1 Cosmological principle

Basic assumption. The Universe on large scales is homogeneous and isotropic. (The "cosmological principle.")

Why make this assumption? Initally a philosophical principle:

- We are not special. – Copernicus
- Mathematically easiest to analyze.
- Restricts possible cosmological models so that (hopefully!) data point to unique solution. Small number of parameters (historically H_0, Ω_m).

But this is science and we have evidence:

- Microwave background is isotropic to a few parts in 10^5 (aside from dipole).
- Distribution of cosmologically distant objects (e.g. quasars) isotropic to a few parts in 10^2.
- Some tests of homogeneity (number counts, spectral distortions, etc.) These require more machinery and we will discuss them later.

What do they mean? Must specify what "homogeneous and isotropic" means in GR. Definition is that (i) the spacetime can be sliced into homogeneous and isotropic 3-surfaces Σ_t, labeled by a time coordinate t; (ii) can define "comoving observers" who see an isotropic Universe whose characteristics depend only on t; (iii) there is a unique comoving observer at each spacetime event. *Subtlety:* uniqueness fails in 3 special cases (Minkowski, de Sitter, anti de Sitter). We won't worry about this but the metric we derive applies to these cases.

2 FRW metric

General form. Can assign labels to each comoving observer and use them as spatial coordinates x^1, x^2, x^3. Then have metric:

$$ds^2 = -F\,dt^2 + 2g_{0i}\,dt\,dx^i + g_{ij}\,dx^i\,dx^j. \tag{1}$$

F, g_{0i}, g_{ij} are functions of x^k, t.

First restrictions on metric: Comoving observer's 4-velocity u^α must be $\perp$ Σ_t (by isotropy). Since x^i fixed $\to$ $u^\alpha = (F^{-1/2}, 0, 0, 0)$, first component from normalization. This is perpendicular to any vector $\mathbf{v}$ tangent to Σ_t, i.e. with $v^0 = 0$. Dot product is

$$\mathbf{u}\cdot\mathbf{v} = g_{\alpha\beta}u^\alpha v^\beta = g_{0i}v^i F^{-1/2}. \tag{2}$$

For this to be zero for all $\mathbf{v}$ we have $g_{0i} = 0$. (Note $F = 0$ would imply singular coordinate system.) Consequence is that $g^{0i} = 0$ and the 3×3 matrix g^{ij} is the inverse of g_{ij}.

Second restriction on metric comes from acceleration of comoving observers, $a^\mu = u^\nu \nabla_\nu u^\mu$. Can work out Christoffel symbols and find spatial components

$$a^i = F^{-1}\Gamma^i_{00} = F^{-1} g^{ij} \left[-\frac{1}{2} \frac{\partial}{\partial x^j}(-F) \right] = g^{ij} \frac{\partial}{\partial x^j} \ln \sqrt{F}. \tag{3}$$

We can lower the indices and get

$$a_j = \frac{\partial}{\partial x^j} \ln \sqrt{F}. \tag{4}$$

Now $\mathbf{a}$ is a vector measurable by the comoving observer so had better not have any spatial components (would pick out a preferred direction). So this equation implies F is spatially constant, i.e. depends only on t. In fact we can get rid of F by redefining:

$$t'(t) = \int \sqrt{F(t)}\, dt \tag{5}$$

and making a change of coordinates to t', x^1, x^2, x^3. The new metric is

$$ds^2 = -dt'^2 + g_{ij}\, dx^i\, dx^j. \tag{6}$$

We will drop the prime. In these coordinates $u^\mu = (1, 0, 0, 0)$. [*Aside:* Can show $\mathbf{a} = 0$.]

Third restriction comes from isotropy of local expansion. Take any purely spatial vector $\mathbf{v}$ (spatial in comoving frame, i.e. $\perp \mathbf{u}$) that is normalized ($v_\mu v^\mu = 1$) and construct

$$H(t, x^i, \mathbf{v}) = v^\mu v_\nu \nabla_\mu u^\nu. \tag{7}$$

(H is just a name now but will become meaningful shortly. But note that this is a component of the gradient of the velocity.) This function can only depend on t. Since $v_0 = v^0 = 0$:

$$H(t, x^i, \mathbf{v}) = v^i v_j \Gamma^j_{i0}. \tag{8}$$

Use

$$\Gamma^j_{i0} = g^{jk} \frac{1}{2} \frac{\partial}{\partial t} g_{ik}, \tag{9}$$

so that

$$H(t, x^i, \mathbf{v}) = v^i v_j g^{jk} \frac{1}{2} \frac{\partial}{\partial t} g_{ik} = \frac{1}{2} v^i v^k \frac{\partial}{\partial t} g_{ik}. \tag{10}$$

This can't depend on the direction of $\mathbf{v}$, so long as it satisfies the normalization condition $v^i v^k g_{ik} = 1$. Thus the 3×3 symmetric matrix $\partial g_{ik}/\partial t$ must be a scalar multiple of g_{ik}.

$$\frac{\partial}{\partial t} g_{ik} = I g_{ik} \tag{11}$$

for some I. Inspection gives $H = I/2$ so we can write

$$\frac{\partial}{\partial t} g_{ik} = 2H g_{ik}, \tag{12}$$

where H depends only on t. If we define

$$a(t) = \exp \int H(t)\, dt \tag{13}$$

then integrating Eq. (12) gives

$$g_{ik}(t, x^j) = \gamma_{ik}(x^j) \exp[2 \int H(t)\, dt] = \gamma_{ik}(x^j)[a(t)]^2. \tag{14}$$

The metric then takes the form

$$ds^2 = -dt^2 + [a(t)]^2 \gamma_{ij}(x^k)\, dx^i\, dx^j, \tag{15}$$

where γ_{ij} is the line element for a homogeneous, isotropic 3-manifold.

Definitions: (Very important!)

- The metric described by Eq. (15) is the *Friedmann-Robertson-Walker (FRW) metric.*
- The function $H(t)$ is called the *Hubble parameter* or Hubble rate or Hubble "constant." (Describes rate of expansion.)
- The function $a(t)$ is called the *scale factor.* (Describes how large the Universe is relative to the epoch at which $a = 1$.)

Spatial metric. We are not done yet since we haven't solved for the 3-dimensional spatial metric γ_{ij}. This metric must be isotropic around every point so we can use spherical polar coordinates (χ, θ, ϕ):

$$ds_3^2 = d\chi^2 + f(\chi)(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{16}$$

The problem is determining which functions $f(\chi)$ give rise to a homogeneous, isotropic 3-manifold.

First choice: $f(\chi) = \chi^2$ corresponds to the usual spherical polar coordinates of Euclidean 3-space. Explicit transformation:

$$\begin{aligned} x^1 &= \chi \cos\theta; \\ x^2 &= \chi \sin\theta \cos\phi; \\ x^3 &= \chi \sin\theta \sin\phi. \end{aligned} \tag{17}$$

We call this model *spatially flat* (or just "flat").

Second choice: Another homogeneous isotropic manifold is the hypersphere of radius R, called S^3. Usual equation for hypersphere is:

$$(y^1)^2 + (y^2)^2 + (y^3)^2 + (y^4)^2 = R^2, \tag{18}$$

Can re-cast in the form of Eq. (16) by switching to polar coordinates:

$$\begin{aligned} y^1 &= R\cos\frac{\chi}{R}; \\ y^2 &= R\sin\frac{\chi}{R}\cos\theta; \\ y^3 &= R\sin\frac{\chi}{R}\sin\theta\cos\phi; \\ y^4 &= R\sin\frac{\chi}{R}\sin\theta\sin\phi, \end{aligned} \tag{19}$$

In this case the metric is

$$ds_3^2 = d\chi^2 + R^2\sin^2\frac{\chi}{R}(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{20}$$

so Eq. (16) works but with $f(\chi) = R^2\sin^2(\chi/R)$. Usually we define the curvature K instead of the radius of curvature R; definition is $K = R^{-2}$. Then

$$f(\chi) = K^{-1}\sin^2(K^{1/2}\chi), \tag{21}$$

where K is the *spatial curvature*. Such a Universe is called *closed*. Properties:

- There is a maximum value $\chi = \pi R = \pi K^{-1/2}$ corresponding to antipodal point/coordinate singularity.
- *Locally* (fixed χ), $f(\chi) \to \chi^2$ as $K \to 0$ ($R \to \infty$) so the flat Universe is a limiting case of closed. But global topology is different from flat case.

Third choice: can analytically continue Eq. (21) to negative K (imaginary R).

$$f(\chi) = (-K)^{-1}\sinh^2[(-K)^{1/2}\chi]. \tag{22}$$

Will still satisfy homogeneity and isotropy. Spatial 3-manifold is called "hyperbolic" space and Universe is called *open*. There is no problem taking $\chi \to \infty$ – no nontrivial topology, antipodal point, etc.

In terms of possible functions $f(\chi)$ this is all there is. (Will be a homework problem.)

Often times $a(t)$ is re-scaled to allow us to set $K = +1$ for closed Universe or $K = -1$ for open. (For completeness $K = 0$ is flat.) We won't do this in this class because it will be more convenient to make $a = 1$ today.

Topology: We've determined all of the possible metrics, but not all possible topologies. For the closed Universe, we can identify antipodal points: take the point (y^1, y^2, y^3, y^4) and decare it to be the *same point* as $(-y^1, -y^2, -y^3, -y^4)$. In polar coordinates, identify:

$$(\chi, \theta, \phi) \leftrightarrow (\pi R - \chi, \pi - \theta, \pi + \phi). \tag{23}$$

This is still globally homogeneous and isotropic because at any point the antipodal point lies at the same distance (πR) in every direction. Maximum unique value of χ is $\pi R/2$. This manifold is called the *projective space* RP^3. It is not simply connected and locally looks like a closed Universe.

Cannot play this game with flat or open Universes because there is no antipodal point. If point O is identified with some other set of points $P_1, P_2, ...$ then there is a closest such point and the direction to it breaks global isotropy. Example is identification in Euclidean space of all points

$$(x^1, x^2, x^3) \leftrightarrow (x^1 + nL, x^2, x^3) \tag{24}$$

where n is an integer. This "cylinder" Universe is globally anisotropic because x^1 direction is preferred. But local isotropy applies and the microwave background might still look isotropic. Constraints on these models are active area of research but we won't pursue them here. (Not "standard.")

Summary:

- Globally homogeneous, isotropic Universes are characterized by a scale factor $a(t)$ and a choice of spatial manifold.
- Allowed choices of spatial manifold are $K > 0$ closed, $K < 0$ open, or $K = 0$ flat; and if $K > 0$ the topology (S^3 or RP^3).
- These statements are independent of GR. (We've only used symmetry so far!)

The metric described by the above picture is:

$$ds^2 = -dt^2 + a^2(t)[d\chi^2 + f(\chi)(d\theta^2 + \sin^2\theta\, d\phi^2)]. \tag{25}$$

Orthonormal frame: We will find it useful to consider a comoving observer to carry a tetrad of unit vectors $\{\mathbf{u}, \mathbf{e}_{\hat{1}}, \mathbf{e}_{\hat{2}}, \mathbf{e}_{\hat{3}}\}$. These vectors are orthonormal in the sense that

$$\begin{aligned} u_\mu u^\mu &= -1, \\ u_\mu (\mathbf{e}_{\hat{i}})^\mu &= 0, \text{ and} \\ (\mathbf{e}_{\hat{i}})_\mu (\mathbf{e}_{\hat{j}})^\mu &= \delta_{\hat{i}\hat{j}}, \end{aligned} \tag{26}$$

i.e. $\mathbf{u}$ is the 4-velocity of the comoving observer and $\{\mathbf{e}_{\hat{i}}\}_{i=1}^3$ are the spacelike basis vectors. These are not unique since $\{\mathbf{e}_{\hat{i}}\}_{i=1}^3$ can be rotated; a convenient choice would be

$$\begin{aligned} u^\mu &= (1, 0, 0, 0) \\ \mathbf{e}_{\hat{1}} &= (0, \frac{1}{a}, 0, 0) \\ \mathbf{e}_{\hat{2}} &= (0, 0, \frac{1}{a\sqrt{f(\chi)}}, 0) \\ \mathbf{e}_{\hat{3}} &= (0, 0, 0, \frac{1}{a\sqrt{f(\chi)}\sin\theta}). \end{aligned} \tag{27}$$

3 Friedmann equations

Ricci tensor: So far we have constructed the form of the metric but we don't know the scale factor $a(t)$. In order to do this we'll need to start using GR and relate $a(t)$ to the matter content of the Universe. So we'll need to find the Ricci curvature tensor $R_{\mu\nu}$ for the metric of Eq. (25). This could be done by brute force but taking advantage of symmetry will help us. The comoving observer can break $R_{\mu\nu}$ into:

- 1 time-time component $R_{\mu\nu}u^\mu u^\nu$;
- 3 time-space components $R_{\mu\nu}u^\mu(\mathbf{e}_{\hat{i}})^\nu$; and
- 6 space-space components $R_{\mu\nu}(\mathbf{e}_{\hat{i}})^\mu(\mathbf{e}_{\hat{j}})^\nu$.

The time-time components form a scalar which have some value, in this case R_{tt}. The time-space components form a vector and by isotropy are zero. The space-space components are restricted by isotropy to the form,

$$R_{\mu\nu}(\mathbf{e}_{\hat{i}})^\mu(\mathbf{e}_{\hat{j}})^\nu = L\delta_{\hat{i}\hat{j}}. \tag{28}$$

So we're searching for the two quantities R_{tt} and L, which can at most be a function of the time t because of homogeneity.

Let's start with R_{tt}. We can begin by using the usual formula for the Christoffel symbols:

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2}g^{\mu\nu}(-g_{\alpha\beta,\nu} + g_{\alpha\nu,\beta} + g_{\beta\nu,\alpha}). \tag{29}$$

And then:

$$R_{\beta\nu} = \Gamma^\alpha_{\beta\nu,\alpha} - \Gamma^\alpha_{\beta\alpha,\nu} + \Gamma^\alpha_{\sigma\alpha}\Gamma^\sigma_{\beta\nu} - \Gamma^\alpha_{\sigma\nu}\Gamma^\sigma_{\beta\alpha}. \tag{30}$$

The tt and $\chi\chi$ components are

$$\begin{aligned} R_{tt} &= -3\frac{\ddot{a}}{a}; \\ R_{\chi\chi} &= a\ddot{a} + 2\dot{a}^2 + 2K. \end{aligned} \tag{31}$$

(Details are for the homework.) Using the orthonormal basis (Eq. 27) the latter implies

$$L = \frac{R_{\chi\chi}}{a^2} = \frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{K}{a^2}. \tag{32}$$

Einstein equations: Now that we have the Ricci tensor, we can construct the Einstein tensor. In the comoving observer's basis $\{\mathbf{u} = \mathbf{e}_{\hat{0}}, \mathbf{e}_{\hat{1}}, \mathbf{e}_{\hat{2}}, \mathbf{e}_{\hat{3}}\}$ the Ricci tensor is

$$R_{\hat{\alpha}\hat{\beta}} \equiv R_{\mu\nu}(\mathbf{e}_{\hat{\alpha}})^\mu(\mathbf{e}_{\hat{\beta}})^\nu = \begin{pmatrix} R_{tt} & 0 & 0 & 0 \\ 0 & L & 0 & 0 \\ 0 & 0 & L & 0 \\ 0 & 0 & 0 & L \end{pmatrix}. \tag{33}$$

The trace of this is $R = -R_{tt} + 3L$. ($-$ because of signature!) The Einstein tensor,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}, \tag{34}$$

is

$$G_{\hat{\alpha}\hat{\beta}} \equiv G_{\mu\nu} (\mathbf{e}_{\hat{\alpha}})^\mu (\mathbf{e}_{\hat{\beta}})^\nu = \begin{pmatrix} \frac{1}{2}R_{tt} + \frac{3}{2}L & 0 & 0 & 0 \\ 0 & \frac{1}{2}R_{tt} - \frac{1}{2}L & 0 & 0 \\ 0 & 0 & \frac{1}{2}R_{tt} - \frac{1}{2}L & 0 \\ 0 & 0 & 0 & \frac{1}{2}R_{tt} - \frac{1}{2}L \end{pmatrix}. \tag{35}$$

Einstein's equation tells us this is $8\pi G T_{\hat{\alpha}\hat{\beta}}$, and $T_{\hat{\alpha}\hat{\beta}}$ has diagonal components ρ, p, p, p where ρ is density and p is pressure. So we can say

$$\begin{aligned} 8\pi G \rho &= \frac{1}{2} R_{tt} + \frac{3}{2} L; \\ 8\pi G p &= \frac{1}{2} R_{tt} - \frac{1}{2} L. \end{aligned} \tag{36}$$

Usually we take two linear combinations of these. From the first equation, divided by 3:

$$\frac{8}{3} \pi G \rho = \left(\frac{\dot{a}}{a} \right)^2 + \frac{K}{a^2}. \tag{37}$$

Taking $-1/6$ times the ρ equation plus $-1/2$ times the p equation:

$$-\frac{4}{3} \pi G (\rho + 3p) = \frac{\ddot{a}}{a}. \tag{38}$$

These are the *Friedmann equations.*

Continuity equations: The pressure and density are in fact not independent functions but are related by a continuity equation,

$$\nabla_\mu T^{\mu\nu} = 0. \tag{39}$$

Or:

$$\partial_\mu T^{\mu\nu} + \Gamma^\mu_{\mu\alpha} T^{\alpha\nu} + \Gamma^\nu_{\mu\alpha} T^{\mu\alpha} = 0. \tag{40}$$

For the FRW universe it's only the $\nu = 0$ (or t) component that is not trivial because of symmetry. Using the comoving frame components:

$$T_{\hat{\alpha}\hat{\beta}} \equiv T_{\mu\nu} (\mathbf{e}_{\hat{\alpha}})^\mu (\mathbf{e}_{\hat{\beta}})^\nu = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}, \tag{41}$$

we can write the contravariant components:

$$\begin{aligned} T^{\mu\nu} &= T_{\hat{\alpha}\hat{\beta}} (\mathbf{e}_{\hat{\alpha}})^\mu (\mathbf{e}_{\hat{\beta}})^\nu \\ &= \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & a^{-2} p & 0 & 0 \\ 0 & 0 & a^{-2} p [f(\chi)]^{-1} & 0 \\ 0 & 0 & 0 & a^{-2} p [f(\chi) \sin^2\theta]^{-1} \end{pmatrix}. \end{aligned} \tag{42}$$

So the continuity equation (40) becomes

$$\dot{\rho} + \Gamma^i_{i0}\rho + \Gamma^0_{00}\rho + \Gamma^0_{00}\rho + \Gamma^0_{\chi\chi}\frac{p}{a^2} + \Gamma^0_{\theta\theta}\frac{p}{a^2 f(\chi)} + \Gamma^0_{\phi\phi}\frac{p}{a^2 f(\chi)\sin^2\theta} = 0. \quad (43)$$

Of the relevant Christoffel symbols:

$$\begin{aligned} \Gamma^0_{00} &= 0 \\ \Gamma^i_{j0} &= H\delta^i_j \\ \Gamma^0_{\chi\chi} &= Ha^2 \\ \Gamma^0_{\theta\theta} &= Ha^2 f(\chi) \\ \Gamma^0_{\phi\phi} &= Ha^2 f(\chi)\sin^2\theta. \end{aligned} \quad (44)$$

So the continuity equation is:

$$\dot{\rho} + 3H\rho + 3Hp = 0. \quad (45)$$

We usually write this as

$$\dot{\rho} = -3H(\rho + p). \quad (46)$$

Relation to first law of thermodynamics: Equation (46) can also be derived from the first law of thermodynamics,

$$dU = dQ - p\,dV, \quad (47)$$

where U is total energy, Q is heat input, and V is volume. In cosmology there is no net heat input into the Universe, $dQ = 0$. Also $U = \rho V$ so:

$$d(\rho V) = -p\,dV. \quad (48)$$

Rearrange:

$$\rho\,dV + V\,d\rho = -p\,dV \quad (49)$$

and isolate $d\rho$:

$$d\rho = -(\rho + p)\frac{dV}{V}. \quad (50)$$

Now $V \propto a^3$ so

$$\frac{dV}{V} = 3\frac{da}{a} = 3\frac{aH\,dt}{a} = 3H\,dt, \quad (51)$$

from which

$$\dot{\rho} = -3H(\rho + p). \quad (52)$$

That this derivation is possible is not surprising since first law of thermodynamics is the continuity equation for energy. (Subtlety is definition of pressure.)

4 Examples - constant equation of state

A common case to consider in cosmology is the case where the pressure/density ratio is constant. We call this ratio the *equation of state* w.

$$w = \frac{p}{\rho}. \tag{53}$$

If we know w then from the continuity equation we can determine how the energy density scales with redshift. The continuity equation gives:

$$\dot{\rho} = -3H(\rho + w\rho) = -3H(1+w)\rho. \tag{54}$$

Since $H = \dot{a}/a$, we find

$$\frac{\dot{\rho}}{\rho} = -3H(1+w) = -3(1+w)\frac{\dot{a}}{a}, \tag{55}$$

which has the solution

$$\rho = \rho_0 a^{-3(1+w)}, \tag{56}$$

where ρ_0 is the density at $a = 1$. (Single constant of integration.)

Simple examples:

- *Nonrelativistic matter:* No pressure, or $|p| \ll \rho = \rho c^2$. Good approximation for baryons and dark matter. Redshifts as $\rho \propto a^{-3}$, so that "total mass" $\rho V \propto \rho a^3$ is conserved. (Sometimes called "dust" but in this course "dust" will mean real dust!)

- *Radiation:* In this course will mean gas of massless particles, such as photons or (at early times) neutrinos. Has $p = \rho/3$. Redshifts as $\rho \propto a^{-4}$. Additional factor of a can be interpreted as loss of energy of photon due to redshifting. (Stretching of wavelength.)

- *Cosmological constant:* By definition

$$T^{\mu\nu} = \frac{\Lambda}{8\pi} g^{\mu\nu}. \tag{57}$$

  Corresponds to $\rho = \Lambda/(8\pi G)$ and $p = -\rho$ so $w = -1$. Energy density remains constant as Universe expands.

The expansion history for a flat universe can be calculated from Friedmann equation (37):

$$\frac{8}{3}\pi G\rho = \left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2}. \tag{58}$$

$K = 0$ so:

$$\frac{8}{3}\pi G\rho_0 a^{-3(1+w)} = \left(\frac{\dot{a}}{a}\right)^2. \tag{59}$$

Let's take the square root

$$\left(\frac{8}{3}\pi G\rho_0\right)^{1/2} a^{-3(1+w)/2} = \frac{\dot{a}}{a} \tag{60}$$

and using $\dot{a} = da/dt$ we can separate variables,

$$\left(\frac{8}{3}\pi G\rho_0\right)^{1/2} dt = a^{3(1+w)/2-1}\, da. \tag{61}$$

Integrate:

$$\left(\frac{8}{3}\pi G\rho_0\right)^{1/2} t = \frac{2}{3(1+w)} a^{3(1+w)/2}. \tag{62}$$

We've left out the integration constant, which is equivalent to setting $t = 0$ at $a = 0$ (the Big Bang). So the scale factor as a function of time is

$$a = \left[\frac{3(1+w)}{2}\right]^{2/3(1+w)} \left(\frac{8}{3}\pi G\rho_0\right)^{1/3(1+w)} t^{2/3(1+w)}. \tag{63}$$

An exception occurs for the cosmological constant because $w = -1$. In this case, Eq. (61) has a logarithmic integral,

$$\left(\frac{8}{3}\pi G\rho_0\right)^{1/2} t = \ln a + \text{const}, \tag{64}$$

so the expansion is exponential,

$$a \propto \exp\left[\left(\frac{8}{3}\pi G\rho_0\right)^{1/2} t\right]. \tag{65}$$

It's customary to write this in terms of Λ where $\rho = \Lambda/(8\pi G)$:

$$a \propto \exp\left[\left(\frac{\Lambda}{3}\right)^{1/2} t\right]. \tag{66}$$

Exponential expansion has the property that the Hubble rate $H = \dot{a}/a$ is a constant,

$$H = \left(\frac{\Lambda}{3}\right)^{1/2}. \tag{67}$$

So if the Universe contains only a cosmological constant then:

- There is no Big Bang ($a = 0$ has no solution for finite t);
- The Univese looks the same at all times (no dependence of spacetime observables on t).

Exercise The Universe looks the same to all freely falling observers – maximally symmetric.

This solution is called *de Sitter spacetime*. It's not the real Universe but it will serve as an approximation during inflation, and in the distant future.