# GIZMO: Cosmic Ray Notes

Philip F. Hopkins[1]

[1] *TAPIR, Mailcode 350-17, California Institute of Technology, Pasadena, CA 91125, USA. E-mail:phopkins@caltech.edu*

**ABSTRACT**

Description of Cosmic Ray (CR) implementation, in the code GIZMO.

**Key words:** methods: numerical — hydrodynamics — cosmology: theory — elementary particles — dark matter

## 1 NUMERICAL METHODS

### 1.1 Non Cosmic-Ray Physics

The simulations in this paper were run with the multi-physics code GIZMO[1] (Hopkins 2015), in its meshless finite-mass MFM mode. This is a mesh-free, finite-volume Lagrangian Godunov method which provides adaptive spatial resolution together with conservation of mass, energy, momentum, and angular momentum, and the ability to accurately capture shocks and fluid mixing instabilities (combining advantages of both grid-based and smoothed-particle hydrodynamics methods). We solve the equations of ideal magneto-hydrodynamics (MHD), as described and tested in detail in Hopkins & Raives (2016); Hopkins (2016), with fully-anisotropic Spitzer-Braginskii conduction and viscosity and other diffusion operators implemented as described in Hopkins (2017). The simulations include the physics of cooling, star formation, and stellar feedback from the FIRE-2 version of the Feedback in Realistic Environments (FIRE) project, described in detail in Hopkins et al. (2017a), but we briefly summarize these before describing the new physics – our treatment of cosmic rays – in more detail. Gravity is solved for gas and collisionless (stars and dark matter) species with adaptive Lagrangian force softening. Gas cooling is followed from $T = 10 - 10^{10}\,\mathrm{K}$ including free-free, Compton, metal-line, molecular, fine-structure, dust collisional, photo-electric and photo-ionization heating by both local sources and a uniform meta-galactic background, and self-shielding. Gas is turned into stars using a sink-particle prescription: gas which is locally self-gravitating at the resolution scale following Hopkins et al. 2013, self-shielding/molecular following Krumholz & Gnedin 2011, Jeans unstable, and denser than $n_{\rm crit} > 1000\,\mathrm{cm}^{-3}$ is converted into star particles on a free-fall time. Star particles are then treated as single-age stellar populations with all IMF-averaged feedback properties calculated from STARBURST99 (Leitherer et al. 1999) assuming a Kroupa (2001) IMF. We then explicitly treat feedback from SNe (both Types Ia and II), stellar mass loss (O/B and AGB winds), and radiation (photo-ionization and photo-electric heating and UV/optical/IR radiation pressure), with implementations at the resolution-scale described in Hopkins et al. (2017a) and Hopkins et al. (2017b).

### 1.2 Cosmic Rays

The implementation of cosmic ray (CR) physics in GIZMO includes fully-anisotropic cosmic ray transport with streaming and advection/diffusion, CR cooling (hadronic and Compton, adiabatic, and streaming losses), injection in SNe shocks, and CR-gas coupling. The CRs are treated as an ultra-relativistic fluid (adiabatic index $\gamma_{\rm cr} = 4/3$) in a "single bin" approximation.[2]

Integrating over the CR distribution function and spectrum, the standard equation for the CR energy density $e_{\rm cr}$ is (McKenzie & Voelk 1982):

$$\frac{\partial e_{\rm cr}}{\partial t} + \nabla\cdot\mathbf{F}_{\rm cr} = \langle\mathbf{v}_{\rm cr}\rangle\cdot\nabla P_{\rm cr} + S_{\rm cr} - \Gamma_{\rm cr} \tag{1}$$

where $P_{\rm cr} \equiv (\gamma_{\rm cr}-1)\,e_{\rm cr}$ is the CR pressure; $S_{\rm cr} \equiv \dot{e}_{{\rm cr},\,*}$ the source term; $\Gamma_{\rm cr} \equiv e_{\rm cr}\,\tilde{\Lambda}_{\rm cr}$ the CR sink/loss (or "cooling") term; and $\langle\mathbf{v}_{\rm cr}\rangle \equiv \mathbf{v}_{\rm gas} + \mathbf{v}_{\rm st}$ is the bulk CR advection velocity, itself de-composed into the gas velocity $\mathbf{v}_{\rm gas}$ and the "streaming velocity" $\mathbf{v}_{\rm st}$. Here $\mathbf{F}_{\rm cr}$ is the CR energy flux which is usually de-composed into $\mathbf{F}_{\rm cr} \equiv \langle\mathbf{v}_{\rm cr}\rangle\,(e_{\rm cr}+P_{\rm cr}) + \mathbf{F}_{\rm di}$, where the first term represents advection and $\mathbf{F}_{\rm di}$ is a diffusive-like flux.

#### 1.2.1 *Transport: Advection & Streaming*

In our method, each mesh-generating point (which defines the gas resolution "elements") represents a finite-volume domain that moves with the fluid velocity $\mathbf{v}_{\rm gas}$ in a quasi-Lagrangian fashion. After operator-splitting the source/injection and loss/cooling terms, it is convenient to re-write the advection and streaming terms in the following Lagrangian, finite-volume form (see e.g. Uhlig et al. 2012):

$$\frac{DE_{\rm cr}}{Dt} = \int_\Omega d^3\mathbf{x}\,\Big\{ -P_{\rm cr}\,(\nabla\cdot\mathbf{v}_{\rm gas}) + \mathbf{v}_{\rm st}\cdot\nabla P_{\rm cr} - \nabla\cdot[\mathbf{v}_{\rm st}(e_{\rm cr}+P_{\rm cr}) + \mathbf{F}_{\rm di}]\Big\} \tag{2}$$

where $D/Dt = \partial/\partial t + \mathbf{v}_{\rm gas}\cdot\nabla$ is the Lagrangian derivative co-moving with the gas, and $E^i_{\rm cr} = \int_{\Omega_i} e_{\rm cr}\,d^3\mathbf{x}$ is the conserved total CR energy in the finite-volume domain $\Omega_i$ belonging to element $i$. Pure advection with the gas is therefore automatically handled. In cosmological simulations, the Hubble flow is included in $\nabla\cdot\mathbf{v}_{\rm gas}$.

The $P_{\rm cr}(\nabla\cdot\mathbf{v}_{\rm gas})$ term represents adiabatic changes to the CR energy via compression/expansion (the "PdV work"), which exchanges energy with gas. We will refer to this as the "adiabatic" term throughout.[3]

The $\mathbf{v}_{\rm st}\cdot\nabla P_{\rm cr}$ term represents "streaming losses," which again exchange energy with the gas. As CRs stream, instabilities excite high-frequency Alfven waves (frequency of order the gyro frequency, well below our simulation resolution limits; see e.g. Wentzel 1968; Kulsrud & Pearce 1969) which are damped and thermalize their energy effectively instantly (compared to our simulation timescales).[4]

Finally, the $\int_\Omega d^3\mathbf{x}\,\nabla\cdot[\mathbf{v}_{\rm st}(e_{\rm cr}+P_{\rm cr}) + \mathbf{F}_{\rm di}]$ term does not change the total CR energy, but represents flux of energy between resolution elements. This can be transformed via Stokes's law into a

[1] A public version of GIZMO is available at `http://www.tapir.caltech.edu/~phopkins/Site/GIZMO.html`

[2] One can think of this either as evolving only the CR energy density at the energies $\sim$GeV which dominate the CR pressure, or, equivalently, approximating the CR energy spectrum as having a universal shape at all positions.

[3] To ensure manifest energy conservation, this is solved when the mesh positions are updated. Using the exact discrete change in the domain volume $\Delta V_i = \int dt \int_{\Omega_i} d^3\mathbf{x}\,(\nabla\cdot\mathbf{v}_{\rm gas})$, we have $\Delta E_{\rm cr} = -P_{\rm cr}\,\Delta V_i$. This is removed from the total energy equation after the hydrodynamic Riemann problem is solved, to determine the total gas energy update.

[4] With the streaming velocity defined below, the streaming loss term can be written $DE_{\rm cr}/Dt = -E_{\rm cr}/\tau_{\rm st}$ with $\tau_{\rm st}^{-1} = (\gamma_{\rm cr}-1)\,|\hat{B}\cdot\hat{\nabla}e_{\rm cr}|^2\,|v_{\rm st}\,\nabla e_{\rm cr}|/e_{\rm cr}$. When this is updated the resulting energy lost $\Delta E_{\rm cr} = \int dt\,\tau_{\rm st}^{-1}\,E_{\rm cr}$ is added to the gas thermal energy.

surface integral, $\int_{\partial\Omega} d\mathbf{A}\cdot[\mathbf{v}_{\rm st}(e_{\rm cr}+P_{\rm cr})+\mathbf{F}_{\rm di}]$, which is then solved via our usual second order-accurate, finite-volume Godunov MFM method (in a manner identical to the hydrodynamic equations, see Hopkins (2015) for details).

We explicitly evolve the conserved quantities $E_{\rm cr}$ and total gas energy $E_{\rm gas}$ which are exchanged (either between gas elements or one another), ensuring manifest total energy conservation.

### 1.2.2 The Streaming Velocity

CRs stream at some speed $v_{\rm st}$ down the local CR phase-space density gradient (which is equivalent in our single-bin approximation to CR pressure or energy density gradient), projected along the magnetic field lines, i.e. $\mathbf{v}_{\rm st}\equiv -v_{\rm st}\,\hat{B}\,(\hat{B}\cdot\hat{\nabla}P_{\rm cr})$ where $\hat{\nabla}P_{\rm cr}=\hat{\nabla}e_{\rm cr}=(\nabla P_{\rm cr})/|\nabla P_{\rm cr}|=(\nabla e_{\rm cr})/|\nabla e_{\rm cr}|$ is the direction of the CR pressure/energy density gradient.

It is generally believed that micro-scale instabilities limit the streaming velocity to approximately the sound speed $c_s$ (in the weak-field, plasma $\beta\gg 1$, regime) or Alfven speed $v_A$ in the low-$\beta$ limit (see Skilling 1971; Holman et al. 1979, or more recently Kulsrud 2005; Yan & Lazarian 2008; Enßlin et al. 2011), so we interpolate between the two by adopting $v_{\rm st}^2=c_s^2+v_A^2$. In our simulations with streaming disabled, we simply set $v_{\rm st}=0$.

### 1.2.3 Diffusive Transport Terms

It is common in the literature to treat $\mathbf{F}_{\rm di}$ in the "zero-th moment" expansion, i.e. approximate it as an anisotropic scalar diffusion with $\mathbf{F}_{\rm di}=-\kappa_{\rm cr}\,\hat{B}\,(\hat{B}\cdot\nabla e_{\rm cr})$. However at high resolution this is problematic for several reasons: (1) it imposes a quadratic timestep criterion ($\Delta t<C_{\rm cour}\,\Delta x^2/\kappa_{\rm cr}$, where $\Delta x$ is the resolution and $C_{\rm cour}$ the Courant factor) which becomes very small; (2) it implies un-physical super-luminal CR transport when the gradient-scale length $e_{\rm cr}/|\nabla e_{\rm cr}|\gtrsim\Delta x$ becomes smaller than $\kappa_{\rm cr}/c\sim 1\,{\rm pc}\,(\kappa_{\rm cr}/10^{29}\,{\rm cm^2\,s^{-1}})$ (typical values considered in this paper); (3) it cannot smoothly handle the transition between streaming and diffusion-dominated regimes; and (4) it encounters the usual difficulties with anisotropic diffusion operators in moving-mesh codes described in Hopkins (2017) (including e.g. difficulty if CRs are "trapped" at local maxima).

We therefore instead follow Jiang & Oh (2018) and solve the two-moment expansion of the Vlasov equation, i.e. explicitly evolve the flux equation (the first moment equation):

$$\frac{1}{\tilde{c}^2}\,\frac{\partial\mathbf{F}_{\rm di}}{\partial t}+\nabla\cdot\mathbb{P}_{\rm cr}=-\frac{(\gamma_{\rm cr}-1)}{\kappa_{\rm cr}}\,\mathbf{F}_{\rm di} \tag{3}$$

where we take $\nabla\cdot\mathbb{P}_{\rm cr}=(\hat{B}\otimes\hat{B})\cdot(\nabla P_{\rm cr})=(\gamma_{\rm cr}-1)\,\hat{B}\,(\hat{B}\cdot\nabla e_{\rm cr})$, and $\kappa_{\rm cr}$ is the same diffusion coefficient. Trivially, we see that in steady-state and/or when $\tilde{c}$ is large, or $\Delta t\gg\kappa_{\rm cr}/\tilde{c}^2$ (or on spatial scales $\gg\kappa_{\rm cr}/\tilde{c}$), this equation becomes $\mathbf{F}_{\rm di}\approx-\kappa_{\rm cr}\,\hat{B}\,(\hat{B}\cdot\nabla e_{\rm cr})$, and we recover the usual diffusion equation. However, this smoothly limits the maximum bulk transport velocity of the CRs to $\tilde{c}$, and makes the timestep criterion $\Delta t<C_{\rm cour}\,\Delta x/\tilde{c}$.[5]

For true micro-physical CR motion, $\tilde{c}\approx c$, the speed of light. However, for our purposes in these simulations – where we only capture bulk CR properties in the fluid limit – it is more convenient to consider this a numerical "nuisance" parameter. As noted above, in steady-state, solutions for the CR properties are completely independent of $\tilde{c}$. Thus we will converge to the same solutions, regardless of $\tilde{c}$, provided it is set to some value faster than the *actual* bulk flow speeds realized in our simulations. We have experimented extensively with this and find that, for the simulations here, values $\tilde{c}\sim 500-1000\,{\rm km\,s^{-1}}$ are sufficient to give converged results.

In some experiments in this paper we wish to compare the results using the simpler pure-diffusion (zeroth-moment) approximation: we then simply assume $\mathbf{F}_{\rm di}\rightarrow-\kappa_{\rm cr}\,\hat{B}\,(\hat{B}\cdot\nabla e_{\rm cr})$ and solve the anisotropic diffusion equation (with the stricter Courant condition) as described in Hopkins (2017).

It is worth noting that our CR treatment is akin to the first-moment or "M1" moments-based method for radiation hydrodynamics, with the "reduced speed of light" $\tilde{c}$ (Levermore 1984), while the "pure diffusion" approximation is akin to flux-limited diffusion (without the limiter).

### 1.2.4 Sources & Injection

We assume CR injection from SNe, with a fixed fraction $\epsilon_{\rm cr}$ ($=0.1$, in our "default" cases) of the initial ejecta energy ($\Delta E_{\rm cr}=\epsilon_{\rm cr}\,E_{\rm SNe}$ with $E_{\rm SNe}\approx 10^{51}$ erg) of every SNe explosion going into CRs. SNe explosions inject thermal and kinetic energy into neighboring gas resolution elements according to the algorithm described in detail in Hopkins et al. (2017b); we therefore reduce the coupled energy by $1-\epsilon_{\rm cr}$ and inject the remaining $\epsilon_{\rm cr}$ energy alongside the metals, mass, and thermal+kinetic energy using the same relative "weights" to determine the CR energy assigned to each neighbor. Likewise the CR flux is updated assuming the CRs free-stream at injection ($\mathbf{F}_{\rm di}\rightarrow\mathbf{F}_{\rm di}+\Delta\mathbf{F}_{\rm di}$ with $\Delta\mathbf{F}_{\rm di}=\Delta e_{\rm cr}\,\tilde{c}\,\hat{r}$ from the source). The injection is therefore operator-split and solved discretely (associated with each SNe).

### 1.2.5 Hadronic & Coulomb Losses ("Cooling")

We adopt the estimate for combined hadronic plus Coulomb losses from Völk et al. (1996) and Ensslin et al. (1997) as synthesized and updated in Guo & Oh (2008):

$$\tilde{\Lambda}_{\rm cr}=7.51\times 10^{-16}\,{\rm s^{-1}}\,(1+0.22\,\tilde{n}_e)\left(\frac{n_{\rm H}}{\rm cm^{-3}}\right) \tag{4}$$

where $n_{\rm H}$ is the hydrogen number density and $\tilde{n}_e$ is the number of free electrons per hydrogen nucleus. Following that work we assume $\sim 1/6$ of the hadronic losses (the term independent of $\tilde{n}_e$) and all Coulomb losses are thermalized and go into heating the gas, adding a volumetric gas heating term

$$Q_{\rm gas}=7.51\times 10^{-16}\,{\rm s^{-1}}\,e_{cr}\,(0.17+0.22\,\tilde{n}_e)\left(\frac{n_{\rm H}}{\rm cm^{-3}}\right) \tag{5}$$

The remaining CR losses are assumed to escape in the form of $\gamma$-rays and other products to which the gas is optically thin.

The loss and heating terms are operator-split and solved together with all other gas heating/cooling terms with our usual fully-implicit cooling scheme described in Hopkins et al. (2017a).

### 1.2.6 Cosmic Ray Pressure on Gas

When solving the gas equations-of-motion (the Riemann problem between neighboring fluid elements), we include the CR pressure in the local strong-coupling approximation, i.e. take the total pressure $P=P_{\rm gas}+P_{\rm cr}$. The effective sound speed of the two-fluid mixture is then given by $c_s^2=\partial P/\partial\rho=(c_s^2)_{\rm gas}+\gamma_{\rm cr}\,P_{\rm cr}/\rho$, where $\rho$ is the gas density.

### 1.2.7 "Isotropic" or "Pure-Hydrodynamic" Runs

By default, we solve the CR equations coupled to the ideal MHD equations, and treat the CR transport (streaming and advection/diffusion) fully anisotropically. However in some tests below we consider "isotropic," pure-hydrodynamic cases. In these cases we disable MHD so simply solve the hydrodynamic equations, and have $v_A=0$ in the streaming velocity, and replace $\hat{B}$ wherever it appears above (representing projection of motion along field lines) with $\hat{\nabla}P_{\rm cr}$.

[5] We adopt $C_{\rm cour}=0.4$ throughout, and have validated stability (as expected) for this value. $\Delta x$ in the Courant condition is defined in the same manner as Hopkins et al. (2017a) as the local mean inter-particle separation (i.e. the equivalent of the grid spacing in a regular-grid code), $\Delta x\equiv(m/\rho)^{1/3}$.

### *1.2.8 The Diffusion Coefficient*

The only remaining un-specified parameter in the CR treatment is the effective diffusion coefficient $\kappa_{\rm cr}$. Our method is sufficiently flexible that this can vary locally, in principle. However since there is still substantial debate about the physical nature and scalings of this diffusivity, and substantial empirical uncertainty regarding its value (even in the Milky Way, only order-of-magnitude constraints exist for the "effective" galaxy-averaged diffusivity of the $\sim$GeV CRs which dominate the energy density), we will take $\kappa_{\rm cr} =$ constant in individual simulations and systematically vary this value.

## ACKNOWLEDGMENTS

Support for PFH was provided by an Alfred P. Sloan Research Fellowship, NSF Collaborative Research Grant #1715847 and CAREER grant #1455342. Numerical calculations were run on the Caltech compute cluster "Wheeler," allocations from XSEDE TG-AST130039 and PRAC NSF.1713353 supported by the NSF, and NASA HEC SMD-16-7592.

## REFERENCES

Ensslin T. A., Biermann P. L., Kronberg P. P., Wu X.-P., 1997, ApJ, 477, 560
Enßlin T., Pfrommer C., Miniati F., Subramanian K., 2011, A&A, 527, A99
Guo F., Oh S. P., 2008, MNRAS, 384, 251
Holman G. D., Ionson J. A., Scott J. S., 1979, ApJ, 228, 576
Hopkins P. F., 2015, MNRAS, 450, 53
Hopkins P. F., 2016, MNRAS, 462, 576
Hopkins P. F., 2017, MNRAS, 466, 3387
Hopkins P. F., Raives M. J., 2016, MNRAS, 455, 51
Hopkins P. F., Narayanan D., Murray N., 2013, MNRAS, 432, 2647
Hopkins P. F., et al., 2017a, MNRAS, submitted, arXiv:1702.06148,
Hopkins P. F., et al., 2017b, MNRAS, in press, arXiv:1707.07010,
Jiang Y.-F., Oh S. P., 2018, ApJ, 854, 5
Kroupa P., 2001, MNRAS, 322, 231
Krumholz M. R., Gnedin N. Y., 2011, ApJ, 729, 36
Kulsrud R. M., 2005, Plasma physics for astrophysics. Princeton, N.J. : Princeton University Press
Kulsrud R., Pearce W. P., 1969, ApJ, 156, 445
Leitherer C., et al., 1999, ApJS, 123, 3
Levermore C. D., 1984, Journal of Quantitative Spectroscopy and Radiative Transfer, 31, 149
McKenzie J. F., Voelk H. J., 1982, A&A, 116, 191
Skilling J., 1971, ApJ, 170, 265
Uhlig M., Pfrommer C., Sharma M., Nath B. B., Enßlin T. A., Springel V., 2012, MNRAS, 423, 2374
Völk H. J., Aharonian F. A., Breitschwerdt D., 1996, Space Sci. Rev., 75, 279
Wentzel D. G., 1968, ApJ, 152, 987
Yan H., Lazarian A., 2008, ApJ, 673, 942