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Notes on Non-Ideal MHD Implementations for Molecular Clouds and Planetary/Stellar Disks

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1 OTHER PHYSICS OF INTEREST

There is a tremendous range of physics for these problems, much of which is available in GIZMO. For example:

(i) **Sink Particles, Star and Planet Formation/Growth/Accretion, and "Feeback" from those Sources:** GIZMO has a tremendous number of modules for all of these processes. For e.g. the cases of most interest for star and/or planet formation, see the description of the `SINGLE_STAR` modules (there are many).

(ii) **Micro-physical conduction & viscosity:** GIZMO can follow isotropic (Navier-Stokes) or anisotropic (Spitzer-Braginskii) conduction and viscosity with either user-specified coefficients or physically self-consistently calculated coefficients based on the local state of the gas. See the code description of these modules (`CONDUCTION`, `VISCOSITY`) for details.

(iii) **Turbulent "mixing," diffusion, conduction, viscosity, and other terms:** GIZMO likewise can follow un-resolved "turbulent mixing" with standard sub-grid-scale treatments of the local effective "eddy diffusivity" (qualitatively following e.g. Smagorinsky 1963). See the code description of these modules (`TURB_DIFF_X`) for details.

(iv) **Cosmic rays:** Most of the cooling/chemistry modules assume a uniform cosmic ray background by default, but if explicit cosmic ray transport is desired, see the `COSMIC_RAYS` modules.

(v) **Shearing Boxes, Driven Turbulent Boxes, and other special boundary conditions:** All available, see the documentation on different types of boxes and boundary conditions.

(vi) **Dust Dynamics:** GIZMO includes detailed models for the dynamics of dust grains, used extensively in various papers. Depending on which Config flags are enabled, we can explicitly follow drag, Lorentz, radiation pressure, and gravitational forces, as well as use the local dust grain density to compute dust-gas heating/cooling terms, opacities, and ionization states for the gas. With `COOLING` enabled, and appropriate flags set, the code will use the local chemistry to self-consistently calculate the drag forces including e.g. Lorentz, Coulomb, Stokes, and Epstein terms appropriate for an arbitrary power-law grain size distribution. See `GRAIN_FLUID` flags.

(vii) **Optically-Thick Cooling & Heating Physics:** At sufficiently high densities, gas becomes optically thick to its own cooling radiation. In simulations with explicit multi-frequency radiative transfer (the `RADTRANSFER` flags), this can be handled explicitly – these modules include sophisticated treatments of different dust, gas, and radiation temperatures. Without explicit radiation transport, but `COOLING` enabled, GIZMO approximates the optically-thick cooling limit following Rafikov (2007) (the relevant approximations are checked against exact results for protoplanetary disks in Chiang & Goldreich 1997; Allen et al. 2007; Zhu et al. 2012).[1]

2 NON-IDEAL MHD COEFFICIENTS

Non-ideal MHD is enabled by the flag `MHD_NON_IDEAL`. The numerical methods for the non-ideal terms are presented and tested in Hopkins (2017). If this flag is set but no cooling and/or chemistry is enabled, the coefficients are set by-hand. If, however, cooling and/or chemistry is enabled (with the `COOLING` master flag, regardless of which sub-module is used), then the relevant coefficients will be dynamically calculated based on the local plasma state of the gas.

Astrophysical non-ideal MHD effects relevant in the limits relevant for GMCs and protostellar/planetary disks (dense gas with very low ionized fraction, so the local "tight coupling" approximation applies) include Ohmic dissipation, the Hall effect, and ambipolar diffusion. All appear as diffusion operators in the induction equation; if we operator-split the ideal MHD term (solved whenever MHD is enabled), we have

$$\frac{d\mathbf{B}}{dt} = -\nabla \times \left[\eta_O \, \mathbf{J} + \eta_H \, \left(\mathbf{J} \times \hat{B} \right) - \eta_A \, \left(\mathbf{J} \times \hat{B} \right) \times \hat{B} \right] \quad (1)$$

where $\mathbf{J} = \nabla \times \mathbf{B}$. The diffusivities $\eta_{O,\,H,\,A}$ govern Ohmic resistivity, the Hall effect, and ambipolar diffusion, respec-

[1] Following Rafikov (2007), each gas element is treated as a midplane element embedded in an optically thick slab with surface density $\Sigma_{\rm slab}$ calculated according to a local Sobolev approximation. We assume it is in LTE, which relates the temperature change at midplane to surface radiation by way of the optical depth, using the equations in Appendix A therein. To good approximation, the heating/cooling rate per element is "capped" at the maximum value $|dE/dt| = \sigma\, T_{\rm mid}^4\, (\Sigma_{\rm slab}/\mu)^{-1}/(1+\kappa_R\,\Sigma_{\rm slab})$, where $T = T_{\rm mid}$ is the element or "midplane" temperature, μ the mean molecular weight, and κ_R the Rosseland mean opacity. We calculate κ_R at low temperatures from the tables in Semenov et al. (2003); at high temperatures ($>1500\,$K) we explicitly tabulate the Thompson, molecular-line, H^- ion, Kramers, and e^- conductivity terms as in Badnell et al. 2005.

tively, and are given by the general expressions:

$$\eta_O \equiv \frac{c^2}{4\pi}\,\frac{1}{\sigma_O} \tag{2}$$

$$\eta_H \equiv \frac{c^2}{4\pi}\,\frac{\sigma_H}{\sigma_H^2+\sigma_P^2} \tag{3}$$

$$\eta_A \equiv \frac{c^2}{4\pi}\,\left[\frac{\sigma_P}{\sigma_H^2+\sigma_P^2}-\frac{1}{\sigma_O}\right] \tag{4}$$

$$\sigma_O \equiv \frac{e\,c}{B}\,\sum_j\,n_j\,|Z_j|\,\beta_j \tag{5}$$

$$\sigma_H \equiv \frac{e\,c}{B}\,\sum_j\,\frac{n_j\,Z_j}{1+\beta_j^2} \tag{6}$$

$$\sigma_P \equiv \frac{e\,c}{B}\,\sum_j\,\frac{n_j\,|Z_j|\,\beta_j}{1+\beta_j^2} \tag{7}$$

$$\beta_j \equiv \frac{|Z|_j\,e\,B}{m_j\,c\,\nu_j} \tag{8}$$

where $\sigma_{O,\,H,\,P}$ are the Ohmic, Hall, and Pedersen conductivities, the index j sums over the different relevant species in the fluid (here ions, electrons, neutrals, and dust grains, i, e, n, g, respectively), $Z_i=+1$, $Z_e=-1$, $Z_n=0$, and Z_g are the mean neutral/electron/ion/grain charges, $m_{i,\,e,\,n,\,g}$ the ion/electron/neutral/grain mass ($m_n=\mu\,m_p$ is calculated using the appropriate mean molecular weight μ for the element abundances, temperature, and molecular fraction determined in the cooling chemistry, and $m_g=(4\pi/3)\,a_g^3\,\bar{\rho}_g$ with a_g the grain radius and $\bar{\rho}_g$ the internal grain material density), $n_{i,\,e,\,n,\,g}$ the number density of each species (with mean number density $n=\rho/(\mu\,m_p)$ and $n_g=m_n\,f_{dg}\,n/m_g$ with f_{dg} the dust-to-gas ratio by mass), m_p is the proton mass, e the electron charge, c the speed of light, and $B=|\mathbf{B}|$ the magnitude of the magnetic field.

At the densities and temperatures of interest, the collision frequencies $\nu_{e,\,i,\,g}$ are given by:

$$\nu_e = 0.051\,n_e\,T_{100}^{-1.5}\,\mathrm{cm^3\,s^{-1}} + \frac{\rho_n\,\left[5.15\,T_{100}^{0.65}+1.06\,T_{100}^{0.5}\right]}{10^9\,(m_n+m_e)}\,\mathrm{cm^3\,s^{-1}} \tag{9}$$

$$\nu_i = 0.051\,\frac{\rho_e}{\rho_i}\,n_e\,T_{100}^{-1.5}\,\mathrm{cm^3\,s^{-1}} + \frac{\rho_n\,\left[1.91\,\left(\frac{m_p}{\mu_{\rm i-H_2}}\right)^{\frac{1}{2}}+0.31\,\left(\frac{m_p}{\mu_{\rm i-He}}\right)^{\frac{1}{2}}\right]}{10^9\,(m_n+m_i)}\,\mathrm{cm^3\,s^{-1}} \tag{10}$$

$$\nu_g = \frac{\pi\,a_g^2\,\delta_{gn}\,\rho_n}{(m_n+m_g)}\,\left(\frac{128\,k_B T}{9\pi\,m_n}\right)^{1/2} \tag{11}$$

where the terms in ν_e and ν_i represent electron-ion, electron/ion-H_2, and electron/ion-He collisions, respectively, with assumed H, He abundances ≈ 0.76, 0.24 (changing this has negligible effects), and $T_{100}\equiv T/100\,K$. In ν_g, $\delta_{gn}\approx 1.3$ is the Epstein coefficient for spherical grains (Liu et al. 2003).

We follow Wurster et al. (2016), who themselves follow Draine & Sutin (1987) and assume the grains have a non-evolving size distribution with primarily collisional+cosmic ray charging, and that the system obeys global charge neutrality and local ionization equilibrium: this allows us to calculate

$$Z_g \equiv -\psi\,\frac{a_g\,k_B\,T}{e^2} \tag{12}$$

$$\psi = \alpha\,\left(\exp\left(\psi\right)-\frac{(m_i/m_e)^{1/2}}{1+\psi}\right) \tag{13}$$

$$\alpha \equiv \frac{\zeta\,e^2\,m_e^{1/2}\,m_g^2}{(8\pi)^{1/2}\,a_g^3\,f_{dg}^2\,(k_B T)^{3/2}\,m_n^2\,n} \tag{14}$$

$$n_i = \frac{\zeta\,n}{k_{ig}\,n_g} \tag{15}$$

$$n_e = \frac{\zeta\,n}{k_{eg}\,n_g} \tag{16}$$

$$k_{ig} \equiv \pi\,a_g^2\,\left(\frac{8\,k_B\,T}{\pi\,m_i}\right)^{1/2}\,(1+\psi) \tag{17}$$

$$k_{eg} \equiv \pi\,a_g^2\,\left(\frac{8\,k_B\,T}{\pi\,m_e}\right)^{1/2}\,\exp\left(-\psi\right) \tag{18}$$

where k_B is the Boltzmann constant, T is the gas kinetic temperature, $k_{ig,\,eg}$ are the coefficients for ion-grain and electron-grain collisions respectively, and ζ is the local cosmic ray background ionization rate.

For the grains, we assume an effective size $a_g=0.1\,\mu m$, material density $\bar{\rho}_g = 3\,\mathrm{g\,cm^{-3}}$ (typical of both silicate and carbonaceous grains), and constant dust-to-metals ratio $f_{dg}=0.01\,(Z/Z_\odot)$. For reasonable values, these choices only weakly influence the coefficients $\eta_{O,H,A}$, compared to the values of n, T, and ζ. We assume that at low temperatures and high densities, the ions are dominated by Mg, so $m_i=24.3\,m_p$ (at high temperatures, lighter ions become important, but this only changes the diffusivities where they are dynamically irrelevant; explicitly assuming $m_i=m_p$ above 200 K, for example, we obtain identical conclusions). This completely determines the non-ideal MHD coefficients, given the MHD state of the gas and ζ. If cosmic rays are being evolved explicitly, ζ is calculated from the local cosmic ray energy density (assuming a universal shape of the spectral distribution for cosmic rays); otherwise we assume a fixed Milky Way (really solar-circle) like background, $\zeta\approx 10^{-17}\,\mathrm{s^{-1}}$, but this can easily be modified where it is assumed in `gradients.c`.

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