1. [15 points] Two identical galaxies are initially at rest, at a large distance from one another. They are spherical, composed solely of identical stars, and their light distributions obey the Sérsic law:

\[ I_m(R) = I(0) \exp(-kR^{1/m}), \]

with Sérsic index \( m \) and effective radius \( R_e \). The galaxies fall together and merge. If the merger product also satisfies the Sérsic law with the same index, what is its effective radius?

2. [40 points] (You may want to refer to Binney & Tremaine, Section 8.1 for a more detailed discussion of dynamical friction.) The derivation of the dynamical friction formula assumes that the subject system is a point mass, but in many cases of interest the subject system is an extended body, such as a cluster or satellite galaxy, characterized by a median radius \( r_h \). If the point of closest approach of the eld star to the center of the subject body is \( < \sim r_h \) then the deection of the eld-star orbit, and its contribution to the drag force, will be smaller than if the subject body were a point of the same total mass.

(a) Argue that the total drag force is largely unaffected by the non-zero size of the subject body if \( r_h < \sim b_{90} \), where \( b_{90} \) is given by:

\[ b_{90} = \frac{G(m + M)}{V_0^2} \]

is the impact parameter at which a background particle will be deflected through 90°.

(b) If \( r_h > \sim b_{90} \), argue that encounters with impact parameter \( < \sim r_h \) make a negligible contribution to the total drag force. Using:

\[ \int_0^{b_{\text{max}}} \int_0 \int_0 b \Delta v \, db \, d\theta \, dz = \frac{MV_0 b_{90}^2}{m + M} \ln(1 + \Lambda^2) \]

argue that in this case the Coulomb logarithm is given by \( \Lambda \approx b_{\text{max}}/r_h \).

(c) Combine these conclusions to argue that the correct value of the Coulomb logarithm for a subject body of median radius \( r_h \) is approximately

\[ \Lambda = \frac{b_{\text{max}}}{\max(r_h, GM/v_{\text{typ}}^2)}, \]

where \( v_{\text{typ}} \) is a typical background particle velocity.

3. [25 points] Spitzer (1956) inferred from absorption lines in the spectra of stars at high Galactic latitudes that the Galaxy is surrounded by gas a the virial temperature with \( n_e T \approx 5 \times 10^8 \text{K m}^{-3} \) near the Sun. Taking the Galactic potential to be spherical with \( v_c = 220 \text{km/s} \) at all
radii, and the gas to be an isothermal mixture of fully ionized hydrogen and helium, with one ion in ten $^4$He, show that the mass of gas inside radius $r$ is

$$M(r) \approx 10^8 M_\odot \left(\frac{10^6 K}{T}\right) \int_0^{r/R_0} dx x^{2-\alpha},$$

(5)

where $\alpha \approx 3.63(10^6 K/T)$. Hence estimate the radius to which this atmosphere must extend if it is to contain that part of the Galaxy’s share of the Local Group’s baryons that is not in Galactic stars.

4. [20 points] a) Derive the scaling of galaxy radius and mass with circular velocity. You may assume that all dark matter halos have the same spin parameter, $\lambda$, the same mean density within their virial radius, have flat rotation curves and that the entire baryonic content of each halo cools and forms a galactic disk. You may ignore the self-gravity of the galaxy and any backreaction on the dark matter.

b) Assume now that supernovae feedback is effective and expels gas from the forming galaxies at a rate $\beta = (V_c/200\text{km/s})^{-2}$ relative to the star formation rate. Assuming again that all gas in each halo cools to form a galactic disk and that feedback can eject the gas at above the escape velocity of the halo (so that it never returns) derive the scaling of galaxy mass with $V_C$. If the probability for gas to be ejected is independent of location within the disk (and therefore independent of the specific angular momentum of the gas) how is the scaling of the disk radius with $V_C$ altered by this feedback?