Ph 236 – Homework 6

Due: Friday, November 18, 2011

1. Gravitational wave detector. [10 points]
MTW Exercise 18.5.

2. Riemann tensor in linearized gravity. [8 points]
MTW Exercise 18.1.

3. Gravitomagnetism. [18 points]
In our derivation of Newtonian gravity, we assumed slowly moving sources, i.e. in the stress-energy tensor we kept terms of order ρ but dropped terms of order ρv. Now let us put the latter back: we will consider terms of order ρv (i.e. the momentum density ${}^{(3)}\boldsymbol{S}$) but not those of order ρv^2. You may continue to assume the source to be time-stationary, i.e. we are interested in considering the gravitational fields of rotating bodies (such as Earth!).

(a) In terms of ρ and S_i, and the inverse-Laplacian ∇^{-2}, find $\bar{h}_{\mu\nu}$ in the Lorentz gauge.

(b) Evaluate the Christoffel symbols ${\Gamma^i}_{00}$ and ${\Gamma^i}_{0j}$ outside the sphere. Show that the acceleration of an object relative to the coordinate system, i.e. d^2x^i/dt^2, is of the form

$$ {}^{(3)}\boldsymbol{a} = -\nabla\Phi + {}^{(3)}\boldsymbol{v} \times {}^{(3)}\boldsymbol{B}. \tag{1} $$

What is the "gravitomagnetic field" ${}^{(3)}\boldsymbol{B}$, and how does it relate to ${}^{(3)}\boldsymbol{S}$? [*Hint*: This should bear some resemblance to the generation of a magnetic field by a current distribution, but will **not** be identical.]

(c) Consider a uniform-density sphere of mass M and radius R, with $M \ll R$, centered at the origin. Suppose that it is in uniform rotation around the 3-axis, with angular velocity $\Omega \ll R^{-1}$ (so that the rotation is far from relativistic). Find, outside the sphere, both its gravitational potential Φ and the gravitomagnetic field.

(d) At an order of magnitude level, what is the ratio of the gravitomagnetic "acceleration" of a satellite in Earth orbit to the Newtonian acceleration?