Ph 236 – Homework 2
Due: Friday, October 14, 2011

1. Ampère's Law. [8 points]
In class we wrote down one of Maxwell's equations, $F^{\alpha\beta}{}_{,\beta} = 4\pi J^\alpha$. Write down the spatial components of this equation and show that it reduces to the familiar Ampère's Law.

2. Conservation of energy and momentum in electrodynamics. [14 points]
MTW Exercise 3.18.

3. Electromagnetic waves. [14 points]
Consider a plane electromagnetic wave in vacuum. Such a wave is of the form

$$F_{\alpha\beta}(x^\mu) = \Re[f_{\alpha\beta}\exp(ik_\mu x^\mu)], \tag{1}$$

where $\Re$ denotes the real part, $f_{\alpha\beta}$ is an antisymmetric tensor amplitude (since it is inside a real part, the tensor amplitude can in principle be complex if the components vary with different phases), and k_μ is the 4-dimensional wave vector.

(a) Show that if all components above are expressed in a Lorentz frame, that the frequency of the wave is $\omega \equiv k_0$ and the 3-dimensional wave vector ${}^{(3)}\boldsymbol{k}$ has components k_i.

(b) Prove that Maxwell's equations are satisfied if and only if $f_{\alpha\gamma}k^\gamma = 0$ and $f_{\alpha\beta}k_\gamma + f_{\beta\gamma}k_\alpha + f_{\gamma\alpha}k_\beta = 0$.

(c) Prove that if $f_{\alpha\beta}$ is nonzero that $k_\gamma k^\gamma = 0$.

(d) Show that the dispersion relation for the wave is $\omega = |{}^{(3)}\boldsymbol{k}|$. What are the phase and group velocities of electromagnetic waves?

(e) Specialize to the case where ${}^{(3)}\boldsymbol{k}$ points along the positive 3-axis. Write down the most general amplitude $f_{\alpha\beta}$ possible. How many independent complex coefficients does it contain?