Problem Set 4
Ay 124: Structure and Dynamics of Galaxies

Handed out: February 25th 2010; Due back: March 11th 2010

1. [25 points] We can measure the diskiness of an isophote in an elliptical galaxy by fitting an ellipse to it, and then writing the surface brightness as a function of angle $\phi$ around this ellipse as a Fourier series,

$$I(\phi) = I_0(a) \left( 1 + \sum_{n=1}^{4} C_n \cos(n\phi) + \sum_{n=1}^{4} S_n \sin(n\phi) \right), \quad (1)$$

where $a$ is the semi-major axis of the ellipse. Assuming that all the $C_n$ and $S_n$ are small, show that

$$\frac{a_4}{a} \approx \frac{C_4}{0.4 \ln 10(d\mu/da)}, \quad (2)$$

where $\mu(a)$ is the galaxy’s surface brightness (measure on a magnitude scale) at distance $a$ down the major axis, and $a_4/a$ is the so-called diskiness parameter.

2. [20 points] Using the Abel integral

$$j(r) = -\frac{1}{\pi} \int_{r}^{\infty} \frac{dI}{dR} \frac{dR}{\sqrt{R^2 - r^2}}; \quad (3)$$

show that the luminosity density follows a power law $j \propto r^{-\alpha}$ then the surface brightness $I$ varies with projected radius $R$ as $I \propto R^{-(1+\alpha)}$.

3. [10 points] The fundamental plane is defined by

$$\langle I \rangle^{0.36} R_e^{-1} \sigma_0^{1.4} = \text{constant}. \quad (4)$$

An edge-on view of the fundamental plane is required with $\log \sigma_0$ plotted horizontally. What linear combination of $\log R_e$ and $\langle I \rangle$ should be plotted vertically?

4. [20 points] The 3D luminosity density of a spiral galaxy disk in cylindrical coordinates $(R, \phi, z)$ can be described by

$$j(R, z) = \begin{cases} j_0 e^{-R/R_d} \text{sech}^2(z/2z_0) & \text{for } R < R_{\text{max}} \\ 0 & \text{for } R > R_{\text{max}}. \end{cases} \quad (5)$$

a) Obtain an expression for the total luminosity, $L$, of the disk. b) Show that the surface brightness of the disk when viewed edge-on is given by

$$I(R, z) = I(0, 0) \frac{R}{R_d} K_1(R/R_d) \text{sech}^2(z/2z_0), \quad (6)$$

where $K_1$ is a modified Bessel function. Determine the dependence of $I(0, 0)$ upon $L$ and the model’s scale lengths.
5. [25 points] A simple model for the disk of the Milky Way consists of an exponential disk \( \Sigma(R) = \Sigma_0 \exp(-R/R_d) \) with \( R_d = 3.5 \text{kpc} \) and \( \Sigma_0 = 2.6 \times 10^8 M_\odot/\text{kpc}^2 \) and a flat rotation curve \( V(R) = 200 \text{km/s} \). If the disk had a Toomre parameter of \( Q = 0.75 \) what range of wavelengths would be unstable at the Solar radius \( (R = 8 \text{kpc}) \)? (You may approximate the disk as a fluid disk. Hint: Use the dispersion relation, stability requires \( \omega^2 > 0 \) so that \( \omega \) is real.) What would be the growth rate at a typical wavelength in this range? (Hint: For perturbations which grow as \( \exp(\omega t) \) the timescale is \( 1/\omega \).)