General-Relativistic Simulations of Stellar Collapse and The Formation of Stellar-Mass Black Holes

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The Core-Collapse Scenario

Evolved Massive Star

M > 8-10 M_{SUN}

H

O/Ne/Mg

Si

Fe-group nuclei

C

10^7 km

10^9 km
The Core-Collapse Scenario

Protoneutron Star, $R \sim 30$ km

“Core Bounce” at nuclear density.

Iron Core

$M > 1.3 - 2.2 \, M_{\text{SUN}}$

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The Supernova Problem

- Shock always stalls due to dissociation & neutrino losses (no ‘prompt’ hydrodynamic explosions).

Animation by Evan O’Connor
The Core-Collapse Scenario

Protoneutron Star, $R \sim 30 \text{ km}$

Accretion

Shock

$\mathbf{L}_\nu$
The Core-Collapse Scenario

Protoneutron Star, $R \sim 30$ km

Supernova Explosion

Shock is revived.
The Core-Collapse Scenario

Protonutron Star, R ~30 km

Shock is revived.

Shock is not revived.

Supernova Explosion

Collapse to Black Hole (Collapsar)
The Core-Collapse Scenario

Protonutron Star, $R \sim 30 \text{ km}$

- Accretion
- Shock
- $L_\nu$

Shock is revived.

Shock is not revived.

Supernova Explosion

Collapse to Black Hole (Collapsar)

This talk!
Core Collapse Timeline

- Energy reservoir: few $\times 10^{53}$ erg (100 B)
- Explosion energy: $\sim$1 B (+ $E_{\text{bind}}$ envelope)

- Time frame for explosion: $\sim$0.3 – 2.0 s after bounce.
- BH formation at baryonic PNS mass $\geq$ 1.8 – 2.5 $M_{\odot}$ (?).
- What stars do/don’t explode?
  - CCSN Mechanism?
  - Conditions for BH formation?
  - Connection to GRBs?
Maximum Neutron Star Mass

- Observation of old NSs – most solid (approved by Jim Lattimer!):
  J1903+0327 (NS in NS/WD or NS/MS system) -> $1.671 \pm 0.008 \, M_{\text{Sun}}$
  (see http://www.stellarcollapse.org/nsmasses)
Maximum Neutron Star Mass

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![Graph showing the relationship between Baryonic Mass and Radius for various models: LS180, LS375, LS220, HShen. The graph also shows the temperature at $0.1 \, \text{MeV}$ in a v-less $\beta$-equilibrium condition.](image-url)
Studying Black Hole Formation

• Protoneutron star collapse is a purely-GR phenomenon.

• Published work:
  
  **1D:** Lagrangian GR neutrino-radiation-hydro, very few detailed models.  
  [Seidel ‘91, Liebendörfer et al. ‘04, Sumiyoshi et al. ‘06,’07,’08, Fischer et al. ‘09]

  **2D/3D:** Collapse of isolated NS or collapsing polytropes.  
  No microphysics/neutrinos. [Baiotti et al. ‘05, ‘06, Shibata & Sekiguchi ‘05]

• **Our new approach:** [O’Connor & Ott 2010, O’Connor & Ott ‘10b (in prep.), Ott et al. 2010b (in prep.)]
  
  (1) Study *systematics* of BH formation in the limiting case of  
  1.5D (spherical symmetry + rotation) using the code **GR1D**.  
  -> Parameter study in EOS, progenitor structure, rotational setup.

  (2) Include efficient microphysics/neutrino-transport technology  
  that can be extended to multi-D.

  (3) Perform 3+1 GR simulations of most interesting cases to study  
  multi-D dynamics and gravitational-wave emission.
GR1D

- **GR1D**: Open-Source 1.5D GR hydrodynamics code.
- **Eulerian Radial-gauge, polar-slicing** (-> Schwarzschild-like coordinates).

\[
\frac{\alpha(r,t)}{\alpha(r,t)} = \exp[\phi(r,t)] , \quad X(r,t) = \left(1 - \frac{2m(r,t)}{r}\right)^{-1/2}
\]

\[
\phi(r,t) = \int_0^r X^2 \left[ \frac{m(r',t)}{r'^2} + 4\pi r'(P + \rho X^2 u' u') \right] \, dr'.
\]

- Choice of coordinates greatly simplifies GR hydro equations (zero shift).
- Disadvantage: Cannot evolve past horizon formation (like May & White, Misner & Sharp, van Riper formulations).

**GR Hydro equations in GR1D**

Implemented as semi-discrete finite-volume scheme with PPM reconstruction, HLLE Riemann solver and Runge-Kutta time integration.

\[
\partial_t \vec{U} + \frac{1}{r^2} \partial_r \left[ \frac{\alpha r^2}{X} \vec{F} \right] = \vec{S}
\]
GR1D: Approximate Rotation

- **Rotation in 1D: “Shellular”** – constant $\Omega$ on spherical shells.

Effective centrifugal force:

$$ f_{\text{cent}} = \frac{2}{3} \omega^2 r $$

$$ \partial_t (S_\phi) + \frac{1}{r^2} \partial_r \left( \frac{\alpha r^2}{X} F_\phi \right) = S_\phi $$

Extra term in equation for $r$ momentum:

$$ S_\phi = \rho h W^2 v_\varphi r, $$

$$ F_\phi = \rho h W^2 v_\varphi rv = S_\phi v, $$

$$ S_\phi = \rho h W^2 \alpha v_\varphi X \left[ 4 \pi r^2 P + \frac{m}{r} \right] + \frac{2}{3} \left( \frac{\rho h W^2 v_\varphi^2}{Xr} \right) $$

+ additional terms in metric quantities due to $\phi$ momentum.

Central density, $40-M_{\odot}$ model put into rotation with increasing initial central angular velocities.

Conservation of baryonic mass and angular momentum.
GR1D: EOS & Microphysics

- Multiple finite-temperature microphysical nuclear EOS:
  H. Shen et al. 1998, Lattimer & Swesty 1991 with K={180,220,375} MeV.
  EOS tables available in HDF5 format on http://www.stellarcollapse.org.

- Neutrinos during collapse: effective \( Y_e(\rho) \) approx. [Liebendörfer ‘05].

- Postbounce: 3-flavor, energy-averaged (gray) neutrino leakage scheme.

- Approximate neutrino heating:

  \[
  Q_{\nu_i}^{\text{heat}}(r) = \int_{\text{heat}} \frac{L_{\nu_i}^{\text{FRF}}(r)}{4\pi r^2} \sigma_{\text{heat,}\nu_i} \frac{\rho}{m_u} X_i \left\langle \frac{1}{F_{\nu_i}} \right\rangle e^{-2\tau_{\nu_i}}
  \]

- Nuclear Reaction Network & consistent multi-species advection.
  (work in progress).
Stellar-Mass Black Hole Formation
Stellar-Mass Black Hole Formation

- There is no direct/prompt black hole formation.

- **Generic:** $M_{IC} = M_{PNS}$ at bounce $\approx 0.5 - 0.7 M_{\odot}$. Set by nuclear physics, electron capture and general collapse hydrodynamics.

- Inner core easily stabilized by stiff core of the nuclear force + nucleon degeneracy. Exception: Very massive stars, $M > \sim 150 M_{\odot}$.
Parameter Study: Progenitor Compactness

\[ \rho_{\text{central}} \left[ 10^{14} \text{ g cm}^{-3} \right] \]

\[ t - t_{\text{bounce}} \left[ \text{s} \right] \]

[O’Connor & Ott 2010b in prep.]
Influence of Rotation

\[ j(r) = \frac{j_{16,\infty}}{1 + \left( \frac{A_{M,\odot}}{r} \right)^{2/7}} \times 10^{16} \text{ cm}^2 \text{ s}^{-1} \]

\[ \Delta j_{16,\infty} = 0.25 \text{ between models} \]

\[ u40WHW02 \text{ LS180} \]

\[ j_{16,\infty} = 0, 1, 2, 3 \]

\( \rho_c [10^{14} \text{ g cm}^{-3}] \)
Prospects for Nonaxisymmetric Instabilities?

- \((T/|W|)_{\text{dynamical}} > \approx 0.27,\)  
  \((T/|W|)_{\text{secular}} > \approx 0.14.\)

- Low-\(T/|W|\) instability: Dynamical shear instability.

Nonaxisymmetric Instability: *Redistribution of angular momentum* \(\rightarrow\) limit on protoneutron star spin.

\[ T/|W| = 0.27 \]

\[ \Delta j_{16,\infty} = 0.25 \]

between simulations

\[ t - t_{\text{bounce}} \left[ \text{S} \right] \]

[C. D. Ott @ GR19, Mexico City, 2010/07/06]
Spin of the nascent BH is limited to $a^* < 1$ by nonaxisymmetric instabilities in the protoneutron star.
• 2 competing GRB central scenarios:
  (1) **Millisecond Magnetars** [Bucciantini, Quataert, Metzger et al. ’07-’10, (Dessart et al. ’08)]
  (2) **Collapsars** [Woosley/MacFadyen et al.]

• -> 3+1 GR approach required to study CCSN-GRB connection
Computational Framework

Zelmani Core-Collapse Simulation Package composed of:

• **Cactus:** Open-source software framework for HPC, developed at the **Center for Computation & Technology** at LSU.

• Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formulation of numerical GR (McLachlan code, open source).

• General-Relativistic Hydrodynamics (GRHydro, open source).

• Adaptive Mesh Refinement (Carpet driver, open source).

• Implementation of GR1D leakage scheme and multiple finite-temperature nuclear EOS (open source/open physics).

• **Code is able to dynamically form black holes and track subsequent accretion evolution.**

• Full 3D simulations scale to O(2048) compute cores. Simplified runs scale to O(10000) cores.
Exploratory Simulations of 3D BH Formation

[Ott et al. 2010 in prep.]

• 3+1 D, but restricted to an octant with symmetry boundary conditions.
• 75-\(M_{\text{Sun}}\) low-metallicity progenitor model of Woosley, Heger, & Weaver ‘02.
• Simplified EOS:
  Piecewise polytrope with thermal component and supernuclear \(\Gamma = 2.4\).
• Approximate neutrino cooling, no neutrino heating:
  \[
  Q_{\nu}^{-} \approx -1.4 \times 10^{20} \left( \frac{T}{2 \text{ MeV}} \right)^6 e^{-\tau} \text{erg g}^{-1} \text{s}^{-1}
  \]
  – Temperature \(T\) via ideal gas of \(n\) and \(p\).
  – Optical depth \(\tau\) : fit from GR1D simulation.
• Moderate rotation, \(\Omega_0 = \{0,1\} \text{ rad/s}\).
  Initial uniform rotation in inner 1 \(M_{\text{Sun}}\).
• 11 levels of AMR.
• Excision of hydrodynamics inside horizon.
Soon to come:

More rapid rotation, full 3D without symmetries.
Gravitational Waves from BH Formation

C. D. Ott @ GR19, Mexico City, 2010/07/06

- 40 M_{Sun}, WW95 model
- Ω_0 = 1.5 rad/s
- a^* = 0.7

(results preliminary)
Time: 75.67 ms
Summary

• No “direct” (or prompt) BH formation in ordinary massive star collapse. A protoneutron star (PNS) phase always precedes BH formation.

• Extensive 1.5D parameter study of BH formation:
  Nonaxisymmetric instability will limit protoneutron star spin and enforce $a^* < 1$ for the nascent BH.

• First 3D simulations of collapse, PNS phase, PNS collapse, and post-BH formation phase -> first gravitational waveforms indicate characteristic GW signature of BH formation.

• Much work ahead: 3D without symmetry constraints, GRMHD, neutrino leakage/transport.
Supplemental Slides
Example: Black Hole Formation in Failing Core-Collapse Supernovae


Simulations and animations by Evan O’Connor
• GR1D reproduces full transport to 25% in terms of $L_\nu$ and to 10% in terms of the time of BH formation, but is roughly 10 times faster.

-> allows for parameter study.
Computational Cost & Scaling

- 9 levels of refinement, each $400^3$ zones, 400 3D grid functions
  -> Memory footprint $> \sim 2 \text{ TB}$ (including inter-process buffers)
- 1 single-zone update: 50 kflop; total timesteps: $\sim 1 \text{ M}$ (fine grid).
  -> $\sim 1500 \text{ Petaflops}$. Factor 5-10 larger with radiation transport.

![Graph showing weak scaling of a 9-level AMR test calculation of the coupled GR + GRHD system, evolving a neutron star.](image)