1. **Ampère’s Law.** [8 points]

In class we wrote down one of Maxwell’s equations, $F_{\alpha\beta,\beta} = 4\pi J^\alpha$. Write down the spatial components of this equation and show that it reduces to the familiar Ampère’s Law.

2. **Conservation of energy and momentum in electrodynamics.** [14 points]

MTW Exercise 3.18.

3. **Electromagnetic waves.** [14 points]

Consider a plane electromagnetic wave in vacuum. Such a wave is of the form

$$F_{\alpha\beta}(x^\mu) = \Re[f_{\alpha\beta}\exp(ik_\mu x^\mu)],$$

where $\Re$ denotes the real part, $f_{\alpha\beta}$ is an antisymmetric tensor amplitude (since it is inside a real part, the tensor amplitude can in principle be complex if the components vary with different phases), and $k_\mu$ is the 4-dimensional wave vector.

(a) Show that if all components above are expressed in a Lorentz frame, that the frequency of the wave is $\omega \equiv k_0$ and the 3-dimensional wave vector $^{(3)}k$ has components $k_i$.

(b) Prove that Maxwell’s equations are satisfied if and only if $f_{\alpha\gamma}k_\gamma = 0$ and $f_{\alpha\beta}k_\beta + f_{\beta\gamma}k_\alpha + f_{\gamma\alpha}k_\beta = 0$.

(c) Prove that if $f_{\alpha\beta}$ is nonzero that $k_\gamma k_\gamma = 0$.

(d) Show that the dispersion relation for the wave is $\omega = |^{(3)}k|$. What are the phase and group velocities of electromagnetic waves?

(e) Specialize to the case where $^{(3)}k$ points along the positive 3-axis. Write down the most general amplitude $f_{\alpha\beta}$ possible. How many independent complex coefficients does it contain?