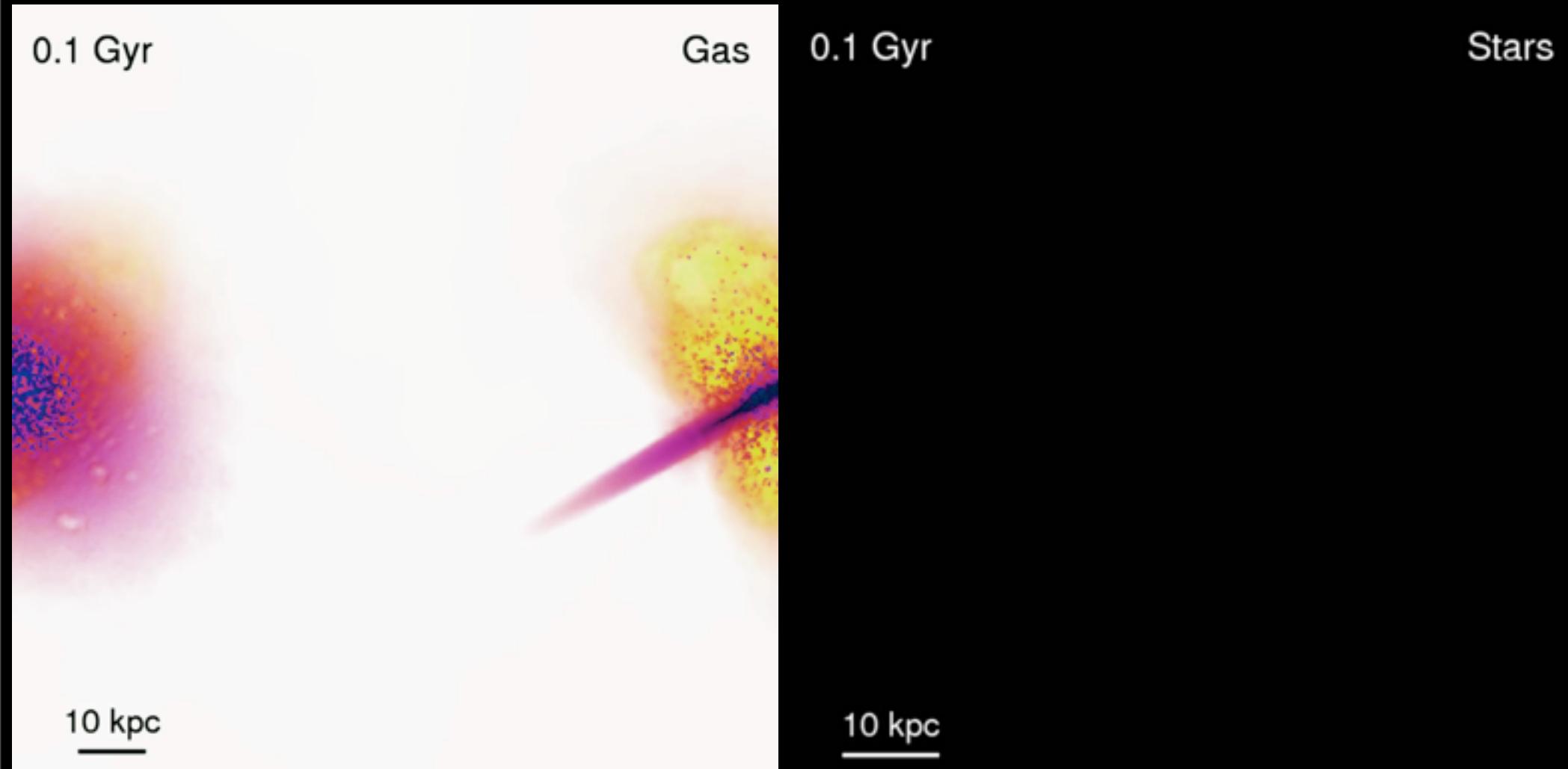


ISM Structure: Order from Chaos

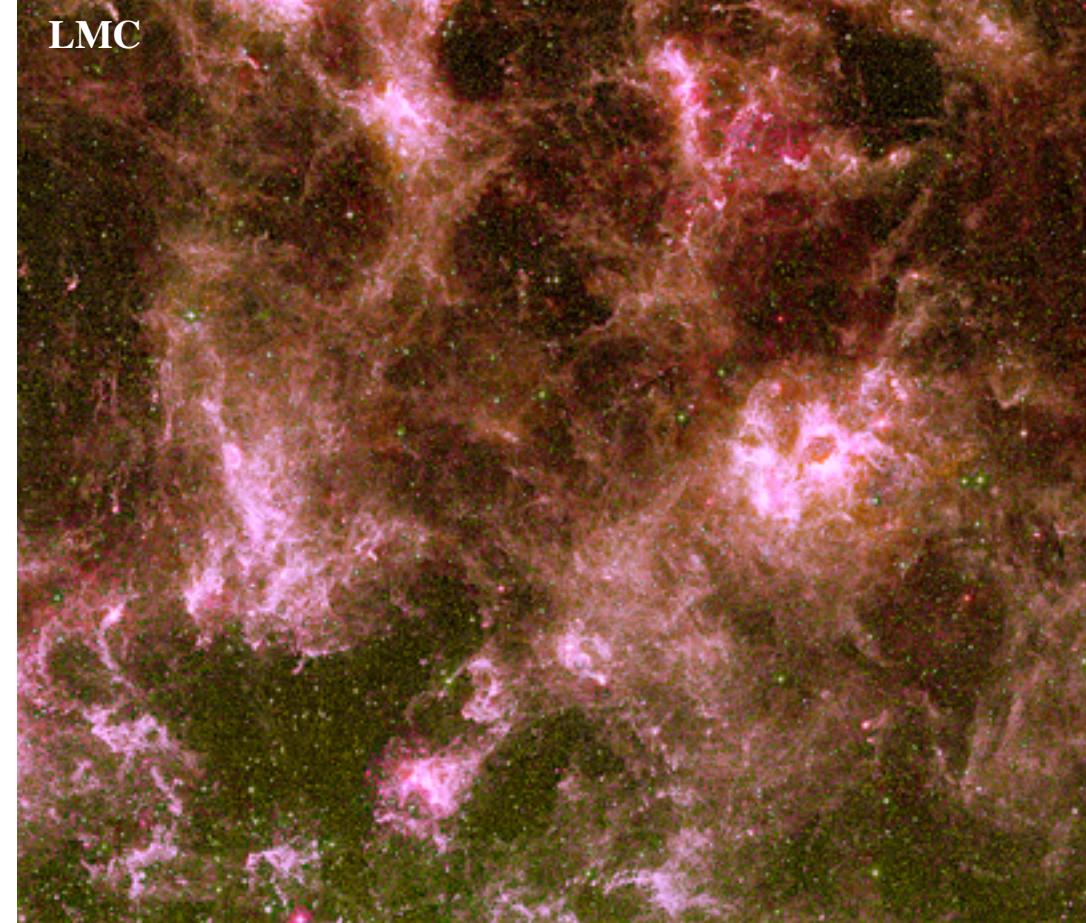
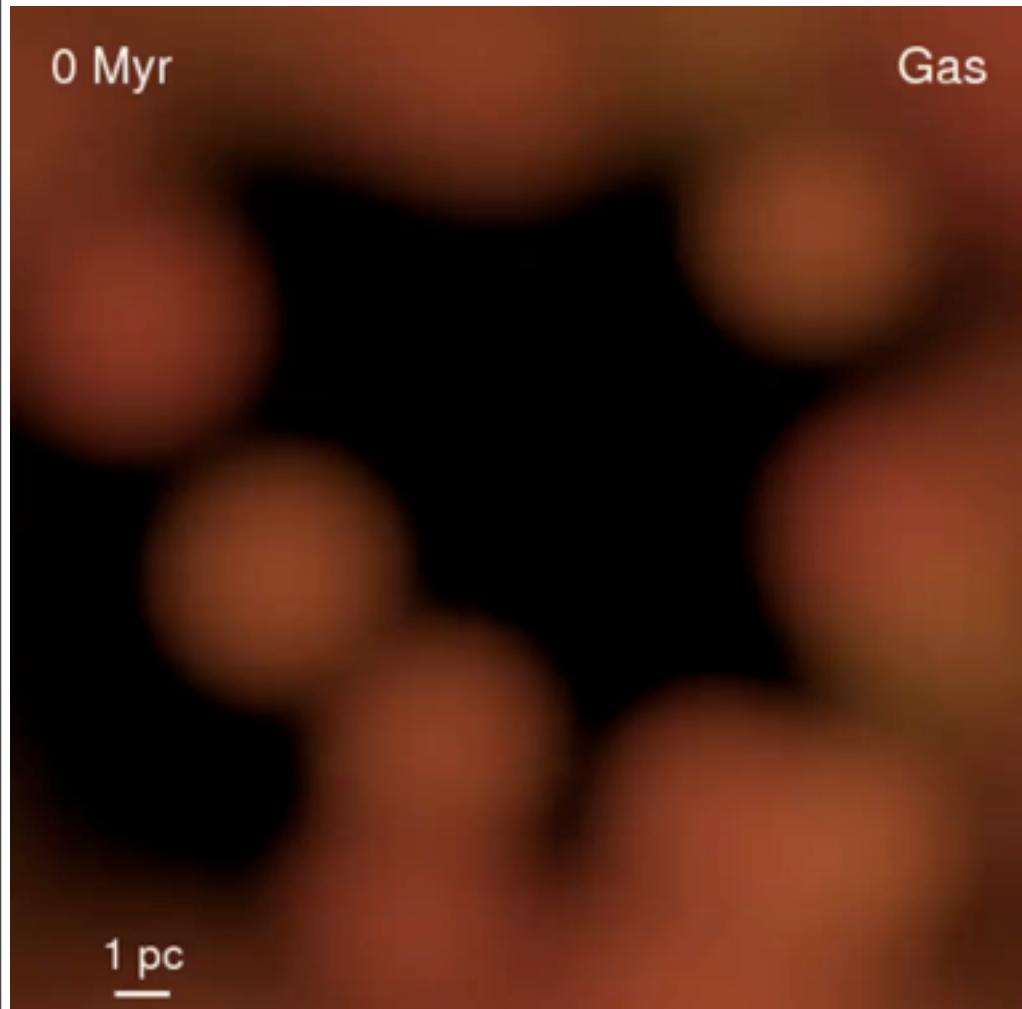


Philip Hopkins

with Eliot Quataert, Norm Murray,
Lars Hernquist, Dusan Keres, Todd Thompson, Desika Narayanan,
Dan Kasen, T. J. Cox, Chris Hayward, Kevin Bundy, & more

The Turbulent ISM

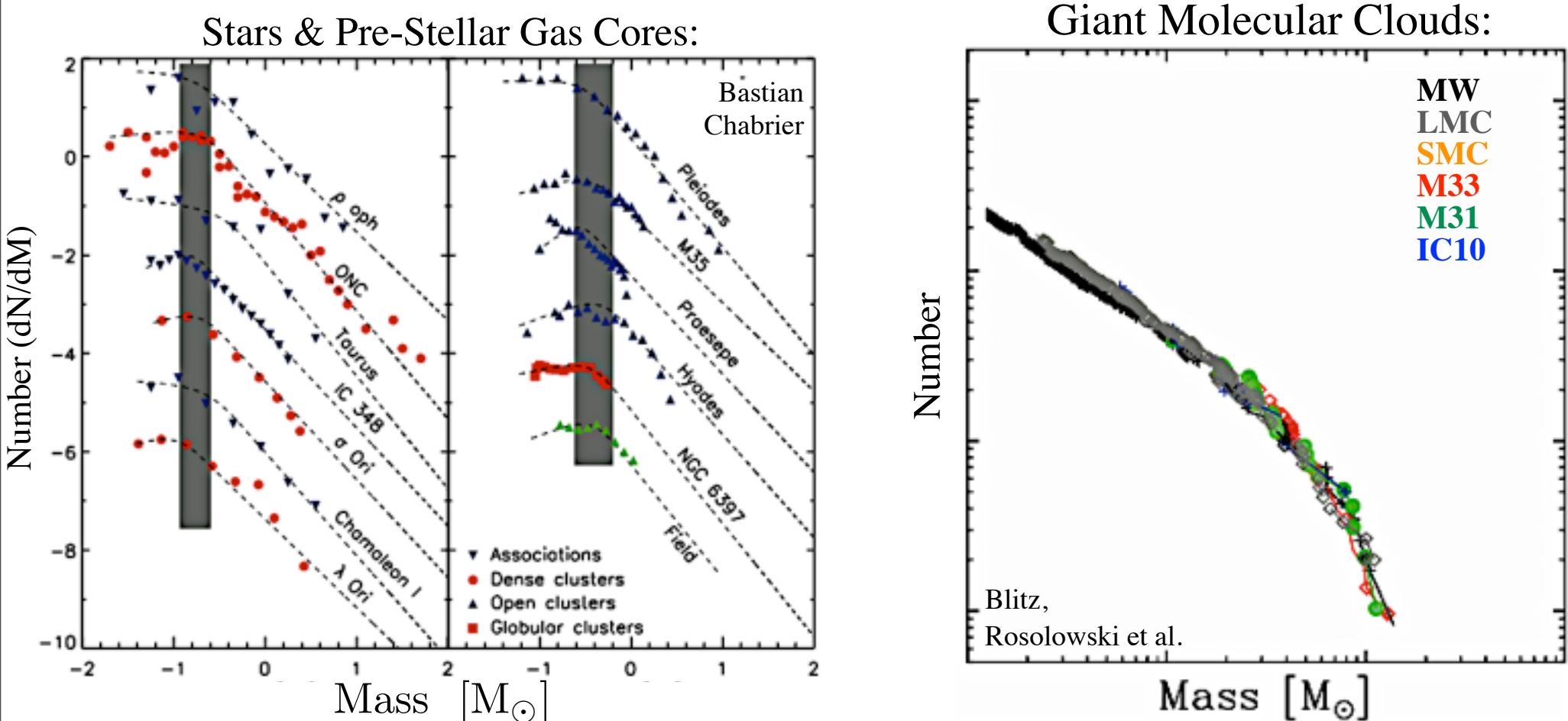
IMPORTANT ON
(ALMOST) ALL SCALES



- Gravity
- Turbulence
- Magnetic, Thermal, Cosmic Ray, Radiation Pressure
- Cooling (atomic, molecular, metal-line, free-free)
- Star & BH Formation/Growth
- “Feedback”: Massive stars, SNe, BHs, external galaxies, etc.

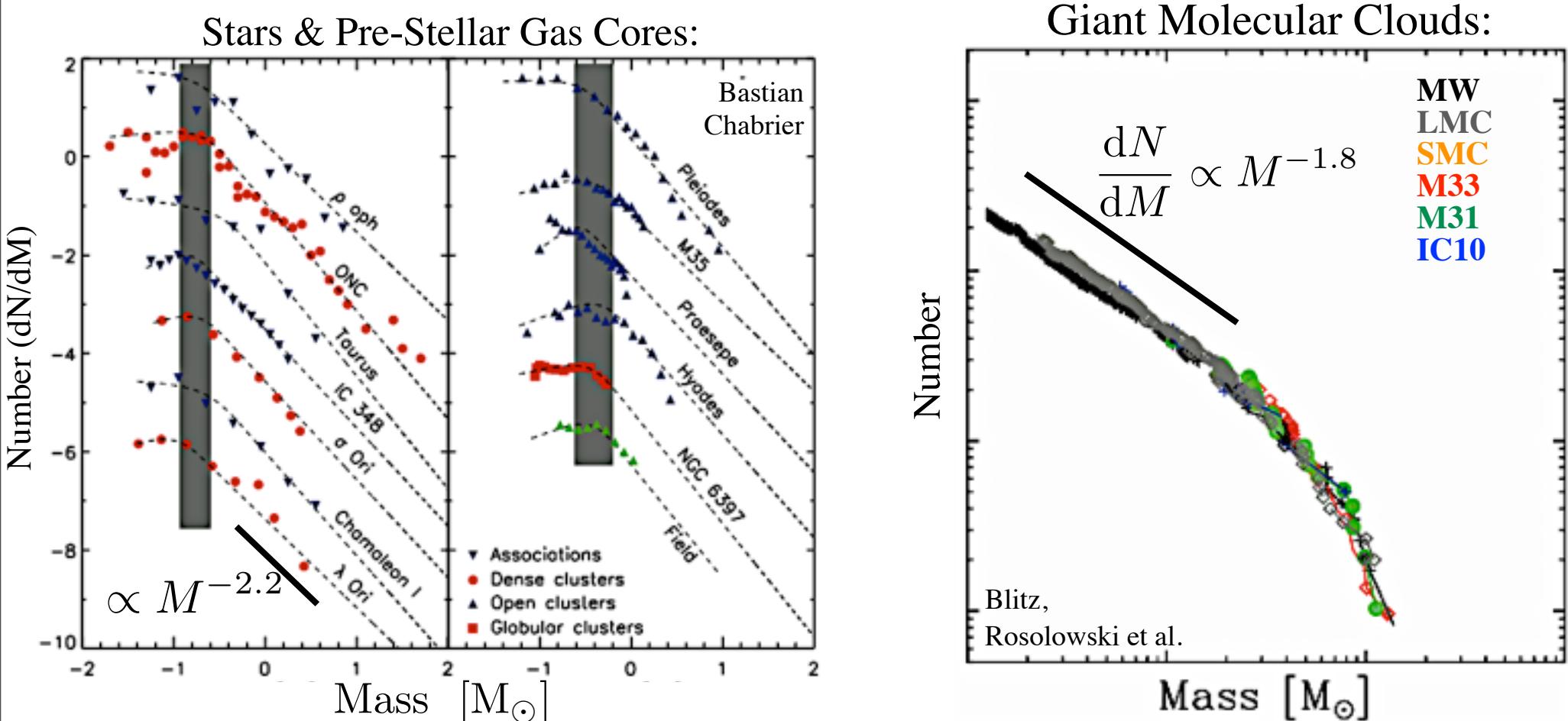
The ISM

YET THERE IS SURPRISING REGULARITY



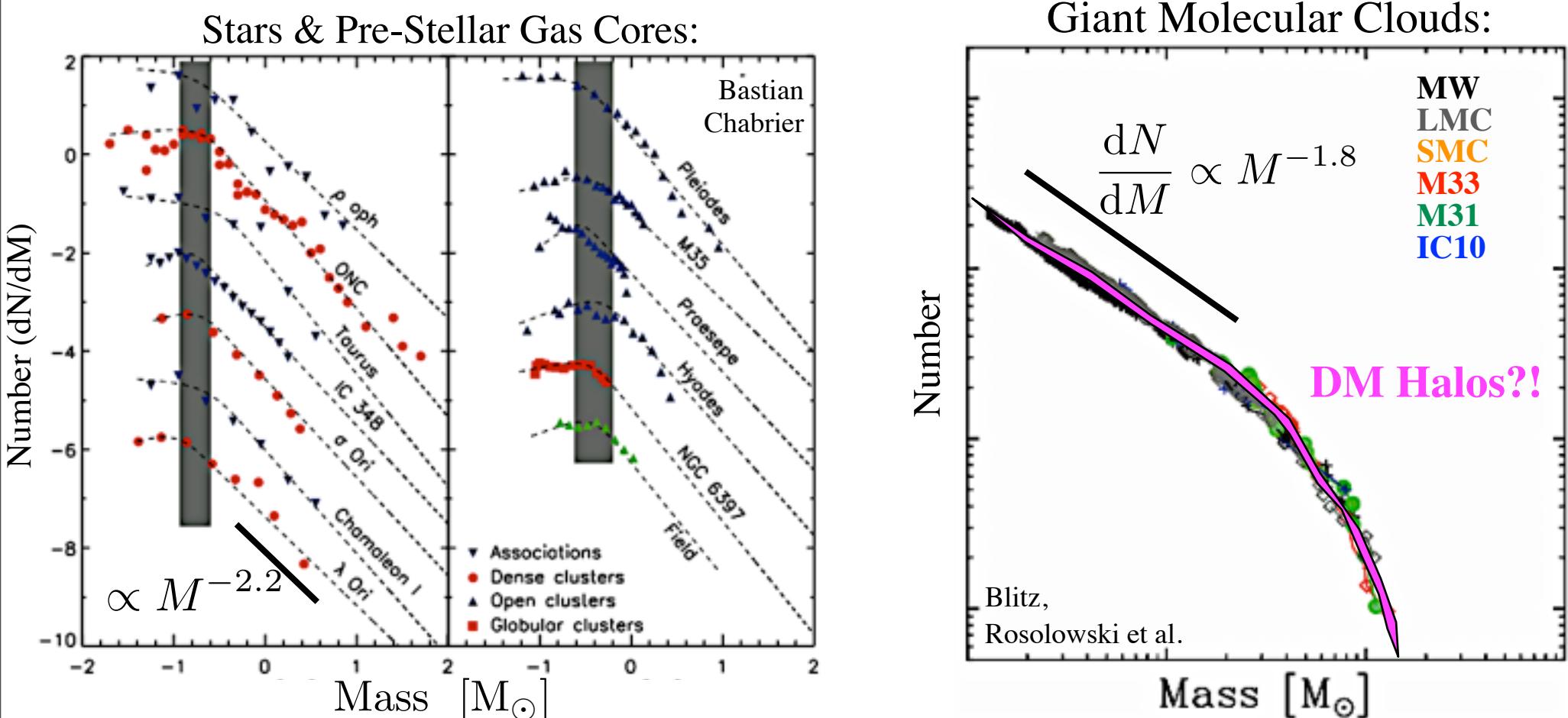
The ISM

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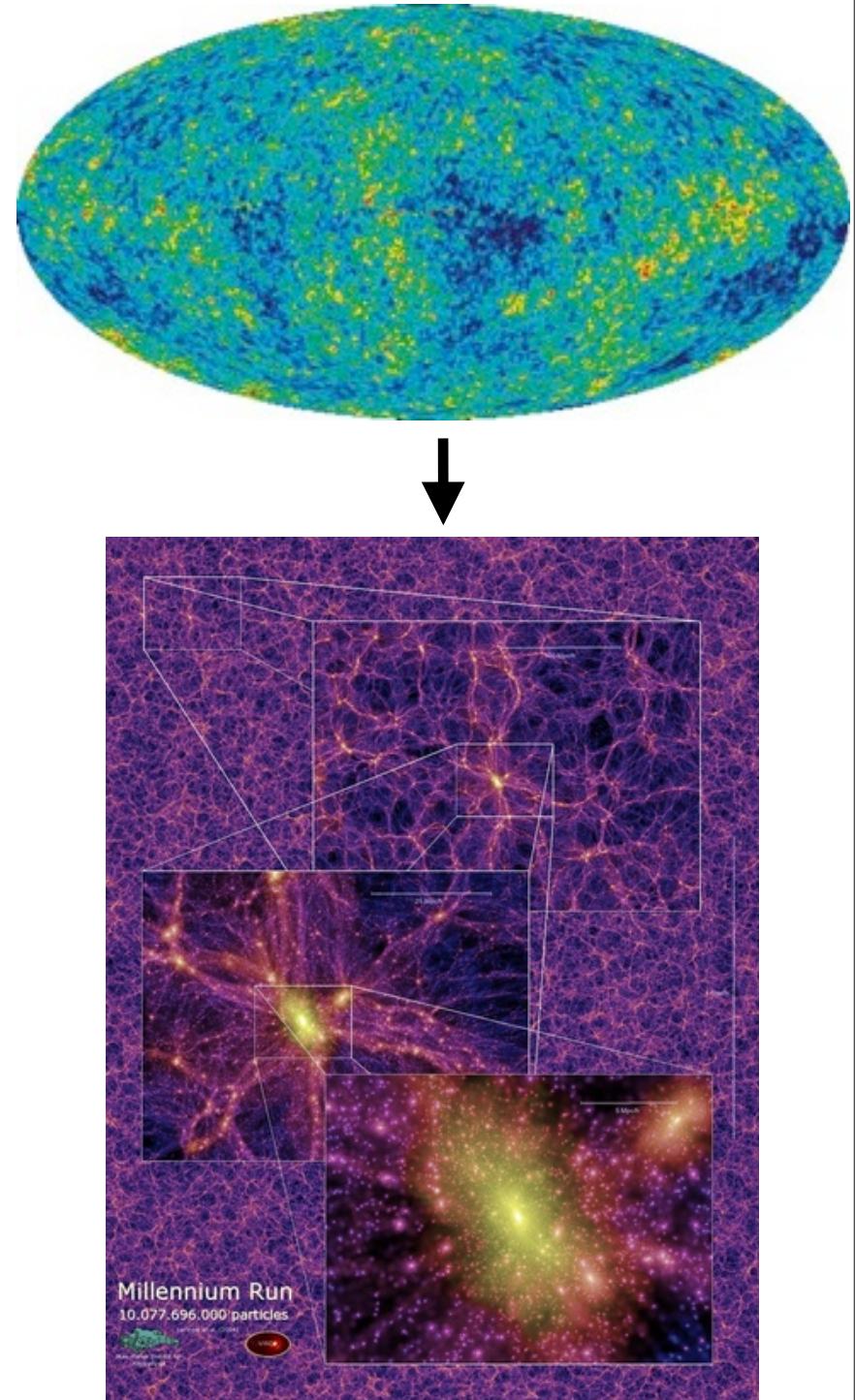
The ISM

YET THERE IS SURPRISING REGULARITY



Extended Press-Schechter / Excursion-Set Formalism

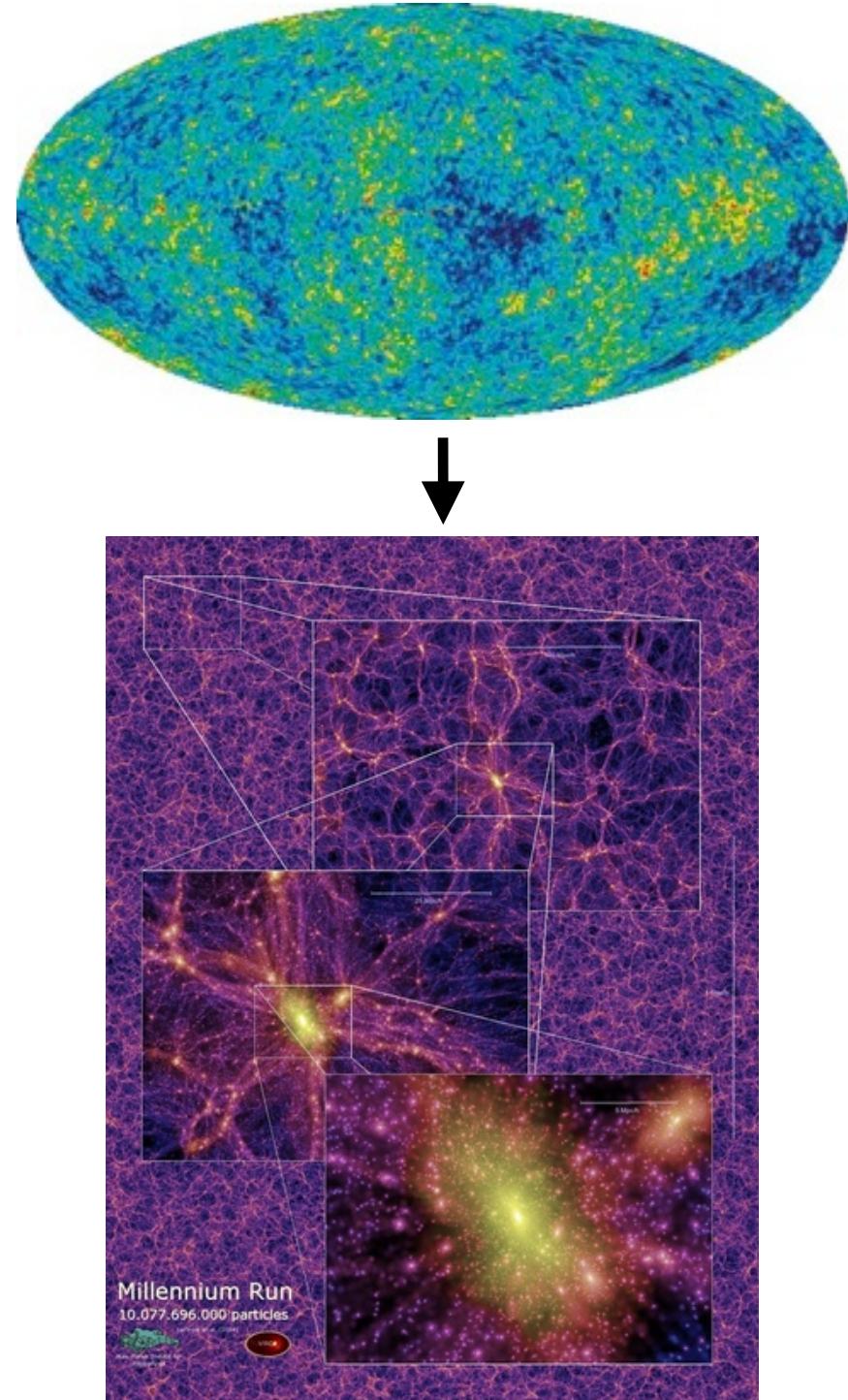
- Press & Schechter '74:
 - r Fluctuations a Gaussian random field
 - Know linear power spectrum $P(k \sim 1/r)$: variance $\sim k^3 P(k)$



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 - Turnaround & gravitational collapse

$$\bar{\rho}(< R \sim 1/k) > \rho_{\text{crit}}$$



Extended Press-Schechter / Excursion-Set Formalism

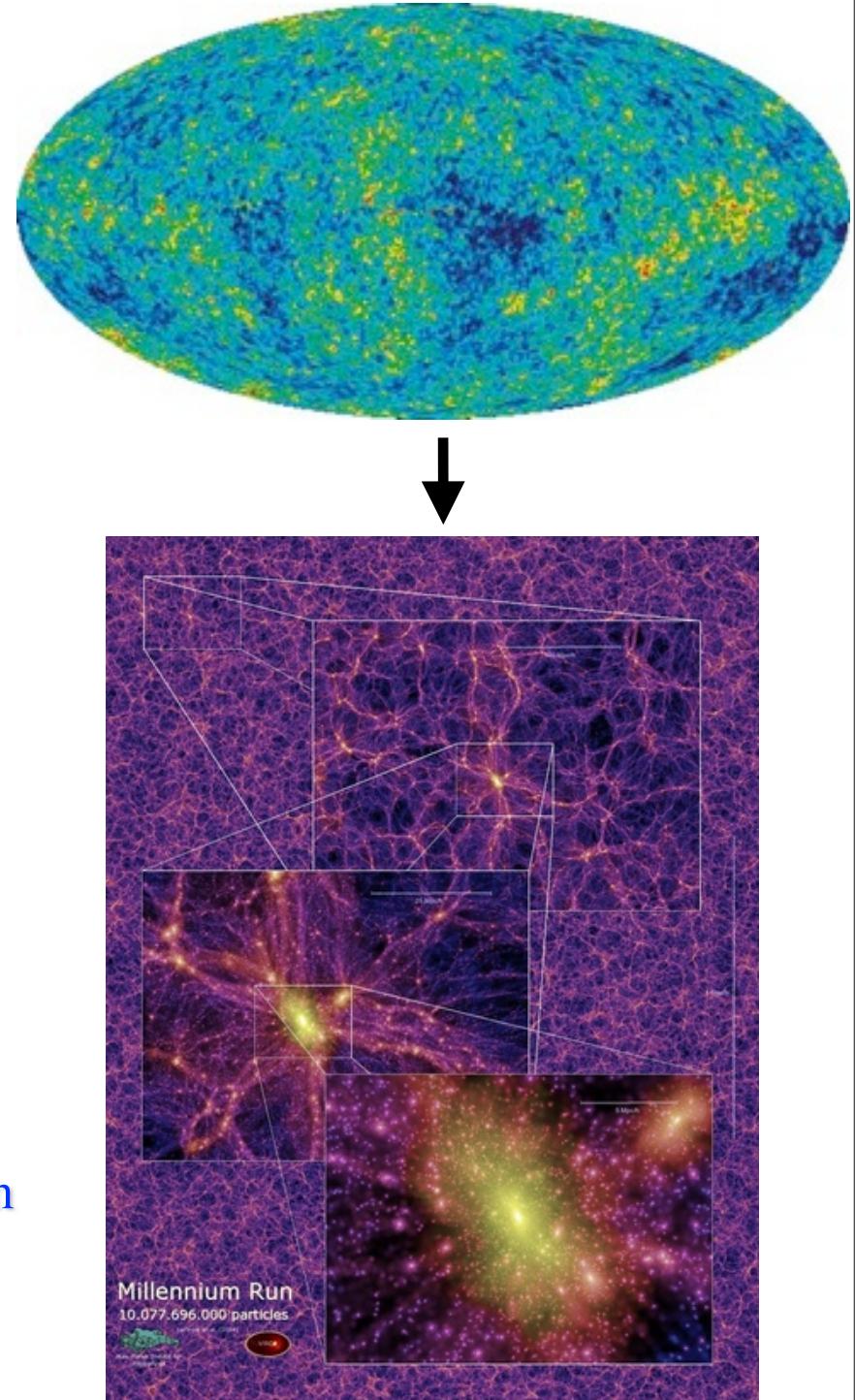
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$$\bar{\rho}(< R \sim 1/k) > \rho_{\text{crit}}$$

- Generalize to conditional probabilities,
N-point statistics, resolve “cloud in cloud” problem
(e.g. Bond et al. 1991)



Turbulence

BASIC EXPECTATIONS

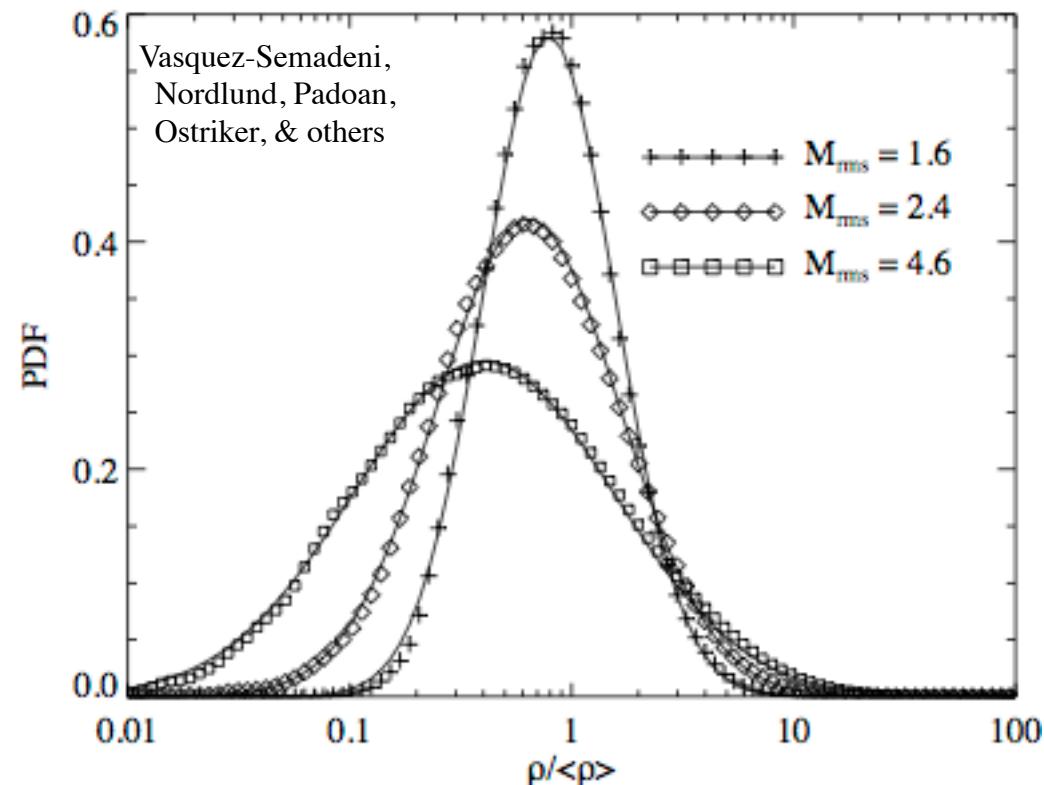
Velocity: $E(k) \propto k^{-p}$ ($k E(k) \sim u_t(k)^2$)

Turbulence

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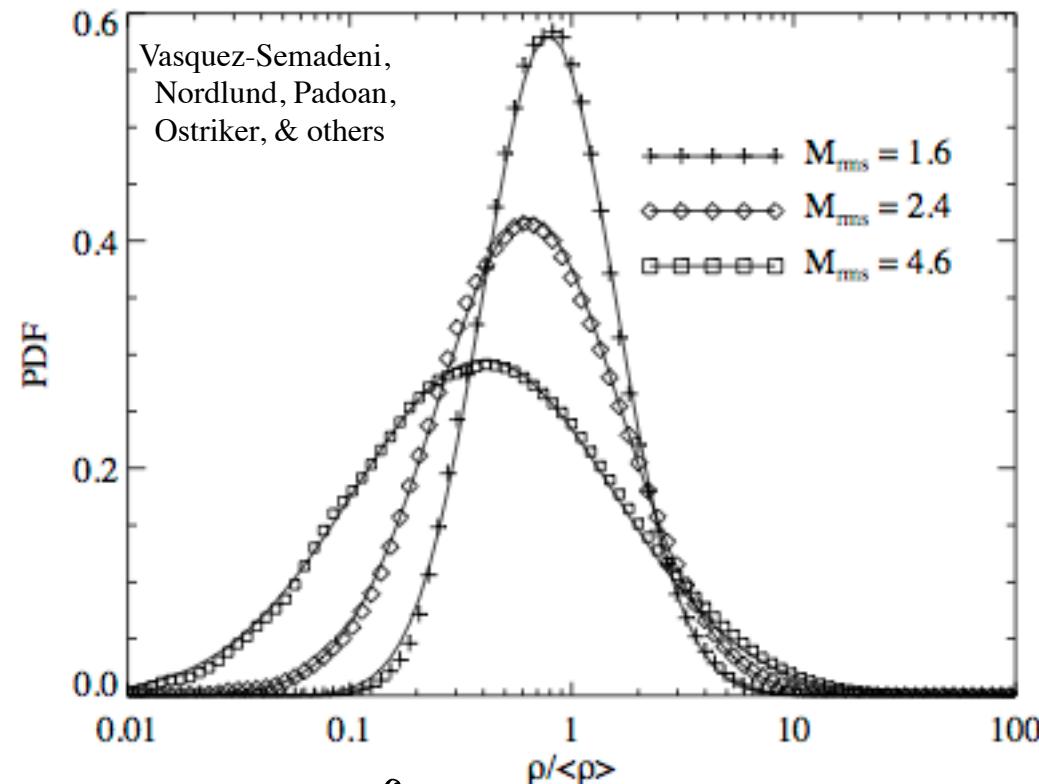


Turbulence

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$$S(R) = \int d \ln k S_k |W(k, R)|^2$$

What Defines a Fluctuation of Interest?

DISPERSION RELATION:

$$\omega^2 = \kappa^2 + c_s^2 k^2 + u_t(k)^2 k^2 - \frac{4\pi G \rho |k| h}{1 + |k| h}$$

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Chandrasekhar '51, Vandervoort '70, Toomre '77

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 $\propto r^{-2}$

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Turbulence

$$\propto r^{p-3} \sim r^{-1}$$

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Angular Momentum Thermal Pressure Turbulence Gravity

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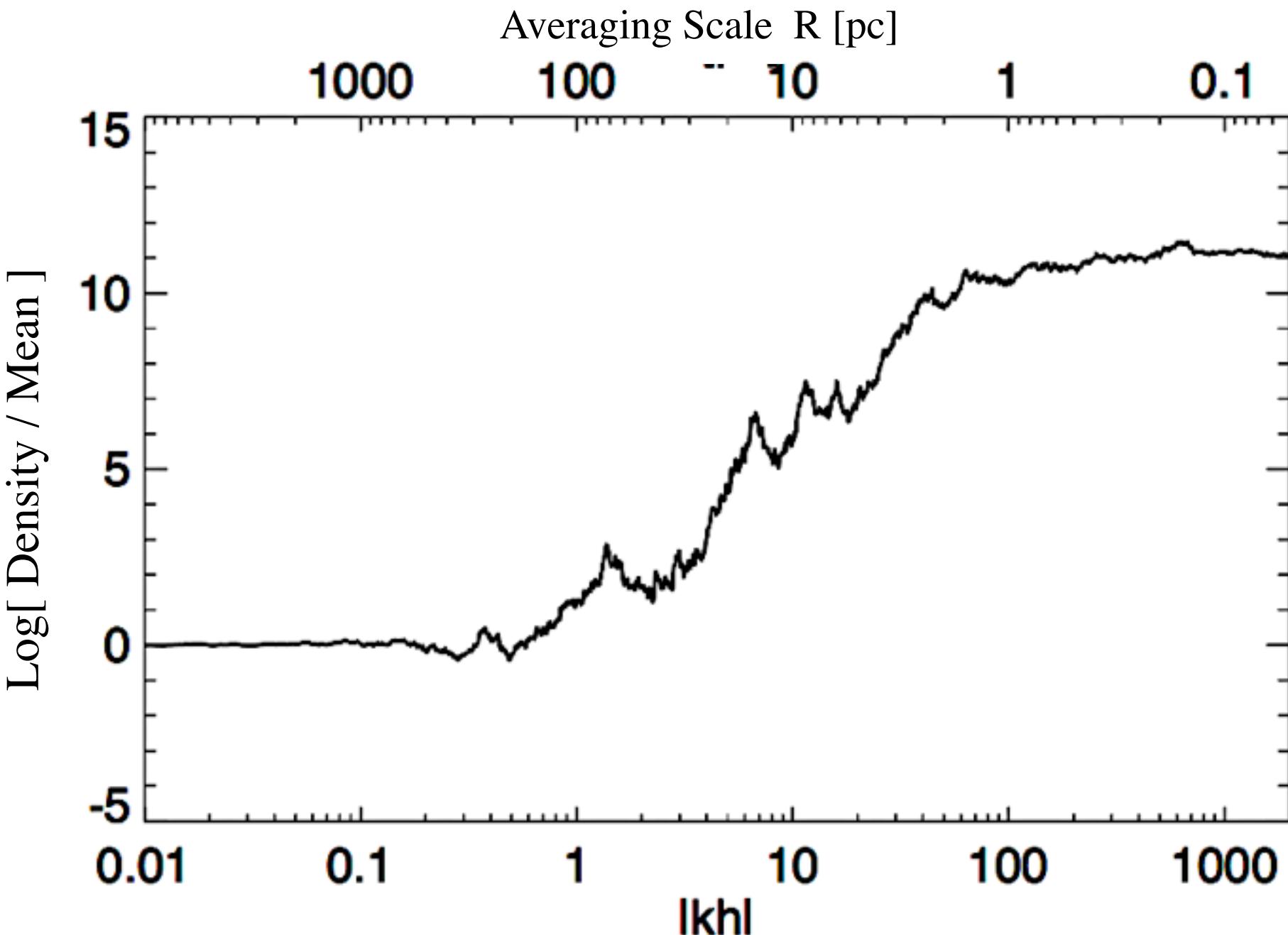
Mode Grows (Collapses) when $\omega < 0$:

$$\rho > \rho_c(k) = \rho_0 (1 + |kh|) \left[(\mathcal{M}_h^{-2} + |kh|^{1-p}) kh + \frac{2}{|kh|} \right]$$

Chandrasekhar '51, Vandervoort '70, Toomre '77

“Counting” Collapsing Objects

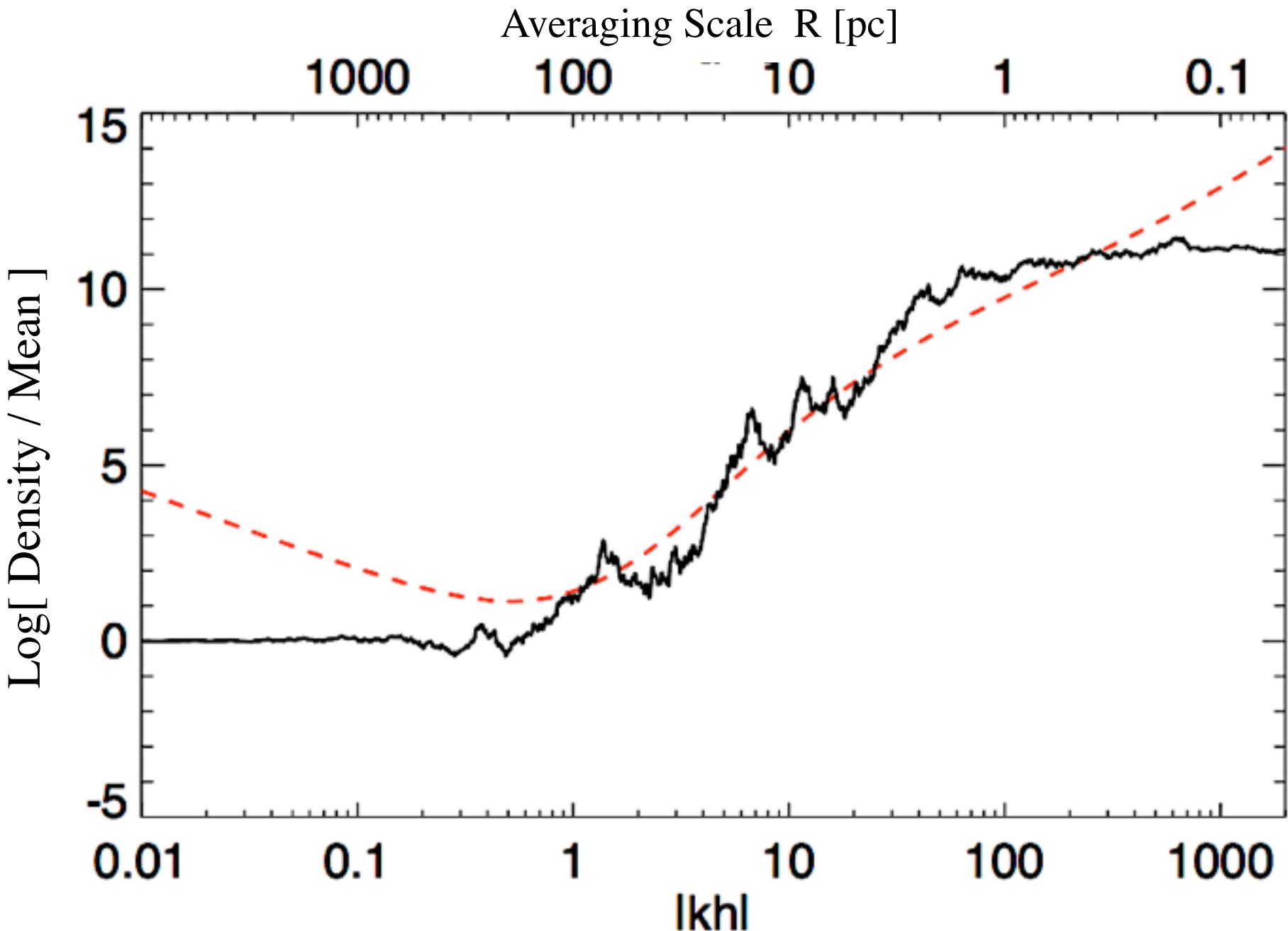
EVALUATE DENSITY FIELD vs. “BARRIER”



“Counting” Collapsing Objects

PFH 2011

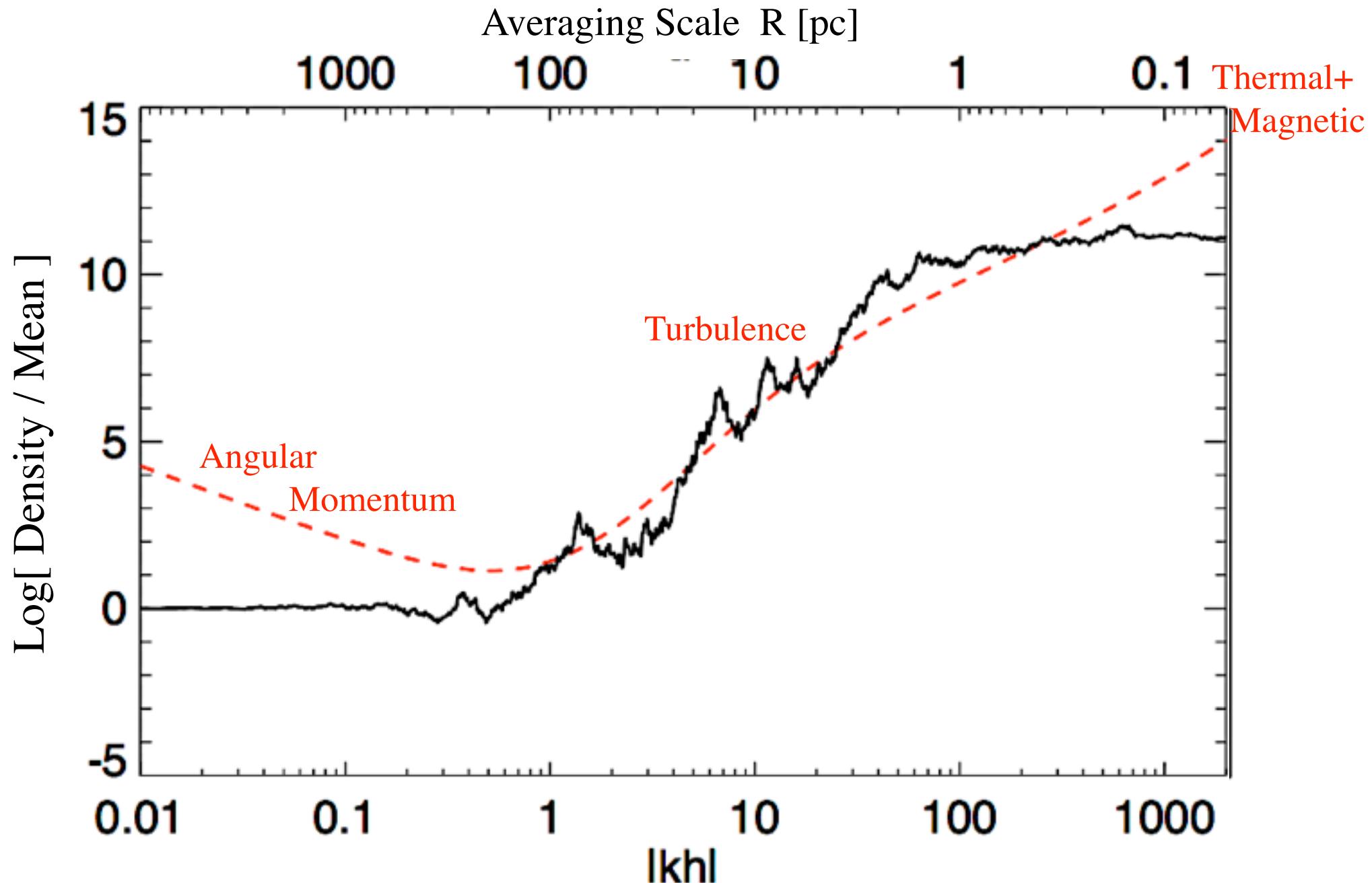
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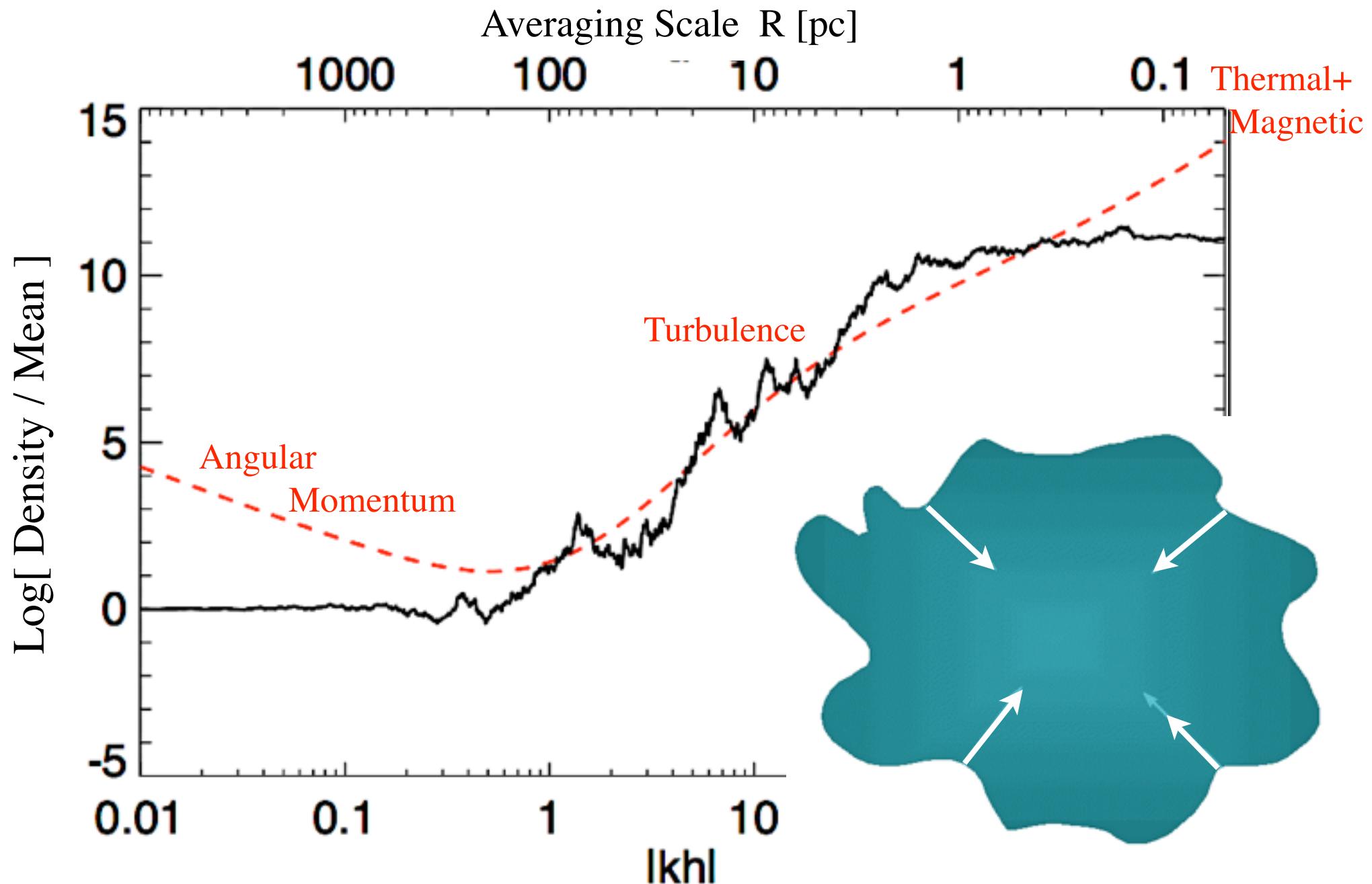
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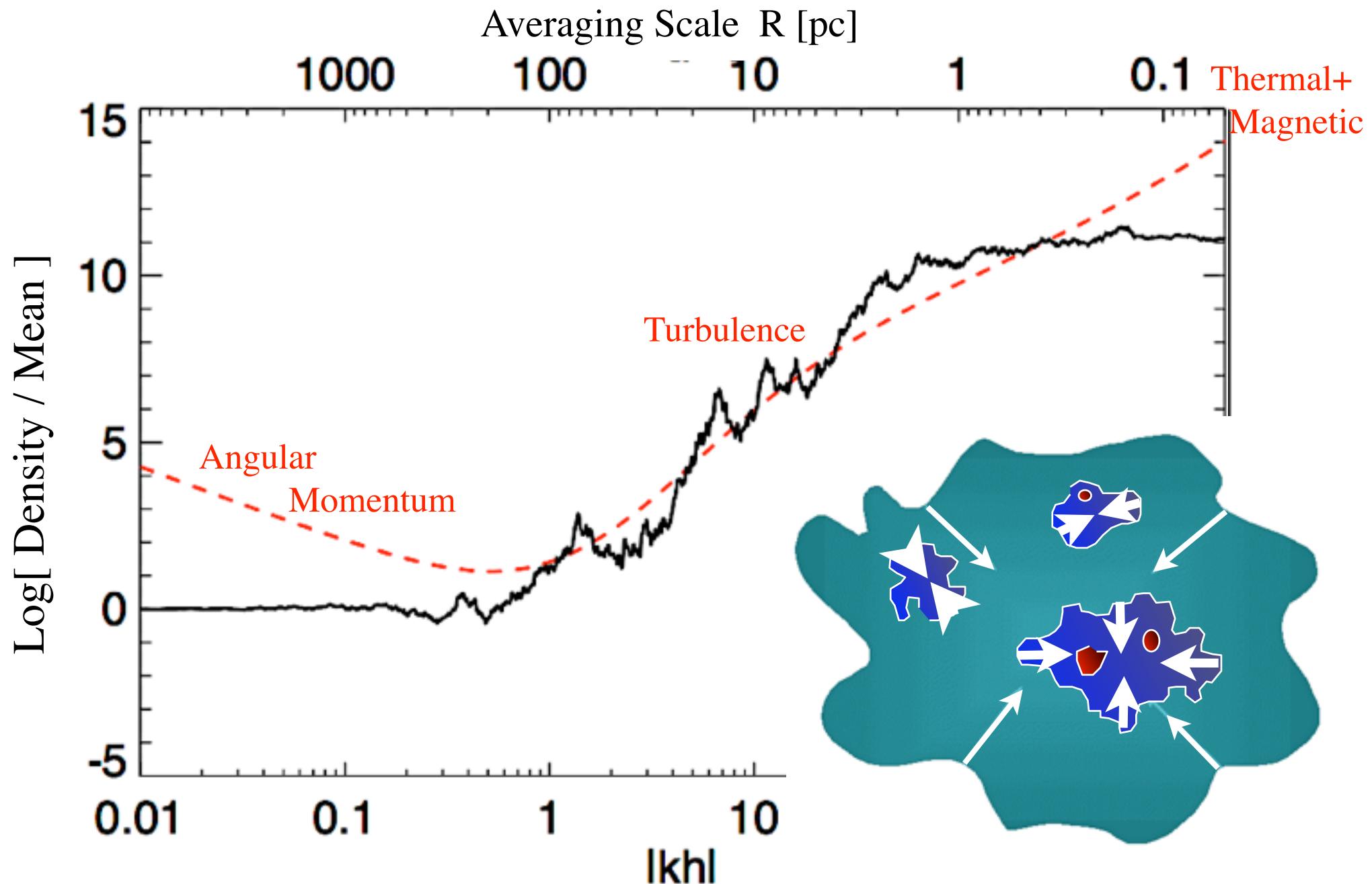
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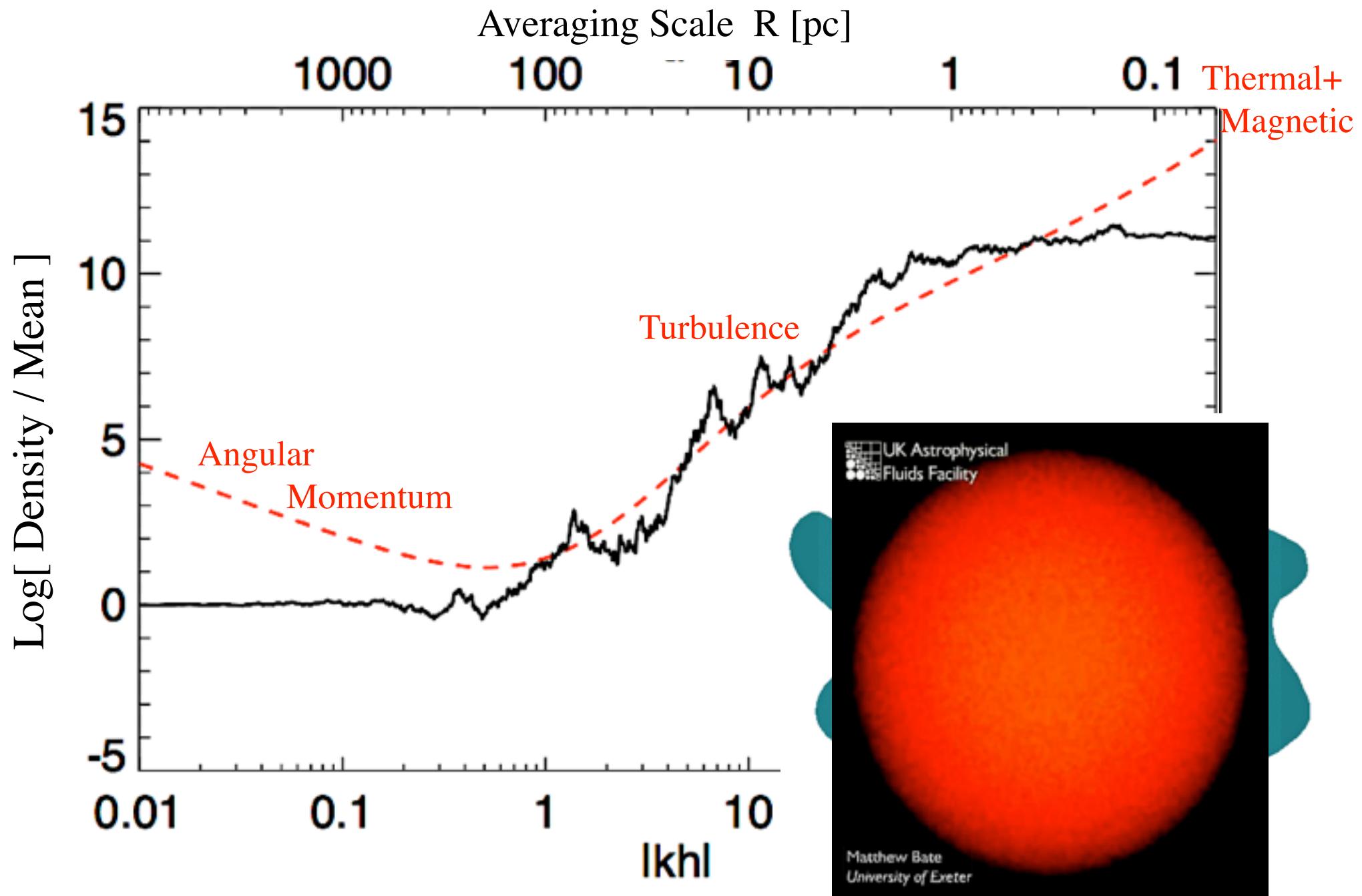
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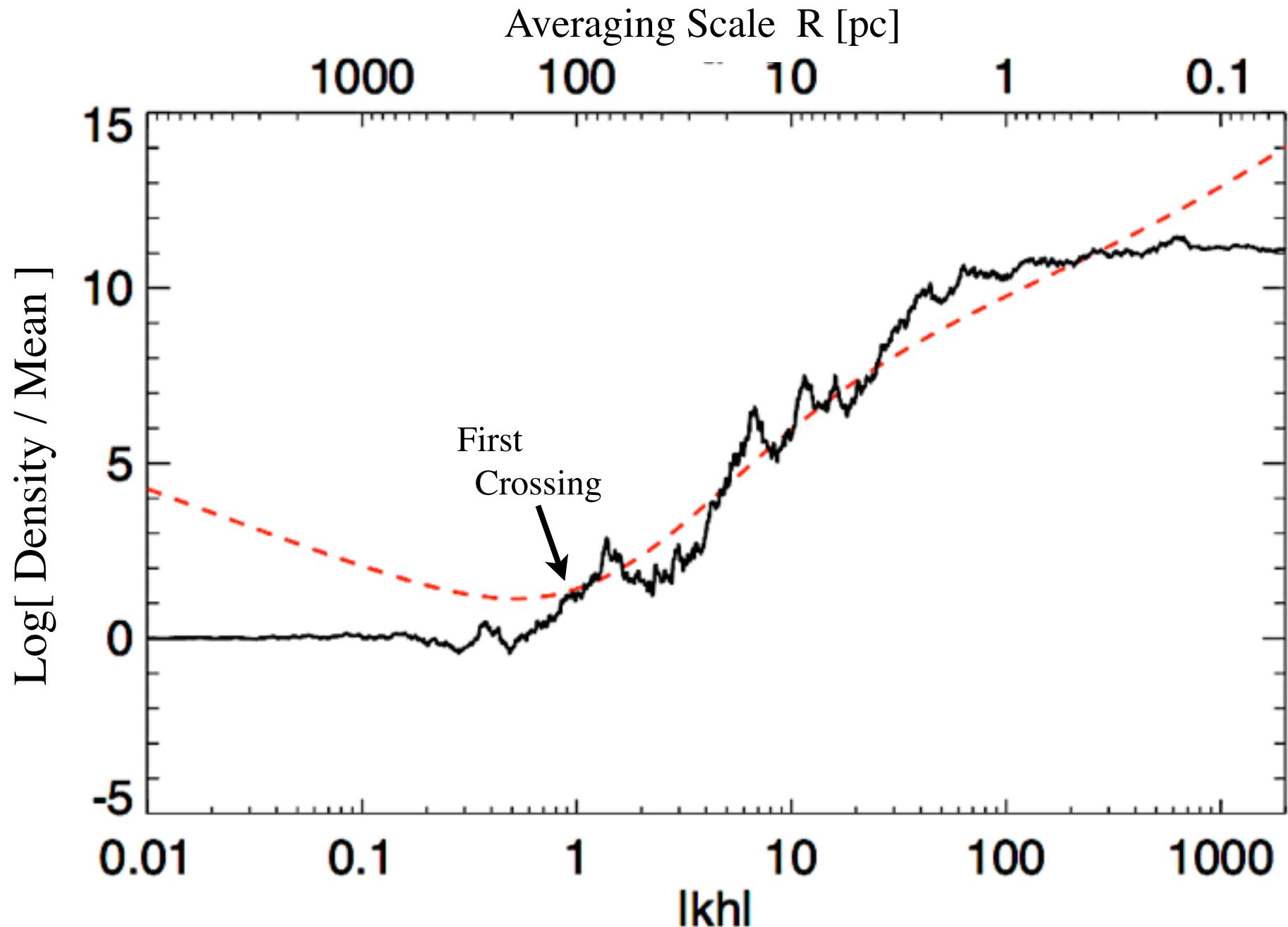
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PFH 2011

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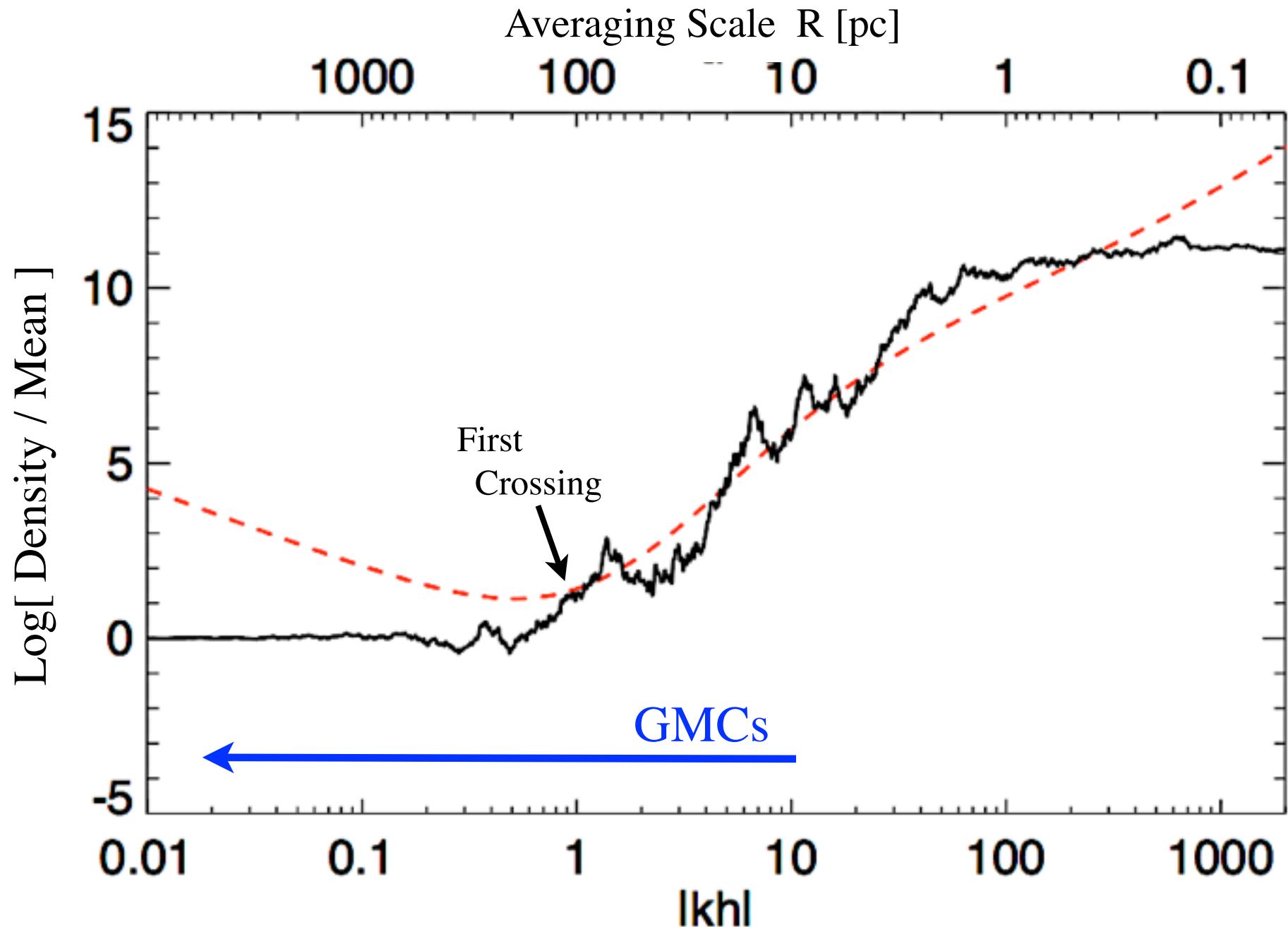
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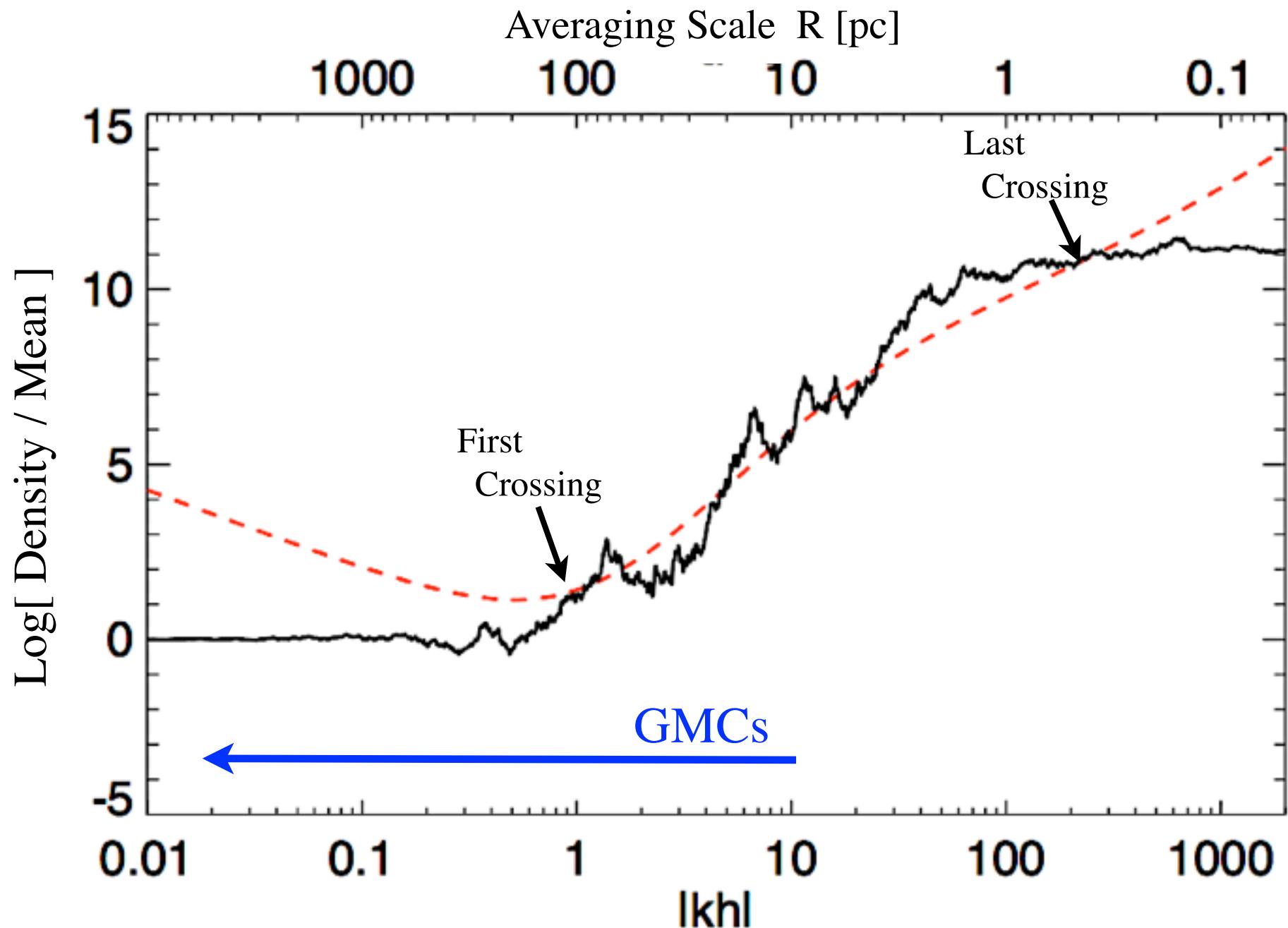
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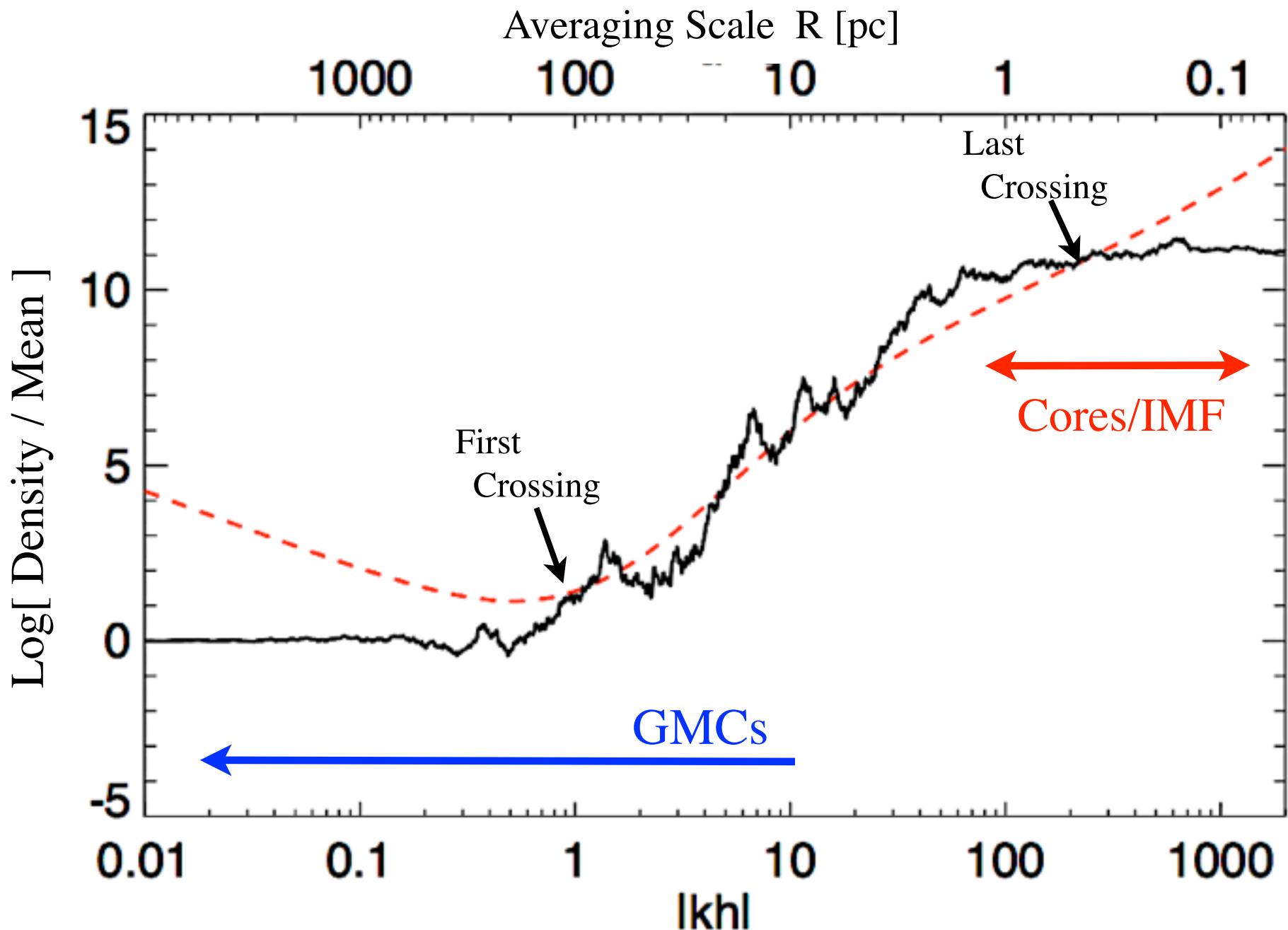
PFH 2011

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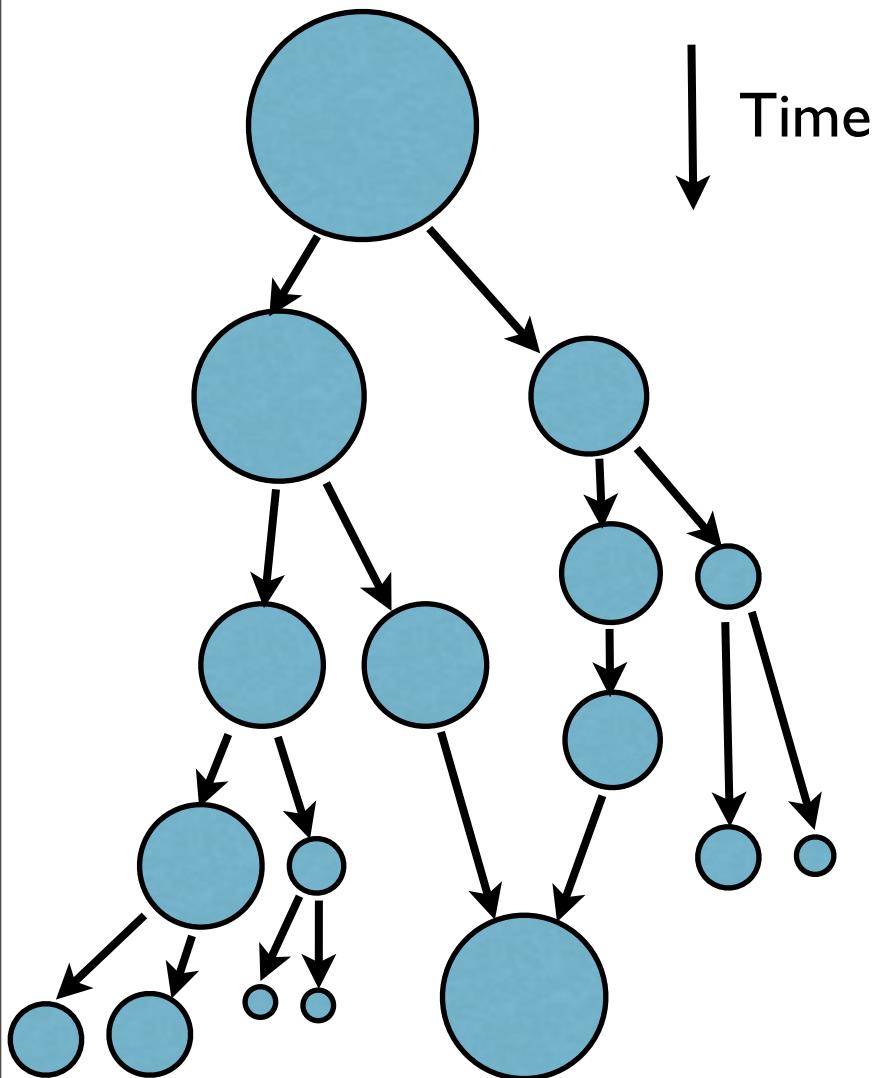
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Evolve the Fluctuations in Time

CONSTRUCT “MERGER/FRAGMENTATION” TREES

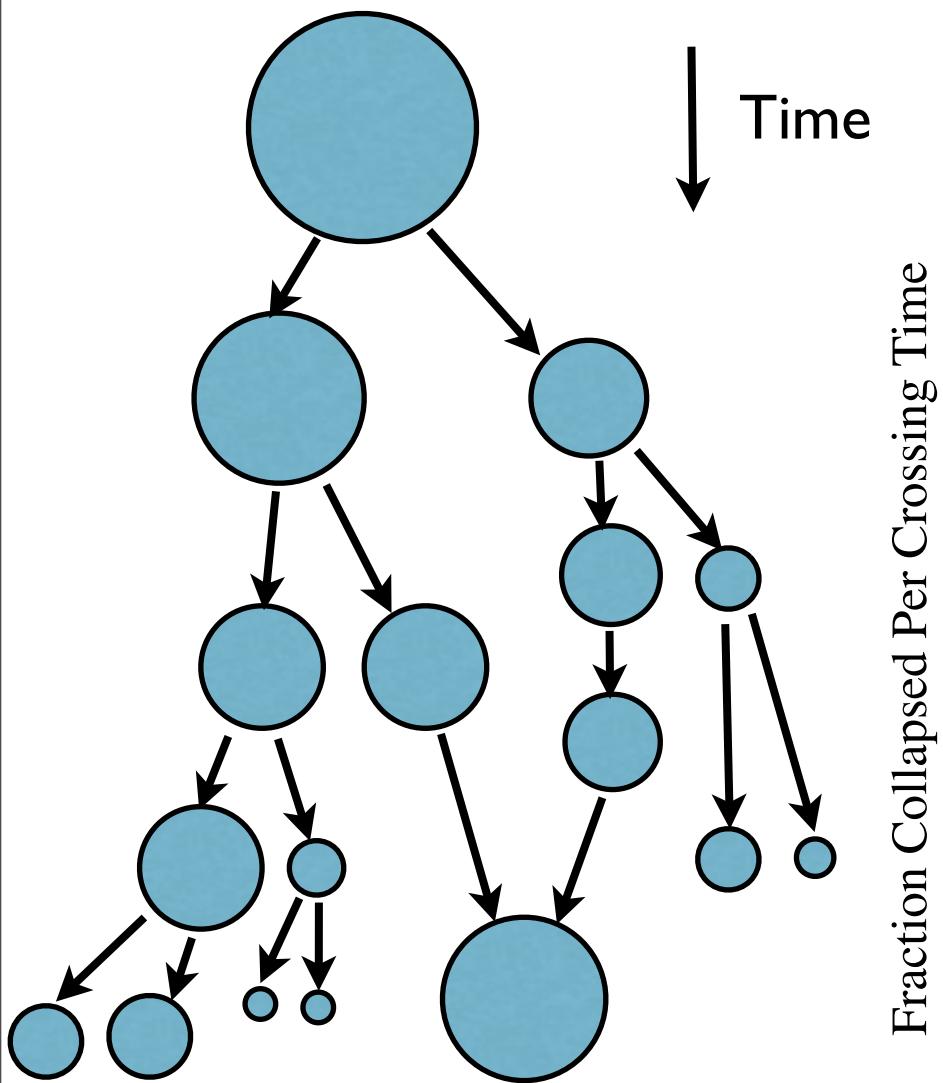
$$p(\delta | \tau) = \frac{1}{\sqrt{2\pi S (1 - \exp [-2\tau])}} \exp \left[- \frac{(\delta - \delta(t=0) \exp [-\tau])^2}{2 S (1 - \exp [-2\tau])} \right]$$



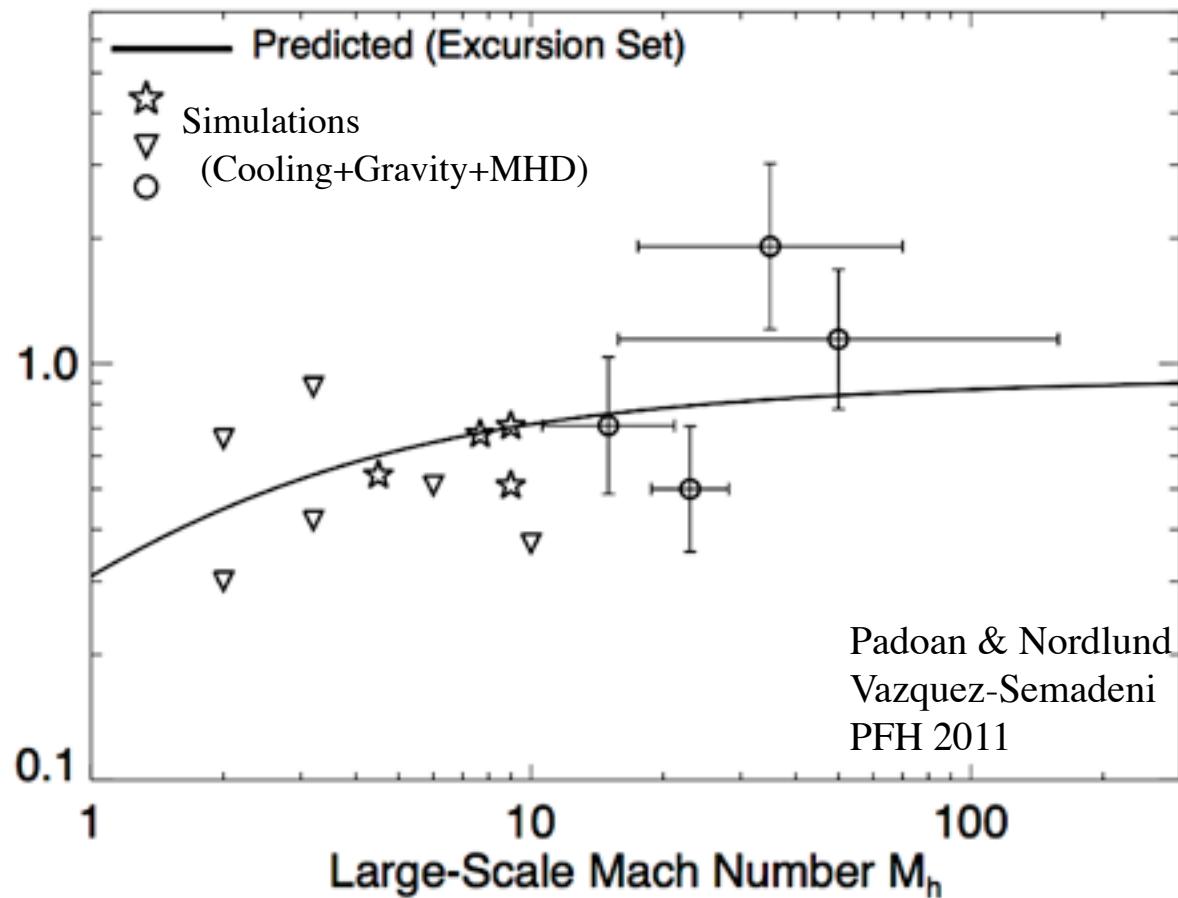
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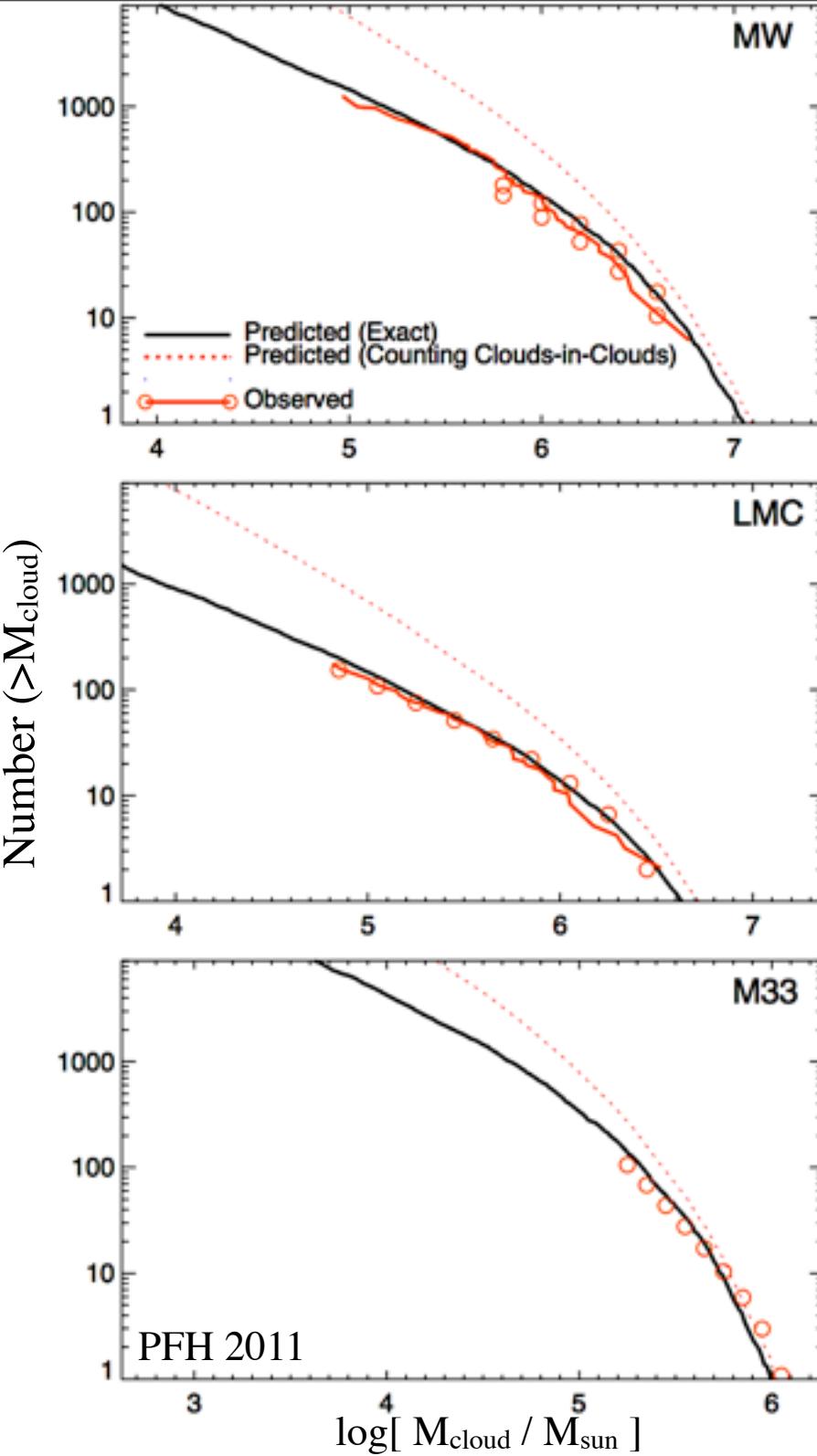


Fraction Collapsed Per Crossing Time



Padoan & Nordlund
Vazquez-Semadeni
PFH 2011

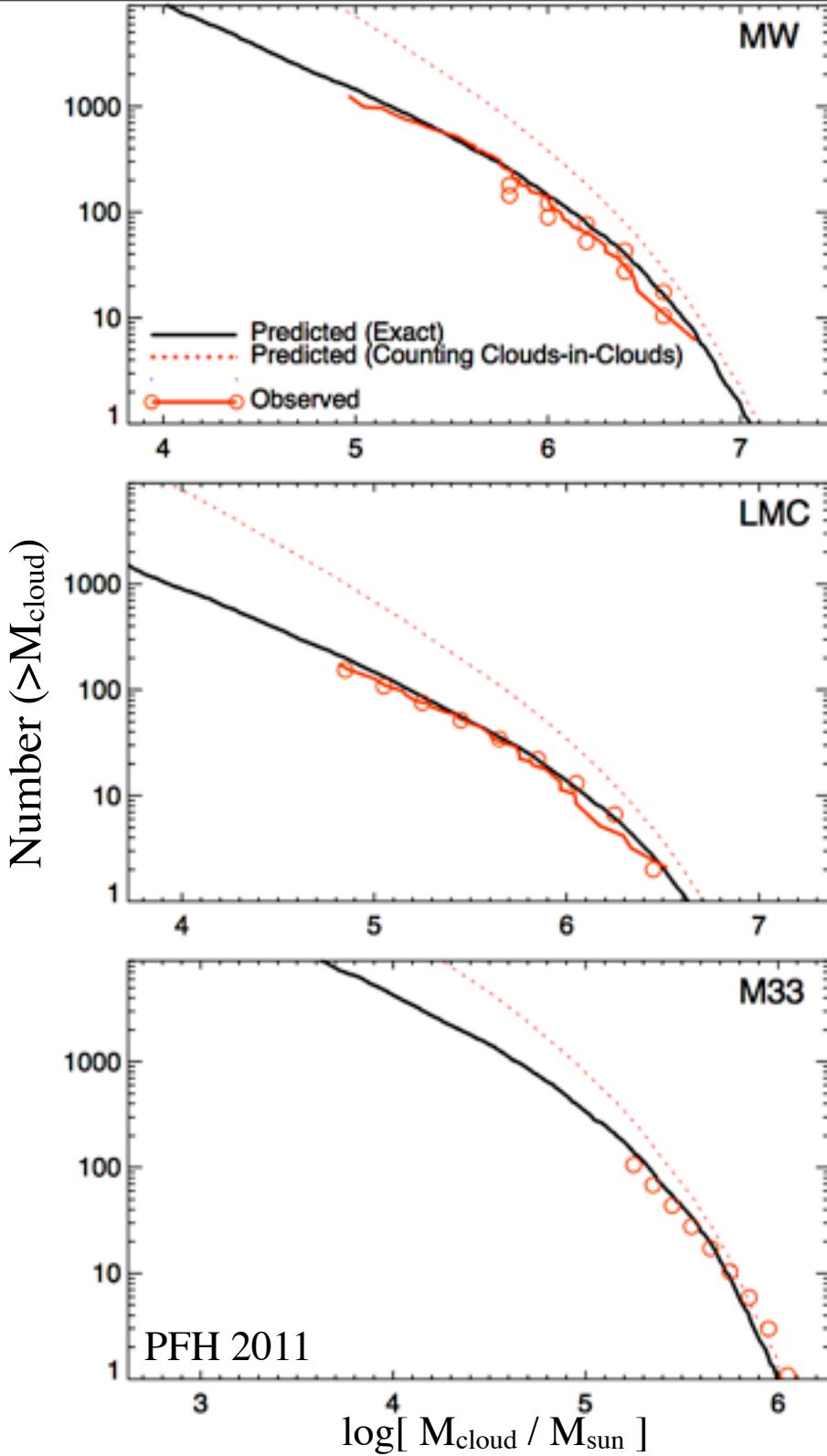
The “First Crossing” Mass Function VS GIANT MOLECULAR CLOUDS



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$$r_{\text{sonic}} \ll r \ll h$$

$$S(r) \sim S_0$$

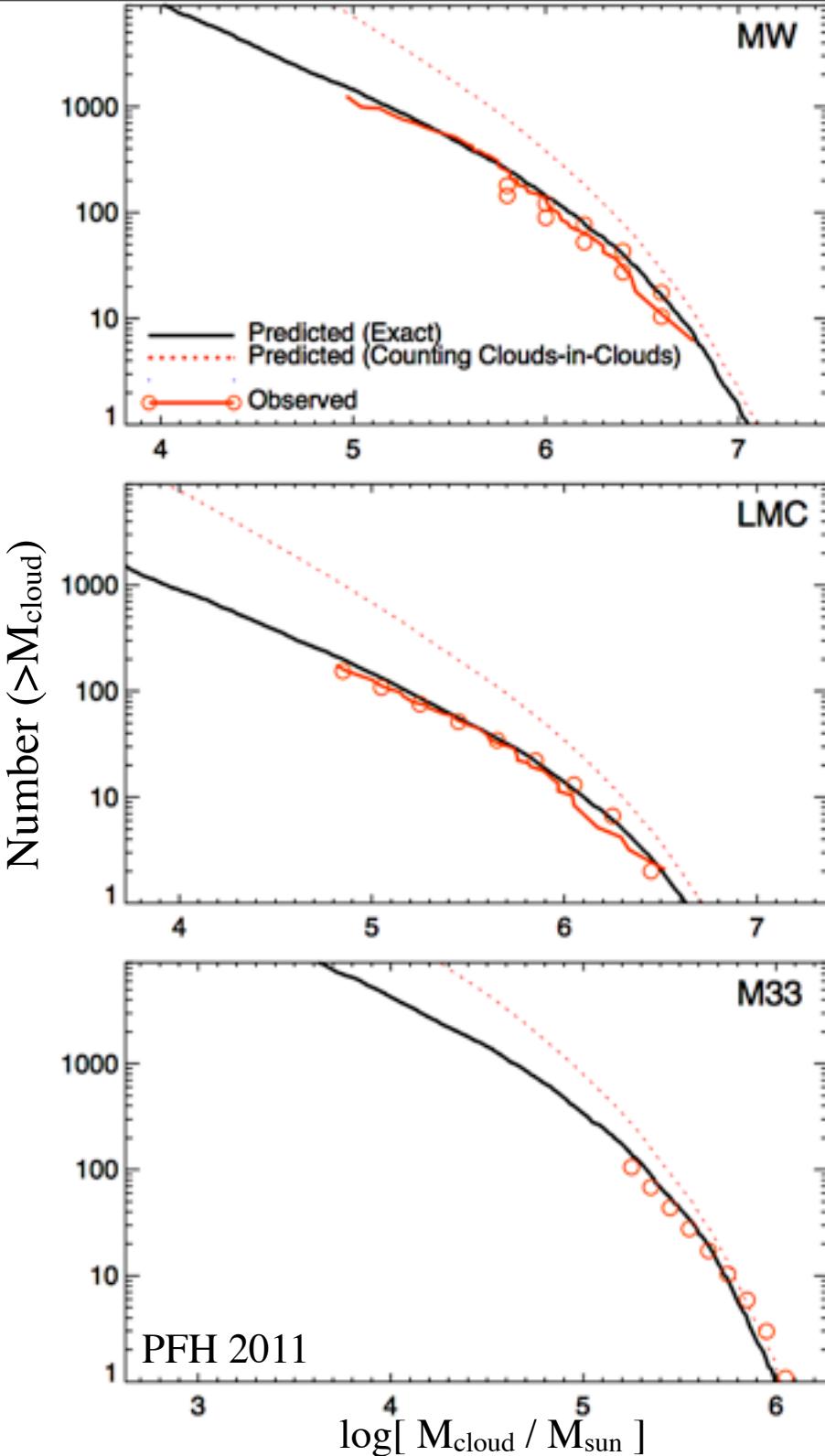


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$$\frac{dn}{dM} \propto M^{-\alpha} e^{-(M/M_J)^\beta}$$



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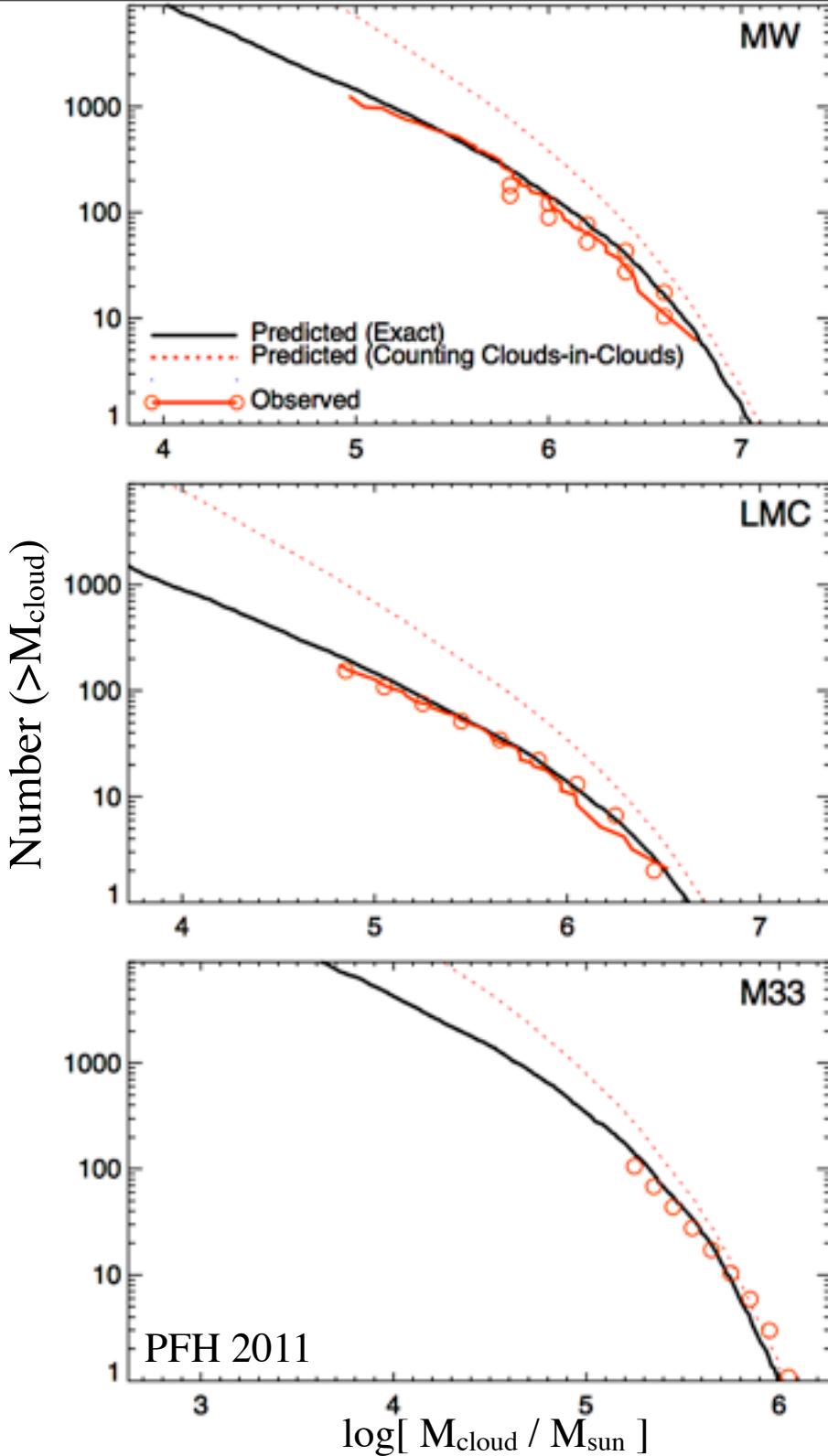
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$$\alpha \approx -2 + \frac{(3-p)^2}{2Sp^2} \ln \left(\frac{M_J}{M} \right)$$

$$\approx -2 + 0.1 \log \left(\frac{M_J}{M} \right)$$

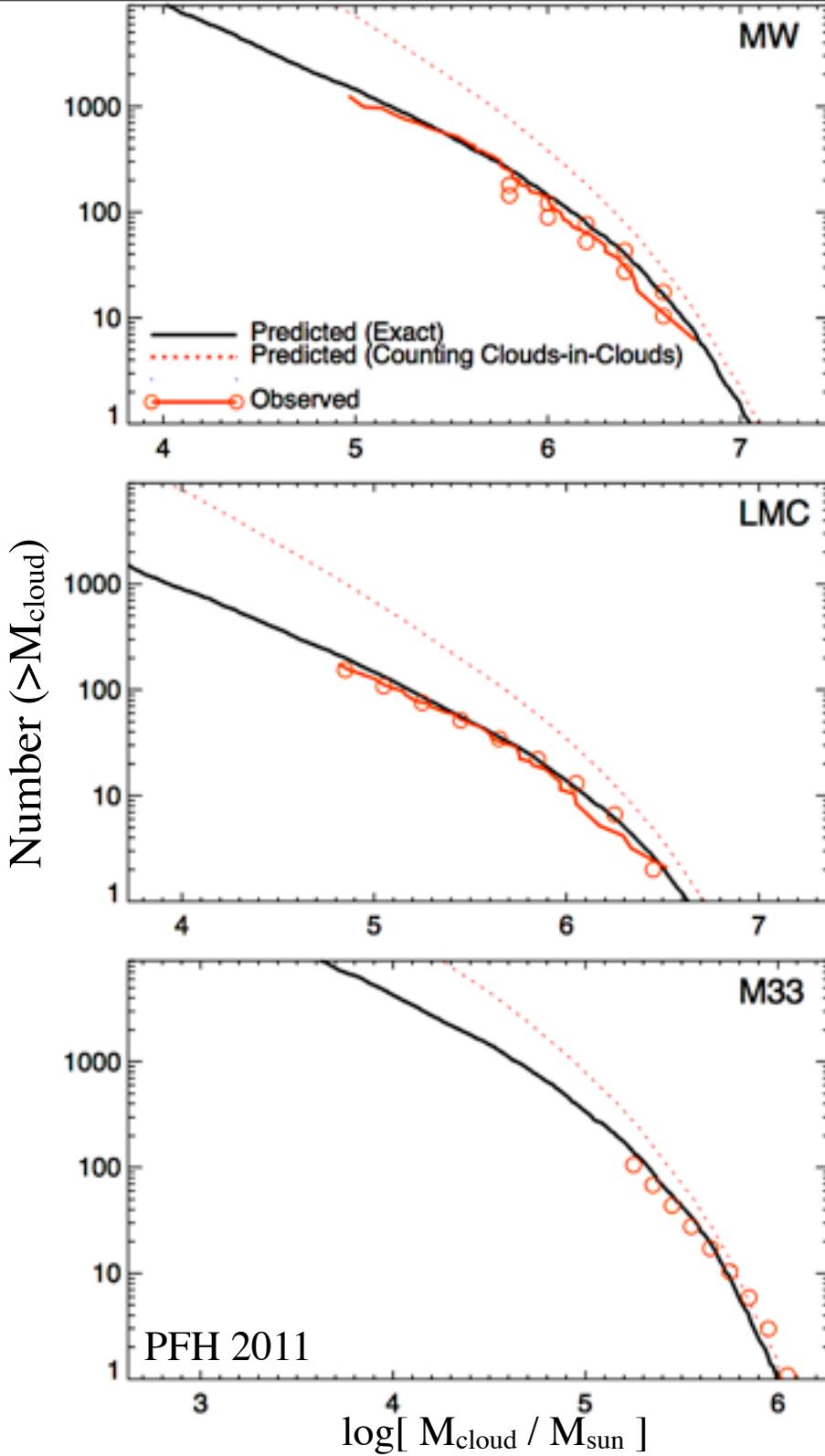
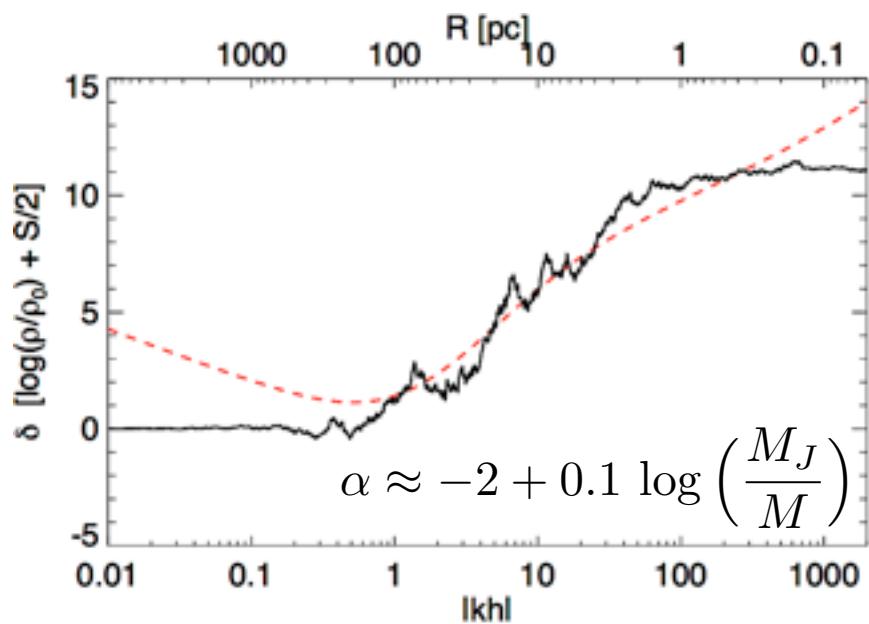


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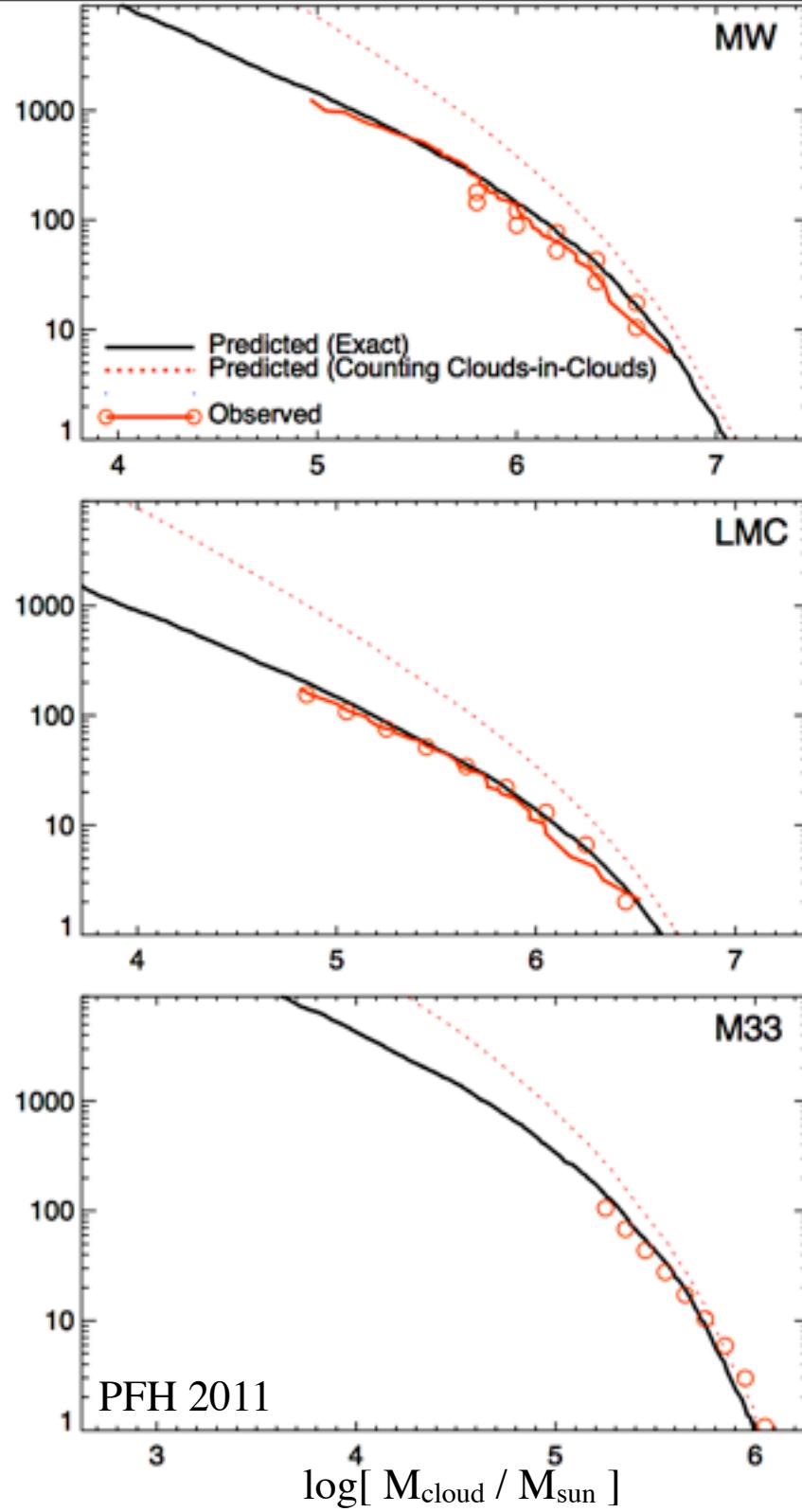
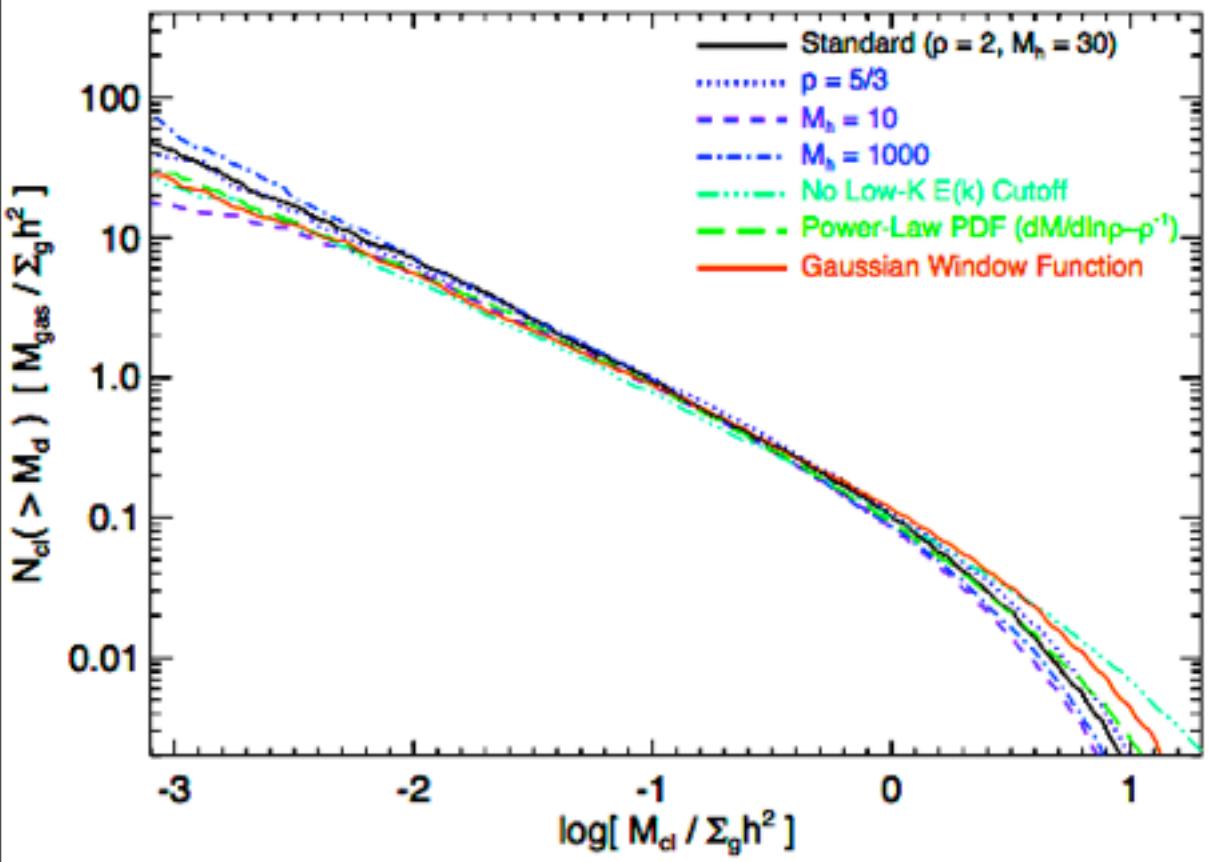


The “First Crossing” Mass Function

VS GIANT MOLECULAR CLOUDS

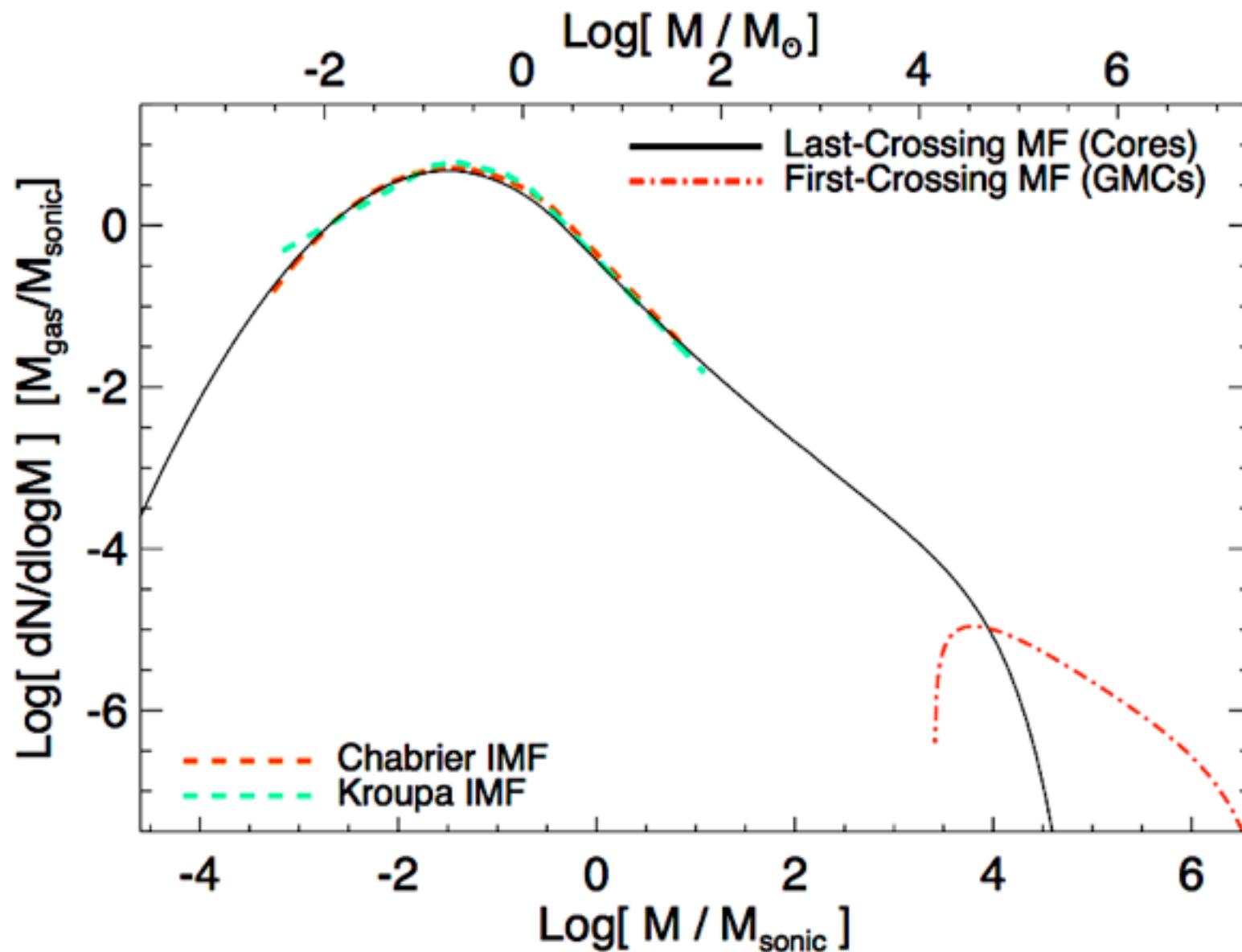
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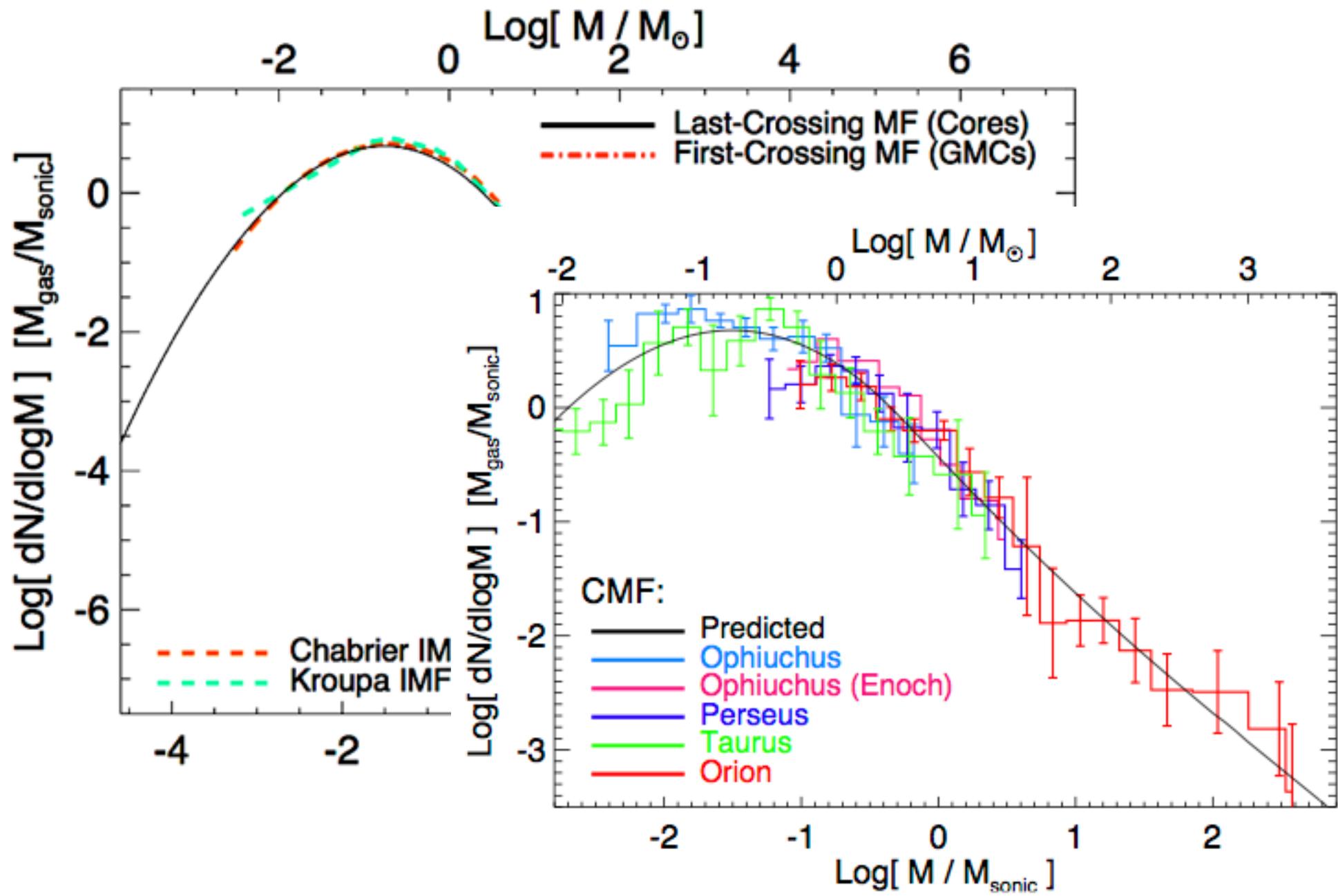
The “Last Crossing” Mass Function

VS PROTOSTELLAR CORES & THE STELLAR IMF



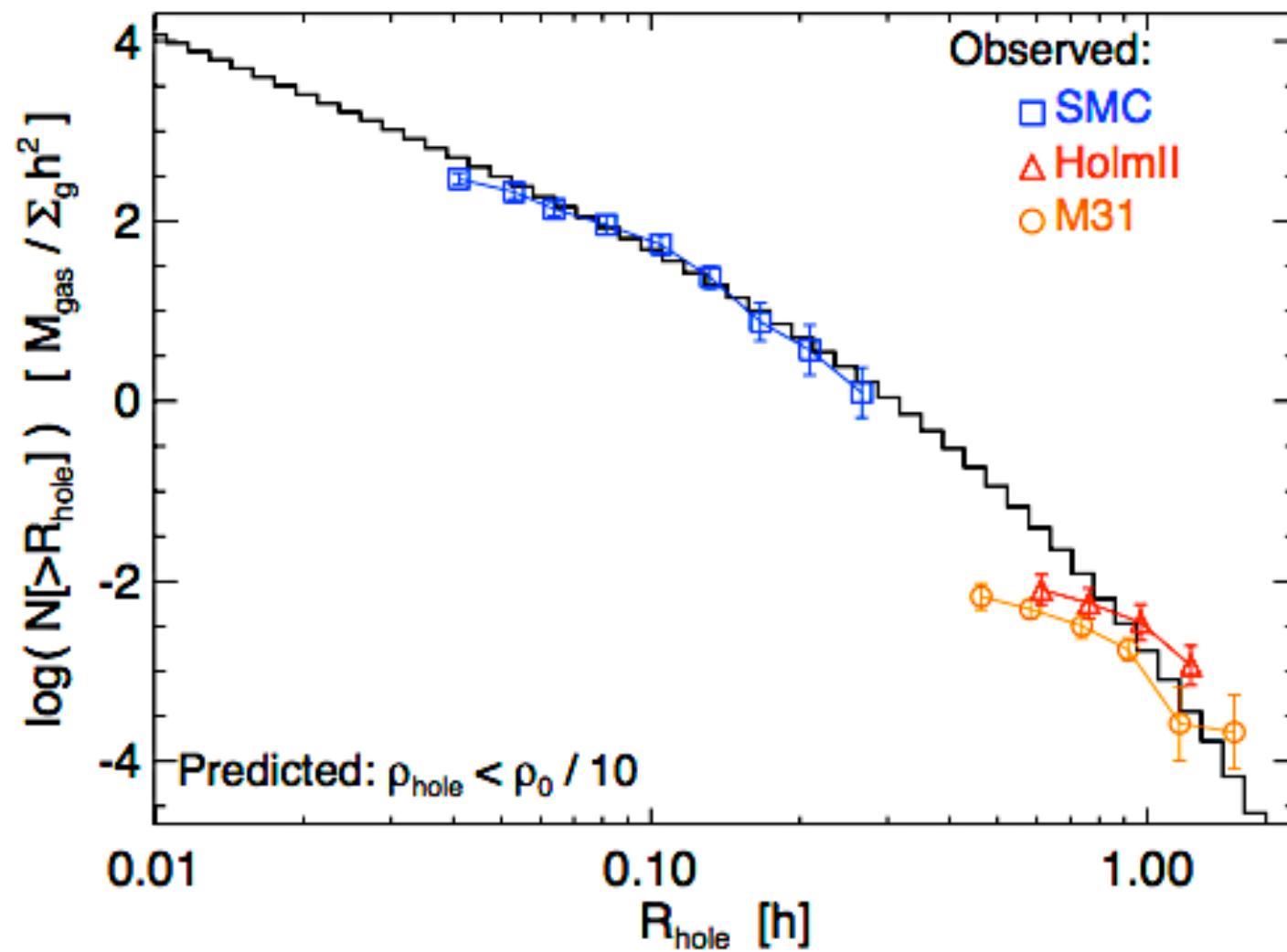
The “Last Crossing” Mass Function

VS PROTOSTELLAR CORES & THE STELLAR IMF



“Void” Abundance

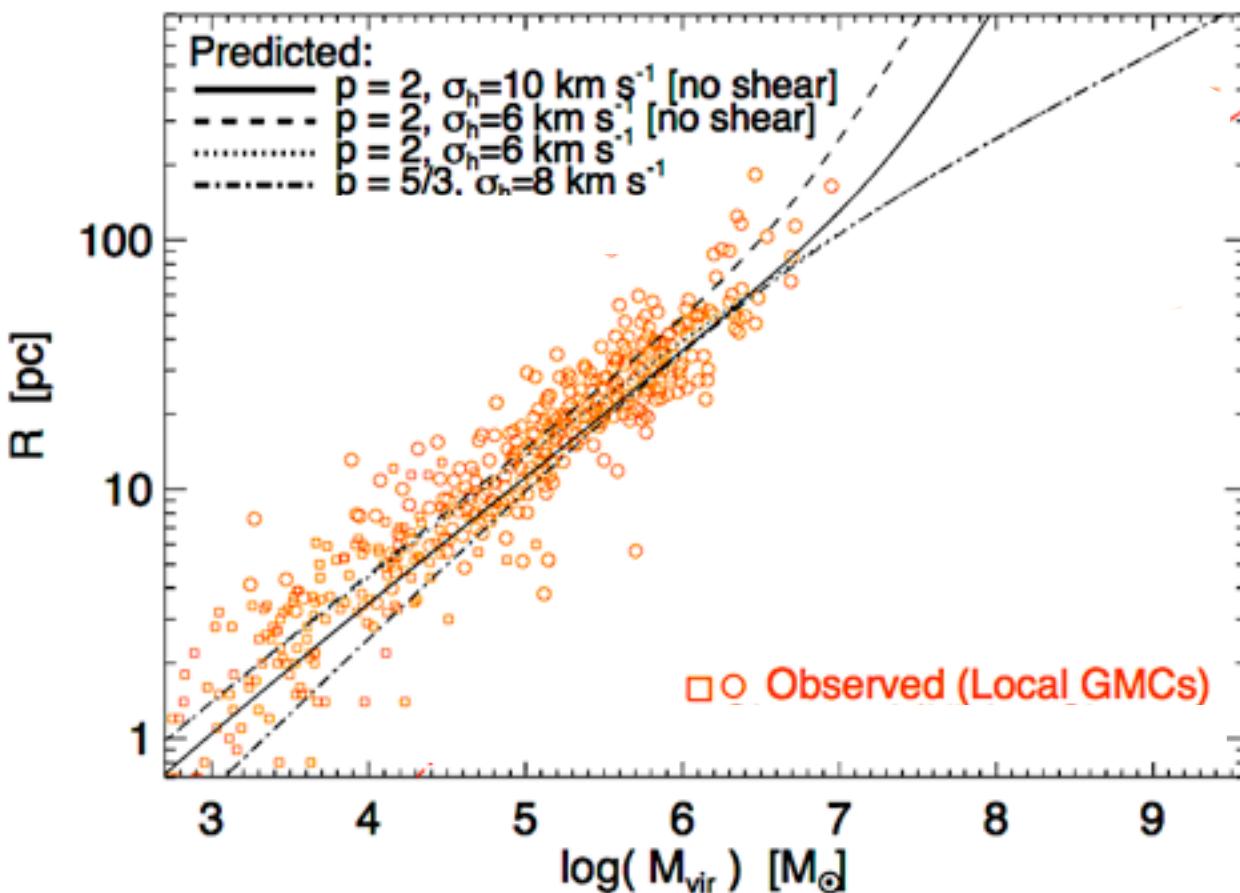
VS HI “HOLES” IN THE ISM



Don't need SNe to “clear out” voids

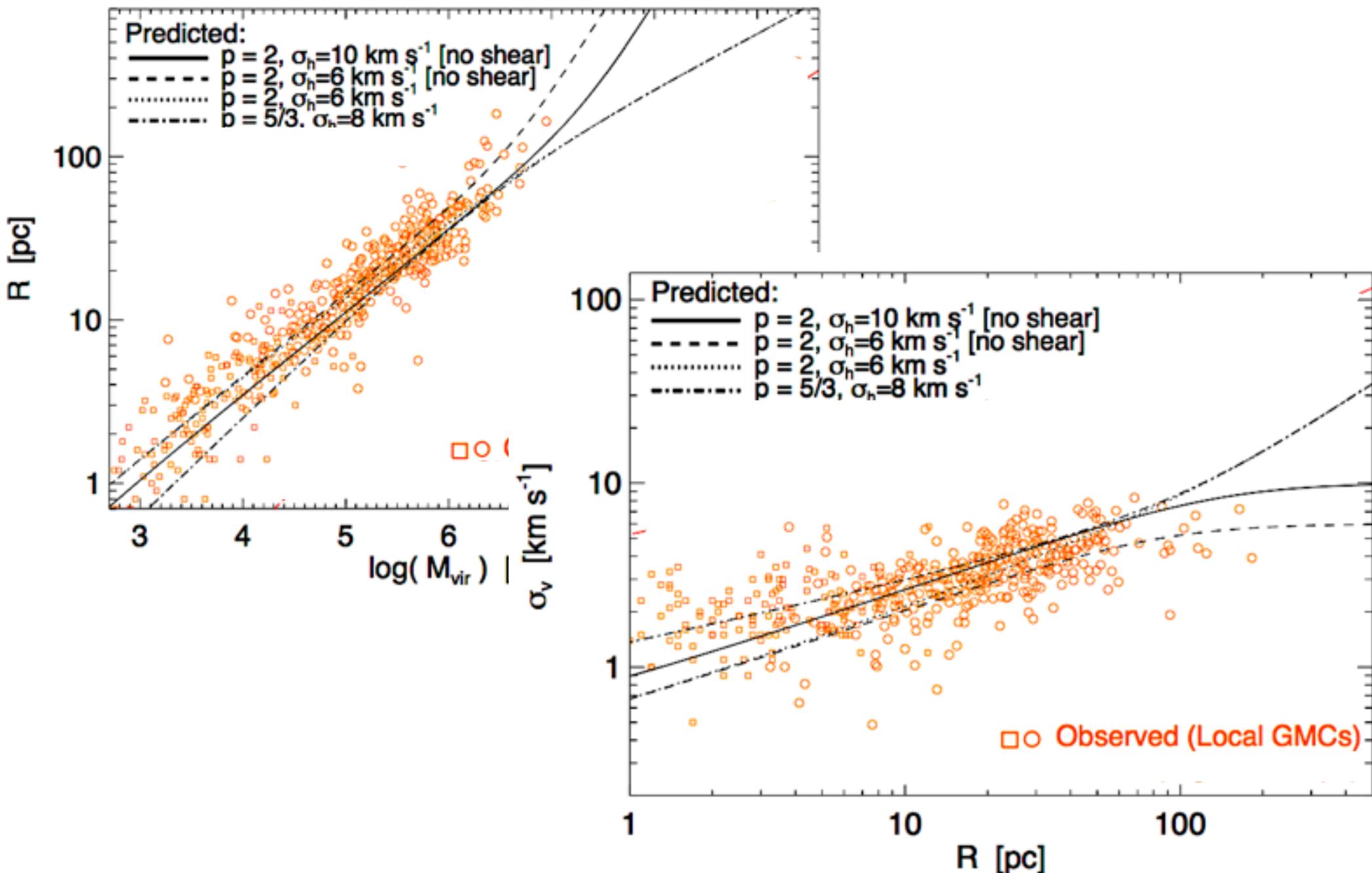
Structural Properties of “Clouds”

LARSON’S LAWS EMERGE NATURALLY



Structural Properties of “Clouds”

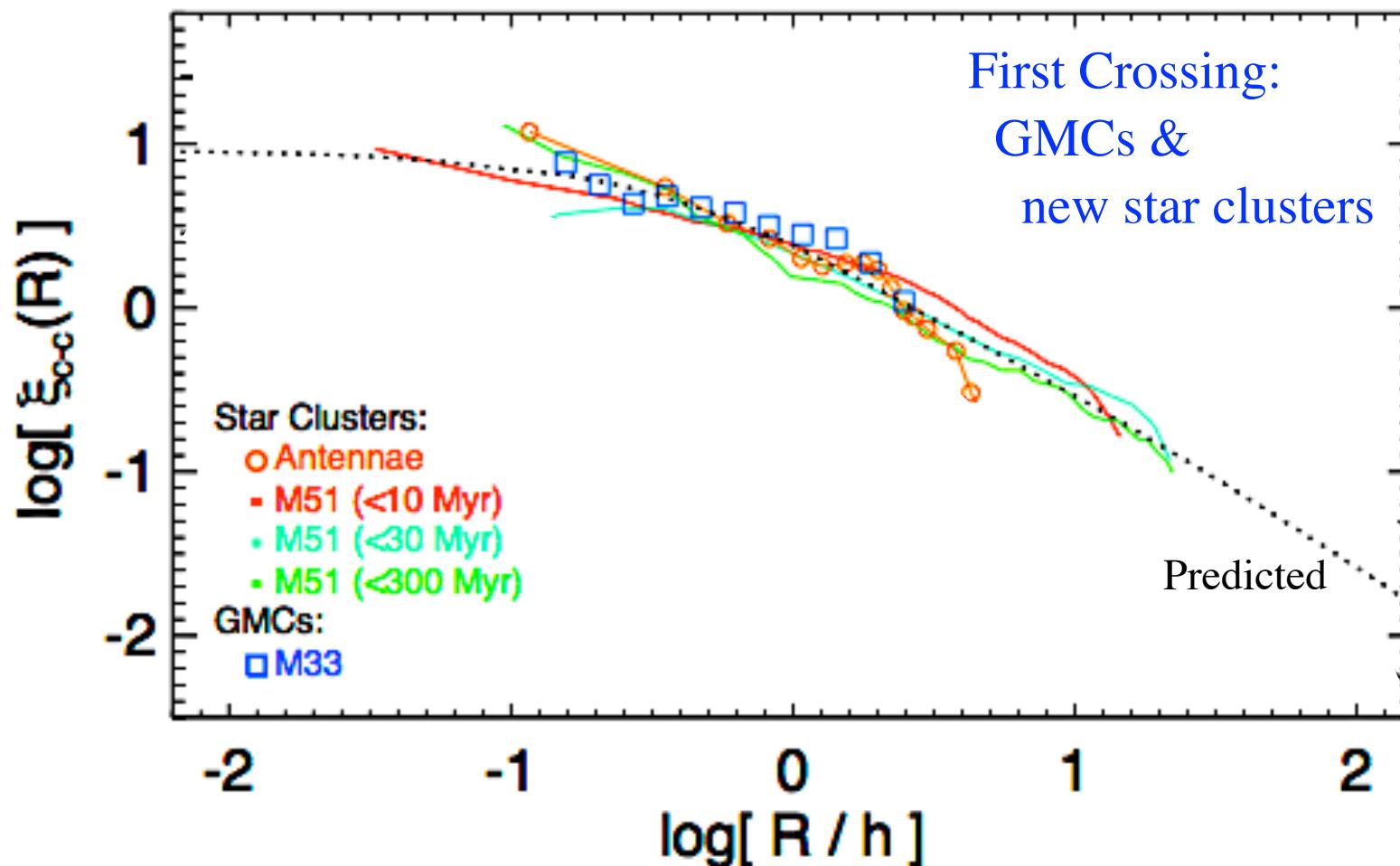
LARSON’S LAWS EMERGE NATURALLY



Clustering

PREDICT N-POINT CORRELATION FUNCTIONS

$$1 + \xi(r | M) \equiv \frac{\langle n[M \mid r' < r] \rangle}{\langle n[M] \rangle}$$

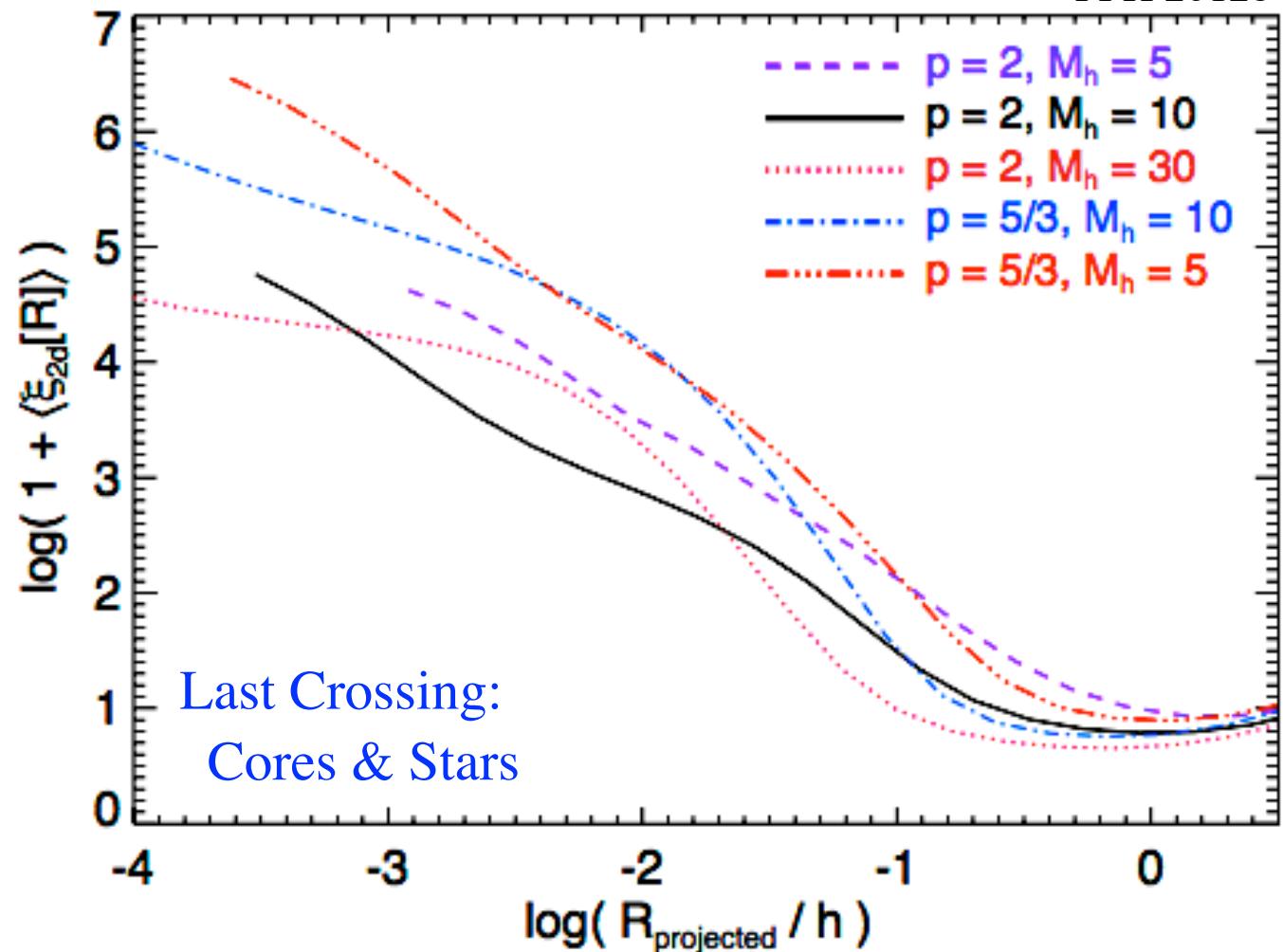


Clustering

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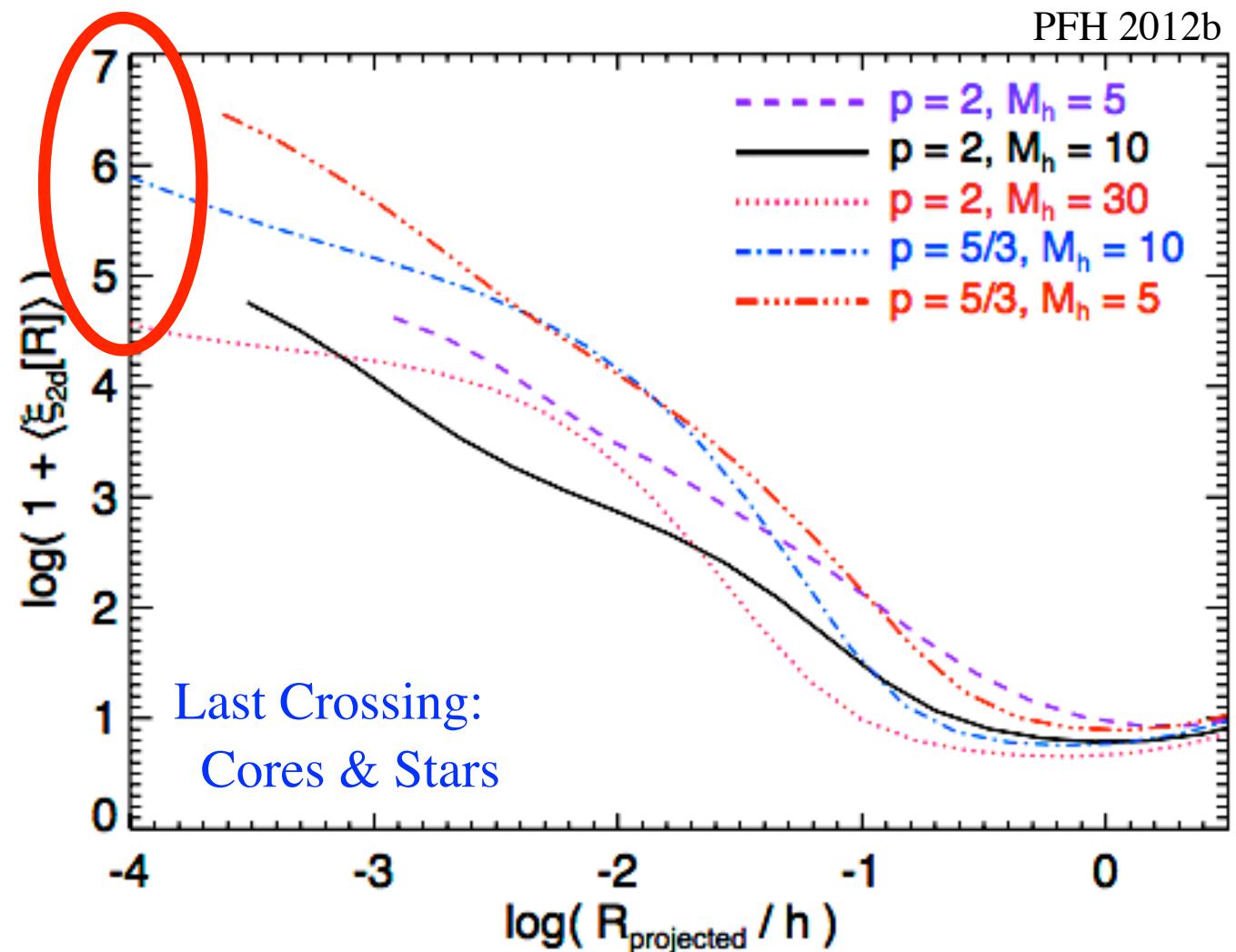
PFH 2012b



Clustering

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Clustering

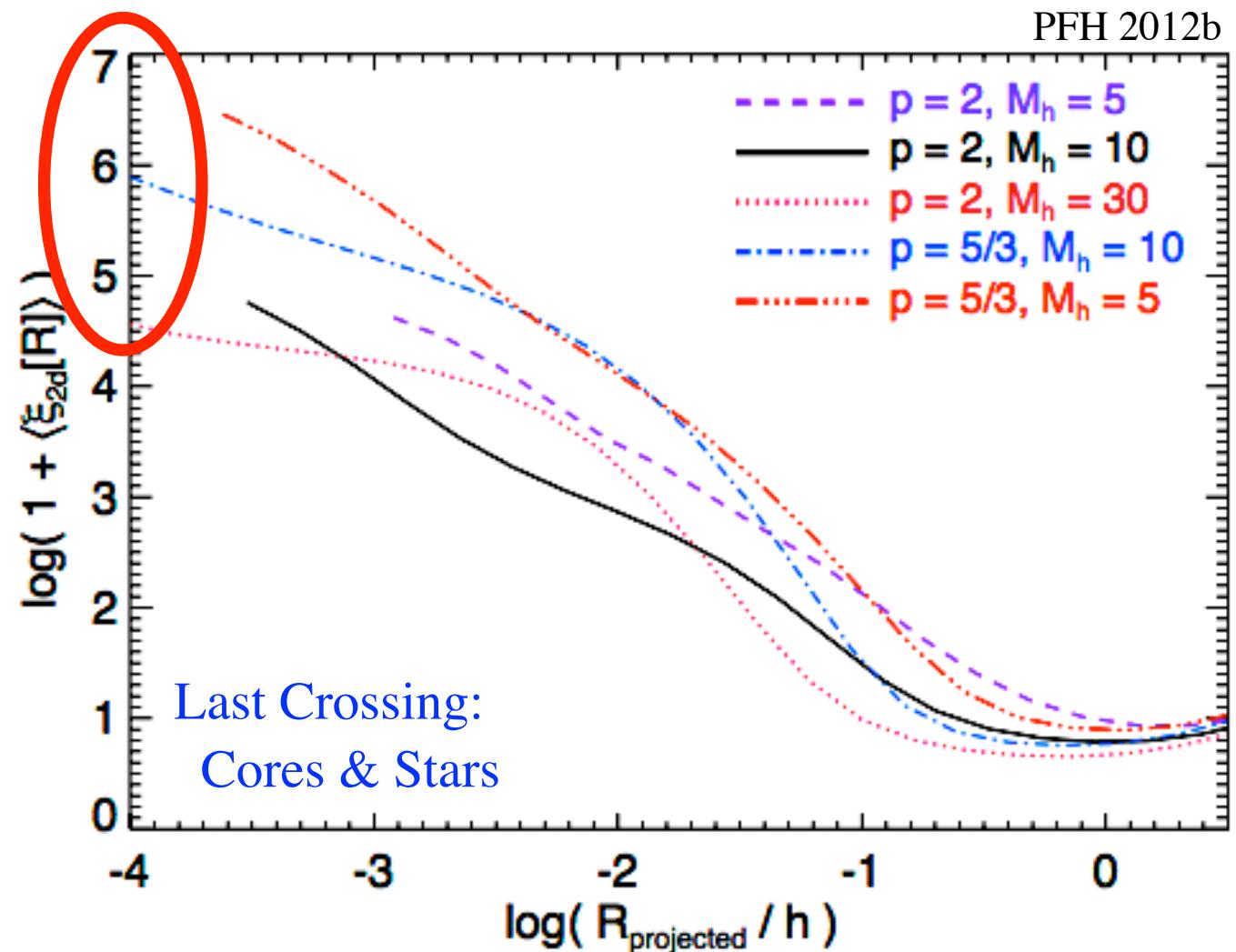
PREDICT N-POINT CORRELATION FUNCTIONS

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Why is Star Formation Clustered?

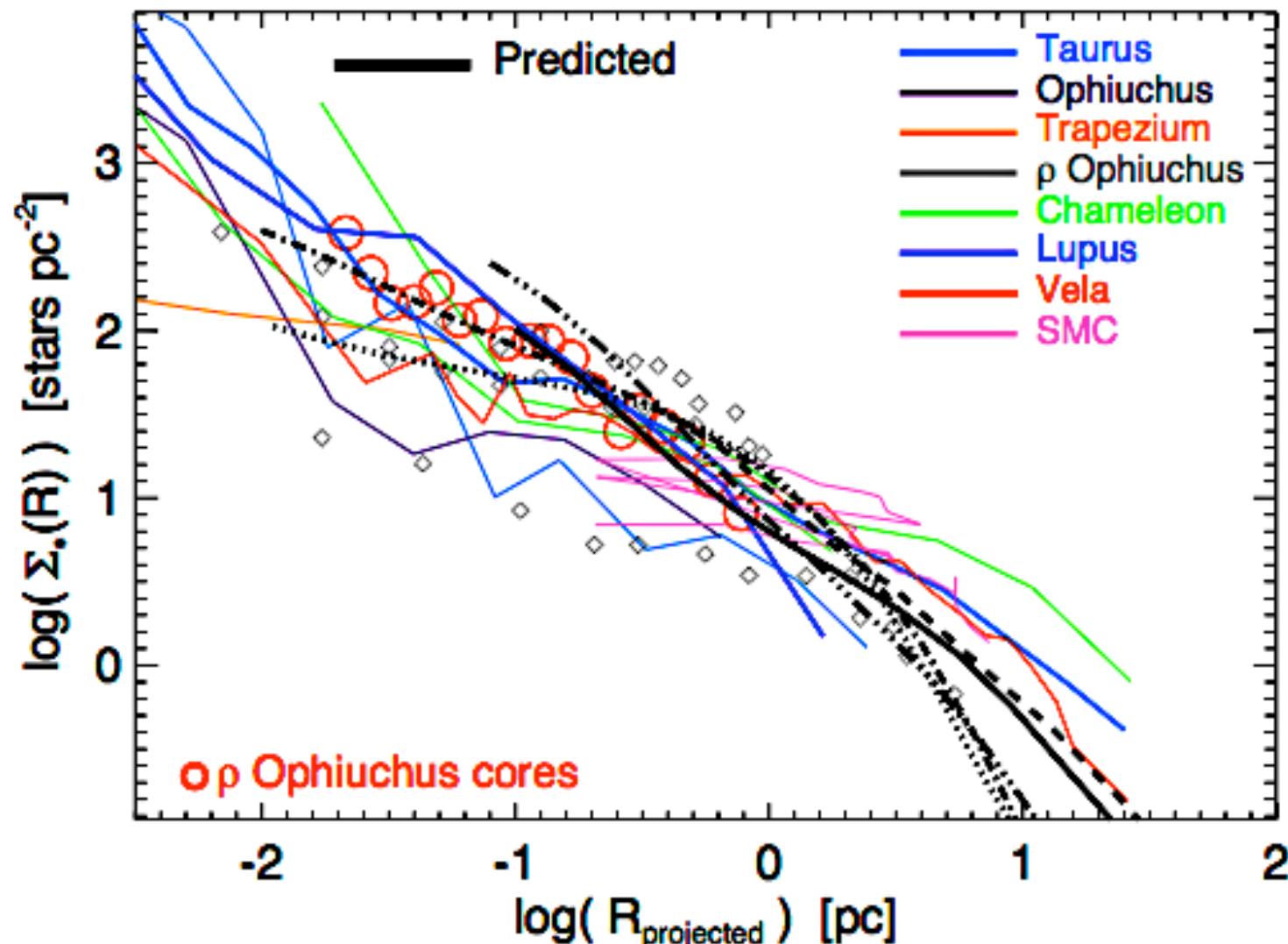
$$S \sim \ln \mathcal{M}(k)^2$$

$$\sim \ln r^{3-p}$$

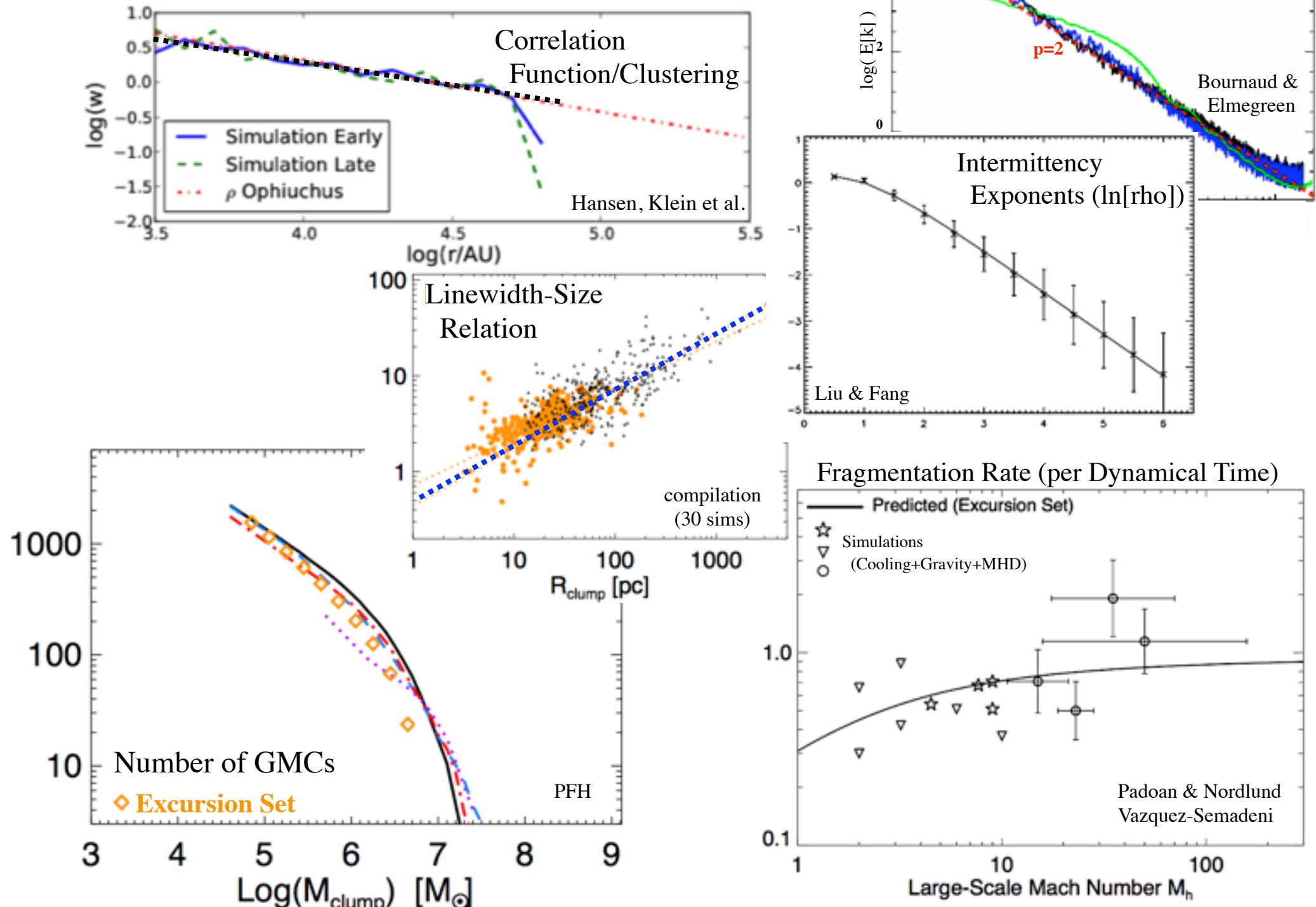


Clustering of Stars: Predicted vs. Observations

PREDICT N-POINT CORRELATION FUNCTIONS

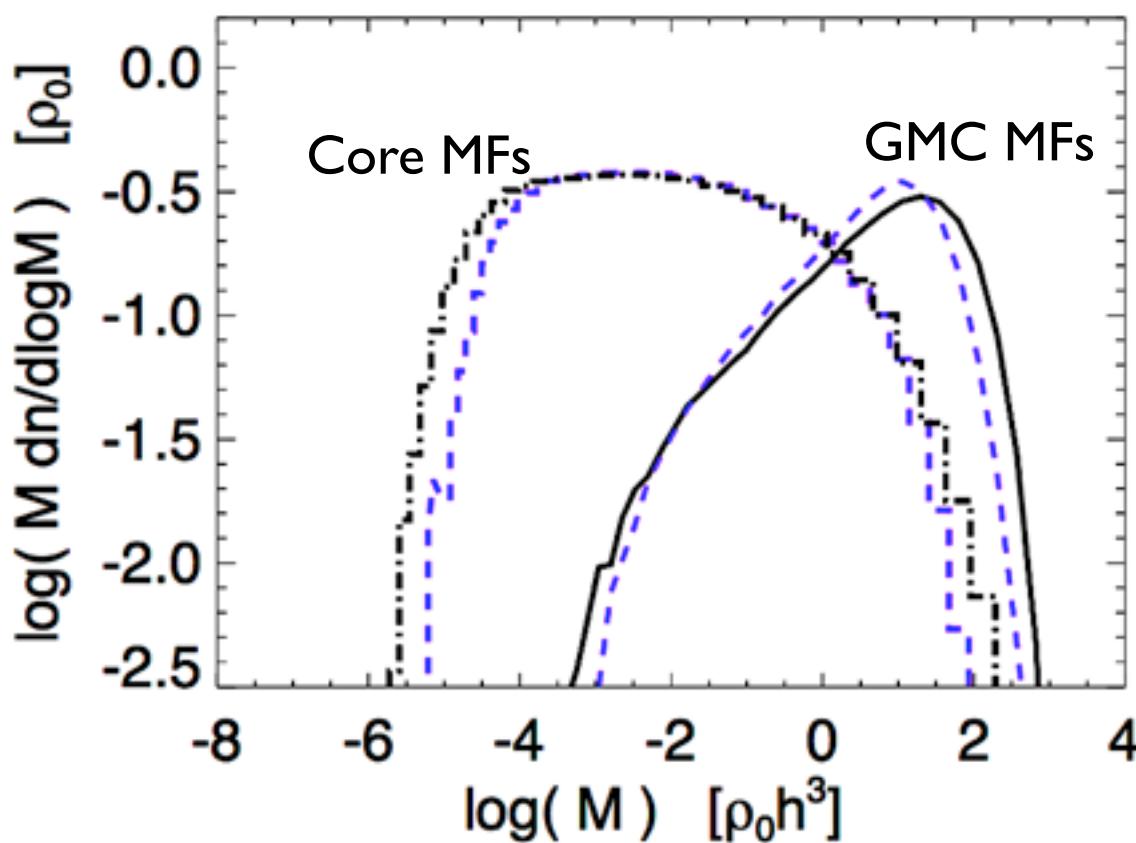
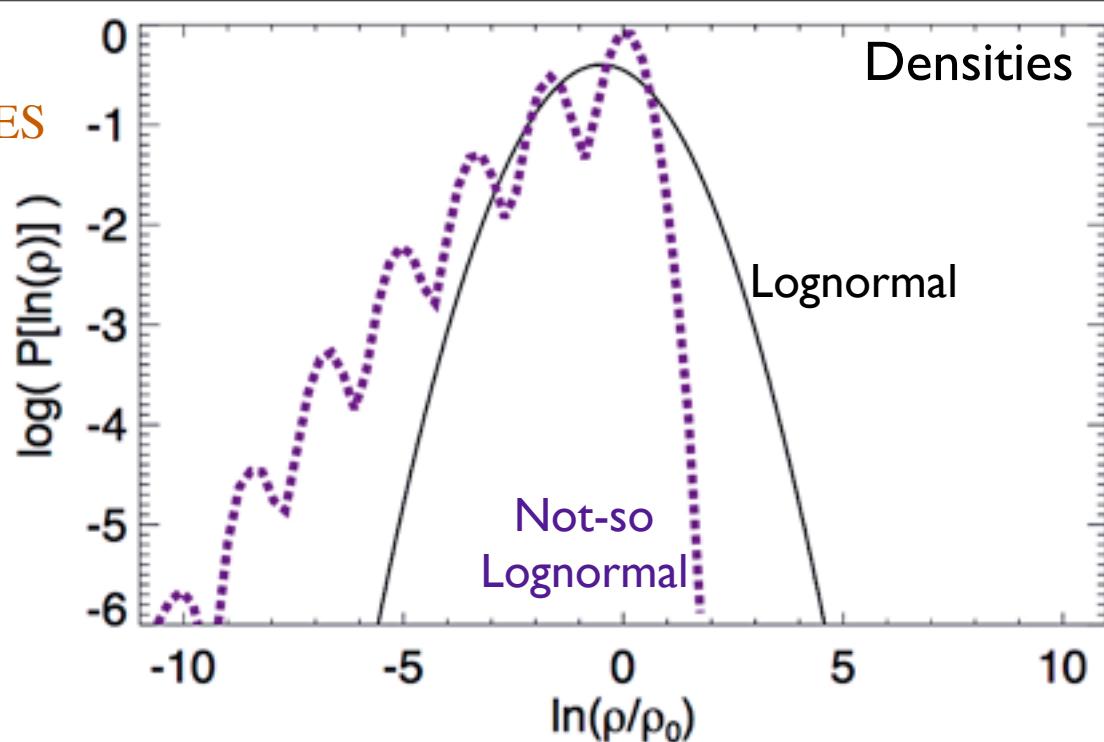


Testing the Analytics vs. NUMERICAL SIMULATIONS



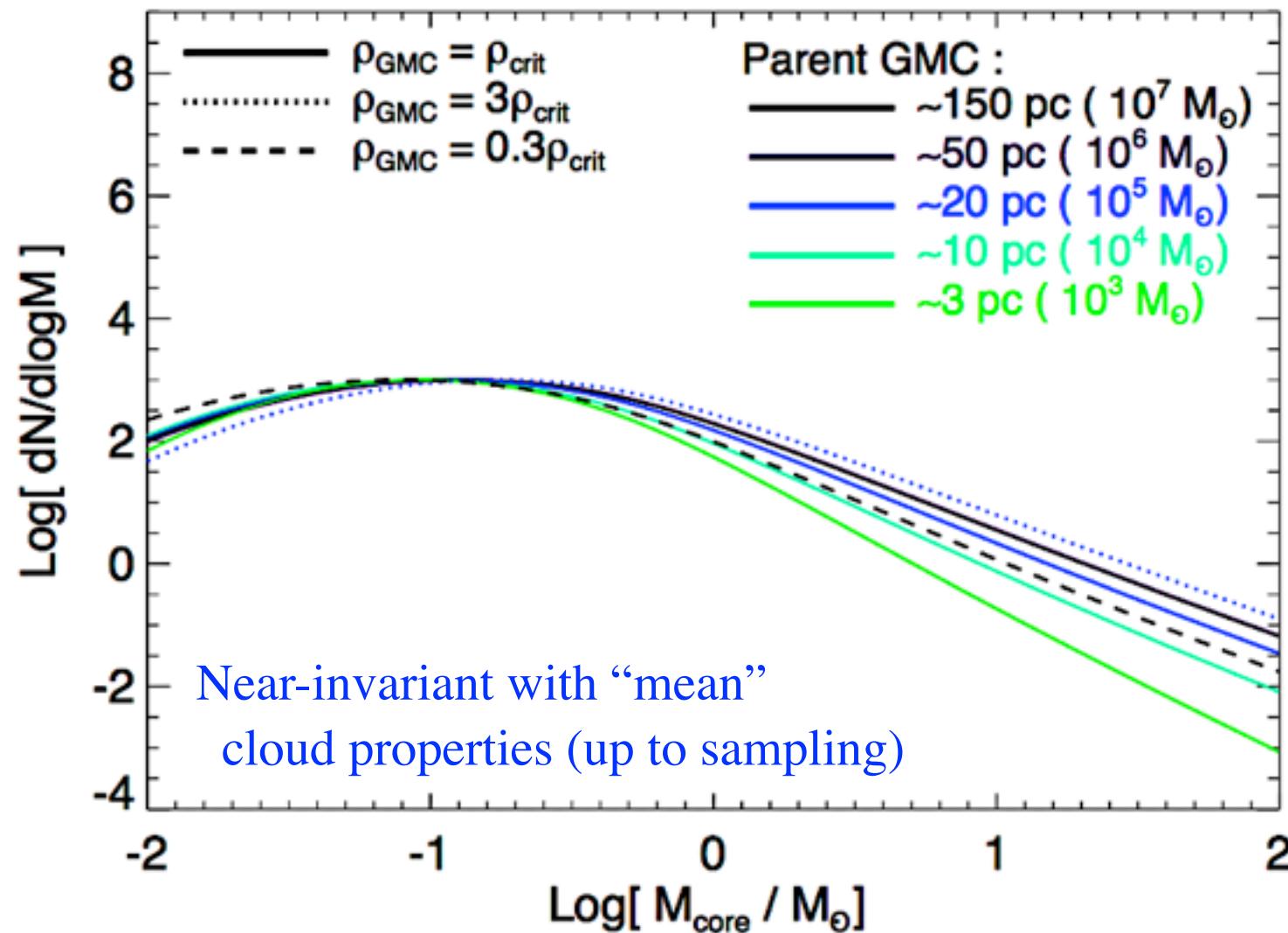
General, Flexible Theory: EXTREMELY ADAPTABLE TO MOST CHOICES

- Complicated, multivariable gas equations of state
- Accretion
- Magnetic Fields
- Time-Dependent Background Evolution/Collapse
- Intermittency
- Correlated, multi-scale driving



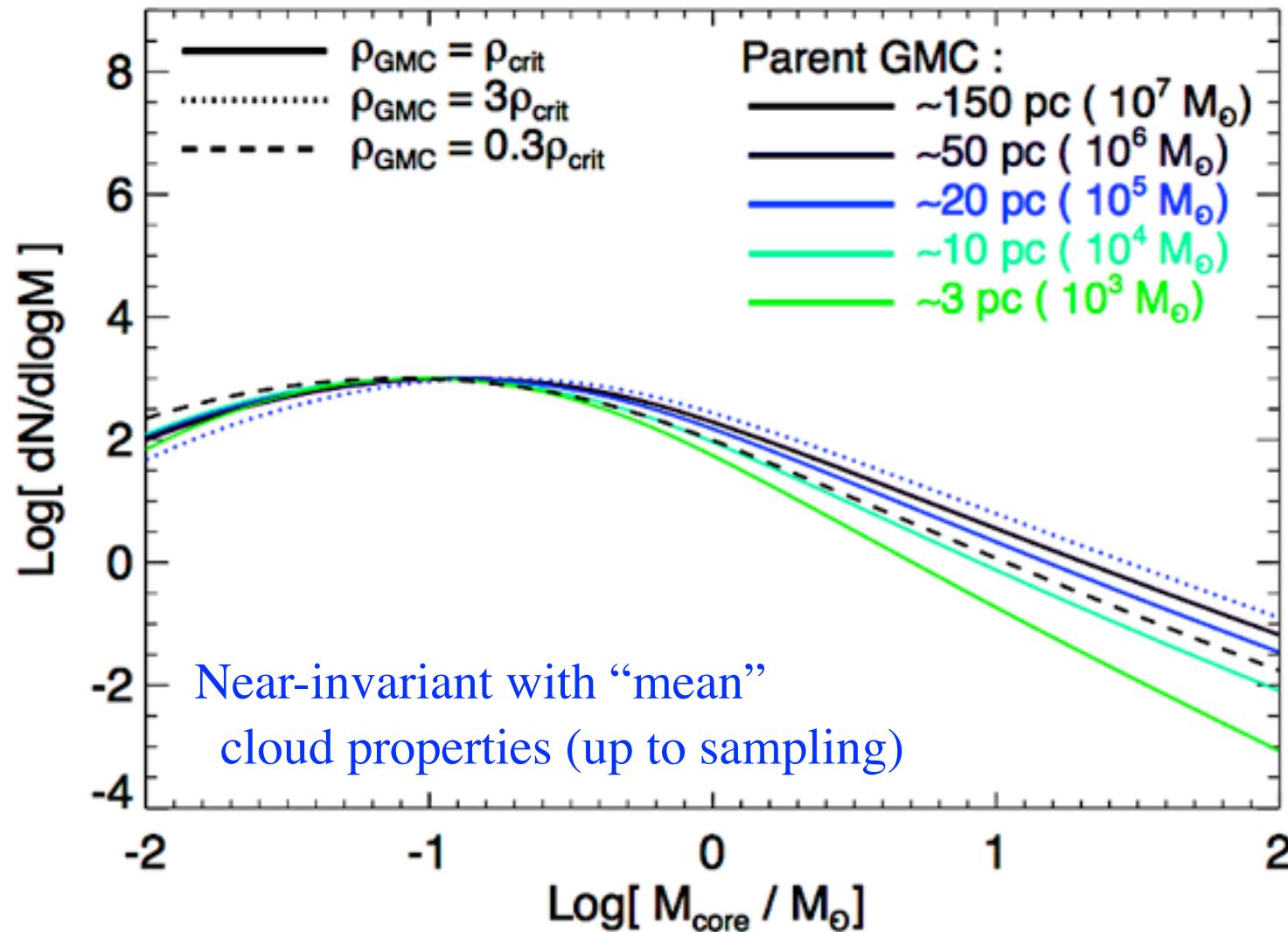
Variation in the Core Mass Function

VS “NORMAL” IMF VARIATIONS



Variation in the Core Mass Function

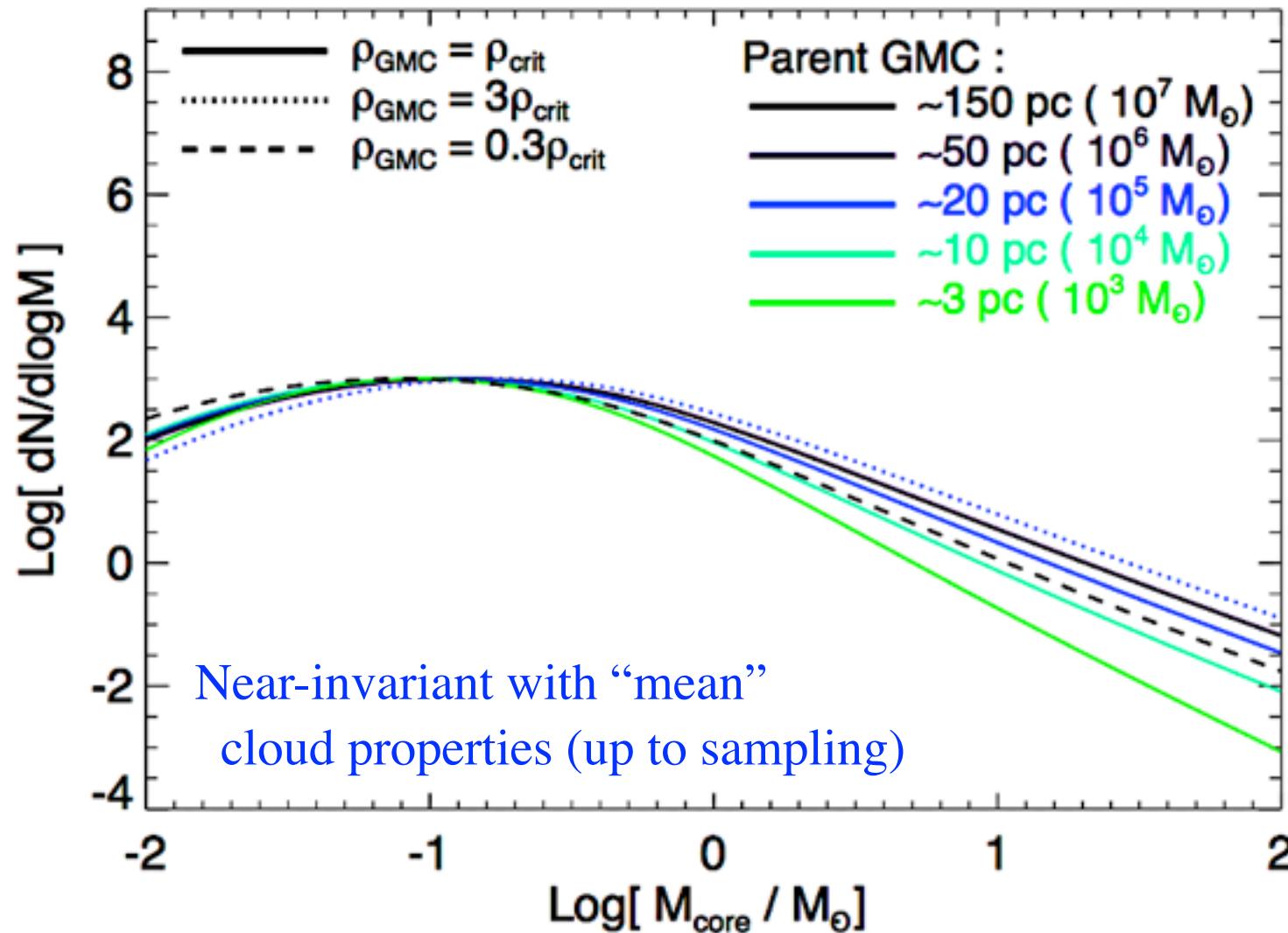
VS “NORMAL” IMF VARIATIONS



$$M_{\text{sonic}} \equiv M(\rho_{\text{crit}} | R_{\text{sonic}}) \sim \frac{c_s^2 R_{\text{sonic}}}{G}$$

Variation in the Core Mass Function

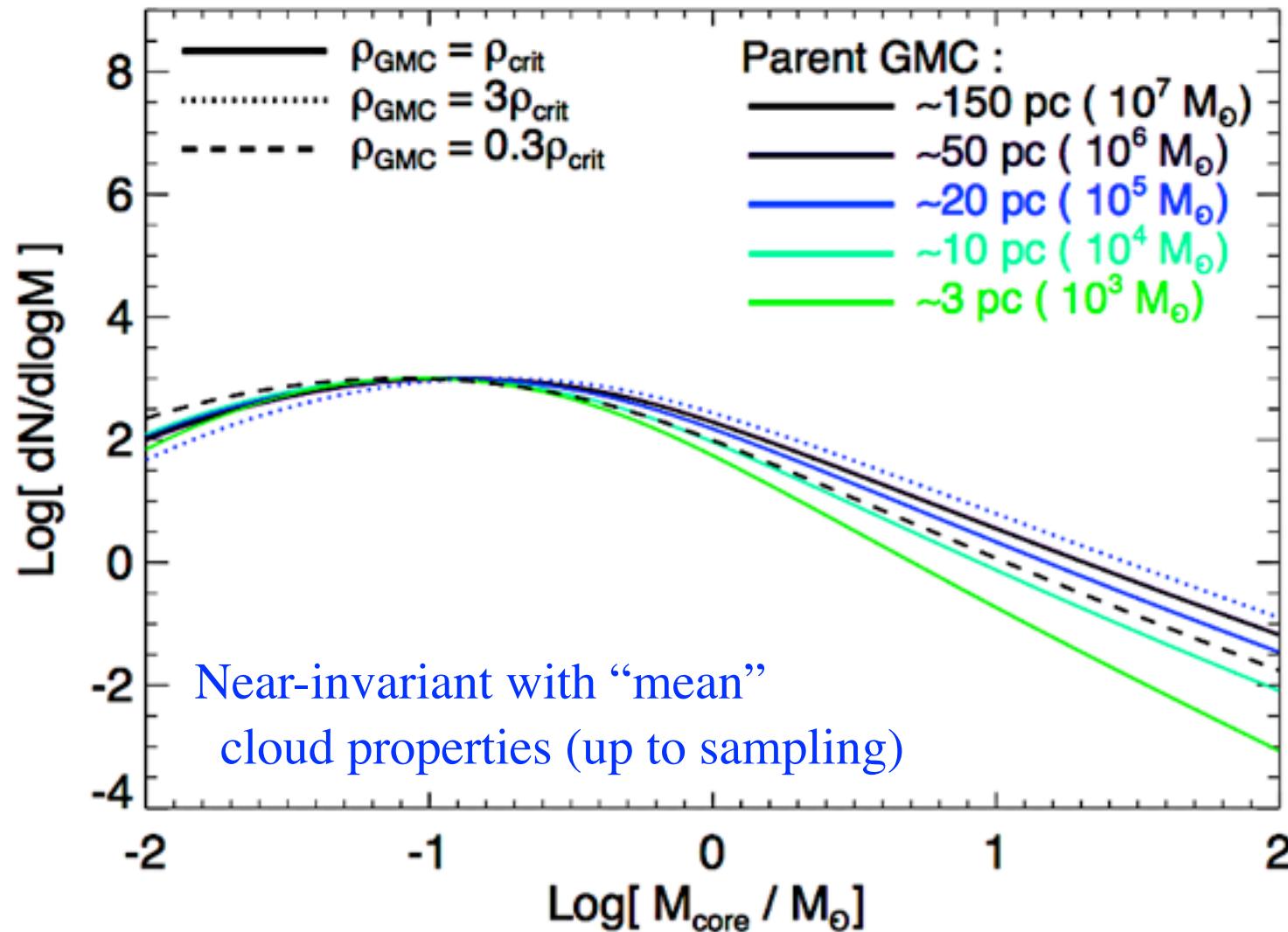
VS “NORMAL” IMF VARIATIONS



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Variation in the Core Mass Function

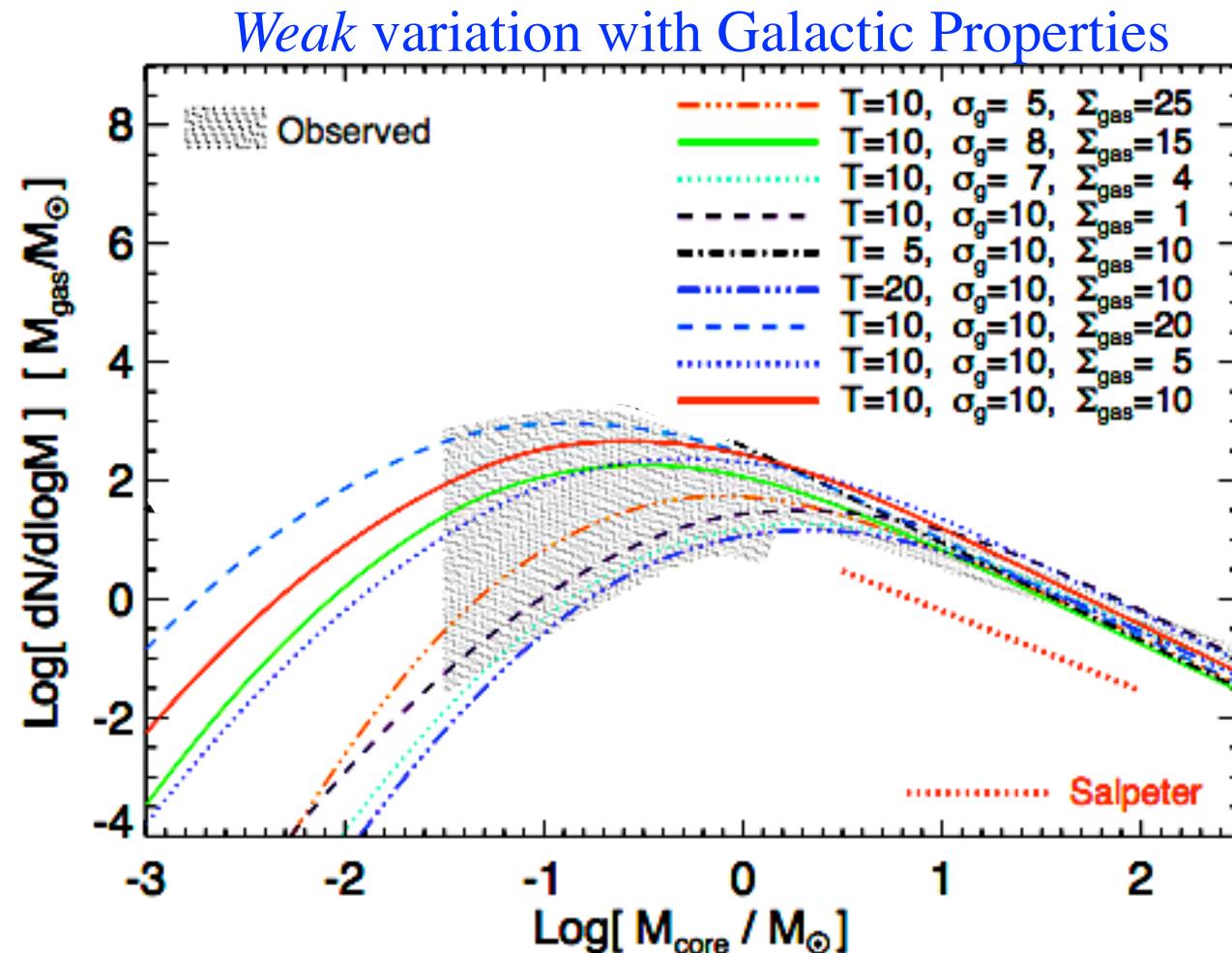
VS “NORMAL” IMF VARIATIONS



$$M_{\text{sonic}} \equiv M(\rho_{\text{crit}} | R_{\text{sonic}}) \sim \frac{c_s^2 R_{\text{sonic}}}{G} \sim M_{\odot} \frac{T_{\min}}{10 K} \frac{R_{\text{sonic}}}{0.1 \text{ pc}} \propto T_{\min}^2 \frac{R_{\text{cl}}}{\sigma_{\text{cl}}^2}$$

Variation in the Core Mass Function

VS “NORMAL” IMF VARIATIONS



$$M_{\text{sonic}} \equiv M(\rho_{\text{crit}} | R_{\text{sonic}}) \sim \frac{c_s^2 R_{\text{sonic}}}{G} \sim M_{\odot} \frac{T_{\min}}{10 K} \frac{R_{\text{sonic}}}{0.1 \text{ pc}} \propto T_{\min}^2 \frac{R_{\text{cl}}}{\sigma_{\text{cl}}^2}$$

MW: $T_{\text{cold}} \sim 10 K$
 $\sigma_{\text{gas}} \sim 10 \text{ km s}^{-1}$
($Q \sim 1$ for $\Sigma_{\text{gas}} \sim 10 M_{\odot} \text{ pc}^{-2}$)

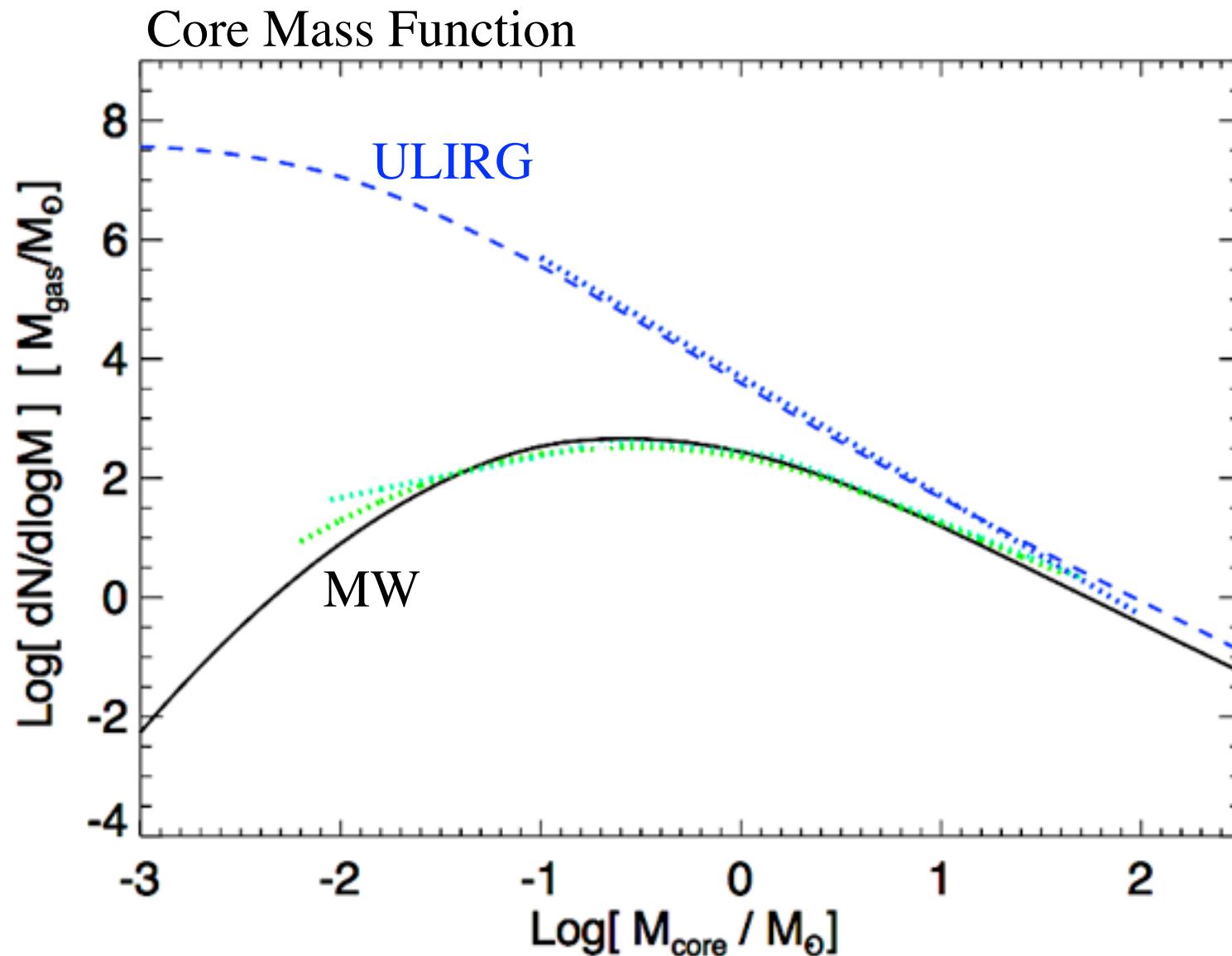


MW: $T_{\text{cold}} \sim 10 K$
 $\sigma_{\text{gas}} \sim 10 \text{ km s}^{-1}$
($Q \sim 1$ for $\Sigma_{\text{gas}} \sim 10 M_{\odot} \text{ pc}^{-2}$)



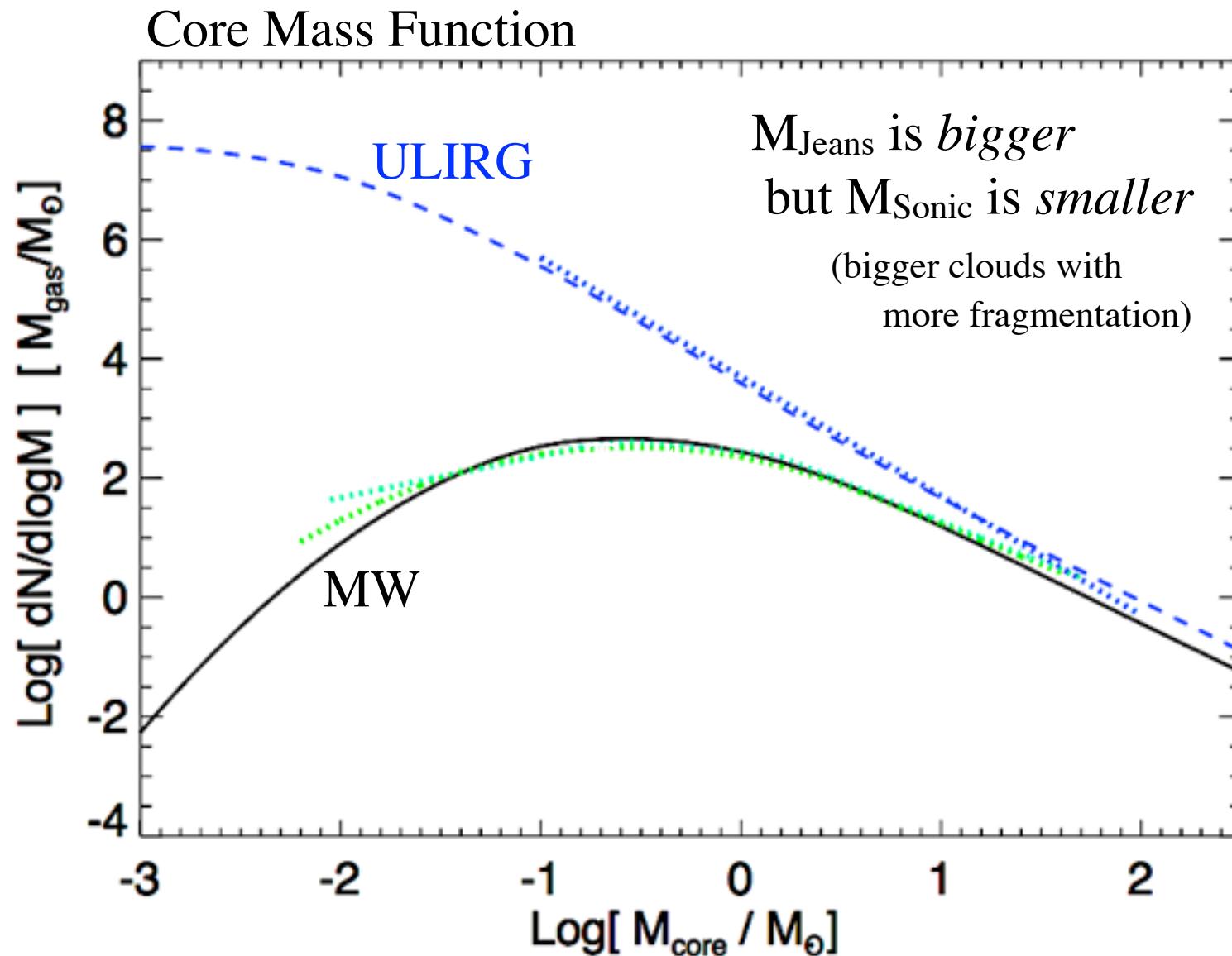
ULIRG: $T_{\text{cold}} \sim 70 K$
 $\sigma_{\text{gas}} \sim 80 \text{ km s}^{-1}$
($Q \sim 1$ for $\Sigma_{\text{gas}} \sim 1000 M_{\odot} \text{ pc}^{-2}$)





BUT, What About Starbursts?

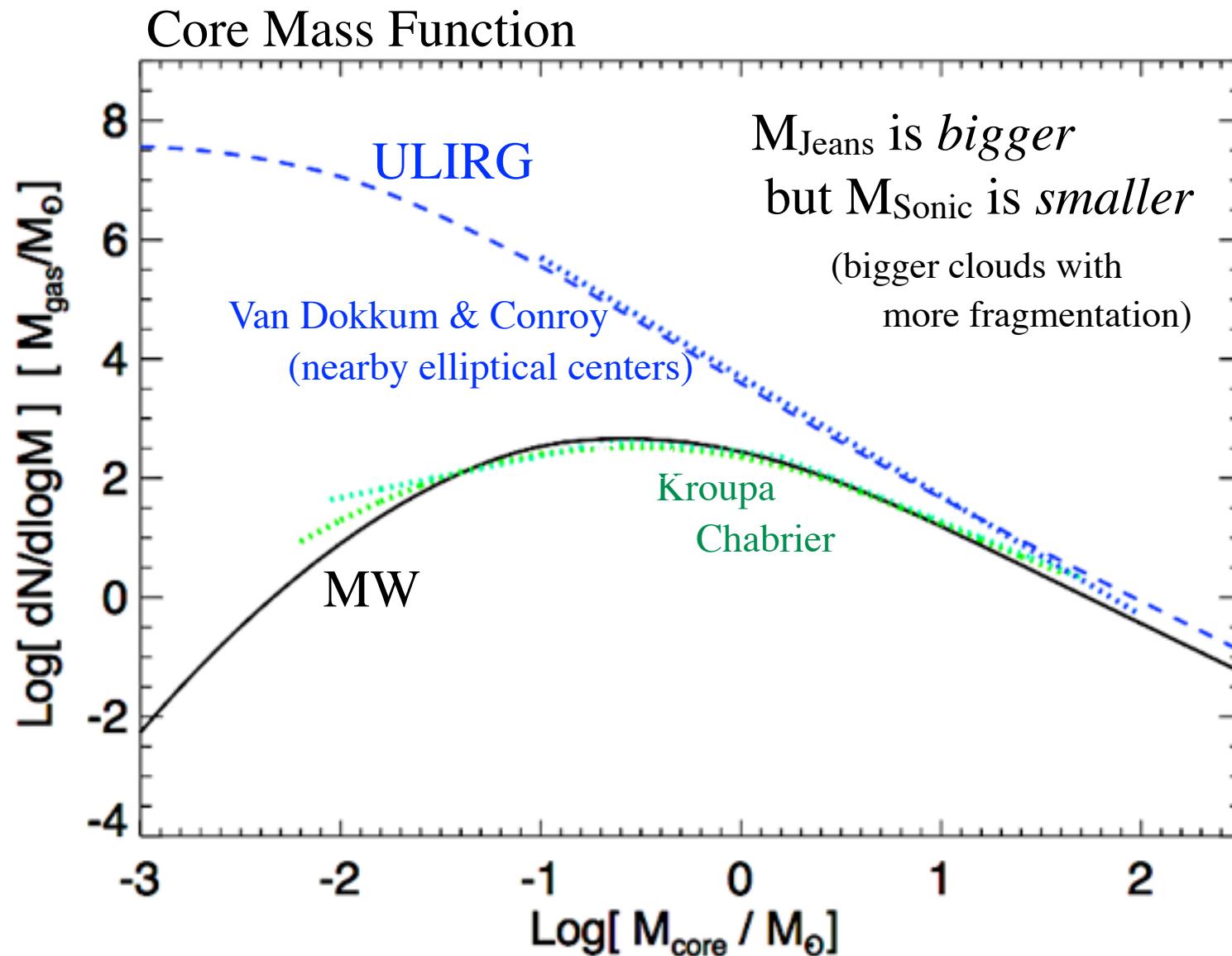
BOTTOM-HEAVY: TURBULENCE WINS!



Mach number in ULIRGs: $\mathcal{M} \gtrsim 100$

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2. Why Doesn't Everything Collapse?

“Top-down” turbulence can’t stop collapse once self-gravitating

Fast Cooling: $\dot{M}_* \sim \frac{M_{\text{gas}}}{t_{\text{freefall}}}$

Summary:

* ISM *statistics* are far more fundamental than we typically assume *

- **Turbulence + Gravity:** ISM structure follows
 - Lognormal density PDF is *not* critical
 - *ANALYTICALLY* understand:
 - GMC Mass Function & Structure (“first crossing”)
 - Core MF (“last crossing”) & Linewidth-Size-Mass
 - Clustering of Stars (correlation functions)
- **Feedback Regulates & Sets Efficiencies of Star Formation**
 - K-S Law: ‘enough’ stars to offset dissipation (set by gravity)
 - Independent of small-scale star formation physics (how stars form)