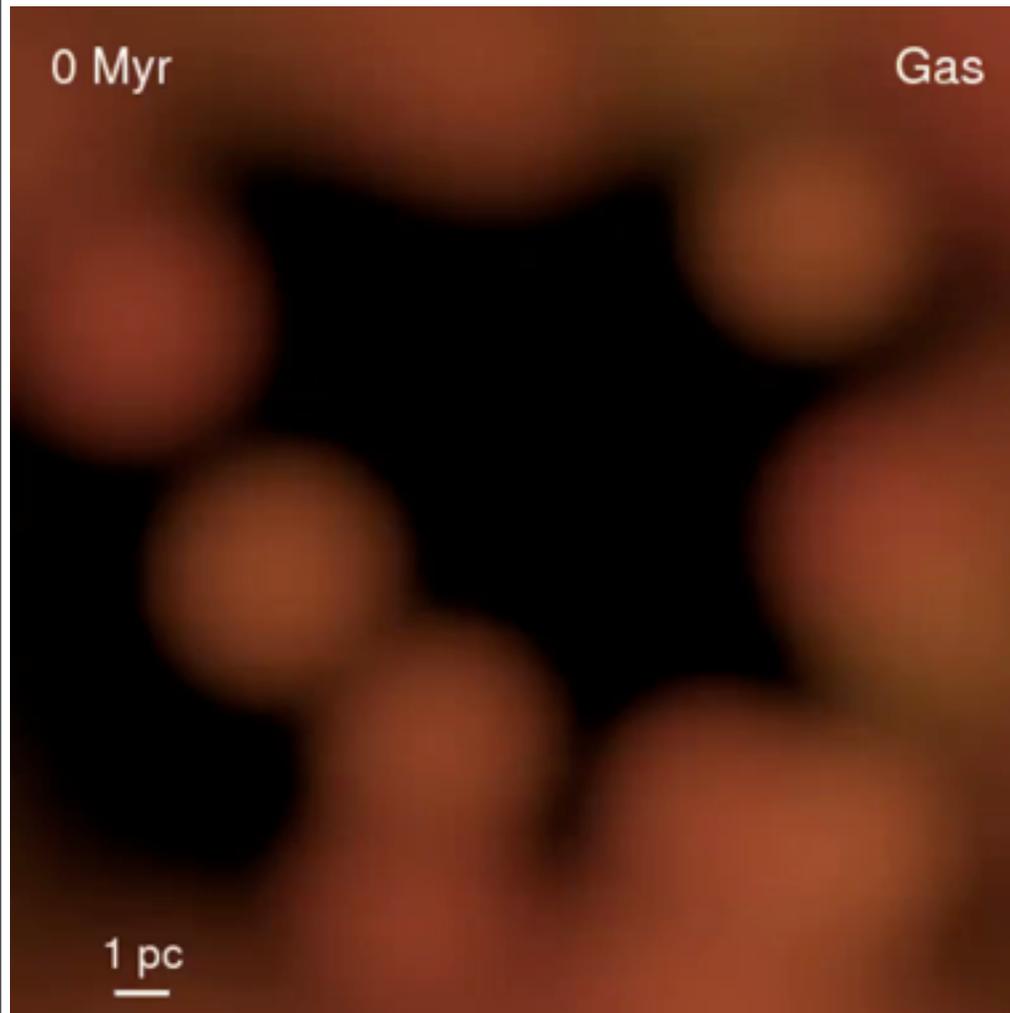
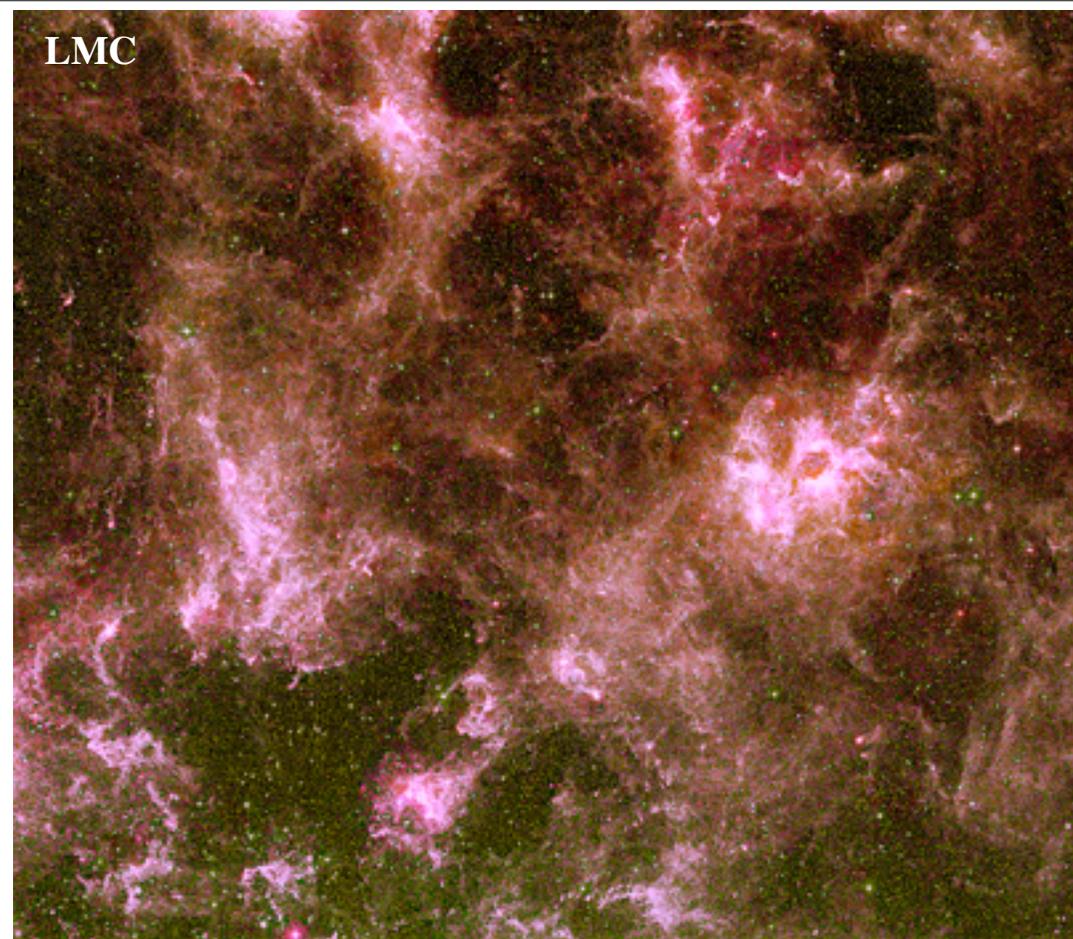


The Turbulent ISM

SUPER-SONIC TURBULENCE
DOMINATES (ALMOST) ALL SCALES

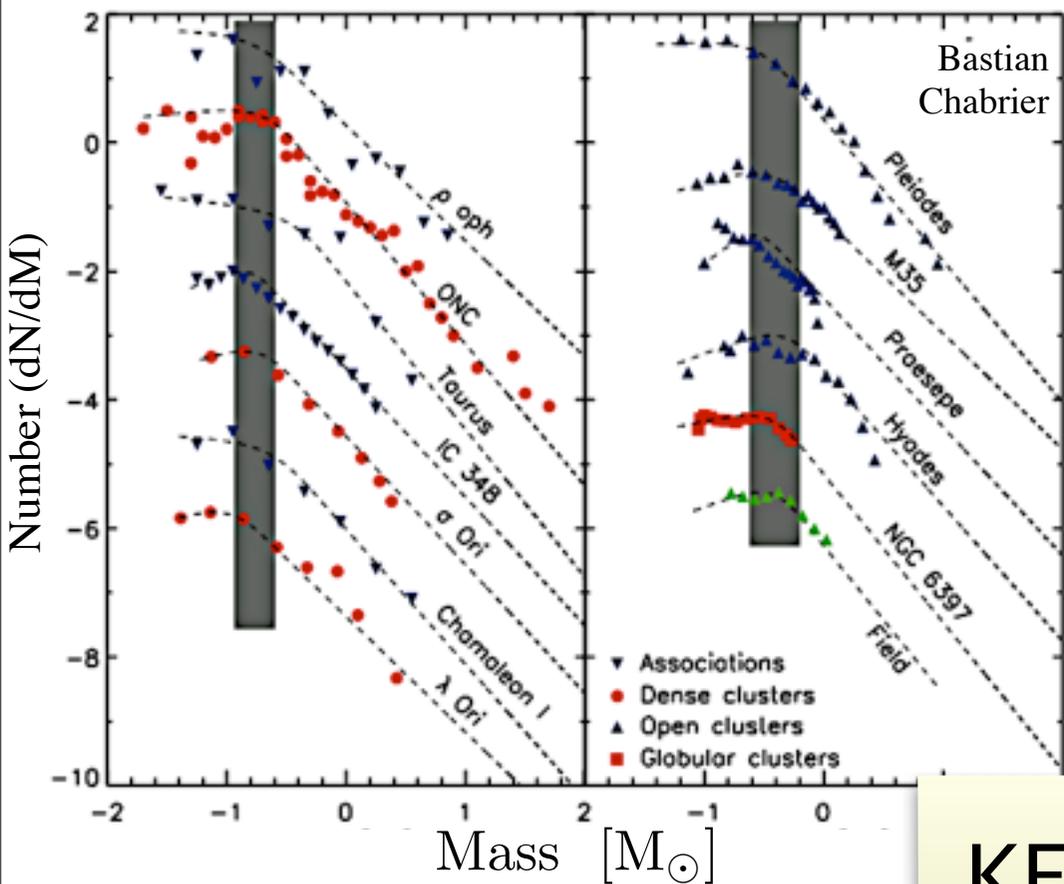


- Gravity
- Turbulence
- Magnetic, Thermal, Cosmic Ray, Radiation Pressure
- Cooling (atomic, molecular, metal-line, free-free)
- Star & BH Formation/Growth
- “Feedback”: Massive stars, SNe, BHs, external galaxies, etc.

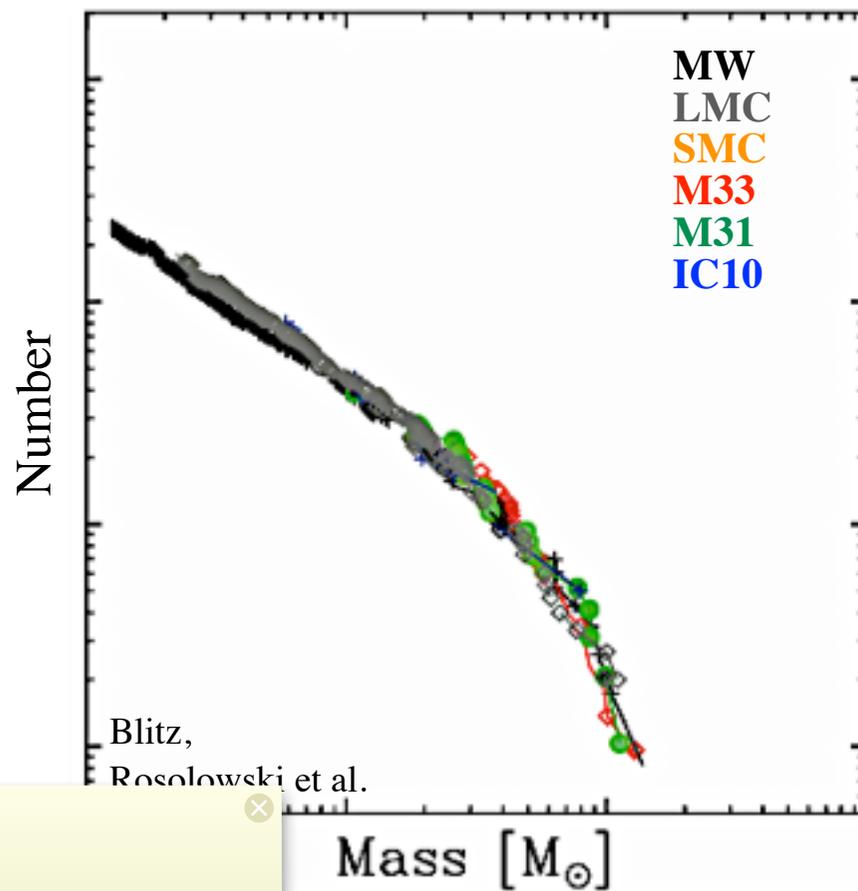
The ISM

YET THERE IS SURPRISING REGULARITY

Stars & Pre-Stellar Gas Cores:



Giant Molecular Clouds:

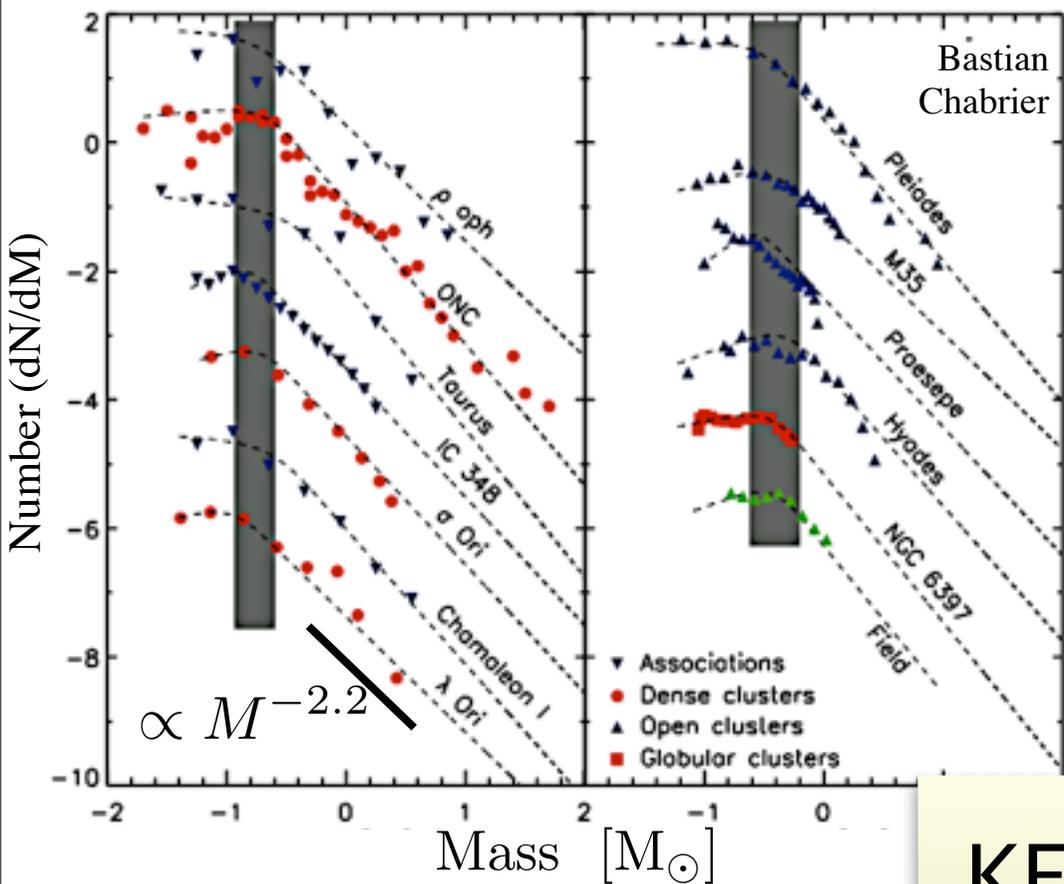


KEEP
PWRLAW
LABELS?

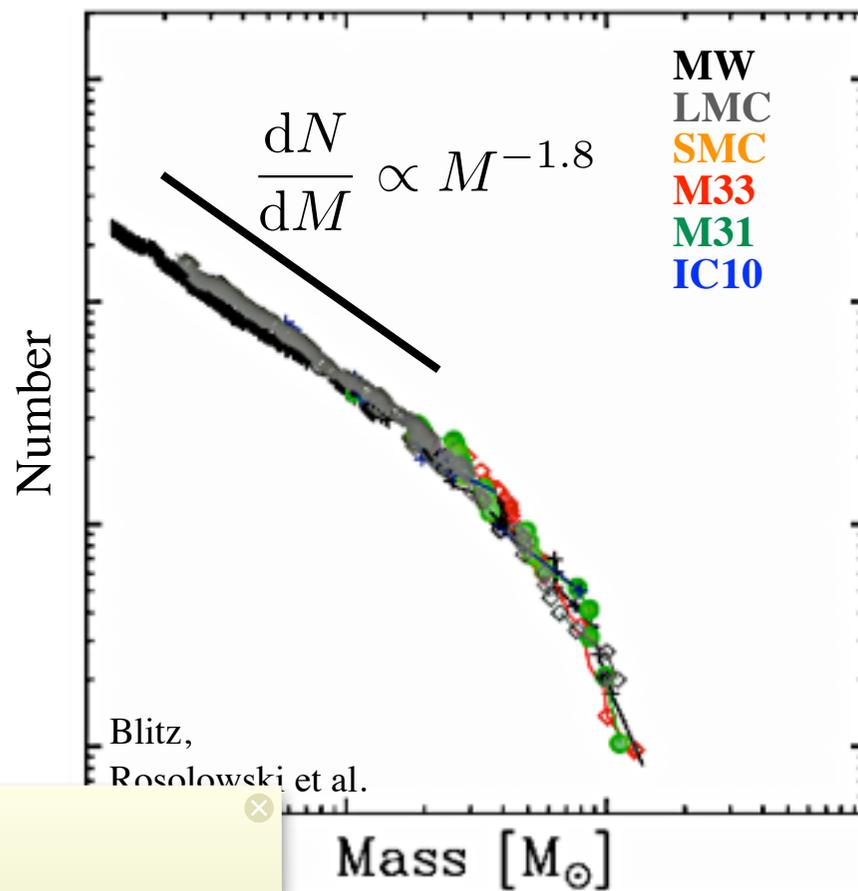
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Super-Sonic Turbulence

BASIC EXPECTATIONS

$$E(k) \propto k^{-p}$$

$$dE \equiv E(k) dk$$

$$(k E(k) \sim u_t(k)^2)$$

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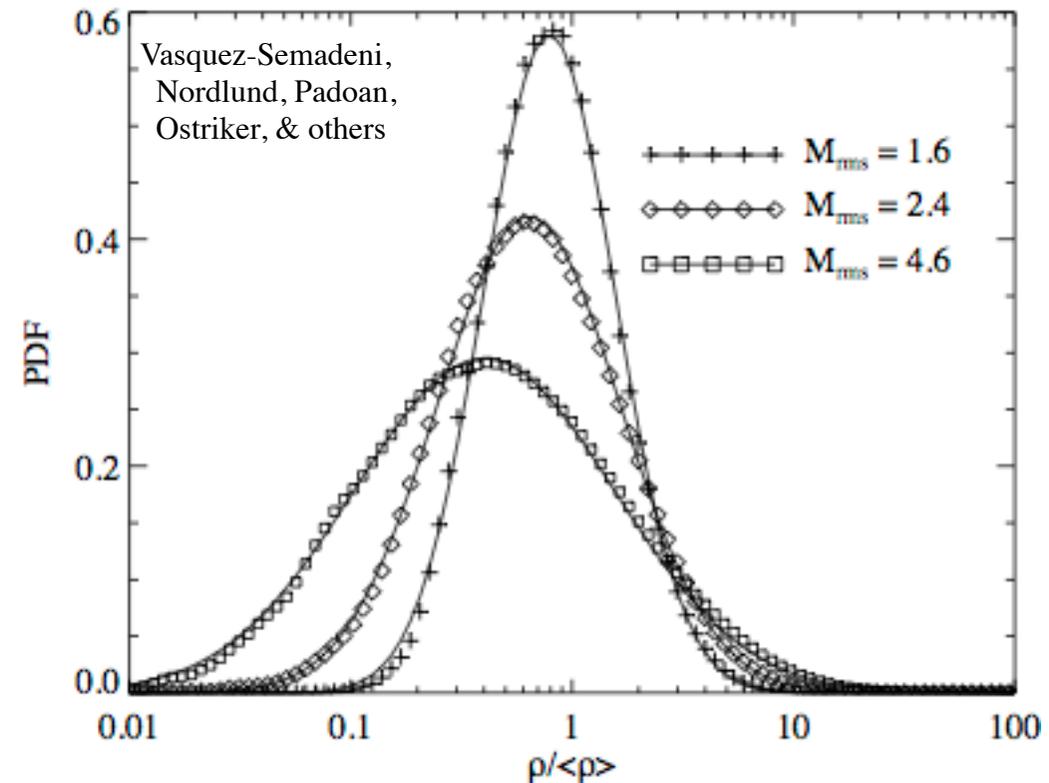
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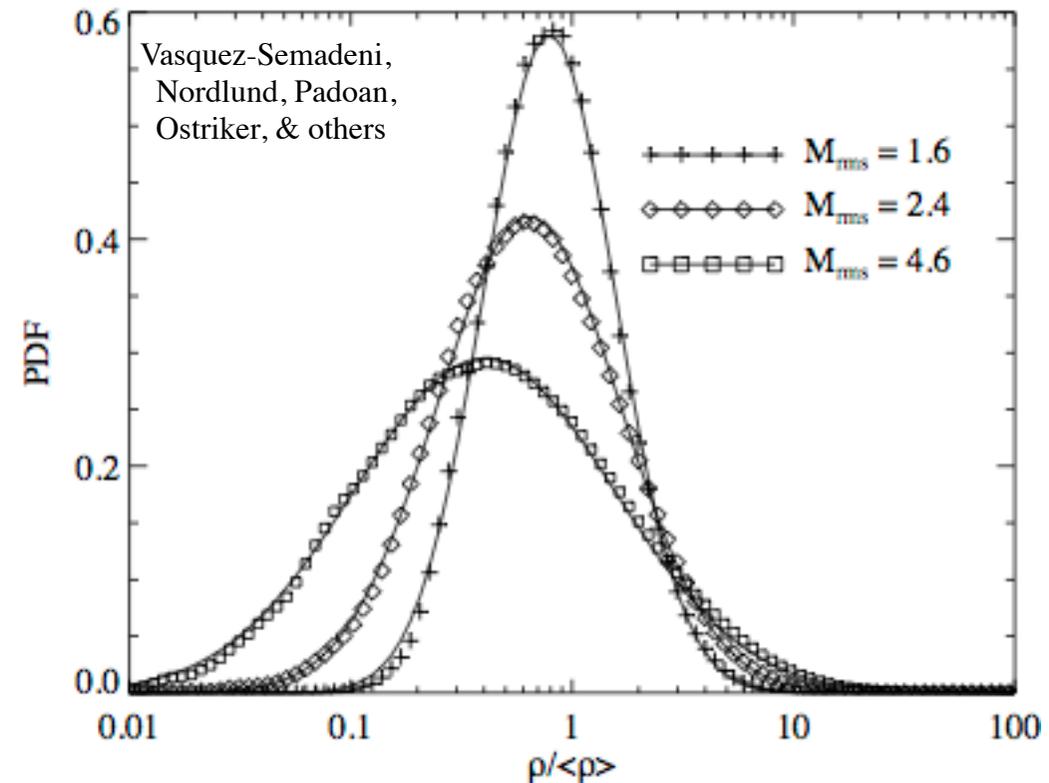
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$$\frac{\partial \ln \rho}{\partial t} = -\mathbf{u} \cdot \nabla \ln \rho - \nabla \cdot \mathbf{u}$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - c_s^2 \nabla \ln \rho$$



Super-Sonic Turbulence

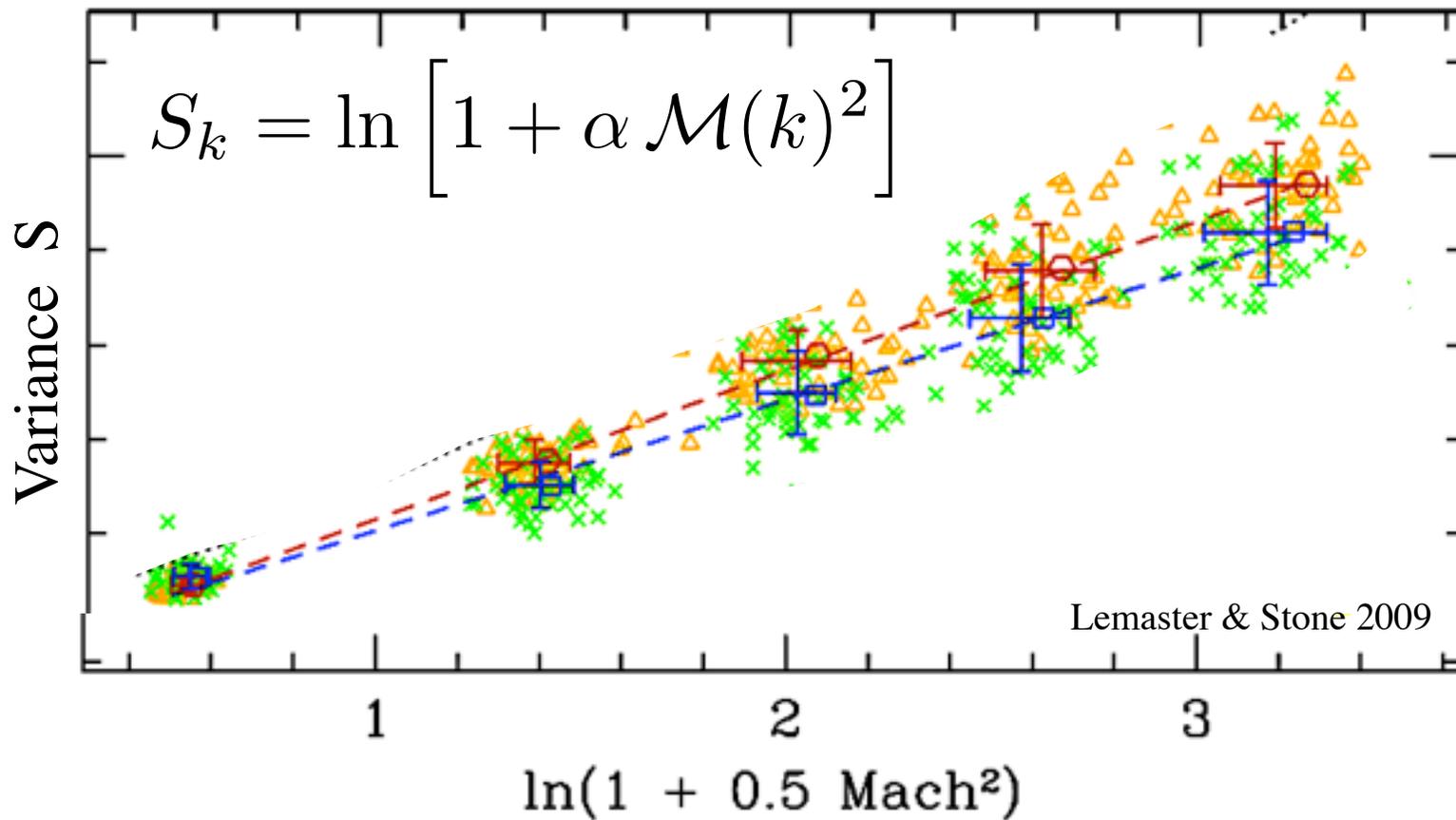
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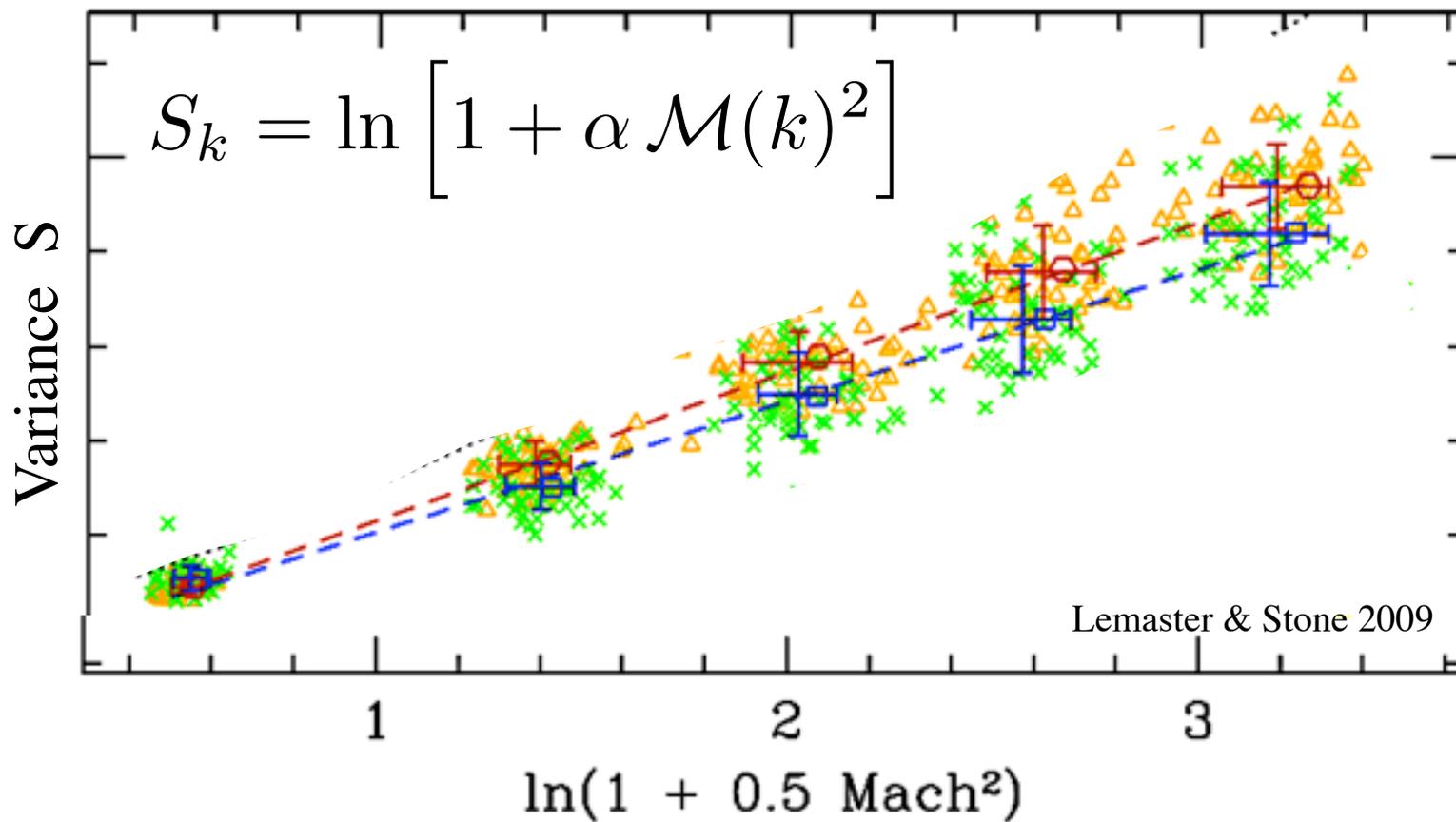
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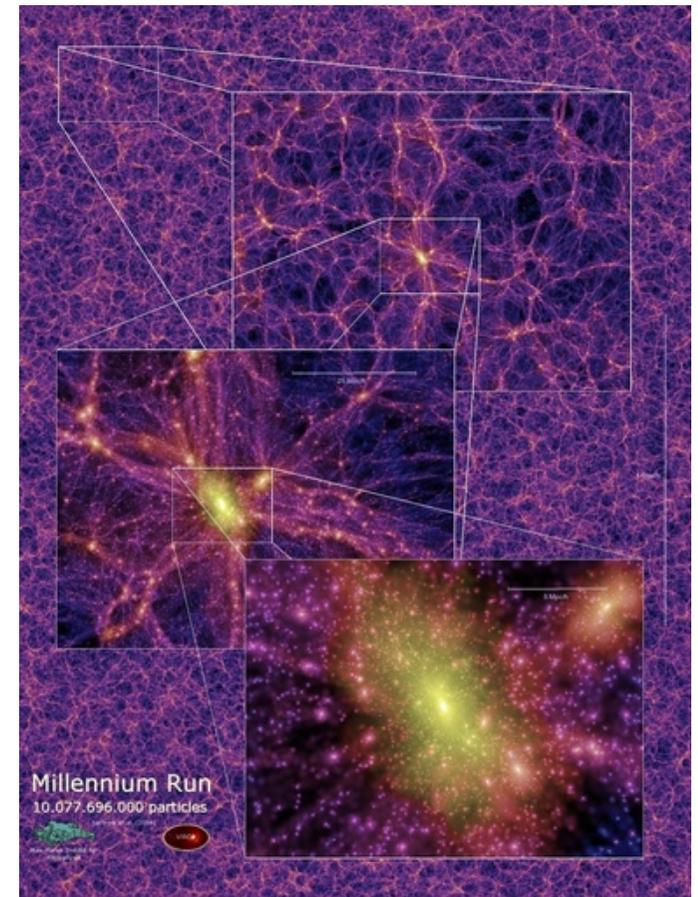
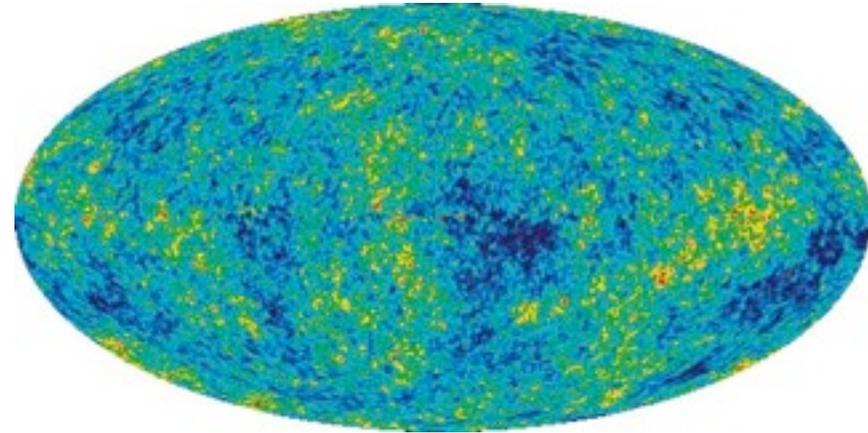
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Extended Press-Schechter / Excursion-Set Formalism

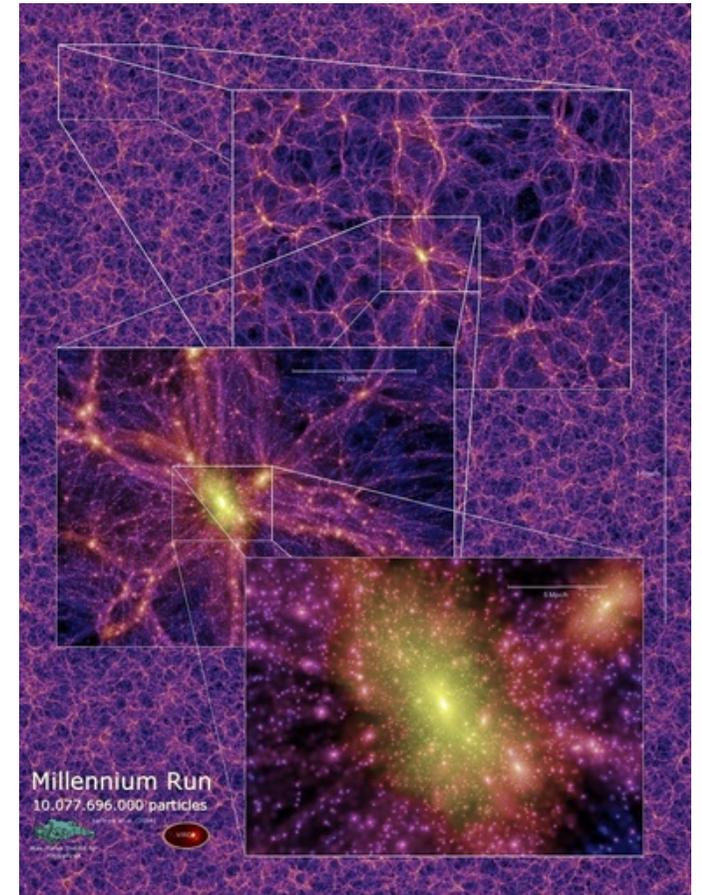
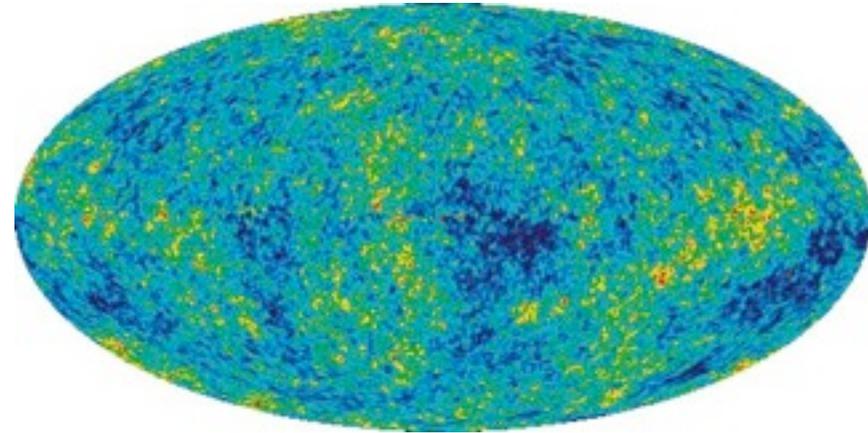
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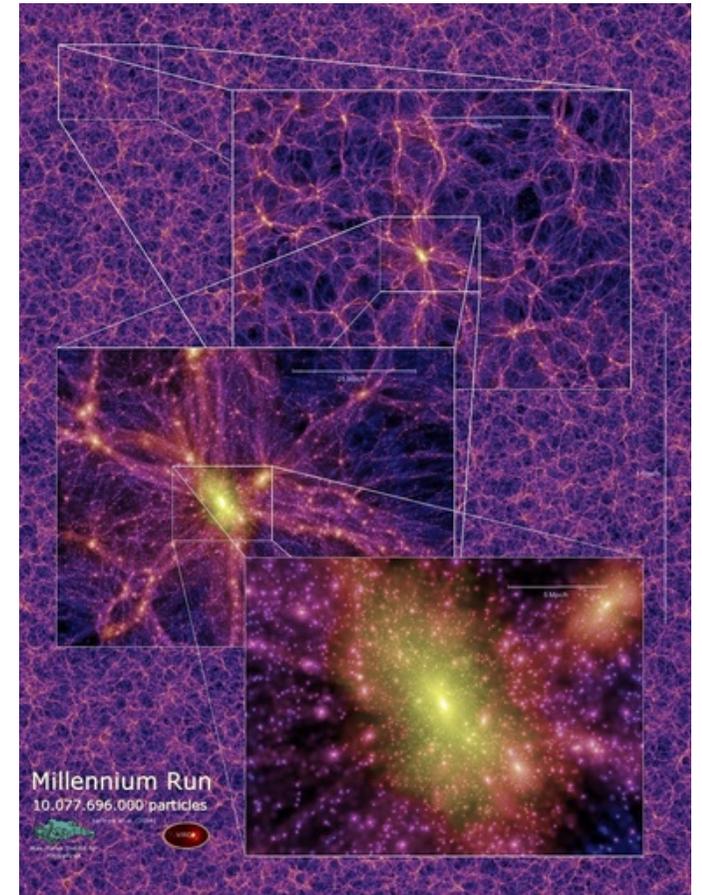
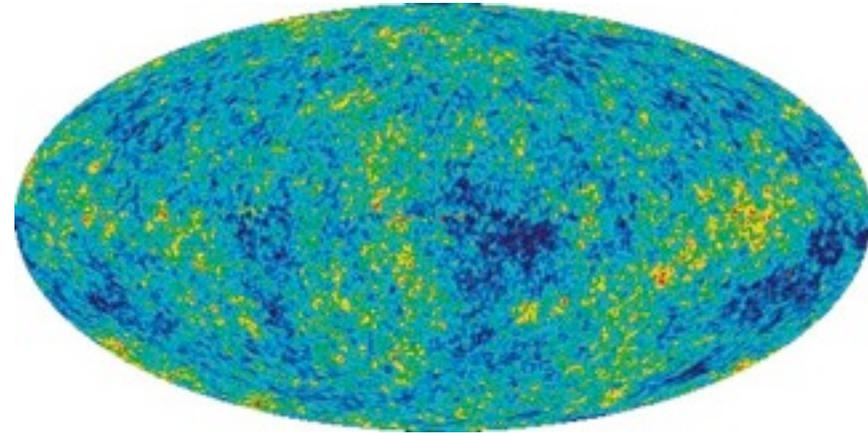
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- Generalize to conditional probabilities,
N-point statistics, resolve “cloud in cloud” problem
(e.g. Bond et al. 1991)



What Defines a Fluctuation of Interest?

DISPERSION RELATION:

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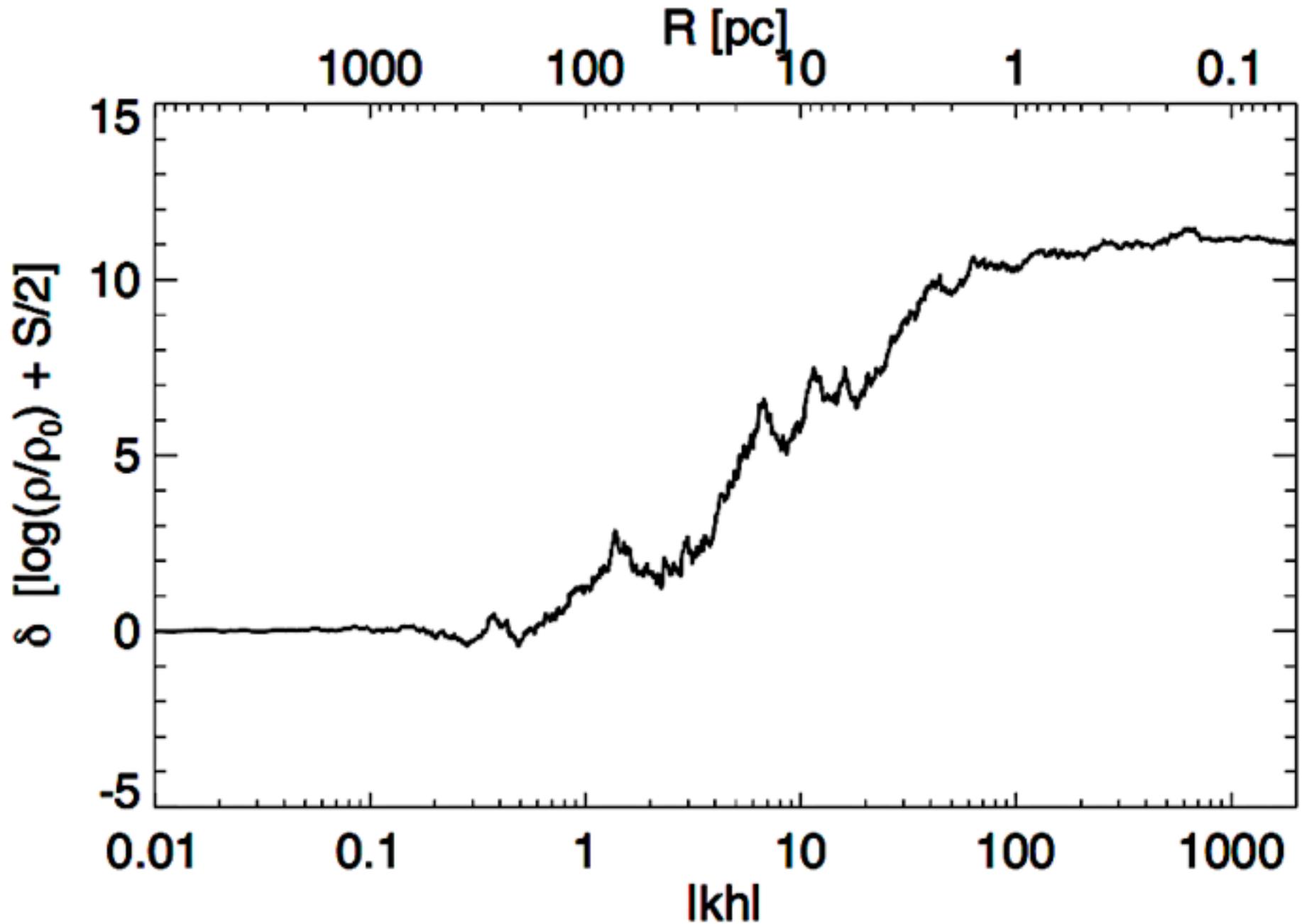
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Mode Grows (Collapses) when $w < 0$:

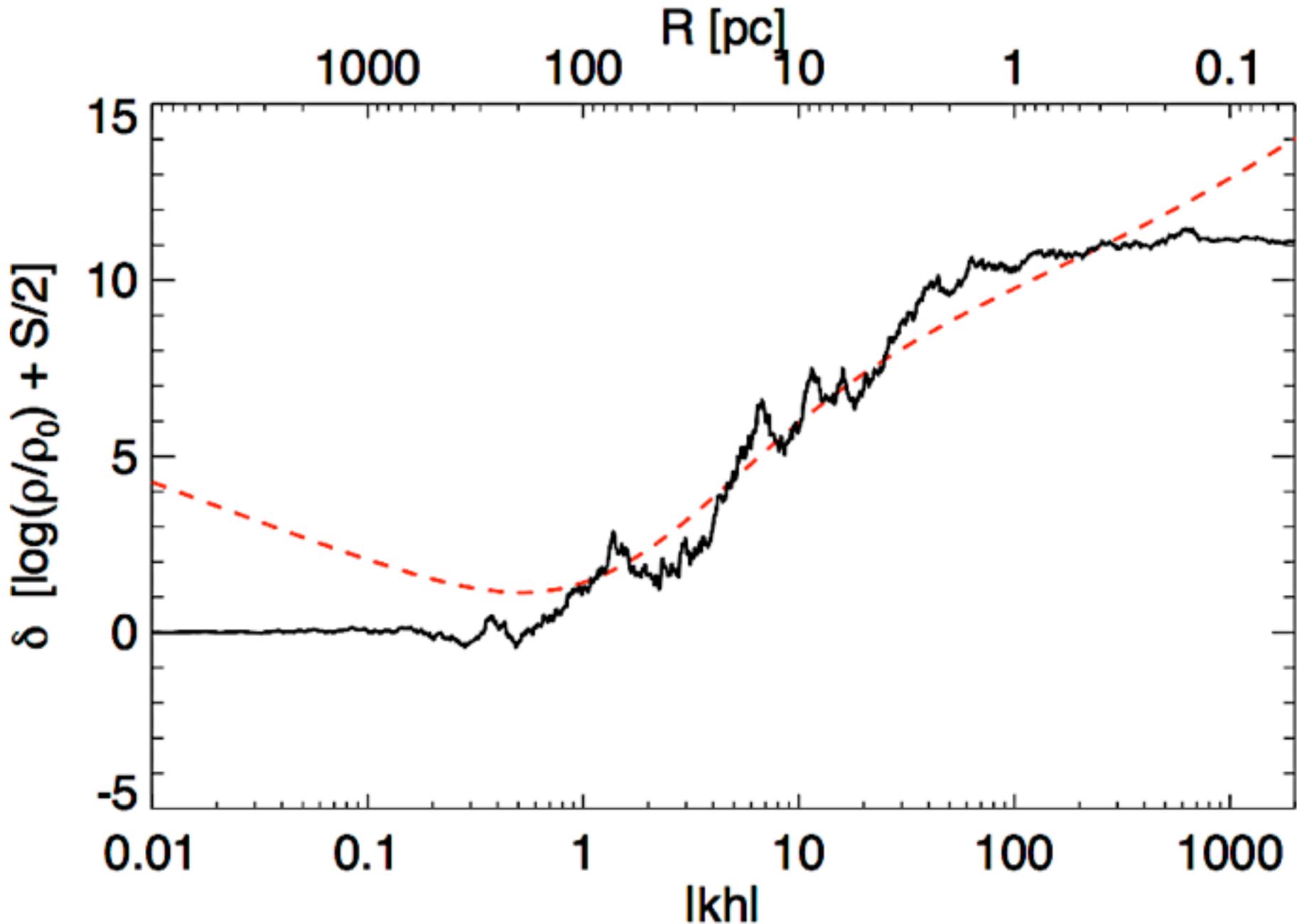
$$\rho > \rho_c(k) = \rho_0 (1 + |kh|) \left[(\mathcal{M}_h^{-2} + |kh|^{1-p}) kh + \frac{2}{|kh|} \right]$$

EVALUATE DENSITY FIELD vs. “BARRIER”



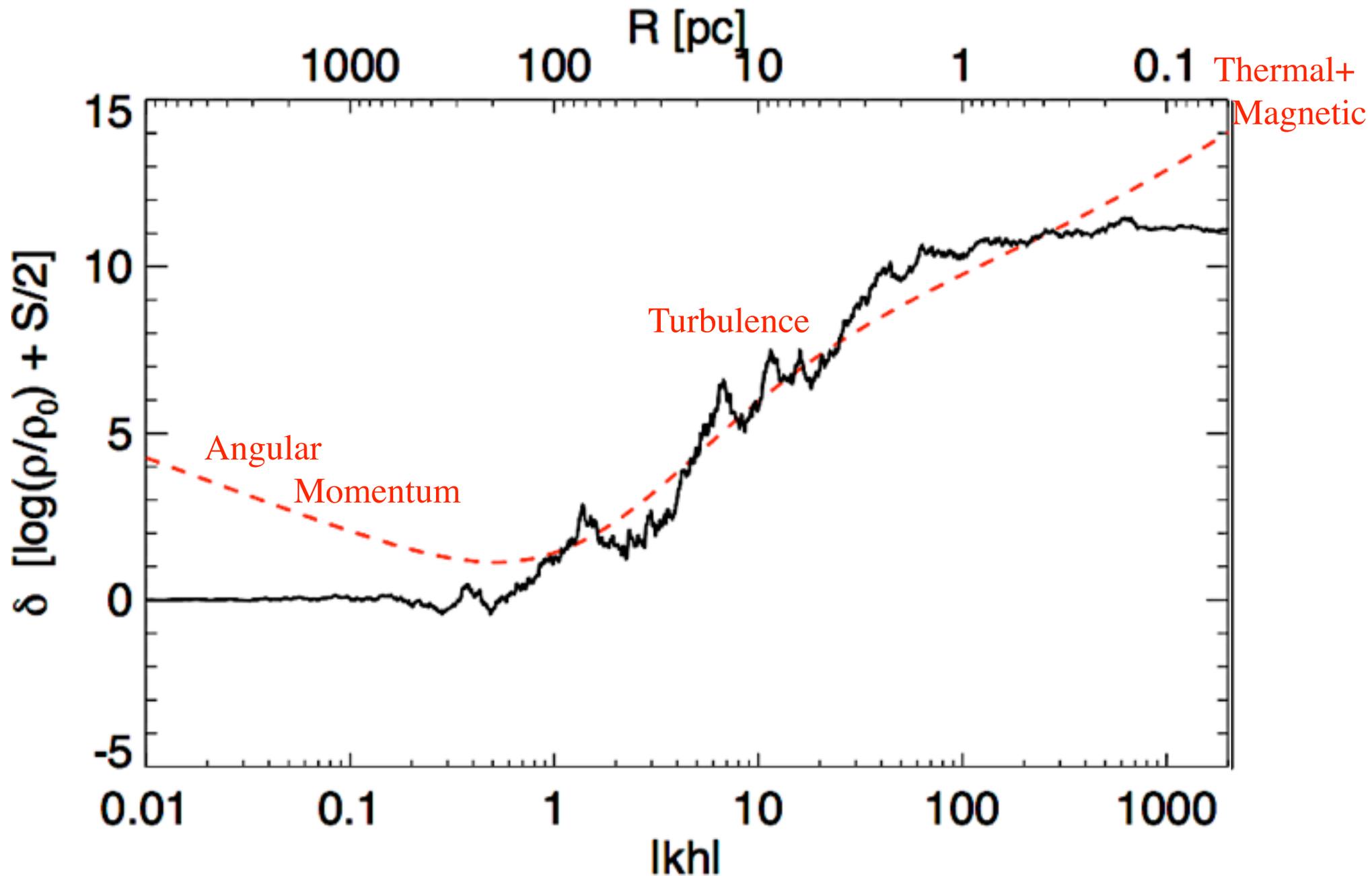
“Counting” Collapsing Objects

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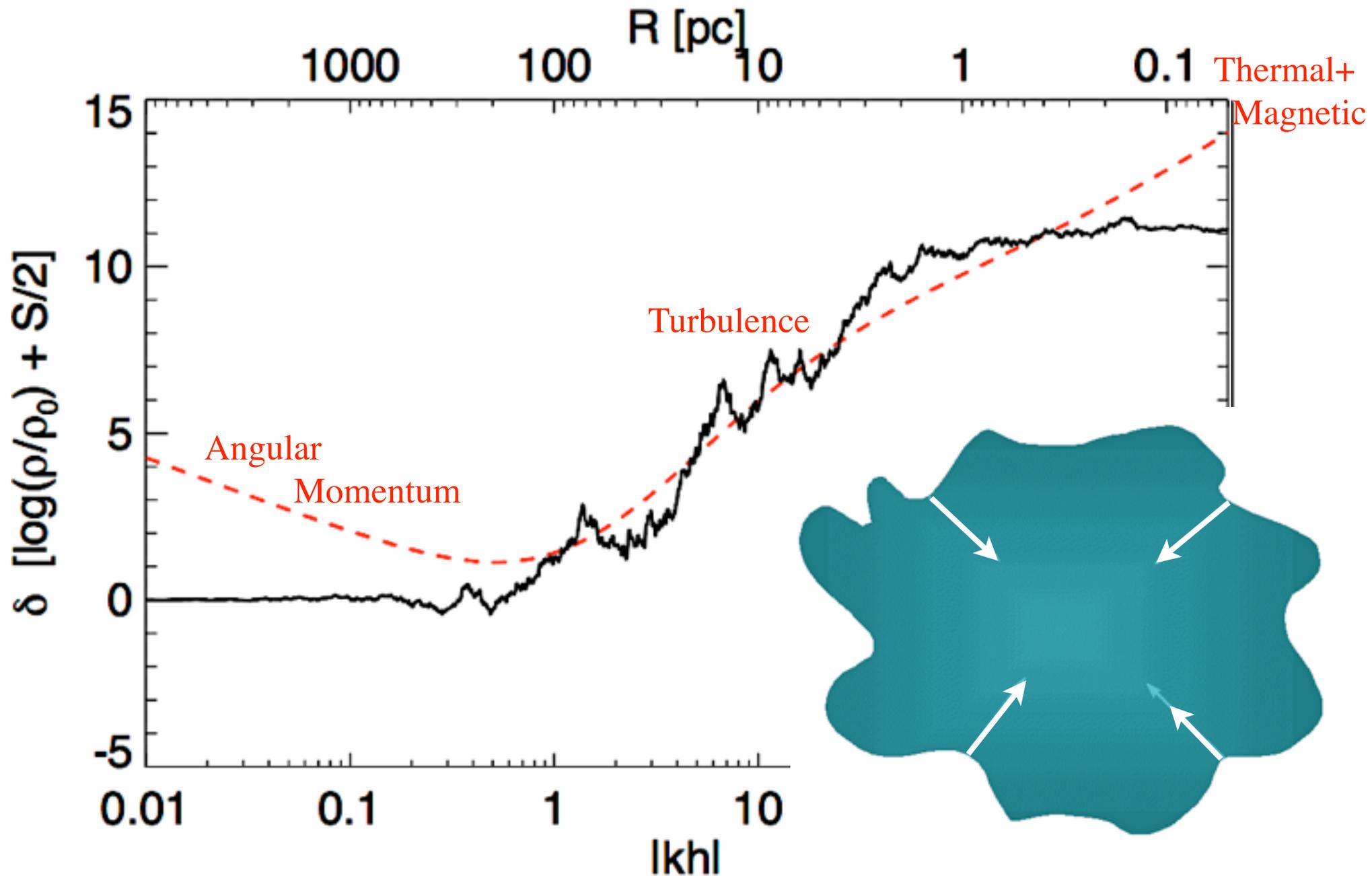
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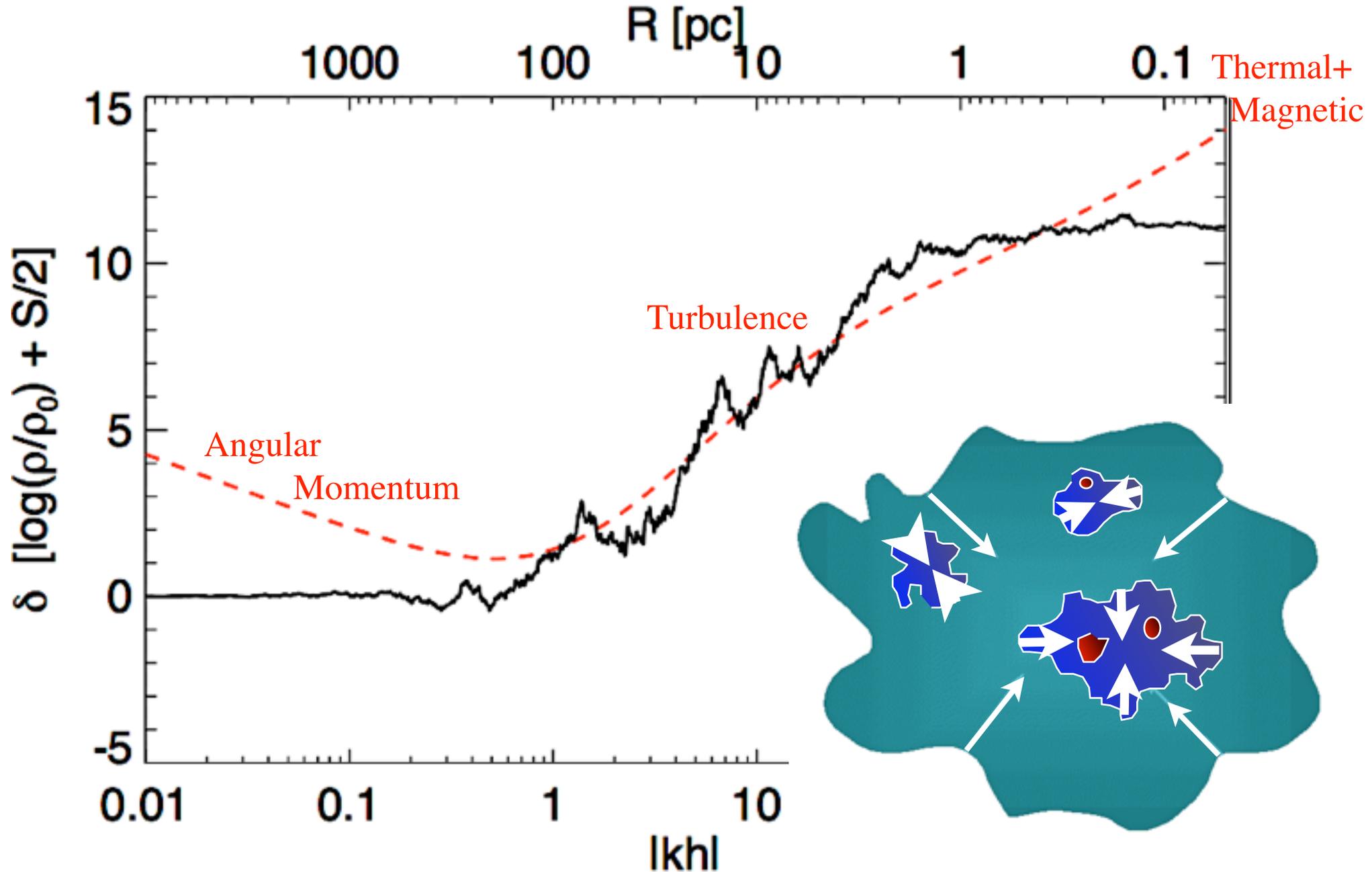
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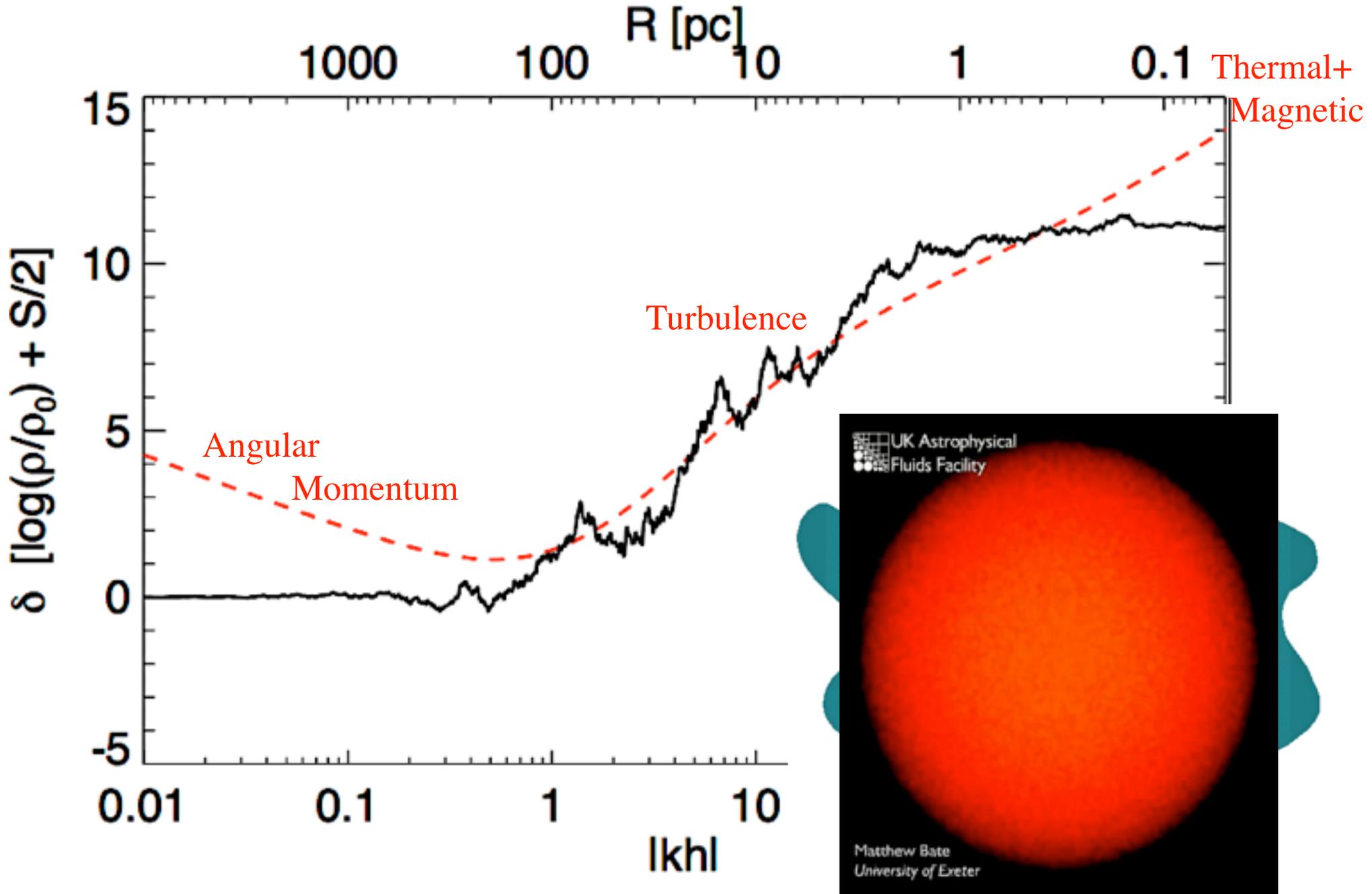
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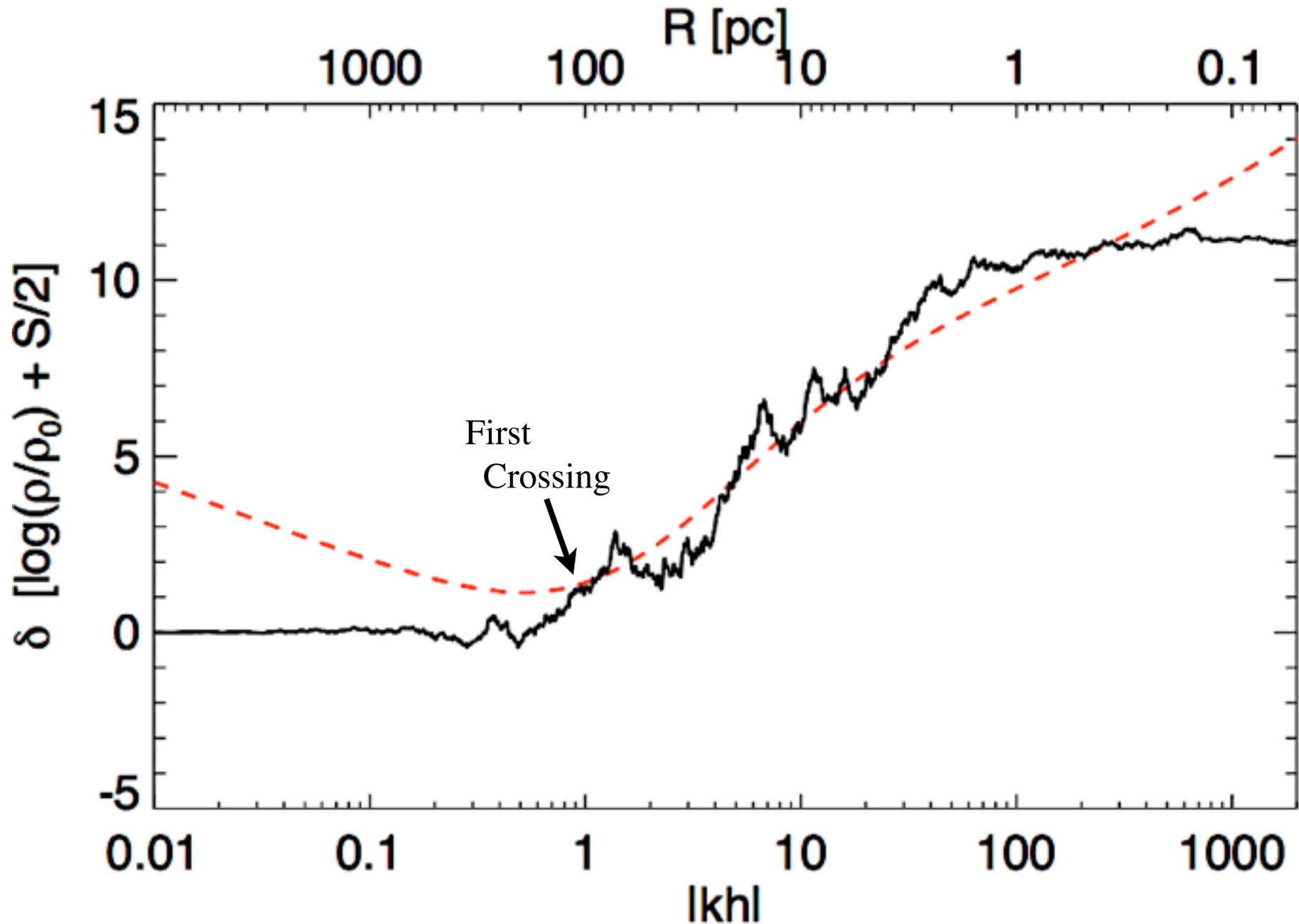
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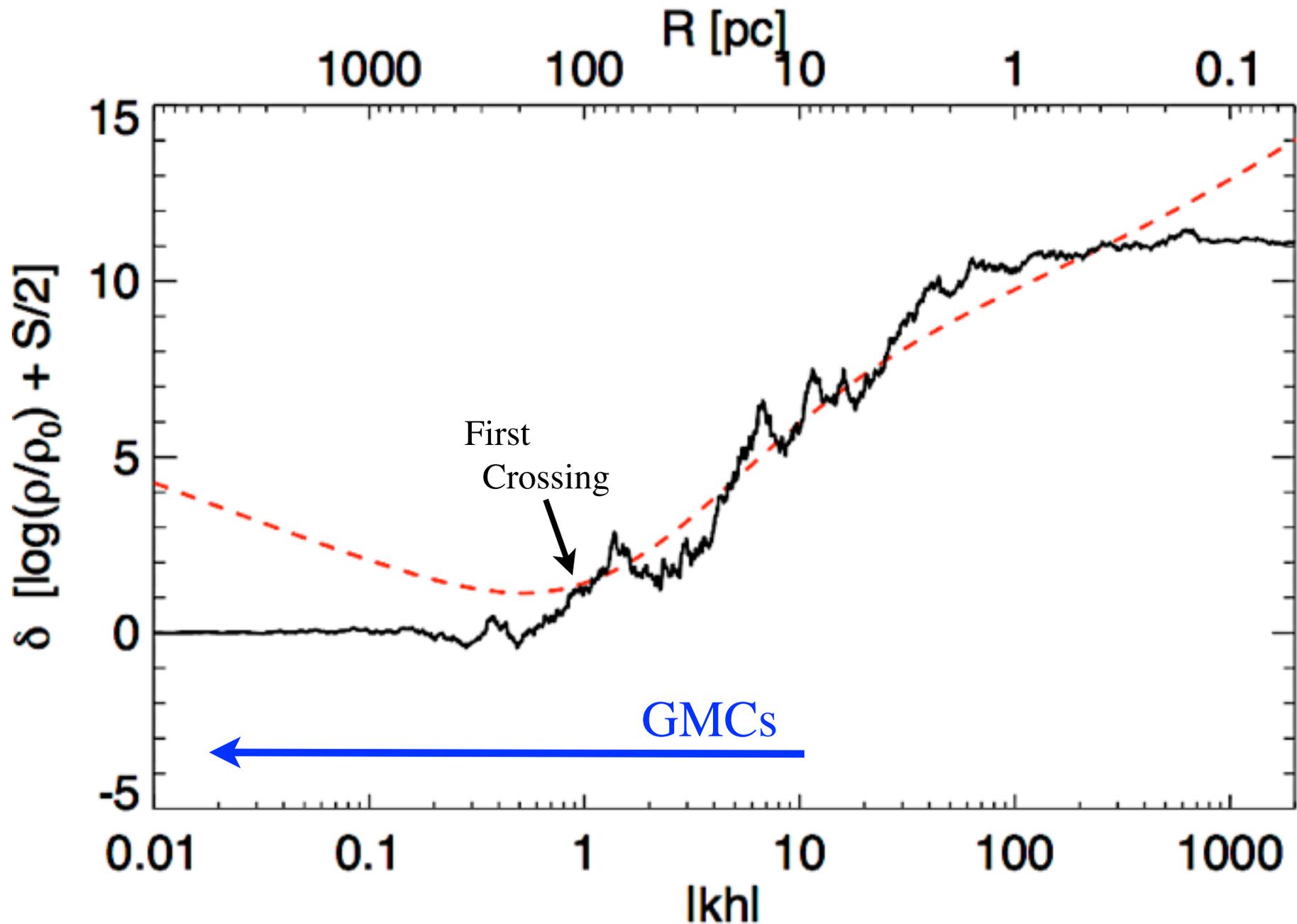
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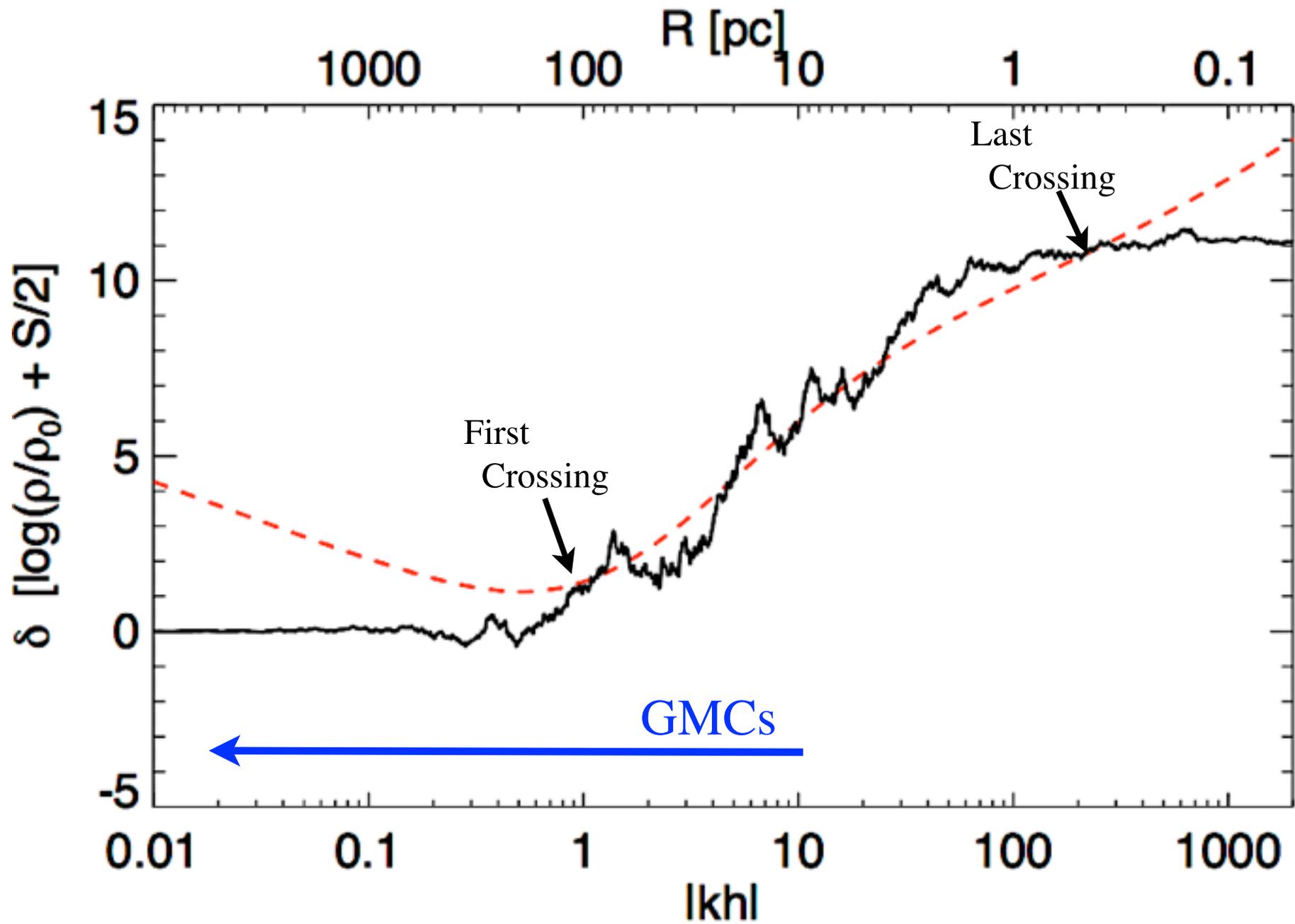
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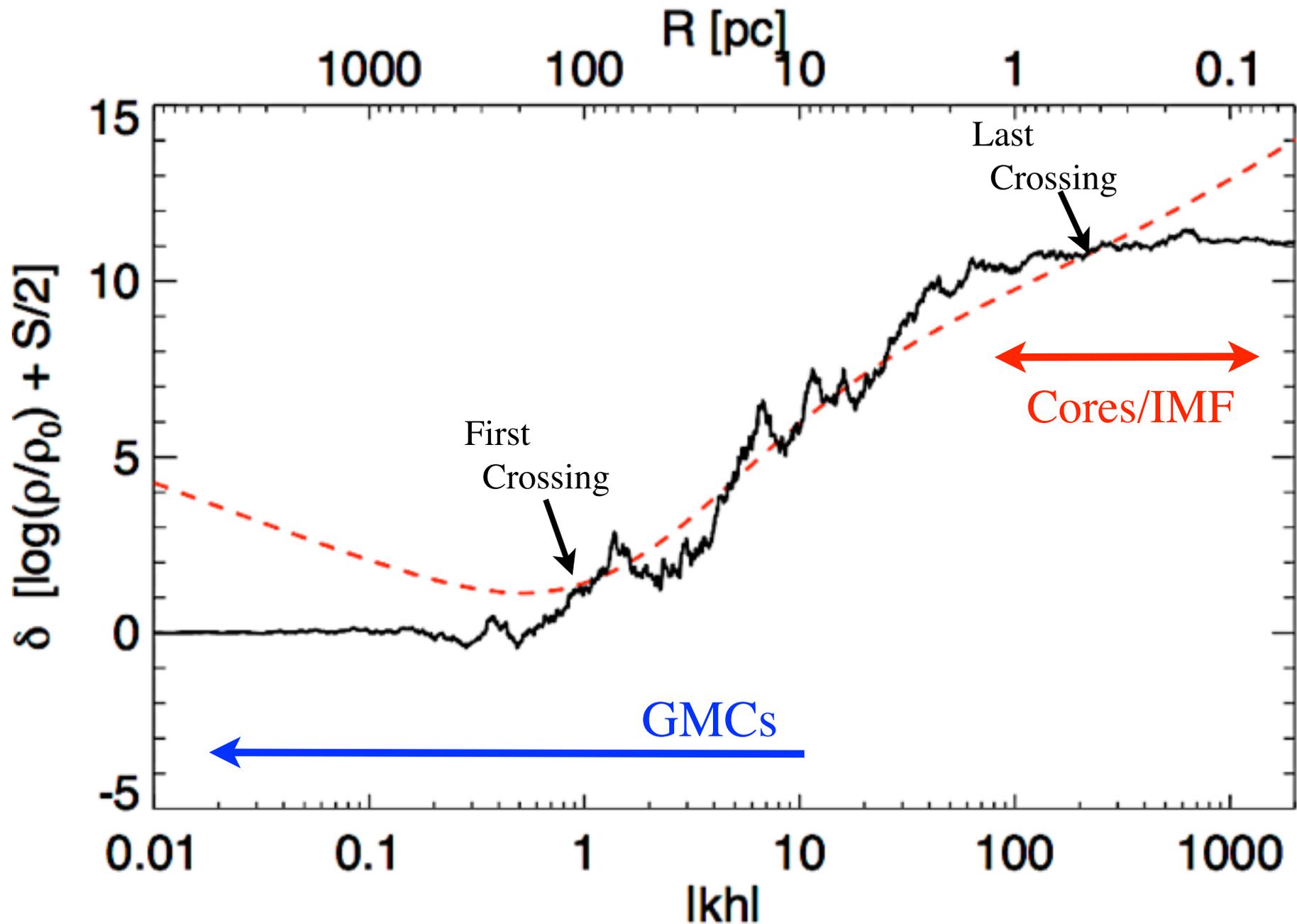
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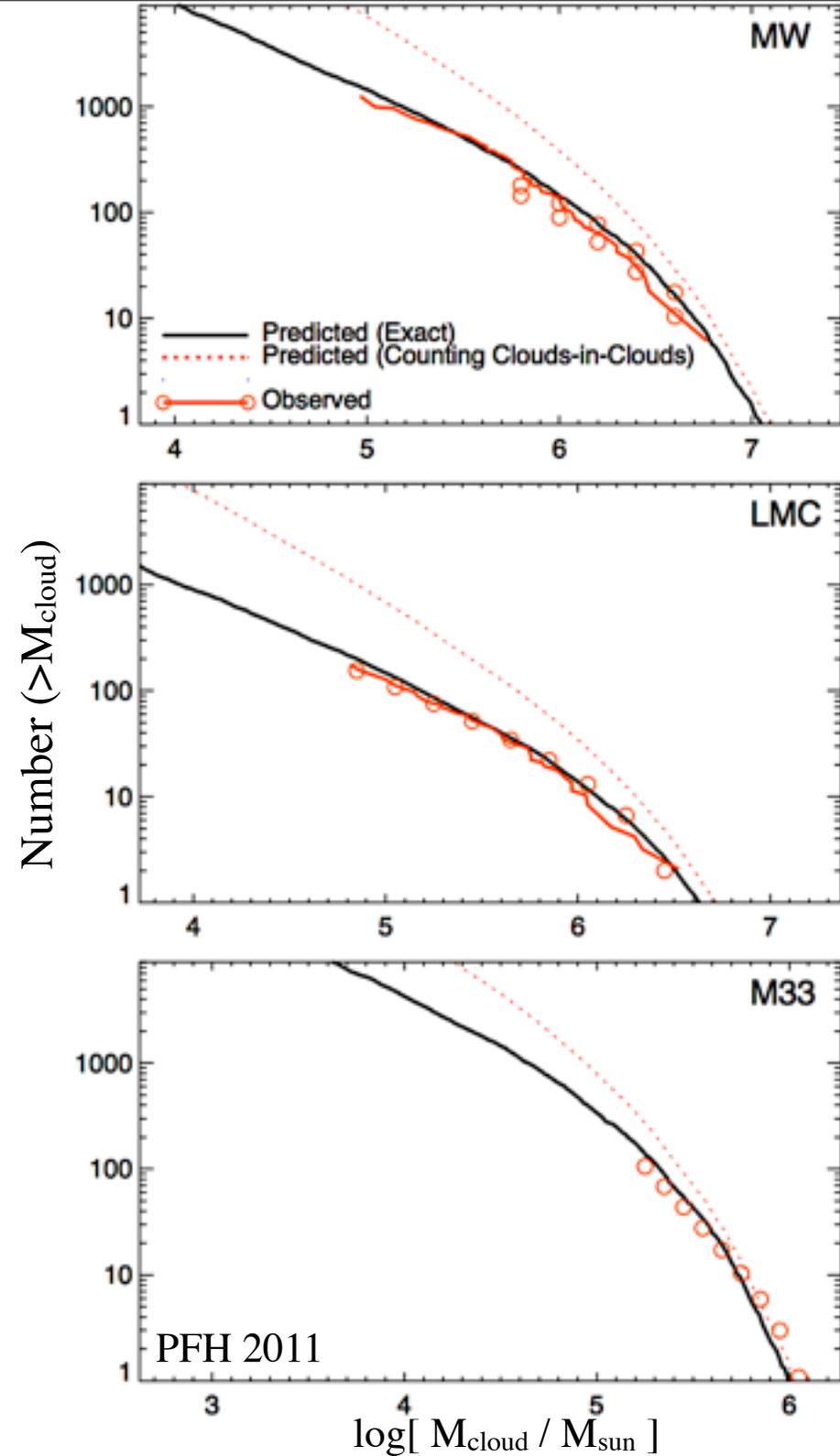


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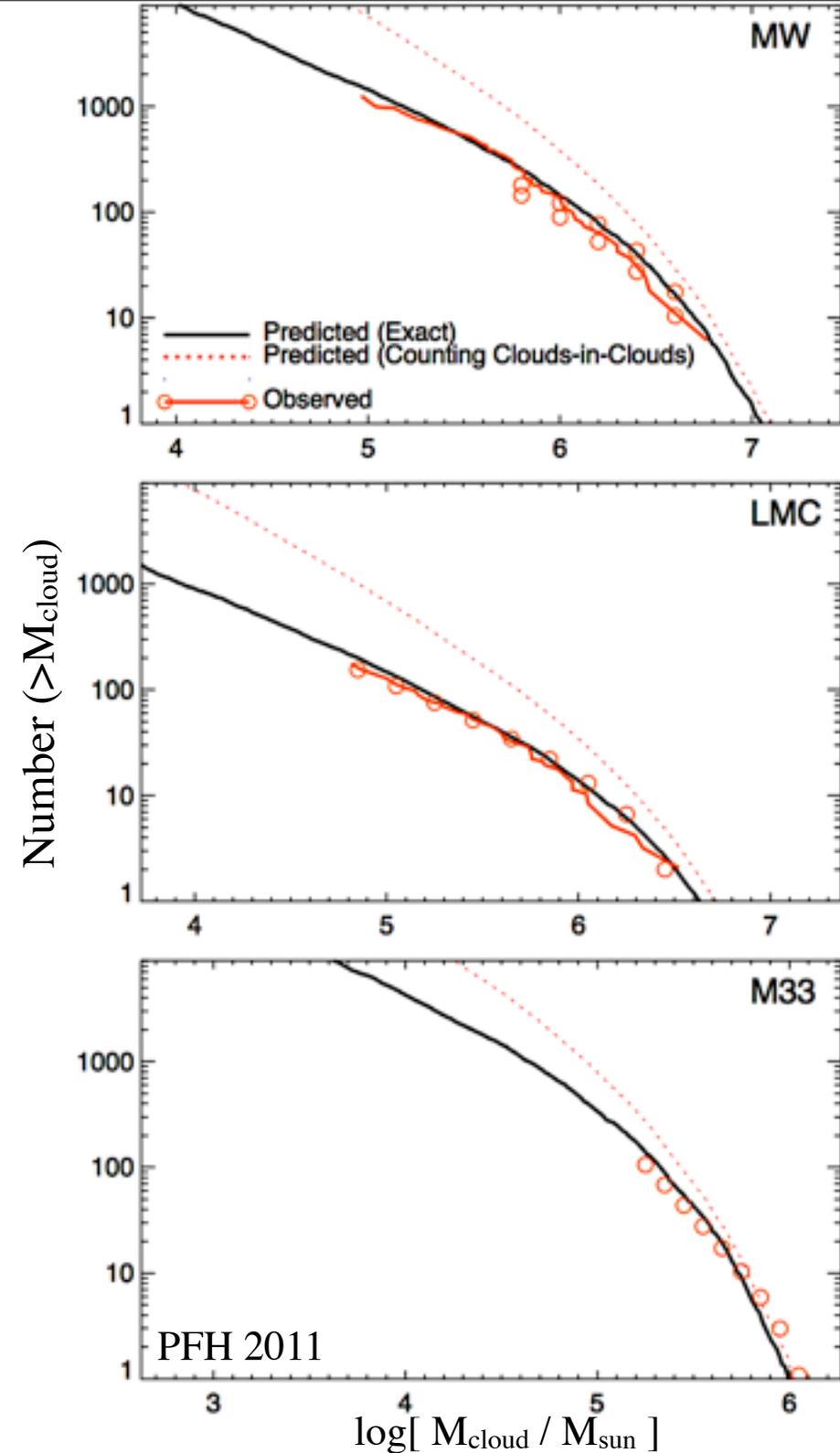
The “First Crossing” Mass Function VS GIANT MOLECULAR CLOUDS



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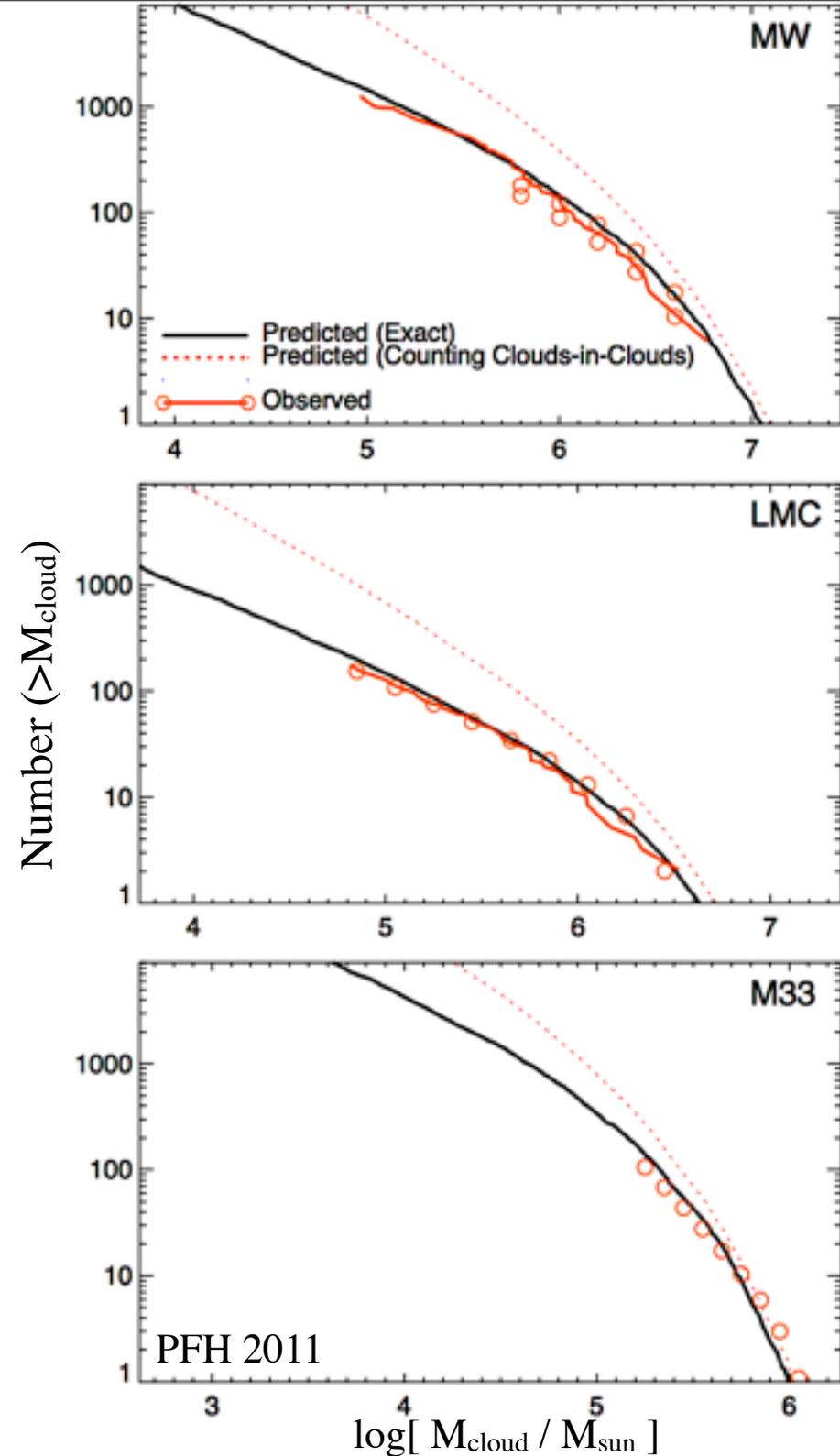


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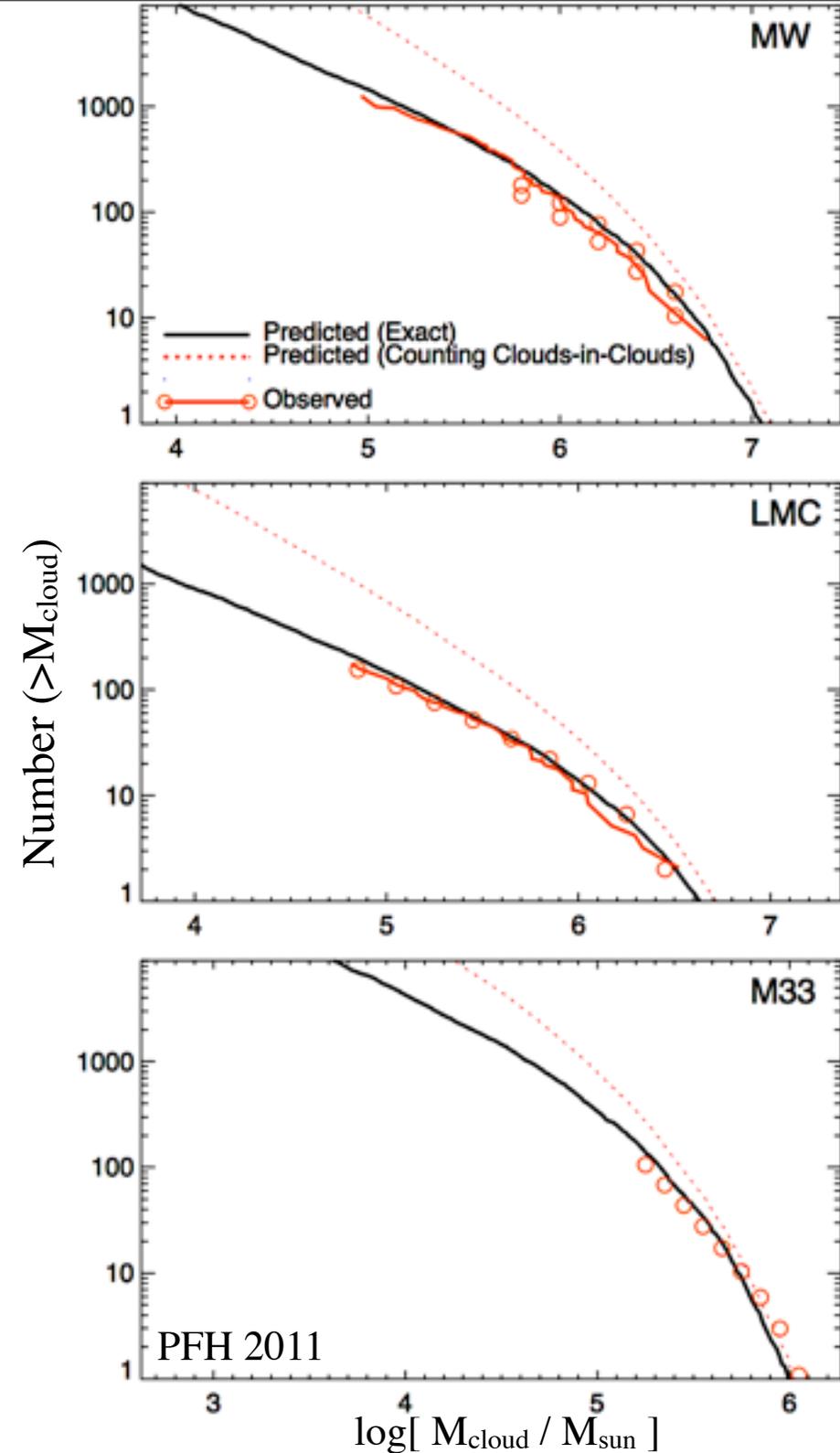
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$$\alpha \approx -2 + \frac{(3-p)^2}{2Sp^2} \ln\left(\frac{M_J}{M}\right)$$

$$\approx -2 + 0.1 \log\left(\frac{M_J}{M}\right)$$

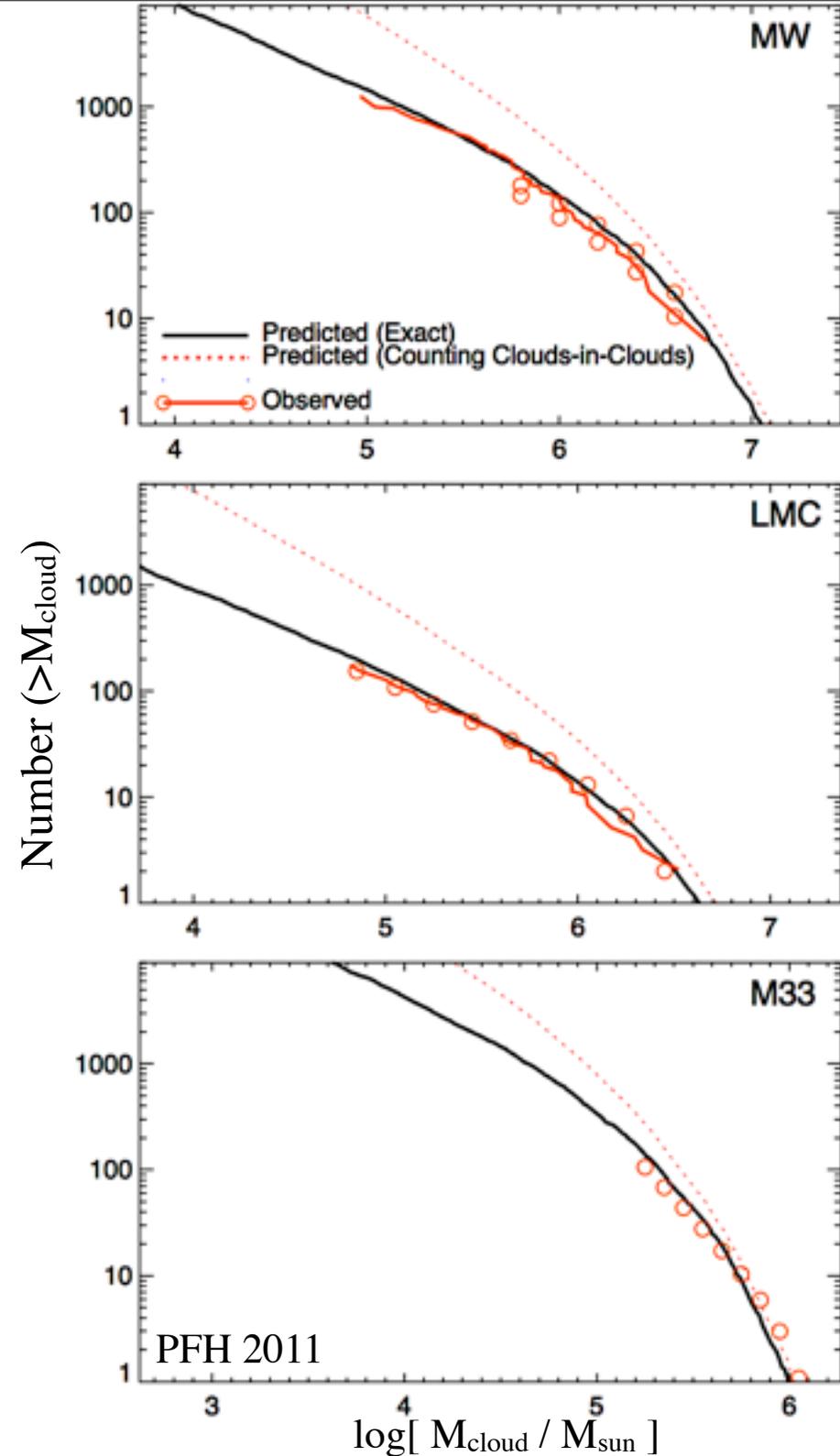
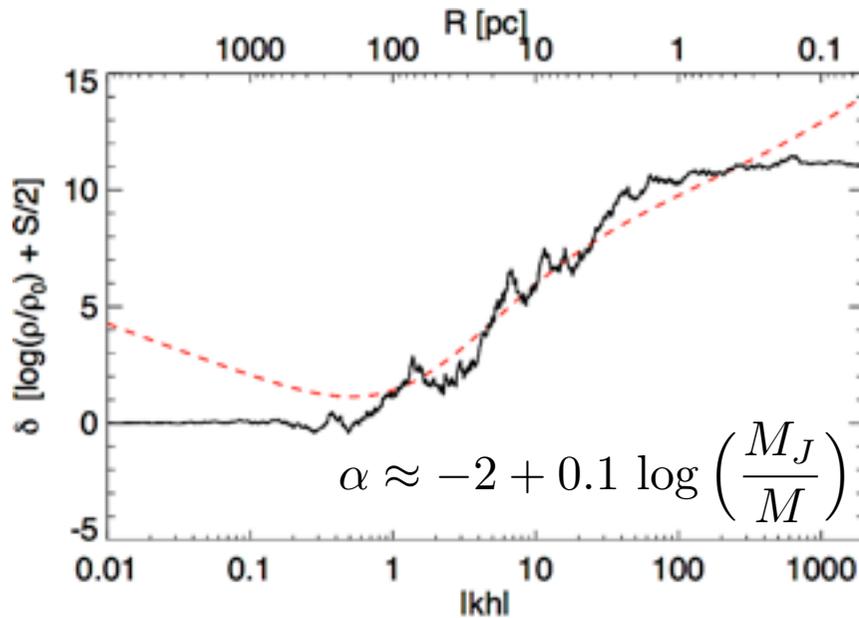


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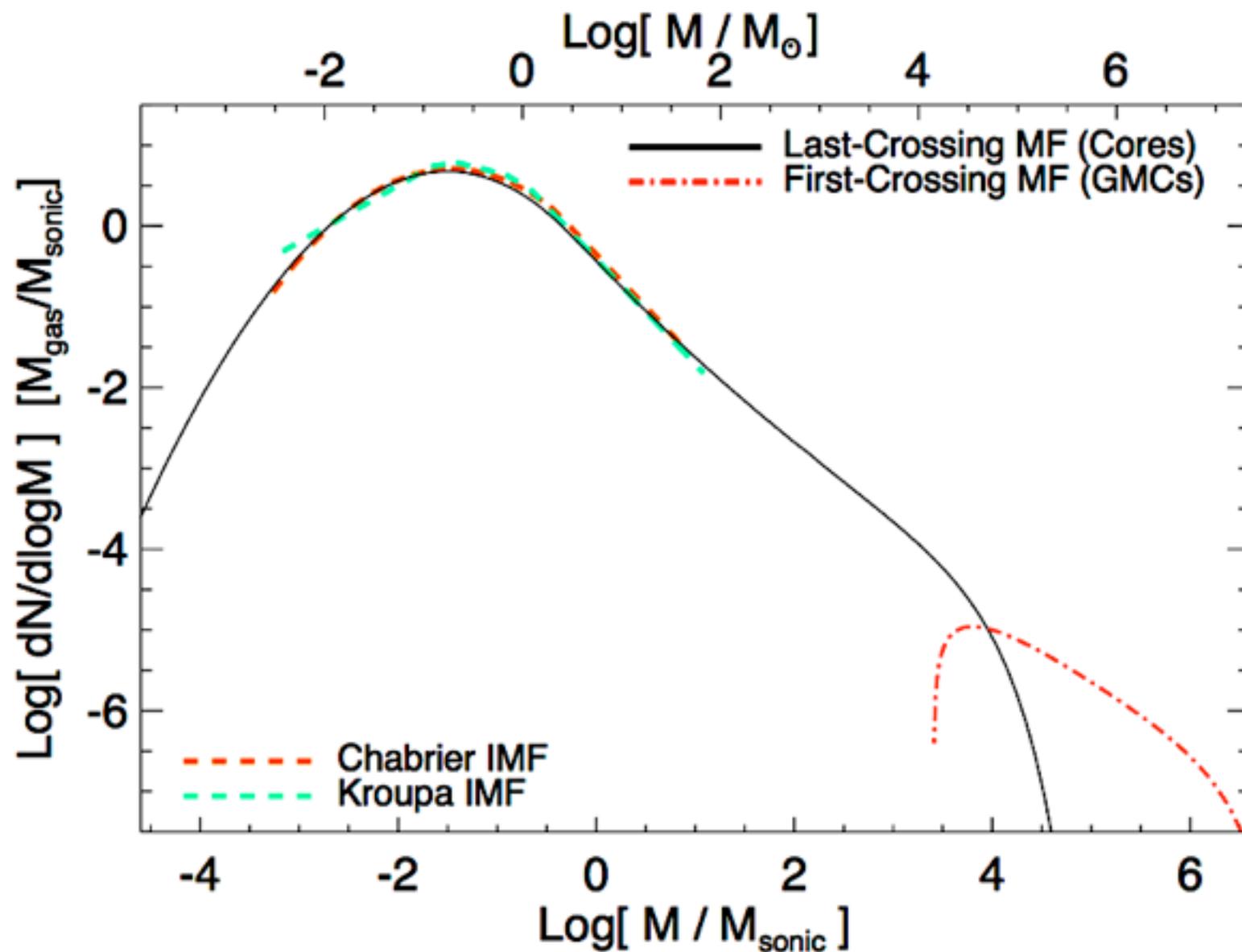
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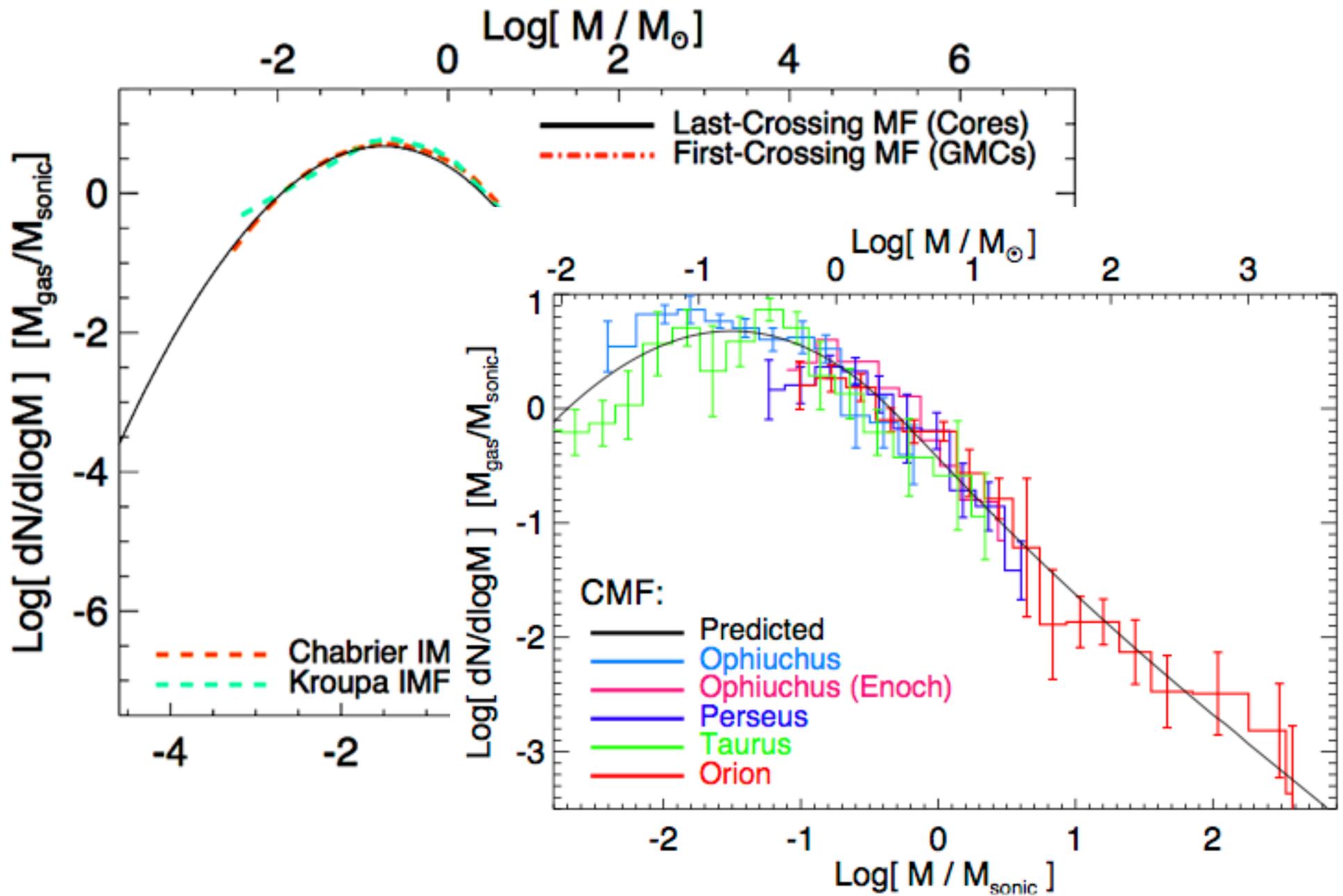
The “Last Crossing” Mass Function

VS PROTOSTELLAR CORES & THE STELLAR IMF



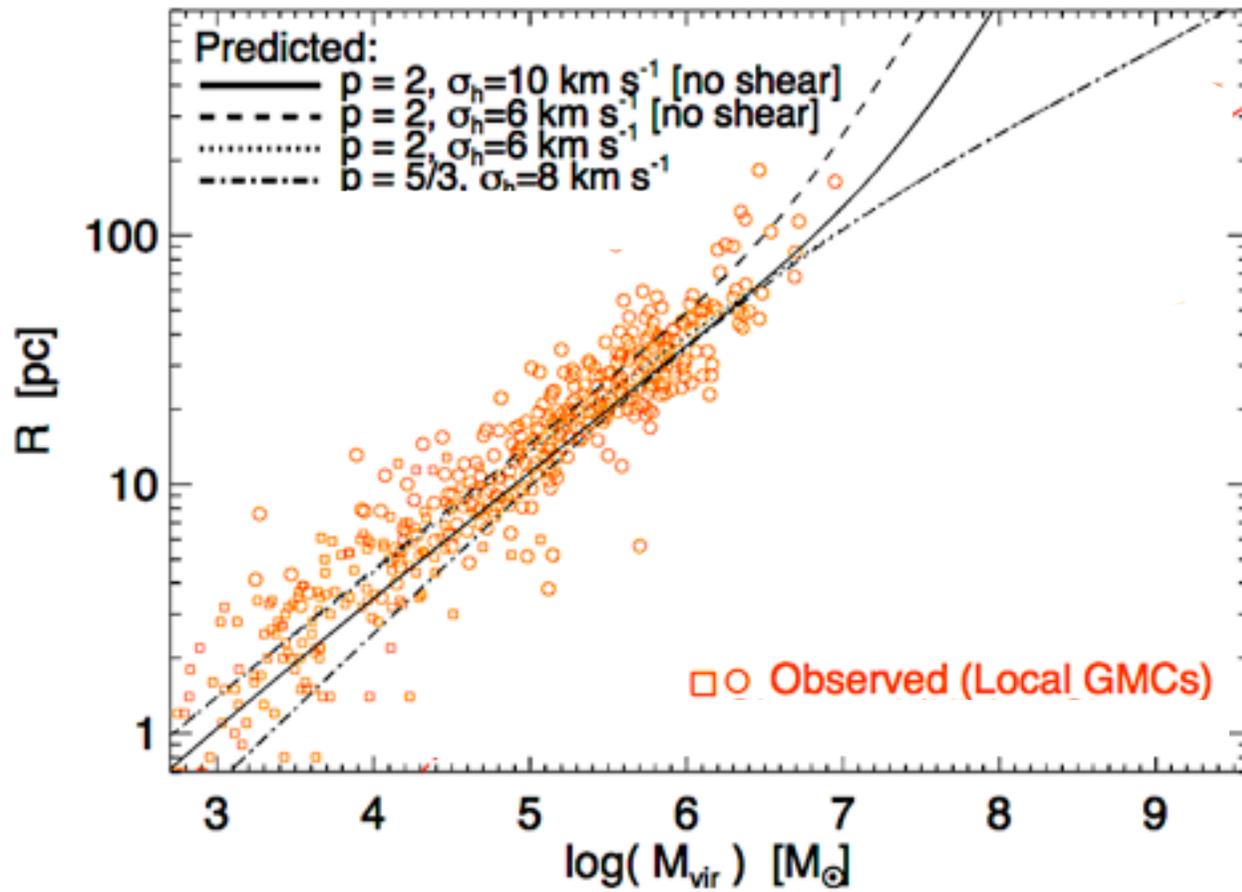
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Structural Properties of “Clouds”

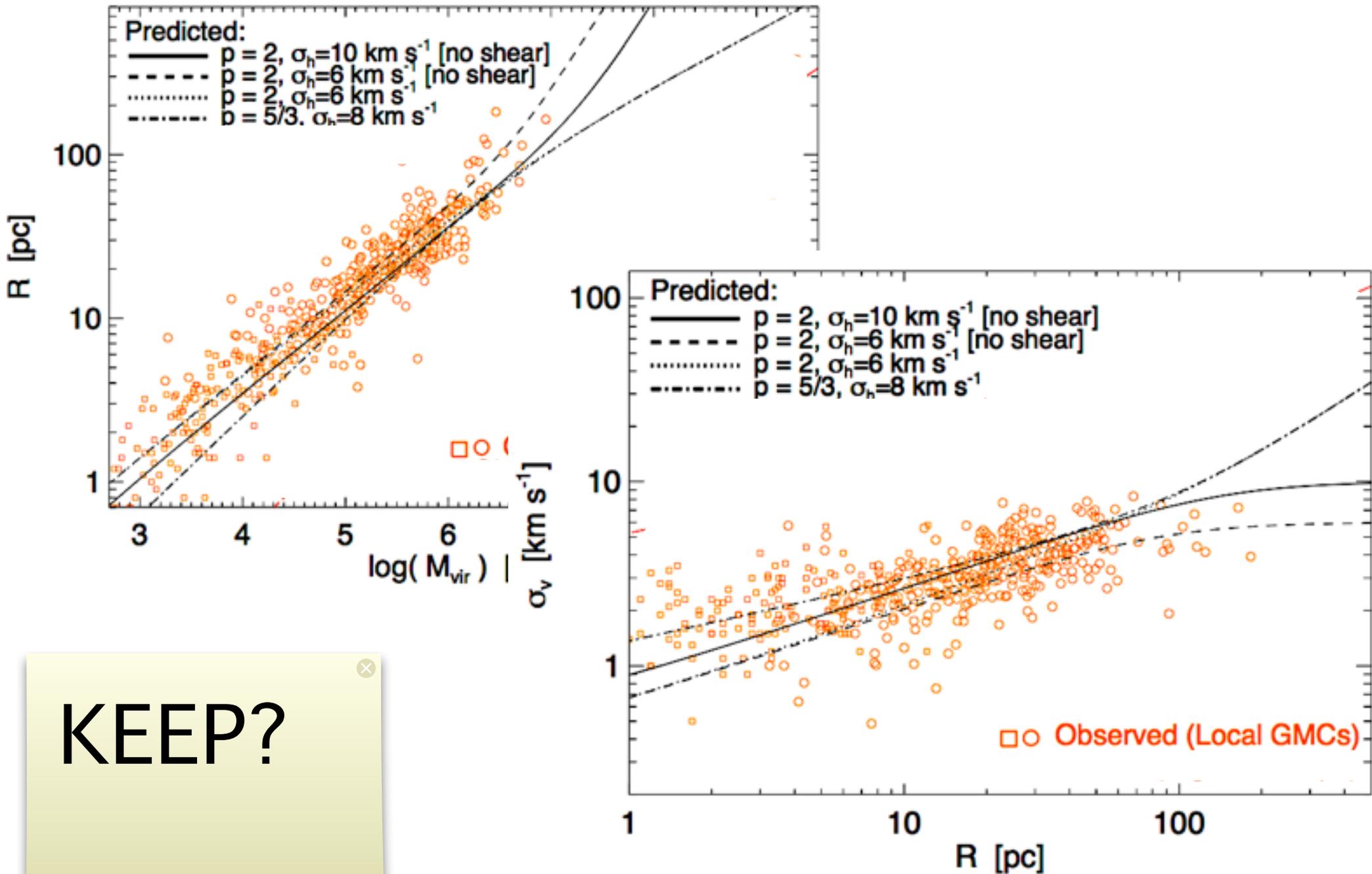
LARSON'S LAWS EMERGE NATURALLY



KEEP?

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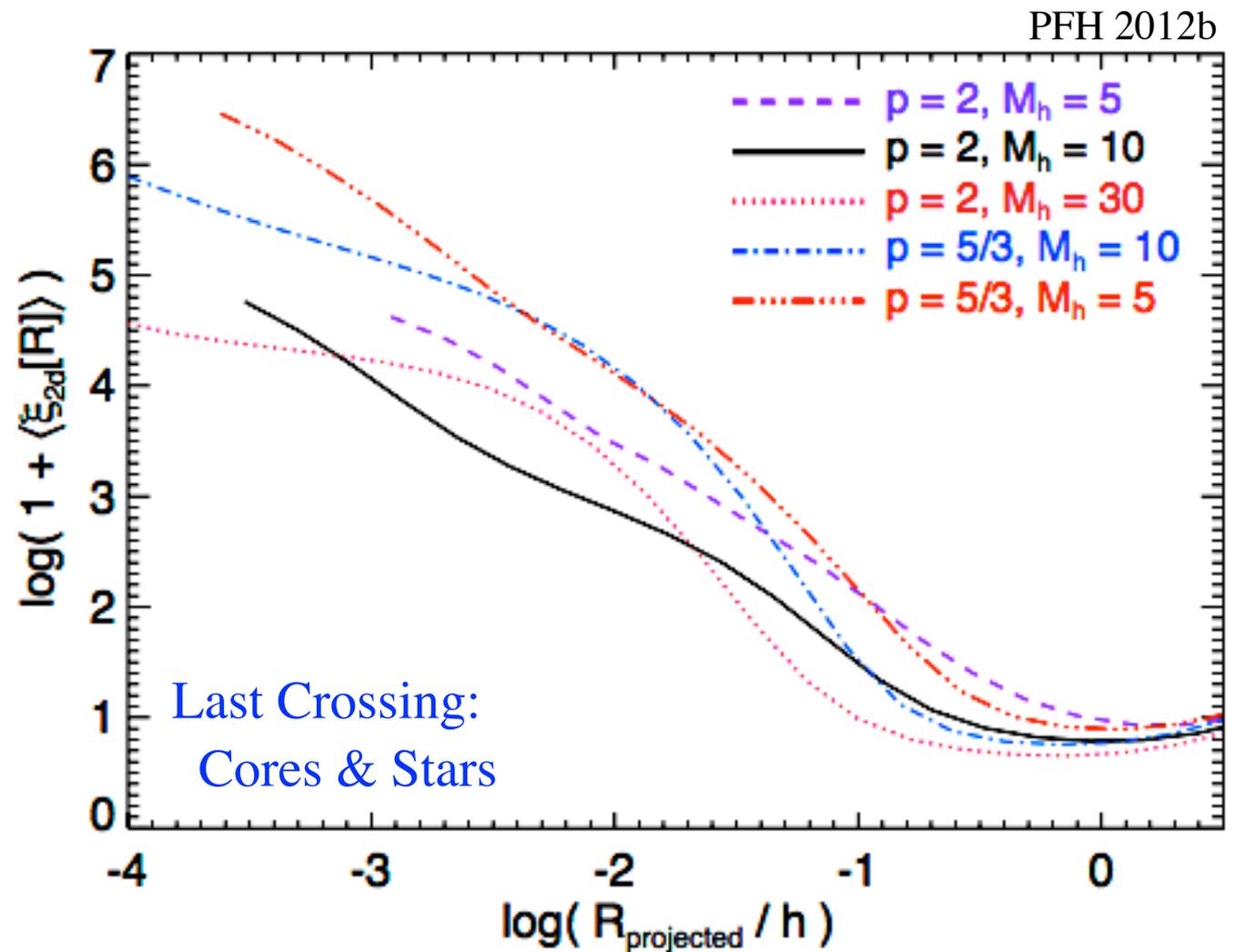
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Clustering

PREDICT N-POINT CORRELATION FUNCTIONS

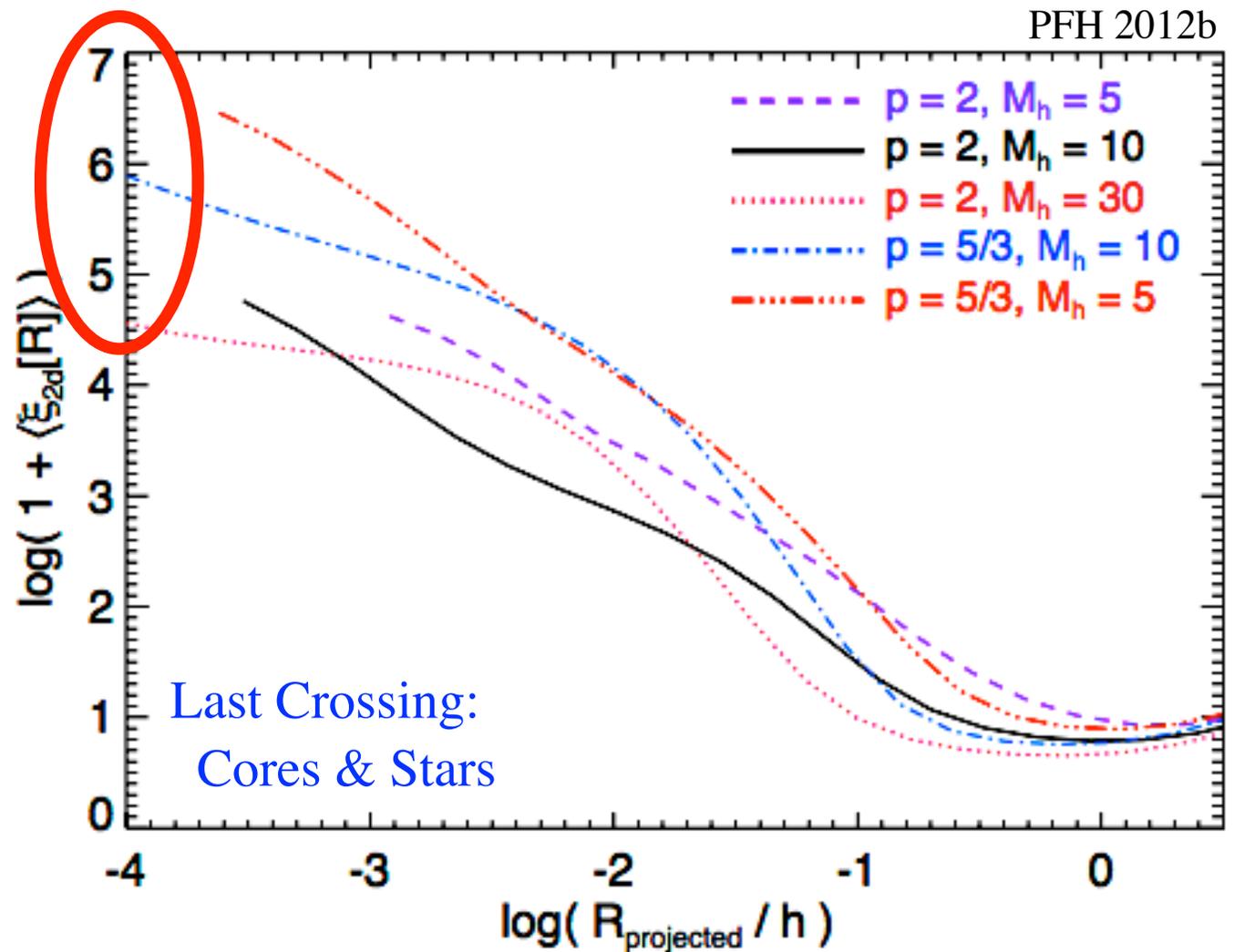
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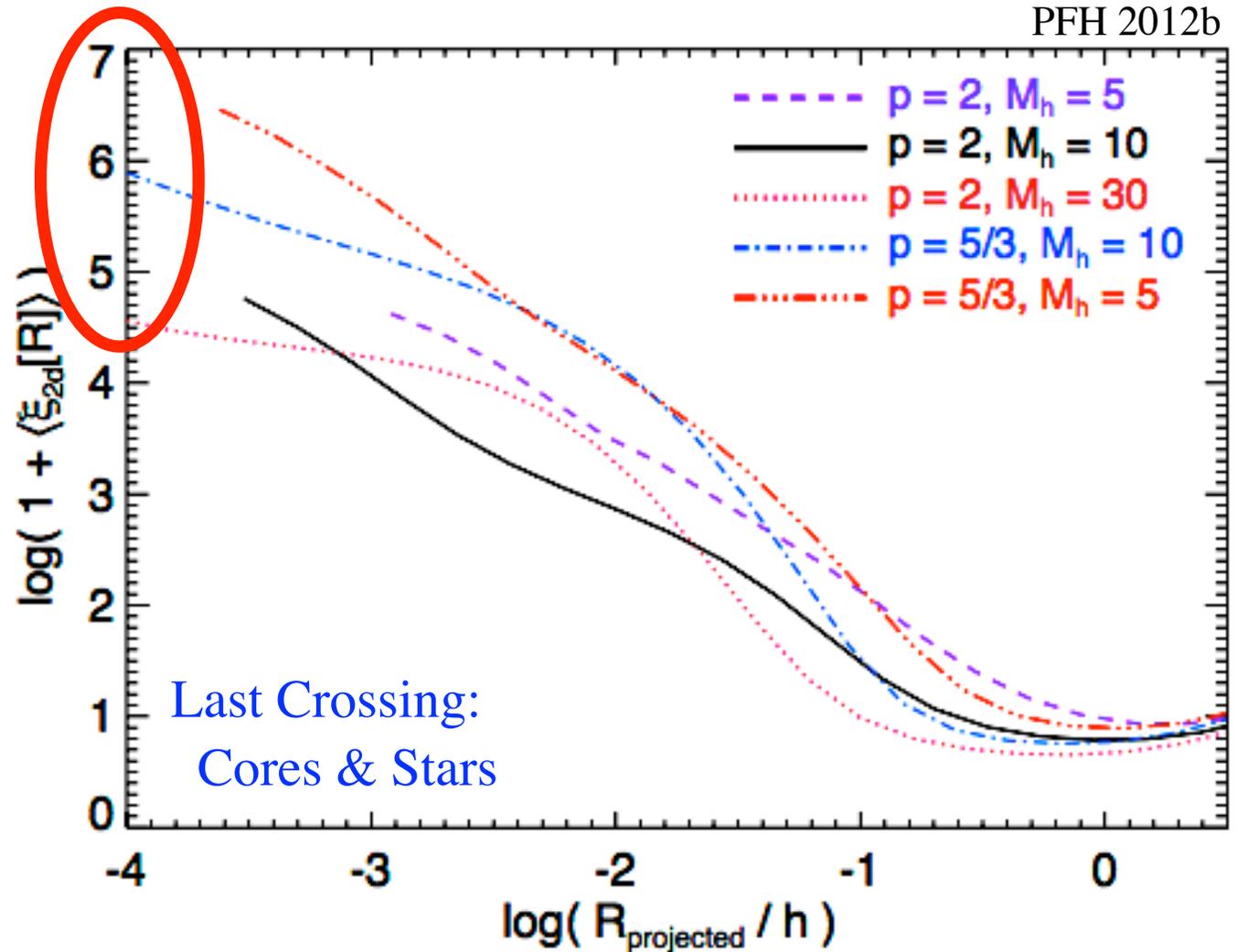
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Why Do Stars
Form in
Clusters?

$$S \sim \ln \mathcal{M}(k)^2 \\ \sim \ln r^{3-p}$$



Evolve the Fluctuations in Time

CONSTRUCT “MERGER/FRAGMENTATION” TREES

$$p(\delta | \tau) = \frac{1}{\sqrt{2\pi S (1 - \exp[-2\tau])}} \exp \left[-\frac{(\delta - \delta(t=0) \exp[-\tau])^2}{2 S (1 - \exp[-2\tau])} \right]$$

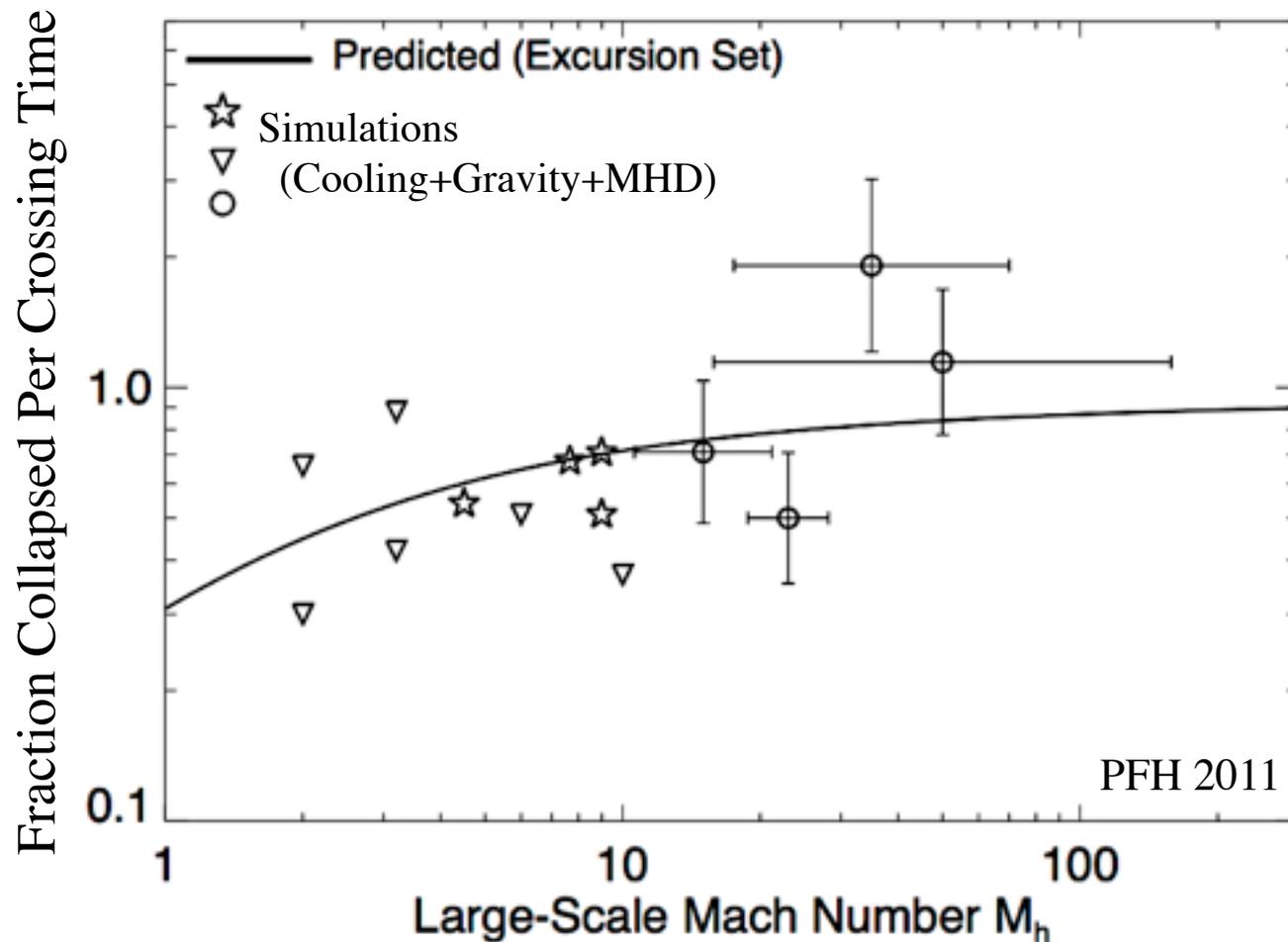
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