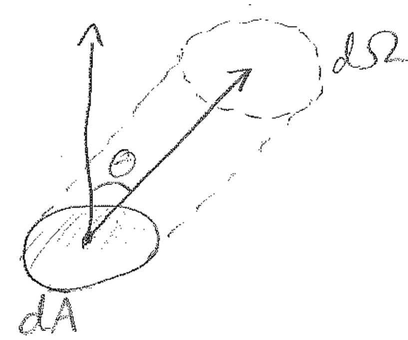
Ay 123 Lecture XIV Stellar Atmospheres

Stellar spectrum encodes information about the atmospheric properties of the star, in particular

- T_{eff}
- surface gravity
- composition (metal abundances)

Extracting info requires modeling of radiative transfer

14.1 Definitions



$d\Omega = \sin\theta \, d\theta \, d\phi$

- Specific intensity, I_ν

  erg cm^{-2} Hz^{-1} srad^{-1} s^{-1}

  $dE_\nu = I_\nu \cos\theta \, dA \, d\nu \, d\Omega \, dt$

  "Radiance"

- Mean intensity, J_ν

  I_ν averaged over 4π srad

  $$J_\nu = \frac{1}{4\pi} \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} I_\nu(\theta,\phi) \, d\phi$$

  erg cm^{-2} Hz^{-1} s^{-1}

- Flux, F_ν

  erg cm^{-2} Hz^{-1} s^{-1}

  Energy flow perpendicular to surface

  $$F_\nu = \int I_\nu \cos\theta \, d\Omega = \int_0^\pi \sin\theta\cos\theta \, d\theta \int_0^{2\pi} I_\nu(\theta,\phi) \, d\phi$$

Can separate in/outgoing flux

$$F_{\nu,out} = \int_0^{\theta=\pi/2} I_\nu \cos\theta \, d\Omega$$

$$F_{\nu in} = -\int_{\theta=\pi/2}^{\theta=\pi} I_\nu \cos\theta \, d\Omega$$

$$F_\nu = F_{\nu out} - F_{\nu,in} = 0 \quad \text{if radiation is isotropic}$$

At surface, $F_{\nu,in} = 0$ and

$$F_\nu = 2\pi \int_0^{\pi/2} \cos\theta \sin\theta \, I_\nu \, d\theta$$

$$= 2\pi I_\nu \quad \text{if } I_\nu \text{ isotropic}$$

I_ν is "surface brightness"

↳ Independent of distance

Other integrals:

$$H_\nu = \frac{1}{4\pi} \int I_\nu \cos\theta \, d\Omega = \frac{F_\nu}{4\pi}$$

$$K_\nu = \frac{1}{4\pi} \int I_\nu \cos^2\theta \, d\Omega = \frac{I_\nu}{3} \quad \text{if isotropic}$$

Energy density:

$$u_\nu = \frac{1}{c} \int I_\nu \, d\Omega = \frac{4\pi I_\nu}{c} \quad \text{if isotropic}$$

Radiation pressure

$$P_\nu = \frac{1}{c} \int I_\nu \cos^2\theta \, d\Omega = \frac{1}{3} u_\nu \quad \text{if isotropic}$$

14.2 Absorption and Emission

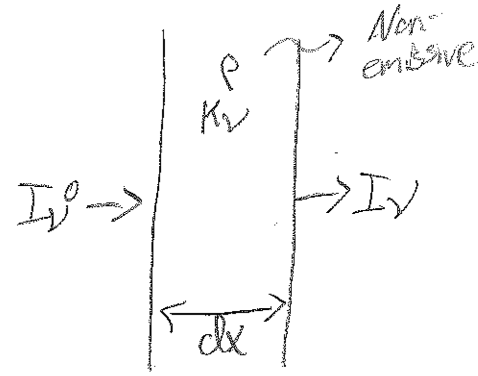
Consider volume dV of star with density ρ. Energy is emitted in all directions in frequency interval $d\nu$

$$dE_\nu = j_\nu \rho \, d\nu \, dV$$

↳ Mass emission coefficient

κ_ν = mass absorption coefficient, $cm^2 g^{-1}$

$$\alpha_\nu = \kappa_\nu \cdot \rho \qquad cm^{-1}$$



$$dI_\nu = -\kappa_\nu \rho I_\nu \, dx$$
$$= -\alpha_\nu I_\nu \, dx$$

$$\Rightarrow \quad I_\nu = I_\nu^0 e^{-\int \alpha_\nu dx}$$

$$\boxed{I_\nu = I_\nu^0 e^{-\tau_\nu}} \qquad \tau_\nu = \int \kappa_\nu \rho \, dx \quad \text{Optical Depth}$$

In general, κ_ν can be decomposed into absorption (α) and scattering (σ)

Kirchoff's Law: For blackbody radiation, source function $S_\nu = \frac{j_\nu}{\kappa_\nu}$ is equal to Planck function $B_\nu(T)$ which is solely function of temperature

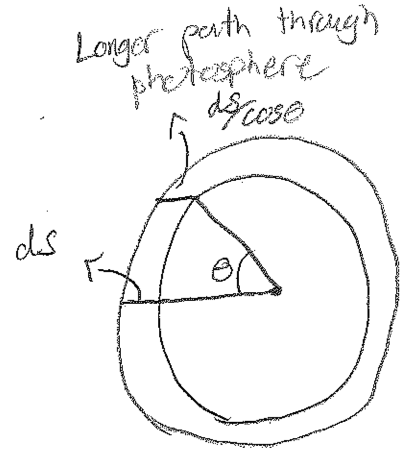
$$S_\nu = \frac{j_\nu \rho dx}{\kappa_\nu \rho dx} = \frac{dI_\nu}{d\tau_\nu} \qquad \text{Specific intensity per optical depth}$$

For atmosphere with pure scattering

$$j_\nu = \sigma I_\nu, \quad K_\nu = \sigma, \quad S_\nu = I_\nu$$

For atmosphere with pure absorption thermal equilibrium,

$$S_\nu = B_\nu$$



14.3 Limb Darkening

Let $I_\nu(\theta, \phi) \to I(\theta)$

$I_\nu(\theta)$ is observed to be larger at small values of θ, Sun appears darker around edges

Empirically, $I_\nu(\theta) = I_\nu(0)\left[1 - A(1-\cos\theta)\right]$

$$I_\nu(0) = \int_0^\infty S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

↳ Spec Intensity at surface $\tau_\nu = 0$

Incorporating angular dependence, $dx \to \frac{dx}{\cos\theta}$

$$I_\nu(\tau_\nu = 0, \theta) = \int_0^\infty S_\nu(\tau_\nu) e^{-\frac{\tau_\nu}{\cos\theta}} \frac{1}{\cos\theta} d\tau_\nu$$

$\mu = \cos\theta$

$$I_\nu(\mu) = \int_0^\infty \underbrace{S_\nu(\tau_\nu)}_{\text{Increases with depth}} \underbrace{e^{-\tau_\nu/\mu}}_{\text{Decreases with depth}} \frac{d\tau_\nu}{\mu}$$

Taylor expansion

$$S_\nu \approx a + b\tau_\nu$$

$$\Rightarrow I_\nu(\mu) = \int_0^\infty (a + b\tau_\nu)\, e^{-\tau_\nu/\mu} \frac{d\tau_\nu}{\mu}$$

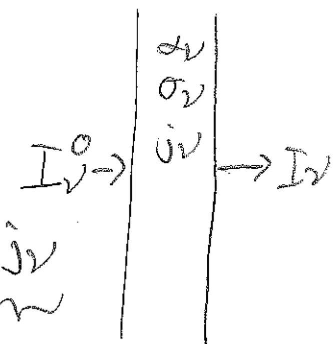
$$= -a e^{-x}\Big|_0^\infty - b\mu \int_0^\infty x e^{-x} dx$$

$$= a + b\mu = a + b\cos\theta$$

Intensity largest at center

- Measurement of b gives $T(\tau)$
- Measurement at different ν gives K_ν
- Basis of model atmospheres
- Few measurements available (eclipsing binaries)

14.4 Radiative Transfer Equation



$$\frac{dI_\nu}{dx} = \underbrace{-(\sigma_\nu + \alpha_\nu) I_\nu}_{\text{Scattering, Absorption out of beam}} + \underbrace{\frac{1}{4\pi}\int \sigma_\nu I_\nu \, d\Omega}_{\text{Scattering into beam}} + \underbrace{j_\nu}_{\text{Emission}}$$

$= -K_\nu I_\nu + j_\nu$ ↳ New includes scattering

↳ $= \sigma_\nu + \alpha_\nu$ (opacity)

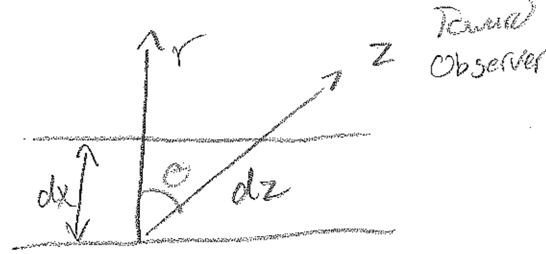
$$\Rightarrow \frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{K_\nu}$$

$$= -I_\nu + S_\nu$$

$$\Rightarrow e^{\tau_\nu}\left(\frac{dI_\nu}{d\tau_\nu} + I_\nu\right) = e^{\tau_\nu} S_\nu = \frac{d}{d\tau_\nu}\left(e^{\tau_\nu} I_\nu\right)$$

$$\Rightarrow I_\nu(\tau_\nu) = \underbrace{I_\nu(0)e^{-\tau_\nu}}_{\substack{\text{Original Intensity}\\ \text{attenuated}}} + \underbrace{\int_0^{\tau_\nu} S_\nu(t_\nu)\, e^{(t_\nu-\tau_\nu)}\, dt_\nu}_{\substack{\text{Integral of source over}\\ \tau_\nu\text{, attenuated by } \tau_\nu - t_\nu\\ \text{optical depth difference between}\\ \text{emission/absorption}}}$$

To incorporate angular dependence, replace $dz = -\frac{dx}{\mu}$



$$\Rightarrow \mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$$

13.5 Eddington Solution

Integrate above

$$\frac{1}{4\pi}\int \frac{dI_\nu(\theta)}{d\tau_\nu}\cos\theta\, d\Omega = \frac{1}{4\pi}\int I_\nu\, d\Omega - \frac{1}{4\pi}\int S_\nu\, d\Omega$$

↳ Isotropic

$$\Rightarrow \frac{dH_\nu}{d\tau_\nu} = J_\nu - S_\nu$$

Integrate again:

$$\frac{1}{4\pi}\int \frac{dH_\nu}{d\tau_\nu}\cos\theta\, d\Omega = \frac{1}{4\pi}\int J_\nu \cos\theta\, d\Omega - \cancelto{0}{\frac{1}{4\pi}\int S_\nu \cos\theta\, d\Omega}$$

$$\Rightarrow \frac{dK_\nu}{d\tau_\nu} = H_\nu$$

$$\Rightarrow K_\nu = H_\nu \tau_\nu + \text{const}$$

Eddington approximation: I_ν independent of θ (okay deep in atmosphere)

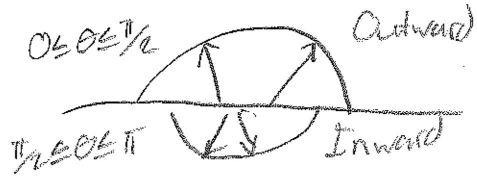
$$\Rightarrow K_\nu \approx \frac{I_\nu}{4\pi}\int \cos^2\theta\, d\Omega \approx \frac{J_\nu}{3}$$

Then

$$\frac{dJ_\nu}{d\tau_\nu} = 3H_\nu$$

$$\Rightarrow J_\nu = 3H_\nu \tau_\nu + \text{const}$$

At surface, use two-moment approximation. Split radiation into inward/outward components, and assume each is isotropic



$$J_\nu^0 = \frac{1}{4\pi}\int_0^{\pi/2}\int_0^{2\pi} I_\nu^0 \sin\theta \, d\theta \, d\phi = \frac{I_\nu^0}{2}$$

$$H_\nu = \frac{1}{4\pi}\int_0^{\pi/2}\int_0^{2\pi} I_\nu^0 \sin\theta\cos\theta \, d\theta \, d\phi = \frac{I_\nu^0}{4}$$

$$\Rightarrow H_\nu^0 = \frac{1}{2} J_\nu^0$$

This determines the constant of integration so that

$$J_\nu(\tau_\nu) = 2H_\nu^0 + 3H_\nu^0 \tau_\nu$$

$$= \frac{F_\nu^0}{2\pi}\left(1 + \frac{3}{2}\tau_\nu\right)$$

$$= \frac{3F_\nu^0}{4\pi}\left(\tau_\nu + 2/3\right)$$

From Planck's Law,

$$J_\nu(\tau) = \frac{\sigma}{\pi} T^4(\tau)$$

$$F^0 = \sigma T_{eff}^4 \rightarrow \text{Definition of Effective Temperature}$$

$$\Rightarrow \frac{\sigma}{\pi} T^4(\tau) = \frac{3\sigma}{4\pi} T_{eff}^4 \left(\tau_\nu + 2/3\right)$$

$$\Rightarrow \boxed{T(\tau) = \left[\frac{3}{4}\left(\tau + 2/3\right)\right]^{1/4} T_{eff}}$$

Note $T = T_{eff}$ at $\tau = 2/3$

Limb darkening:

$$I_\nu(0,\theta) = \int J_\nu e^{-\tau_\nu/\cos\theta} \frac{d\tau_\nu}{\cos\theta}$$

$$= \frac{F_\nu^0}{2\pi} + F_\nu^0 \int_0^\infty \frac{3}{4\pi} \tau_\nu e^{-\tau_\nu/\cos\theta} \frac{d\tau}{\cos\theta}$$

$$= \frac{F_\nu^0}{2\pi}\left(1 + \frac{3}{2}\cos\theta\right)$$

So

$$\frac{I_\nu(0,\theta)}{I_\nu(0,0)} = \frac{2}{5} + \frac{3}{5}\cos\theta$$

Results assume I_ν isotropic which is not perfect approximation.
When ν dependence is dropped and Rosseland mean opacity is used, this is called grey atmosphere.

Computational models without approximations are used these days.

Absorption/Emission Lines

Radiative Transfer Solution:

$$I_\nu(\tau_\nu) = I_{\nu,0} e^{-\tau_\nu} + S_\nu\left(1 - e^{-\tau_\nu}\right)$$

Assuming S_ν constant along path

For $\tau_\nu \ll 1$, $I_{\nu,0} < S_\nu$

$$\Rightarrow I_\nu(\tau_\nu) \simeq S_\nu \tau_\nu$$

$\Rightarrow$ Emission Line spectrum

For $\tau_\nu \gg 1$

$$I_\nu(\tau_\nu) \simeq S_\nu \simeq B_\nu(T)$$ Blackbody spectrum

Now consider case where S_ν not constant along path, or where $I_{\nu,0} \neq 0$. In limit of $\tau_\nu \ll 1$, $e^{-\tau_\nu} \simeq 1 - \tau_\nu$

$$\Rightarrow I_\nu \simeq I_{\nu,0} + \tau_\nu\left(S_\nu - I_{\nu,0}\right)$$

For $S_\nu < I_{\nu,0}$,

$$I_\nu \simeq I_{\nu,0} - \left(I_{\nu,0} - S_\nu\right)\int \rho \kappa_\nu \, dx$$

Smaller intensity for larger absorption
$\Rightarrow$ Absorption Lines

- Occurs when temperature decreases with height in a star so that $S_\nu < I_{\nu,0}$

For $S_\nu > I_{\nu,0}$,

$$I_\nu \simeq I_{\nu,0} + \left(S_\nu - I_{\nu,0}\right)\int \rho \kappa_\nu \, dx$$

Larger intensity for larger absorption
$\Rightarrow$ Emission Lines

- Occurs when temperature <u>increases</u> with height. This occurs in stellar coronae, and stars irradiated by binary companion

14.6 Model Atmospheres

We have $T(\tau)$, can use HSE to get $P(\tau)$, $\rho(\tau)$

$$\frac{dP}{dr} = -\rho g$$

$$\frac{dP}{d\tau}\frac{d\tau}{dr} = -\rho g \Rightarrow \frac{dP}{d\tau}(-\rho K) = -\rho g$$

$$\Rightarrow \frac{dP}{d\tau} = \frac{g}{K} \quad \begin{matrix} g \rightarrow \sim \text{constant} \\ K \rightarrow K(\rho, T) \end{matrix}$$

In low-mass stars, K dominated by H^- ion

$$K_\nu \propto \underbrace{P_e}_{\text{electron pressure}} \alpha_\nu(H^-) + \alpha_\nu(H)$$

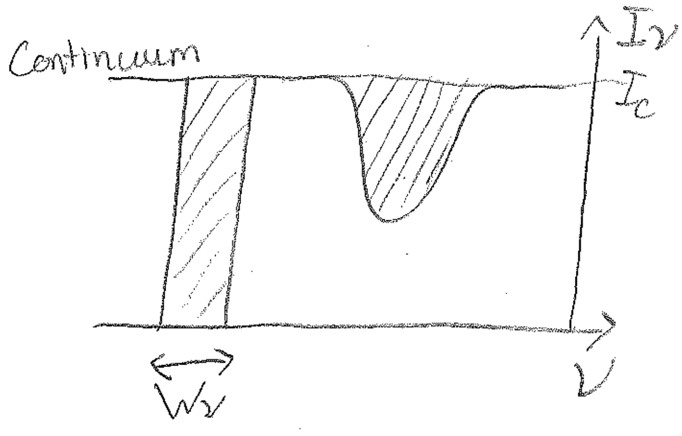
Iterate to solve for K, P_e, P_{gas}, equilibrium requires $F_\nu(\tau_\nu) =$ constant

14.7 Absorption line spectra

Purpose of model atmosphere is to compute emergent spectrum and absorption lines, so that stellar properties, abundances can be measured using "curve of growth" analysis.

Equivalent width of absorption line

$$W_\nu = \int \frac{I_c - I_\nu}{I_c} d\nu$$



We would like to know column density of absorbing atoms

$$N = n \; h \rightsquigarrow \text{column height}$$
↳ # density

$$I_\nu = I_c e^{-hK_\nu \rho} = I_c e^{-hN\alpha_\nu}$$

What is α_ν? Depends on atomic/quantum physics

Natural broadening

- Absorption lines not infinitely sharp because of Heisenberg's uncertainty principle:

$$\Delta E \, \Delta t \gtrsim \hbar/2$$

↳ line-width in energy

↳ Life time of electron in energy level $\Delta t = \frac{1}{\gamma}$

- Stronger lines (large γ) are broader

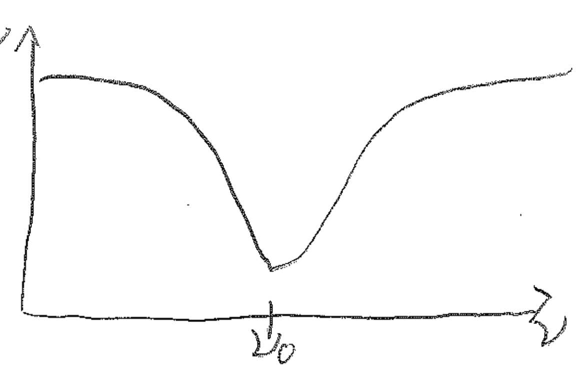
$$\alpha_\nu = \frac{e^2}{2mc} \frac{\gamma}{2\pi} \frac{1}{(\nu-\nu_0)^2 + (\gamma/4\pi)^2}$$

$$\int \alpha_\nu \, d\nu = \frac{\pi e^2}{mc} f$$
↳ "Oscillator Strength" Transition probability

$$W_\nu = hN \frac{\pi e^2}{mc} f$$

- f and γ determined via W_ν measurement in lab
  - function of ρ, T

- Natural broadening produces Lorentzian line profile



Doppler Broadening

- Lines broadened by Doppler shift due to thermal velocities of atoms

Maxwellian velocity distribution, Gaussian radial velocity distribution

$$f(v) = \frac{1}{\sqrt{\pi}} \frac{1}{v_t} e^{-v^2/v_t^2} \qquad v_t = \sqrt{\frac{2k_BT}{m}}$$

Doppler shift $\Delta\nu = \nu_0 \frac{v}{c}$

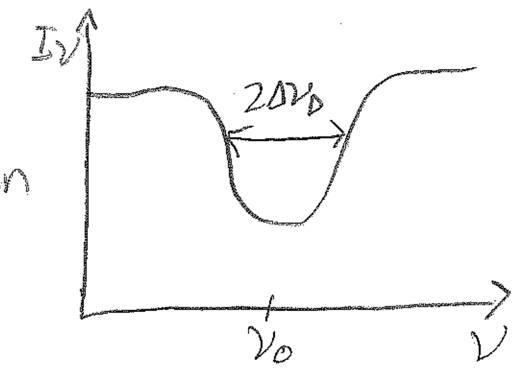
$$\Rightarrow \alpha_\nu = \int_0^\infty \alpha\left(\nu - \nu\frac{v}{c}\right) f(v)\, dv$$

$$= \frac{1}{\sqrt{\pi}} \frac{1}{\Delta\nu_D} \int_0^\infty \alpha(\nu - \nu') e^{-\nu'^2/\Delta\nu_D^2}\, d\nu'$$

with $\Delta\nu_D = \nu_0 \frac{v_t}{c}$

$$\Rightarrow \alpha_\nu \propto \frac{1}{\Delta\nu_D} e^{-(\Delta\nu/\Delta\nu_D)^2} \quad \text{Gaussian}$$

$$\Delta\nu = \nu - \nu_0$$



Full line profile

- Convolution of Lorentzian/Gaussian

$$\alpha_\nu = \frac{\pi^{1/2} e^2}{mc} f \frac{1}{\Delta\nu_D}\left[\frac{x}{\pi}\int_{-\infty}^{\infty} \frac{e^{-(v-u)^2}}{u^2 - x^2}\, du\right] \qquad x = \frac{\gamma}{4\pi\nu_D}$$

Voigt Function $H(x, v)$ $\qquad v = \frac{\Delta\nu}{\Delta\nu_D}$

Curve of Growth

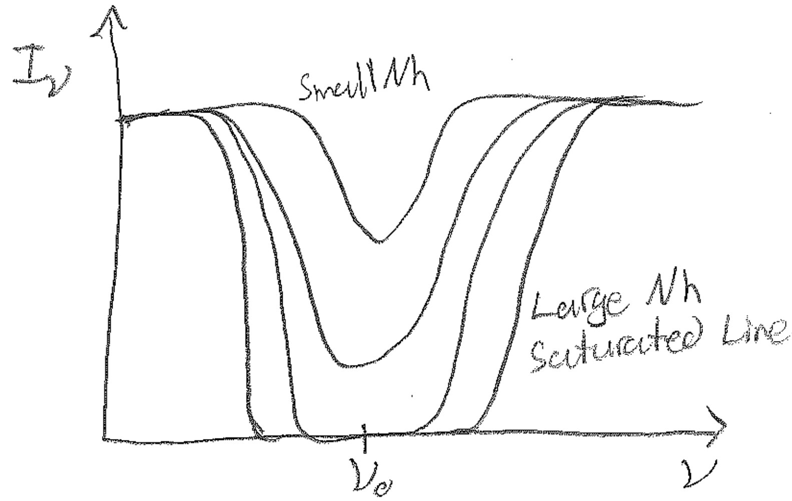
Arises from dependence of W_ν on Nh.

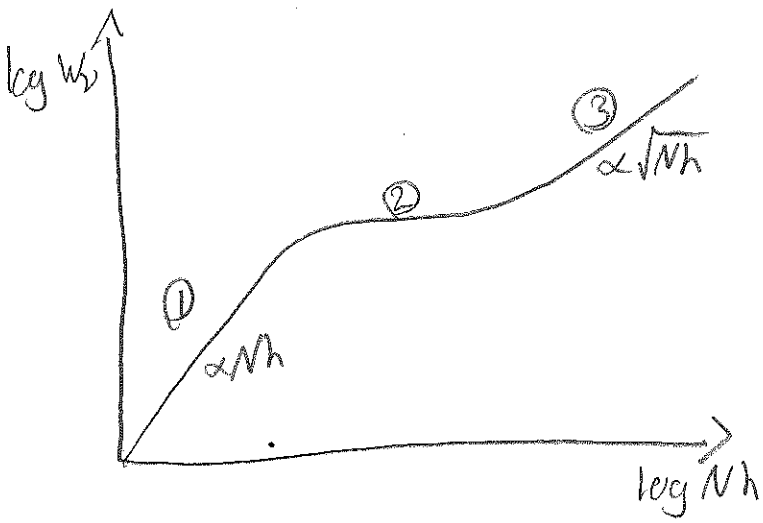
Weak lines: Optically thin, $\tau \ll 1$

$$\Rightarrow W_\nu = \int_0^\infty N h \alpha_\nu \, d\nu$$
$$= N h \int_0^\infty \alpha_\nu \, d\nu \quad \propto N h$$

Stronger lines:

- W_ν increases slower with Nh, because most photons have already been absorbed $\Rightarrow$ line saturation
- Only photons from line wings can be absorbed
- For large Nh, $W_\nu \propto \sqrt{Nh}$



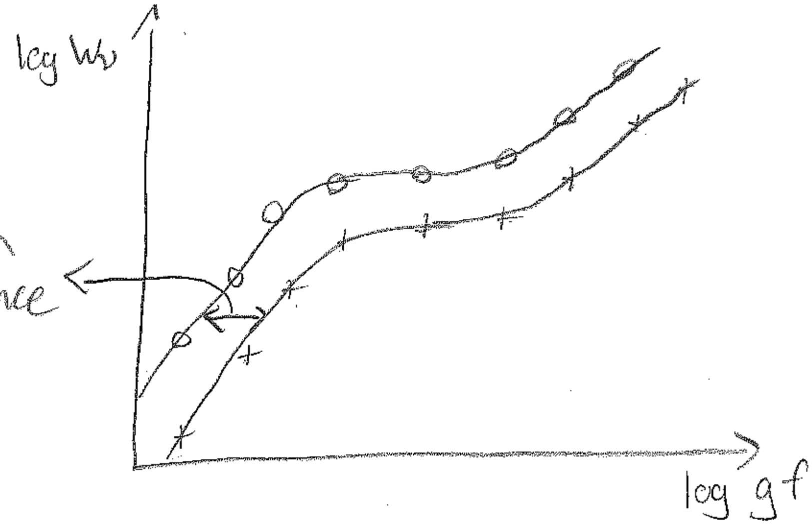


1. Optically thin, linear
2. Flat, start of saturation
3. Square root, damping wings

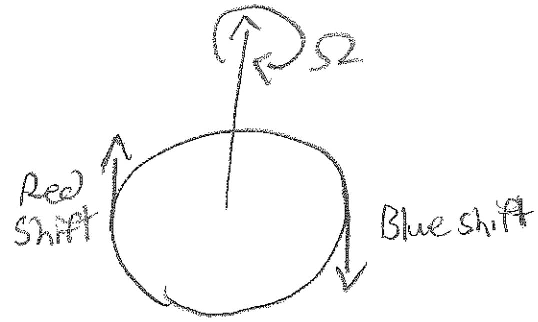
<u>Abundances</u>

- Consider many different Fe I lines w/ different oscillator strengths (but all have same Nh)
- Plot curve of growth for different lines

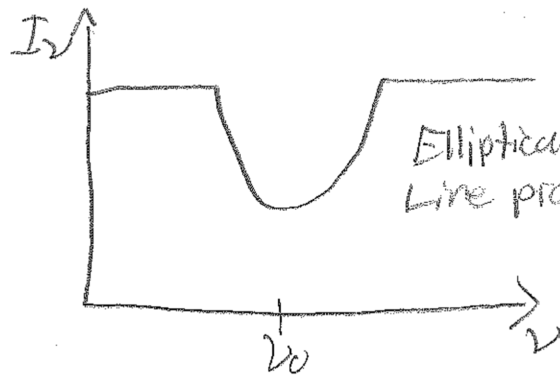
Shift relative to Sun tells relative abundance



Rotational line broadening



- Doppler shift due to rotational velocity distorts line profile
- $\Delta\nu \sim \nu_0 \frac{V_{rot} \sin i}{c}$ ~> inclination



Turbulent line broadening

- "micro/macro" turbulence
- $\Delta\nu \sim \nu_0 \frac{V_{turb,rms}}{c}$

Collisional/Pressure broadening

- Collisions between atoms knock e^- out of energy levels
- Collisions between atoms knock e⁻ out of energy levels
- Shorter effective Δt for lifetime, still yields Lorentzian profile

$$\gamma \rightarrow \gamma_{natural} + 2\gamma_{col}$$

$$\gamma_{col} = V_T \, n \, \sigma_{col}$$

σ_{col} ~> collision cross section

n ↳ # density

Narrower lines for larger (lower g) stars like RGB, AGB stars

- Measurement of γ_{col}, T_{eff} gives ρ at photosphere, P
- Pressure at photosphere $P_\nu = \frac{2}{3}\frac{g}{K_\nu}$ yields surface gravity
- Used to measure log g for some strong lines in low-mass stars

Stark broadening

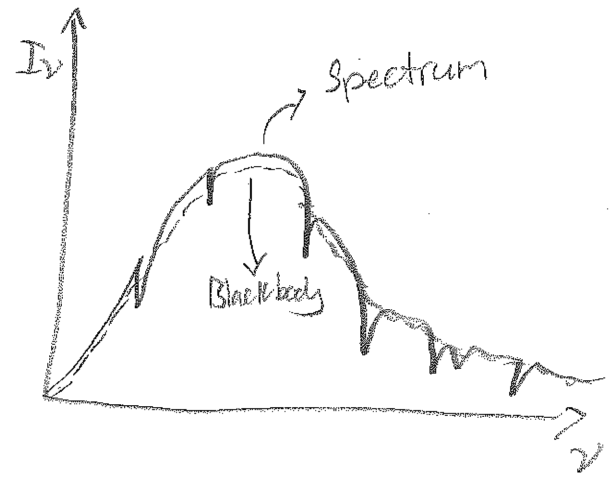
- Electric field splits atomic lines
- E field from nearby ions effectively broadens spectral lines
- Important at high densities, i.e. in white dwarf atmospheres
- Very broad lines in WD spectra

Zeeman Splitting

- Magnetic field splits atomic lines
- Allows for measurement of magnetic field at stellar photosphere

Measuring surface temperature/gravity

- T_{eff} can be approximately measured by fitting blackbody to continuum



- Can be better measured from W_ν of lines with different sensitivity to temperature

- Saha Equation for ionization:

$$\frac{n_+ n_e}{n_0} = \left(\frac{2\pi m_e k_B T}{h^2}\right) e^{-\chi/k_B T}$$

- Relative populations of n_+, n_0 exponentially sensitive to T
- Relative W_ν of n_+, n_0 transitions can be used to measure T

Pressure:

- n_e dependent on T_{eff}, but also dependent on density
  $n_e \propto \rho \propto P \propto g$. Larger n_e for larger g.
  $\Rightarrow$ Smaller n_+/n_0 for larger g at fixed T
  $\Rightarrow$ Smaller $\frac{W_\nu(n_+)}{W_\nu(n_0)}$ for larger g at fixed T