Ph 236 – Homework 8
Due: Friday, January 13, 2012

Note — This is a half-homework.

1. Conservation laws. [18 points]

(a) Consider a spacetime with a metric $g_{\mu\nu}$ that does not explicitly depend on $x^0 = t$. Show that the vector ξ^μ with contravariant components $(1, 0, 0, 0)$ is a Killing field. Explain why in this case p_0 is conserved.

(b) Let's try an alternative, canonical derivation of the same conservation law. Consider a particle of mass μ and write down for it the action

$$S = -\mu \int d\tau, \tag{1}$$

where τ is proper time; the geodesics must be derivable from this action principle. Then it should have the Lagrangian

$$L = -\mu \frac{d\tau}{dt}. \tag{2}$$

Write this Lagrangian as a function of $x^i(t)$ and $\dot{x}^i(t)$. Use the standard canonical procedure to derive the Hamiltonian, and show that it leads to the conservation law $p_0 = \text{constant}$.

(c) For a particle moving in the nearly Newtonian spacetime,

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Phi)[(dx^1)^2 + (dx^2)^2 + (dx^3)^2], \tag{3}$$

evaluate p_0 to lowest order, i.e. to order Φ in the potential and to second order in the velocity relative to the Newtonian coordinate system. If Φ is time-independent this should correspond to a conservation law – which one is it?