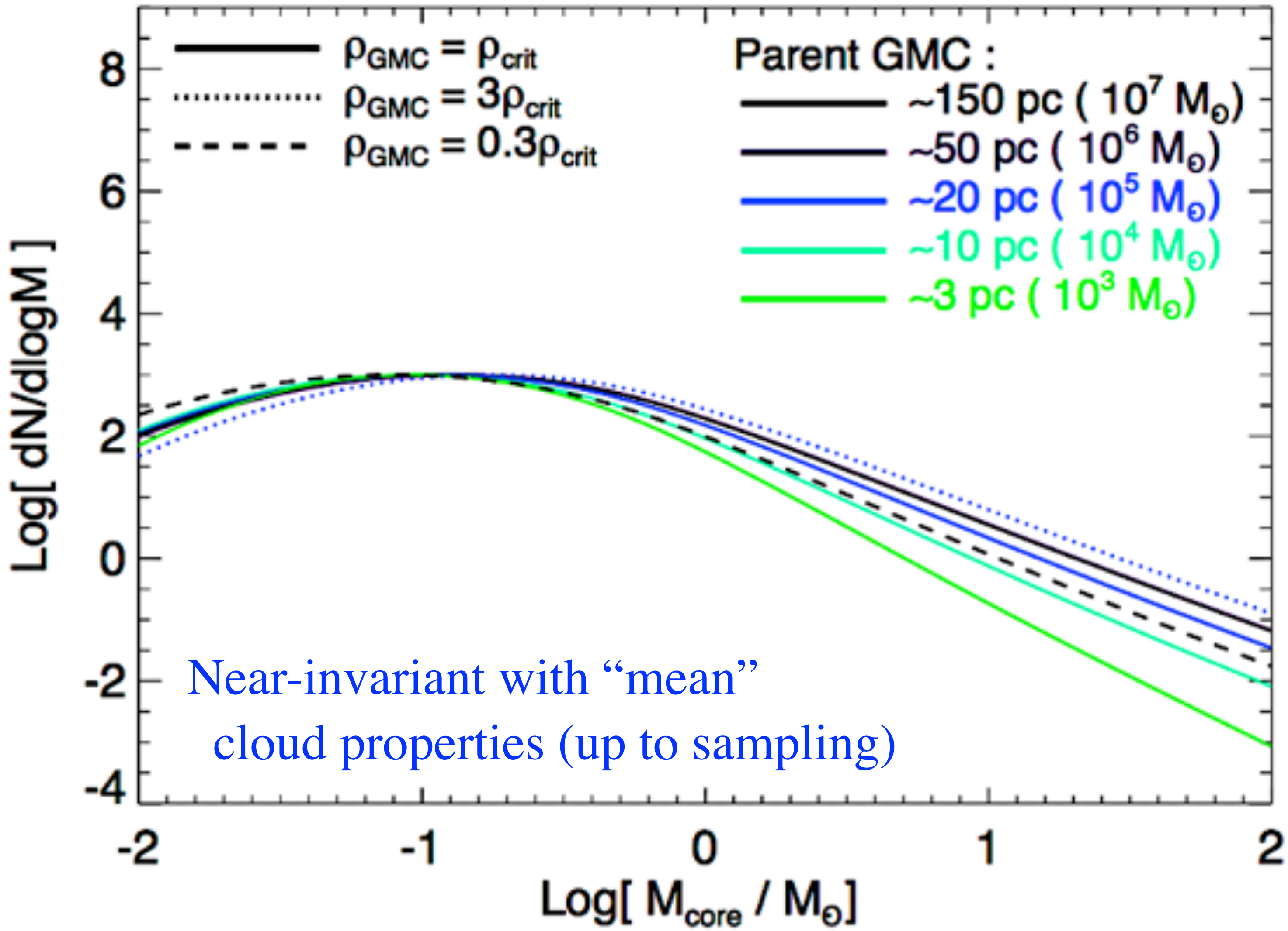


Variation in the Core Mass Function



Variation in the Core Mass Function

VS "NORMAL" IMF VARIATIONS



Variation in the Core Mass Function

VS "NORMAL" IMF VARIATIONS

Variation in the Core Mass Function

VS "NORMAL" IMF VARIATIONS

Jeans Length & Mass: $\ell_{\rm Jeans} \sim \dfrac{c_s}{\sqrt{G\,\rho}}$ $\qquad M_{\rm Jeans} \sim \dfrac{c_s^3}{G^{3/2}\,\rho^{1/2}}$

Variation in the Core Mass Function

VS "NORMAL" IMF VARIATIONS

Jeans Length & Mass: $\ell_{\rm Jeans} \sim \dfrac{c_s}{\sqrt{G\,\rho}}$ $\qquad M_{\rm Jeans} \sim \dfrac{c_s^3}{G^{3/2}\,\rho^{1/2}}$

Sonic Length & Mass: $R_{\rm Sonic} \sim R(\mathcal{M}=1) \sim h_{\rm disk}\,\mathcal{M}(h_{\rm disk})^{-2}$

Variation in the Core Mass Function

VS "NORMAL" IMF VARIATIONS

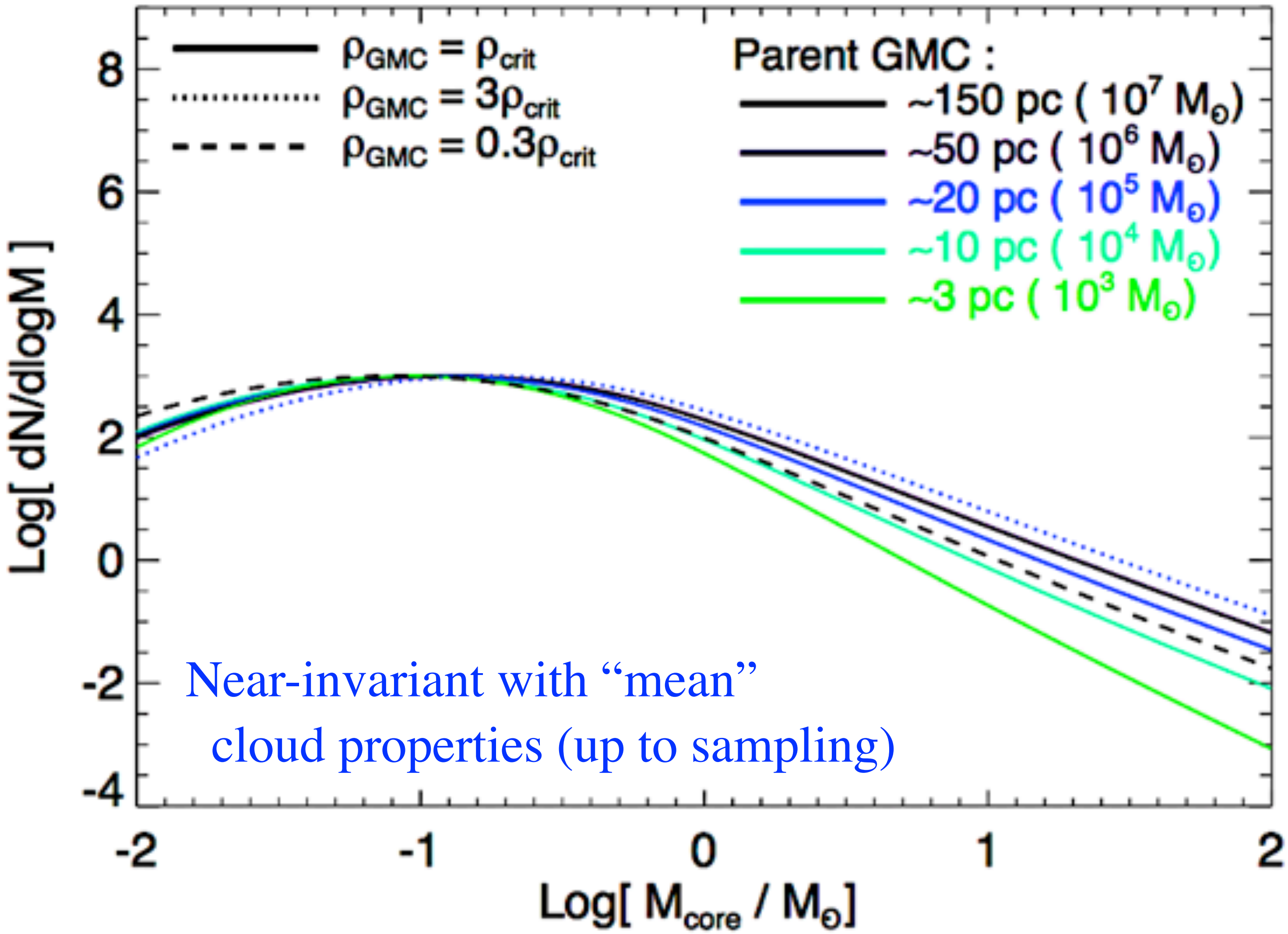
Jeans Length & Mass: $\ell_{\rm Jeans} \sim \dfrac{c_s}{\sqrt{G\,\rho}}$ $\qquad M_{\rm Jeans} \sim \dfrac{c_s^3}{G^{3/2}\,\rho^{1/2}}$

Sonic Length & Mass: $R_{\rm Sonic} \sim R(\mathcal{M}=1) \sim h_{\rm disk}\,\mathcal{M}(h_{\rm disk})^{-2}$

$$M_{\rm sonic} \sim \frac{c_s^2\,R_{\rm sonic}}{G} \sim \frac{c_s^4}{G^2\,Q_{\rm disk}\,\Sigma_{\rm disk}}$$

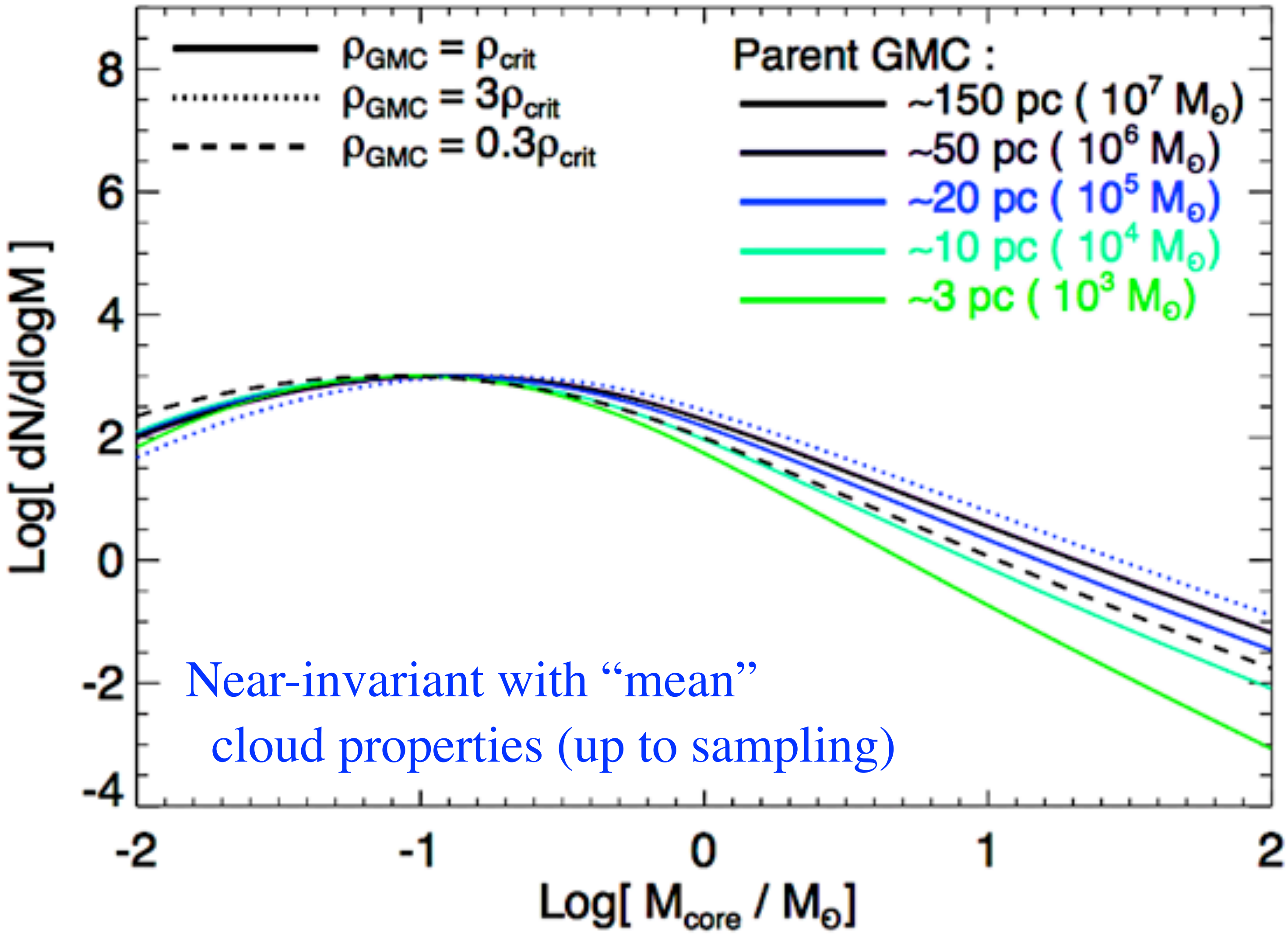
Variation in the Core Mass Function

VS "NORMAL" IMF VARIATIONS



Variation in the Core Mass Function

VS "NORMAL" IMF VARIATIONS

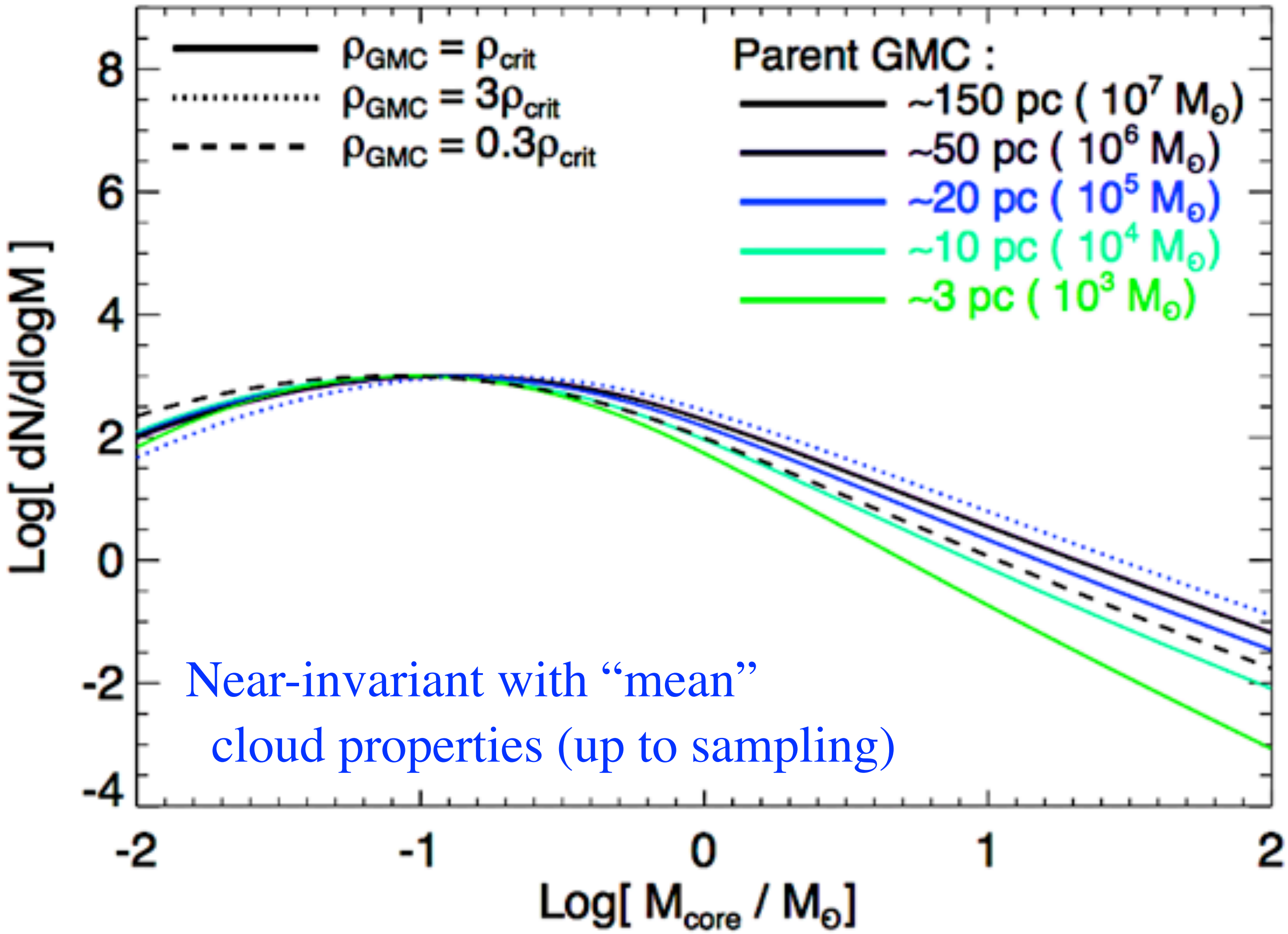


Near-invariant with "mean" cloud properties (up to sampling)

$$M_{\rm sonic} \equiv M(\rho_{\rm crit}\,|R_{\rm sonic}) \sim \frac{c_s^2\,R_{\rm sonic}}{G}$$

Variation in the Core Mass Function

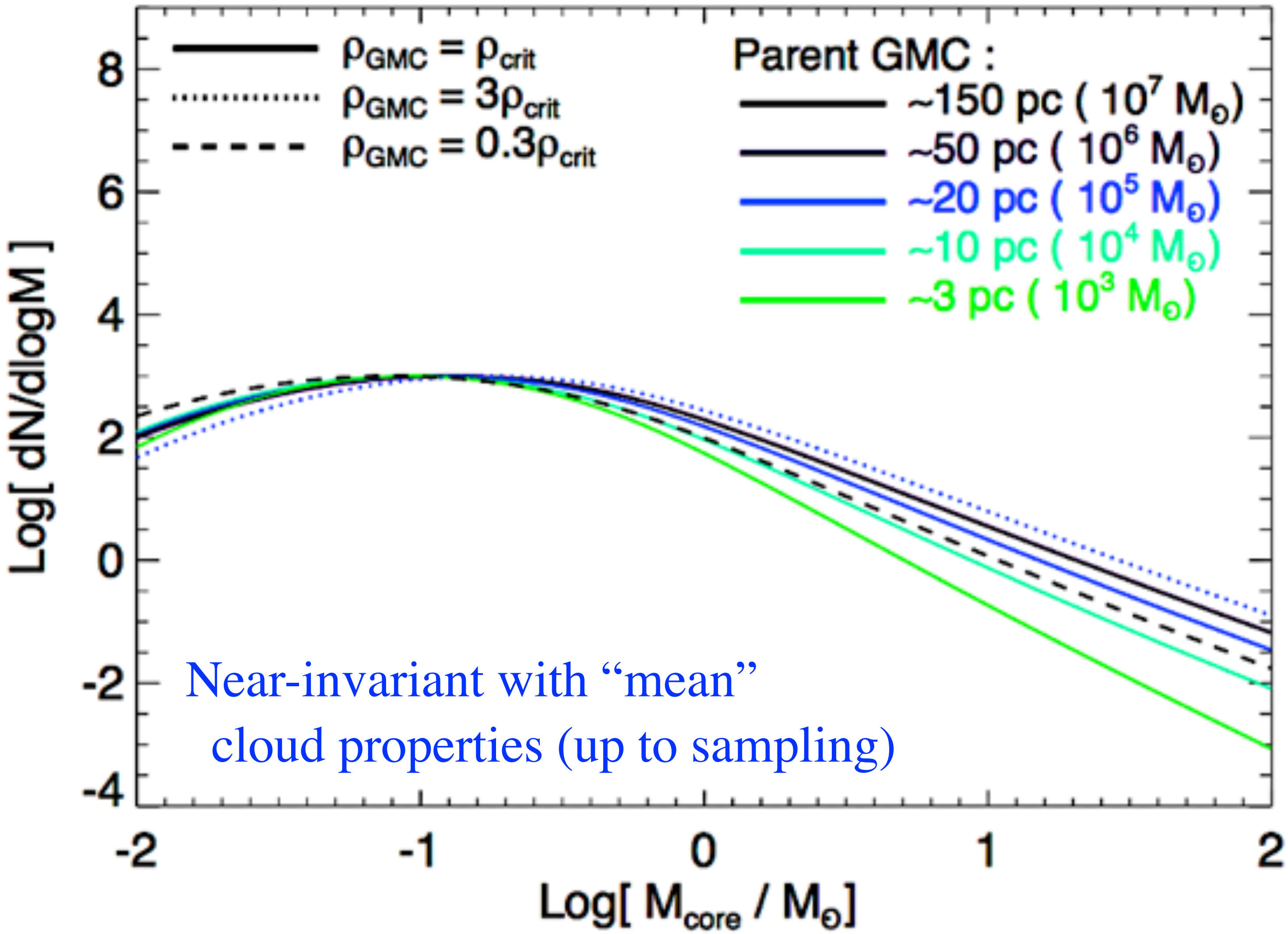
VS "NORMAL" IMF VARIATIONS



$$M_{\rm sonic} \equiv M(\rho_{\rm crit}\,|R_{\rm sonic}) \sim \frac{c_s^2\,R_{\rm sonic}}{G} \sim M_\odot\,\frac{T_{\rm min}}{10\,K}\,\frac{R_{\rm sonic}}{0.1\,{\rm pc}}$$

Variation in the Core Mass Function

VS “NORMAL” IMF VARIATIONS

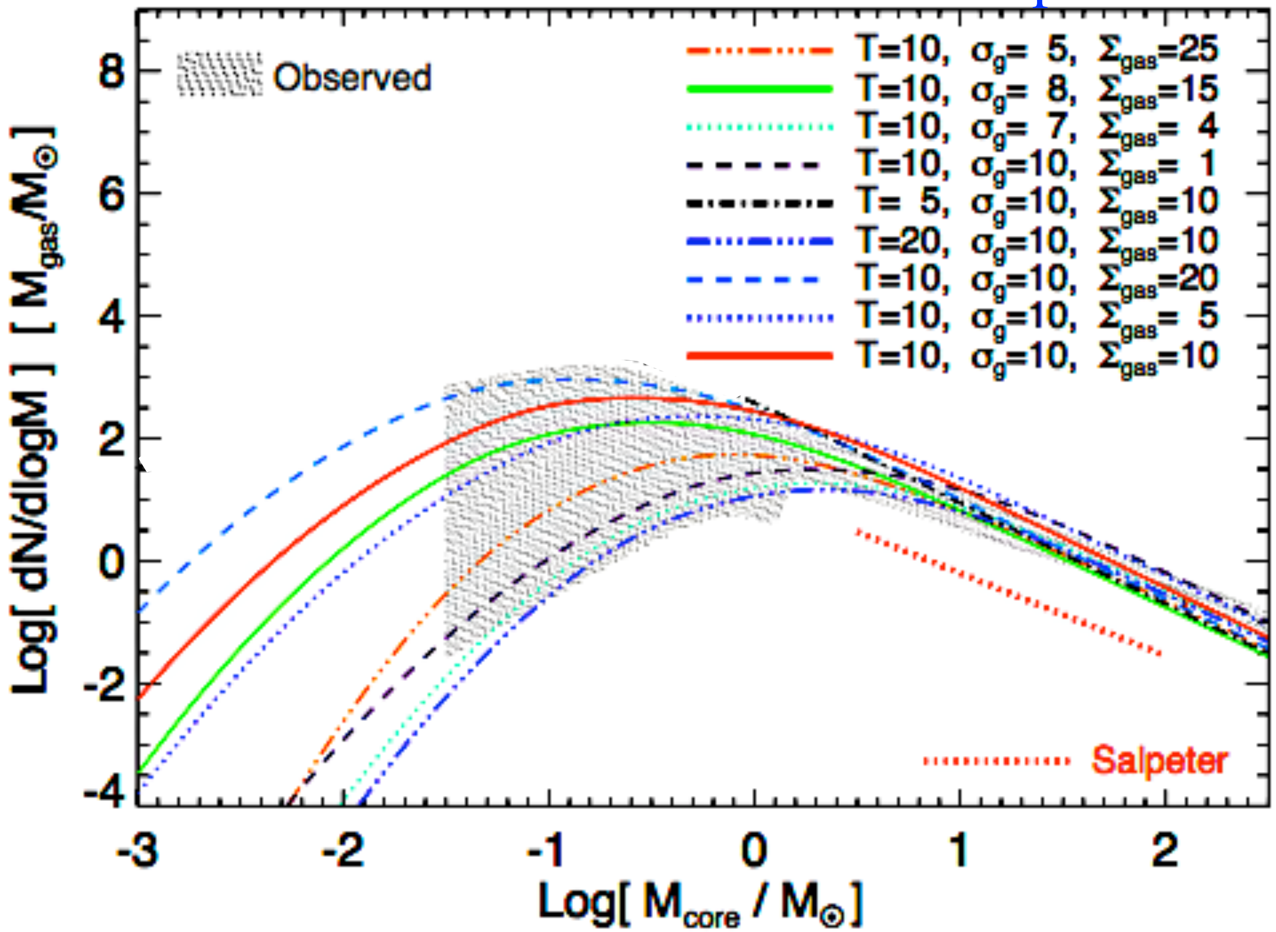


$$M_{\rm sonic} \equiv M(\rho_{\rm crit}\,|R_{\rm sonic}) \sim \frac{c_s^2\,R_{\rm sonic}}{G} \sim M_{\odot}\,\frac{T_{\rm min}}{10\,K}\,\frac{R_{\rm sonic}}{0.1\,{\rm pc}} \propto T_{\rm min}^2\,\frac{R_{\rm cl}}{\sigma_{\rm cl}^2}$$

Variation in the Core Mass Function

VS "NORMAL" IMF VARIATIONS

Weak variation with Galactic Properties



$$M_{\rm sonic} \equiv M(\rho_{\rm crit}\,|R_{\rm sonic}) \sim \frac{c_s^2\,R_{\rm sonic}}{G} \sim M_{\odot}\,\frac{T_{\rm min}}{10\,K}\,\frac{R_{\rm sonic}}{0.1\,{\rm pc}} \propto T_{\rm min}^2\,\frac{R_{\rm cl}}{\sigma_{\rm cl}^2}$$

BUT, What About Starbursts?

MW: $T_{\rm cold} \sim 10\,K$

$\sigma_{\rm gas} \sim 10\,{\rm km\,s^{-1}}$

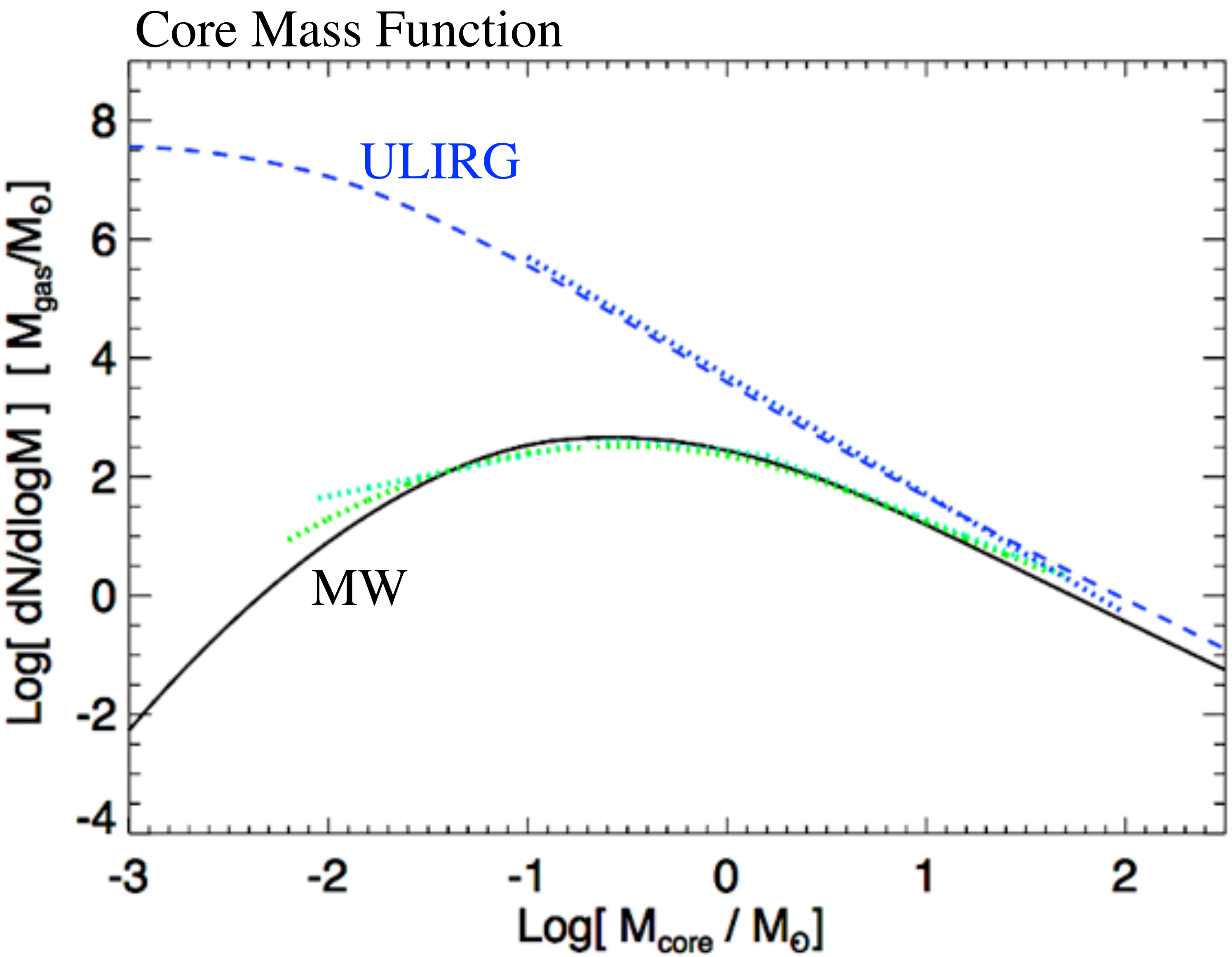
$(Q \sim 1 \text{ for } \Sigma_{\rm gas} \sim 10\,M_{\odot}\,{\rm pc^{-2}})$

BUT, What About Starbursts?

MW: $T_{\rm cold} \sim 10\,K$
$\sigma_{\rm gas} \sim 10\,{\rm km\,s^{-1}}$
$(Q \sim 1 \text{ for } \Sigma_{\rm gas} \sim 10\,M_{\odot}\,{\rm pc^{-2}})$

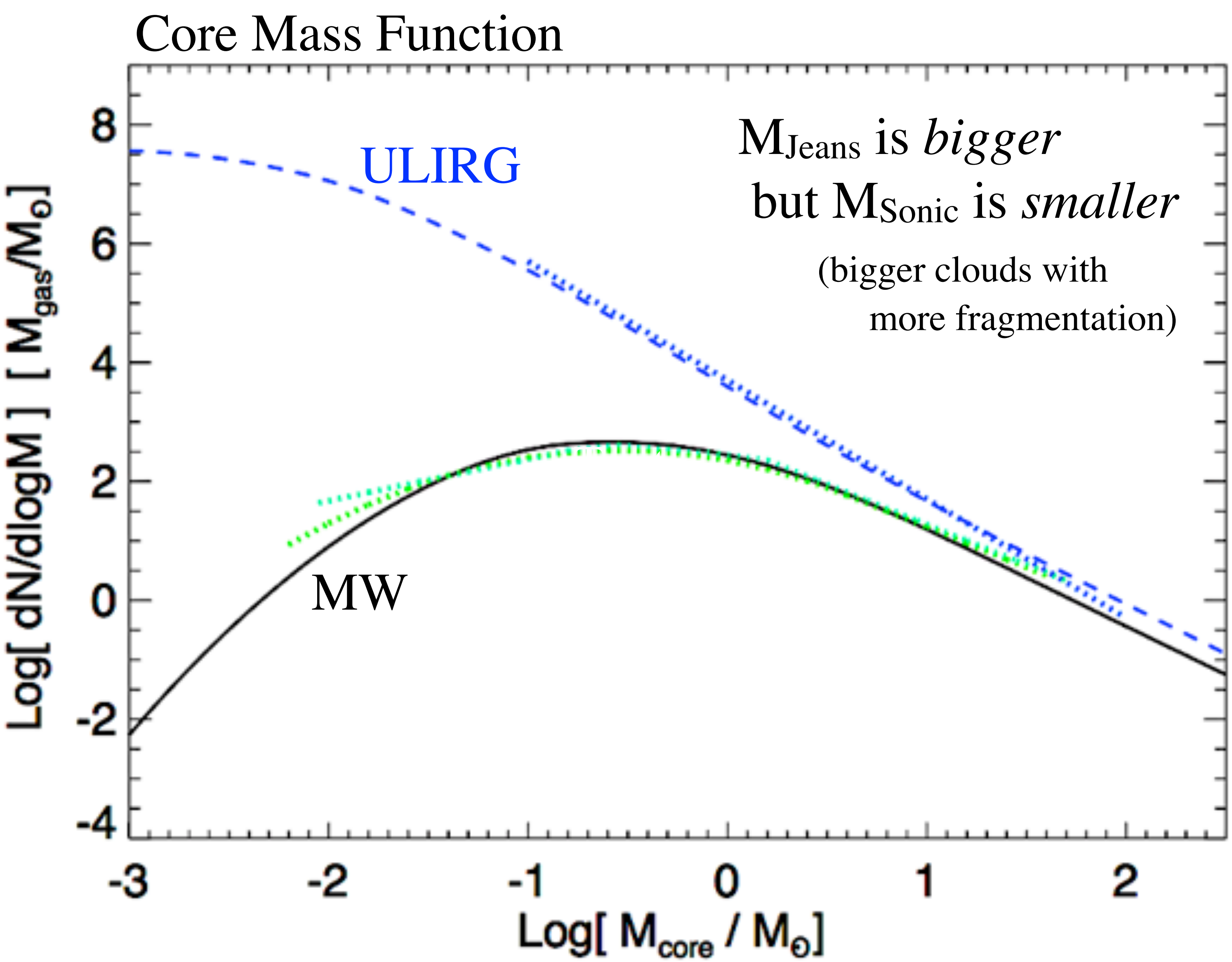
ULIRG: $T_{\rm cold} \sim 70\,K$
$\sigma_{\rm gas} \sim 80\,{\rm km\,s^{-1}}$
$(Q \sim 1 \text{ for } \Sigma_{\rm gas} \sim 1000\,M_{\odot}\,{\rm pc^{-2}})$

BUT, What About Starbursts?



BUT, What About Starbursts?

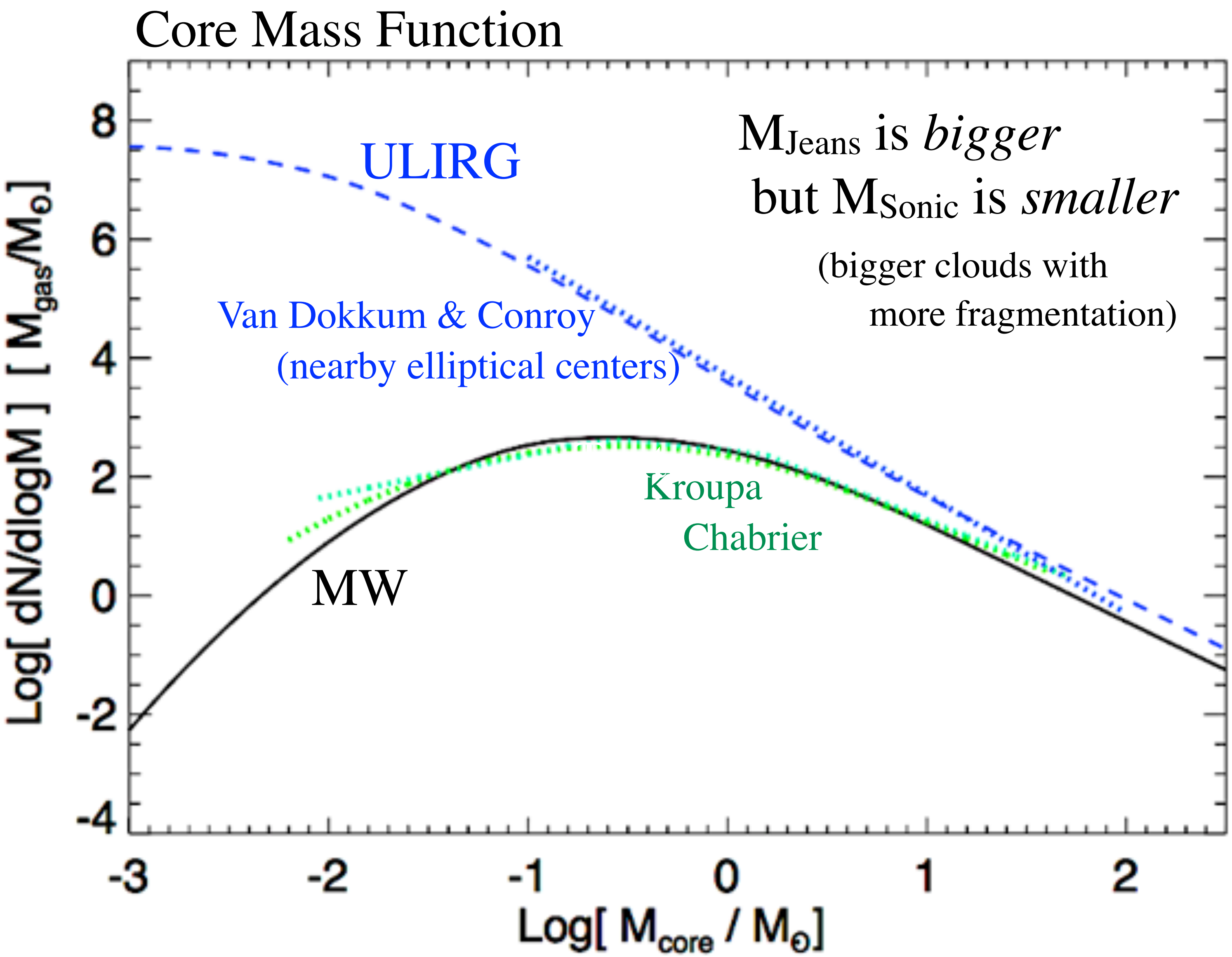
BOTTOM-HEAVY: TURBULENCE WINS!



Mach number in ULIRGs: $\mathcal{M} \gtrsim 100$

BUT, What About Starbursts?

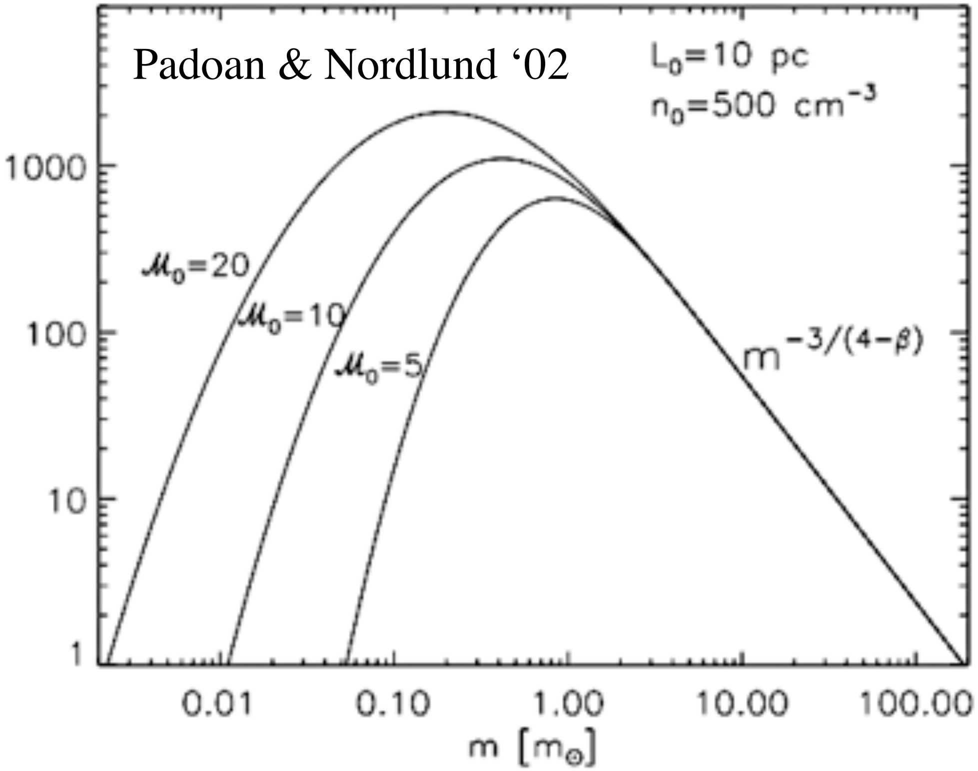
BOTTOM-HEAVY: TURBULENCE WINS!

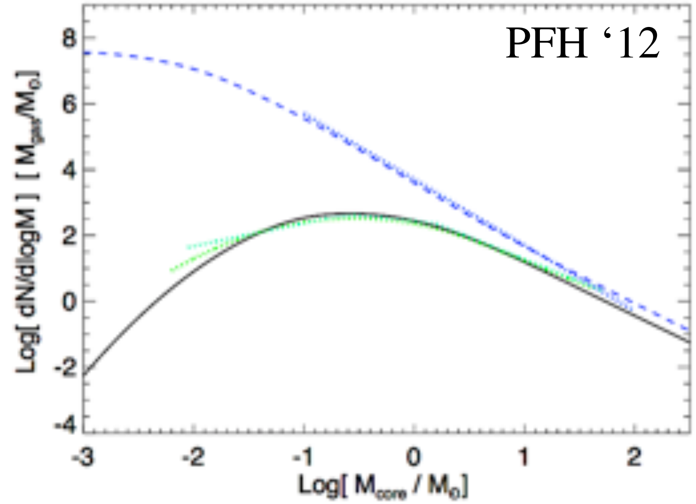


Mach number in ULIRGs: $\mathcal{M} \gtrsim 100$

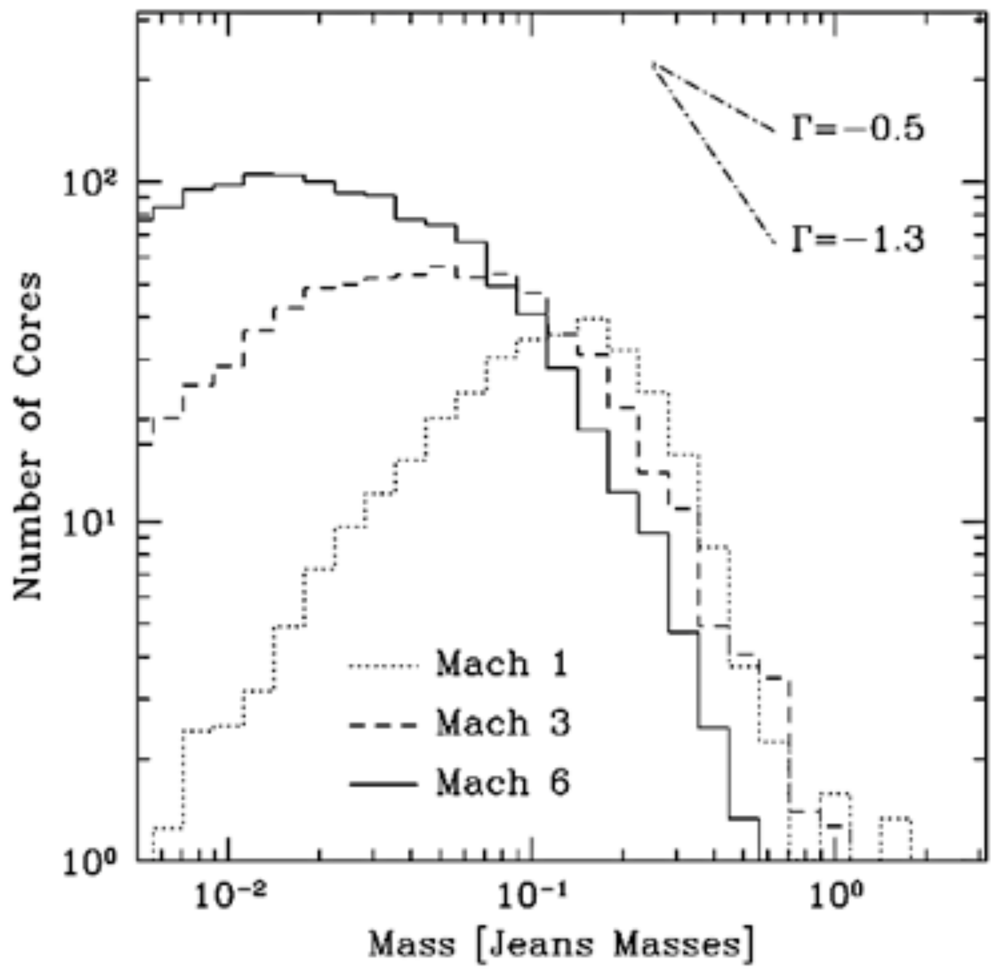
BUT, What About Starbursts?

BOTTOM-HEAVY: TURBULENCE WINS!





Ballesteros-Paredes '06



Open Questions:

1. What Maintains the Turbulence?

Open Questions:

1. What Maintains the Turbulence?

Efficient Cooling: $$\dot{P}_{\rm diss} \sim \frac{M_{\rm gas}\, v_{\rm turb}}{t_{\rm crossing}}$$

Open Questions:

1. What Maintains the Turbulence?

Efficient Cooling: $\dot{P}_{\rm diss} \sim \dfrac{M_{\rm gas}\, v_{\rm turb}}{t_{\rm crossing}}$

2. Why Doesn't Everything Collapse?

Open Questions:

1. What Maintains the Turbulence?

Efficient Cooling: $\dot{P}_{\rm diss} \sim \dfrac{M_{\rm gas}\, v_{\rm turb}}{t_{\rm crossing}}$

2. Why Doesn’t Everything Collapse?

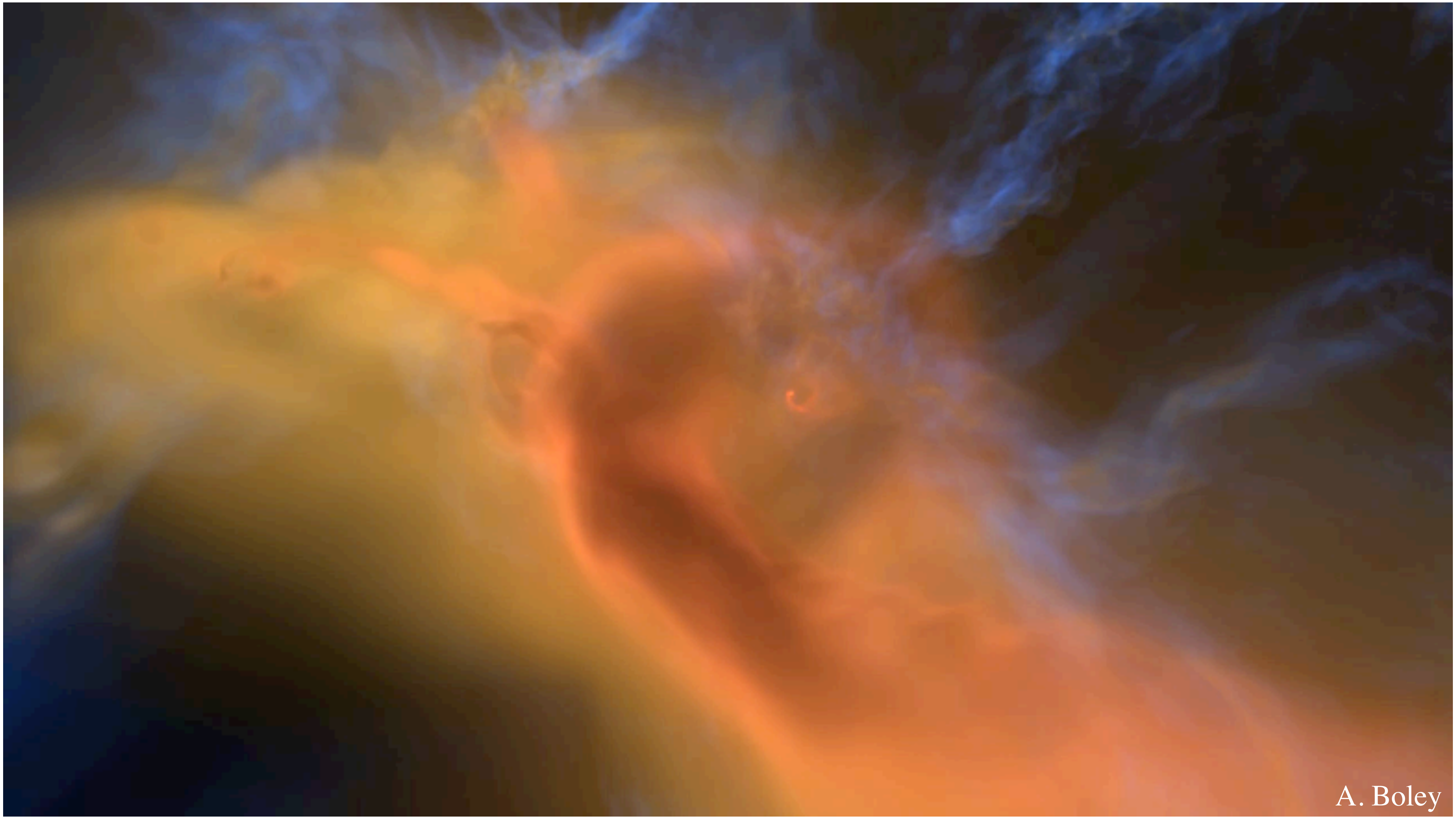
“Top-down” turbulence can’t stop collapse once self-gravitating

Fast Cooling: $\dot{M}_{*} \sim \dfrac{M_{\rm gas}}{t_{\rm freefall}}$

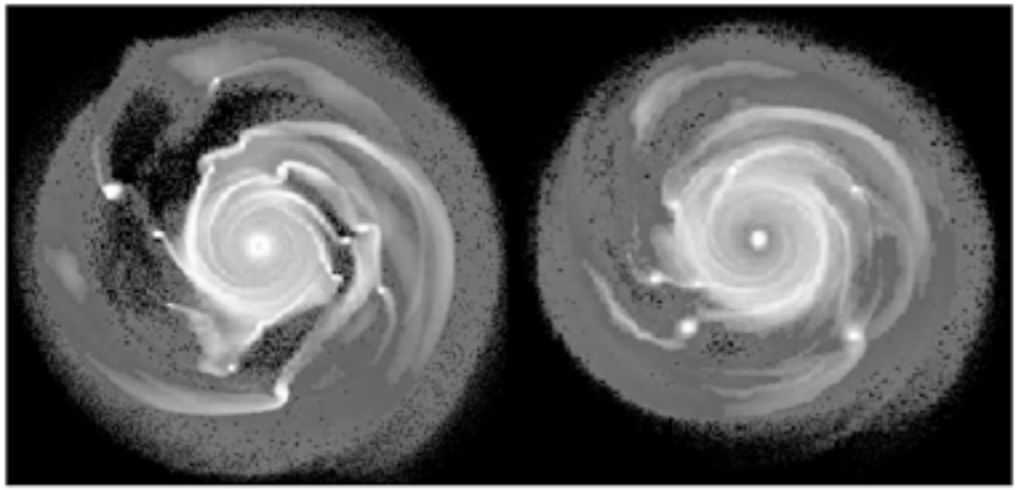
What About Planets?

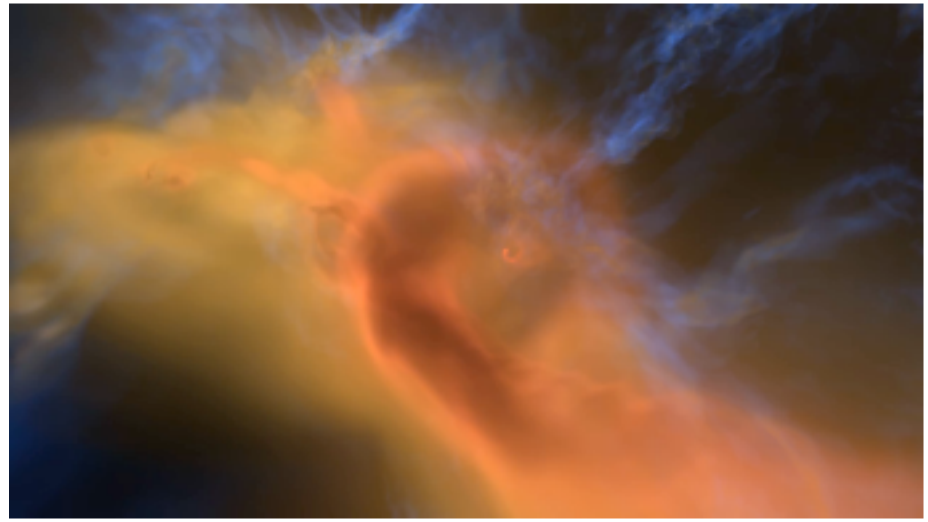
Planet Formation?

- Two channels:
  - (1) "Core accretion"
  - (2) "Direct Collapse"



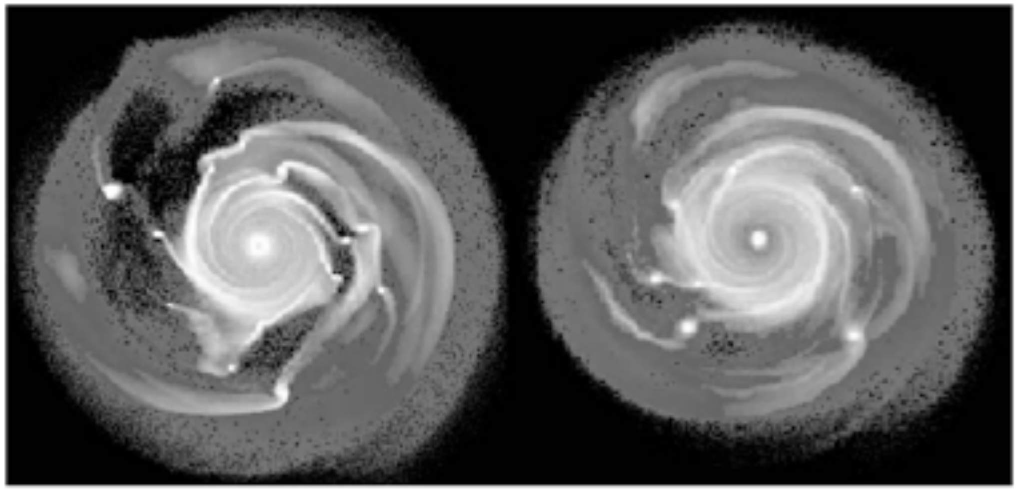
Standard (Toomre) Criterion for Direct Collapse:

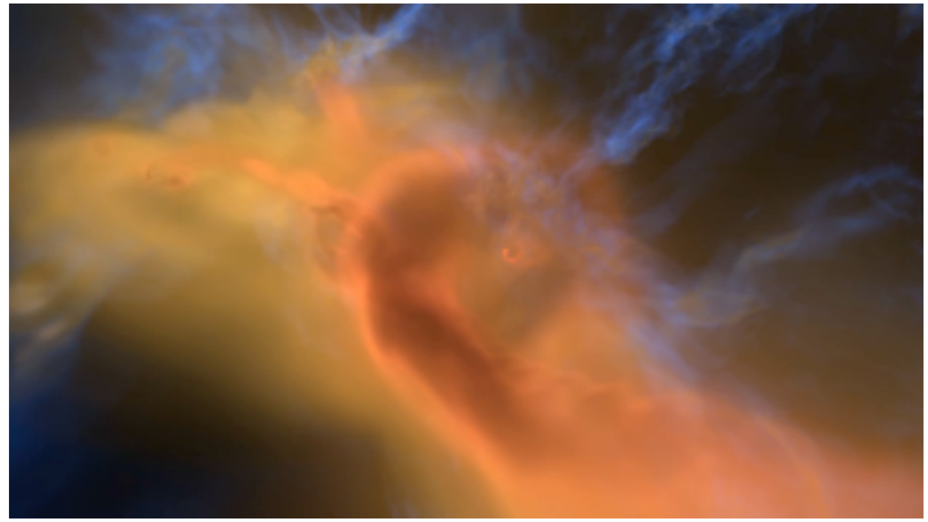




$$Q = \frac{c_s\,\Omega}{\pi\,G\,\Sigma_{\rm gas}} \sim \frac{1}{\rho}\,\frac{M_*}{r_*^3}$$

Standard (Toomre) Criterion for Direct Collapse:





$$Q = \frac{c_s\,\Omega}{\pi\,G\,\Sigma_{\rm gas}} \sim \frac{1}{\rho}\,\frac{M_*}{r_*^3}$$

$$Q \sim 100\left(\frac{\Sigma_{\rm gas}}{\Sigma_{\rm MMSN}}\right)^{-1} r_{*,\,\rm AU}^{-1/4}$$

But, What if the Disks Are Turbulent?

But, What if the Disks Are Turbulent?

- Need density fluctuation: $$\frac{\rho}{\langle\rho\rangle} \gtrsim \frac{1}{\langle\rho\rangle}\frac{M_*}{r_*^3} \sim Q$$

But, What if the Disks Are Turbulent?

- Need density fluctuation: $$\frac{\rho}{\langle\rho\rangle} \gtrsim \frac{1}{\langle\rho\rangle}\frac{M_*}{r_*^3} \sim Q$$

- Turbulence:: stochastic fluctuations with $$\sigma_{\ln\rho} \approx \sqrt{\ln\left(1+\mathcal{M}^2\right)} \sim \mathcal{M}$$

But, What if the Disks Are Turbulent?

- Need density fluctuation: $$\frac{\rho}{\langle\rho\rangle} \gtrsim \frac{1}{\langle\rho\rangle}\,\frac{M_*}{r_*^3} \sim Q$$

- Turbulence:: stochastic fluctuations with $$\sigma_{\ln\rho} \approx \sqrt{\ln\left(1+\mathcal{M}^2\right)} \sim \mathcal{M}$$

- So, at any instant, in a given region: $$P_\rho \sim \mathrm{erfc}\left[\frac{\ln Q}{\sqrt{2}\,\sigma_{\ln\rho}}\right]$$

But, What if the Disks Are Turbulent?

- Need density fluctuation: $$\frac{\rho}{\langle\rho\rangle} \gtrsim \frac{1}{\langle\rho\rangle}\,\frac{M_*}{r_*^3} \sim Q$$

- Turbulence:: stochastic fluctuations with $\sigma_{\ln\rho} \approx \sqrt{\ln\left(1+\mathcal{M}^2\right)} \sim \mathcal{M}$

- So, at any instant, in a given region: $$P_\rho \sim \mathrm{erfc}\left[\frac{\ln Q}{\sqrt{2}\,\sigma_{\ln\rho}}\right]$$

- Q~100, *M*~0.1 :: $P_p \sim 10^{-7}$ is small!

But, What if the Disks Are Turbulent?

But, What if the Disks Are Turbulent?

- Most unstable wavelength ("size" of regions) : $\sim h$

But, What if the Disks Are Turbulent?

- Most unstable wavelength ("size" of regions) : $\sim h$
- So have $N_{\rm volumes} \sim \left(\frac{r_*}{h}\right)^2$ independent "samples" (at a given time)

But, What if the Disks Are Turbulent?

- Most unstable wavelength ("size" of regions) : $\sim h$
- So have $N_{\rm volumes} \sim \left(\frac{r_*}{h}\right)^2$ independent "samples" (at a given time)
- Turbulence evolves stochastically with coherence time ~ eddy turnover time:

$$t_{\text{``reset''}} \approx t_{\rm cross}({\rm turb}) \approx t_{\rm dyn} = \Omega^{-1} \sim {\rm yr}$$

But, What if the Disks Are Turbulent?

- Most unstable wavelength ("size" of regions) : $\sim h$
- So have $N_{\rm volumes} \sim \left(\frac{r_*}{h}\right)^2$ independent "samples" (at a given time)
- Turbulence evolves stochastically with coherence time ~ eddy turnover time:

$$t_{\rm ``reset"} \approx t_{\rm cross}({\rm turb}) \approx t_{\rm dyn} = \Omega^{-1} \sim {\rm yr}$$

- And disks have a long lifetime $t_{\rm disk} \sim {\rm Myr}$

so "resample" it $\frac{t_{\rm disk}}{t_{\rm dyn}}$ independent times

But, What if the Disks Are Turbulent?

- Most unstable wavelength ("size" of regions) : $\sim h$
- So have $N_{\rm volumes} \sim \left(\frac{r_*}{h}\right)^2$ independent "samples" (at a given time)
- Turbulence evolves stochastically with coherence time ~ eddy turnover time:

$$t_{\rm ``reset"} \approx t_{\rm cross}({\rm turb}) \approx t_{\rm dyn} = \Omega^{-1} \sim {\rm yr}$$

- And disks have a long lifetime $t_{\rm disk} \sim {\rm Myr}$

  so "resample" it $\dfrac{t_{\rm disk}}{t_{\rm dyn}}$ independent times

$$P_{\rm tot} \sim \left(\frac{t_{\rm disk}}{t_{\rm dyn}}\right)\left(\frac{r_*}{h}\right)^2 {\rm erfc}\left[\frac{\ln Q}{\sqrt{2}\,\sigma_{\ln \rho}}\right]$$

But, What if the Disks Are Turbulent?

- Most unstable wavelength ("size" of regions) : $\sim h$
- So have $N_{\rm volumes} \sim \left(\frac{r_*}{h}\right)^2$ independent "samples" (at a given time)
- Turbulence evolves stochastically with coherence time ~ eddy turnover time:

$$t_{\text{``reset''}} \approx t_{\rm cross}({\rm turb}) \approx t_{\rm dyn} = \Omega^{-1} \sim {\rm yr}$$

- And disks have a long lifetime $t_{\rm disk} \sim {\rm Myr}$

so "resample" it $\dfrac{t_{\rm disk}}{t_{\rm dyn}}$ independent times

$$\gtrsim 1 \text{ for } \begin{array}{c} Q \sim 100 \\ \mathcal{M} \gtrsim 0.1 \end{array}$$

But, What if the Disks Are Turbulent?

- Most unstable wavelength ("size" of regions) : $\sim h$
- So have $N_{\rm volumes} \sim \left(\frac{r_*}{h}\right)^2$ independent "samples" (at a given time)
- Turbulence evolves stochastically with coherence time ~ eddy turnover time:

$$t_{\text{“reset”}} \approx t_{\rm cross}({\rm turb}) \approx t_{\rm dyn} = \Omega^{-1} \sim {\rm yr}$$

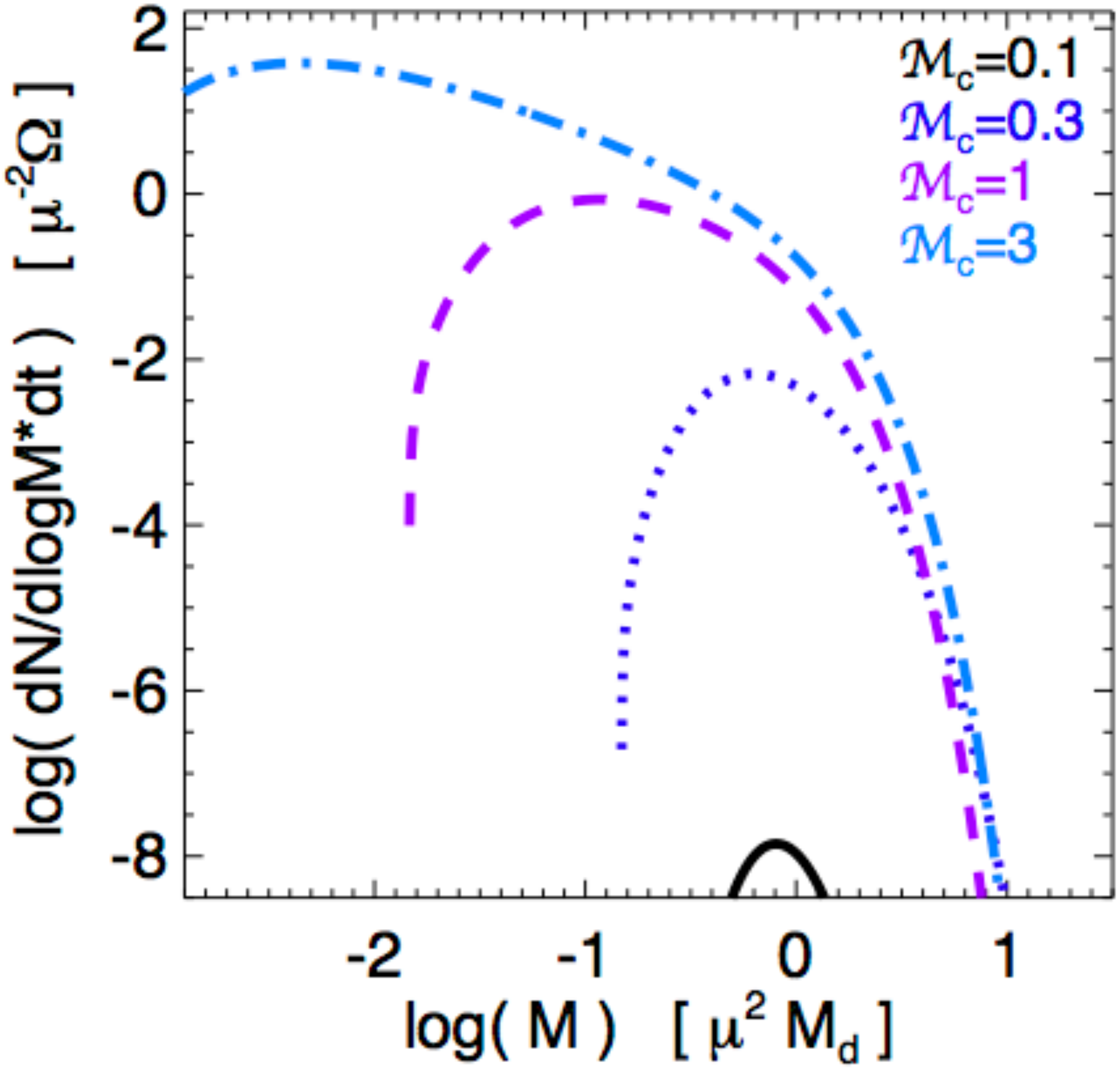
- And disks have a long lifetime $t_{\rm disk} \sim {\rm Myr}$

  so "resample" it $\frac{t_{\rm disk}}{t_{\rm dyn}}$ independent times

$$P_{\rm tot} \sim \left(\frac{t_{\rm disk}}{t_{\rm dyn}}\right)\left(\frac{r_*}{h}\right)^2 {\rm erfc}\left[\frac{\ln Q}{\sqrt{2}\,\sigma_{\ln\rho}}\right] \gtrsim 1$$ for $$Q \sim 100 \qquad \mathcal{M} \gtrsim 0.1$$

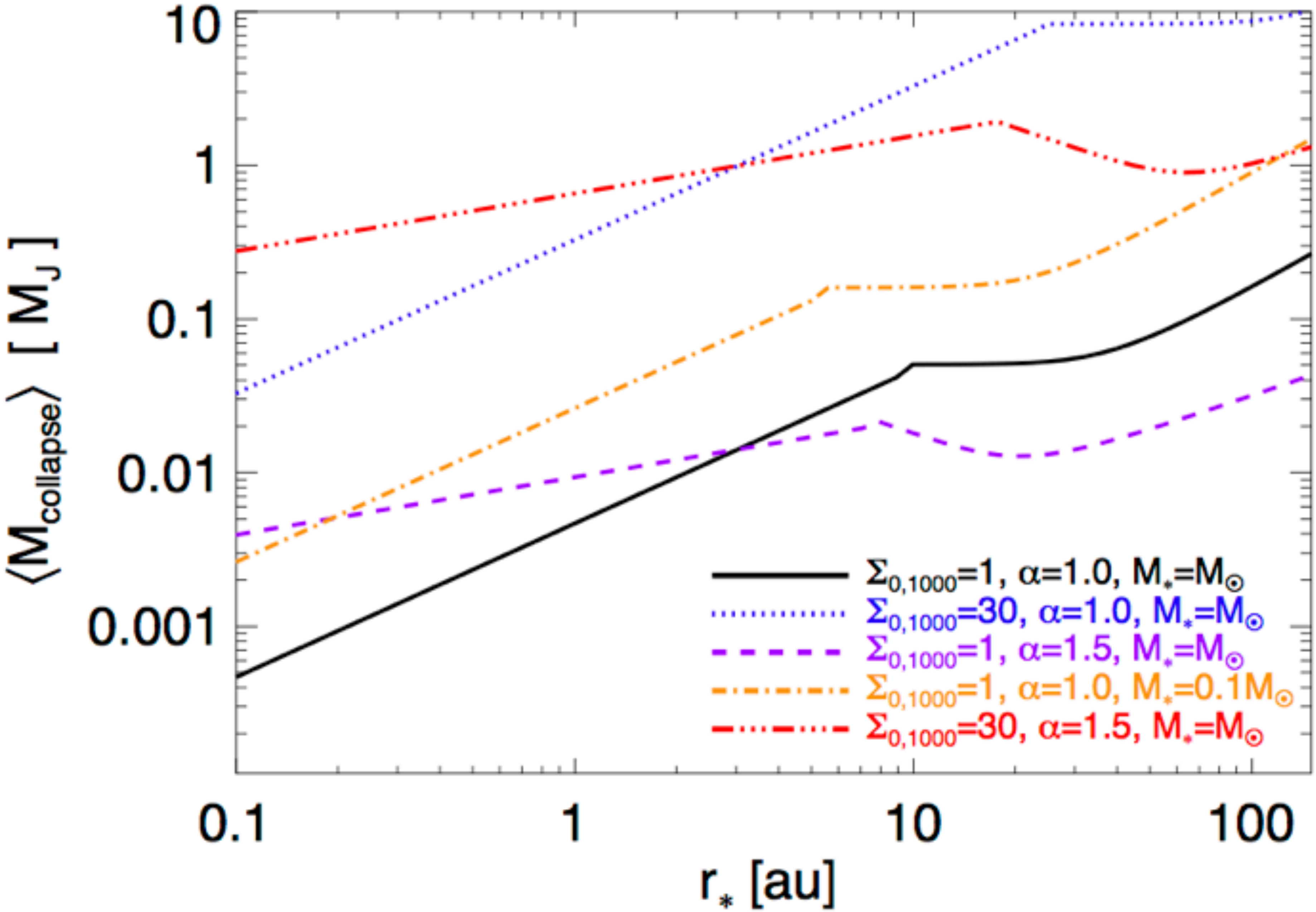
Mass Function of “Stochastic” Direct Collapse Events

RIGOROUSLY CALCULATE RATE OF EVENTS VS MASS



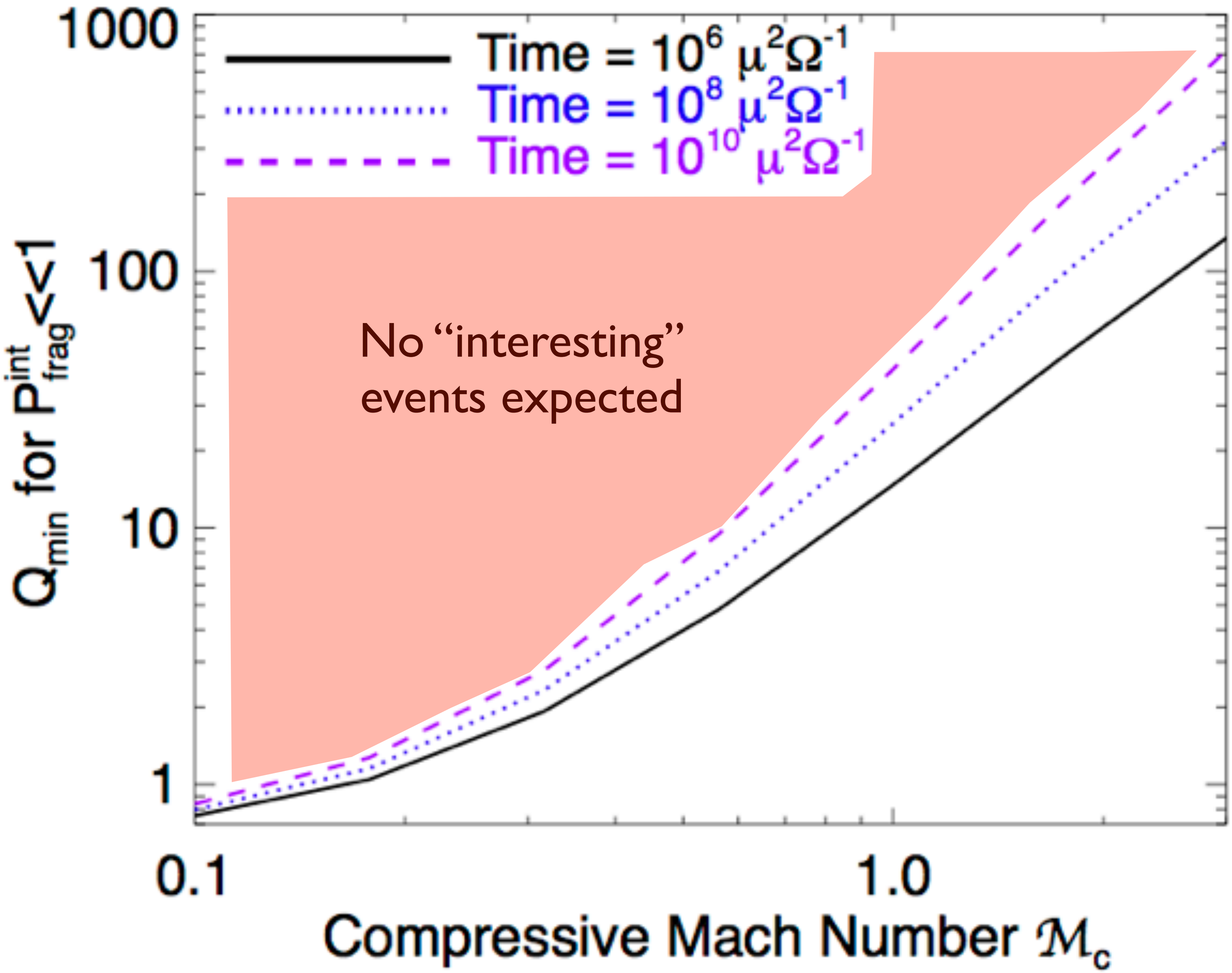
Mass Function of “Stochastic” Direct Collapse Events

RIGOROUSLY CALCULATE RATE OF EVENTS VS MASS



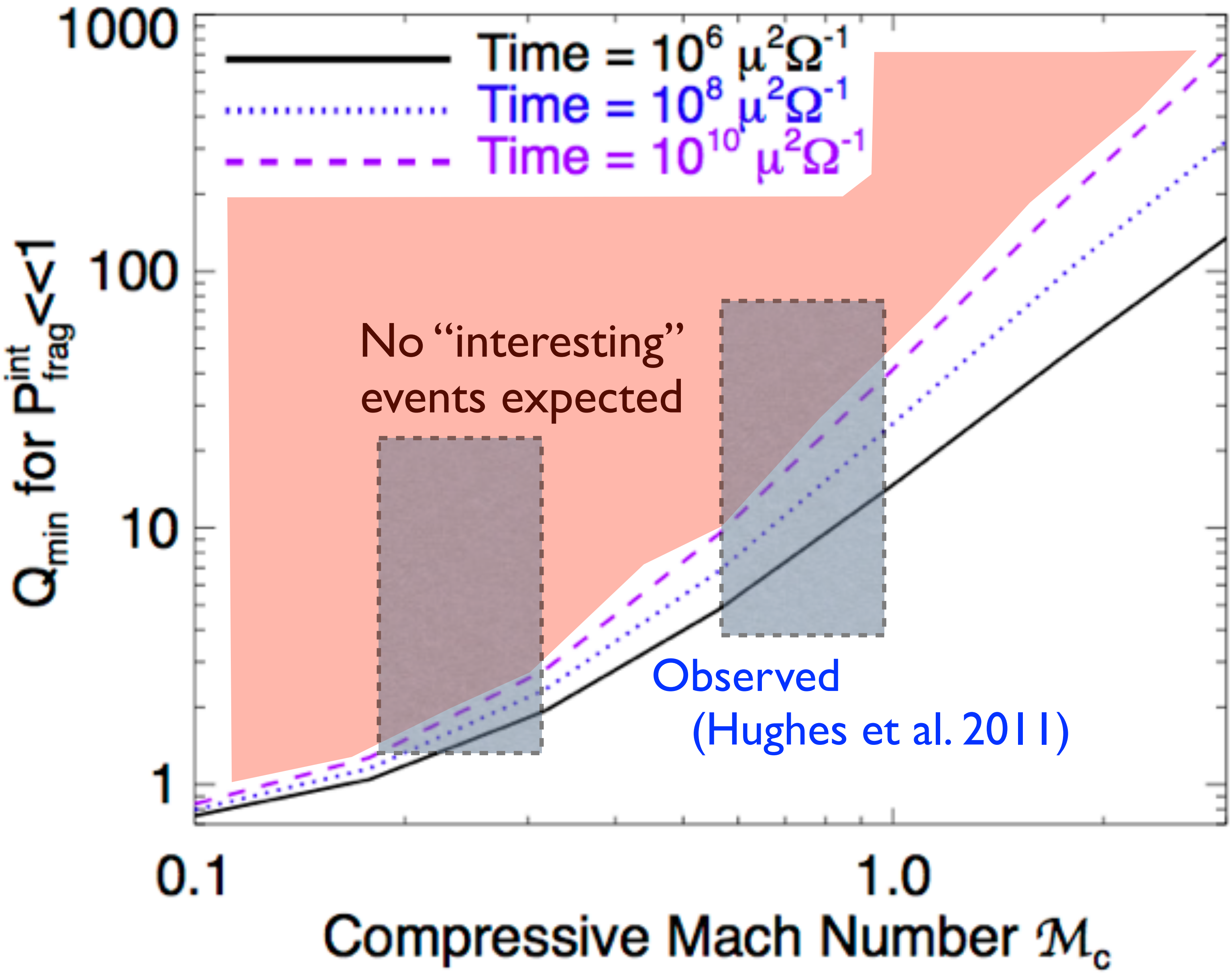
What is the real "threshold" for an event?

(FOR A GIVEN DISK LIFETIME)



What is the real “threshold” for an event?

(FOR A GIVEN DISK LIFETIME)

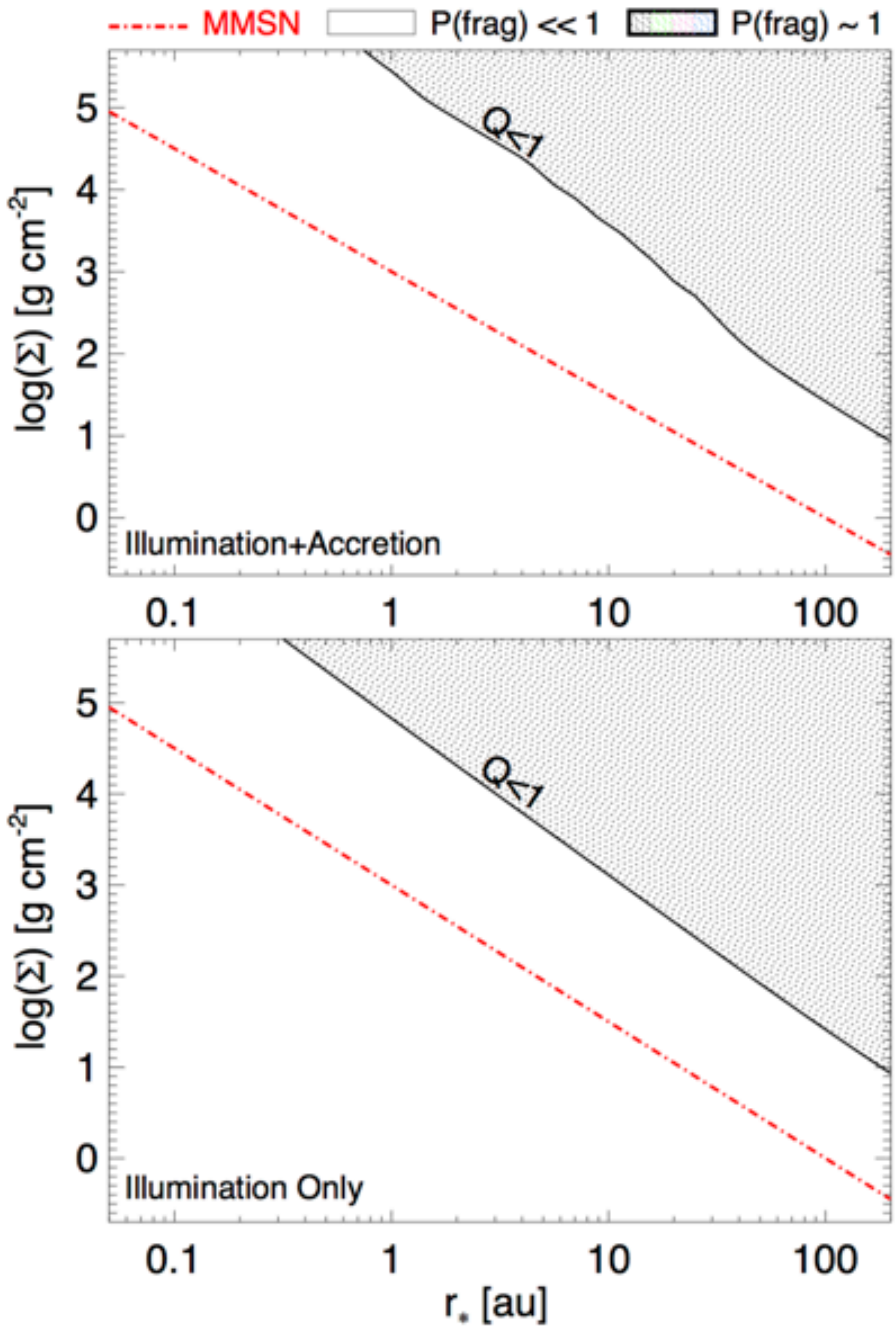


What is Theoretically Expected?

"TYPICAL" DISK AROUND
A SOLAR-TYPE STAR

- Self-consistently calculate temperatures, etc.

- Different drivers of turbulence:
  - Convection
  - Magneto-Rotational Instability
  - "Gravito-Turbulence"

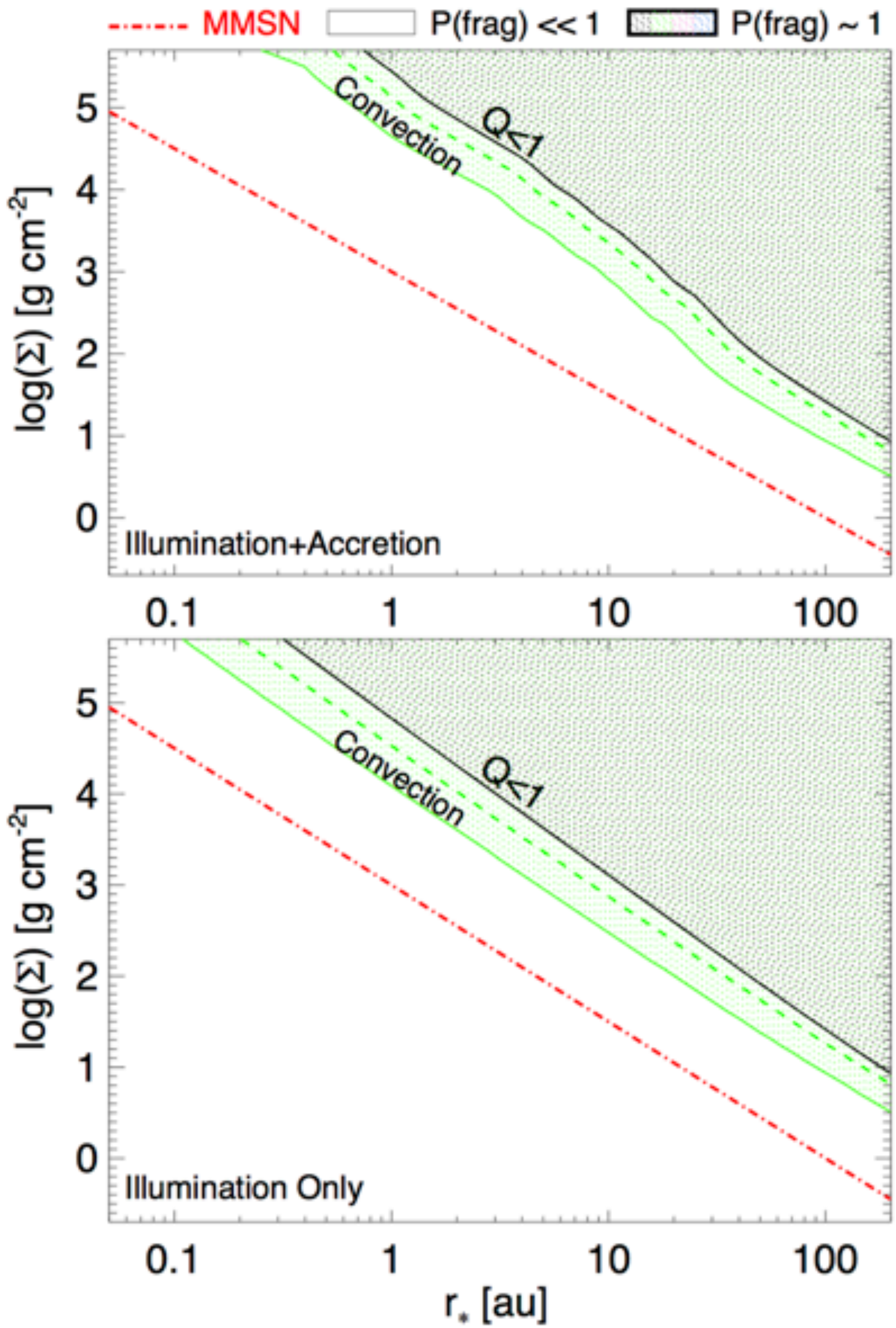


What is Theoretically Expected?

“TYPICAL” DISK AROUND
A SOLAR-TYPE STAR

- Self-consistently calculate temperatures, etc.

- Different drivers of turbulence:
  - Convection
  - Magneto-Rotational Instability
  - “Gravito-Turbulence”

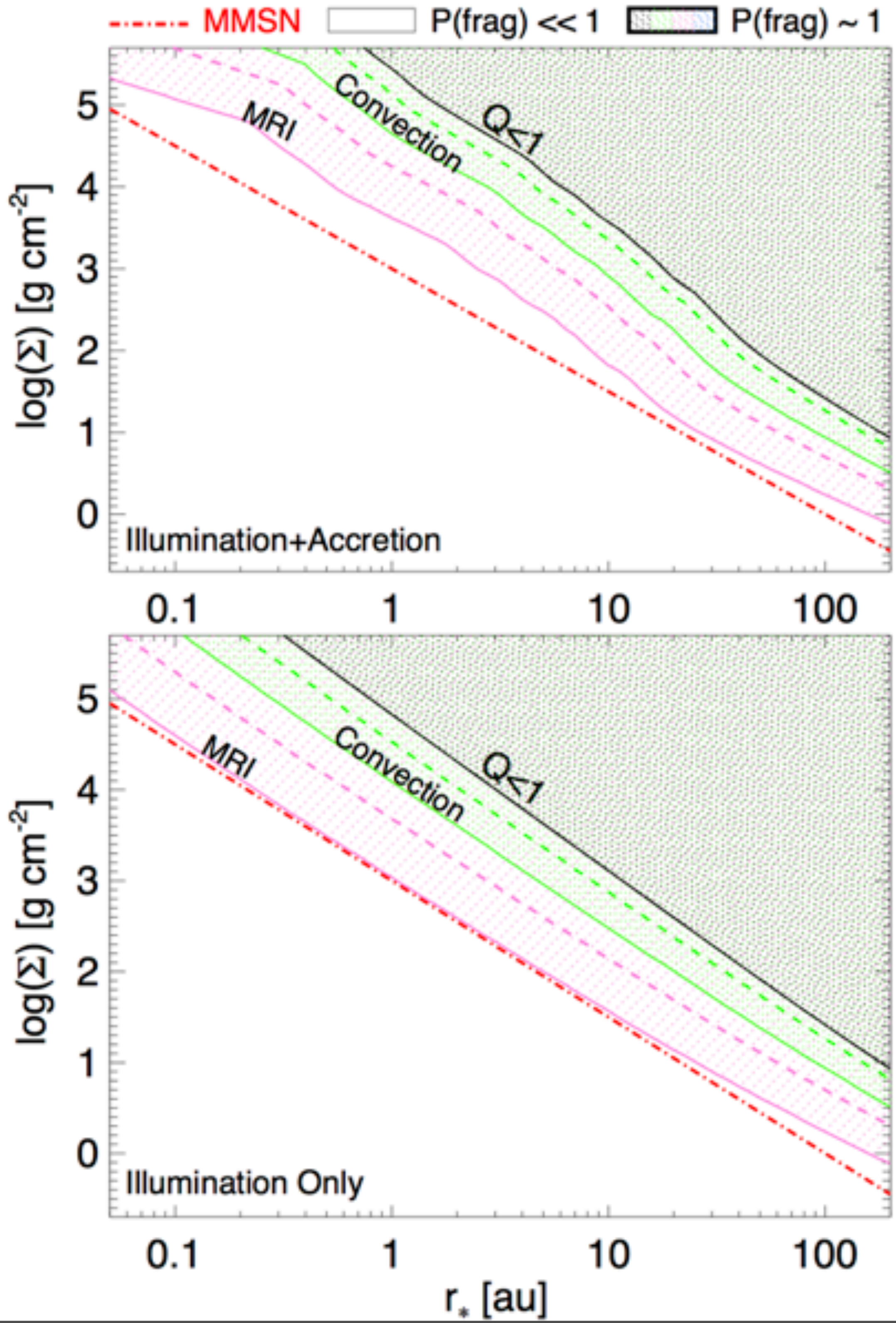


What is Theoretically Expected?

"TYPICAL" DISK AROUND A SOLAR-TYPE STAR

- Self-consistently calculate temperatures, etc.

- Different drivers of turbulence:
  - Convection
  - Magneto-Rotational Instability
  - "Gravito-Turbulence"

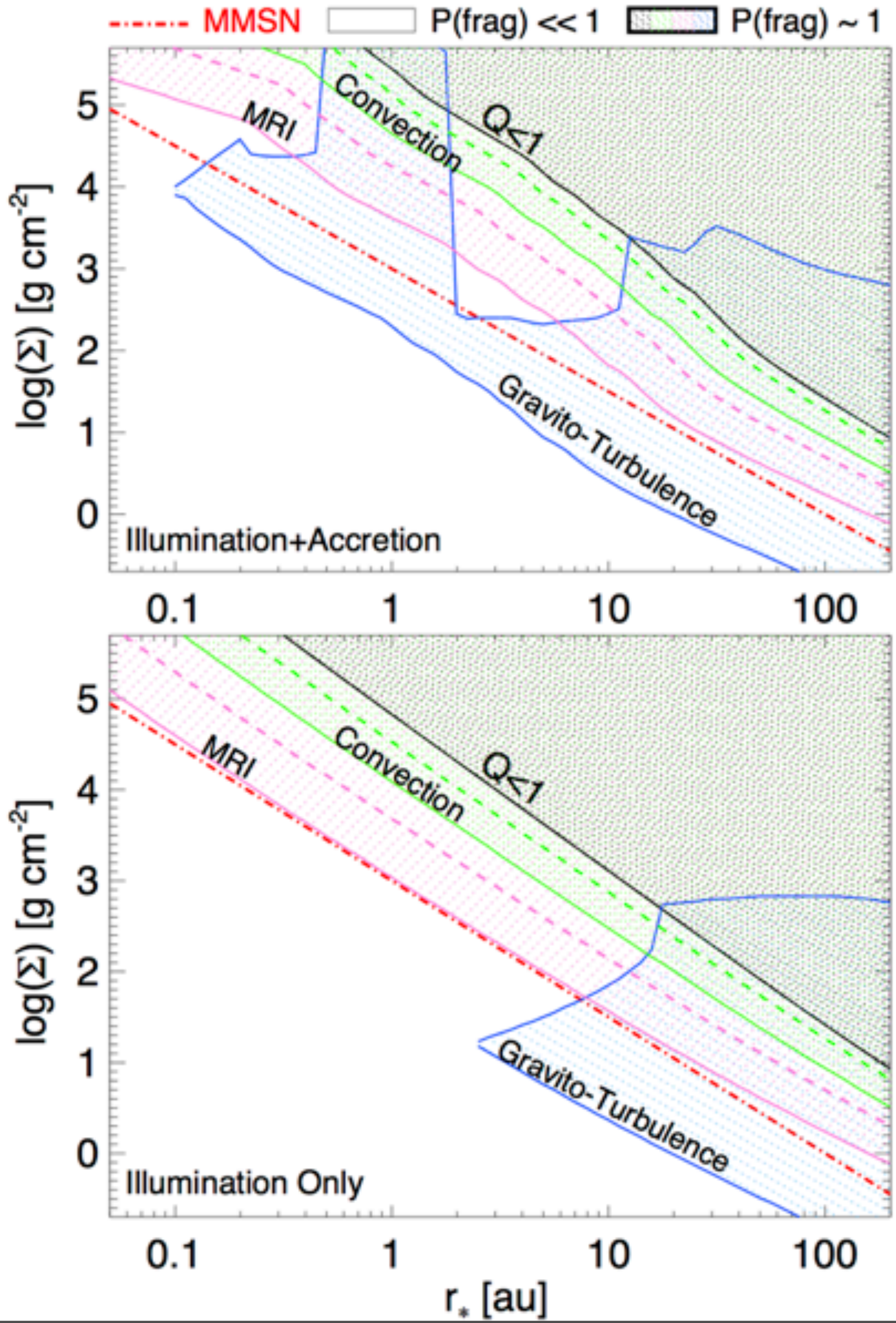


What is Theoretically Expected?

"TYPICAL" DISK AROUND
A SOLAR-TYPE STAR

- Self-consistently calculate temperatures, etc.

- Different drivers of turbulence:
  - Convection
  - Magneto-Rotational Instability
  - "Gravito-Turbulence"



What If The Statistics Aren't Gaussian?

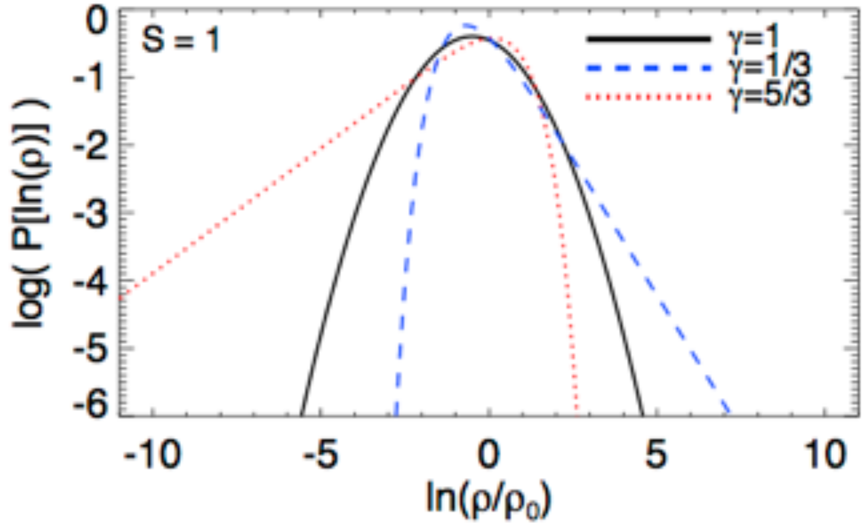
What If The Statistics Aren’t Gaussian?

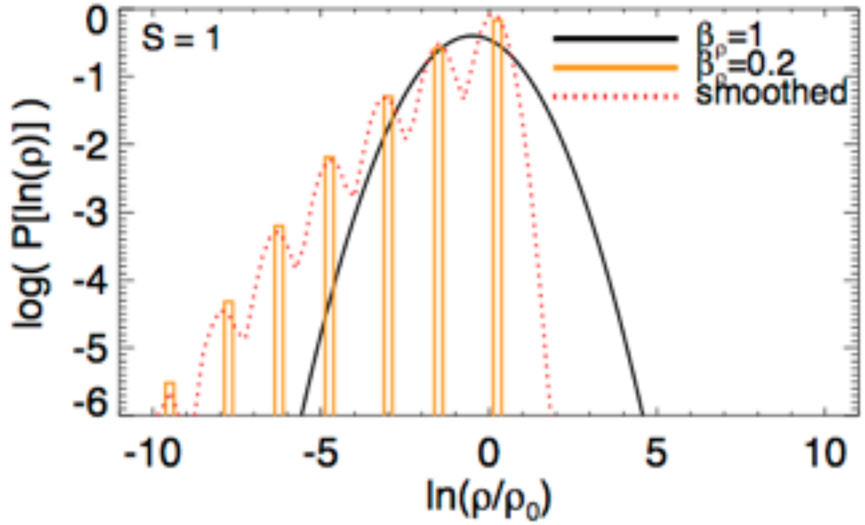
... actually, they never are, and that’s great!

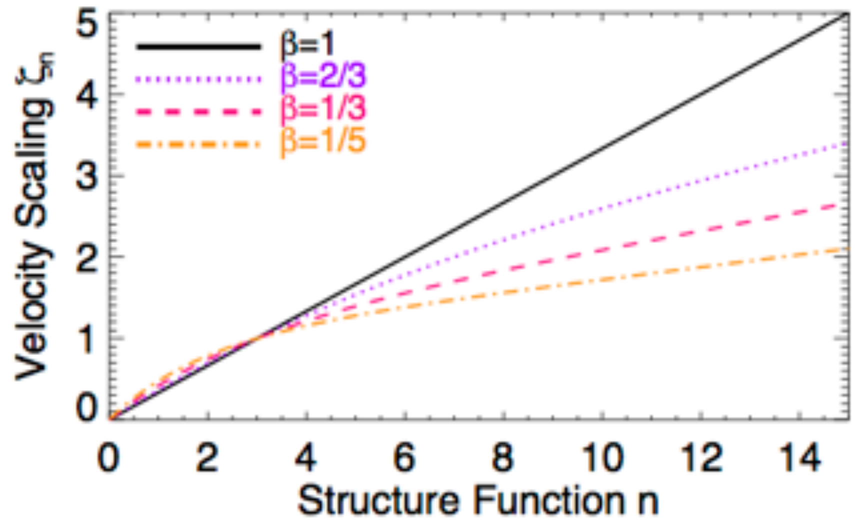
Many kinds of Non-Gaussianity Appear:

BUT THESE ARE TRACTABLE!

- Non-isothermal equations of state
- Long-range forces (gravity)
- Intermittency
  (non-self similarity)
  in the turbulence

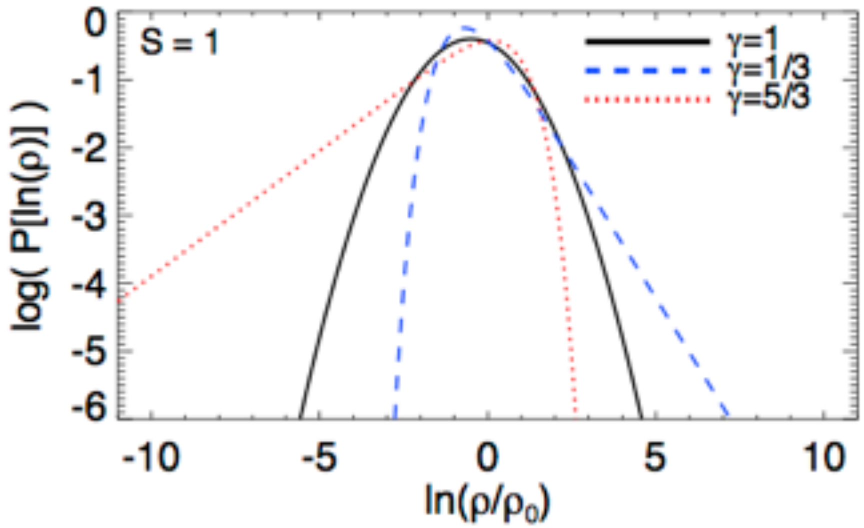


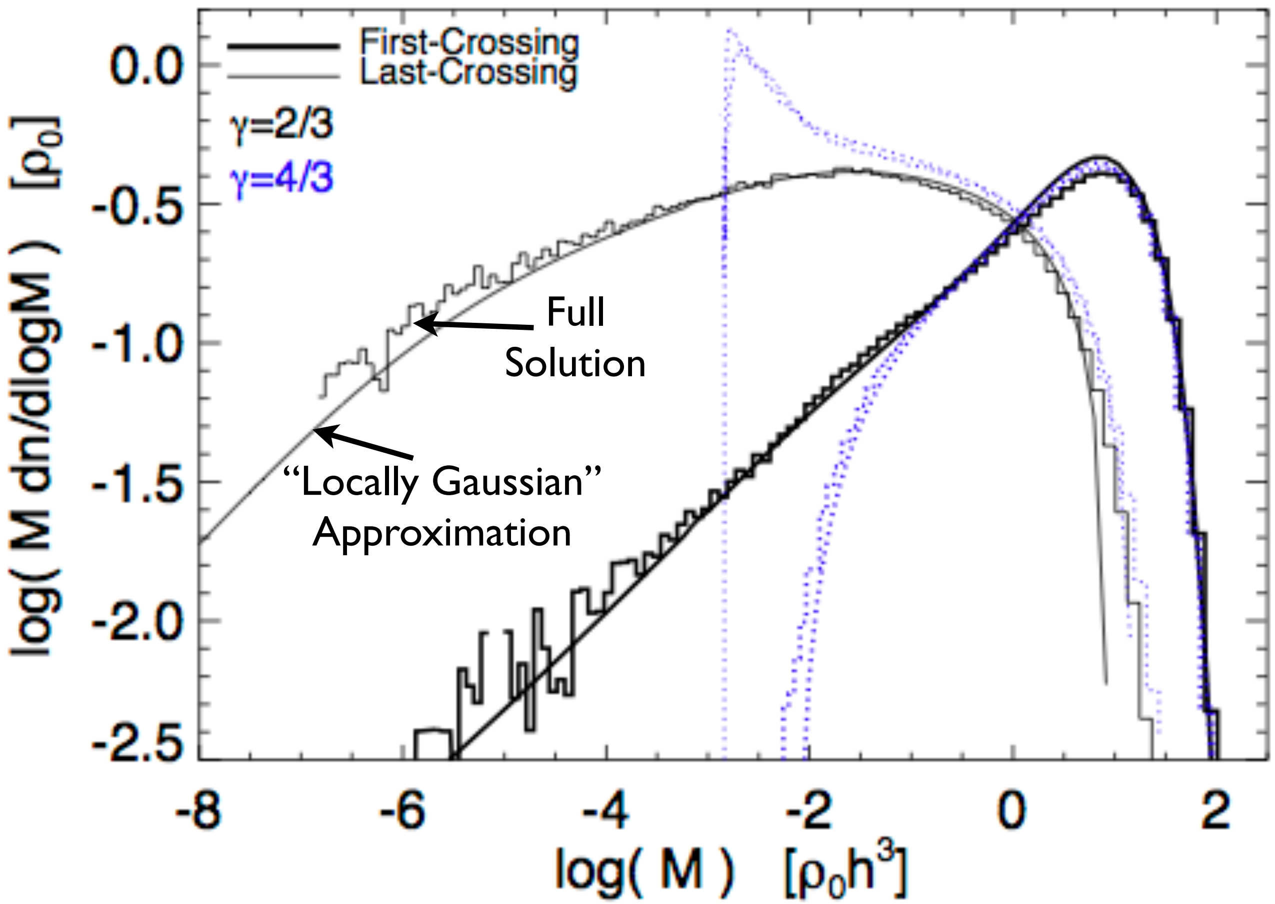




Example: Non-isothermal equations of state

APPLY COSMOLOGICAL METHODS FOR
NON-GAUSSIAN FIELDS

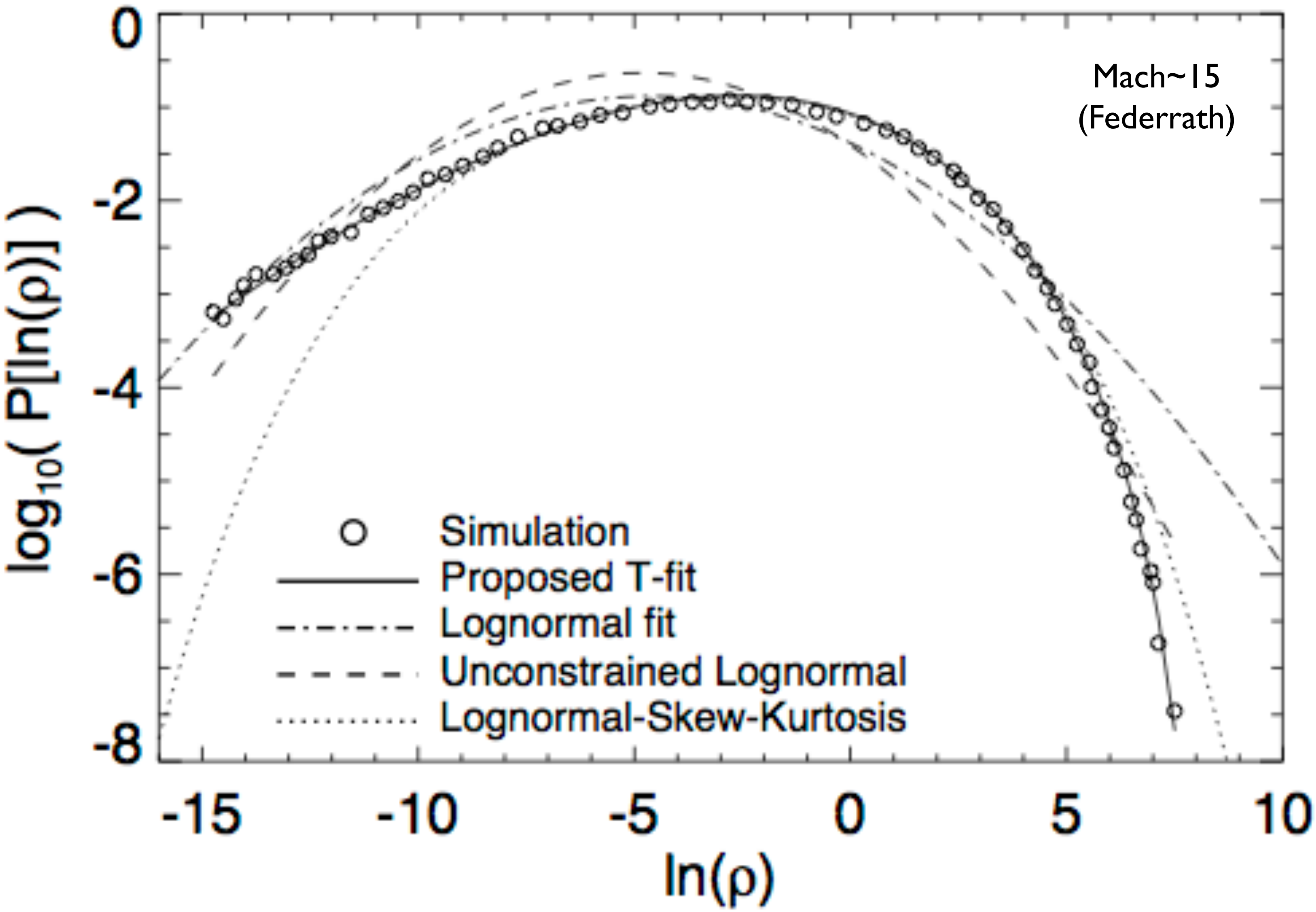




More Interesting: Even Isothermal Gas is Not Lognormal!

MASS CONSERVATION & INTERMITTENCY PREVENT IT

- **Lognormal:** $\mathrm{d}p(\ln\rho \,|\, R) = \dfrac{1}{\sqrt{2\pi\, S(R)}} \exp\left[\dfrac{-(\ln\rho - \langle \ln\rho\rangle)^2}{2\, S(R)}\right]$



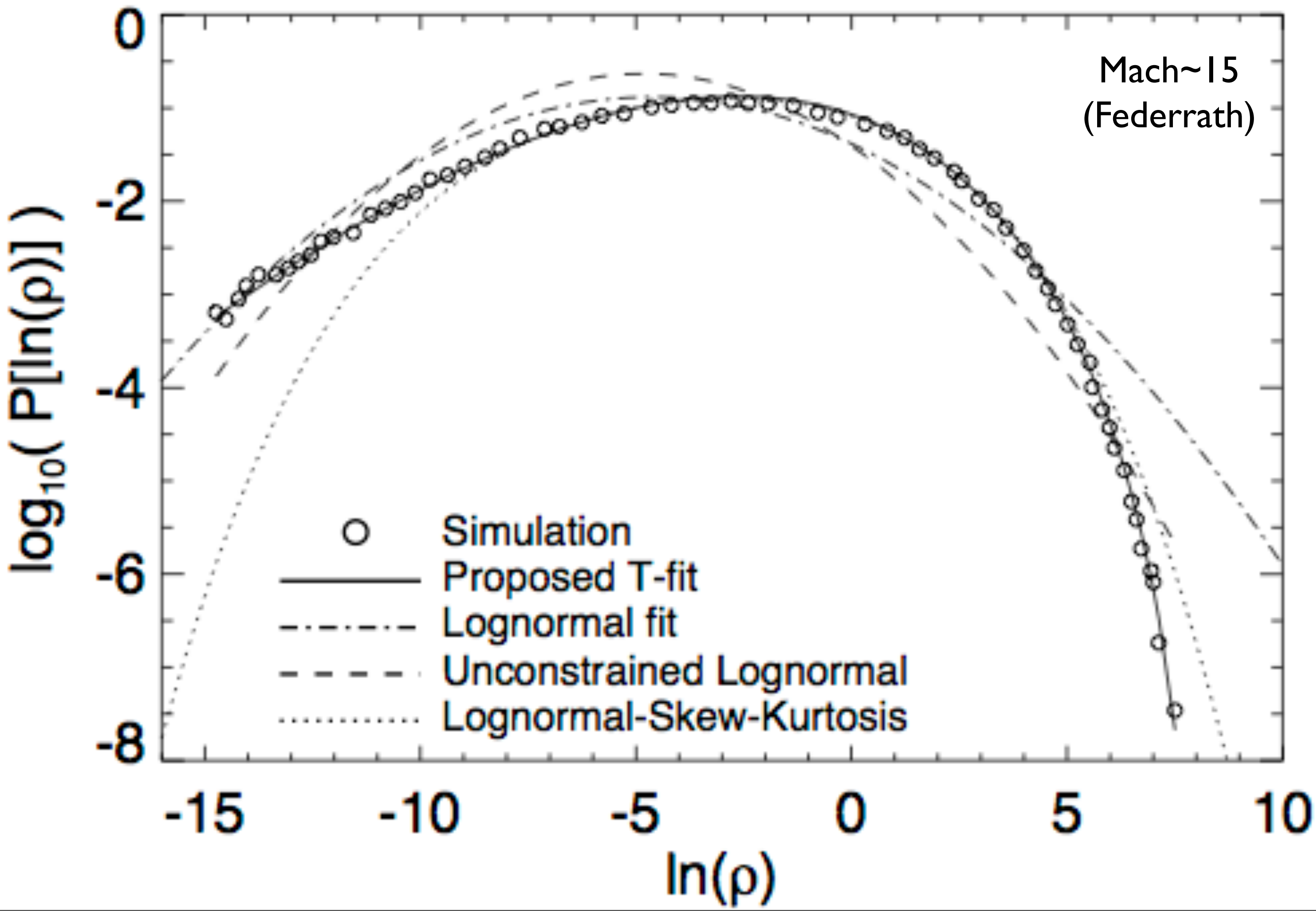
More Interesting: Even Isothermal Gas is Not Lognormal!

MASS CONSERVATION & INTERMITTENCY PREVENT IT

- T-Function: $P_V(\ln\rho)\,\mathrm{d}\ln\rho = I_1(2\sqrt{\lambda u})\exp[-(\lambda+u)]\sqrt{\frac{\lambda}{u}}\,\mathrm{d}u$

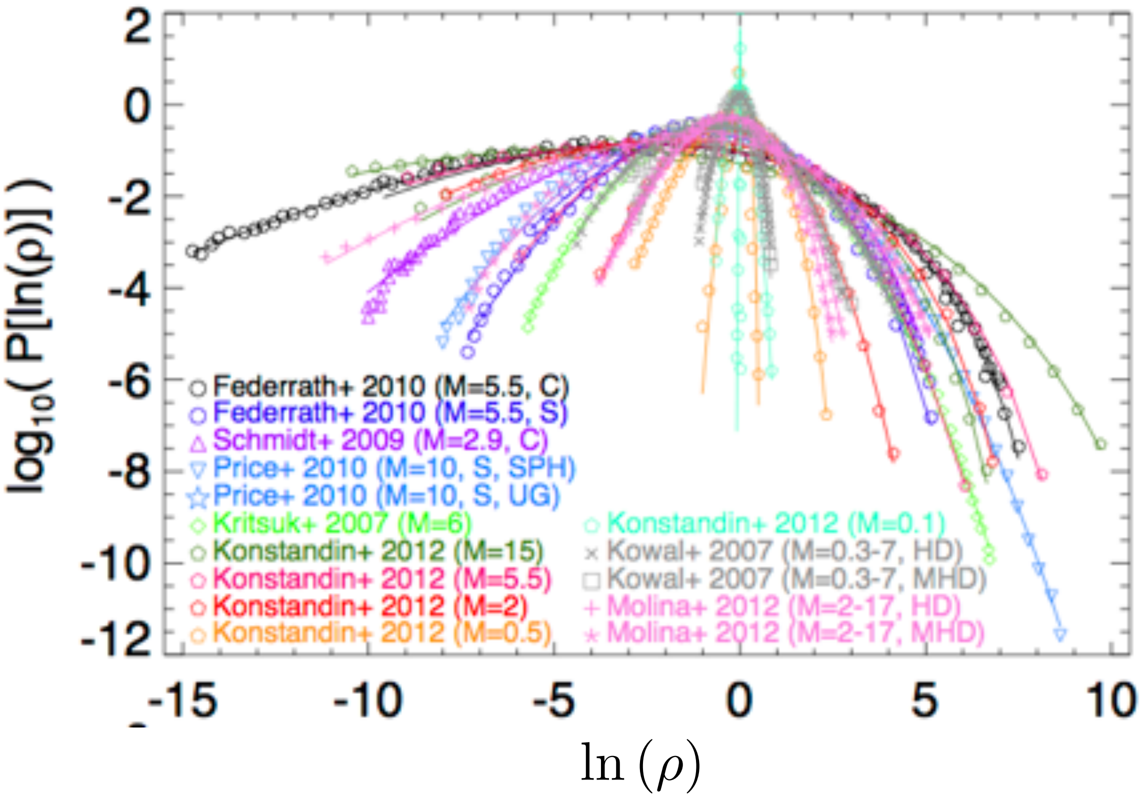
  (Castaing 1996)

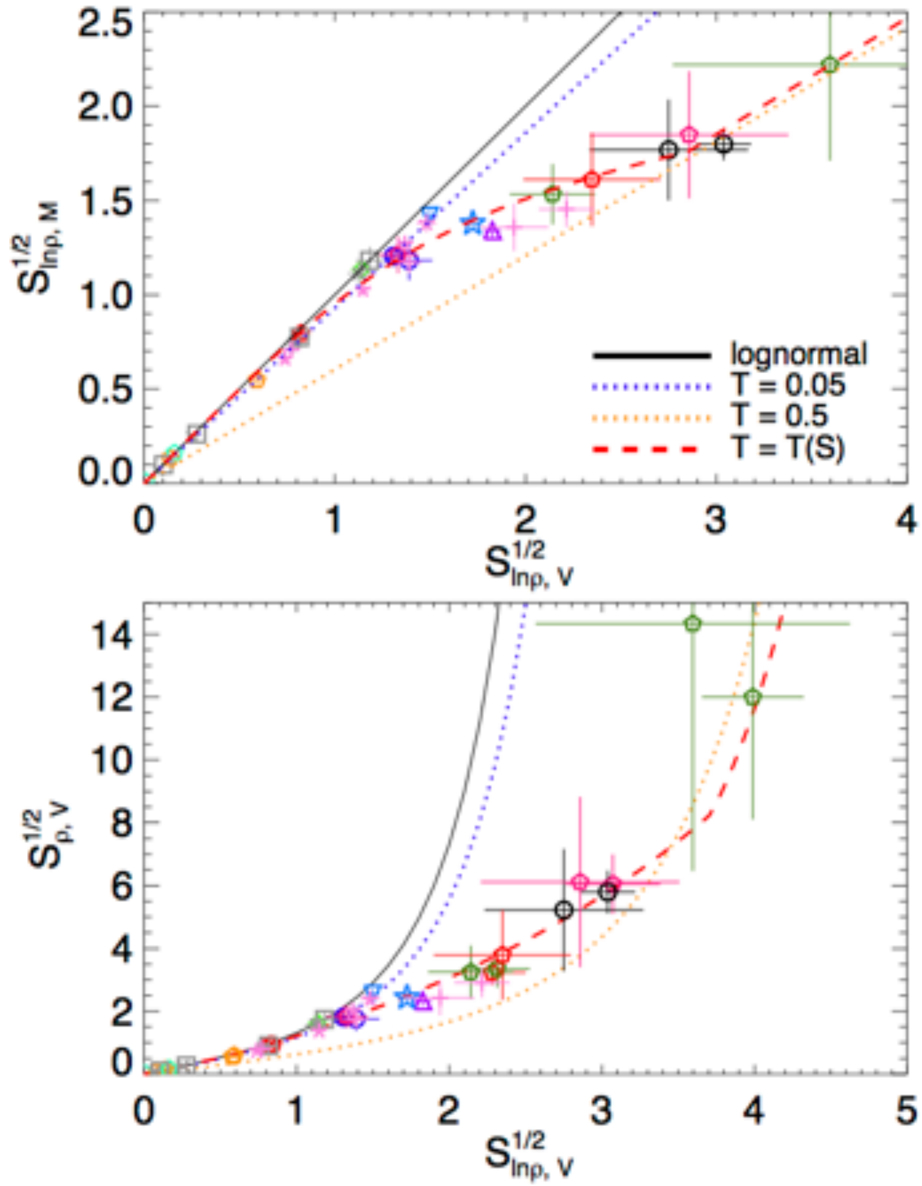
$$= \sum_{m=0}^{\infty} \frac{\lambda^m e^{-\lambda}}{m!} \frac{u^{m-1} e^{-u}}{(m-1)!}\,\mathrm{d}u \qquad (u \propto \ln\rho)$$



More Interesting: Even Isothermal Gas is Not Lognormal!

EXPLAINS MANY DISCREPANCIES IN SIMULATIONS & METHODS

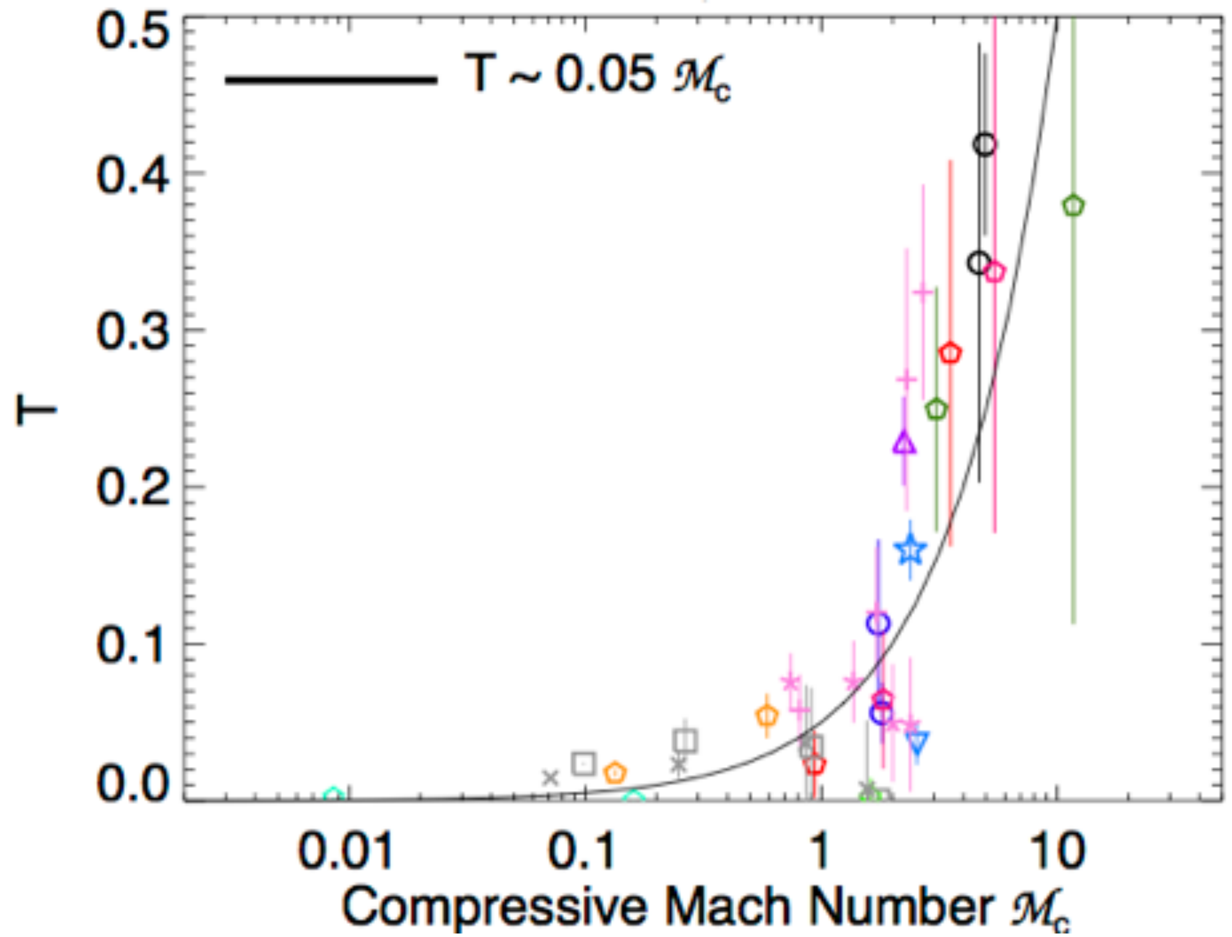




More Interesting: Even Isothermal Gas is Not Lognormal!

MASS CONSERVATION & INTERMITTENCY PREVENT IT

- Parameter $T = 0\text{-}1$ represents the "degree of intermittency"
  - *Fundamental* parameter of multi-fractal/cascade models of turbulence



- Same values for *T* derived from density PDF or velocity statistics

Summary:

*** ISM *statistics* are far more fundamental than we typically assume ***

- **Turbulence + Gravity:** ISM structure follows
    - Lognormal density PDF is ***not*** critical
    - *ANALYTICALLY* understand:
        - **GMC Mass Function & Structure** ("first crossing")
        - **Core MF** ("last crossing") & **Linewidth-Size-Mass**
        - **Clustering** of Stars (correlation functions)

- **Planet Formation in Direct Collapse:**
    - Modest turbulence (Mach >0.1) is sufficient for ~1 event(s)
    - Applies to grains as well?

- **Non-Gaussian Statistics**: not dominant in calculations above
    - But very interesting probes of the structure of turbulence!
    - Indicates Mach-density connection generalizes over entire cascade