Technical document on Challenge1B1.3

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I. INTRODUCTION

We have simplified the waveform model by decomposing it in the form:

$$h \sim \sum A_{mn}(t) e^{i(n\Phi + 2\tilde{\gamma} + m\alpha)}$$

where Φ, γ and α are the orbital phase, perihelion precession phase and orbital plane precession phase respectively. This expression was then approximated by truncating the series in n and expanding the amplitudes in eccentricity and further truncating these series. The detector modulation is included using the full response function but with a low order (linear) interpolation to construct the TDI streams. The overlap between the approximated signal and the true signal (produced by synthetic LISA) is typically above 90%. The waveform generation code is not fully optimized and takes about 1 min/waveform for a $10M_\odot$ plus $10^6 M_\odot$ inspiral.

For the search, we primarily used a stochastic Metropolis-Hastings search that employed several proposal distributions and included simulated annealing. The search was carried out in several steps which we describe in the following.

II. FIRST STEP

The first step involved running a Metropolis-Hastings search that had multiple chains of two different kinds:

- A search that draws proposal parameter sets at plunge uniformly from the prior range and then generates the waveform by integrating backwards from the plunge. This search used two types of waveform filter: six-month long waveforms ending at plunge and six-month long waveforms ending six months before plunge. These chains tended to lock quickly onto portions of the signal, from which we were able to readily identify the harmonic composition of the true signal. This is discussed further below.

- A Metropolis-Hastings Monte-Carlo search employing simulated annealing and time annealing. This was the same method we used for Challenge 1.3 and is described in the technical document for that Challenge. For Round 1.3B, we changed the implementation of the two annealing schemes but the basic idea was the same. The plunge time is constrained automatically using time slides. These chains are long and run slowly since the main proposal distribution uses normal jumps based on a Fisher Matrix that must be computed numerically.

In the previous challenge we found that the search chains could quite easily find one or two dominant harmonics, but the chain would then get stuck, locked onto those harmonics and subsequently merely refining the extrinsic parameters to improve the fit. To constrain the source parameters, we must find correctly the three fundamental frequencies of the signal and their first (and second for $M > 5 \times 10^6$) derivatives. The Metropolis-Hastings Monte-Carlo (MHMC) chains are able to find a few harmonics very well but have problems to subsequently move away from these points and find the other (missing) harmonics. On the other hand, the search using uniform jumps is able to find other harmonics since different chains can lock on to *different* harmonics.

III. SECOND STEP

Combining the uniform jumps and MHMC chains helps us to identify several harmonics. We identify harmonics by taking the parameters of each point in the chain that is above a certain SNR threshold and computing the overlap of each harmonic of that template with the data. The SNR accumulation as a function of frequency allows us to identify which harmonics of the template have matched the signal's harmonics and over which time range. We have found that the uniform jumps can give us more information than the MHMC chains at this stage, since they find more harmonics, albeit at lower SNR and for shorter duration. We found that this harmonic analysis also provided a decent constraint on plunge time.

IV. THIRD STEP

As a third step we try to make use of the harmonic information by running two other types of Monte Carlo chains. The first type of chain is constrained to only make jumps that preserve the frequency composition of the template at a specified time, using the constraints derived at the second stage. These chains also enforce a constraint on the plunge time. This search helps us to find points in parameter space which lock to (almost) all the signal harmonics and to adjust the extrinsic parameters to build up significant SNR.

This constrained search would be the last step if our estimate of the frequencies at the given time was exact (since the chains will evolve to find the first and second derivatives). In principle we can include additional constraints on the derivatives of the frequencies if we are able to measure the same harmonics at different instances of time.

In practice, the frequency determination at step two is not exact, which limits how close to the true parameters the chain can get. For this reason, we also run another set of chains that take the initial guess for the fundamental frequencies and the phase of the dominant harmonic at the fiducial time and then make small jumps in the fundamental frequencies and in the derivative of the dominant frequency, while maintaining the constraint on plunge time. The idea of this second search is to refine the frequency estimates, which can then be fed back into the constrained search described in the preceding two paragraphs. Due to a lack of time this was implemented hurriedly and runs extremely slowly. At the time of submission we knew from the harmonic analysis that we had not reached the true parameter points, but we submitted the highest SNR points obtained for each source by that time. We hope that with further optimization and more run-time, the chains will reach the true point.