1 Big Bang Nucleosynthesis: Overview

A few seconds after the Big Bang, almost all of the energy density in the Universe was in photons, neutrinos, and e^+e^- pairs, but some was in the form of baryons. We thus come to the subject of BBN: the production of the light elements in the first few minutes after the Big Bang. We will discuss the subject in 3 phases:

- The determination of the neutron:proton ratio.
- Fusion and radioactive decay to produce D, ^{3}He, ^{4}He, ^{7}Li.
- The observation and interpretation of light element abundances.

See also Dodelson §3.2 & 1.3.

2 The $n : p^+$ ratio

Neutrons and protons are interconverted by weak interactions:

$$\begin{aligned} n &\leftrightarrow p^+ + e^- + \bar{\nu}_e \\ n + \nu_e &\leftrightarrow p^+ + e^- \\ n + e^+ &\leftrightarrow p^+ + \bar{\nu}_e. \end{aligned} \tag{1}$$

Equilibrium physics. Let's examine how these reactions play out before e^+e^- annihilation ($T \geq 200$ keV) and when the weak interactions are fast (turns out to be $T > 1$ MeV). The electron chemical potential is negligible in this case, and neutrino chemical potential is essentially zero (in standard model!), so we should have

$$\mu_n = \mu_p. \tag{2}$$

The chemical potential is related to abundance for a nonrelativistic species (recall $T \ll m_p, m_n$):

$$\mu_X = m_X + T \ln \left[\frac{n_X}{g_X} \left(\frac{2\pi}{m_X T} \right)^{3/2} \right], \tag{3}$$

where g_X is the degeneracy ($2s + 1$; 2 for n or p^+). The equilibrium condition then gives

$$\frac{n_n}{n_p} = e^{-(m_n - m_p)/T} = e^{-\mathcal{Q}/T}. \tag{4}$$

We've defined $\mathcal{Q} = m_n - m_p = 1.293$ MeV. So at high temperatures ($T \gg \mathcal{Q}$) and in thermal equilibrium there are the same number of neutrons as protons. As T drops we have fewer neutrons, and eventually in thermal equilibrium they all go away.

Non-equilibrium physics. But the real Univese is not in thermal equilibrium and we'd better explore the consequences. Let's define a fraction X_n of the baryons to be neutrons and $X_p = 1 - X_n$ to be protons. Then we have

$$\dot{X}_n = -\lambda_{np} X_n + \lambda_{pn}(1 - X_n), \tag{5}$$

where λ_{np} and λ_{pn} are the $n \to p^+$ and $p^+ \to n$ conversion rates (units are 1/sec). Without doing any work, we know that $\dot{X}_n = 0$ in thermal equilibrium, i.e. if $X_n/(1 - X_n) = e^{-\mathcal{Q}/T}$. Therefore:

$$\lambda_{pn} = \lambda_{np} e^{-\mathcal{Q}/T} \tag{6}$$

and

$$\dot{X}_n = \lambda_{np}[-X_n + e^{-\mathcal{Q}/T}(1 - X_n)]. \tag{7}$$

The transition rate λ_{np} from the 2-body reactions is

$$\lambda_{np} = n(\nu_e)\langle\sigma(n\nu_e \to p^+ e^-)v\rangle_{\rm th} + n(e^+)\langle\sigma(ne^+ \to p^+\bar{\nu}_e)v\rangle_{\rm th}, \tag{8}$$

where the averages are thermal and v is the velocity of the initial particle (≈ 1 since they are relativistic). The averaged cross sections are an exercise in weak interaction theory (not discussed in this course since it requires QFT). The result is (for relativistic electrons):

Number density,

$$n(\nu_e) = \frac{3\zeta(3)}{4\pi^2}T^3; \quad n(e^+) = \frac{3\zeta(3)}{2\pi^2}T^3. \tag{9}$$

Cross sections,

$$\begin{aligned} \langle\sigma(n\nu_e \to p^+ e^-)v\rangle_{\rm th} &= \frac{510\pi^2}{3\zeta(3)\tau_n \mathcal{Q}^5}(12T^2 + 6\mathcal{Q}T + \mathcal{Q}^2). \\ \langle\sigma(ne^+ \to p^+\bar{\nu}_e)v\rangle_{\rm th} &= \frac{255\pi^2}{3\zeta(3)\tau_n \mathcal{Q}^5}(12T^2 + 6\mathcal{Q}T + \mathcal{Q}^2). \end{aligned} \tag{10}$$

(These are written in terms of neutron lifetime τ_n because they contain the same matrix element.)

As an example, at 1 MeV these numbers are:

- Densities: $n(\nu_e) = 1.2 \times 10^{31}\,\text{cm}^{-3}$; $n(e^+) = 2.4 \times 10^{31}\,\text{cm}^{-3}$.
- Cross sections: $\langle\sigma(n\nu_e \to p^+e^-)v\rangle_{\rm th} = 7 \times 10^{-32}\,\text{cm}^{-3}\,\text{s}^{-1}$; $\langle\sigma(ne^+ \to p^+\bar{\nu}_e)v\rangle_{\rm th} = 4 \times 10^{-32}\,\text{cm}^{-3}\,\text{s}^{-1}$.
- Neutron-proton conversion rate, $\lambda_{np} = 1.7\,\text{s}^{-1}$.
- Equilibrium ratio, $(n : p^+)_{\rm eq} = 1 : 3.6$.
- Age of Univese: $t = 0.74\,\text{s}$.

One can see that the conversion time λ_{np}^{-1} is comparable to the age of the Univese at a temperature of ~ 1 MeV. At later times, $T \propto t^{-1/2}$, and $\lambda_{np} \propto T^3 \propto t^{-3/2}$, so the neutron-proton conversion time $\lambda_{np}^{-1} \propto t^{3/2}$ becomes longer than the age of the Universe. Therefore we get *freeze-out* – the reaction rates become slow and the $n : p^+$ ratio goes to a constant.

The freeze-out calculation will be a homework exercise; the answer is $n : p^+ = 0.15$. Note that this doesn't depend on numbers like the baryon density, etc.

After freeze-out the neutron abundance continues to decline because it is unstable with a lifetime of $\tau_n = 886\,\mathrm{s}$ to:

$$n \to p^+ + e^- + \bar{\nu}_e. \tag{11}$$

Thus the actual neutron abundance is

$$X_n = 0.15 e^{-t/\tau_n}. \tag{12}$$

We'll see that BBN occurs at a time of $\sim 200\,\mathrm{s}$ after the Big Bang so $X_n = 0.12$ then.

3 Fusion and light nuclei

Baryon abundance. To discuss formation of the light nuclei we'll need to know the baryon abundance. Cosmologists usually report this in terms of the fraction of critical density Ω_b. The density of baryons today is then:

$$\rho_{b0} = \frac{3\Omega_b H_0^2}{8\pi G} = 1.879 \times 10^{-29} \Omega_b \left(\frac{H_0}{100\,\mathrm{km/s/Mpc}} \right)^2 \ \mathrm{g/cm^3}. \tag{13}$$

Cosmologists usually define the dimensionless number h by

$$h = \frac{H_0}{100\,\mathrm{km/s/Mpc}}, \tag{14}$$

which is about 0.7 (recall lectures on H_0). Then the baryon density today is

$$\rho_{b0} = 1.879 \times 10^{-29} \Omega_b h^2 \ \mathrm{g/cm^3}, \tag{15}$$

or dividing out by the nucleon mass the number density today is

$$n_{b0} = 1.13 \times 10^{-5} \Omega_b h^2 \ \mathrm{cm^{-3}}. \tag{16}$$

This is today, at a CMB temperature of $T = 2.73\,\mathrm{K}$. If we use the fact that (after e^+e^- annihilation) the temperature decreases as a^{-1} and the baryon density decreases as a^{-3}, we find

$$n_{b0} = 5.6 \times 10^{20} \Omega_b h^2 T_9^3 \ \mathrm{cm^{-3}}, \tag{17}$$

where T_9 is the temperature in GigaKelvin.

The baryon density is small (e.g. $\Omega_b h^2 = 0.02229 \pm 0.00073$ from the CMB data, Spergel et al 2007 ApJS 170, 335). We'll see that BBN occurs at $T_9 \sim 1$ so the baryon density is $\sim 10^{19}$ baryons per cm^3 – less than the density of air!

Equilibrium nucleosynthesis theory. We discussed the chemical potential of a species, μ_X, above in Eq. (3). If a nucleus with Z protons, N neutrons,

Nucleus	$g(^AZ) = 2I + 1$	$B(^AZ)$, MeV	Decay mode
^{2}H	3	2.22	stable
^{3}H	2	8.48	$\beta \rightarrow^3$He (12.6 yr)
^{3}He	2	7.72	stable
^{4}He	1	28.30	stable
^{6}Li	3	31.99	stable
^{7}Li	4	39.25	stable
^{7}Be	4	37.60	electron capture to ^{7}Li (53 days)
^{12}C	1	92.16	stable

and baryon number $A = Z + N$ is in equilibrium, it should have chemical potential:

$$\mu(^AZ) = Z\mu_p + N\mu_N. \tag{18}$$

We define the binding energy of the nucleus to be the difference between the mass of its protons and neutrons, and the nucleus itself:

$$B(^AZ) = Zm_p + Nm_n - m(^AZ). \tag{19}$$

The abundance by mass $X(^AZ) = An(^AZ)/n_b$ in equilibrium is then:

$$X(^AZ) = \frac{g(^AZ)}{2^A} A^{5/2} \left(\frac{2\pi}{m_{\rm nuc}T}\right)^{(3/2)(A-1)} n_b^{A-1} X_p^Z X_n^N e^{B(^AZ)/T}. \tag{20}$$

(The 2^A is from proton and neutron spin states.) The proton and neutron abundances $X_{p,n}$ are determined by fixing the total number of protons and neutrons. If this equation holds, we say that we have *nuclear statistical equilibrium* or NSE.

Note that formation of heavy nuclei is favored at low temperatures (exponential factor) and high density (n_b^{A-1} factor).

Because we have the baryon density as a function of temperature, for a given n/p ratio we can immediately work out the equilibrium abundances as a function of time. Simplify to:

$$X(^AZ) = \frac{g(^AZ)}{2^A} A^{5/2} (9.3 \times 10^{-14} \Omega_b h^2 T_9^{3/2})^{A-1} X_p^Z X_n^N e^{B(^AZ)/T}. \tag{21}$$

The exponential has to be very large in order to overcome the tiny factor $9.3 \times 10^{-14}\Omega_b h^2$ and for this reason nuclei don't form until the temperature falls well below 1 MeV.

In NSE, half of the neutrons are absorbed into ^{4}He at $T_9 = 3.3$, or $t = 16$ s. (Deuterium is never favored by NSE, e.g. at 16 s we have $X_{D,NSE} = 6 \times 10^{-12}$.) Half of the helium is burned to ^{12}C at $T_9 = 1.12$, or t =140 s. But NSE doesn't apply at low temperatures because the reaction rates are too slow.

Deuterium and ^{4}He production. At the low densities of BBN the 4-body reaction:

$$p^+ + p^+ + n + n \rightarrow^4 \text{He} \tag{22}$$

is very slow. The only way to make helium is to first make deuterium:

$$p^+ + n \leftrightarrow^2 \mathrm{H} + \gamma, \tag{23}$$

then build $A = 3$ nuclei by 2-body reactions:

$$\begin{aligned} {}^2\mathrm{H} + p^+ &\leftrightarrow {}^3\mathrm{He} + \gamma \\ {}^2\mathrm{H} +^2 \mathrm{H} &\leftrightarrow {}^3\mathrm{H} + p^+ \\ {}^2\mathrm{H} +^2 \mathrm{H} &\leftrightarrow {}^3\mathrm{He} + n. \end{aligned} \tag{24}$$

(The ^{2}H$-{}^2$H reaction is 5 orders of magnitude faster than ^{2}H-p$^+$ but there's more p^+ than ^{2}H.) Finally we make ^{4}He:

$$\begin{aligned} {}^3\mathrm{H} + p^+ &\leftrightarrow {}^3\mathrm{He} + n \\ {}^3\mathrm{H} +^2 \mathrm{H} &\leftrightarrow {}^4\mathrm{He} + n \\ {}^3\mathrm{He} +^2 \mathrm{H} &\leftrightarrow {}^4\mathrm{He} + p^+. \end{aligned} \tag{25}$$

Deuterium has a binding energy of 2.22 MeV. Consequences are (1) its equilibrium abundance is very small, and (2) it is easily destroyed by photons so Eq. (23) remains in equilibrium. (Photodisintegration is faster than Hubble expansion or any competing D destruction process.) Before helium forms, when $X_p = 0.88$ and $X_n = 0.12$, this abundance is

$$X({}^2\mathrm{H}) = 4 \times 10^{-14} \Omega_b h^2 T_9^{3/2} e^{25.8/T_9}. \tag{26}$$

At late times ($T_9 < 1.5$, $t > 40$ s), $A = 3$ nuclei are thermodynamically favored over protons and neutrons, and ^{4}He is favored over ^{3}H,He, so once an $A = 3$ nucleus is formed by Eq. (24) it has negligible probability of going back to deuterium. Most burn to ^{4}He. Thus during this era, ^{4}He builds up in accordance with

$$\dot{X}({}^4\mathrm{He}) = 2X({}^2\mathrm{H}) X_p n_b \langle \sigma v \rangle_{Dp} + 2X({}^2\mathrm{H})^2 n_b \langle \sigma v \rangle_{DD}. \tag{27}$$

(The 2 is because these are mass fractions, not number fractions, and ^{4}He is twice as massive as ^{2}H.)

This situation comes to an end at $T_9 \approx 0.8$, $t = 200\,\mathrm{s}$ (homework exercise!), when the production of ^{4}He exhausts the supply of neutrons. The final abundance of ^{4}He is then:

$$X({}^4\mathrm{He})_{\mathrm{final}} = 2X_{n,\mathrm{initial}} \approx 0.24, \tag{28}$$

where the factor of 2 is because each gram of neutrons yields two grams of ^{4}He (the protons provide the other gram). The final quantity of ^{4}He depends somewhat on $\Omega_b h^2$ because at higher $\Omega_b h^2$ the neutrons are converted to ^{4}He earlier and have less time to decay, hence more ^{4}He is produced.

After the neutrons run out the deuterium abundance falls as it burns (Eq. 24) mainly to ^{3}He and then to ^{4}He. The peak abundance is ~ 0.01. Because the fusion rates decline sharply with temperature some ^{2}H, ^{3}H, and ^{3}He is left unburned. Any remaining ^{3}H decays to ^{3}He. Final abundances are:

- $X(^2\text{H}) \sim 5 \times 10^{-5}$. Often quoted as D:H ratio by number, $\sim 3 \times 10^{-5}$.
- $X(^3\text{He}) \sim 3 \times 10^{-5}$. Or: ^{3}He:H$\sim 10^{-5}$.

More ^{2}H and ^{3}He are produced at lower densities because it is harder to destroy them.

Heavier elements. Some $A = 7$ nuclei are produced by the following reactions:

$$
\begin{aligned}
{}^3\text{He} + {}^4\text{He} &\leftrightarrow {}^7\text{Be} + \gamma \\
{}^3\text{H} + {}^4\text{He} &\leftrightarrow {}^7\text{Li} + \gamma.
\end{aligned} \tag{29}
$$

The two nuclei can be interconverted by the reaction

$$ {}^7\text{Be} + n \leftrightarrow {}^7\text{Li} + p^+ \tag{30} $$

and destroyed by

$$ {}^7\text{Li} + p^+ \rightarrow 2\,{}^4\text{He}. \tag{31} $$

At high baryon densities ($\Omega_b h^2 \geq 0.01$) more ^{7}Be than ^{7}Li is produced because the last reaction depletes ^{7}Li. The ^{7}Be eventually captures an electron (when the temperature of the Universe gets low enough!) and decays to ^{7}Li:

$$ {}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e. \tag{32} $$

Total predicted yield is small: by mass $X(^7\text{Li})=3 \times 10^{-9}$, by number ^{7}Li:H$\sim 4 \times 10^{-10}$. The ^{7}Be yield increases with $\Omega_b h^2$.

Nuclei with $A > 7$ are not produced in significant quantities:

- ^{8}Be unstable, decays to $2\,^4$He with half-life of 10^{-12} s.
- Proton/neutron capture rates insufficient for $^7\text{Li}\rightarrow{}^8\text{Li}\rightarrow{}^9\text{Be}$.
- Triple-alpha reaction, $3\,^4\text{He}\rightarrow{}^{12}$C+photons, too slow at low density.

See e.g Fig. 1.8 of Dodelson for abundance predictions.

4 Observational tests

Now we'd like to know how well the observed abundances of the light nuclei match the predictions. In addition to measurement errors, we must be aware of three additional types of error:

- *Reaction rate uncertainties* – many reaction rates are poorly determined.
- *Astration* – has the matter being observed been processed in stars, which could alter abundances?
- *Fractionation* – have the elements, or even isotopes of the same element, been separated by chemical or physical processes?

Helium-4. The ^{4}He abundance is usually measured from emission line strengths in ionized gas clouds. Since ^{4}He is also produced in stars, one plots $X(^4\text{He})$ as a function of oxygen abundance and extrapolates to zero oxygen. Olive & Skillman (2004, ApJ, 617, 29) find $0.232 < X(^4\text{He}) < 0.258$. From WMAP baryon abundance, prediction is $0.2482 \pm 0.0003 \pm 0.0006$.

The central value is lowered to 0.2468 if the short neutron lifetime, 880 s, from Serebrov et al 2005 (Phys Lett B 605,72) is correct. (Shorter neutron lifetime means faster weak rates so the freeze-out of $n : p^+$ ratio occurs later. Fewer neutrons then means less ^{4}He.) In either case there is agreement with the observations.

Deuterium. The ^{2}H abundance (D:H ratio by number) depends strongly on location:

- Planets: D:H=150 ppm on Earth; $\sim 1\%$ on Venus; 10–40 ppm on Jupiter.
- ISM, based on UV absorption: $\sim$10–40 ppm, highly variable.
- Intergalactic Lyman-α absorption (H 1216Å line, $1s \to 2p$, ^{1}H and ^{2}H split due to reduced mass effect): 16–40 ppm.

The intergalactic absorption is probably the most reliable indicator of primordial abundance since deuterium is burned in stars. The value on terrestrial planets is significantly affected by fractionation.

Compare to prediction from WMAP baryon abundance: $25.7^{+1.7}_{-1.3}$ ppm.

Helium-3. This is hard to measure, and hard to interpret since ^{3}He can be either created or destroyed in stars (^{2}H is only destroyed). Bania et al 2002 (Nature 415, 54) provide upper limit of ^{3}He:H ratio (by number) of 15 ppm using the $^3\text{He}^+$ hyperfine transition (wavelength 3.5 cm). Compare to prediction $10.5 \pm 0.3 \pm 0.3$.

Lithium. Lithium is produced both in BBN and by cosmic ray spallation, and is destroyed in stars ($^7\text{Li}+p^+$ reaction). Typically observed through 6708Å doublet ($1s^2 2s - 1s^2 2p$ transition of neutral Li) and 6104Å triplet ($1s^2 2p - 1s^2 3d$) in absorption in stellar atmospheres. Can separate ^{6}Li vs. ^{7}Li based on line shape. The BBN prediction is that, by number, $^7\text{Li:H}=(4.4 \pm 0.3) \times 10^{-10}$.

Observed that for low-metallicity stars, there is a plateau in ^{7}Li:H versus metal abundance, at values $(1.3 \pm 0.2) \times 10^{-10}$. (Factor of 3 discrepancy!) Potential systematics:

- Destruction of ^{7}Li in stars?
- Stellar atmosphere modeling?
- Rate for important destruction mechanism grossly underestimated, or production overestimated?

Any solution based on stellar astrophysics must also account for small scatter ($\sim$7%) in ^{7}Li:H ratio at given metallicity (Fe:H).

The problem is made worse by the observation of ^{6}Li (e.g. Asplund et al 2006 ApJ 644, 229), with abundance $^6\text{Li:H} \sim 10^{-11}$. BBN should produce essentially no ^{6}Li ($\sim 10^{-14}$). This raises tensions:

- Maybe ^{6}Li was produced by cosmic ray interactions before the Galaxy was formed? Accelerate ^{4}He, impact on ^{4}He to make Li. But also makes ^{7}Li problem worse.
- If ^{7}Li was burned during stellar evolution, ^{6}Li should have been destroyed – $^6\text{Li}+p^+ \rightarrow ^4\text{He}+^3\text{He}$ is faster than burning ^{7}Li.

This remains an outstanding problem in cosmology. The most exciting possibility is that there is new physics involved, but the case is far from solid.