Ph 236 – Homework 11
Due: Friday, February 3, 2012

1. Perihelion precession in general gravity theories. [14 points]
Suppose that we replaced the exterior solution of the spherical star with the following alternative metric:

$$ds^2 = -\left[1 - 2\frac{M}{r} + 2\beta\frac{M^2}{r^2} + \mathcal{O}(r^{-3})\right] dt^2 + \left[1 + 2\gamma\frac{M}{r} + \mathcal{O}(r^{-2})\right] dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{1}$$

where γ and now β are dimensionless coefficients that depend on the theory of gravity. We discussed γ last term in the context of Shapiro delay and deflection of light experiments, and found $\gamma = 1$ for GR.

(a) What is the GR result for β?

(b) Consider the orbit of a test particle in this metric. Find the leading-order (in powers of $1/r$) form of the effective potential.

(c) What is the prediction for the perihelion precession of Mercury in this theory? How does it depend on β and γ?

2. Contribution of other planets to perihelion precession. [10 points]
The relativistic perihelion precession is just one term contributing to the precession of orbit of Mercury. The largest non-GR effect is the pull of the other planets on Mercury's orbit.

To make this problem tractable: A planet in an orbit far outside that of Mercury will produce a Newtonian tidal field acting on the Sun-Mercury system. As long as the outer planet is not in a resonance with Mercury (this is a valid assumption in the Solar System), you may estimate the time-averaged effect of the outer planet by treating its mass $m_{\rm op}$ as smeared into a ring of circumference $2\pi a_{\rm op}$ where $a_{\rm op}$ is that planet's mean distance from the Sun. This contribution leads to a potential of the form

$$\Phi(r) = -\frac{M_\odot}{r} + \text{const} + \frac{1}{2}kr^2 + ... \tag{2}$$

for some constant k.

(a) Find k in terms of $m_{\rm op}$ and $a_{\rm op}$.

(b) What is the implied precession rate?

(c) Numerically evaluate (b) and determine which planet(s) dominate(s) the perturbation to Mercury.

3. Muons in ultradense matter. [8 points]
Consider the model discussed in class where ultradense matter consists purely of degenerate neutrons, protons, electrons, and (possibly) muons. If these particles were noninteracting, what would be the critical density above which muons appear even at $T = 0$? (Numerically evaluate your expression.)